A COURSE IN

ELECTRICAL MACHINE DESIGN

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1

Principles of Electrical Machine Design

1·1 Design of Machines. Design may be defined as a creative physical realization of theoretical concepts. Engineering design is application of science, technology and invention to produce machines to perform specified tasks with optimum economy and efficiency.

Engineering is the economical application of scientific principles to practical design problems. If the items of cost and durability are omitted from a problem, the results obtained have no engineering value. The problem of design and manufacture of electric machinery is to build, as economically as possible, a machine which fulfils a certain set of specifications and guarantees Thus **design** is subordinated to the question of economic manufacture.

The major considerations to evolve a good design are :

1. Cost 2. Durability 3. Compliance with performance criteria as laid down in specifications.

In most of the situations it becomes difficult to design a machine which meets all the performance indices and also satisfies the cost and durability criteria because these requirements are usually conflicting. It is impossible to design a machine which is cheap and is also durable at the same time. This is because a machine which is to have a long life span must use high quality materials and advanced manufacturing techniques which obviously make it costly. The performance indices have to be met for certain. However, a compromise between cost and durability can be had. A good design is one where the machine has reasonable operating life, say between 20 to 30 years and has a low initial cost. This is particularly true of motors, especially induction motors used for general purpose applications. However, large synchronous machines and transformers which are used in power systems must be designed with reliability and durability in operation as the major considerations, with less emphasis on initial cost.

1·2 Design factors. The mechanical force required for movement in rotating electrical machines can be produced both by electrostatic and electromagnetic fields since both the fields store energy. In electrostatic machines, the energy density is limited by the dielectric strength of the medium used. For example, if air is used the dielectric medium, the maximum value of electric intensity that can be used is 3 MV/m (on account of dielectric break down) which corresponds to an energy density of about 40 J/m^3. In electromagnetic machines, magnetic effect is used for production of force and there is no comparable restriction in magnetic fields. However, the maximum value of flux density that can be used is about 1·6 Wb/m^2, because beyond this value magnetic saturation sets in the ferromagnetic materials required to complete the magnetic circuit of the machine. This limits the energy density in the air gap to about 1 MJ/m^3. This energy density is approximately 25,000 as much for electric fields.

Thus, at voltages that can be developed and used by normal means, the forces produced by electrostatic effects are very weak. On the other hand, a small current can produce large mechanical forces by electromagnetic means and therefore all the modern electrical machines are electromagnetic type.

However, electrostatic considerations cannot be omitted altogether in electrical machines, especially in high voltage transformers, and synchronous and induction machines,

the electrostatic field has to be considered as regards its secondary influence on the operating characteristics, apart, of course, from the question of insulation.

The basic structure of an electromanegtic rotating electrical machine is shown in Fig. 1·1. The machine consists of the following parts :

1. **Magnetic Circuit.** It provides the path for the magnetic flux and consists of air gap, stator and rotor teeth, and stator and rotor cores (yokes).

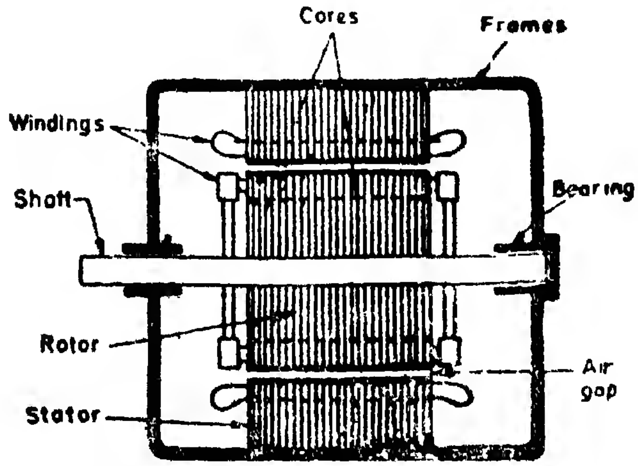


Fig. 1·1. Basic constructional details of a rotating machine.

2. **Electric Circuit.** It consists of stator and rotor windings. The winding of a transformer or a rotating machine conveys electrical energy to or from the working region and is concerned with production of emf and development of magneto-mechanical force. Windings are formed from suitably insulated conductors.

3. **Dielectric Circuit.** The dielectric circuit consists of insulation required to isolate one conductor from another and also the windings from the core. The insulating materials are essentially non-metallic and may be organic or inorganic, natural or synthetic.

4. **Thermal Circuit.** The thermal circuit is concerned with mode and media for dissipation of heat produced inside the machine on account of losses.

5. **Mechanical parts.** The important mechanical parts of a machine are its frame, bearings and shaft.

In a design problem one has to deal with numerous electric, magnetic, dielectric, thermal and mechanical quantities which are interrelated in very complex ways. Adequate space, consistent with both cost and performance criteria, has to be found for these quantities in a machine. A successful design brings out an economic compromise for space occupied by iron, copper (aluminium), insulation and coolant (which may be air, hydrogen, water or oil).

1·3 Limitations in design. Apart from availability of suitable materials, facilities available for manufacture of required machine parts and facilities required for transportation, the following considerations impose limitation on design.

1. **Saturation.** Electromagnetic machines use ferro-magnetic materials. The maximum allowable flux density to be used is determined by the saturation level of the ferromagnetic material used. A high value of flux density results in increased excitation resulting in higher cost for the field system.

2. **Temperature rise.** The most vulnerable part of a machine is its insulation. The operating life of a machine depends upon the type of insulating materials (insulants) used in its construction and the life of the insulating in turn materials depends upon the temperature rise of the machine. If an insulating material is operated beyond the maximum allowable temperature, its life is drastically reduced.

Proper cooling and ventilation techniques are required to keep the temperature rise within safe limits. The coolant (cooling medium) flowing along proper paths picks up the heat from the machine parts for dissipation outside the machine thereby preventing the temperature rise to become excessive.

3. Insulation. The insulating materials used in a machine should be able to withstand the electrical, mechanical and thermal stresses which are produced in the machine. The mechanical strength of insulation is particularly important in the case of transformers. Large axial and radial forces are produced when the secondary winding of a transformer is short circuited with the primary on. Therefore, while designing insulation for a transformer, due consideration must be given to the capability of the insulation to withstand large mechanical stresses (which may be either compressive or tensile) that are produced under short circuit conditions as apart from electrical and thermal breakdown considerations.

The type of insulation is decided by the maximum operating temperature of the machine parts where it is put.

The size of insulation is not only decided by the maximum voltage stress but also by the mechanical stresses produced. For example, for the same operating voltage, thicker insulation has to be used for large sized conductors than for smaller sized ones.

4. Efficiency. The efficiency of a machine should be as high as possible to reduce the operating costs. In order to design a highly efficient machine, the magnetic and electric loadings used should be small and this requires the use of large amount of material (both iron as well as copper/aluminium). Therefore, the capital cost of a machine designed for high efficiency is high while its running cost is low.

5. Mechanical parts. The construction of an electrical machine has to satisfy numerous technological requirements. The construction should be as simple as possible and also it is technologically good if it is carried out using simple and economical means with as little labour as possible. But the technological techniques should be consistent with the requirements of performance, reliability and durability.

The design of mechanical parts is particularly important in the case of high speed machines. For example, while designing a turbo-alternator, the rotor slot dimensions are so selected that the mechanical stresses at the bottom of rotor teeth do not exceed the maximum allowable limit.

In induction motors, the length of air gap is kept as small as mechanically possible in order to have a high power factor. The length of air gap and also that of the size of the shaft are mainly decided by the mechanical considerations. The size of the shaft should be such that it does not give rise to excessive unbalaced magnetic pull (U.M.P.) when deflected. The shaft of induction motors should be short and stiff so that there is no significant deflection of the shaft and hence the unbalanced magnetic pull is small and is within manageable proportions.

In large machines, the size of the shaft is decided by considering the critical speed which depends on the deflection of the shaft.

Bearings of rotating machines are subjected to the action of rotor weight, external loads, inertia forces due to unbalanced rotors and forces on account of unbalanced magnetic pull. The type of bearings to be used in a machine are decided by considering the above mentioned forces and also the type of construction whether the machine is horizontally or vertically mounted.

6. Commutation. The problem of commutation is important in the case of commutator machines as commutation conditions limit the maximum output that can be taken from a machine. For example, at present the maximum power output of a single unit d.c. machine is approximately 10 MW and this limitation is solely on account of commutation difficulties.

7. Power factor. Poor power factor results in larger values of current for the same power and, therefore, larger conductor sizes have to be used.

This problem of power factor is particularly important in the case of induction motors. The size and hence cost of induction motors can be reduced by using a high value of flux density in the air gap but this results in saturation in iron parts of machine and consequently a poor power factor. Thus the value of flux density used depends upon the power factor and hence power factor becomes a limiting factor. In fact, the length of air gap to be provided in an induction motor is primarily determined by power factor considerations.

8. Consumer's Specifications. The limitations imposed by consumer's specifications on the design of electric machinery are obvious. The specifications as laid down in the consumer's order have to be met and the design evolved should be such that it satisfies all the specifications and also the economic constraints imposed on the manufacturer.

9. Standard specifications. These specifications are the biggest strain on the design because both the manufacturer as well as the consumer cannot get away from them without satisfying them.

1·4. Modern trends in design of electric machines. The design of electrical machines is both a science and an art. A *"science"*, because it follows established and universally accepted physical and mathematical principles which have been verified by the experimental methods and an *"art"* in that the knowledge of these principles is often insufficient to produce a correct and economic design. This can be achieved by correct decisions based upon judgement and intution, and through understanding of the subject.

The design of electrical machines consists essentially of the solution of many complex and diverse engineering problems and normally these problems are loosely interrelated to a lesser or a greater degree. In fact the design of electrical machines presents a mathematically indeterminate problem with many solutions as the number of equations is less than the number of unknowns. The design of electrical machines at the first onset requires the choice of principal constructional scheme appropriate to the desired machine performance and types of construction of its basic machine parts. Preliminarily it is also required to choose a system of cooling and ventilation for the machine, materials for its magnetic system, insulating materials, the conductor materials and the like.

It is very rare indeed, that in practice one starts designing a machine right from fundamentals without any help from experience already gained in designing machines. The design of insulation, cooling and ventilation and mechanical parts is a matter of application of some empirical relationships obtained after years of practice. Calculations are then made whereby the main dimensions of active parts of machine, winding details and other parameters are determined. Therefore, the machine designer starts with a number of known parameters like basic electromagnetic and constructional data and performs a series of mathematical operations that may or may not involve logical decisions to arrive at one or more than one acceptable solutions.

The overall design process, right from the specification requirements to the determination of machine dimensions and other items of information required for the manufacture of both static and rotating machines, may be considered as a single engineering problem or several interconnected engineering problems.

The process of design of a single machine may be divided into three major design problems, such as :

(*i*) electromagnetic design, (*ii*) mechanical design, and (*iii*) thermal design.
These problems may be solved separately and the results combined later on. Each of these three major problems may further be broken down into simple and loosely related elements. Each element may be considered as a separate problem and as such this procedure involves solution of elements many times over.

The other aspect of the modern day design of electrical machines is designing a number of machines, all of which form part of a single system. For example, generators, motors and transformers form a part of an electromechanical energy network. The different machines of such a system are interconnected and react upon each other, sometimes considerably and on occasions disasterously. Therefore, the machines for such a system cannot be designed in isolation and the designs of all the machines have to be completed concurrently since the design of one machine depends upon that of the others. The problem thus is that of optimization of the system.

Sometimes it is desired to design a series of machines having different ratings to fit into a single frame size. In this case, the finished designs of machines must be produced in groups, where all designs within a group are interdependent. This again is an optimisation problem because frame sizes have to be optimum giving due weightage to designs of all the machines in a series or a group.

Therefore, the optimal solution involves iterations wherein values of variables are changed to satisfy both the performance and cost constraints It is clear that the design of electrical machines is an iterative process wherein the assumed data may have to be varied many times to arrive at the desired design. The evolution of design to meet the specified optimum criteria is a matter of long and tedious iterations and this fact has led to the application of fast digital computers to the design of electrical machines and transformers.

The digital computer has completely revolutionalized the field of electrical machine design. The computer aided design has the advantages of eliminating tedious and time consuming hand calculations thereby releasing the designer from numerical drudgery to allow him devote time to grapple with physical and logical ideas This accelerates the design process enormously.

Also the use of computer makes possible more *'trial'* designs, and enables sophisticated calculations to be made without intolerable tedium and excessive time. It makes possible the checking of data at every stage, reduces empiricism, readily handles non-linearities and incorporates the designers *'know-how'*.

In addition, computer aided design permits more detailed and precise functional relationships which give rise to possibility of new and comprehensive design procedures.

1·5. Modern machine manufacturing techniques. It is well known that electric power has wide and diverse applications in all fields of human activity. The utilization of electric power is going up day by day and this calls for increased generating capacity and obviously construction of new electric power stations. This necessitates and will continue to necessitate a great need for a variety of electric machines over a wide range of power outputs. The growing needs of both electric generation and electric utility industries have to be met by continuously expanding electrical machine manufacturing industry applying modern manufacturing techniques. The modern trends in electrical machine manufacturing industry are discussed below :

1. The modern electrical machines are characterized by a very wide range of power outputs. The power range varies from a fraction of a watt to several hundreds of megawatt in a single unit. Thus the ratio of power output of the smallest machine to that of the largest machine is $1 : 10^{10}$.

The range of rotational speeds of electrical machines is very wide. One machine may have a speed of few revolutions per second while that of another may be several thousand revolutions per second. The large varied fields of applications, and wide range of both power output and speed of operation of electrical machines has led to a variety of types of construction.

The type of construction adopted depends upon the power output of the machine and also on its rotational speed. Thus a classification of electrical machines by construc-

tional features and their subdivision on the basis of power outputs and rotational speeds can be made.

No universally accepted classification of electrical machines based upon constructional features and power outputs exists as yet. However, a broad classification of electrical machines can be made because machines having power outputs within a particular range have some typical common constructional features. A suggested classification on the basis may be :

(*i*) **Small size machines.** Electrical machines having a power output upto about 750 W may be called small machines.

(*ii*) **Medium size machines.** Electrical machines having power outputs ranging from a few kilowatt up to approximately 250 kW may be classified as medium size machines.

(*iii*) **Large size machines.** Electrical machines with power outputs in the range of 250 kW up to about 5000 kW are classified as large size machines. The machines are usually designed and manufactured as a series and have a definite power output range.

(*iv*) **Larger size machines.** These machines are manufactured on special orders from customers to meet their specific demands. Therefore, large machines are designed on individual basis. The power outputs of these machines may be as high as hundreds of megawatt.

The type of construction to be adopted is considerably influenced by the operating speed of the machine. For example, low speed machines (below 250 rpm) have large diameter and small axial length, while high speed machines (3000 rpm and over) have small diameter and a long core length. Commercially available large machines have rotational speed in the range of 400—1500 rpm while medium size machines have a speed range of 2,000 to 2,500 rpm.

2. The second important feature of modern electrical machine manufacture is the trend to build machines which are smaller in size and therefore involve the use of lesser material, and at the same time have the same efficiency and overload capacity. The increase in power ratings using smaller size coupled with good overall performance has been possible only due to the following technological advancements.

(*i*) There has been considerable developments and refinement in the techniques relating to construction and arrangement of conductors and some other parts of the machine and this has resulted in drastic reduction in stray load losses.

(*ii*) There has been a vast development in the cooling and ventilation systems for machines. The new methods are much more effective for dissipation of heat generated inside the machine.

3. The third important factor in the manufacture of modern machines is the use of magnetic materials which have a high permeability, a low iron loss and a high mechanical strength. The materials permit use of the high values of flux density and therefore results in reduction in the size of the machine and promotes the extention of power output.

4. There has been a significant improvement in the insulating materials and newer materials are increasingly being used in the present day machines. These insulating materials are able to withstand much higher temperatures. Since, the rating of the machine mainly depends upon the insulating materials used in it, greater outputs are possible with the use of these insulating materials. In other words, the use of better class of insulating material allows the machine sizes to be used for the same output power ratings.

5. Modern machine building marked with use of higher electro-magnetic loadings for active parts and increased mechanical loadings for construction materials.

6. In order to expedite the process of machine manufacture at reduced cost, different improved and refined manufacturing techniques are used for individual machine parts.

7. Modern electrical machines have a wide field of applications. They are used in varied environments and under different operating conditions. The design of the machine and its manufacture should be such that it (the machine) operates satisfactorily under the desired environmental conditions.

1.6. Basic principles. It has been explained in the preceding sections that the design of insulation, ventilation and cooling systems and of mechanical parts is based upon knowledge derived from decades of experience and practice. However, the design of both electric and magnetic circuits is based upon some well established basic laws

The action of electromagnetic machines can be related to three basic principles which are :

(*i*) Induction, (*ii*) Interaction and (*iii*) Alignment.

1. Faraday's law of magnetic induction. This law states that emf induced in a closed electric circuit is equal to the rate of change of flux linkages.

$$\text{Flux linkages } \psi = N\phi \qquad \text{..(1.1)}$$

where N is the number of turns in a coil and ϕ is the flux linking with all of them.

In most cases, the flux ϕ does not link with all the turns or alternatively all the turns do not link with the same flux. Under these circumstances, the summation of all products of magnetic flux with complete turns of the magnetic circuit gives the total value of flux linkages ψ.

The total flux linkages thus are :

$$\psi = N_1\phi_1 + N_2\phi_2 + \ldots\ldots + N_n\phi_n = \sum_{k=1}^{n} N_k\phi_k \qquad \text{...(1.2)}$$

where N_k is the number of turns which link with flux ϕ_k.

In case there is a change in the value of the flux linkages of the coil, an induced emf is produced in it whose value is given by :

$$e = -\frac{d\psi}{dt} \text{ volt.} \qquad \text{...(1.3)}$$

The negative sign in Eqn. 1.3 indicates that the direction of the induced emf is such that the current produced by it opposes the change in flux linkages.

The change in flux linkages can be caused in three ways.

(*i*) The coil is stationary with respect to flux and the flux varies in magnitude with respect to time.

(*ii*) The flux is constant with respect to time and is stationary and the coil moves through it.

(*iii*) Both the changes mentioned above occur together *i.e.* the coil moves through a time varying field.

In the method outlined in (*i*) where the coil is stationary and the flux is time varying, an emf called **transformer** (or **pulsational**) **emf** is produced. Since no motion is involved, there is no energy conversion and the process that really takes place **is energy transference.** This principle is used in transformers which employ stationary coils and time varying fluxes for transfer of energy at one level to another.

In (*ii*) the flux cutting rule can be employed to illustrate the emf generated in a conductor moving in a constant stationary field. The emf generated in a conductor of length moving at right angles to a uniform, stationary, time invarying magnetic field is given by :

$$e=-Blv \quad \text{volt} \qquad \ldots(1\cdot4)$$

where B=flux density, Wb/m²(T) ;

l=length of conductor, m ;

and v=linear velocity of conductor, m/s.

The generated emf in this case is called a **"motional emf"** because it is caused due to motion of a conductor. Since motion is involved in the production of this emf, the process involves electromechanical energy conversion. This principle is utilized in rotating machines like d.c., induction and synchronous machines.

In (*iii*) a conductor or a coil moving across a stationary time varying magnetic field (flux) and therefore both transformer as well as motional emfs are produced in the conductor or the coil. This process thus involves both transfer and conversion of energy. This principle is utilized in the working of commutator machines.

2. Biot Savart's law. This law gives the value of force produced on account of **interaction** between a magnetic field and a current carrying conductor. The electromagnetic force is given by :

$$f_e=Bli \sin\alpha \quad \text{newton} \qquad \ldots(1\cdot5)$$

where B=flux density, Wb/m² (T) ;

l=length of conductor, m ;

i=current carried by conductor, A ;

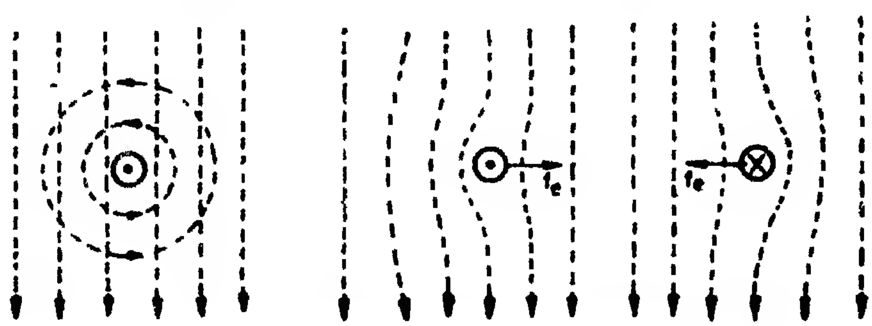
α=angle between the direction of current and the direction of magnetic field.

The direction of the force produced is perpendicular to both current and magnetic field.

In electrical machines, the magnetic field is radial in the air gap *i.e.* the conductors and the magnetic field are perpendicular to each other and thus $\alpha=90°$.

$$\therefore \quad f_e=Bli \quad \text{newton} \qquad \ldots(1\cdot6)$$

In Fig. 1·2 (*a*), B represents the flux density of an undisturbed (original) magnetic field. The introduction of a current carrying conductor introduces a new magnetic field. The original field and the field due to conductor combine to produce a resultant field as shown in Fig. 1·2 (*b*). The resultant field is distorted in the neighbourhood of the conductor, the resultant flux density being greater on one side and lesser on the other and this results



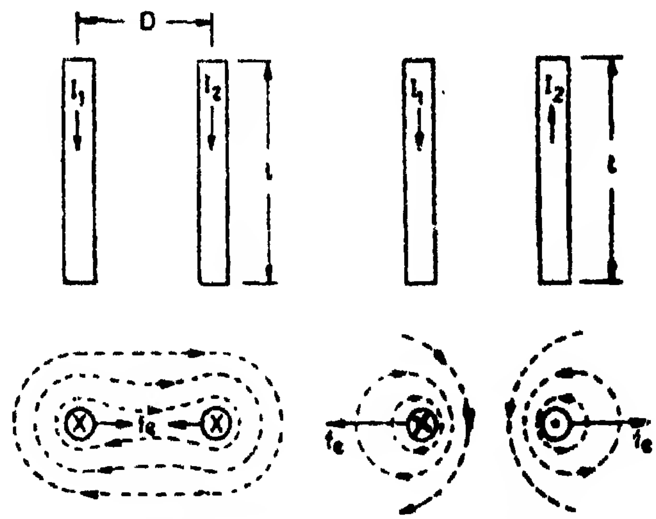
(*a*) (*b*) (*c*)

Fig. 1·2. Force on a current carrying conductor situated perpendicular to a magnetic field (interaction law)

in production of an electromagnetic force in the direction indicated. In case the increase in flux density on one side is equal to the reduction on the other side, the electromagnetic force is given by Eqn. 1·6.

When either the direction of the current or the direction of the magnetic field is reversed, the direction force acting on the conductor is reversed. However, if the directions of both the current as well as the magnetic field are reversed, the direction of the force produced remains unaltered. Fig. 1·2 (*c*) shows the effect of reversing the current when the direction of the field is unchanged. It is clear that under these conditions the direction of force produced is reversed.

Biot Savart's law can be applied to determine force between two current carrying conductors. Fig 1·3 shows two parallel current carrying conductors of l separated by a distance D and situated in a medium of permeability μ. The two currents are I_1 and I_2. In Fig. 1·3 (*a*) the two currents flow in the same direction while in Fig. 1·3 (*b*) they flow in the opposite direction. The resultant magnetic fields are also shown. It is clear that when conductors carry current in the same direction, there is a force of attraction between them, while there is a force of repulsion between them if they currents in the opposite direction.



(*a*) Attraction (*b*) Repulsion

Fig. 1.3. Force between current carrying conductors.

The value of the flux density, at the position of conductor carrying current I_2, due to current I_1 is :

$$B=\mu H=\mu\,\frac{I_1}{2\pi D}$$

$\therefore$ Electromagnetic force

$$F=BIl=\mu\frac{I_1}{2\pi D}\,I_2 l$$

$$=\frac{\mu l}{2\pi D}\,I_1\,I_2 \text{ newton} \qquad \ldots(1\cdot7)$$

Alignment. If a magnetic field exists in an ambient low permeability medium like air and if a piece of high permeability material is placed in this field, the latter experiences a force which tries to align it with the direction of field in such a way that it occupies a position of minimum reluctance. The principle of production of force due to alignment is used to reluctance motors.

2

Electrical Engineering Materials

2·1. Electrical Conducting Materials. Materials serving as electrical conductors can be divided into main groups, namely :

1. **High Conductivity Materials.** These materials are used for making all types of windings required in electrical machines, apparatus and devices, as well as for transmission and distribution of electric energy. These materials should have the least possible resistivity.

2. **High Resistivity Materials (Alloys).** These materials are used for making resistances and heating devices.

2·2. High Conductivity Materials. The fundamental requirements to be met by high conductivity materials are :

(*i*) highest possible conductivity (least resistivity),

(*ii*) least possible temperature co-efficient of resistance,

(*iii*) adequate mechanical strength, in particular, high tensile strength and elongation characterizing to a certain degree of the flexibility, *i.e.* absence of brittleness,

(*iv*) rollability and drawability which is important in the manufacture of wires of small and intricate sections,

(*v*) good weldability and solderability which ensure high reliability and low electrical resistance of the joints,

and (*vi*) adequate resistance to corrosion.

The above requirements vary with the purpose. For example, it is desirable to make the windings of electrical machines of least possible resistivity even at the expense of slight loss in tensile strength.

The need for high mechanical strength and flexibility is self-evident. Even where conductors operate without significant mechanical loading, as in coils of various electrical apparatus, the operations involved in making these coils require that the wires have definite mechanical properties. It is, therefore, essential to know the values of resistivity, specific weight, density, resistance temperature coefficient, co-efficient of thermal expansion, thermal, conductivity specific heat and tensile strength of conducting materials used in electrical machines.

2·2·1. Copper. Copper is the most widely used electrical conductor combining, as it does, high electrical conductivity with excellent mechanical properties and relative immunity from oxidation and corrosion under service conditions. It is highly malleable and ductile metal. It can be cast, forged, rolled, drawn, machined. Mechanical working hardens it but annealing restores it to soft state.

Most electrical machines employ windings of annealed high conductivity copper. The International Annealed Copper Standard (IACS) has at 20°C, a resistivity $0{\cdot}017241 \times 10^{-6}\ \Omega$ m, a resistance temperature co-efficient 0·00393 per °C and a tensile strength 220—250 MN/m^2.

Hard drawn copper wires are used in electrical machines as wire drawing increases the mechanical strength although the resistivity also increases a little.

Copper is universally used for windings of electrical machines because it is easily workable without any possibility of fracture. Further it can be soldered easily which simplifies the jointing process.

The properties of annealed copper are given in Table 2·1.

Table 2·1

Characteristic	*Copper*	*Aluminium*
(*i*) Density, kg/m^3	8900	2700
(*ii*) Melting point, °C	1083	660
(*iii*) Thermal conductivity W/m—°C	350	200
(*iv*) Resistivity, Ω m	0·01724 × 10^{-6}	0·0287 × 10^{-6}
(*v*) Resistance temperature co-efficient at 20°C, —1°C	[illegible]	0·0039
(*vi*) Co-efficient of thermal expansion at 20°C/°C	[illegible]6·7 × 10^{-6}	25·5 × 10^{-6}
(*vii*) Specific heat, J/kg-°C	390	920
(*viii*) Specific strength MN/m^2	220—250	

The sizes of copper conductors as per Indian standard specifications and British standard specifications are given in chapter 18.

2.2.2. Aluminium. It is predicted that at the present rate of consumption of copper existing deposits will be exhausted within a period of about 40 years and with constantly accelerated rate of consumption, this period may be as short as possible. The only freely available conductor material capable of replacing copper for the windings of electrical machines is aluminium.

Aluminium is joining ever increasing applications for a number of economic and engineering reasons, primarily the high demand for conductor materials which cannot be met by copper production alone. Therefore, aluminium which is the conductor material next to copper is used. Also aluminium is available in abundance on earth's surface Pure aluminium is softer than copper and therefore, can be rolled into thin sheets (foils). Aluminium cannot be drawn into very fine wires on account of its low mechanical strength.

In replacing copper conductors with aluminium ones in electrical machines due account should be taken of their differences in resistivity, density and mechanical strength.

The following analysis helps in the expansion of copper and aluminium conductors used in electrical machines.

Let us consider a conductor having length l, area a, resistivity ρ and mass density g. The conductor is carrying a current I.

Copper loss in conductor

$$P_c = I^2R = (\delta a)^2 \frac{\rho l}{a} = \delta^2 \rho l a \qquad \text{...(2·1)}$$

where δ=current density

∴ Copper loss per unit volume (specific copper loss)

$$p_c = \frac{\delta^2 \rho l a}{l a} = \delta^2 \rho \quad \text{...(2·2)}$$

and copper loss per unit mass

$$p_c' = \frac{\delta^2 \rho}{g} \quad \text{...(2·3)}$$

Therefore, specific copper loss varies directly as resistivity and inversely as mass density. The average temperature of conductors in electrical machines is approximately 75°C. The resistivity of copper is $0·021 \times 10^{-6}$ Ω m at 75°, while that of aluminium is $0·034 \times 10^{-6}$ Ωm. The ratio of resistivities is 0·034/0·021=1·62. Therefore, for the same volume of conducting material, the I^2R losses in a machine aluminium conductors are 1·62 times that in a machine using copper conductors. The rating of a machine is determined by the temperature rise which in turn is directly proportional to losses. Thus for the same temperature rise (and hence for the same loss), the current rating of a machine using aluminium conductors is $\frac{1}{\sqrt{1·62}}$=0·78 times that of a machine using copper conductors. Hence, the use of aluminium conductors results in reduction of rating by 22%.

The above is true of standard industrial tranformers, motors and generators. However, large transformers, motors and generators differ radically from most industrial machines in that each is tailor made for a particular application, thereby permitting design modification to offset to some extent the disadvantage of higher resistivity of copper. For example, for the same loss and same length of the conductor, the aluminium conductor should have an area 1·62 times that of copper which means an increase of space by about 60%. Thus in rotating electrical machines where the slot width is fixed, the slot depth has to be increased by about 60% to accommodate deep aluminium conductors. If round conductors are used, the area of aluminium wire is $\sqrt{1·62}$=1·27 times that of copper wire to give the same loss. However, since the density of copper is 8900/2700=3·3 times that of copper, the weight of aluminium conductor to give the same loss is 1·62/3·3 =0·49 times that of copper.

The comparison of copper and aluminium used as conductors is given in Table 2·2. This is valid for equal resistance per unit length.

Table 2·2. Comparison of Aluminium and Copper Wires

Item	*Copper*	*Aluminium*
Cost	1	0·49* p_c/p_a
Cross-section	1	1·62
Diameter	1	1·27
Volume	1	2·04
Weight	1	0·49
Breaking strength	1	0·64

* p_c=unit price by weight of copper.
p_a=unit price by weight of aluminium.

Aluminium was first used, instead of copper, in the armature and field windings of motors, generators etc., and in transformers built in Germany during 1914—18. Although it is claimed that such machines compete in efficiency and price with copper wound machines, aluminium is, however, at an obvious disadvantage wherever the space for windings is limited. When using aluminium winding wires, machines have to be redesigned for larger slots to accommodate aluminium wire if it is to have the same resistance as the copper wire it replaces.

For induction motors with power outputs up to 100 kW, aluminium can be used as material for bars and squirrel cage. The squirrel cage rotor windings can be fabricated by brazing slot bars to end rings or by integral casting with silicon-aluminium alloy (6—12% Si) of resistivity $0{\cdot}04 \times 10^{-8}$—$0{\cdot}05 \times 10^{-8}\,\Omega$ m. In the latter case bars and rings are obtained by pouring molten aluminium into assembled rotor core. Die cast aluminium windings are extensively used for rotors of induction motors.

Super-enamelled aluminium wires are used for stator windings of small induction motors.

Aluminium is also used for windings of transformers. Aluminium when adopted as a conductor material in small transformers, decreases the overall cost of the transformer. But when used in large transformers it gives increased size and cost. A new development in the field of transformer manufacture is the use of **foil** type windings. This is because aluminium can be rolled to thinner and more flexible sheet than copper. Thin sheets of aluminium (foils) are used to make bobbin type of coils of one turn per layer. Foil type windings are often used for low voltage windings of small and medium rated transformers.

Aluminium is now used for the manufacture of transformer tanks because of its light weight. Also use of aluminium tanks in place of steel tanks reduces the stray load loss.

Aluminium oxidizes very quickly in normal atmospheric conditions and acquires a thin film of oxide Al_2O_3, which effectively protects it from further oxidation. By reason of the high melting point of the aluminium oxide coating (of the order of 2000°C) and the rapidity which a freshly exposed aluminium surface becomes oxidized, aluminium wire cannot be soldered by conventional means. Now-a-days aluminium is successfully soldered by using special ultrasonic soldering irons. The jointing of connectors in an aluminium winding offers no difficulties.

2·2·3. Iron and Steel. Steel alloyed with chromium and aluminium is used for making starter rheostats where lightness combined with robustness and good heat dissipation are important considerations.

Cast iron is used in the manufacture of resistance grids to be used in the starters of large motors.

2·2·4. Alloys of Copper

1. **Bronze.** Copper base alloys containing tin, cadmium, berryllium and certain other metals are generally called 'bronzes' and used as high conductivity materials. All bronzes possess high mechanical strength as compared with copper, but have higher resistivities.

Beryllium Copper. It has been found that the addition of [illegible] 2½ per cent berylium to copper makes a hard alloy which is capable of being rolled and formed into springs and contact strips. Therefore, it is used for current carrying springs, brush holders, sliding contacts and knife switch blades. Its resistivity is 3 to 6 times that of copper.

Cadmium Copper. Alloys containing 1·1 per cent cadmium give wires which are stiffer, harder, and of higher tensile strength than hard-drawn copper. It is used for making contact wires and commutator segments. Cadmium copper is also used for cage windings because it can be flame brazed without deterioration.

2. **Brass.** This finds a very wide application in electrical engineering field. It generally contains 66 per cent copper and 34 per cent zinc. It has greater mechanical strength and wear resistance than copper, but considerably lower conductivity (high resistivity). Brass is easily shaped by press forming methods, lends itself to deep drawing, has good weldability, and solderability, and is fairly resistant to corrosion. Therefore, it has gained wide use in the manufacture of electrical apparatus as current carrying and structural materials.

3. **Copper silver alloy**. This alloy contains 99·10% copper and 0·06 to 0·1 per cent silver. It has a resistivity $0·01814 \times 10^{-8}$ Ωm. Silver bearing copper is used in turbo-alternators because of its resistance to thermal shortening and creep.

Table 2·3 shows properties of copper alloys.

Table 2·3. Properties of Copper Alloys

Material	*State*	*Resistivity (times copper)*	*Tensile strength, MN/m²*
Cadmium Copper (0·9% Cd)	Annealed	1·05	Upto 300
	Hard	1·1 to 1·2	Upto 715
Bronze (Cd 0·8% Sn 0·6%)	Annealed	1·65 to 1·8	Upto 280
	Hard	1·8 to 2	Upto 715
Beryllium Copper (Be 2·25%)	Annealed	6·0	480 to 590
	Ageing at 350°C	3·3	90 to 1080
Brass (Cu 70% Zn 30%)	Annealed	4·0	340 to 510
	Hard	4·0	Upto 860

2·3 Materials of High Resistivity. Conductors of high resistance are used where it is actually desired to dissipate electrical energy as heat *i.e.*, in starting and regulating devices for motors etc. In such cases it is usual to call the high resistance conductors as resistors, resistance coils, resistance elements or heating elements.

Materials of high resistivity are primarily alloys of different metals. Among metals that have been particularly important as basic materials for making these alloys, the following may be mentioned : nickel, silver and iron. They can be classified according to their purpose. Three categories are :

(*i*) The first group consists of materials used in precision measuring instruments and in making standard resistances and resistance boxes.

(*ii*) The second group consists of materials from which resistance elements are made for all kinds of rheostats and similar control devices.

(*iii*) The third group consists of materials suitable for making high temperature elements for electric furnaces, heating devices and loading rheostats.

2·3·1. Materials used for Precision Work. An important requirement imposed on high resistivity materials intended for use in precision electrical instruments and for making standard resistances is stability of resistance over the period of time (no tendency to age) and during fluctuations of temperature. The latter implies that the material should have a low resistance temperature co-efficient. The thermo-electro-motive force resulting from

contact of material with copper should be minimum so as not to introduce errors into measurements. Cost is not of much importance for these materials.

The most important material used for this class is Manganin. Its classical composition is Cu 86 per cent, Mn 12 per cent and Ni, 2 per cent. Nickel serves to reduce thermal emf of contact with copper to a very low value, of about 1·0 μV/°C. This alloy has a resistivity of $0{\cdot}43\times10^{-6}$ Ω m and a resistance temperature coefficient of the order of 1×10^{-5} per °C.

2·3·2 Materials used for Rheostats. The resistance materials used in making rheostats can have a large thermo-emf and a large resistance temperature co-efficient. But this material should meet special requirements such as a high permissible working temperature and low cost, the latter being dictated by the fact that these materials are required in large quantities in widely used devices and equipment where large changes in resistance are allowed.

The principal alloy in this group is constantan, consisting of 60 to 65 per cent copper and 40 to 35 per cent Nickel. Sometimes small amounts of Manganese and Iron are also included. Soft constantan wire has resistivity of 0·46 to $0{\cdot}53\times10^{-6}$ Ω m and hard constantan wire, 0·46 to $0{\cdot}53\times10^{-6}$ Ω m. The resistance temperature coefficient is near zero. It has a thermo-emf of 39 μ V/°C with respect to copper. Constantan can be safely used upto a temperature of 500°C. A voltage (maximum) of one volt per turn should be used when designing constantan wire rheostats.

2·3·3 Materials used for Heating Devices. The primary requirement for high temperature resistance alloys intended for use in electric furnaces and heating devices is a high working temperature. This requirement is satisfied by a material which has a sufficiently high melting point and is either non-corrosive or forms a surface layer of dense, high melting oxide protecting it from further corrosion. Platinum is an incorrodible material with a high melting point (1710°C) with a resistivity $0{\cdot}117\times10^{-6}$ Ω m. Platinum is used in laboratory electric furnaces with a working temperature of 1300°C.

The most extensively used high working temperature resistance materials are alloys of nickel, chromium and iron called Nichrome and alloy of aluminium, iron and chromium. The quality of these alloys, especially the working temperature strongly depends upon the chromium content. The presence of chromium ensures a high melting point of oxide coating. The resistivity of Nichrome varies from 1·1 to $1{\cdot}27\times10^{-6}$ Ω m.

Nichrome is available as round wire and strip, cold drawn with an oxidized surface and also hot rolled with scale covered surface. The optimum working temperature for Nichrome wire is 900° to 1000°C.

The resistance wire data is given in Table 2·4 on page 16.

2·4. Electrical Carbon Materials. Electrical carbon materials are manufactured from graphite and other forms of carbon coal etc. The conductivity of carbon used is slightly less than metals and alloys and therefore it is used for making brushes for electrical machines.

Brush carbons are often graphited *i.e.* heat treated to increase the size of crystals. This raises the conductivity of the brushes and reduces their hardness. Carbon brushes should acquire a mirror smooth surface soon after being put to service. This is because they ride on commutators of electrical machines and, therefore, unless their surface is smooth, they will cause wear of commutator and will wear it down rapidly. The characteristics of different kinds of brushes are given in Table 2·5. on page 17.

2·5. Super-Conductivity. Very few practical electromechanical devices are built using only current carrying conductors and no iron to act as magnetic circuit because of the relatively small forces or torques that can be produced per unit of machine volume. This is because of weak magnetic fields owing to absence of iron. While there is no limit on the value of flux density that can exit in air or space but in order to produce a strong field in

Table 2·4. Resistance Wire Data

SWG. No.	Diameter mm	Area mm^2	Resistance per metre Ω		Current for temperature, A					
					100°C		200°C		300°C	
			Ferry	Nichrome	Ferry	Nichrome	Ferry	Nichrome	Ferry	Nichrome
10	3·25	8·3	0·059	0·132	20·1	16·0	30·8	30·4	40·0	49·7
11	2·95	6·82	0·0722	0·16	18·5	14·1	28·1	26·4	36·4	43·5
12	2·64	5·48	0·0896	0·199	14·8	12·25	22·4	22·7	29·0	37·3
13	2·34	4·29	0·133	0·255	12·6	10·4	18·8	19·2	24·5	31·7
14	2·03	3·24	0·149	0·336	10·5	8·7	15·5	15·75	10·1	25·9
15	1·83	2·63	0·184	0·417	9·3	7·5	13·4	13·7	17·4	22·25
16	1·63	2·07	0·234	0·525	8·1	6·4	11·5	11·75	15·1	18·75
17	1·42	1·59	0·305	0·685	7·0	5·3	9·8	10·0	13·0	15·5
18	1·22	1·17	0·416	0·93	5·75	4·3	8·2	8·3	11·0	12·6
19	1·02	0·811	0·597	1·35	4·6	3·4	6·7	6·6	9·2	10·0
20	0·914	0·657	0·74	1·66	4·1	2·9	6·0	5·75	8·3	8·6
21	0·813	0·519	0·945	2·1	3·6	2·4	5·4	4·8	7·4	7·4
22	0·711	0·397	1·235	2·75	3·1	1·9	4·6	4·1	6·5	6·3
23	0·61	0·292	1·675	3·74	2·7	1·5	4·0	3·3	5·5	5·2
24	0·559	0·245	2·0	4·43	2·4	1·3	3·55	2·78	5·0	4·45
25	0·508	0·203	2·42	5·39	2·18	1·13	3·2	2·48	4·06	3·95
26	0·457	0·164	2·99	6·64	2·0	0·99	2·9	2·17	3·6	3·5
27	0·417	0·136	8·58	7·98	1·82	0·9	2·68	1·95	3·21	3·14
28	0·376	0·111	4·33	9·9	1·66	0·8	2·42	1·75	2·85	2·8
29	0·345	0·0937	5·22	11·65	1·65	0·75	2·22	1·6	2·58	2·55
30	0·315	0·0779	6·26	[illegible]	1·4	0·68	2·0	1·44	2·3	2·3
31	0·295	0·0682	7·02	16·0	1·3	0·64	1·81	1·35	2·13	2·15

Table 2·5. Characteristics of Carbon Brushes

S. No.	*Type of Brush Material*	*Specific Resistance* Ωm	*Scleroscope Hardness*	*Transverse strength* MN/m²	*Current density* A/cm²	*Contact Drop* (V)*	*Co-efficient of Friction***	*Pressure* KN/m²	*Peripheral Speed* m/s
1.	Carbon	25·5—89	45—85	24—69	5·5—7	Medium-High	Medium-High	13·8—41·5	10—20·5
2.	Carbon Graphite	12·5—63·5	40—75	20·5—51·5	7—8	Low-Medium	Low-Medium	13·8—41·5	15—25·5
3.	Graphite	7·5—46·5	10—30	6·5—31	6—9·5	Low-Medium	Low-Medium	8·5—27·5	45·5—61
4.	Graphite (Resin bonded)	12·5—1270	10—35	6·5—34·5	1·5—9·5	Medium-very high	Low-Medium		45·5—60
5.	Electrographite	10—89	12—75	10—69	5·5—11	Medium-High	Low-Medium	10—34·5	15—46
6.	Metal Graphite	0·05—2·5	7—35	17—69	11—23·5	Very low-Low	Very low-Medium	10—27·5	15—30·5

*High : over 2·5 V, Medium : 1·8—2·5 V ; Low : 1—1·18 V : Very Low : Below IV.

**High : over 0·26 ; Medium : 0·2—0·26 : Low : 0·15—0·2 : Very Low 0·15 and below

air, the conductors have to carry a large value of current. The high value of this current can be obtained by :

(*i*) adopting a large conductor area and thus using a small value of current density,

(*ii*) adopting a small conductor area and thus using a large value of current density.

Now, if a low value of current density is used, the conductor area has to be large which results in large volume of machine. On the other hand if a high value of current density is used, smaller conductor areas are obtained. This no doubt results in reduction in the volume of machine, but small conductor area results high resistance for the machine and, therefore, the I^2R loss is high resulting in large temperature rise.

Thus, it is clear that the value of flux density is limited by current density in conductors producing the field. With copper or aluminium conductors at normal operating temperature current densities must generally be limited to about 1 to 10 A/mm^2.

It is also clear from above that the limitations are not imposed by current density as such but by the excessive temperature rise produced on account of high value of resistance due to use of small conductor areas.

The resistance also depends upon the value of resistivity. Therefore, if we have a material of zero resistivity its resistance will be zero (and hence I^2R loss will also be zero) irrespective of the value of conductor area. Thus with such a conductor we can use a very high value of current density (and hence a very high value of flux density in air) giving very small machine volume and there will be an added advantage of absence of iron parts in the machine.

Materials exhibiting zero value of resistivity are known as **'superconductors'**. A large number of metals become super-conducting below a particular temperature characteristic of the particular metal. This temperature is known as the transition temperature. Table 2·6 gives the transition temperatures.

Table 2·6. Transition Temperatures

Metals	*Transition Temperature* K	*Compounds*	*Transition Temperature* K
Aluminium	1·18	Pb_2 Au	7·0
Cadmium	0·52	Sn Sb	3·9
Indium	3·41	Cu_2 S	1·6
Lead	7·19	Mo N	12·0
Mercury	4·15	ZrC	2·3
Tin	3·72		
Uranium	0·80		
Zinc	0·86		

Super conducting compounds and alloys do not necessarily have components which are themselves superconducting.

It is interesting to note that metals which are very good conductors at room temperature *i.e.* copper, silver and gold etc. do not exhibit superconducting properties, on the other hand the metals and alloys which are bad conductors at room temperature have super-conducting properties.

Superconductivity will disappear if the temperature of material is raised above its critical temperature or if a sufficiently strong magnetic field or current density is employed. This critical magnetic field i.e. the field at which superconductivity vanishes and the critical current density are a function of temperature and are low for high temperatures. The transition from the superconducting state to conducting state is reversible.

Coming back to electrical machines, with the introduction of superconducting materials, much higher current densities are possible and practical machines working at low temperatures (below the transition temperature of the materials used) may be developed. Supercooled coils can produce flux densities of 10 Wb/m^2 or higher. In comparison, it is only possible (with great difficulty) to produce a flux density of 0·1 Wb/m^2 in the absence of iron parts, by using normal coils at room temperature. Therefore, it is clear from above that with the development of superconductors, the machine sizes may be considerably reduced.

Superconducting transformer windings and rotor windings of large alternators have been developed but the experiments show that they are not yet economically feasible. This is because the advantages of superconductivity must be balanced against the capital, operating and energy-loss costs of providing it.

2·6. Magnetic Materials. All magnetic materials possess magnetic properties to a greater or a lesser degree. The magnetic properties of materials are characterized by their relative permeability. In accordance with the value of relative permeability, materials may be divided into three broad classes.

1. **Ferromagnetic materials.** The relative permeabilities of these materials are much greater than unity and these permeability values are dependent upon the magnetizing force.

2. **Paramagnetic materials.** These materials have their relative permeabilities only slightly greater than unity. The value of susceptibility is thus positive for these materials.

3. **Diamagnetic materials.** These materials have their relative permeabilities slightly less than unity. In both Paramagnetic and Diamagnetic materials the value of permeability is independent of the magnetizing force.

When a paramagnetic or a diamagnetic material is placed in a magnetic field, the distortion of field is negligible and, therefore, the force exerted by the magnetic field is small.

When a ferromagnetic material is placed in a magnetic field, there is considerable distortion and, therefore, the force exerted is very large. This property makes ferromagnetic materials very useful for electrical engineering applications. Iron, Nickel, Cobalt, and many of their alloys are ferromagnetic. To these substances can be added certain other ferromagnetic alloys and compounds containing aluminium, chromium, manganese, copper and silver.

2·7. Types of Magnetic Materials.

1. **Soft magnetic materials.** The hysteresis loss depends upon the area of hysteresis loop. For this reason, magnetic cores used in alternating magnetic fields are made from materials whose hysteresis loops are more or less narrow [see Figs. 2·1 (a) and (b)]. These materials are called soft magnetic materials.

Soft magnetic materials are used in the manufacture of eletrical machines, transformers and many kinds of electrical apparatus, instruments and devices.

2. **Hard magnetic materials.** Materials with broad hysteresis loops [Fig. 2·1 (c)] are called hard magnetic materials. These materials are used in certain types of electrical machines of low power rating, and in all kinds of instruments and devices requiring permanent magnets which set up magnetic fields of their own.

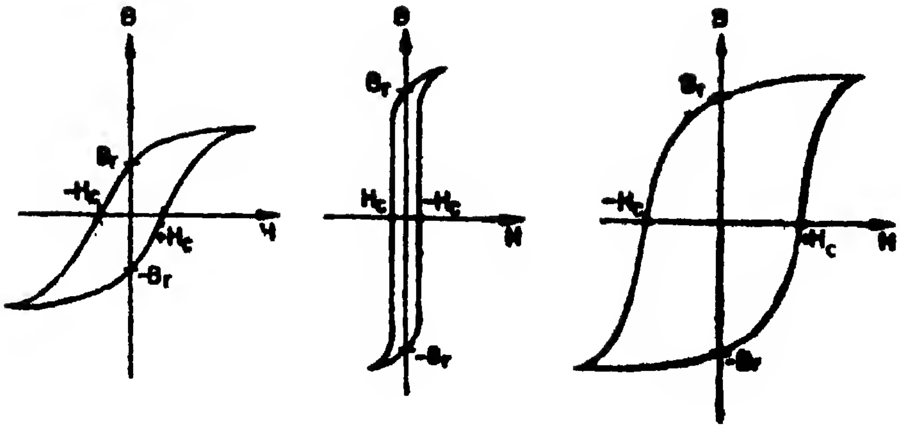


Fig. 2·1. Hysteresis loops.

2.8 Soft Magnetic Materials. Soft magnetic materials employed in commercial practice may be considered under the following three classifications :

(*i*) Solid core materials (*ii*) Electrical sheet and strip

(*iii*) Special purpose alloys.

2·8·1. Solid core materials. These materials are normally used for parts of magnetic circuits carrying steady flux such as cores of d.c. electromagnets, relays, and field frames of d.c. machines. The basic requirement used for steady magnetic fields is high permeability, particularly at high values of flux densities. For majority of uses it is also desirable that hysteresis be low. The principal materials used are soft iron, relay steel, cast steel, cast iron and ferro-cobalt (an alloy of 35 per cent cobalt and 65 per cent iron).

1. **Iron, low carbon, and silicon steel.** Technically pure iron is widely used in many kinds of electrical apparatus and instruments as cores and pole shoes for eletromagnets, components for relays, electrical instruments and other devices.

The most harmful impurity is carbon which sharply raises the coercive force and hysteresis loss. Silicon produces a harmful effect on very pure iron. But with the presence of oxygen in the iron, silicon is helpful as it increases the grain size. This increase in grain size improves the soft magnetic properties. During mechanical working the soft magnetic properties of the materials deteriorate and, therefore, it should be annealed in order to restore it to its initial state.

Low carbon electrical steel and iron have a comparatively low resistivity. Electrolytic iron for example has a resistivity of 0·1 ohm per m and mm². Because of their low resistivity, these materials have large eddy current loss if they are operated at high flux densities in alternating current fields. This limits their use to fields carrying steady flux or weak alternating current fields.

2. **Cast iron.** It has a low relative permeability and is used principally in field frames when cost is of primary importance and extra weight is not objectionable.

3. **Gray cast iron.** It is magnetically inferior to wrought iron or steel but is used to a limited extent because of ease of casting complex shapes.

Cast iron usually contains from 2·7 to 3·6 per cent carbon, 2·0 to 2·7 per cent silicon. Phosphorus ranges from 0·15 to 0·5 per cent and sulphur under 0·15 per cent.

4. **Cast Steel.** Cast steel is extensively used for those portions of magnetic circuit which carry steady flux and need superior mechanical quantities. Good quality cast steel has :

Carbon 5·5%, Silicon 0·2%, Manganese 2·5%, Phosphorus 0·08% and Sulphur 0·05%.

5. **Soft Steel.** Rolled and welded frames of soft steel plates are now widely used in place of cast steel.

6. **Ferro-cobalt.** It is characterized by very high permeability in the upper part of the normal induction range. It has its saturation flux density 10 per cent higher than that of pure iron. Its cost is relatively high and its use is limited to pole pieces where a high value of induction (flux density) is desired.

2·8·2. Sheet Steels.

Electrical Steel Sheets (Non Oriented Steel). In the early days of electrical industry the sheet material used for the magnetic circuits of electrical machines and the cores of the transformers was iron with a low content of carbon and other impurities. This had one major disadvantage—that of 'ageing'. **Ageing** is the term used to denote the deterioration of magnetic performance in service, caused by increase in co-ercive force and hysteresis loss which in turn caused commulative overheating and subsequent breakdown. At the beginning of the present century, however, it was discovered that a great improvement in magnetic properties could be obtained by alloying silicon with iron. At present the laminations used in electrical machines and in transformers working at or near supply frequencies are made of silicon steel in which the content of silicon lies from about 0·3% to 4·5% by weight.

Addition of silicon virtually eliminates ageing problem, gives reduced hysteresis loss and increased resistivity thereby reducing eddy current loss. Unfortunately the addition of silicon has two serious drawbacks as the percentage of silicon increases, it is found that there is some loss of permeability at higher flux densities and loss of ductility. Therefore, above about 5% silicon content, the resulting alloy is very brittle and cannot be punched or sheared.

The electrical sheet steels available vary from those having a low silicon content and therefore possess good permeability at high flux densities (high saturation flux densisy), high ductility but high losses, to those having a high silicon content and subsequently lower permeability, lower ductility but also lower losses. These electrical sheet steels may be manufactured by either hot rolling or cold rolling.

In rotating electrical machines, it is desirable to work the iron parts at higher values of flux density in order to achieve a higher output to weight ratio. The magnetic material therefore should have a high saturation flux density and hence the presence of silicon is a disadvantage. Therefore in rotating electrical machines we use steels of low silicon content and these steels are termed as **"dynamo grade steel"**

In small machines where iron losses are of secondary importance materials with 0·5% silicon or less are employed. On the other hand in turbo-generators the highest possible efficiency is desired and therefore in order to keep down iron losses, high silicon content laminations (upto 4·5% silicon) are used.

Sheet steels possessing higher silicon content (4—5% silicon) called **"transformer grade steels"** are mainly used in transformers as not much importance is attached to the magnetizing current. This steel is called **high resistance steel** (h. r. s.) on account of its high resistivity and consequently low eddy current loss.

The various grades of electrical sheet steel (Non-oriented Steel) are specified in IS : 648—1970. This steel is used for machines and transformers working at power frequency. The steel may be cold or hot rolled.

IS 648—1970 specifications for Non-oriented Electrical Steel Sheets for magnetic circuits specifies nominal thickness, length and width maximum core loss (at flux densities of 1 Wb/m^2 and 1·3 Wb/m^2) and silicon contents for various grades.

The standard grades and their characteristics are listed in Table 2·7. on pages 22, 23.

Table 2·7. Characteristics of Non-oriented Electrical Sheets for magnetic circuits

Grade	Density kg/m³	Silicon content percent	Thickness (mm)	Maximum iron losses at 50 Hz		B/H Characteristics		Resistivity Ωm	Resistivity Ω m Temperature coefficient of resistance
				1·0 Wb/m² (W/kg)	1·3 Wb/m² (W/kg)	H A/m	B Wb/m²		
Group A									
						400	1·31		
			0·40	—	4·92	800	1·46		
360	7800	0·30	0·50	3·60	5·58	1600	1·54	$0·15 \times 10^{-6}$	$33·8 \times 10^{-4}$
			0·63	4·30	6·50	4000	1·67		
			1·00	5·50	—	8000	1·78		
						24000	1·98		
						400	1·27		
			0·40	—	4·12	800	1·38		
300	7750	0·85	0·50	3·00	4·76	1600	1·50	$0·21 \times 10^{-6}$	$25·0 \times 10^{-4}$
			0·63	3·59	5·64	4000	1·64		
			1·00	4·62	—	8000	1·76		
						24000	1·96		
						400	1·30		
			0·40	—	3·70	800	1·38		
260	7700	1·75	0·50	2·60	4·12	1600	1·48	$0·33 \times 10^{-6}$	$16·5 \times 10^{-4}$
			0·63	2·84	4·70	4000	1·60		
			1·00	3·94	—	8000	1·72		
						24000	1·94		
						400	1·30		
			0·40	—	3·40	800	1·38		
230	7650	2·50	0·50	2·30	3·75	1600	1·48	$0·41 \times 10^{-6}$	$13·7 \times 10^{-4}$
			0·63	2·51	4·21	4000	1·60		

			1·00	3·08	—	8000	1·72		
						24000	1·92		
						400	1·28		
			0·40	—	2·93	800	1·37		
180	7600	3·00	0·50	1·80	3·22	1600	1·46	—	—
			0·63	2 13	3·64	4000	1·58		
			1·00	—	—	8000	1 70		
						24000	1·90		
Group B									
						400	1·28		
			0·35	1·40	2·36	800	1 36		
140	7550	3·50	0·40	—	2·51	1600	1·44		
		to	0·45	—	2·67	4000	1·56		
		4·00	0·50	—	2·82	8000	1·67		
						400	1·28		
			0·35	1·30	2·20	800	1·36		
130	7550	4·00	0·40	—	2·36	1600	1·44	—	—
			0·45	—	2·51	4000	1·56		
			0·50	—	2·67	8000	1·67		
Group C									
						400	1·30		
						800	1·41		
120(92)*	7560	4·00	0·35	1·20	2·03	1600	1·49	$0{\cdot}60\times10^{-4}$	$7{\cdot}5\times10^{-4}$
			0·50	1·54	2·51	4000	1·59		
						8000	1·70		
						24000	1·90		
						400	1·26		
108(86)	7550	4·00	0·35	1·08	1·90	100	1·35		
			0·50	—	—	1600	1·43	$0{\cdot}60\times10^{-4}$	$7{\cdot}5\times10^{-4}$
						4000	1·55		
						400	1·26		
99(80)	7580	4·00	0·35	0·99	1·76	800	1·35	$0{\cdot}60\times10^{-4}$	$7{\cdot}5\times10^{-4}$
			0·50	—	—	1600	1·47		
						4000	1 55		
			0·35	0·92	1 63				
92(74)	7500	4·00	0·50	—	—	—	—	$0{\cdot}60\times10^{-4}$	$7{\cdot}5\times10^{-4}$

*The figures within brackets indicate the equivalent grade numbers of Precision Pressing Division of M/s. Guest Keen, Williams

The preferred lengths and widths are given in Table 2·8.

Table 2·8. Preferred lengths and widths of non-oriented steel sheet

Length mm	3000	2500	2000	1500
Width mm	1000	1000	1000	750

Cold Rolled Grain Oriented Steel. (c.r.o.s.) By far the largest portion of magnetizing mmf in a normal rotating machine is required for the air gap, but if iron is worked too far in saturation region of its B—H curve the mmf required for iron parts of magnetic circuit would be very large. Since in order to keep down the size of the machine it is necessary to use a high flux density, the natural out-come is to develop a material which allows iron parts to be worked at high flux densities without requiring a large mmf and without causing excessive iron loss.

In ordinary hot rolled sheets the constituent crystals are disposed in a random fashion. The crystallographic axes of the individual crystals do not take up any special alignment with respect to the direction of rolling, with sheet surface or with each other. As a result, hot rolled sheets, have a low value of permeability.

The rolling direction of the material is magnetically superior to the other directions in the sheet and this property led to the introduction of "cold rolled grain oriented sheet steel". This steel is manufactured by a series of cold reductions and intermediate annealing. This cold reduced material has strong directional magnetic properties, the rolling direction being the direction of highest permeability. This direction is also the direction of lowest hysteresis loss. This type of material is suitable for use in transformers and also for large turbo-alternators since the axis of the core can be made to correspond with the rolling direction of the sheet and therefore full use is made of high permeability low loss direction of the sheet.

The directional variations of iron loss and magnetizing current of a typical specimen of cold rolled grain oriented steel are given in Fig. 2·2. The iron loss and magnetizing current in the direction of rolling for a given peak flux density are each taken as unity. For example, relative iron loss and magnetizing current are respectively 3·6 and 120 for magnetization on an axis at 60° to the direction of rolling. The use of c.r.o.s. is highly advantageous if its directional properties are properly utilized. This steel can be worked to higher flux densities than h. r. s.

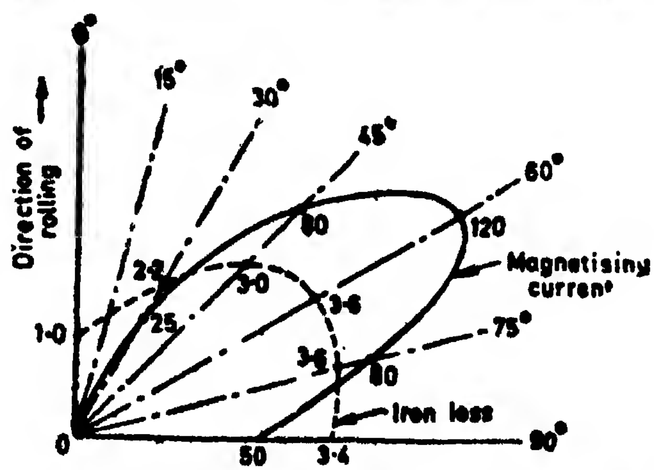


Fig. 2·2. Directional properties of cold-rolled grain oriented transformer steel.

Cold rolled grain oriented steels, though costly, give much reduced iron loss and much better magnetization curve than hot rolled steels. Therefore it is possible to employ a substantially higher flux density in the core for a given magnetizing current.

I.S. 3024—1965 (Amendment 1966). Specifications for Electrical steel sheets (oriented) gives the specifications for cold rolled grain oriented steel.

Thickness of lamination = 0·33 mm

Density = 7650 kg/m³

The guaranteed maximum iron loss and magnetising VA at 50 Hz and $B_m = 1\cdot$ Wb/m², for various grades are as shown in Table 2·9.

Table 2·9. Specific iron loss and VA/kg for different grades of c. r. o. s.

Grade	W/kg	VA/kg
136(62)	1·36	2·65
123(56)	1·23	2·09
112(51)	1·12	1·66
101(46)	1·01	1·33

The figures within brackets indicate the corresponding grades used by Precision Pressing Division of M/s Guest, Keen, Williams.

The B—H curves and loss curves of various grades of laminations are given in Chapter 4.

2·8·3. Special Purpose Alloys. In order to obtain high flux densities in weak magnetic fields in, say instrument transformers, use is made of special magnetic alloys having high initial and maximum permeability. A group of iron alloys containing Nickel between 30 to 90 percent with possible addition of molybdenum and chromium when given appropriate treatment during manufacture show very high permeablilities at low flux densities and much lower losses than iron. The important alloys in this category are Permalloy and Mumetal.

1. **Mumetal.** It has a lower permeability but has high electrical resistance so that eddy current losses are lower. It contains a certain amount of copper.

2. **Permalloys.** According to the nickel content, commercially produced permalloys can be divided into two groups : high nickel and low nickel.

High Nickel Permalloy. It is alloyed with small amounts of molybdenum, molybdenum with copper or molybdenum with chromium, the nickel content being as high as 80 percent. It has high initial and maximum permeability and high resistivity. This makes it suitable for magnetic amplifiers, weak current transformers, cored induction coils communication and control equipments.

Low Nickel Permalloy. It contains nickel from 38 to 50 per cent with additions of manganese, silicon and chromium. It has lower permeability than high nickel variety but has higher resistivity. It is used for making cores of transformers, induction coils and chokes and communication equipment.

3. **Superpermalloy.** It consists of iron and nickel alloyed with copper and molybdenum. This alloy is distinguished by its high purity. It has a very high initial relative permeability of upto 100,000.

4. **Perminvar.** It is often necessary to have a magnetic material in which the permeability is independent of the field strength. Such materials find applications in certain kinds of chokes and current transformers. One of these alloys is perminvar an alloy of iron, nickel and cobalt. Its use is limited by high cost and difficulties in its manufacture.

5. **Permendur.** For cores and poles of magnetic circuits whose purpose is to provide a rather strong magnetic field in the air gap of apparatus such as electromagnets, oscilographs, microphones etc., it is necessary to have a material capable of setting up a flux density much greater than that obtained with electrical steel. An alloy meeting above requirement is permendur. It contains 49 per cent cobalt and 2 per cent vanadium, and 49 per cent iron.

Table 2·10 shows the properties of some high permeability alloys.

Table 2·10. Properties of Special Purpose Alloys

Properties	*Mumetal*	*Permendur*
Initial relative permeability	40,000	1000
Maximum relative permeability	120,000	7000
Saturation Density, Wb/m^2	0·8	2·36
Remanence from Saturation (Wb/m^2)	0·47	1·5
Co-ercive force, A/m	0·2	16·0
Co-efficient of linear expansion, /°C	13×10^{-6}	9×10^{-6}
Resistivity, Ωm	$0·6 \times 10^{-6}$	$0·47 \times 10^{-6}$
Density, kg/m^3	8800	8050
Specific heat (J/kg-°C)	439	—

2·9. Insulating Materials. Insulating materials or Insulants are extremely diverse in origin and properties. They are essentially nonmetallic, are organic or inorganic, uniform or heterogeneous in composition, natural or synthetic. Many of them are of natural origin as, for example, paper, cloth, paraffin wax and natural resins. Wide use is made of many inorganic insulating materials such as glass, ceramics and mica. Many of the insulating materials are man-made products manufactured in the form of resins, insulating films etc. In recent years wide use is made of new materials whose composition and properties place them in an intermediate position between inorganic and organic substances. These are the synthetic organo-silicon compounds, generally termed as silicones.

2·10. Electrical Properties of Insulating Materials. There are many properties which determine the suitability of a material for use as an insulating material.

Resistivity or specific resistance, electric strength or breakdown voltage, permittivity and dielectric hysteresis.

An ideal insulating material should have :

(*i*) high dielectric strength, sustained at elevated temperatures,

(*ii*) high resistivity or specific resistance

(*iii*) low dielectric hysteresis,

(*iv*) good thermal conductivity,

(*v*) high degree of thermal stability *i.e.* it should not deteriorate at high temperatures.

In addition to above the material should have good mechanical properties such as ease of working and application should be able to withstand moisture, it should be non-hygroscopic vibration, abrasion and bending. Also it should be able to withstand chemical attack, heat and other adverse conditions of service.

In addition to above electrical properties, we must also consider the mechanical properties of the material, and its ability to withstand moisture, chemical attack, heat, and other conditions of proposed service.

Unfortunately, the electrical properties of insulating materials vary widely with many factors, including :

(i) dimensions of test piece,

(ii) r.m.s. value, wave form and frequency of impressed voltage,

(iii) temperature and moisture content of test piece,

(iv) mechanical pressure on test piece.

The data obtained by laboratory tests on specimens may be taken as typical value and a high factor of safety must always be provided in electrical insulation to allow for effect of moisture, heat, mechanical stress, and abnormal electrical voltage gradients owing to surges. Allowance also must be made for burrs, sharp edges which produce intense local voltage gradients.

2·11. Temperature Rise and Insulating Materials. Every electrical machine is a power converting device. A generator converts mechanical power into electrical power, a motor converts electrical power into mechanical power and a transformer converts electrical power at one voltage to electrical power at another voltage. During these processes of power or energy conversion, some waste in energy is inevitable. In electrical machines the loss in energy occurs in electric circuits and in portions of magnetic circuit which carry varying flux. Further losses occur in machine parts subjected to mechanical friction.

The losses produced in the machine are converted into heat energy, as a result of which the various parts of the machine are heated, *i.e.*, their temperature rises above that of the surroundings. It is important to note that the losses are mainly produced in the active parts of the machine *i.e.*, in the iron parts which carry flux and the conductors which carry current. Thus the heat energy appears mainly in the active parts resulting in increase in temperature of iron and copper above that of the ambient medium.

In order to ensure reliable and satisfactory operation of electrical machines, the heating of every part must be controlled within certain definite limits. The losses in an electrical machine are of importance not because they constitute a source of inefficiency, but, because most of them result in temperature of windings rising appreciably. This rise in temperature effects the insulating materials put to isolate the windings from the iron parts. It is essential to ensure the reliability of winding insulation as the insulating materials being to deteriorate at relatively small temperatures. There is always a safe maximum temperature to which a particular insulating meterial can be subjected to without losing its effectiveness and as the temperature rise in a machine depends upon the losses which in turn depend upon the output of the machine, it is obvious, that the maximum allowable load on machine would be determined, first of all by maximum permissible temperature of the insulating materials used in it.

As stated above, each insulating material has a certain permitted limiting temperature at which it may operate reliably for a sufficiently long period compatible with operation of the machine. We can consider the following example for illustrating the impor tance of temperature rise.

The life of class A insulating materials can be expressed by an empirical relation,

$$T_{life}=72\times10^{8}\exp[-0{\cdot}09\,\theta]$$

where, T_{life}=life of insulating material in years,

and θ=maximum temperature to which the material can be continuously subjected, °C.

Thus if it is operated at θ=90°C, it can safely operate for about 22 years but its life span would be cut down to approximately half (11·6 years) if the maximum temperature is raised to 97°C. The life is cut down to about 36 days when operated at 150°C and to

about 10 hours if operated at 200°C. Fig. 2·3 shows the relationship between temperature and life of class A insulating material. Hence it is clear that even if we operate the

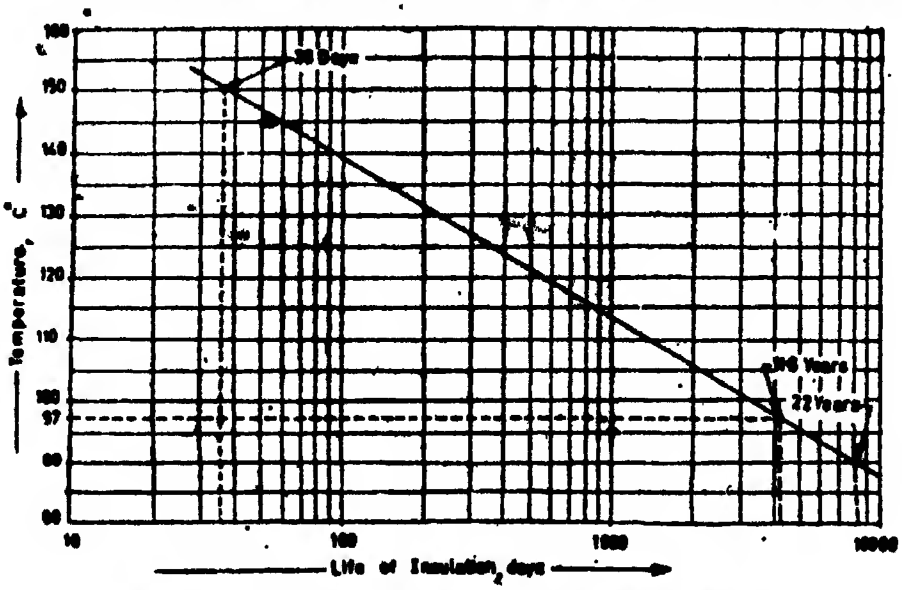


Fig. 2·3. Temperature rise-life curve for class A insulation.

machine a little above the maximum allowable temperature, it seriously affects the life of insulating materials. As the machine operation depends upon the condition of insulation, it is evident that the insulating materials must be worked within safe temperature limits.

In addition to affecting the insulation, an excessive temperature rise may also adversely influence the mechanical operating conditions of a given machine part. Thus, for example, the commutator may lose its original dimensions or the soldered joints between the commutator and the windings may open out or the bearings may break down. However, these problems are relatively minor and can be solved by designing the machine properly and operating it under correct running conditions.

It does not follow from above that the solution to the insulation problems or the other mechanical and electrical problems created by the temperature rise, lies in operating the machine with low temperature rise. Such a machine could easily built if low values for electricland magnetic loanings are used, but this would result in a very heavy and costly machine. Therefore, the correct solution is to build a machine which extensively utilizes its materials of construction and has the required efficiency and a sufficiently long span of life.

The heat produced in a machine depends upon the losses but the presence of ambient air, a cooling or a ventilating system tends to take away the heat produced. The temperature rise actually obtaining depends upon the relation between cooling conditions and amount of heat produced. The final temperature rise is reached when the rates of heat production and dissipation become equal. Thus, other things being equal, the temperature rise would be higher if the cooling conditions are worse and *vice versa*. Consequently the temperature rise in a machine is inseparable from the problem of cooling and ventilation.

The temperature rise in a machine can be kept within safe limits by properly designing its ventilation system. A higher output can be taken from a given machine frame by having a good ventilation system and proper insulating materials.

In recent years these problems have acquired a great importance in connection with intensification of machine utilization and a great deal of attention has been paid to obtaining increased output for a given weight of materials by improvements in ventilation and the heat resisting properties of insulating materials.

2·12. Classification of Insulating Materials. Classification of insulating materials for electrical machinery and apparatus in relation to their thermal stability are given in Indian Standard Publication No. 1271—1958.

This publication covers an extremely wide range of insulating materials and will benefit the reader to work to them when available.

The classification covers seven classes of insulating materials generally used in electrical machinery and apparatus in relation to their thermal stability in service.

Classification. The recognised classes of insulating materials and the temperature assigned to them are as follows :

Class	Temperature
Y (formely 0)	90°C
A	105°C
E	120°C
B	130°C
F	155°C
H	180°C
C	above 180°C

Class of materials having a temperature limit lower than that for class Y, is not included in this classification. Since the materials falling in class are not widely used as insulation for windings for machines, transformers or switchgear. Insulation may be grouped into the following recognised classes :

Class Y. This insulation consists of materials, or combinations of materials, such as cotton, silk and paper without impregnation. Other materials or combinations of materials can be included in this class, if by experience or accepted tests they can be shown to be capable of operating at class Y temperatures.

Class A. This insulation consists of materials or combinations of materials such as cotton, silk and paper when suitably impregnated or coated when immersed in a dielectric liquid such as oil. Other materials or combinations of materials may be included in this class, if by experience or accepted tests they can be shown to be capable of operation at class A temperatures.

Class E. This insulation consists of materials or combinations of materials which by experience or accepted tests can be shown to be capable of operating at class E temperature (materials possessing a degree of thermal stability allowing them to be operated at a temperature 15°C higher than class A materials).

Class B. This insulation consists of materials or combinations of materials such as mica, glass fibre, asbestos, etc., with suitable bonding substances. Other materials or combinations of materials, not necessarily inorganic, may be included in this class, if by experience or accepted tests they can be shown to be capable of operation at class B temperatures.

Class F. This insulation consists of materials or combinations of materials, such as mica, glass, fibre, asbestos, etc., with suitable bonding substances as well as other materials or combinations of materials, not necessarily inorganic, which by experience or accepted tests can be shown to be capable of operation at class F temperatures (materials possessing a degree of thermal stability allowing them to be operated at a temperature 25°C higher than class B materials).

Class H. This insulation consists of materials, such as silicon elastomer and combinations of materials such as mica, glass fibre, asbestos, etc., with suitable bonding subs-

tances, such as appropriate silicon resins. Other materials or combinations of materials may be included in this class, if by experience or accepted tests they can be shown to be capable of operation at class H temperature.

Class C. This insulation consists of materials or combinations of materials such as mica, porcelain, glass and quartz with or without an inorganic binder. Other materials or combinations of materials may be included in this class, if by experience or accepted tests they can be shown to be capable of operation at temperatures above the class H limit. Specific materials or combinations of materials in this class will have a temperature limit which is dependent upon their physical, chemical and electrical properties.

The examples of various classes of insulating materials are :

Class Y. Cotton, silk, paper, cellulose, wood etc., neither impregnated nor immersed in oil.

Materials of class Y are unsuitable for electrical machinens and apparatus as they deteriorate rapidly and are extremely hygroscopic.

Class A. Materials of class Y impregnated with natual resins cellulose esters, insulating oils, etc. Also included in this class are laminated wood, varnished paper.

Class E. Synthetic resin enamles, cotton and paper laminates with formaldehyde bonding, etc.

Class B. Mica, glass fibre, asbestos with suitable bonding substances ; built up mica, glass fibre, and asbestos laminates.

Class F. Materials of class B with bonding materials of higher thermal stability.

Class H. Glass fibre and asbestos materials, and built up mica, with silicon resins.

Class C. Mica, ceramics, glass, quartz without binders or with silicon resins of higher thermal stability.

Class C materials are not directly involved in machine design.

2.13. Insulating Electrical Materials Used in Modern Electric Machines

Mica. Mica is used in its virgin form or sheet state is difficult to work. Therefore, it is used in the form of sheets of splittings with shellac, bitumin or synthetic or polyester bending.

Micafolium. It is a wrapping consisting of mica splittings which are to paper and air dried. It may be moulded directly to on conductors then rolled and compressed between heated plates to solidify the material and to expel air.

Fibrous glass. It is made from material which is free alkali metal oxides which may form a surface coating that may attack the glass silicates. Glass does not absorb moisture volumetrically, but may attract it by capillary action between the fine filaments. Tapes and clothes woven from continuous filament yarns of glass have a high resistivity, thermal conductivity and tensile strength and form a good class B insulation. The space factor of this insulating material is good but the material is prone to abrasive damage. This glass-silk coverings are used for wires of field coils or mush windings of induction motors.

Asbestos. This material is mechanically weak, even when woven with cotton fibres, and is a poorer insulating materials as compared with fibre glass. Laminates of asbestos with synthetic resins have good mechanical strength and thermal resistivity. Asbestos in the form of wire and strip coverings have resilience and abrasion resistance. However, the space factor is low.

Cotton fibre. Fibre cotton woven from acetylates cotton, recently developed, have remarkable resistance to heat, "tendering". They are much less hygroscopic then ordinary cotton materials.

Polyamides. Polymides in the form of nylon tapes have high mechanical strength and have a good space factor on account of their thinness. Nylon film is one of the few plastic films having adequate resistance to temperature and can withstand tearing.

Synthetic-resin enamels. These enamels of the vinyl-acetate or nylon types have an excellect smooth finish and have been used for much windings, with considerable improvement in winding times and length of mean turn. They also give good binding to windings.

Slot-lining materials. These materials in the past have been mica composites. However, the mica content is easily damaged in the forming. In small motors a two-ply varnished cotton cloth bended to pressboard is found satisfactory. On the other hand, three-ply material may serve for heavier windings.

Wood. Wood, the form of synthetic-resin-impregnates compresses laminations, has proven itself to be a robust and accurate materials for packing blocks, coil supports and spacers.

Silicones. Silicones are semi-inorganic materials with a basic structures of alternate silicon and oxygen atoms. They are extremely resistant to heats. They act binders in Class H insulation and permit their continuous operation at 180°C. Even when disintegrate by excessive temperatures, the resisue is the insulator silica. Silicones are water-repellent and anti-corrosive. They are used in dry (oil-less) transformers, traction motors, mill motors and miniature aircraft machines operating over a winding temperature range of 200 to —40°C. They have a high thermal conductivity, improved heat transfer co-efficient. which facilitate heat dissipation from conductors.

Epoxide thermosetting resins. These materials have assumed importance in casting potting, laminating-adhesive and varnishing applications, and in the encapsulation of small transformers.

Synthetic Resin. Bonded paper, cotton and glass-fibre with synthetic resin laminates have good electrical and mechanical properties as sheets large cylinders and tubes.

Petroleum-based mineral oils. They are extensively used in the cooling and insulation of immersed transformers. The characteristics of importance are chemical stability, expansion coefficient, resistance to sludging by oxidation, and viscosity. Their electric strength is good when they are clean and moisture proof.

Askarels. They are synthetic non-flammable insulating liquids which, when decomposed by an electric arc, evolve only non-explosive gases. The commonest askarel is a 60/40 mixture of hexachlorodiphenyl trichlorovenzine giving a low pour point and a satisfactory viscosity/temperature characteristics.

2·14. Applications of Insulating Materials. We will consider some important materials in general use in electrical machines and apparatus, such as those employed for the insulation of :

(*i*) wires for magnet coils and windings of machines.

(*ii*) laminations.

(*iii*) machines and transformers.

2·14·1. Insulating Materials for Wires. Small round wires are used in enormous uantities for the coil qwindings of instruments, electromagnetic apparatus and electrical machines. The principal requirements of the insulating materials to be used in these cases are flexibility, thinness, rapidity of application, ability to withstand stresses and abrasive actions during the process of winding. The materials in general use are enamel, cotton, rayon, silk and fibrous class.

Enamel Covering. This consists of a thin film of either oil base or synthetic base varnish applied by drawing the wire through a trough of varnish and then through a heated

chamber so as to bake the varnish covering into a tough and elastic film of high dielectric strength. The process is rapid and cheap and therefore, enamelled wire is used almost universally for the small motors and industrial apparatus.

The commonly used enamels and their applications are given below :

Refrigerator motors—Polyvinylformal, Polyesterimide

General motors—Polyesterimide (theic)

Oil cooled transformers—Polyamideimide

Cokes—Polyesterimide (theic)

Due to thinness of varnish coating (0·0!5 to 0·075 mm) a high space factor is obtained for coil windings, which results in a saving in wire and reduced overall dimensions for the coil compared with other forms of covering.

Cotton Covering. It consists of a number of cotton threads wound helically on the wire in a lapping machine. Cotton covered wire is manufactured either with a single layer (single cotton covering S.C.C.) or two layers (double cotton covering D.C.C.). The thickness of cotton covering varies from 0·05 to 0·2 mm. D.C.C. wires are largely used for coils of field magnets, armatures and a.c. motors, as the cotton covering can withstand rougher handling than enamel covering and it provides a cushioning effect or the bedding of turns of the several layers. The coils are impregnated with varnish to improve dielectric strength and the heat dissipating qualities.

Silk Covering. It is used as covering for wires as it gives a high space factor. At present, owing to its high price, it is not normally used.

Fibrous Glass Covering. It consists usually of a double lapping of threads of continuous glass fibres. The thickness of double lapping for round wires is about 0·15 to 0·2 mm. Varnishing is essential to enable the covered wire to be handled and manipulated during winding. Impregnation with varnish increases the resistance of wire to abrasion and prevents absorption of moisture.

Fibrous glass coverings are employed for windings which are required to operate in the class B temperature range.

Asbestos Coverings. This covering was formally used for round wires when class-B insulation was required. But such coverings have been superseded by coverings of fibrous glass. The advantages of fibrous glass are its lower moisture, absorption and considerably high space factor.

2·14·2. Insulating Materials for Laminations. The core stacks in modern machines are subjected to high pressures during assembly and therefere to avoid metal to metal contact, laminations must be well insulated. The main requirements of a good lamination insulation are homogeneity in thin layers, toughness and high resistivity. The following are the common insulating materials for laminations :

1. **Insuline.** This is a kaolin (China) mixture which sprayed on to one or both sides of the lamination. The total thickness of coating per lamination is 0·01 to 0·025 mm.

2. **Oxide.** A natural oxide coating is formed on the sheets during the hot rolling process. But this insulation cannot be depended upon as it may be inadequate. Extra oxide coating, with a resistance about ten times as high as the resistance of natural oxide coating, is applied. This process is termed as "steam blueing".

3. **Varnish.** This is the most effective type of insulation now available. It makes the laminations rust proof and is not effected by the temperatures produced in electrical machines. Varnish is usually applied to both sides of lamination to a thickness of about 0·006 mm on plates of 0·35 mm thickness. Varnish gives a stacking factor of about 0·95.

2·14·3. Insulating Materials for Machines. D.C. and A.C. motors and generators for industrial purposes are usually insulated with class A or E materials, but turbo-alternators, traction motors and aircraft machines are insulated with class B materials to enable higher operating temperatures to be used for the purpose of obtaining larger output from a given frame size. Class E insulation it commonly used for induction motors.

Materials used for round wires are also employed for the insulation of rectangular wires and may be applied by either lapping or braiding.

Rectangular (bar) conductors, such as used for armatures of large machines are insulated by taping with the following materials.

1. **Class A Materials.** They include tapes and flexible sheet materials, the former being employed for the taping of rectangular conductors, armature and field coils of d.c. machines and stator and rotor of a.c. machines.

Cotton and Oiled Cambric Tapes. These tapes are usually 0·125 to 0·25 mm thick. Cotton tapes are impregnated to prevent absorption of moisture. Oiled Cambric tape (Empire cloth) is cut from a roll of fine cotton fabric which have been treated with linseed oil and oxidised. Its dielectric strength is 40 kV/mm.

Tough Fibrous Materials. These are used for slot linings of low voltage machines in thicknesses from 0·25 to 0·5 mm. Typical examples are latheroid, horn fibre, pressboard etc. manufactured from cellulose and rag and treated with oil.

Nylon and Terylene. These materials have high tensile strength and good dielectric properties (about 80 kV/mm for varnished paper).

2. **Class B Materials.** They are generally used as tapes. Typical materials are fibrous glass, asbestos and mica.

Fibrous Glass Tape. It is used in thickness from 0·075 to 0·275 mm. It is woven from continuous filament yarn and is impregnated with varnish before use. The dielectric strength is 40 kV to 48 kV per mm. The type of varnish determines the safe operating temperature as glass fibre alone can stand a temperature of 250°C.

Asbestos Tapes. Owing to superiority of fibrous glass tape, asbestos in the form of paper finds only limited application. They are useful where cushioning effect is required as in the inerturn insulation of bare wound on edge windings such as are employed in salient pole alternators and the interpoles of large d.c. machines.

Mica. Mica is used in the following forms :

(*i*) built up sheets, rings and cones, and (*ii*) tapes.

Mica is used in the form of splittings or flakes (0·0125 to 0·025 mm in thickness) which in the manufacture of sheet (called micanite), are mechanically shifted to the required thickness on to a thin supporting paper, cemented or bonded with shellac or a synthetic resin, and hot pressed to remove the solvent and consolidate the whole into a hard solid plate.

Mica tape consists of thin mica splittings or flakes bonded to a backing of thin high-grade paper, cotton cloth or fibrous glass to give mechanical strength. In some cases a thin paper is applied to the mica splittings after they have been laid on the backing paper.

When paper or cotton is employed as the backing material the thickness should be reduced to the minimum required for the mechanical strength, as at the upper limit of the Class B temperature these materials will ultimately be changed.

Mica tape is largely employed for taping armature and field coils of traction motors (in cases when fibrous glass tape is not employed) and for taping the stator coils of

high-voltage alternators. The dielectric strength is of the order of 40 to 48 kV/mm, and that of splittings alone of 80 to 160 kV/mm according to the quality.

Varnishes and impregnating compounds are important insulating materials for electrical machines. They are employed for the dual purpose of moisture-proofing and increasing the dielectric strength of fibrous insulating materials. High voltage multi-turn coils are pressure impregnated in vacuum to eliminate voids, the presence of which would increase the electric stress on the insulation and reduce the thermal conductivity of the coils.

Formerly, varnishes were made from natural gums or resins (with suitable solvents) and oxidizing oils (*i.e.* linseed oil). Although such varnishes are still employed for repair work, synthetic varnishes of the phenol-formaldehyde type are preferred by large manufacturers, as the polymerization, which takes place during baking, produces uniform hardening throughout, instead of only on the surface when the hardening occurs by evaporation or oxidization.

The new silicon varnishes are more heat resistant and waterproof than other varnishes, and enable machines with fibrous glass and other inorganic insulation to operate at temperatures of 200—250°C.

Laminates of paper, cotton cloth, asbestos, etc., formed by bonding layers of these materials with synthetic resin are employed for the terminal boards of d.c. and low voltage a.c. machines, and other parts requiring a rigid insulator with a flat surface.

Porcelain or moulded insulators are employed for insulating the terminals of high voltage machines.

2·14·4. Insulating Materials for Transformers. Fibrous (Class A) materials are usually employed for both air-cooled and oil-cooled transformers, but the high-grade asbestos paper tape previously mentioned is used in some air-cooled transformers when they are required to operate in the Class B temperature range for the purpose of reducing their weight (*i.e.* in portable welders and in aircraft).

Cotton or oiled cambric is used for taping the coils of air-cooled transformers (Class A temperature limits) ; the coils being impregnated after taping.

Synthetic-resin-bonded paper, treated pressboard or similar material is used for the insulation between core and coils and also between the primary and secondary windings.

High grade manilla paper tape is usually employed for insulating the conductors (except in cases where these are already insulated) of oil cooled transformers, and cotton tape for the external taping of the coils and bindings.

High quality synthetic-resin bonded paper, or alternatively a high grade presspaper (Elephantide), in the form of cylinders and flanges, is employed for the insulation between core and coils, and also between the primary and secondary windings.

Pressboard or presspaper is employed as spacers (to provide ducts for the oil), packing between coils, barriers between coils and tank, etc.

3

Heating and Cooling of Electrical Machines

MODES OF HEAT DISSIPATION

3·1. Heat Dissipation. The process of energy transfer in the case of transformers and electro-mechanical energy conversion in the case of rotating electrical machines involves currents in the conductors, and fluxes in the ferromagnetic parts. Thus there are I^2R losses in windings and core losses in the ferromagnetic cores. In addition losses occur in tank walls, end plates and covers on account of leakage flux. The losses appear as heat and therefore the temperature of every affected part of the machine rises above the ambient medium which is normally the surrounding air. The heated parts of an electrical machine dissipate heat into their surroundings by conduction, and convection assisted by radiation from the outer surfaces.

3·1·1. Conduction. This mode of dissipation of heat is important in the case of solid parts of machine like copper, iron and insulation. Consider two points in an electric circuit having potentials V_1 and V_2, the current flowing between them is,

$$I=\frac{(V_1-V_2)}{R}$$

where R is the electrical resistance of the conducting medium between them. Similarly we can write the equation for heat flow by conduction between two surfaces separated by a heat conducting medium, as :

$$Q_{con}=\frac{\theta_1-\theta_2}{R_\theta} \quad \text{...(3·1)}$$

where Q_{con}=heat dissipated by conduction, W ;

θ_1, θ_2=temperatures of two bounding surfaces, °C ;

R_θ=thermal resistance of the conducting medium, thermal ohm (or in °C/W.)

Thermal Resistance. The thermal ohm is defined as the thermal resistance which causes a drop of 1°C per watt of heat flow. Eqn. 3·1 permits heat conduction problems to be solved by methods of calculation similar to those used in electric circuits. The thermal resistance, like electrical resistance, can be written as :

$$R_\theta=\frac{\rho t}{S} \quad \text{...(3·2)}$$

$$=\frac{t}{\sigma S} \quad \text{...(3·3)}$$

ρ=thermal resistivity of material, Ω (thermal) m or °C$-$m/W ;

σ=$1/\rho$=thermal conductivity, W/°C$-$m ;

t=length of medium, m ;

S=area of surface separated by the medium, m^2

Eqn. 3·1 can be written as :

$$Q_{con}=\frac{S(\theta_1-\theta_2)}{\rho t} \text{ W} \qquad \ldots(3\cdot4)$$

Heat dissipated per unit surface area by conduction is :

$$q_{con}=\frac{\theta_1-\theta_2}{\rho t} \text{ W/m}^2 \qquad \ldots(3\cdot5)$$

The temperature difference across the conducting medium

$$\theta=\theta_1-\theta_2$$

$$=Q_{con}\, R_\theta \qquad \ldots(3\cdot6)$$

$$=Q_{con}\left(\frac{\rho t}{S}\right) \qquad \ldots(3\cdot7)$$

Considering Eqn. 3·7, we find that a material having a large value of thermal resistivity will dissipate less amount of heat or alternatively for the dissipation of same heat the temperature rise will be larger.

Table 3·1 gives the values of thermal resistivities of different materials.

Table 3·1. Thermal Resistivities

Material	*Thermal Resistivity (ohm metre)*	*Material*	*Thermal Resistivity (ohm metre)*
Air (Still)	20	Asbestos	4
Cotton Cloth	14	Empire cloth	
Micanite	8	Mica	3
Compressed paper	8	Sheet steel	
Paper	7·5	along laminations	0·02
Transformer coil	6·25	across laminations	0·05 to 0·1
Pressboard	6	Brass	0·01
Varnished cloth	5	Aluminium	0·005
Mica tape	1·6 to 6·6	Copper	0·0026

From Table 3·1, we have $\rho=20$ for air and 7·5 for paper. Air has a greater thermal resistivity than paper and thus the presence of air pockets in the insulation of a machine would have disastrous effects on heat dissipation, resulting in large temperature rises.

Let us consider the case of a heated surface where the coolant (cooling medium) is a fluid and takes away the heat by conduction. Perfect contact between the heated surface and the coolant at the outer face is rare. Therefore, the temperature difference θ is mainly dependent upon the fluid flow conditions *i.e.* whether the flow is stream line or turbulent and upon the condition of the surface. There may be interfering oxide films, gas bubbles in liquids, and a stagnant surface layer of fluid and therefore, the thermal conductivity of coolant is much smaller than that of metals.

The heat conducted across the interface is :

$$q_{con}=\rho_c(\theta_s-\theta_f) \text{ W/m}^2 \qquad \ldots(3\cdot8)$$

where θ_s and θ_f are the temperatures of surface and fluid respectively and ρ_c has a value of 30–1500 for oil and 10–50 for ai

Example 3·1. *A copper bar 12 mm in diameter is insulated with micanite tube which fits tightly around the bar and into the rotor slot of an induction motor. The micanite tube is 1·5 mm thick and its thermal resistivity is 8 Ωm. Calculate the loss that will pass from copper bar to iron if a temperature difference of 25°C is maintained between them. The length of bar is 0·2 m.*

Solution. Consider Fig. 3·1.

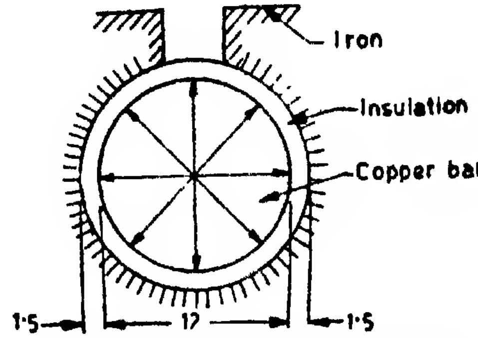


Fig. 3·1. Conduction from copper bar (All dimensions in mm)

Area of insulation in the path of heat flow

S=periphery of micanite at mean radius × length

$=\pi(12+15)\times10^{-3}\times0{\cdot}2$

$=8{\cdot}48\times10^{-3}\ \text{m}^2.$

From Eqn. 3·2, thermal resistance of micanite tube,

$$R_\theta=\frac{\rho t}{S}=\frac{8\times1{\cdot}5\times10^{-3}}{8{\cdot}48\times10^{-3}}$$

$=1{\cdot}415\ \Omega$

∴ From Eqn. 3·1 heat dissipated,

$$Q_{con}=\frac{\theta_1-\theta_2}{R_\theta}=\frac{25}{1{\cdot}415}=17{\cdot}67\ \text{W}.$$

Example 3·2. *The thermal conductivity of assembled armature laminations is 20 times as great along the direction of laminations as in the direction across the laminations. Calculate the loss that will be conducted across the laminations in a stack 40 mm thick and 6000 mm² in cross section with a difference of 20°C. Given that a difference of 5°C will cause 25 W to be conducted through a cross section of 2500 mm² in area and 20 mm thick measured along the laminations.*

Solution. Given :

$Q_{con}=25$ W, $t=20$ mm$=20\times10^{-3}$ m

$S=2500\ \text{mm}^2=25\times10^{-4}\ \text{m}^2$, and $\theta=\theta_1-\theta_2=5°C$.

This data refers to a direction along the laminations.

From Eqn. 3·4,

$$Q_{con}=\frac{S(\theta_1-\theta_2)}{\rho t}\quad\text{or}\quad 25=\frac{25\times10^{-4}\times5}{\rho\times2\times10^{-2}}$$

∴ Thermal resistivity along the direction of laminations

$\rho=0{\cdot}025\ \Omega$ m.

Hence thermal resistivity across the laminations$=20\times0{\cdot}025=0{\cdot}5\ \Omega$ m.

$$\left(\text{It is given that ratio }\frac{\text{resistivity across the laminations}}{\text{resistivity along the laminations}}=20\right).$$

Applying Eqn. 3·4, heat conducted across the laminations

$$Q_{con}=\frac{S(\theta_1-\theta_2)}{\rho t}$$

$$=\frac{6000\times10^{-6}\times20}{0{\cdot}5\times40\times10^{-3}}=6\ \text{W}.$$

3·1·2. Radiation. The heat dissipated by radiation from a surface depends its temperature and its other characteristics like colour, roughness etc.

For the case of a very small spherical radiating surface inside a large and or black spherical shell, the heat radiated per unit surface is given by Stefan Boltzmann law :

$$q_{rad}=5{\cdot}7\times10^{-8}\,e(T_1^4-T_0^4)\ \text{W/m}^2 \qquad \ldots(3{\cdot}9)$$

where T_1, T_0=absolute temperatures of the emmitting surface and the ambient medium respectively, K ;

θ_1, θ_0=temperatures of the emitting surface and the ambient medium respectively, °C ;

e=co-efficient of emissivity ;

=1 for perfect black bodies and is always less than unity for others.

Table 3·2 gives the co-efficient of emissivity and absorption factor for different types of surfaces.

Table 3·2. Emissivity and Absorption Factors

Surface	*Emissivity* e	*Absorption factor* e_a
Aluminium	0·10	0·15
Copper	0·15	—
Steel		
Rough	0·24	—
Sheet	0·55	—
Metal Paint Aluminium	0·55	0·55
Lead Paints		
White	0·90	0·25
Grey	0·95	0·75

Eqn. 3·9 can be written as :

$$q_{rad}=5{\cdot}7\times10^{-8}\,e(T_1-T_0)[T_1^3+T_1^2T_0+T_1T_0^2+T_0^3] \qquad \ldots(3{\cdot}10)$$

Now $(T_1-T_0)=[(273+\theta_1)-(273+\theta_0)]=\theta_1-\theta_0=\theta$

represents the temperature rise of the body, above the ambient medium.

The term $(T_1^2+T_1^2T_0+T_1T_0^2+T_0^3)$ varies relatively little within conventional temperature limits for electrical machines and so Eqn. 3·9 can be written as :

$$q_{rad}=5{\cdot}7\times10^{-8}\,eK_r(T_1-T_0)$$
$$=5{\cdot}7\times10^{-8}\,eK_r(\theta_1-\theta_0)\ \text{W/m}^2$$

where $K_r=[T_1^2+T_1^2T_0+T_1T_0^2+T_0^3]$.

The value of 'e' for cast iron or steel surface, varnished insulation etc. is between 0·90 to 0·97. Assuming a value of 0·83 for safety, we have :

$$q_{rad}=5{\cdot}7\times10^{-8}\times0{\cdot}83\,K_r(\theta_1-\theta_0)$$
$$=4{\cdot}8\times10^{-8}\,K_r(\theta_1-\theta_0)=\lambda_{rad}(\theta_1-\theta_0)$$
$$=\lambda_{rad}.\theta\ \ \text{W/m}^2 \qquad \ldots(3{\cdot}11)$$

where $\lambda_{rad}=4.8\times10^{-8}K_r$

=modified specified heat dissipation or emissivity measured in W/m^2 at a temperature rise of 1°C. Its value depends upon the temperature θ_1 and θ_0 and hence is not constant.

Table 3·3 gives the value of λ_{rad} with respect to temperature rises and temperatures of ambient medium.

Table 3·3. Specific Heat Dissipation (Radiation) in W/m^2—°C

Temperature rise θ°*C*	*Temperature of ambient medium* θ_0°*C*	
	20	30
5	5·03	5·55
20	5·42	5·98
40	6·05	6·59
80	7·30	7·98

The total heat dissipated by radiation is :

$$Q_{rad}=q_{rad}\times S=\lambda_{rad}\,\theta\,S \text{ watt} \quad ...(3\cdot12)$$

In electrical machines and transformers radiation does not normally occur by itself and in almost every case it is accompanied by convection. Therefore, the following expression may be conveniently used,

$$q_{rad}=2\cdot9\,e\theta^{1\cdot17} \quad \text{W/m}^2 \quad ...(3\cdot13)$$

Examining Table 3·2, we find that the value of co-efficient of emissivity for dull metallic point is 0·9 while for polished metal it is 0·15. This means that the specific heat dissipation (W/m^2—°C) due to radiation for surfaces painted with dull metallic paints is large. Hence all the electrical machines are painted with dull metallic paints (usually grey in colour) in order to have a large heat dissipation due to radiation. This keeps the temperature rise of the machines to a low value.

Some electrical apparatus, particularly transformers are used for outdoor duty. They may absorb radiant heat from the sun by **Insolation.** The earth's outer atmosphere receives about 1·3 kW/m^2 and under favourable climatic and air conditions, about two third of this energy may reach the earth's surface. In case the absorption factor of a surface is high, (approaching unity) it may absorb large energy by insolation thereby its temperature rises by a few degrees. However, if the absorption factor is small, the sun's radiation is re-emitted.

Example 3·3. *A heat radiating body can be assumed to be spherical surface with co-efficient of emissivity=0·8. The temperature of the body is 60°C and that of the walls of the room, in which it is placed, is 20°C. Find the heat radiated from the body in watt per square metre.*

Solution.

We have $e=0\cdot8,\ T_1=273+60=333$ K

$T_0=273+20=293$ K.

Using Eqn. 3·9,

Heat radiated $q_{rad}=5\cdot7\times10^{-8}\times0\cdot8\,(333^4-293^4)$

$=224\cdot6$ W/m^2.

Example 3·4. *A 250 V, 1 kW, single element resistor is made from 0·2 mm thick nickel chrome strip. The temperature rise of strip is not to exceed 300°C over the ambient temperature of 30°C. Calculate the length and width of the strip. Assume, emissivity = 0·9 radiating efficiency = 0·75, resistivity of nickel chrome 1×10^{-6} Ωm.*

Solution. Temperature of strip $\theta_1=300+30=330°C$.

Temperature of ambient medium $\theta_0=30°C$.

Absolute temperature of strip $T_1=330+273=603$ K.

Absolute temperature of ambient medium

$$T_0=30+273=303 \text{ K}.$$

Co-efficient of emissivity $=0·9$.

Effective co-efficient of emissivity, $e=0·75\times0·9=0·675$.

From Eqn. 3·9, heat dissipated by radiation

$$q_{rad}=5·7\times10^{-8}\, e\,(T_1^4-T_0^4)=5·7\times10^{-8}\times0·675\,(603^4-303^4)$$
$$=4760 \text{ W/m}^2.$$

$$\text{Resistance of strip } R=\frac{\text{voltage}^2}{\text{power}}=\frac{(250)^2}{1000}=62·5\ \Omega$$

Let, l = length of strip, m; w = width of strip, m ;

and t = thickness of strip, m.

Area of cross section of strip $=w\times t=w\times0·2\times10^{-3}=0·2w\times10^{-3}$ m².

Resistance of strip $R=\dfrac{\rho l}{wt}$

$$=\frac{1\times10^{-6}\,l}{0·2\times10^{-3}\,w}=\frac{5l}{w}\times10^{-3}\ \Omega$$

∴ $(5\,l/w)\times10^{-3}=62·5$ or $l/w=12500$.

Heating dissipating surface of strip, neglecting edges, $S=2\,lw$.

Total heat radiated $=q_{rad}\times S=4760\times2lw=9520\,lw$

∴ $9520\,lw=1000$ or $lw=0·105$

But $l/w=12500$ ∴ $l^2=0·105\times12500=1312·5$

or $l=36·2$ m

and $w=\dfrac{0·105}{36·2}\text{ m}=2·9$ mm

3·1·3. Convection. Heat dissipation by convection is classified into two categories : natural and artificial.

1. Natural Convection. Liquid and gas particles near heated body become lighter and rise, giving place to cooler particles which in turn get heated and rise. This natural process, due to changes in fluid density is known as natural convection.

The heat dissipated per unit surface by natural convection is given by

$$q_{conv}=K_c\,(\theta_1-\theta_0)^n \text{ W/m}^2 \quad ...(3·14)$$

where K_c = a constant depending on the shape and dimensions of hot body

n = a constant depending upon shape and dimensions of hot body. Its value lies between 1 and 1·25.

θ_1 = temperature of emitting surface, °C ;

θ_0 = temperature of ambient medium, °C.

Taking $n=1$,

heat dissipated per unit area by convection,

$$q_{conv}=K_c\,(\theta_1-\theta_0)=\lambda_{conv}\,\theta \qquad ...(3\cdot15)$$

where λ_{conv}=specific heat dissipation or emissivity due to convection measured in W/m^2—°C.

Total heat dissipated by convection,

$$Q_{conv}=q_{conv}\,S=\lambda_{conv}\,\theta S \text{ watt} \qquad ...(3\cdot16)$$

Convection is a complicated phenomenon and heat convected depends upon many variables such as (*i*) power density (*ii*) temperature difference between heated surface and coolant (*iii*) height, orientation, configuration and condition of heated surface (*iv*) thermal resistivity, density, specific heat, viscosity and co-efficient of volumetric expansion of fluid, and (*v*) gravitational constant.

In or er to find the heat convected the following formulae have been developed for the temperature range and structure of machines and transformers. For vertical planes of height not less than 1 metre and with a temperature difference θ between the surface and the ambient air of pressure p atmosphere,

$$q_{conv}=2\,p^{1/2}\,\theta^{5/4}\text{ W/m}^2 \qquad ...(3\cdot17)$$

For vertical tubes of diameter d metre,

$$q_{conv}=1\cdot3d^{-1/4}\theta^{5/4}\text{ W/m}^2 \qquad ...(3\cdot18)$$

2. **Artificial Convection.** In modern machines heat is removed by artificial circulation of cooling medium. For example, a transformer tank may be cooled by blasting air on it or a turbo-alternator may be cooled by circulating hydrogen. This is known as cooling by artificial convection. The usual method employed for cooling of machines is by blasting air on heated surfaces ; these surfaces may be open or closed. The increase in heat dissipation by air blasts is due to increase in convection. The problem of calculation of heat dissipation by artificial convection is very complex as it mainly depends on the constructional features of the machine. These constructional details are different for every machine and so no exact relationship can be given for artificial convection. However, one of the most widely used formulae for air blasts on open surfaces is :

$$\lambda'_{conv}=\lambda_{conv}\,(1+K_v\sqrt{V})\text{W/m}^2\text{—°C} \qquad ...(3\cdot19)$$

where λ'_{conv}=specific heat dissipation of a blasted surface.

λ_{conv}=specific heat dissipation by natural convection.

V=relative velocity of cooled surface and air blast, m/s

K_v=a constant, depending upon whether the blast is uniform or non-uniform,

=1·3 for uniform blasts.

The value of K_v comes down to even 0·5 for non-uniform blasts.

3·2. Newton's Law of Cooling. Losses are produced in various parts of electrical machines due to which the machine temperature rises. The machine attains a steady temperature rise after some time. At this temperature the heat produced in the machine is equal to the heat leaving its surface by radiation and convection.

It has been found earlier that the heat dissipated by radiation is given by relation :

$$Q_{rad}=\lambda_{rad}\,\theta S \text{ watt}$$

if temperature rise remains within normal conventional limits for electrical machines. We have also seen that if the temperature rise varies between moderate limits, the heat dissipated by convection is given by :

$$Q_{conv}=\lambda_{conv}\,\theta S \text{ watt}$$

∴ Total heat radiated by radiation plus convection is :

$$Q = Q_{rad} + Q_{conv} = \lambda_{rad}\,\theta S + \lambda_{conv}\,\theta S$$

$$= (\lambda_{rad} + \lambda_{conv})\,\theta S = \lambda\theta S \text{ watt} \qquad \ldots(3\cdot20)$$

where $\lambda = \lambda_{rad} + \lambda_{conv}$ ...(3·21)

= specific heat dissipation emissivity due to radiation plus convection

Equation 3·20 represents the Newton's Law of Cooling. It should be noted that Newton's Law of Cooling is strictly correct for cases where the body is acted upon by a uniform current of air. It is therefore applicable to natural cooling only for a restricted range of temperature, though the results become quite accurate if λ is understood to be strictly constant only for a given temperature rise.

Table 3·4 gives the total specific loss dissipation due to convection plus a radiation at 40°C.

Table 3·4. Emissivities of various Surfaces at 40°C
(Specific loss dissipation due to convection plus radiation)

Surface	λ W/m²–°C
Polished metal	8·2
Tarnished metal	9·1
Aluminium Paint	10·8
Oil Paint	13·0
Shellac Varnish	13·5
Cotton Tape	12·4
Varnished Tape	15·0

3·3. Ambient Temperature. The usage of the term "*ambient temperature*" is imbiguous. As far as radiation is concerned, the ambient temperature is the temperature of sky and the ground to which heat is radiated by the hot bodies. For other forms of heat dissipation to the atmosphere, the ambient temperature is the temperature of the bulk of the air at a distance remote enough to be affected by the thermal field of the heated body. This distance is usually a few metre. The use of air temperature as ambient temperature is justified only where most of heat is dissipated by convection.

TEMPERATURE GRADIENT

3·4. Internal Temperatures (Hot Spot Temperatures). The loss in electrical machines occurs inside the iron cores and windings. This loss, which is produced inside, is dissipated to the surface from where it is taken away by the cooling medium. This internal flow of heat, from the parts in which it is actually generated to the cooling surfaces from which it is transferred to the coolant, is important in determining the hot spot temperatures and the temperatures to which the insulating materials would be subjected.

If the cross section of a coil or a core, in which the electrical losses occur, is very large or if the insulation around the coil or the core is very thick, there is always a danger of exceptionally high internal temperatures developing, even when the temperature of the external surface is below the maximum specified limit. In order that there should not be any injury to the insulating materials we must determine the temperature of the hottest spot, a place where the local temperature is the highest.

Thus, the problem is to determine approximately the difference in temperature between the outside surface from which heat is carried away and the hottest spot inside the windings, from which the heat must travel through the conductors and insulation before it can be dissipated away. The problem is not so simple as it may seem to be, because of the complexity of nature of machine and also because the designer has to rely upon certain empirical formulae and experimental results.

3·4·1. Calculation of Internal Temperature. Fig. 3·2 represents a very large plate of thickness 't', consisting of a homogeneous material. Assuming the length and width of the plate to be very large as compared with its thickness, the heat flow need be considered only across its thickness (in the direction shown in Fig. 3·2). The heat flow is from centre outwards towards the two surfaces which are assumed to be at the same temperature.

Suppose

l=length of plate, m,
w=width of plate, m;
ρ=thermal resistivity of material along the direction of heat flow, Ω m;
q=heat produced per unit volume, W/m²;
θ=temperature rise, °C.

Consider an elementary strip of thickness dx at a distance x from the centre and let $d\theta$ be the temperature difference across the walls of this strip. Heat to be dissipated across this strip.

$$Q_x = \text{heat per unit volume} \times \text{volume} = q \times lwx$$

Temperature difference between the walls of this strip is :

$$d\theta = \text{heat conducted} \times \text{thermal resistance of the strip}$$

$$= qlwx \times \frac{\rho dx}{lw} = q\rho x \; dx.$$

∴ The difference in temperature between the centre and any point at a distance x from the centre along the path of heat flow,

$$\theta = \int_0^x q\rho x \; dx = \frac{q\rho x^2}{2}. \qquad \ldots(3\cdot22)$$

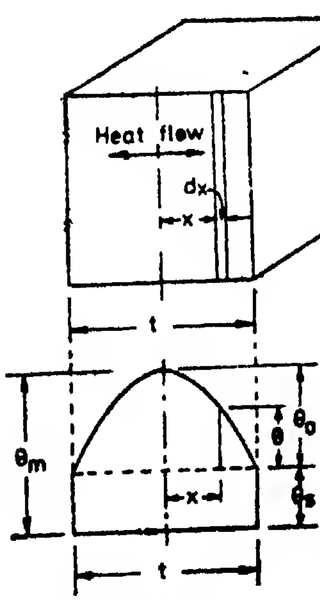


Fig. 3·2. Temperature Gradient.

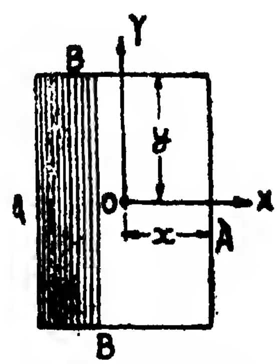


Fig. 3·3. Built up laminated core.

Thus the temperature difference curve is a parabola as shown in Fig. 3·2.

From Eqn. 3·22, the difference in temperature between the centre of the plate and the outer surface (*i e.* $x=t/2$) is,

$$\theta_s=\frac{q\rho t^2}{8}\ ^\circ\mathrm{C} \qquad \ldots(3{\cdot}23)$$

Hence temperature of the hottest spot (centre of plate in this case).

$$\theta_m=\theta_s+\theta_s=\frac{q\rho t^2}{8}+\theta_s \qquad \ldots(3{\cdot}24)$$

where θ_s = temperature of the surface.

3·4·2. Temperature Gradients in Cores. The built up core of an electrical machine is shown in Fig 3·3. It consists of steel laminations insulated from each other by varnish. The core is subjected to alternating magnetization and therefore there is iron loss due to which heat is produced.

The hottest part of the core is at O and heat generated at the centre is to be conducted to surfaces A and B. The path of heat flow along X axis is across the laminations while the heat flow along Y axis is along the laminations.

(*i*) Consider that all the flows along the direction OX.

Therefore from Eqn. 2·22, temperature difference between O and A :

$$\theta_{OA}=\frac{q\ \rho_x\ x^2}{2} \qquad \ldots(3{\cdot}25)$$

where ρ_x is the thermal resistivity across the laminations.

(*ii*) Considering the total heat to flow along the laminations (along OY),

Temperature difference between O and B :

$$\theta_{OB}=\frac{q\ \rho_y\ y^2}{2} \qquad \ldots(3{\cdot}26)$$

where ρ_y is the thermal resistivity along the laminations.

The value of the thermal resistivity along the laminations is low as compared with that acroses the laminations.Therefore at first sight it would indicate that all the heat should be taken along the laminations in order to keep down the internal temperatures or indicates that axial ventilation wherein air is blown across the laminations would be most effective. However, in practice nearly all electrical machines have radial ventilating ducts and their use can be explained with the help of the following example.

Example 3·5. *The thermal resistivity of a stack of laminations measured along the laminations is 0·02 Ωm. The iron loss is 40 kW/m³. Calculate the value of temperature difference between the hot spot and the outside surface for the following cases :*

(*i*) *Length of stack measured along the laminations=0·2 m.*

Length of stack measured across the laminations=0·1 m.

(*ii*) *Length of stack measured along the laminations=1 m.*

Length of stack measured across the laminations=0·1 m.

The thermal resistivity across the laminations is 40 times that along the laminations.

Comment upon the results.

Solution. (*i*) Refer to Fig. 3·3 we have,

$x=0{\cdot}05$ m, $y=0{\cdot}1$ m.

ρ_y=thermal resistivity along $OY=0{\cdot}02\ \Omega$m.

ρ_x=thermal resistivity along $OX=40\times0{\cdot}02=0{\cdot}8\ \Omega$m.

Considering the total heat to flow along the laminations.

Temperature difference between O and B

$$\theta_{OB}=\frac{q\rho_y y^2}{2}=\frac{40{,}000\times0{\cdot}02\times0{\cdot}1^2}{2}=4°\text{C}.$$

Considering total heat to flow across the laminations.

Temperature difference between O and A,

$$\theta_{OA}=\frac{q\rho_x x}{2^2}=\frac{40{,}000\times0{\cdot}8\times0{\cdot}05^2}{4}=40°\text{C}$$

(*iii*) $x=0{\cdot}05$ m, $y=0{\cdot}5$ m.

Considering the heat flow along the laminations

$$\theta_{OB}=\frac{40{,}000\times0{\cdot}02\times0{\cdot}5^2}{2}=100°\text{C}$$

Considering the heat flow across the laminations

$$\theta_{OA}=\frac{40{,}000\times0{\cdot}8\times0{\cdot}05^2}{2}=40°\text{C}.$$

Comments. From the above example it is clear that only with shallow cores it is possible to consider dissipation of heat from edges of the laminations as in this case the temperature difference between the hot spot and outer surface is not excessive if the heat flow is taken along the laminations. But with long cores, internal temperatures would tend to be too excessive if heat flow is taken along the laminations. Thus, in machines with long cores it is necessary to take heat across the laminations and dissipated at the ventilating ducts even though the thermal resistivity across the laminations is higher. Hence, machines with long cores require radial ventilation.

Example 3·6. *A transformer core of plate width 0·5 m and with a stacking factor of 0·94, has a uniformly distributed core loss of 3W/kg. The thermal conductivity of the steel is 150 W/°C—m and a surface temperature is 40°C. Estimate the temperature of the hot spot if the heat flow is (i) all to one of the core, (ii) one half to the surface of each end. The heat flow is assumed to be along laminal. The density of steel plate is 7800 kg/m³.*

Solution. Core loss per unit volume $q=3\times0{\cdot}94\times7800=22000$ W/m³

Thermal resistivity $\rho=\dfrac{1}{\text{thermal conductivity}}=\dfrac{1}{150}$

Temperature of the hot spot $\quad\theta_o=\dfrac{q\rho x^2}{2}+\theta_s$

(*i*) If heat is taken all to one end $x=t=0{\cdot}5$ m.

$$\therefore\quad \theta_m=22000\times\frac{1}{150}\times\frac{(0{\cdot}5)^2}{2}+40=58{\cdot}3.$$

(*ii*) If heat is taken to both the directions $x=t/2=0{\cdot}25$ m.

$$\therefore\quad \theta_m=22000\times\frac{1}{150}\times\frac{(0{\cdot}25)^2}{2}+40=44{\cdot}6°\text{C}.$$

3·4·3. Heat Flow in Two Dimensions. The applications of principles applied in Art. 3·4·1 and 3·4·2 to practical problems is complicated by the fact that heat does not travel along parallel paths and the dissipating surfaces are not homogenous. In actual practice the heat flow is in different directions and the windings and cores have insulation in addition to copper and iron respectively. The thermal resisitivity of built up windings and cores depends upon relative thickness of insulation to copper or iron.

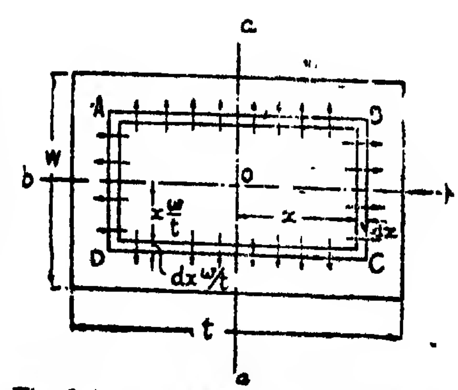


Fig. 3·4. Heat Flow in two dimensions.

Consider a coil or a core having a large axial length as compared with its width or thickness. Fig. 3·4 shows a section through the coil. The whole of the outside surface is supposed to be maintained at a constant temperature. In other words, it is assumed that there is a constant difference of temperature between the hottest spot (O in this case) and any point on the outer surface.

Let l=length of coil m;
w=width of coil m ;
t=thickness of coil, m ;
ρ_y=thermal resistivity along aa, Ω m ;
ρ_x=thermal resistivity along bb, Ω m ;
q=heat produced per unit volume, W/m³.

It can be assumed that the heat travels outward through walls of successive imaginary spaces of rectangular section. $ABCD$ is the boundary of one such rectangular space, the wall of which have thickness dx in the direction bb and $dx\ w/t$ in the direction aa.

Thermal resistance of horizontal elementary strips

$$=\frac{\rho_y\ dx\ w/t}{l(AB+CD)}=\frac{\rho_y\ w\ dx}{4l\ xt}$$

Thermal resistance of vertical elementary strips

$$=\frac{\rho_x\ dx}{l(AD+BC)}+\frac{\rho_x\ tdx}{4lwx}$$

These two thermal paths are in parallel.

Total thermal resistance of walls

$$d_R\theta=\frac{\frac{\rho_y\ w\ dx}{4lxt}\cdot\frac{\rho_x\ t\ dx}{4l\ xw}}{\frac{\rho_y\ w\ dx}{4\ lxt}+\frac{\rho_x\ t\ dx}{4lxw}}$$

$$=\frac{\rho_x\ \rho_y\ tw\ dx}{4lx(\rho_y w^2+\rho_x t^2)}$$

Let q_x be the heat produced in $ABCD$

$$\therefore \qquad q_x=ql\left(2x.2x\ \frac{w}{t}\right)=4qlx^2\ \frac{w}{t}$$

Temperature difference between inner and outer walls of $ABCD$.

$d\theta$=heat loss in space $ABCD\times$thermal resistance of walls

$$=4\ qlx^2\frac{w}{t}\cdot\frac{\rho_x\ \rho_y\ tw\ dx}{4lx\ (\rho_y\ w^2+\rho_x\ t^2)}$$

$$=\frac{qw^2\ \rho_x\ \rho_y\ x\ dx}{[\rho_y w^2+\rho_x t^2]}$$

Difference of temperature between O and outer surface

$$\theta=\frac{qw^2\ \rho_x\ \rho_y}{(\rho_y w^2+\rho_x t^2)}\int_0^{t/2} x dx=\frac{qw^2t^2\ \rho_x\ \rho_y}{8(\rho_y w^2+\rho_x t^2)} \qquad \ldots(3\cdot27)$$

Let Q be the total heat produced in the coil

$\therefore \qquad Q=ql\ wt$

Putting this value of Q in Eqn. 3·27 we get

$$0=\frac{Q}{8l}wl\frac{\rho_x\,\rho_y}{(\rho_y w^2+\rho_x t^2)}=\frac{Q}{8l\left(\frac{w}{t\,\rho_x}+\frac{t}{w\,\rho_y}\right)} \quad ...(3·28)$$

3·4.4. Thermal Resistivity of Winding. ρ_y and ρ_x are the effective resistivities along the directions aa and bb. These values of thermal resistivities include effects of both copper (or iron as the case may be) and insulation. Their values not only depends upon the thermal resistivities of copper and insulation but not on their relative thicknesses.

Let a, b=thickness of copper per metre along aa, and bb respectively

ρ_c, ρ_i=thermal resistivity of copper and insulation respectively

$R_{\theta ac}, R_{\theta ai}$=thermal resistance of copper and insulation respectively along aa,

$R_{\theta a}$=total thermal resistance along aa.

Since both copper and insulation are in series in the path of heat flow.

$$R_{\theta a}=R_{\theta ac}+R_{\theta ai} \quad ...(i)$$

But
$$R_{\theta a}=\frac{\rho_y\times w}{t\times l},\quad R_{\theta ac}=\frac{\rho_c\times w\times a}{t\times l}$$

and
$$R_{\theta ai}=\frac{\rho_i(w-w\times a)}{t\times l}.$$

Putting these values in (i), we have :

$$\rho_y\times w=\rho_c\times w\times a+\rho_i(w-w\times a)$$

or
$$\rho_y=\rho_c\times a+\rho_i(1-a)$$

The resistivity of copper is negligible as compared with that of insulation.

$\therefore$
$$\rho_y=\rho_i(1-a)$$

and similarly
$$\rho_x=\rho_i\,(1-b).$$

If $a=b$ i.e. copper per metre is the same in both the directions,

$\therefore$
$$\rho_x=\rho_y=\rho_i\,(1-a) \quad (3·29)$$

Generalizing the above equations, we have,

$$\rho_e=\rho_i\,(1-a) \quad ...(3·30)$$

where ρ_e=effective thermal resistivity of winding (i.e. copper plus insulation)

ρ_i=thermal resistivity of insulation

a=length of copper per metre of winding thickness.

Eqn. 3·30 can be written as,

$$\rho_e=\rho_i\,\left(1-S_f^{\frac{1}{2}}\right) \quad ...(3·31)$$

where
$$S_f=\text{space factor}=\frac{\text{copper area}}{\text{total winding area}}$$

$$=\frac{a\times a}{1\times 1}=a^2$$

$\therefore$
$$a=S_f^{\frac{1}{2}} \quad ...(3·32)$$

Example 3·7. *A field coil has a cross section of 100×50 mm² and its length of mean turn is 1 m. Estimate the hot spot temperature above that of the outer surface of the coil if the total loss in the coil is 120 W. Assume : Space factor=0·56.*

Thermal resistivity of insulating material=8 Ω m.

Solution. From Eqn. 3·31 effective thermal resistivity,

$$\rho_e=\rho_i\,\left(1-S_f^{\frac{1}{2}}\right)=8\left(1-0·56^{\frac{1}{2}}\right)=2\ \Omega\text{m}.$$

Volume of coil$=100\times10^{-3}\times50\times10^{-3}\times1=5\times10^{-3}$ m³.

$$q=\frac{120}{5\times10^{-3}}=24\times10^{3}\ \text{W/m}^3.$$

Assuming equal inward and outward heat flows and applying Eqn. 3·23, hot spot temperature rise

$$\theta_0=\frac{q\rho t^2}{8}=\frac{24\times10^3\times2\times(50\times10^{-3})^2}{8}=15°\text{C}.$$

Example 3·8. *The copper loss in a winding, 25 mm thick radially is 20W/kg. The thermal resistivity of paper insulation used in this coil is 8 Ω m and the copper space factor is 0·7. The coil is mounted on a former of infinite thermal resistivity. Calculate the maximum temperature difference between the coil surface and the winding.*

One cubic metre of copper weighs 8900 kg.

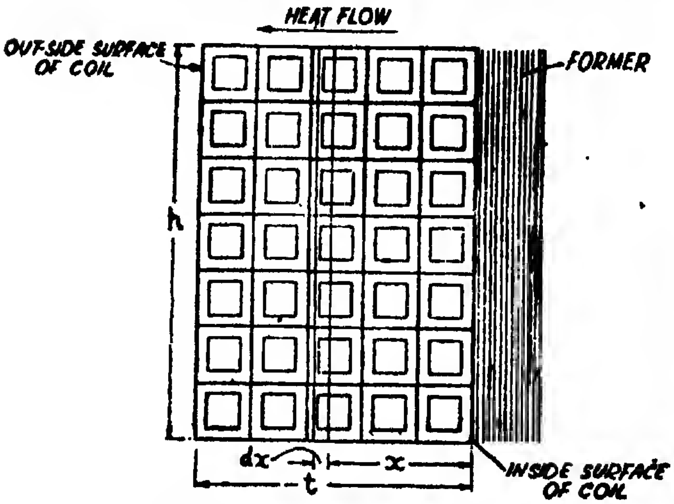


Fig. 3·5. Coil with an insulating cylinder of infinite thermal resistivity.

Solution. Fig. 3·5 shows the arrangement of the coil. As the former has infinite thermal resistivity, the flow of heat in the direction of the former is zero.

Taking the outer surface of former as reference and considering an elementary strip of thickness dx at a distance x.

Heat to be dissipated through this strip $Q_x=qxhw$

where q=heat produced per unit volume.

Thermal resistance of strip

$$dR=\frac{\rho_e dx}{w\times h}$$

where ρ_e=effective thermal resistivity along the direction of heat flow

∴ Temperature across the strip $d\theta=Q_x\times dR$

$$=qxhw\times\frac{\rho_e dx}{wh}=q\,x\rho_e\,dx.$$

The maximum temperature difference is between inner and outer surfaces of coil, and by integrating the above expression, its value is,

$$\theta=\int_0^t q\;x\;\rho_e\;dx=\frac{q\rho_e t^2}{2}.$$

We have $\rho_e=\rho_i(1-S_f^{1/2}),=8(1-0{\cdot}7^{1/2})=1{\cdot}31\ \Omega\text{m}.$

$q=0{\cdot}7\times20\times8900=124600\ \text{W/m}^3$

From the relation derived above,

$$\theta=\frac{q\rho_e t^2}{2}=\frac{124600\times1{\cdot}31\times(25\times10^{-3})^2}{2}=51°\text{C}.$$

3·4·5. Temperature Gradients in Conductors placed in Slots. The conductors placed in slots carry current and therefore heat is produced in them on account of copper loss. In order to assess the temperature in the centre of the core, we consider the following two cases.

(*i*) Considering the slot insulation to be comparatively very thick as compared with that on the end connections, the heat produced in the embedded portion of the conductor is conducted along its length to the end windings. Fig. 3·6 shows a conductor placed in a slot. It is desired to find the temperature difference between its centre O and the overhang.

Let I_z=current carried by the conductor, A ;

L=length of the embedded conductors, m ; a_z=area of the conductor, m²,

ρ_c=thermal resistivity of conductor, Ωm ;
δ=current density in the conductor, A/m^2 ;
ρ=electrical resistivity of conductor, Ωm.

Consider a strip of width dx at a distance x from O.

∴ Heat conducted through the strip,

$$Q_s = I^2R \text{ loss between } O \text{ and strip}$$
$$= I_s^2 \frac{\rho x}{a_s}$$

Thermal resistance of the strip

$$dR_s = \frac{\rho_c dx}{a_s}$$

From Eqn. 3·6, the temperature across the strip

$$d\theta = I_s^2 \frac{\rho x}{a_s} \cdot \frac{\rho_c dx}{a_s} = \rho \frac{I_s^2}{a_s^2} \rho_c \, x \, dx$$

∴ Temperature difference between O and overhang

$$\theta = \int_0^{L/2} \rho \times \frac{I_s^2}{a_s^2} \rho_c x dx$$

$$= \rho \frac{I_s^2}{a_s^2} \rho_c \frac{L^2}{8} \text{ °C} \quad ...(3·33)$$

$$= \rho \delta^2 \rho_c \frac{L^2}{8} \text{ °C} . \quad ...(3·34)$$

(*ii*) Consider that the overhang is considerably hot and therefore the heat produced in the embedded portion of the conductor is to be conducted through the slot insulation to the iron core. We have now to compute the temperature difference between copper and the surrounding iron.

Referring to Fig. 3·7.

Let W_s=width of the slot, m ;
d_s=depth of slot, m ;
t=thickness of insulation, m ;
ρ_i=thermal resistivity of insulation ; Ωm.

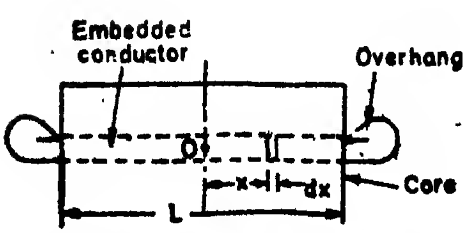


Fig. 3·6. Temperature gradient across the length of embedded conductor.

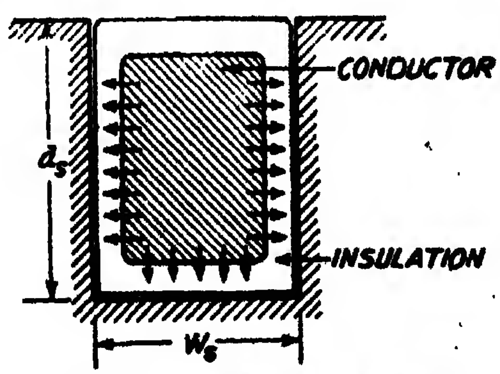


Fig. 3·7. Temperature gradient across the insulation of conductor embedded in a slot.

Heat produced in the conductor $Q=I_s^2\frac{\rho L}{a_s}$.

Area to the path of heat flow$\simeq L(2d_s+W_s)$.

Thermal resistance of insulation $R_s=\frac{\rho_i t}{L(2d_s+W_s)}$

$\therefore$ From Eqn. 3·6, temperature gradient across the insulation,

$$\theta=I_s^2\frac{\rho L}{a_s}\times\frac{\rho_i t}{L(2d_s+W_s)}$$

$$=I_s^2.\frac{\rho}{a_s}..\frac{\rho_i t}{(2d_s+W_s)}\ ^\circ\text{C}. \qquad \ldots(3\cdot35)$$

Putting $I_s=\delta a_s$, we have :

$$\theta=\delta^2\ a_s\ \rho\frac{\rho_i t}{(2d_s+W_s)}\ ^\circ\text{C} \qquad \ldots(3\cdot36)$$

Heating of Turbo-alternator-Rotors. The temperature difference between the conductors in a slot and its iron walls is :

$$\theta=I_s^2\frac{\rho}{a_s}.\ \frac{\rho_i t}{(2d_s+W_s)}\ ^\circ\text{C}. \quad \text{(See Eqn. 3·35)}$$

We can neglect the term W_s for the case of strip copper laid flat in the rotor slots of turbo-alternators because the heat will not travel down the layers of insulation.

$\therefore$ For turbo-alternator rotors,

$$\theta=I_s^2\frac{\rho}{a_s}\ \frac{\rho_i t}{2d_s} \qquad \ldots(3\cdot37)$$

Example 3·9. *Calculate the temperature difference between the centre of the embedded portion of a conductor and the overhang. The length of the machine is 0·5 m and the current density in the conductors is 4 A/mm². The thermal resistivity of copper is 0·0025 Ωm. Assume that the total heat produced is conducted along the length of the conductor, and the electrical resistivity of conductor is 0·021×10⁻⁶ Ωm.*

Solution. From Eqn. 3·34,

$$\theta=\rho\delta^2\rho_c\ \frac{L^2}{8}$$

$$=0\cdot021\times10^{-6}\times(4\times10^6)^2\times0\cdot0025\times\frac{0\cdot5^2}{8}=26\cdot3\ ^\circ\text{C}.$$

as $\qquad \delta=4\text{A/mm}^2=4\times10^6\ \text{A/m}^2$

Example 3·10. *A 6600 volt alternator has open slots each containing 4 conductors. Each conductor has a cross section of 6×8 mm² and carries a current of 200 A. The insulation between the conductors and the slot walls is 3 mm thick and has a thermal resistivity of 3 Ωm. The length of slot portion of conductor is 0·3 m. Calculate the temperature difference between the conductor and the slot walls if :*

(i) the coils fit tightly in the slots

(ii) there is an air space 0·5 mm thick between the coils and the slot walls.
Air has a thermal resistivity of 20 Ωm.

Take the resistivity of copper as 0·021×10⁻⁶ Ωm.

Solution. Total copper loss in each slot

$$=4\times200^2\times\frac{0\cdot021\times10^{-6}\times0\cdot3}{6\times8\times10^{-6}}=21\ \text{W}.$$

The area presented to the path of heat flow is calculated at the centre of insulation thickness.

$$S=0\cdot3\times[(8+3)+2(4\times6+1\cdot5)]\times10^{-3}=18\cdot6\times10^{-3}\ m^2.$$

From Eqn. 3·7, temperature difference across the insulation

$$\theta=Q_{con}.\frac{\rho t}{S}=21\times\frac{3\times3\times10^{-3}}{18\cdot6\times10^{-3}}=10\cdot16\ °C.$$

Therefore 10·16°C is the temperature between the coil and the slot walls if the coils tightly fit into the slots.

(*ii*) The calculation of area presented to the path of heat flow through air is based upon the outer dimensions of the insulated coil.

$\therefore\quad S=93\times[(8+2\times3)+2(4\times6+3)]\times10^{-3}=204\times10^{-3}\ m^2.$

Thickness of air space $=0\cdot5$ mm. $=0\cdot5\times10^{-3}$ m.

Thermal resistivity of air $=23\ \Omega m.$

$\therefore$ Temperature difference across the air space

$$=21\times\frac{20\times0\cdot5\times10^{-3}}{20\cdot3\times10^{-3}}=10\cdot34°C.$$

The insulation and the air space are in series in the path of heat flow. Hence, total temperature difference between the conductor and the slot wall $=10\cdot16+10\cdot34=20\cdot5°C.$

This shows that even a small air space between the coil and the slot walls may result in serious rise in temperature of the conductors.

Example 3·11. *The inner dimensions of the former of field coil of a d.c. generator are 150×250 mm². The former is 2·5 mm thick. Calculate the heat conducted across the former from winding to core if there is an air space 1·0 mm. wide between the former and the pole core. The thermal conductivity of former and air is 0·166 and 0·05 W/m-°C respectively. The winding height is 200 mm and the temperature rise is 40°C.*

Solution. Area of path of heat flow $S=2\ (150+250)\times200\ mm^2=0\cdot16\ m^2.$

From Eqn. 3·3, thermal resistance of former

$$=\frac{t}{\sigma S}=\frac{2\cdot5\times10^{-3}}{0\cdot166\times0\cdot16}=0\cdot094\ \Omega.$$

Thermal resistance of air space $=\dfrac{1\cdot0\times10^{-3}}{0\cdot05\times0\cdot16}=0\cdot125\ \Omega.$

The former and the air space offer thermal resistance to the heat flow from winding to pole core and their thermal resistances are in series.

$\therefore$ Total thermal resistance to heat flow $R_\theta=0\cdot094+0\cdot125=0\cdot219\ \Omega.$

Heat conducted $Q_{con}=\dfrac{\theta_1-\theta_2}{R_\theta}=\dfrac{40}{0\cdot219}=182\cdot6$ W.

COOLING OF ROTATING ELECTRICAL MACHINES

3·6. Cooling and Associated Terminology. Indian Standards Specification IS 6362—1971 "Designation of Methods of Cooling of Rotating Electrical Machines" defines terms connected with cooling of Rotating Electrical Machines. Some of the commonly used terms are explained below :

1. Cooling. A process by means of which heat resulting from losses occuring in a machine is given up to a primary coolant by increasing its temperature. The heated

primary coolant may be replaced by new coolant at lower temperature or may be cooled by a secondary coolant in some form of heat exchanger.

2. **Primary Coolant.** A medium (liquid or gas) which, by being at a lower temperature than a part of a machine and in contact with it, removes heat from that part.

3. **Secondary Coolant.** A medium (liquid or gas) which, being at a lower temperature than the primary coolant, removes the heat from the primary coolant in a heat exchanger.

4. **Heat Exchanger.** A component intended to transfer heat from one coolant to another while keeping the two coolants separate (*i.e.* air cooled heat exchanger, water cooled heat exchanger, double wall, ribbed tubes, etc.)

5. **Inner Cooled (Direct Cooled) Winding.** A winding which has either hollow conductors or tubes which form an integral part of the winding, through which the coolant flows.

6. **Open Circuit Cooling.** A method of cooling in which the coolant is drawn from medium surrounding the machine, passes through the machine and then returns to the surrounding medium.

7. **Closed Circuit Cooling System.** A method of cooling in which a primary coolant is circulated in a closed circuit through the machine, and if necessary through a heat exchanger. Heat is transferred from the primary coolant to the secondary coolant either through the structural parts or in the heat exchanger.

8. **Dependent Circulating Circuit Component.** A separate component in the coolant circulating circuit which is dependent for its operation on the operation of the main machine.

9. **Independent Circulating Circuit Component.** A separate component in the coolant circulating circuit which is independent of the operation of the main machine.

10. **Integral Circulating Circuit Component.** A component in the coolant circulating circuit which forms part of the machine, and which can be replaced only by partially dismantling the main machine.

11. **Machine Mounted Circulating Circuit Component.** A component in the coolant circulating circuit which is mounted on machine, and forms part of it, but which can be replaced without disturbing the main machine.

12. **Separately Mounted Circulating Circuit Component.** A component in the coolant circulating circuit which is associated with a machine, but which is not mounted on or integral with it.

3·7. **Methods of Cooling.** The factor which determines the size of a machine for a given duty is the temperature rise which occurs as a result of the various losses in the machine. Maximum allowable values of temperature rise in various parts of a machine have been standardized by International Electro-technical Commission. It is possible that, as new insulating materials suitable for withstanding higher temperatures are developed, these values will be revised upwards ; but for the immediate future, the greatest gains of output from a given size of machine are likely to arise from improvements in the cooling techniques.

Small electrical machines in the fractional horse power range may be cooled by natural means. In this method no external devices are used and the cooling is done by natural radiation and convection assisted by random air currents set up by rotor where the frames are open. But all modern electrical machines are characterized by large losses per unit area of surfaces of the machine, which dissipate heat into the ambient medium and hence artificial cooling methods are necessary in order to avoid excessive temperature rises during machine operation.

In most cases the cooling of electrical machines is done by air streams and this cooling is called "**Ventilation**". In high speed machines such as turbo-alternators, hydrogen is used for cooling. There are machines in which water is used for cooling.

3·8. **Cooling System.** According to IS : 4722-1968 "Specification for Rotating Electrical Machines" the cooling systems are classified into three types depending upon the origin of cooling.

1. **Natural Cooling.** The machine, is cooled by natural air currents set up either by rotating parts or due to temperature differences. The machine thus is cooled without the use of a fan by the movement of air and radiation.

2. **Self Cooling.** The machine is cooled by cooling air driven by a fan mounted on the rotor or one driven by it.

3. **Separate Cooling.** The machine is cooled either by a fan not driven by its shaft, or it is cooled by a cooling medium other than air put into motion by means not belonging to the machine.

The cooling of machines according to the manner of cooling is of following types :

1. **Open Circuit Ventilation.** The heat is given up directly to the cooling air through the machine which is being continuously replaced.

2. **Surface Ventilation.** The heat is given up by the cooling medium from the external surface of a totally enclosed machine.

3. **Closed Circuit Ventilation.** The heat is transferred to the cooling medium through an intermediate cooling medium circulating in a closed circuit through the machine and a cooler.

4. **Liquid Cooling.** Parts of the machine carry water or another kind of liquid flowing through them, or they are immersed into a liquid.

5. **Inner Cooling of Windings.** This is of two types :

(i) **Inner Gas Cooling.** One or all windings are cooled by a gas, for instance hydrogen, flowing internally through the conductors or coil.

(ii) **Inner Liquid Cooling.** One or all windings are cooled by a liquid, for instance water, flowing internally through the conductors or coils.

3·9. **Enclosures for Rotating Electrical Machines.** The problem of ventilation in rotating electrical machines is closely linked with the types of enclosures used. The various types of enclosures* used are :

1. **Open Machine.** One in which there is no restriction to ventilation other than that necessitated by good mechanical construction.

2. **Open Pedestal Machine (OP).** An open machine which has pedestal bearing supported independently of the machine frame.

3. **Open End-Bracket Machine (OEB).** An open machine having-end brackets of which the bearings form an integral part.

4. **Protected Machine (P).** A machine in which the internal rotating parts and live parts are protected mechanically from accidental or inadvertent contact, while ventilation is not materially impeded.

5. **Screen-Protected Machine (SP).** A protected machine in which the ventilating openings are not less than 64·5 mm² in area. Such protection may be provided by

*IS : 4722-1968 "Specification for Rotating Electrical Machine". All types of enclosures, other than those as given in 1, 2, 3 are such that internal rotating and live parts are protected against accidental or inadvertent contact.

screens of wire mesh, expanded metal, perforated metal or other suitable covers. The use of openings smaller than 64·5 mm^2 is not recognized, as such openings are liable to become closed in service.

6. **Drip-Proof Machine (DP)**. A protected machine in which the opening for ventilation are so protected as to exclude vertically falling water or dirt.

7. **Splash-Proof Machine (SPLP)**. A protected machine in which the ventilating openings are so constructed that drops of liquid or solid particles falling on or reaching any part of them machine at any angle between the vertically downward direction and 100° from that direction cannot enter the machine, whether the machine is running or at rest, by splashing, or otherwise, either directly or by striking and running along a surface.

8. **Hose-Proof Machine (HSP)**. A protected machine so enclosed as to exclude water whether the machine is running or at rest, when washed by a hose having a 9·5 mm diameter nozzle with a maximum pressure of 3·5 kg/cm^2 for a period not exceeding 30 seconds, from a minimum distance of 1·8 metres.

9. **Pipe-Ventilated or Duct-Ventilated Machine**. A machine in which there is a continuous supply or fresh ventilating air, the frame being so arranged that the ventilating air may be conveyed to and/or from the machine through pipes or ducts attached to the enclosing case.

(*a*) A pipe-ventilated or duct-ventilated machine may be one of the following three types :

(*i*) With provision for inlet duct only.

(*ii*) With provision for inlet and outlet ducts.

(*iii*) With provision for outlet duct only.

(*b*) A pipe-ventilated or duct-ventilated machine may be cooled by one of the following means :

(*i*) Self-ventilation (PV)

(*ii*) Forced-draught with air supplied by external pressure (PVFD)

(*iii*) Induced draught with air drawn through the machine by external means (PVID).

10. **Totally Enclosed Machine (TE)**. A machine so constructed that the enclosed air has no connection with the external air but is not necessarily 'air-tight'.

11. **Totally Enclosed Fan-Cooled Machine (TEFC)**. A totally enclosed machine with cooling augmented by a fan, driven by the motor itself, blowing external air over the cooling surface and/or through the cooling passages, if any, incorporated in the machine.

12. **Totally Enclosed Separately Air-Cooled Machine (TESAC)**. A totally-enclosed machine with cooling augmented by a separately-driven fan blowing external air over the cooling surfaces and/or through the cooling passages, if any, incorporated in the machine.

13. **Totally Enclosed Water or other Liquid-Cooled Machine (TEWC)**. A totally enclosed machine with cooling augmented by water-cooled or other liquid-cooled surfaces embodied in the machine itself.

14. **A Totally Enclosed Closed Air Circuit Machine**. A totally enclosed machine having special provision for cooling the enclosed air by passing it through its own cooler, usually external to the machine, The cooler may be of any recognised form using

(*i*) Air (CACA) (*ii*) Water (CACW)

(*iii*) Other suitable cooling medium.

15. **Totally Enclosed Closed Gas Circuit Machine (CGGW).** A totally enclosed machine cooled by gas other than air, the cooling gas being circulated through associated water-cooled gas coolers.

16. **Weather-Proof Machine (WP).** A machine so constructed that it can work without further protection from weather conditions specified by the purchaser.

17. **Watertight Machine (WT).** A machine so constructed that it will withstand, without damage or sign of leakage, complete immersion in water to a depth of not less than 1 m, or subjection to an external water pressure of 0·1 kg/cm² for a period of one hour. The test for watertightness shall be made with the machine stationary and the temperature of the machine shall not exceed the temperature of the water in which it is immersed.

18. **Submersible Machine.** A machine capable of working for an indefinitely long period when submerged under a specified head of water.

19. **Flame Proof Machine (FLP).** A machine which complies with the requirements of IS : 2148—1962 "Specification for flame-proof enclosures of electrical apparatus".

3·10. Induced and Forced Ventilation

Both self ventilation and separate ventilation may be subdivided into two categories :

(*i*) Induced ventilation, (*ii*) Forced ventilation.

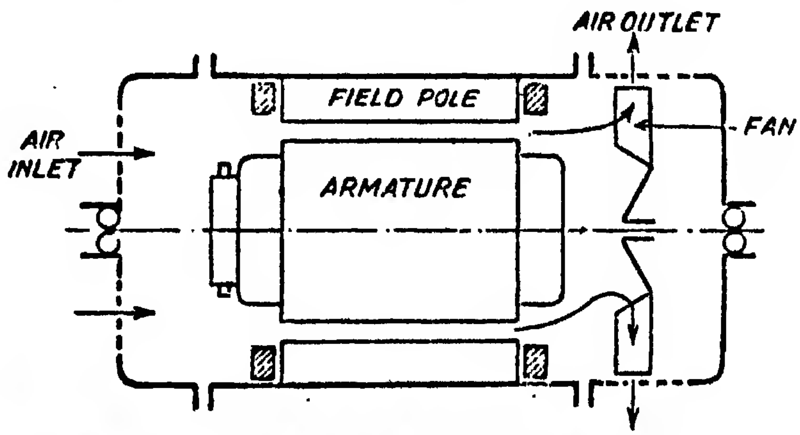


Fig. 3·8. Induced ventilation with internal fan.

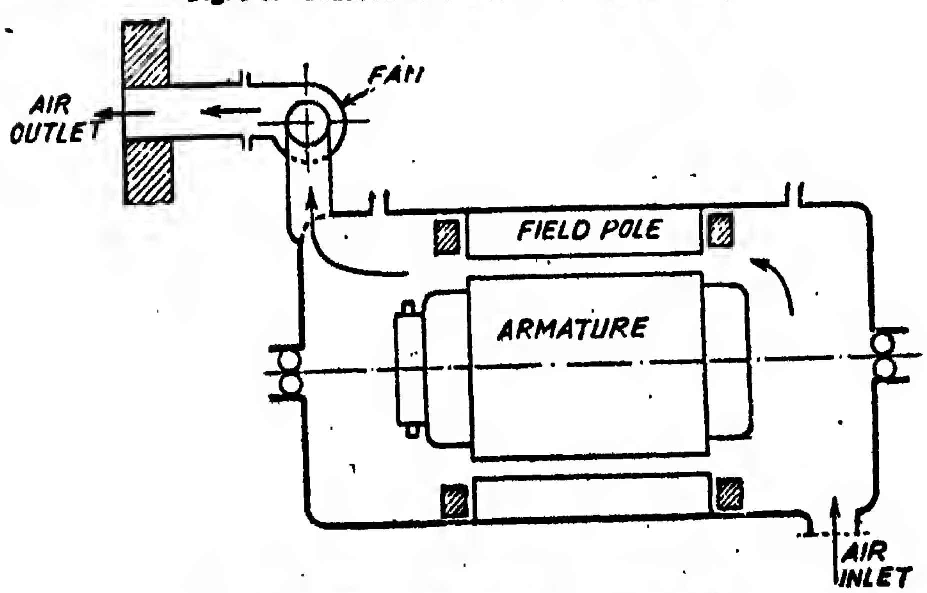


Fig. 3·9. Induced ventilation with external fan.

The ventilation of the machine is induced if the fan produces a decreased pressure of air inside the machine causing the air to be sucked into the machine under the external atmospheric pressure. The air is then pushed out by the fan into the atmosphere. Figs. 3·8 and 3·9 show the induced ventilation using respectively internal and external fans.

The ventilation of machine is said to be forced if the fan sucks the air from the atmosphere and forces it into the machine, from where it is then pushed out to the atmosphere (Figs. 3·10 and 3·11).

Induced self ventilation is most commonly used in machines of small and medium power outputs. In this case the built in fan is mounted on the shaft within the end shield

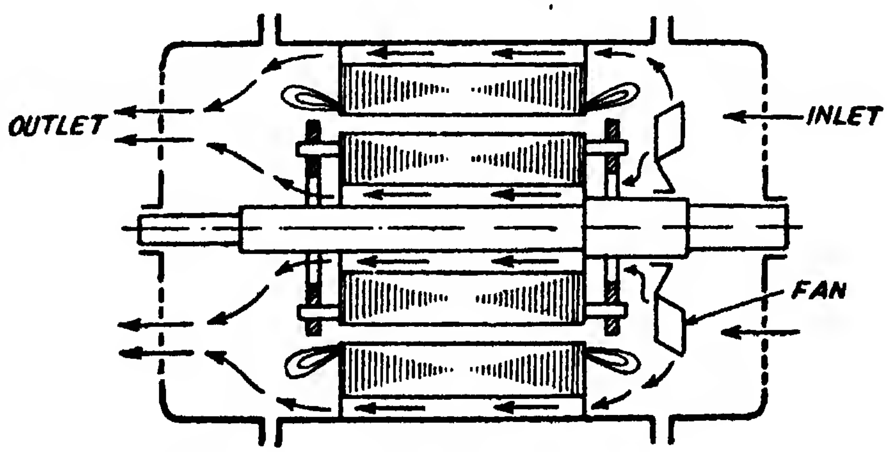


Fig. 3·10. Forced ventilation with internal fan.

on the driving end. In induced ventilation cold air enters the machine whereas with forced ventilation the temperatrure of the cooling air rises on account of losses in the fan. Thus the amount of air required to cool the machine is higher with forced ventilation. However in the past few years quiet operation of electrical machines has become an important requirement. The fan is one of the major sources of noise especially when the inside diameter of the end shield is nearly equal to the outside diameter of fan. Low noise

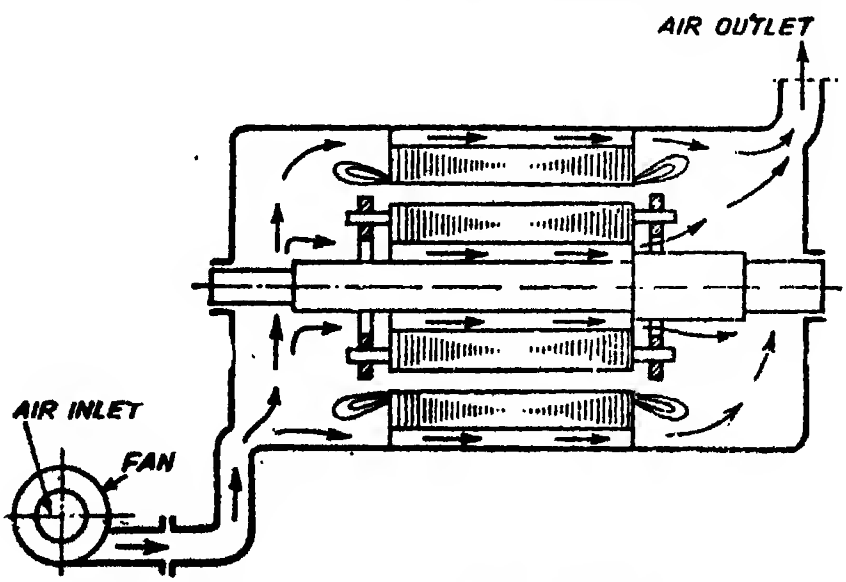


Fig. 3·11. Forced ventilation with external fan.

level machines are designed with smaller specific loadings and consequently losses per unit dissipating surface are small. This permits the use of fans with smaller diameters. In such cases it is possible to use forced self ventilation (Fig. 3·10). In induction motors having radial ducts in the stator and rotor, forced self ventilation is adopted as a rule. There are electrical machines in which a combination of radial and axial forced self ventilation is also used.

3·11. Radial and Axial Ventilation. The ventilating systems can be classified into three types depending upon how the air passes over the heated machine parts, as :

(*i*) Radial, (*ii*) Axial, (*iii*) Combined Radial and Axial.

3·11·1. Radial Ventilating System. This system is most commonly employed because the movement of rotor induces a natural centrifugal movement of air which may be augmented by provisions of fans if required. Fig. 3·12 (*a*) shows the radial system for machines with small core lengths. The end shields are shaped to guide air on the coverhang and then on to the back of the core. This method is suitable for machines upto about 20 kW. For larger machines, which have large core lengths, the core is subdivided in order to provide radial ventilating ducts as shown in Fig. 3·12 (*b*). The air now passes radially through these ducts, the path of air in the ducts being parallel to that over the overhang. The core is normally divided into stacks 40 to 80 mm thick, with ventilating

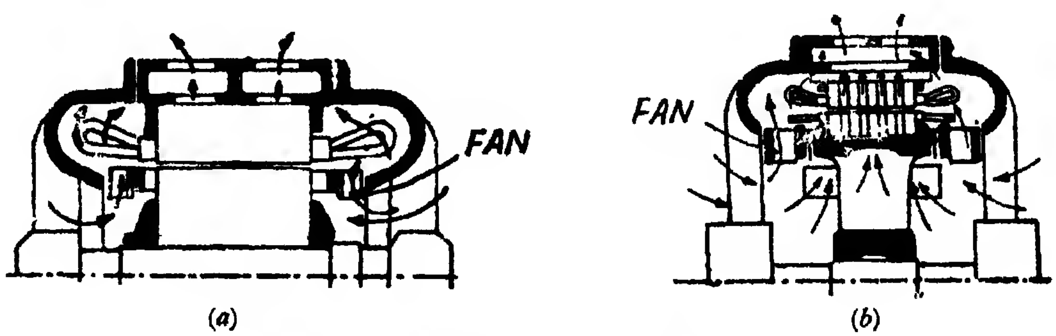


Fig. 3·12. Radial systems of ventilation.

ducts of width 10 mm in between them. The use of 10 mm wide radial ventilating ducts has been a universal practice until recently. However, it has been found that with air flowing up the radial ducts only a layer 3 mm wide in contact with the core walls is effective in cooling and the middle 4 mm wide band contributes very little to cooling of the core. Therefore some firms have started manufacturing machines with radial ventilating ducts of 6·5 mm width. The advantages of radial system are : (*i*) minimum energy losses for ventilation (*ii*) sufficiently uniform temperature rise of machine in the axial direction. The disadvantages are : (*i*) it makes the machine lengths larger as space for ventilating ducts has to be provided along the core length, (*ii*) the ventilating system sometimes becomes unstable in respect to quantity of cooling air flowing.

In radial ventilation, a high rate of heat dissipation is possible in the air gap between stator and rotor on account of high air speeds accompanied by turbulence.

3·11·2. Axial Ventilating System. The axial ventilating system as employed in induction machines is shown in Figs. 3·13 (*a*) and (*b*). In this case axial ducts are provided. If the axial ducts are arranged on the rotor, it is known as simple axial system while if the axial ducts are provided on both stator and rotor, it is called a double axial system. This system of ventilation is suitable for machines of medium output and high speed. This is because in high speed machines, a solid rotor construction with restricted spider is used in order to avoid centrifugal stresses and this restricts the provision of radial ventilating ducts and hence axial ventilation has to be used. In order to increase the cooling surface holes may be punched where considerable heat dissipation occurs. This no doubt improves cooling ; but requires a large core diameter for the increased core depth.

The disadvantages of axial ventilation are (*i*) non uniform heat transfer—it is clear from Fig. 3·13 that the temperature of the end from which the air leaves is higher than that of the end where the air enters. This is because the air gets hot in its passage from the entering end to the leaving end, (*ii*) increased iron loss—the provision of axial ventilating

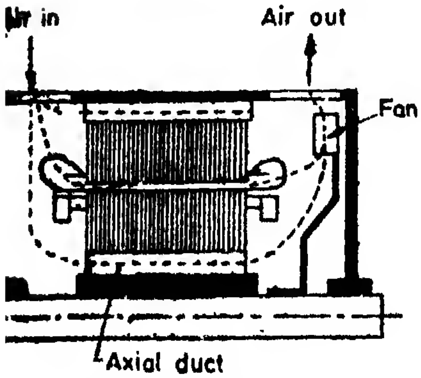


Fig. 3·13. Axial system of ventilation.

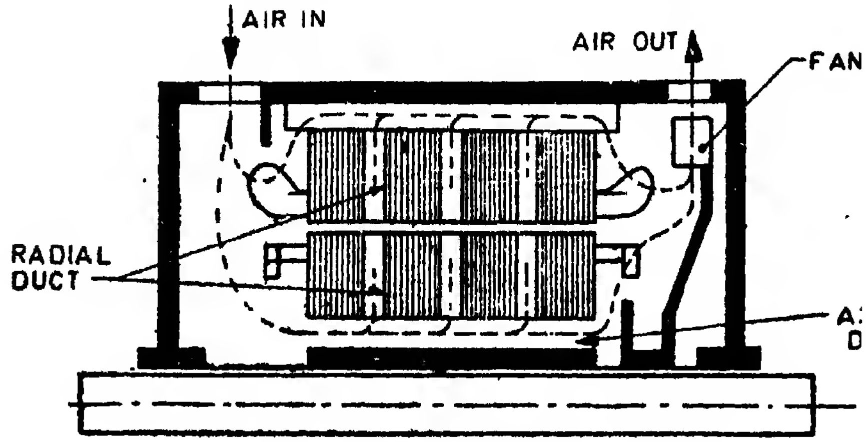


Fig. 3·14. Combined axial and radial ventilation

ducts behind the slots of the stator reduces the amount of iron giving rise to increased flux density in the stator core ; this increases the iron loss. However in a large number of cases this loss is more than compensated by improved cooling.

3·11·3 Combined Axial and Radial Ventilating System. This method is usually employed for large motors and small turboalternators. This is because on these machines the area of axial ducts required to carry sufficient quantity of cooling air becomes excessive giving rise to a large iron loss and therefore a mixed axial and radial system has to be used. Fig. 3·14 shows the mixed system of ventilation. The air is drawn into the machine from one end and is encouraged to pass through the ducts by baffling the fan end of the rotor spider. The rotor mounted fan forces out the air.

3·12. Cooling of Totally Enclosed Machines. Totally enclosed machines are used in applications where the use of open and protected machines is inadmissible *i.e.* in cases where the air contains objectionable impurities like explosive gases and acid fumes that may destroy the insulation. In a totally enclosed machine the inside of machine can have no air connection with the outside and all the heat developed inside should be dissipated into the surroundings from the external surface of the frame. If this heat is dissipated by natural radiation and convection the cooling is not very effective and thus the rating of the machine is low. Hence the use of totally enclosed machines, except when they are absolutely necessary, is quite uneconomical as such machines are always heavier and more expensive than open or protected machines. Totally enclosed machines are of two types :

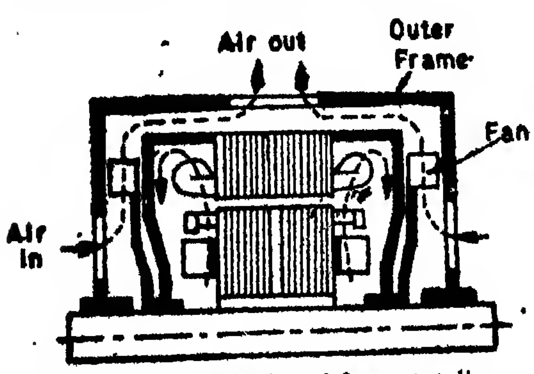


Fig. 3·15. Self ventilated frame totally enclosed machine.

1. **Ventilated Frame Machines.** In self ventilated frame totally enclosed machines a fan is mounted on the shaft outside the working parts of the machine. This fan blows air over the caracass through a space between the main housing and a thin cover plate. (Fig. 3·15). Fig. 3·16 shows another arrangement. The fan pushes the air along the surface of the machine which is provided with ribs. The fan is enclosed by another cover in order to secure required direction of air flow. This type of ventilation is effective for machines with power outputs upto 25 kW. In larger machines (output power upto 200 kW) it is necessary to

provide in addition, circulation of air inside the machine to improve heat dissipation. This is done by providing another fan inside the machine (Fig. 3·17) which may

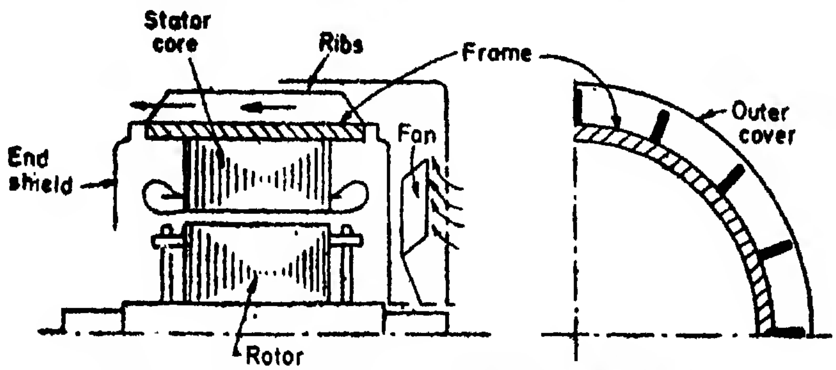


Fig. 3.16. Self ventilated frame totally enclosed machine.

circulate air in the inner portion. The advantage of using an internal fan is that it avoids temperature gradients across the air gap. It should be noted that the air inside acts as the **primary coolant** and the air outside (blown by the outer fan) acts as **secondary coolant.**

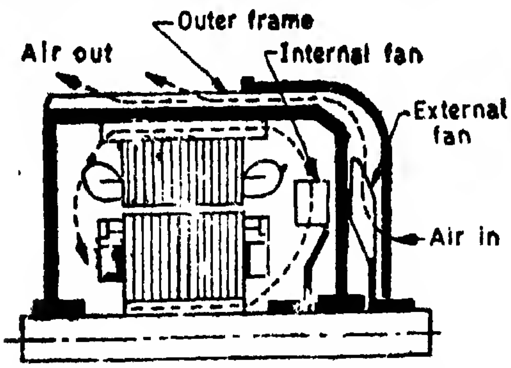


Fig. 3·17. Self ventilated totally enclosed machines. (with external fan)

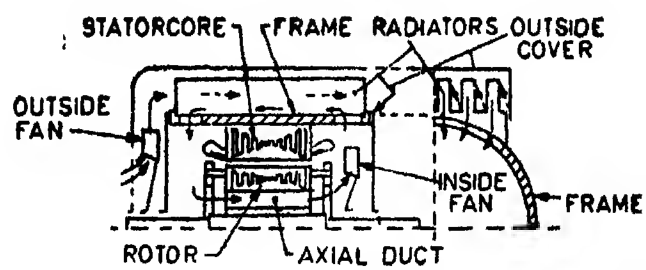


Fig. 3·18. Ventilated radiator totally enclosed machine.

2. **Ventilated Radiator Machines.** In such machines an internal fan circulates air inside the machine. An external fan sucks the hot air from inside and pushes it into the radiators (heat exchangers) provided on the frame of the machine as shown in Fig. 3·18. The hot air is cooled in the radiators by fans which are not an integral part of the machine and is then fed back to the machine. This type of cooling allows totally enclosed induction machines to be built upto 5 MW in a single unit.

At higher ratings the outside air may be cooled by water if there is a convenient source (of water).

3·13. Cooling Circuits. Both self and independent ventilation may be of two types, (*i*) open circuit ventilation, and (*ii*) closed circuit ventilation.

In the open circuit ventilation cold air is drawn in and forced out after passing over the heated machine parts. Thus fresh air is continuously drawn in and expelled. In large machines which require large volumes of air, it is necessary to clean air with suitable filters in order to prevent clogging of cooling ducts with dust. It is also necessary to dry the air to eliminate moisture. The filters have to be frequently cleaned. The filters also increase resistance to air flow and thus additional fan power is required to calculate air.

It is clear that if open circuit ventilation is used for the largest machines which require many tons of cooling air per hour the operating costs will be prohibitive. Also no filter removes all the dust from the ventilating air and therefore for these reasons, open circuit ventilation is not used for the largest size machines.

A complete means of securing clean cooling air is the closed circuit ventilation system. It is a system where the same volume of air passes through a closed circuit. This circuit consists of various passages in the machine, together with a chamber in the foundations containing the fans, air coolers, drying agents, etc. The hot air coming from

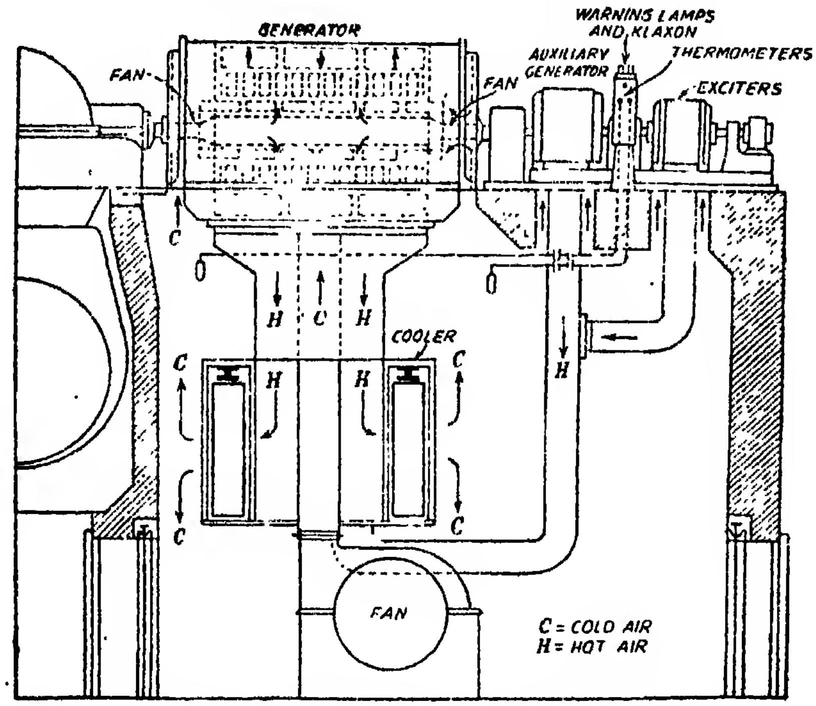


Fig. 3·19. Closed circuit ventilation of an air cooled turbo-alternator.

the outlets in the machine frame is passed through a water cooled air cooler where the air is cooled. The cool air is then returned to the inlet openings of the machine where it is recirculated. Fig. 3·19 shows closed circuit ventilation of an air cooled turbo-alternator.

3·14. Cooling of Turbo-alternators. The problem of cooling of turbo-alternators is one of the most complex problems in electrical engineering since being high speed machines their dimensions as compared with those of hydro-electric generators for instance, are much smaller. In fact the problems associated with cooling arise because turbo-alternators are characterized by long core lengths and small diameters. The varions methods for cooling turbo-alternators are described below.

3·15. Air cooled Turbo-alternators. The cooling of turbo-alternators by air is used for small units. The common notion that the air cooled turbo-alternator is getting obsolete is not correct. In fact gas turbine generating units rated at 17·5 MW, 25 MW and 35 MW which are getting into prominence as emergency auxiliaries in large power stations are air cooled machines. Also there are a number of 30 MW and 60 MW air cooled machines which are yet in service. The various methods used for air cooling of turbo-alternators are :

1. **One sided Axial Ventilation.** Machines for power outputs upto 3 MW permit the use of one sided parallel axial ventilation. In this method the machine is supplied with air by propeller fan and the air enters the machine from one side and leaves from the other. The disadvantage of one sided axial ventilation in long machines is the great difference in the temperature rise of the winding along the length of the machine.

2. **Two sided Axial Ventilation.** In this method the air is forced through the machine from both the sides. The two sided system of ventilation has the advantage that the end windings on both sides of machine have the same temperature rise. This method can be used for machines of rating upto 12 MW.

3. **Multiple Inlet System.** The above methods of axial ventilation cannot be used for turbo-alternators having long core lengths because there is a difficulty in supplying the central parts of the core with requisite volume of cool air. This is because with long core the air gets hot before it reaches the central portions. Thus in larger machines multiple inlet system is adopted.

In a multiple inlet system the outer stator casing is divided into a number of compartments, these being used alternatively as inlet and outlet chambers as shown in Fig. 3·20. In the inlet chambers the air is directed radially inwards while in the outlet chambers the air is directed radially outwards. Air is forced under pressure into the stator

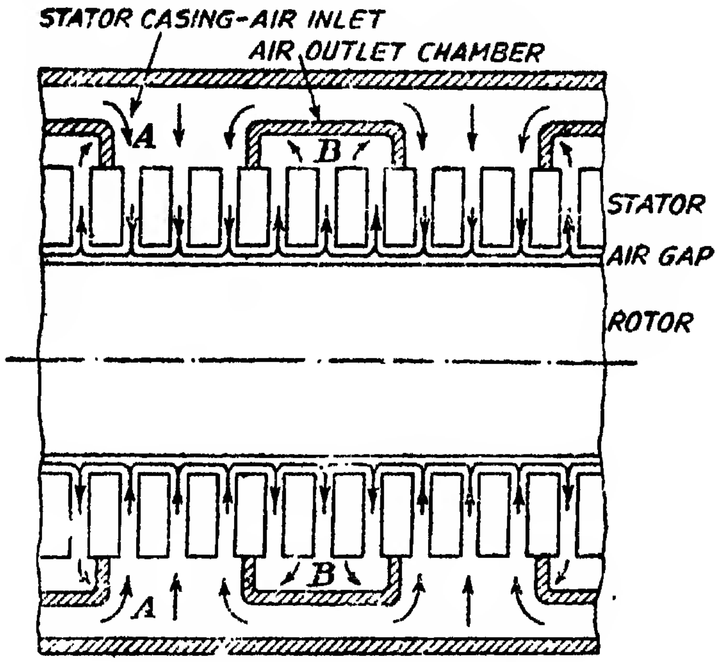


Fig. 3·20. Multiple inlet system.

casing where it enters the inlet chamber *A* from where it flows radially inwards into the stator ducts. Part of this air passes through the axial ducts, the remainder flowing along the air gap. It then passes radially outwards in the adjacent sections of the core to the outlet chambers as at '*B*'.

The air is drawn from the outlet chambers and is sent to the coolers where it is cooled and is recirculated. This method can be used for machines of rating upto 60 MW.

3·16. **Hydrogen Cooling and Hydrogen Cooled Turbo-alternators.** When the problem of increasing generator rating was tackled it became clear that the air cooled machines did not provide the necessary scope for progress. Building of air cooled turbo-alternators above 50 MW rating presented serious ventilation difficulties, not only in circulating the requisite quantity of air through the machine but also because the high fan power required to circulate the air. An idea of the quantities involved can be had from the data of a 60 MW machine which has a total loss of 1000 kW and requires about 150 tons of air per hour and a fan power of about 100 kW. Thus the air circulating fan power required for this duty is high and on this score alone the air cooled generator can be ruled out when it becomes necessary to increase generator ratings. Evidently to push up generator ratings an alternative cooling medium had to be found. That medium was Hydrogen.

The utilization of hydrogen for the cooling of electrial machines gives the following advantages :

3.16.1 Advantages of Hydrogen Cooling

1. Increased Efficiency. An increase in efficiency results from reduction in the ventilation losses (like windage) which are a major portion of the total losses in a high speed machine. This is because the density of hydrogen is only 0.07 times the density of air and therefore the power required to circulate hydrogen should be about 1/14 of the power required for an equivalent quantity of air. In practice the power required is about 1/10 of that for air when hydrogen is 98%.

Calculations show that in 50 MW and 100 MW turbo-alternators with hydrogen cooling the efficiency is raised by approximately 0.8 per cent at full load attaining values of 99.0 to 99.2 per cent.

2. Increase in Rating. Hydrogen has a heat transfer co-efficient 1.5 times and its thermal conductivity is 7 times that of air. Consequently when hydrogen is used as a coolant, the heat is more readily taken up from the machine parts and given out. Therefore heat generated is more effectively removed and the active materials can be loaded more than is possible with air cooling.

The high value of thermal conductivity of hydrogen, the temperature gradients across the film barrier between cooled surface and coolant and also across the coolant are reduced. On account of the above two factors, an increased output can be taken from a given frame size if hydrogen cooling is used.

If conventional hydrogen cooling at 105 kN/m^2 (a little above atmospheric to avoid air leakage into hydrogen) is applied to a generator designed for air cooling, the overall advantage could be an increase in rating by 20-25 per cent. If the gas pressure is raised to 300 kN/m^2 (nearly three atmoshpere) the rating could be increased by about 35 per cent above the rating as an air cooled ganerator. This is because at higher pressures the co-efficient of heat transfer increases (for hydrogen it is 3.65 times that of air at a pressure of 300 kN/m^2) and consequently the temperature gradient across film barrier reduces (see Fig. 3.25) allowing the machine rating to be increased. With conventional hydrogen cooling it is possible to increase the rating of a single unit to 200 MW.

3. Increase in Life. The life of a machine is mainly the life of winding insulation and air pockets in insulation can be sources of such high local temperatures that there is always the risk of insulation breakdown and fire. The thermal conductivity of hydrogen is nearly seven times that of air and is of the same order as the winding insulation. When, therefore, there are pockets filled with hydrogen, heat conduction through them will be as good as through winding insulation and consequently high local temperature rises are not there.

When air is used as a coolant the high voltage winding can fail due to destructive action of corona discharge. This is because air contains oxygen and nitrogen and during corona discharge ozone, nitric acid and other chemical compounds are formed which attack oragnic material in the insulation. On the other hand when hydrogen is used as coolant, sufficient oxygen to form ozone and destructive compounds in the event of corona discharge is not present. The danger from this source is, therefore, greatly reduced. Hence the use of hydrogen as a coolant greatly increases the life of machine.

4. Elimination of Fire Hazard. The outbreak of fire inside the machine is impossible as hydrogen does not support burning.

5. Smaller Size of Coolers. The size of coolers require to cool the gas are smaller in size when hydrogen is used as a coolant.

6. Less Noise. The noise produced by a hydrogen cooled machine is less as the rotor moves in a medium of smaller density.

3.16.2. Hydrogen Cooling System. Hydrogen when mixed with air forms an explosive mixture over a very wide range (4% to 76%) of hydrogen in air. Therefore the

frame of hydrogen cooled machines has to be made strong enough to withstand possible internal explosions without suffering serious damage. All joints in cooling circuits are made gas tight and oil film shaft seals are used to prevent leakage of hydrogen. Fig. 3·21 shows two types of shaft seals. The seals must accommodate axial expansion of the rotor

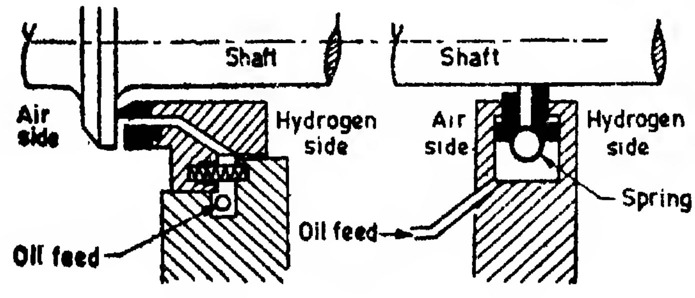


Fig. 3·21. Hydrogen shaft seals.

shaft and the stator frame. Fig. 3·21 (*a*) shows the thrust face seal, the seal bearing is a small thrust bearing held by springs to a collar machined on the rotor shaft. The seal oil is forced in a groove in the bearing face. In Fig. 3·21 (*b*) the seal rings are two short bearings fitting closely to the rotor shaft but held apart by a greater spring. Sealing oil is forced into the space between the two bearing rings and passes in two axial directions along the shaft. A part of the oil flows towards the inner (gas) side and other part to the air side. The oil that flows towards air mixes with air, while the oil that goes towards hydrogen is collected and degassed.

The risks of explosion in the machine casing are reduced by maintaining the hydrogen above atmospheric pressure so that any leakage is from machine to atmosphere where the hydrogen can be quickly dissipated. Initially a pressure of 105 kN/m^2 (little above atmosphere) was used but as advantages of higher pressures (like increase in rating) became apparent pressures in modern conventially cooled turbo-alternators have been increased to 200—300 kN/m^2.

Fans mounted on the rotor circulate hydrogen through the ventilating ducts and internally arranged gas coolers. The gas pressure is maintained by an automatic regulating

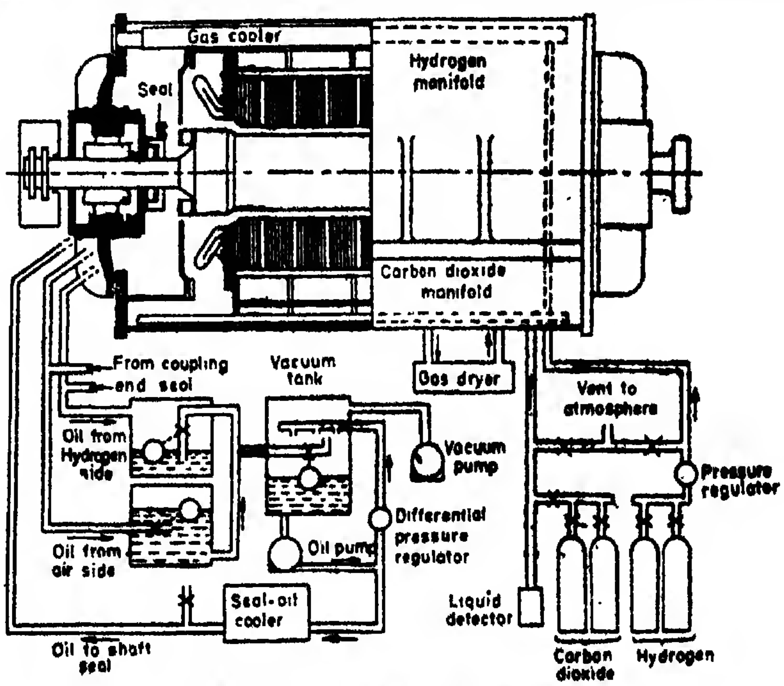


Fig. 3·22. Hydrogen cooling circuit.

and reducing valve controlling the supply from gas cylinders. When filling and emptying the casing of machine, an explosive hydrogen air mixture must be avoided, so that air is first displaced by carbon dioxide gas before hydrogen gas is admitted. The process is reversed when emptying the machine. It is usual to provide a drier to take up water vapour entering through seals. The purity of hydrogen is checked by measuring its thermal conductivity. Fig. 3·22 shows the machine and the auxiliary equipment required for hydrogen cooling.

3·17. Direct Cooling and Direct Cooled Turbo-alternators. The terms conventional cooling and direct cooling are explained first.

Conventional Cooling. Conventionally cooled machines dissipate their losses to a coolant which is entirely outside the coil insulation. The cooling methods described above are all conventional methods.

Direct Cooling. Direct cooling is the process of dissipating the armature and field winding losses to a cooling medium circulating within the winding insulation wall. Machines cooled in this manner are also called "supercharged", "inner cooled" or "conductor cooled" by various manufacturers. In this method the coolant either is in direct contact with conductor copper or is separated only by materials having negligible thermal resistance.

3·17·1. Advantages of Direct Cooling

1. The attempt is always to increase the rating. One way of doing it is by increasing the hydrogen pressure in conventional cooling. Rotor copper heating is the most serious limitation to the output from a conventionally cooled generator and increasing the hydrogen pressure beyond 300 kN/cm^2 gauge showed little increase in machine capability (see Fig. 3·31). It is therefore logical to adopt direct cooling of rotor conductors in order to increase the machine rating.

2. Fig. 3·23 shows the stator of a large turbo-alternator. It is provided with both axial as well as radial ventilating ducts through which hydrogen (or air) is forced. Rotors of large turbo-alternators are also provided with comparable ducting as shown in Fig. 3·23. In this case hydrogen (or air) under pressure is forced through a venti-

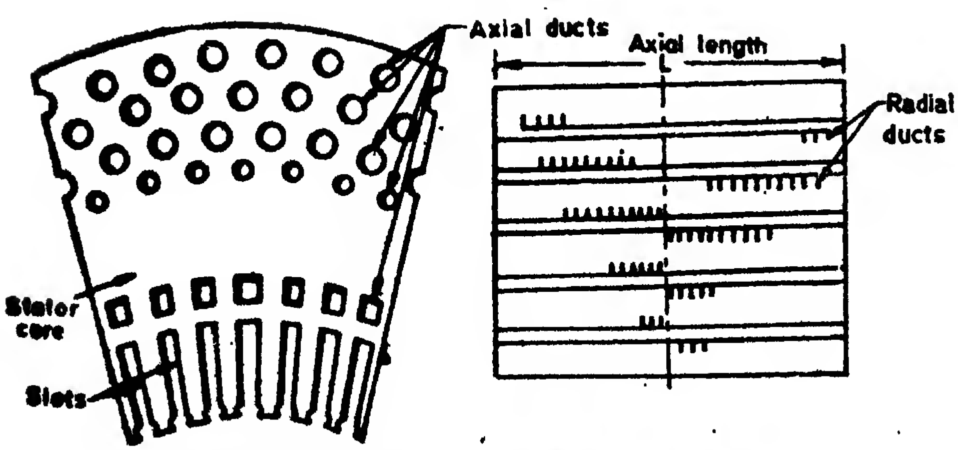


Fig. 3·23. Statator limitation of large turbo-alternators.

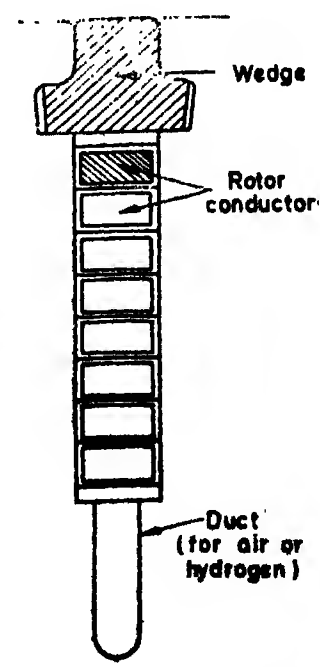


Fig. 3·24. Rotor slot of turbo-alternators.

lating duct provided below the bottom of the rotor slot This arrangement is suitable upto ratings of about 100 MW but above this rating the temperature gradient across the conductor insulation becomes high necessitating the use of direct cooling. The components of the temperature rise in a conventionally hydrogen cooled turbo-alternator are shown in Fig. 3·25. It can be seen that a considerable temperature drop exists between the winding conductor and the coolant in a conventionally cooled machine. The temperature gradients in this drop are across the slot insulation, across the iron and across the heat transfer barrier between coolant and the cooled suface. Since with direct cooling the coolant comes in direct contact with the conductor, the temperature gradients across slot insulation, teeth and surface barrier are almost completely eliminated

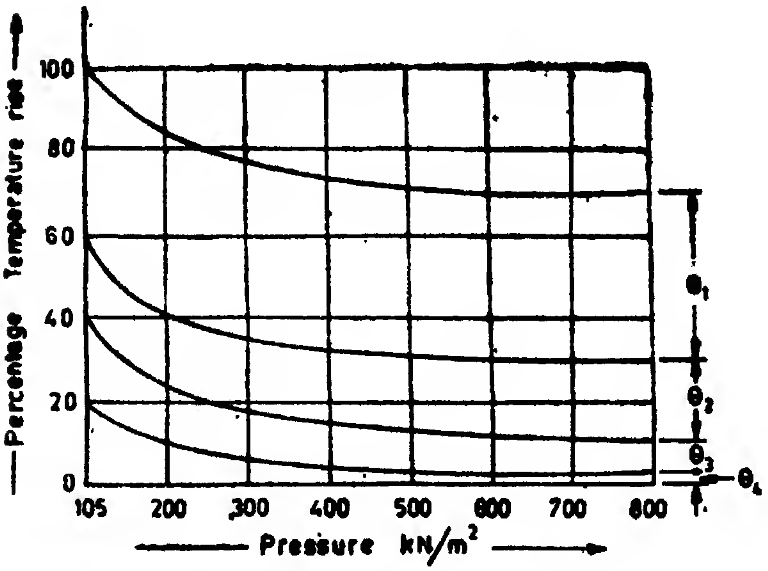


Fig. 3·25. Components of temperature rise in a conventially hydrogen cooled Turbo-alternator.

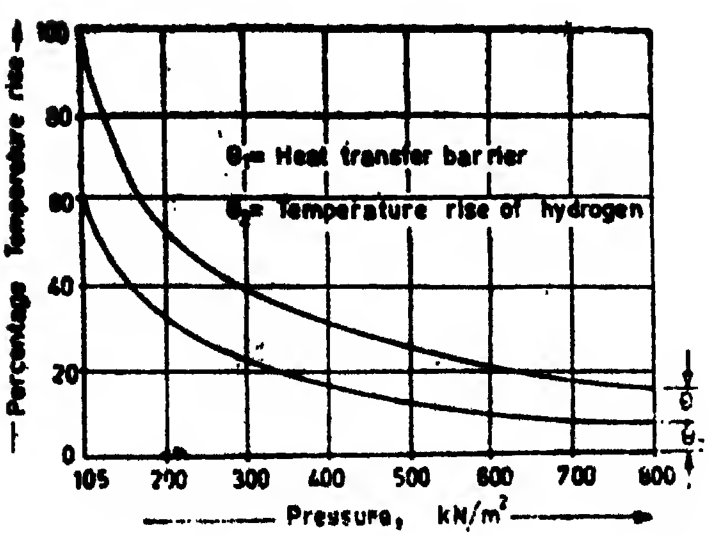


Fig. 3·26. Components of temperature rise in a direct hydrogen cooled Turbo-alternator.

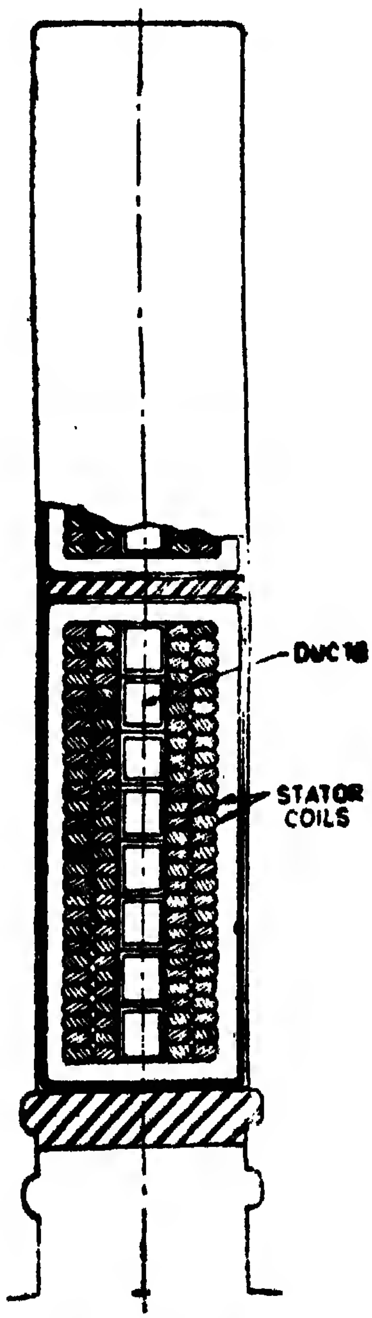


Fig. 3·27. Stator slot of direct hydrogen cooled Turbo-alternator.

with the result that winding temperatures are considerably brought down. Fig. 3·26 sho the components of temperature rise of direct cooled (hydrogen) turbo-alternator. It is th evident that much higher output can be taken from the machine with direct cooling.

3·17·2. Coolants used for Direct Cooling. The coolants normally used a hydrogen, water and oil.

1. **Hydrogen**. In direct cooled system using hydrogen, both stator and rotor co ductors are made hollow. Hydrogen is pumped through these conductors from one end the other. Fig. 3·27 shows slot with solid sub-conductors about the central cooling tub Fig. 3·28 shows a rotor slot having hollow conductors. The rotor conductors consist rectanglar tubes which are ventilated by a cooling circuit separate from that of t stator. The hydrogen gas is admitted to the tubes through flexible insulating conne tions at the ends from a centrifugal impeller mounted on the out board end of t rotor shaft. The rotor slot tubes are electrically connected at the overhang by suitab shaped copper bars to form inlet and outlet parts. The hollow conductors a made of hard drawn silver bearing copper, with synthetic resin bonded glass-cloth lamina insulation. It is possible to build machines upto ratings of about 300 MW by emplo ing direct hydrogen cooling.

2. **Water**. The transition from direct hydrogen cooled to direct water coole turbo-alternator stator windings was a logical step for two reasons. Firstly, as t ratings of turbo-alternators increase, more space has to be provided for the conducto for the flow of requisite quantity of cooling gas. Secondly, mechanical limitation on rotor di meter makes it necessary to increase the physical size of the machine by increasing its lengt and to circulate hydrogen through very long conductor lengths requires high pressure head Water as a coolant has suppersede l hydrogen not only because of its superior heat transfe capacity but also because the viscosity of pure water is very small and it is possible t maintain flow in small tubes without building up dangerously high pressure heads. Th advantages of water as coolant are :

1. The power required to circulate a given coolant is a function of mass o coolant and the pressure head required. The mass of water required to remove a give heat loss is greater than that of hydrogen, but because of its (water) low viscosity the pumpi power required is small.

2. There is no temperature difference between cooling water and the conductor and therefore, in water cooled generators it is possible to use a higher value of specific electric loading than is possible with hydrogen cooling.

With direct water cooling it is possible to have ratings of about 500 MW.

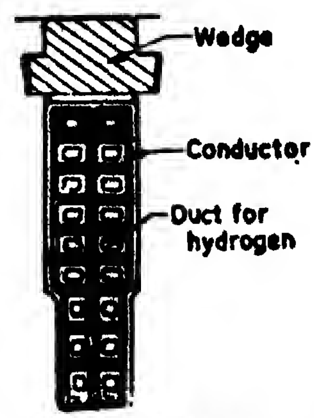


Fig. 3·28. Rotor slot of a direct hydrogen cooled turbo-alternator.

3·17·3. Direct Cooling System. Turbo-alternators of the highest possible ratings so far contemplated are likely to use hydrogen cooled stator cores and direct water cooled stator and rotor windings.

The resistivity, and hence effectiveness of water as a coolant depends upon its purity. The resistivity of water should not be less than 2000 Ωm. Therefore plants must be installed to provide distilled water. Connection of water pipes, which are at earth potential, with the high tension armature presents one of the main problems of water cooling. The connectors must provide a high resistance path between armature and water manifold. A plastic tubing known as polytetrafluoroethylene is used for this purpose.

Fig. 3·29 shows the stator slot of a direct water cooled turbo-alternator. Direct water cooling of rotor winding presents

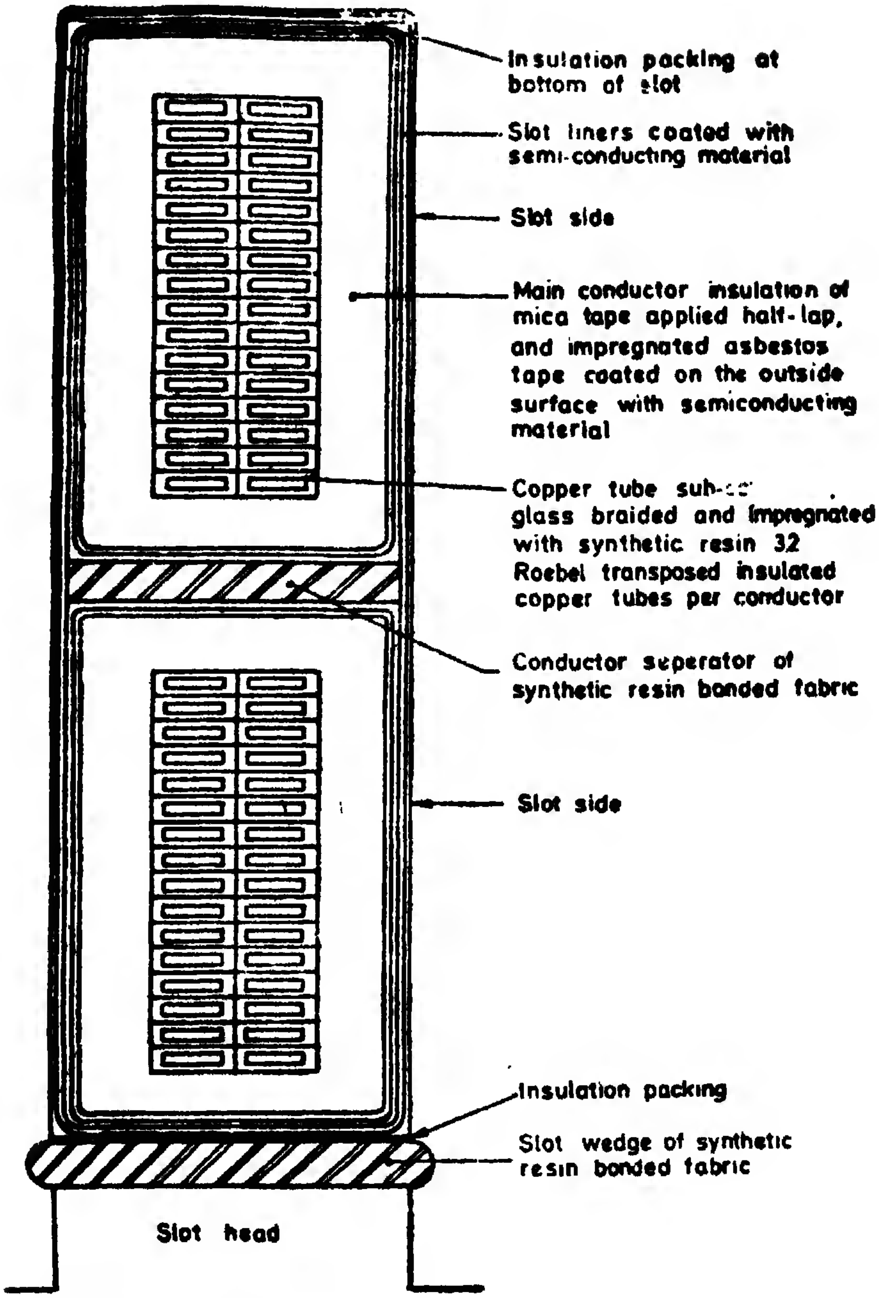


Fig. 3·29. Stator slot of direct water cooled turbo-alternator.

greater mechanical difficulties. Fig. 3·30 shows the rotor slot of direct water cooled turbo-alternator. The slot width is reduced near the bottom in order to increase the tooth width so that the stresses due to centrifugal force are kept within limits.

An inlet seal is attached to the outboard (non coupling) end to enable the water to enter the bore of the shaft. Water travels axially to a point in the plane of the overhang, where its flow becomes radial into manifolds which extend axially under the overhang. The shaft bore is lined with stainless steel to prevent corrosion and contamination of water. Water is transferred from the stainless steel manifolds to the winding by flexible insulating connectors. Water enters at the coil ends and leaves at the mid point of each coil side. The water outlet is a simple box that surrounds the

shaft. Water can be circulated through holes in the slot wedges which are used as damper windings in a turbo-alternator.

The maximum water temperatures generally used are of the order of 60 to 70°C. These temperatures are based on a cooling water temperature not exceeding 40°C at the rear water inlet manifold. The highest temperature rise of water flowing through the copper is, therefore, only 30°C. The water pumping pressure is 300 kN/cm² and the velocity of water hardly exceeds 1·5 m/s. The maximum permissible speed of water is 2·5 m/s. The speed is kept low in order to avoid erosion and cavitation.

Reservoir tanks maintain the necessary water level. The water is tested for its purity and the purity is adjusted by a demineralization unit. The water is cooled in heat exchangers and is filtered before being circulated into the conductors by pumps.

Oil. High grade transformer oil is an effective coolant and is being used in U.S.A. for direct cooling of stator conductors. But oil has a flash point which can be reached by machine under fault conditions and therefore it can be damaging to armature insulation should there be a leakage in the internal cooling system.

Water is preferred in U.K. because it has no fire risk, and if it leaks into winding, the insulation can be restored by simple drying process, provided there is no actual winding failure.

Fig. 3·31 shows the relative current carrying capacities of armature conductors with direct hydrogen, water and oil cooling.

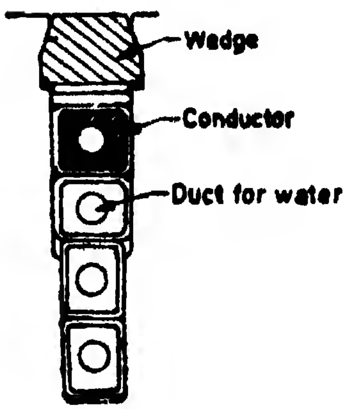


Fig. 3·30. Rotor slot of direct water cooled turbo-alternator.

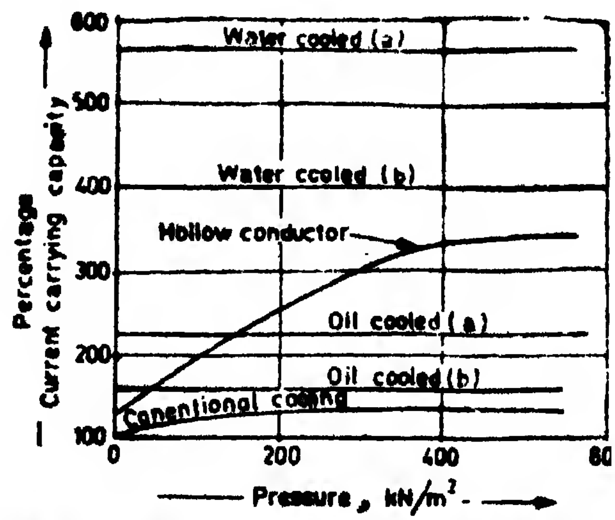


Fig. 3·31. Relative current carrying capacities of turbo-alternators with different methods of cooling.

3·18. Superconducting Rotor Windings. Superconductivity and its advantages have been explained on page 15. Since the stator windings carry alternating currents and consequently cannot be made superconducting. However, the rotor carries a steady field and therefore, superconductivity can be used with advantage to reduce the excitation loss which is quite high in large turbo-alternators. Lorch has proposed a cooling scheme for a 660 MW turbo-alternator which uses a conventional water cooled stator winding and a superconducting rotor winding. In this design the rotor body is non magnetic and is a hollow glass cylinder reinforced by epoxy resin with steel ends. This construction is adopted because a normal steel forging would conduct too much heat from the bearings (produced on account of bearing friction loss) and would offer no magnetic advantage at the very high flux densities used for making superconductivity possible. The rotor is slotted and the supercooled coils are placed in slots and wrapped with glass epoxy resin. The rotor windings are cooled to a temperature of about 5K by a flow of helium entering and leaving the rotor on the axis at the outboard. The rotor windings are shielded from harmful alternating magnetic fields of stator with the help of a rotating copper cylinder shell. The shell is thermally insulated from the rotor by a sealed stainless-steel vacuum enclosure maintained at a temperature

between 80 and 135 K. Liquid nitrogen is passed through a system of stationary cylinders kept within the stator bore to form a cold screen at about 77 K. The gap between the stationary and the rotating parts is filled with hydrogen at a pressure of 0·1 mm of mercury in order to reduce the windage loss. Hydrogen is used because in addition to being light (which reduces windage loss), has a good thermal conductivity. Therefore the complexity of supplying liquid nitrogen to rotor is avoided. The stator as well as the exciter are of the conventional design. However, the exciter of a supercooled machine has a much lower rating on account of drastic reduction in rotor copper loss.

The exciter has to supply 2·5 MW for only a very short duration (a few seconds) in order to build up the intense magnetic field. The field density used is between 2 to 5 Wb/m². At a flux density of 2 Wb/m², it becomes necessary to remove the stator teeth. At very high flux densities like 5 Wb/m², the stator core and teeth become redundant but the use of high flux densities requires a massive magnetic shield round the stator in order to localize the strong magnetic field. The use of non-metallic rotor body and non-magnetic stator results in reduction of synchronous reactance and increase in transient and subtransient constants. On account of small value of synchronous reactance, there will be extremely large mechanical forces developed when the generator is short circuited. Therefore the problem of securing rotor winding to the non-metallic rotor body on a non-magnetic stator against very large short circuit forces are likely to be great.

3·19. Cooling of Water Wheel Generators The simplest form of cooling used for both suspended and umbrella type low speed water wheel generators is the closed circuit ventilation. The machines are installed under the floor of the machine room. The water coolers for air are mounted at outlet openings of the stator frame as shown in Fig. 3·32. Circulation of air is caused by rotation of the rotor poles and by means of fans mounted on both sides of the rotor.

The generators using closed circuit ventilation are usually made fire-proof. The arrangements to make them fire-proof consists of a system of pipes placed concentric with end windings in the generator. In case of fire inside the machine, water or gaseous carbonic acid is injected inside automatically through pipes in the generator.

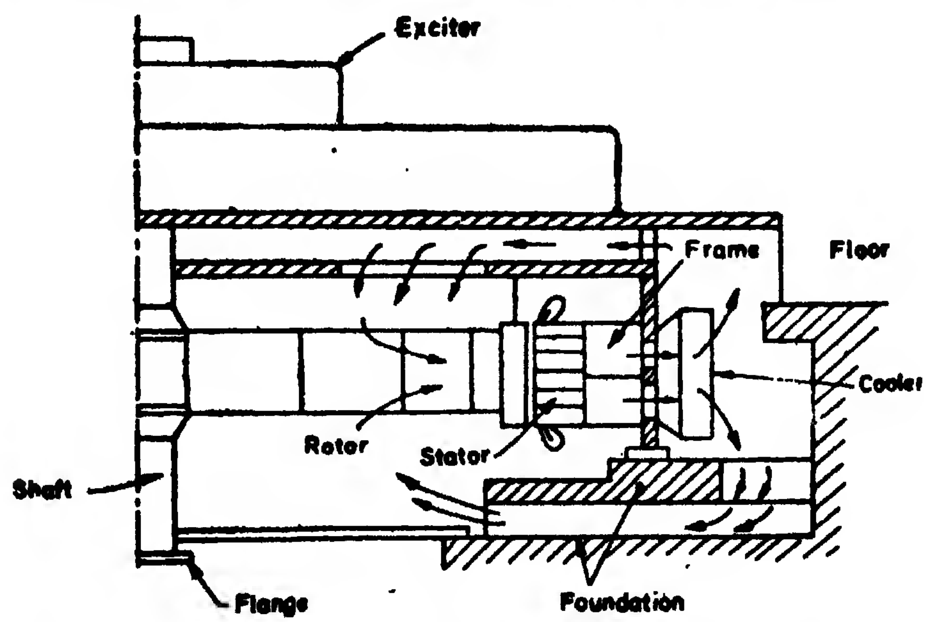


Fig. 3·32. Closed circuit ventilation of water wheel generators with water air coolers mounted at outlets of the stator frame.

3·19·1. Direct Water Cooled Rotor Windings. Large water wheel generators use direct water cooled stator and rotor windings. A typical arrangement for direct water cooling of rotor windings of salient pole generators is shown in Fig. 3·33. The interpolar

space which is normally open to cooling air in a conventional machine, is closed to give more copper space (required by hollow field conductors) thereby given low field copper loss. The mass of water required for direct cooling is only 1/4 of that of air for atmospheric pressure for the same temperature rise.

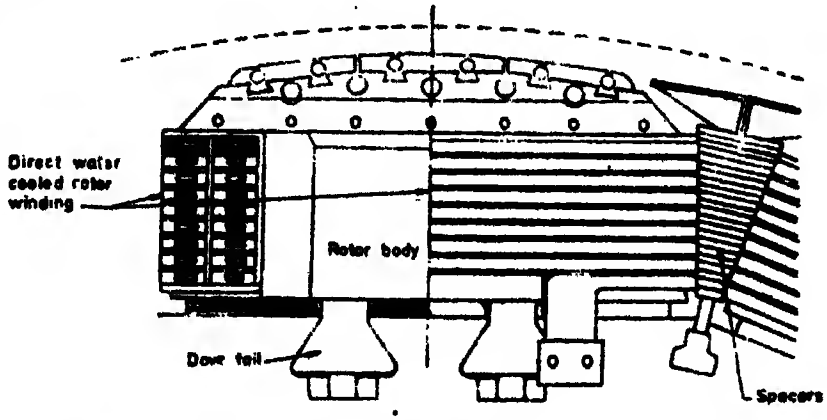


Fig. 3·33. Direct water cooling of salient pole rotor winding.

3·20. Quantity of Cooling Medium (Coolant). The quantity of cooling medium required to absorb losses of machines can be calculated with the help of expressions derived below :

Let Q=loss to be dissipated, kW ;

θ_i=inlet temperature of cooling medium, °C ;

θ=temperature rise of cooling medium, °C ;

H=barometric height, mm of mercury ;

P=pressure, N/m².

3·20·1. Air. Heat to be carried away$=Q$ kW$=Q\times10^3$ W$=Q\times10^3$ J/s.

Let c_p=specific heat of air at constant pressure, J/kg-°C.

We have, heat=weight × specific heat × temperature rise.

Heat per second=weight of air in kg/s × specific heat in J/kg-°C × temperature rise in °C

$\therefore$ Weight of air$=\dfrac{Q\times10^3}{c_p\theta}$ kg/s.

Let volume of 1 kg air at N.T.P. (0°C *i.e.*, 273 K, and 760 mm of mercury) be V m³.

$\therefore$ Volume of air required at N.T.P.$=\dfrac{Q\times10^3}{c_p\theta}V$ m³/s

At actual working conditions the temperature of air is θ_i°C or (θ_i+273) K and the pressure is H mm of mercury. Applying Kelvins' law, we have :

$$\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}$$

Volume of air under actual working conditions is therefore :

$$V_a=\frac{Q}{c_p\theta}V\times\frac{\theta_i+273}{273}\times\frac{760}{H}\times10^3 \text{ m}^3/\text{s} \quad \ldots(3·38)$$

The values for day air are :

c_p=0·2375 cal/gm-°C=995 J/kg-°C.

and V=0·775 m³.

Substituting these values in Eqn. 3·38

$$V_a = \frac{Q}{995\,\theta} \times 0{\cdot}775 \times \frac{\theta_i + 273}{273} \times \frac{760}{H} \times 10^3 \text{ m}^3/\text{s}.$$

$$= 0{\cdot}78\,\frac{Q}{\theta} \times \frac{\theta_i + 273}{273} \times \frac{760}{H} \text{ m}^3/\text{s}. \quad \text{...(3·39)}$$

Power required for blowing air at a rate V_a m³/s at a pressure of P N/m², $= PV_a$ watt.

∴ Power required by fan blowing air

$$P_f = P\frac{V_a}{\eta_f} \times 10^{-3} \text{ kW} \quad \text{...(3·40)}$$

where η_f = fan efficiency.

Typical values for a.c. generators are :

$P = 1000$ to 2000 N/m² or higher for closed circuit ventilation

$\theta = 20°$C and $\theta_i = 25°$C

$\eta_f = 0{\cdot}2$ to $0{\cdot}4$.

3·20·2. Hydrogen. Specific heat of hydrogen at constant pressure $= 3{\cdot}4$ cal/gm—°C and 1 kg of dry gas occupies a volume of 11·3 m³ at 273 K and 760 mm of mercury.

Proceeding as in the case of air :

Volume of hydrogen required for Q kW loss with a temperature rise θ°C,

$$V_H = 0{\cdot}8\,\frac{Q}{\theta}\,\frac{(\theta_i + 273)}{273}\cdot\frac{760}{H} \text{ m}^3/\text{s} \quad \text{...(3·41)}$$

H for hydrogen is 2000—2500 mm.

3·20·3. Water. We have,

Q kW $= 1000\,Q$ J/s.

The specific heat of water is $4{\cdot}18 \times 10^3$ J/kg-°C and so the volume of water required per second for absorbing Q kW loss with a temperatare rise of θ°C,

$$V_w = \frac{1000\,Q}{4{\cdot}18 \times 1000} \text{ kg/s}$$

$$= \frac{0{\cdot}24\,Q}{\theta} \text{ l/s} \quad \text{...(3·42)}$$

3·19·4. Oil. The amount of oil required per second for a dissipation of Q kW loss with a temperature rise of θ°C.

$$V_o = \frac{0{\cdot}24\,Q}{(0{\cdot}35 \text{ to } 0{\cdot}5)\theta} \text{ l/s.} \quad \text{...(3·43)}$$

Example 3·12. *A 50 MVA turbo-alternator has a total loss of 1500 kW. Calculate the volume of air required per second and also the fan power if the temperature rise in the machine is to be limited to 30°C. The other data given is :*

Inlet temperature of air $= 25°C$, *Barometric height* $= 760$ *mm of mercury,*
Pressure $= 2$ *kN/m², Fan efficiency* $= 0{\cdot}4$.

Solution. Given :

$Q = 1500$ kW, $\theta_i = 25°$C, $\theta = 30°$C, $H = 760$ mm.

In the absence of other data we assume that specific heat of air at constant pressure $c_p = 995$ J/kg—°C and volume of 1 kg of air at N.T.P. is $V = 0{\cdot}775$ m³.

$\therefore$ Using Eqn. 3·39, volume of air

$$V_a = 0{\cdot}78 \times \frac{1500}{30} \times \frac{25+273}{273} \times \frac{760}{760} = 42{\cdot}6 \text{ m}^3/\text{s}.$$

From Eqn. 3·40,

Fan power $\quad P_f = P\dfrac{V_a}{\eta_f} \times 10^{-3} = \dfrac{2{,}000 \times 42{\cdot}5}{0{\cdot}4} \times 10^{-3} = 212{\cdot}5$ kW.

Example 3·13. *A turbo-alternator runs on test at a continuous rated load of 30 MVA with a power factor of 0·8. The following cooling air measurements are taken :*

Volume of cooling air measured at intake $= 30$ *m³/s,*
Intake air temperature $= 15°C$,
Outlet air temperature $= 45°C$,
Barometric reading $= 750$ *mm of mercury.*

(a) Find the efficiency of the machine, taking the specific heat of air at constant pressure as 1000 J/kg-°C, and the volume of 1 kg of air at 0°C and a pressure of 760 mm mercury as 0·78 m³.

(b) Calculate the amount of cooling water in litre per second to cool the air, assuming the temperature rise of water to be 8°C.

Solution. (*a*) Given :

$$\theta = 45 - 15 = 30°\text{C},$$
$$\theta_i = 15°\text{C},\ H = 750 \text{ mm of mercury},\ V_a = 30 \text{ m}^3/\text{s}$$
$$c_p = 1000 \text{ J/kg—°C},\ V = 0{\cdot}78 \text{ m}^3.$$

From Eqn. 3·38, volume of air

$$V_a = \frac{Q}{c_p\theta}V \times \frac{\theta_i+273}{273} \times \frac{760}{H} \times 10^3 \text{ m}^3/\text{s}.$$

or

$$\text{loss } Q = V_a\frac{c_p\theta}{V} \times \frac{273}{\theta_i+273} \times \frac{H}{760} \times 10^{-3} \text{ kW}$$

$$= 30 \times \frac{1000 \times 30}{0{\cdot}78} \times \frac{273}{15+273} \times \frac{750}{760} \times 10^{-3} = 1080 \text{ kW}.$$

Output $\quad = 30 \times 10^3 \times 0{\cdot}8 = 24 \times 10^3$ kW.

$$\therefore \quad \text{Efficiency} = \frac{\text{output}}{\text{output}+\text{losses}} = \frac{24000}{24000+1080} = 95{\cdot}7\ \%$$

(*b*) Losses $\quad Q = 1080$ kW

Temperature rise of water $\theta = 8°$C.

From Eqn. 3·42, amount of water,

$$V_w = \frac{0{\cdot}24Q}{\theta} = \frac{0{\cdot}24 \times 1080}{8} = 32{\cdot}4 \text{ l/s}.$$

Example 3·14. *The losses of a 60 MW hydrogen cooled alternator on full load amount to 750 kW. The flow of hydrogen from the coolers is 10 m³/s at 2000 mm of mercury gauge pressure above atmosphere, which is 760 mm. The temperature of hydrogen leaving the coolers is 25°C. Determine the temperature rise of hydrogen assuming specific heat of hydrogen at constant pressure to be 12540 J/kg—°C and weight of 11·2 m³ of hydrogen at 0°C and 760 mm of mercury to be 1 kg.*

Solution. Hydrogen enters the machine after leaving the coolers.

$\therefore$ Volume of hydrogen entering the machine

= volume of hydrogen leaving the coolers = 10 m³/s

Eqn. 3·38, has been derived for air but in fact it is a general equation which is applicable to any gaseous cooling medium with relevant quantities substituted.

∴ Volume of hydrogen

$$V_H = \frac{QV}{c_p\theta} \times \frac{\theta_i+273}{273} \times \frac{760}{H} \times 10^3 \text{ m}^3/\text{s}.$$

∴ Temperature rise

$$\theta = \frac{QV}{c_p V_H} \times \frac{\theta_i+273}{273} \times \frac{760}{H} \times 10^3$$

$$= \frac{750 \times 11\cdot2}{12540 \times 10} \times \frac{25+273}{273} \times \frac{760}{2760} \times 10^3 = 20°\text{C}.$$

Example 3·15. *The total losses in a 40 MVA transformer are 200 kW. The oil for cooling the transformer is circulated by a pump. The oil after taking up heat from the tranformer tank goes to the radiators where it is cooled by circulation of water (Fig. 3·34). Calculate the amount of oil which is raised by 20°C in its passage through the tank. Also calculate the amount of water required for cooling oil in radiators. The rise in temperature of cooling water is 10°C. Assume that 20 percent of losses are dissipated by tank walls.*

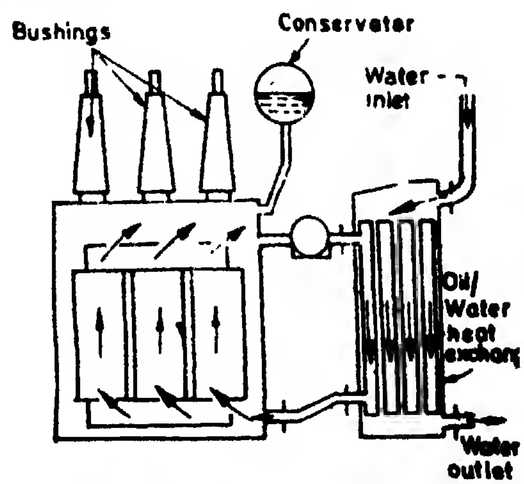


Fig. 3·34. Oil forced water cooled transformer.

Solution.

Amount of oil :

Heat taken up by oil $= 0\cdot8 \times 200 = 160$ kW

∴ $Q = 160$ kW and $\theta = 20°$C

From Eqn. 3·43, oil required

$$V_o = \frac{0\cdot24 \times 160}{0\cdot4 \times 20}$$

$= 4\cdot8$ l/s (assuming $c_p = 0\cdot4$)

Amount of water : $Q = 160$ kW, $\theta = 10°$C

From Eqn. 3·42, water required

$$V_w = \frac{0\cdot24 \times 160}{10} = 3\cdot84 \text{ l/s}.$$

Example 3·16. *A 15 MVA transformer has an iron loss of 80 kW and a copper loss of 120 kW at full load. The tank dimensoons are 3·5×3·0×1·4 metre. The transformer oil is cooled by 3 litre of water per second passed through a cooling coil. Estimate the average temperature rise of the tank if the difference of temperature of water at the inlet and the outlet is 15°C. The specific loss dissipation from tank walls is 10 W/m²—°C.*

Solution

Total losses $= 120 + 80 = 200$ kW.

Out of this loss of 200 kW, some loss is dissipated by tank walls and the rest is taken up by circulating water.

Rise in temperature of water $= 15°$C

Amount of water per second $V_w = 3$ litre.

From Eqn. 3·42, amount of water $V_w = \dfrac{0\cdot24\, Q}{\theta}$

or heat taken away by water $Q = \dfrac{V_w \theta}{0\cdot25} = \dfrac{3 \times 15}{0\cdot24} = 187\cdot5$ kW.

The rest of the loss is to be dissipated by tank walls.

Loss dissipated by walls$=200-187.5=12.5$ kW.

Area of tank walls (neglecting top and bottom surfaces)

$$S=2\times 3.5(3+1.14)=30.8 \text{ m}^2.$$

From Eqn. 3.20, temperature rise of tank

$$\theta=\frac{Q}{S\lambda}=\frac{12.5\times 100}{30.8\times 10}=40.6°\text{C}.$$

Example 3.17. *A 500 MW direct water cooled turbo-alternator has a stator copper loss of 800 kW. The water inlet temperature is 38°C and the outlet temperature is 68°C. Calcuate amount of water required per second. Also calculate the area of water duct in each subconductor if there are 48 slots with 2 conductors per slot and each conductor is subdivided into 32 sub-conductors. The velocity is not to exceed 1.0 m/s*

The pumping pressure is 300 kN/m², calculate the power of water pump if its efficiency is 0.6.

Solution. Temperature rise of water $\theta=68-38=30°$C.

The volume of water required

$$V_w=\frac{0.24\,Q}{\theta}=\frac{0.24\times 800}{30}=6.4 \quad \text{l/s.}=6.4\times 10^{-3} \text{ m}^3/\text{s}$$

Total number of stator conductors$=2\times 48=96$.

Total number of subconductors $=96\times 32$.

Volume of water required for each sub-conductor

$$=\frac{6.4}{96\times 32}=0.00208 \text{ l/s}$$

Area of each duct $=\dfrac{\text{volume/second}}{\text{velocity}}=\dfrac{0.00208\times 10^{-3}}{1}\text{ m}^2\simeq 2\text{mm}^2$

Pumping power $=\dfrac{300\times 10^3\times 6.4\times 10^{-3}}{0.6}\times 10^{-3}=3.2$ kW.

TEMPERATURE RISE—TIME CURVES

3.21. Theory of Solid Body Heating. The temperature of a machine rises when it is run under steady load conditions starting from cold conditions. The temperature at first increases at a rate determined by power wasted. As the temperature rises, the active parts of the machine dissipate heat partly by conduction, partly by radiation, and in most cases, largely by means of air cooling. The higher the temperature rise, the greater would be the effect of these methods of cooling. Therefore, as the temperature rises, its rate of increase falls off owing to better heat dissipating conditions. As shown later, the temperature-time curve is exponential in nature.

The temperature of any part of a machine, not only depends on the heat produced in itself but also on heat produced in other parts. This is because there is always a heat flow from one part to another ; for example, the heat produced in the part of the winding embedded in the slot flows partially through the insulation to the laminations partially to the end windings. Thus the end windings have to transfer to the air, not only the heat produced in them but also a part of the heat produced in the slot portion of the winding.

Electrical machines are not homogeneous bodies. Their parts are made up of different materials like copper, iron and insulation. These materials have different thermal resistivities and due to this, it is rather difficult to calculate the temperature of a part of a machine. However, it is worthwhile taking theory of heating of homogeneous bodies as the basis for analysing the process of machine heating. The results obtained from such a theory are applicable to a certain degree, to the different parts of machine as a whole.

Let Q=power loss or heat developed, J/s, or W ;

G=weight of active parts of machine, kg ;

h=specific heat, J/kg-°C ;

S=cooling surface, m^2 ;

λ=specific heat dissipation, W/m^2-°C ;

$c=1/\lambda$=cooling co-efficient, °C-m^2/W ,

θ=temperature rise at any time t, °C ;

θ_m=final steady temperature rise while heating, °C ,

θ_n=final steady temperature rise while cooling, °C

θ_i=initial temperature rise over ambient medium, °C

T_h=heating time constant, s ;

T_c=cooling time constant, s ;

t=time, s.

3·21·1. Heating. Considering the conditions at any time t from start, heat energy developed in the body during an infinitely small time dt.

$$=\text{heat energy developed per second}\times dt=Qdt \qquad ...(3\cdot44)$$

If during this period dt the temperature of the body rises by $d\theta$, the heat energy stored in the body,

$$=\text{weight of body}\times\text{specific heat}\times\text{difference in temperature}$$

$$=G\,h\,d\theta. \qquad ...(3\cdot45)$$

If in the process of heating, the temperature of the surface rises by θ over the ambient medium, at the instant considered, the heat energy dissipated by the body into the ambient medium due to radiation, conduction and convection,

$$=\text{specific heat dissipation}\times\text{surface}\times\text{temperature rise}\times\text{time}$$

$$=\lambda\times S\times\theta\times dt=S\,\lambda\,\theta\,dt. \qquad ...(3\cdot46)$$

As the heat developed in the machine is equal to the heat stored in the parts plus the heat dissipated, we have from Eqns. 3·44, 3·45 and 3·46,

$$Q\,dt=G\,h\,d\theta+S\,\lambda\,\theta\,dt \qquad ...(3\cdot47)$$

or

$$dt=\frac{d\theta}{\frac{Q}{Gh}-\frac{S\lambda}{Gh}\theta} \qquad ...(3\cdot48)$$

Solving the differential equation 3·44,

$$t=\frac{Gh}{S\lambda}\log_e\left(\frac{Q}{Gh}-\frac{S\lambda}{Gh}\theta\right)+K \qquad ...(3\cdot49)$$

where K is the constant of integration.

The value of K is found by applying the boundary condition,

when $t=0$, we have $\theta=\theta_i$.

Putting this in Eqn. 3·49, we have :

$$0=-\frac{Gh}{S\lambda}\log_e\left(\frac{Q}{Gh}-\frac{S\lambda}{Gh}\theta_i\right)+K$$

or

$$K=\frac{Gh}{S\lambda}\log_e\left(\frac{Q}{Gh}-\frac{S\lambda}{Gh}\theta_i\right).$$

Substituting this value of K in Eqn. 3·49,

$$t=-\frac{Gh}{S\lambda}\log_e\left(\frac{Q}{Gh}-\frac{S\lambda}{Gh}\theta\right)+\frac{Gh}{S\lambda}\log_e\left(\frac{Q}{Gh}-\frac{S\lambda}{Gh}\theta_i\right)$$

$$=-\frac{Gh}{S\lambda}\log_e\frac{\frac{Q}{S\lambda}-\theta}{\frac{Q}{S\lambda}-\theta_i} \qquad \ldots(3{\cdot}50)$$

The machine reaches a final steady temperature rise θ_m when $t=\infty$. Under this condition there is no further temperature rise and the rates of heat production and dissipation are equal. This means

$$d\theta=0 \quad \text{or} \quad Ghd\theta=0$$

when the machine attains final steady temperature rise.

From Eqn. 3·47, for $\theta=\theta_m$,

$$Qdt=S\,\lambda\,\theta_m\,dt$$

or

$$\theta_m=Q/S\lambda \qquad \ldots(3{\cdot}51)$$

$$=Qc/S \qquad \ldots(3{\cdot}52)$$

Putting $\theta_m=\frac{Q}{S\lambda}$ in Eqn. 3·50, we have

$$t=\frac{Gh}{S\lambda}\log_e\frac{\theta_m-\theta}{\theta_m-\theta_i} \qquad \ldots(3{\cdot}53)$$

The term $Gh/S\lambda$ has the dimensions of time and is called the **heating time constant** T_h.

$$\therefore \quad T_h=Gh/S\lambda \qquad \ldots(3{\cdot}54)$$

Putting this value of $T_h=\frac{Gh}{S\lambda}$ in Eqn. 3·53,

$$t=T_h\log_e\left[\frac{\theta_m-\theta}{\theta_m-\theta_i}\right] \qquad \ldots(3{\cdot}55)$$

or

$$\frac{\theta_m-\theta}{\theta_m-\theta_i}=e^{-t/T_h}$$

or

$$\theta=\theta_m(1-e^{-t/T_h})+\theta_i e^{-t/T_h} \qquad \ldots(3{\cdot}56)$$

If the machine starts from cold conditions,

$\theta_i=0$. (No temperature rise over the ambient medium)

$$\therefore \quad \theta=\theta_m(1-e^{-t/T_h}) \qquad \ldots(3{\cdot}57)$$

Relation 3·57 is the equation of temperature rise with time. The temperature rise-time curve is exponential in nature as shown in Fig. 3·35.

Heating Time Constant. Consider Eqn. 3·57,

$$\theta=\theta_m(1-e^{-t/T_h}).$$

Putting $t=T_h$ in the above expression, we have,

$$\theta=\theta_m(1-e^{-1})=0{\cdot}632\,\theta_m.$$

Thus we can define the heating time as the time taken by the machine to attain 0·632 of its final steady temperature rise. The heating time constant of a machine is the index of time taken by the machine to attain its final steady temperature rise.

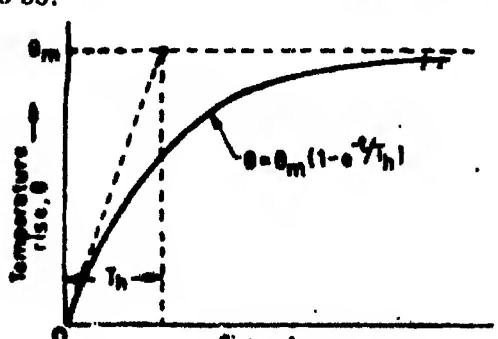


Fig. 3·35. Heating curve.

Considering the relationship $T_h = Gh/S\lambda$ we conclude that the time constant is inversely proportional to λ (specific heat dissipation). λ has a large value for well ventilated machines and thus the value of their heating time constant is small. The value of heating time constant is large for poorly ventilated machines.

Since the volume of machine and hence its weight increases in proportion to the third power, and the surface area in proportion to second power of its linear dimensions, the heating time constant of a machine increases as first power of linear dimensions. Because of this large sized machines have large heating time constants.

The heating time constant of a well-ventilated induction motor of about 20 kW rating may be of the order of minutes while that of large or totally enclosed machines may reach several hours or even days. The heating time constant of conventional electrical machines is usually within the range of ½ to 3-4 hour.

Final Steady Temperature Rise. Considering the expression $\theta_m = Q/S\lambda$, it is clear, other things being equal, the final steady temperature rise is directly proportional to the losses. It is also evident that the final steady temperature rise is inversely proportional to surface area and specific heat dissipation. Thus for the same loss, the machine would attain a higher temperature rise if its dissipating surface is small or, if its ventilation is poor.

3·21·2. Cooling. The equation for cooling can be derived by considering Eqn. 3 47.

$$Q\,dt = Gh\,d\theta + S\lambda\theta\,dt$$

The solution of above equation is :

$$t = -\frac{Gh}{S\lambda}\log_e\left[\frac{Q}{Gh} - \frac{S\lambda}{Gh}\,\theta\right] + K$$

The value of K is obtained by putting boundary conditions,

when $t=0$, $\theta=\theta_i$. From this we have,

$$0 = -\frac{Gh}{S\lambda}\log_e\left[\frac{Q}{Gh} - \frac{S\lambda}{Gh}\,\theta_i\right] + K$$

or

$$K = \frac{Gh}{S\lambda}\log_e\left[\frac{Q}{Gh} - \frac{S\lambda}{Gh}\,\theta_i\right]$$

Substituting this value of K and proceeding as in the case of heating, we get,

$$\theta = \theta_m\,(1 - e^{-t/T_c}) + \theta_i e^{-t/T_c} \quad \text{...(3·58)}$$

where

$$\theta_m = Q/S\lambda \quad \text{...(3·59)}$$

$$= Qc/S \quad \text{...(3·60)}$$

and

$$T_c = Gh/S\lambda \quad \text{...(3·61)}$$

The value of λ under cooling conditions is usually different from that under heating conditions and so the heating and cooling time constants of a machine may have different values.

If a machine is shut down, no heat is produced and so its final steady temperature rise when cooling is zero or $\theta_m = 0$. Und these conditions Eqn. 3·62 thus reduces to

$$\theta = \theta_i e^{-t/T_c} \quad \text{...(3·62)}$$

It is clear from Eqn. 3·6 [illegible] g curve is also exponential in nature as shown in Fig. 3·36. Eqn. 3·62 is a[illegible] hines which are shut down while Eqn. 3·5[illegible] is applied to machines allowe[illegible] tial removal of load.

Cooling Time Constant. Consider the relation,

$$\theta=\theta_i\, e^{-t/T_c}$$

Putting $t=T_c$, we have

$$\theta=\theta_i\, e^{-1}=0.368\ \theta_i.$$

Thus we can define the cooling time constant as the time taken by the machine for its temperature rise to fall to 0·368 of its initial value.

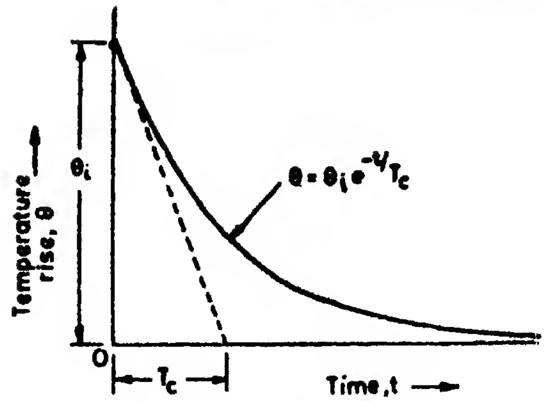


Fig. 3·36. Cooling curve.

As stated earlier the cooling time constant may be different from heating time constant for the same machine, as the ventilation conditions in the two cases may not be the same. The cooling time constant is usually larger owing to poorer ventilation conditions when the machine cools. In self cooled motors the cooling time constant is about 2–3 times greater than the heating time constant because cooling conditions are worse at stand still.

Final Steady Temperature Rise. Considering the relation $\theta_m=Q/S\lambda$, the final steady temperature rise when the machine is cooling is directly proportional to losses. When the machine is shut down, $Q=0$ and so the final temperature rise is zero, *i.e.* the machine finally comes to ambient temperature conditions.

Example 3·18. *A field coil has a heat dissipating surface of 0·15 m² and a length of mean turn of 1 m. It dissipates loss of 150 W, the emissivity being 34 W/m²—°C. Estimate the final steady temparature rise of the coil and its time constant if the cross section of the coil is 100×50 mm². Specific heat of copper is 390 J/kg—°C. The space factor is 0·56. Copper weighs 8900 kg/m³.*

Solution.

Volume of copper $=1\times100\times50\times0.56\times10^{-6}=2.8\times10^{-3}$ m³.

Weight of copper $G=2.8\times10^{-3}\times8900=24.92$ kg.

Final steady temperature rise, from Eqn. 3·51,

$$\therefore \quad \theta_m=\frac{Q}{S\lambda}=\frac{150}{0.15\times34}=29.4°\text{C}.$$

Heating time constant, from Eqn. 3·54,

$$T_h=\frac{Gh}{S\lambda}=\frac{24.92\times390}{0.15\times34}=1906\ \text{s}.$$

Example 3·19. *The exciting coil of an electromagnet has a cross section of 120×50 mm² and a length of mean turn of 0·8 m. It dissipates 150 W continuously. Its cooling surface is 0·125 m² and specific heat dissipation is 30 W/m²—°C. Calculate the final steady temperature rise of the coil surface. Also calculate the hot spot temperature rise of the coil if the thermal resistivity of insulating material used is 8 Ωm. The space factor is 0·56.*

Solution. From Eqn. 3·51, final steady temperature rise of coil surface

$$\theta_m=\frac{Q}{S\lambda}=\frac{150}{0.125\times30}=40°\text{C}.$$

From Eqn. 3·31, effective thermal resistivity

$$\rho_e=\rho_i\,(1-S_f^{1/2})=8(1-0.56^{1/2})=2\ \Omega\ \text{m}.$$

Loss $$q=\frac{150}{120\times50\times10^{-6}\times0.8}=31250\ \text{W/m}^3$$

From Eqn. 3·23,

temperature difference between coil surface and hot spot

$$\theta_o = \frac{q\rho t^2}{8} = \frac{31250 \times 2 \times (50 \times 10^{-3})^2}{8} = 19{\cdot}5°\text{C}$$

$\therefore$ Temperature rise of hot spot $= 40 + 19{\cdot}5 = 59{\cdot}5°$C.

Example 3·20. *A 1000 A shunt consists of 8 strips of nickel-alloy connected in parallel, each having a cross section of 25×2 mm². The normal voltage drop is 75 mV. The alloy used has the following data : resistivity$=0{\cdot}4 \times 10^{-6}$ Ω m, specific heat 500 J/kg—°C, specific gravity$=8000$ kg/m³, rate of heat dissipation$=100$ W/m²—°C. Determine the maximum temperature rise and the time taken to reach 99 per cent of maximum value.*

Solution. Suppose l is the length of each strip expressed in m.

Area of each strip $a = 2 \times 25$ mm² $= 50 \times 10^{-6}$ m².

Resistance of each strip $= \dfrac{\rho l}{a} = \dfrac{0{\cdot}4 \times 10^{-6} \times l}{50 \times 10^{-6}} = 0{\cdot}008\ l$

There are 8 strips in parallel

$\therefore$ Resistance of shunt $R_{sh} = \dfrac{0{\cdot}008\ l}{8} = 0{\cdot}001\ l$

Voltage drop across shunt $= I_{sh} \times R_{sh} = 1000 \times 0{\cdot}001\ l = 75 \times 10^{-3}$ (given)

$\therefore$ Length of each strip $l = 0{\cdot}075$ m $= 75$ mm.

Volume of 8 strips $= 8 \times 75 \times 50 \times 10^{-9} = 0{\cdot}03 \times 10^{-3}$ m³

$\therefore$ Weight of strips $G = 8000 \times 0{\cdot}03 \times 10^{-3} = 0{\cdot}24$ kg.

Total heat dissipating surface

$$S = 8 \times 2(2+25) \times 75 \times 10^{-6} = 0{\cdot}0324 \text{ m}^2.$$

$\therefore$ Time constant $T_h = \dfrac{Gh}{S\lambda} = \dfrac{0{\cdot}24 \times 500}{0{\cdot}0324 \times 100} = 37$ s.

Loss in strips $Q = 1000 \times 75 \times 10^{-3} = 75$ W.

Maximum steady temperature rise $\theta_m = \dfrac{Q}{S\lambda} = \dfrac{57}{0{\cdot}0324 \times 100} = 23{\cdot}15$°C.

Suppose t is the time required to reach 99 per cent of the final steady temperature.

From Eqn. 3·57,

$$\theta = \theta_m(1 - e^{-t/T_h})$$

or $\theta/\theta_m = 0{\cdot}99 = (1 - e^{-t/37})$ or $t = 170{\cdot}4$ s.

Example 3·21. *The heat dissipating surface of a 7·5 kW. totally enclosed induction motor can be approximated as a cylinder of 0·6 m in diameter and 0·9 m in length. The motor can be considered to be made up of a homogeneous material weighing 375 kg and having a specific heat of 725 J/kg—°C. The specific heat dissipation from its surface is 12 W/m²—°C.*

Find (i) the temperature rise of the machine at full load if the efficiency is 90 per cent. Also find the thermal time constant of the machine and (ii) the rating of the machine for the same temperature rise if the machine were screen protected with a specific heat dissipation of 25 W/m²—°C. Assume the same value for efficiency.

Solution. Total heat dissipating surface

$S =$ outer cylindrical surface $+ 2 \times$ end surface

$= \pi \times 0{\cdot}6 \times 0{\cdot}9 + 2 \times \pi/4 \times 0{\cdot}6^2 = 2{\cdot}26$ m².

Efficiency $= \dfrac{\text{output}}{\text{output} + \text{losses}}$ or $0{\cdot}9 = \dfrac{7500}{7500 + \text{losses}}$

$\therefore$ Losses $Q = 833$ W,

From Eqn. 3·51, final study temperature rise,

$$\theta_m = \frac{Q}{S\lambda} = \frac{833}{2\cdot25\times12} = 30\cdot85°C.$$

From Eqn. 3·54,

Time constant $T_h = \frac{Gh}{S\lambda} = \frac{375\times725}{2\cdot26\times12} = 10025$ s $= 2\cdot79$ hour.

(*ii*) We have, for screen protected machine,

$$\theta_m = 30\cdot85°C,\ S = 2\cdot26\ m^2,\ \lambda = 25\ W/m^2-°C$$

∴ From Eqn. 3·51, allowable losses

$$Q = \theta_m S\lambda = 30\cdot85\times2\cdot26\times25 = 1743\ W.$$

∴ Efficiency $= 0\cdot9 = \frac{\text{output}}{\text{output}+1743}$

or output $= 15687$ W $= 15\cdot7$ kW.

Example 3·22. *The temperature rise of a transformer is 25°C after one hour and 37·5°C after two hours of starting from cold conditions. Calculate its final steady temperature rise and the heating time constant. If its temperature falls from the final steady value to 40°C in 1·5 hour when disconnected, calculate its cooling time constant. The ambient temperature is 30°C.*

Solution.

When Heating. Since the transformer starts from cold conditions therefore its temperature rise is given by Eqn. 3·57,

$$\theta = \theta_m(1 - e^{t/T_h})$$

Now we have $\theta = 25°C$ at $t = 1$ hour and $\theta = 37\cdot5°C$ at $t = 2$ hours.

Putting these values in Eqn. 3·57, $25 = \theta_m(1 - e^{-1/T_h})$ ...(*i*)

$$37\cdot5 = \theta_m(1 - e^{-2/T_h}) \quad ...(ii)$$

From (*i*) and (*ii*)

$$\frac{(1-e^{-2/T_h})}{(1-e^{-1/T_h})} = \frac{37\cdot5}{25} = 1\cdot5$$

or $1 + e^{-1/T_h} = 1\cdot5$ or $e^{-1/T_h} = 0\cdot5$

or Heating time constant $T_h = 1\cdot44$ hour.

From (*i*), $25 = \theta_m(1 - e^{-1/T_h}) = \theta_m(1 - 0\cdot5)$

∴ Final steady temperature rise $\theta_m = 25/0\cdot5 = 50°C$.

When Cooling. Temperature rise after 1·5 hour, $\theta = 40 - 30 = 10°C$. Since the transformer is disconnected its final stedy temperature rise when cooling $\theta_m = 0$.

Initial temperature rise is $\theta_i = 50°C$

From Eqn. 3·62, we have, $\theta = \theta_i e^{-t/T_c}$ or $10 = 50e^{-1\cdot5/T_c}$

or cooling time constant $T_c = 0\cdot932$ hour.

Example 3·23. *The initial temperature of a machine is 40°C. Calculate the temperature of the machine after 1 hour if its final steady temperature rise is 80°C and the heating time constant is 2 hours. The ambient temperature is 30°C.*

Solution. Initial temperature rise $\theta_i = 40 - 30 = 10°C$.

From Eqn. 3·56, $\theta = \theta_m(1 - e^{-t/T_h}) + \theta_i e^{-t/T_h}$

Temperature rise after one hour

$$\theta = 80(1 - e^{-1/2}) + 10e^{-1/2} = 37\cdot54°C$$

∴ Temperature of machine after one hour $= 37\cdot54 + 30 = 67\cdot54°C$.

Example 3·24. *A 400 kVA transformer has its maximum efficiency at 80 per cent of full load. During a short full load heat run the temperature rise after one hour and two hours is observed to be 24°C and 34°C, respectively. Find the thermal time constant and final steady temperature rise of the transformer.*

If, by use of a fan, the cooling is improved so that rate of heat dissipation per unit area per degree rise in temperature is increased by 15 per cent find the new kVA rating possible (a) for the same final temperature rise as before, (b) if the allowable temperature rise is taken as 50°C.

Solution. From Eqn. 3·57, we have temperature rise

$$\theta=\theta_m(1-e^{-t/T_h})$$

Now $\theta=24$ at $t=1$ hour and $\theta=34$ at $t=2$ hours

$\therefore$ $24=\theta_m(1-e^{-1/T_h})$ and $34=\theta_m(1-e^{-2/T_h})$

From above, $e^{-t/T_h}=0·417$

or thermal time constant $T_h=1·14$ hour.

We have $24=\theta_m(1-e^{-1/1·14})$ or $24=\theta_m(1-0·417)$

$\therefore$ Final steady temperature rise at full load (400 kVA), $\theta_m=41·2$°C.

The maximum efficiency occurs at 80 per cent of full load. This means that the copper losses at 80 per cent full load are equal to the iron losses.

Suppose P_c and P_i are respectively the copper losses at full load and iron losses. Since copper losses vary as square of the load and the iron losses are constant at all loads, we have :

$(0·8)^2P_c=P_i$ or $P_c=1·563\ P_i$.

Total losses at full load for an output of 400 kVA$=P_c+P_i=2·563\ P_i$.

When a fan is used :

(*a*) Final steady temperature rise $\theta_m=Q/S\lambda$ (See Eqn. 3·51)

$\therefore$ Allowable losses $Q=\theta_m\ S\ \lambda$.

Thus the allowable losses are proportional to product of allowable maximum steady temperature rise, surface area and specific heat dissipation. Since the final steady temperature rise θ_m is the same, the surface area S is constant and the heat dissipation per unit area per degree temperature rise λ is increased by 15 per cent, the new allowable losses are 1·15 times the allowable losses with 400 kVA output.

Allowable losses $=1·15\times2·563\ P_i=2·947\ P_i$

Allowable copper losses $=2·947\ P_i-P_i=1·947\ P_i$

$$=\frac{1·947}{1·563}P_c=1·246\ P_c$$

Suppose the new output is x times 400 kVA.

Therefore the copper losses at this output are $x^2\ P_c$ as the copper losses at 400 kVA are P_c. Thus we have,

$x^2\ P_c=1·246\ P_c$ or $x=1·116$

$\therefore$ New output $=1·116\times400=446·4$ kVA.

(*ii*) The maximum allowable temperature rise with 400 kVA output is 41·2°C. The maximum allowable temperature rise is now raised to 50°C. The surface area is the same but the new heat dissipation is 1·15 times that with 400 kVA.

Therefore new allowable losses

$$=\frac{50}{41·2}\times1\times1·15\times\text{allowable losses with 400 kVA}$$

$$=1·4\times2·563\ P_i=3·577\ P_i$$

Allowable copper losses $=3{\cdot}577\ P_i-P_i=2{\cdot}577\ P_i$.

$$=\frac{2{\cdot}577}{1{\cdot}563}\ P_c=1{\cdot}65\ P_c.$$

Suppose the new output is x times kVA.

$\therefore\quad x^2P_c=1{\cdot}65\ P_c \quad$ or $\quad x=1{\cdot}284$

$\therefore\quad$ New output $=1{\cdot}284\times400=513$ kVA

3·21. Salient Features of Heating Curves.

3·21·1. Heating time constant

Consider Eqn. 3·57, $\quad \theta=\theta_m\,(1-e^{-t/T_h})$

putting $\quad t=T_h$, we get $\theta=0{\cdot}632\ \theta_m \qquad t=2T_h,\ \theta=0{\cdot}865\ \theta_m$

$\qquad\qquad t=3T_h,\quad \theta=0{\cdot}950\ \theta_m \qquad t=4T_h,\ \theta=0{\cdot}982\ \theta_m$

From above it is clear that the machine attains practically its final temperature rise after a time interval equal to four times its heating time constant.

2. Differentiating Eqn. 3·57, we have

$$\frac{d\theta}{dt}=\frac{\theta_m e^{-t/T_h}}{T_h}$$

At $\quad t=0,\quad \dfrac{d\theta}{dt}=\dfrac{\theta_m}{T_h}$ ...(3·63)

From above it is clear that if the initial rate of change of temperature is maintained, the machine would reach its final steady temperature rise in a time equal to its heating time constant. This furnishes another definition for heating time constant as : the time taken by the machine to reach its final steady temperature rise if the initial rate of change of temperature rise is maintained (see Fig. 3·35).

3·21·2. Relation between dθ/dt and θ and Estimation of Heating Time Constant.

Differentiating Eqn. 3·57, we have : $\quad \dfrac{d\theta}{dt}=\dfrac{\theta_m\ e^{-t/T_h}}{T_h}$

but according to the same equation : $\quad e^{-t/T_h}=\dfrac{\theta_m-\theta}{\theta_m}$

$$\therefore\quad \frac{d\theta}{dt}=\frac{\theta_m}{T_h}\cdot\frac{\theta_m-\theta}{\theta_m}=\frac{\theta_m-\theta}{T_h}$$

or $\quad T_h=\dfrac{\theta_m-\theta}{d\theta/dt}$ ...(3·64)

Eqn. 3·64 is the basic relationship for the graphical determination of heating time constant. Consider now a triangle ABC in Fig. 3·37 where AC is a part of the tangent to the heating curve at some point A corresponding to time t and temperature rise θ.

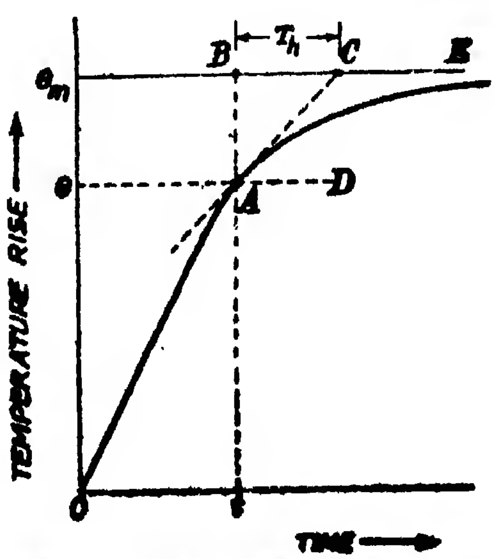


Fig. 3·37. Determination of heating time constant.

For this triangle $BC=\dfrac{AB}{\tan\ \angle BCA}$

but $\quad AB=\theta_m-\theta$ and $\angle BCA=\angle CAD$.

$\angle CAD$ represents the slope of the tangent to the curve at point A. Thus the tangent of this angle is derivative of function at the given point, *i.e.*

$\tan\ \angle CAD=d\theta/dt$

From above, as $\angle CAD=\angle BCA$,

$\therefore\quad \tan\ \angle BCA=d\theta/dt$

or $$BC=\frac{\theta_m-\theta}{d\theta/dt}$$ ...(3·65)

From the comparison of relationships 3·64 and 3·65, it follows that,

$$BC=T_h$$ ...(3·66)

Thus the heating time constant T_h may be determined as the sub-tangent BC subtended by a tangent to the heating curve at any point on a straight line BCE of the final steady temperature rise.

3·21·3. Determination of Final Steady Temperature Rise. We have, from Eqn. 3·64,

$$\theta_m-\theta=T_h\frac{d\theta}{dt}.$$

For small increments, $\Delta\theta/\Delta t$ can be subsituted for $d\theta/dt$ in the above expression

or $\theta_m-\theta=T_h\,(\Delta\theta/\Delta t)$

Taking $\Delta t=$ a constant,

we have, $\theta_m-\theta=K\times\Delta\theta$ where $K=T_h/\Delta t=$ a constant.

Thus the difference $\theta_m-\theta$ for $\Delta t=$ constant, is a linear function of temperature increment $\Delta\theta$.

The above conclusion leads to the following method for determination of θ_m.

Consider any section RP on the heating curve shown in Fig. 3·38. Divide the section into equal intervals.

$$\Delta t_1=\Delta t_2=\Delta t_3=\ldots\ldots\ldots=\Delta t.$$

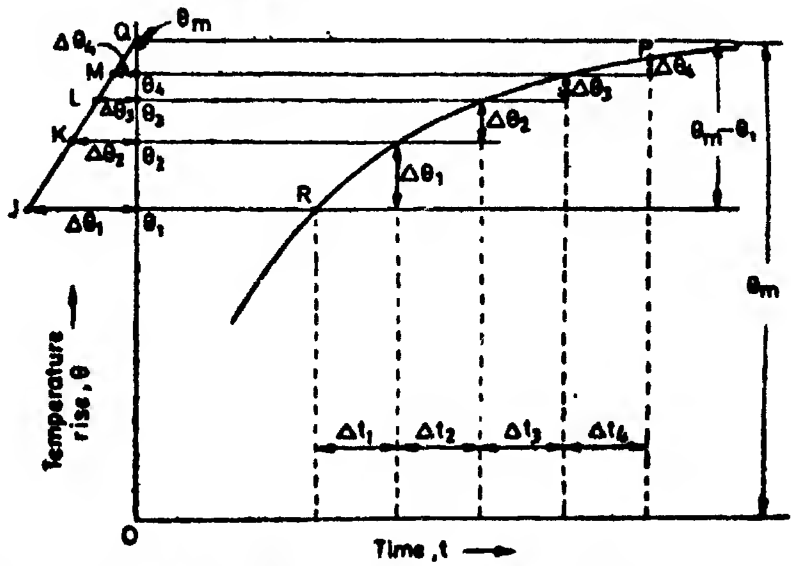


Fig. 3·38. Determination of maximum steady temperature rise from short heat run.

Find the corresponding temperature increments $\Delta\theta_1$, $\Delta\theta_2$, $\Delta\theta_3$ ....etc. Plot the increments of $\Delta\theta$ obtained for corresponding temperature difference values $(\theta_m-\theta_1)$, $(\theta_m-\theta_2)$, $(\theta_m-\theta_3)$......etc., parallel to X axis and to the left of coordinates. Draw a line through the points J, K, L, M of the plotted increments $\Delta\theta_1$, $\Delta\theta_2$, $\Delta\theta_3$......etc. This line intersects Y axis at Q. The value of θ_m is igven by OQ.

Example 3·25. *The field coils of a d.c. machine are excited with 2·5 A at 230 V. The weight of copper in the coils is 60 kg. Estimate the rate at which the temperature will begin to rise when the coils are excited from cold conditions. Specific heat of copper is 390 J/kg—°C.*

Solution. Loss $Q=230\times2{\cdot}5=575$ W.

We have, $\theta=\theta_m(1-e^{-t/T_h})$

or $$\frac{d\theta}{dt}=\frac{\theta_m}{T_h}e^{-t/T_h})$$

Rate of change of temperature rise at $t=0$ is

$$\frac{d\theta}{dt}=\frac{\theta_m}{T_h}$$

But $\theta_m=\frac{Q}{S\lambda}$ (see Eqn. 3·51)

and $T_h=\frac{Gh}{S\lambda}$ (see Eqn. 3·54)

$\therefore$ Rate of change of temperature at $t=0$ is :

$$\frac{d\theta}{dt}=\frac{\theta_m}{T_h}=\frac{Q}{S\lambda}\times\frac{S\lambda}{Gh}=\frac{Q}{Gh}$$

$$=\frac{575}{60\times390}=0{\cdot}0246\ ^\circ\text{C/s}.$$

Example 3·26. *Measurements made from a temperatere rise time curve of a transformer in which the loss dissipated is constant, show that the rate of change of temperature rise is 0·375°C per minute and 0·227°C per minute when the temperature rise is 29·1°C and 46·7°C respectively. Find the final steady temperature rise and the heating time constant of the transformer.*

Solution. From Eqn. 3·64, heating time constant

$$T_h=\frac{\theta_m-\theta}{d\theta/dt}.$$

Substituting the given data in the expression we have

$$T_h=\frac{\theta_m-29{\cdot}1}{0{\cdot}375}=\frac{\theta_m-46{\cdot}7}{0{\cdot}227}$$

From above,

Final steady temperature rise $\theta_m=73{\cdot}65$°C

and heating time constant $T_h=118{\cdot}8$ minute's$\simeq$2 hours.

Example 3·27. *The heat run on a d.c. motor gave the following results :*

Time, minutes,	*0*	*15*	*30*	*45*	*60*	*75*
Temperature, °C,	*50*	*56·6*	*61·8*	*65·8*	*69*	*71·2*

Calculate the final steady temperature rise and the time constant of the machine if the ambient temperature is 30°C.

Solution.

The temperature rise-time data is tabulated below after subtracting the ambient temperature from the temperature of the machine.

Time, minutes	0	15	30	45	60	75
Temperature rise, °C	20	26·6	31·8	35·8	39	41·2

The temperature rise-time curve *RP* is plotted in Fig. 3·39. The time interval between *R* and *P* is divided in 5 equal parts *i.e.* $\Delta t=15$ minutes. From the graph,

$\Delta\theta_1=6{\cdot}6$°C $\quad\Delta\theta_2=5{\cdot}2$°C $\quad\Delta\theta_3=4{\cdot}0$°C

$\Delta\theta_4=3{\cdot}2$°C $\quad\Delta\theta_5=2{\cdot}2$°C

The increments $\Delta\theta_1$, $\Delta\theta_2$, $\Delta\theta_3$, $\Delta\theta_4$ and $\Delta\theta_5$ are plotted against the temperature rises 20, 26·6, 31·8, 35·8 39 °C respectively. These are shown as points *J, K, L, M* and *N* on the diagram. A straight line is drawn through these points intersecting the *Y*-axis at *Q* (at 50°C) and therefore $\theta_m=50$°C.

A tangent is drawn to the curve at point *A* intersecting the final steady temperature line *QBC* at *C*. A perpendicular is drawn from *A* on the final steady temperature rise line intersecting it at *B*.

Heating time constant $T_h=BC=61$ minutes.

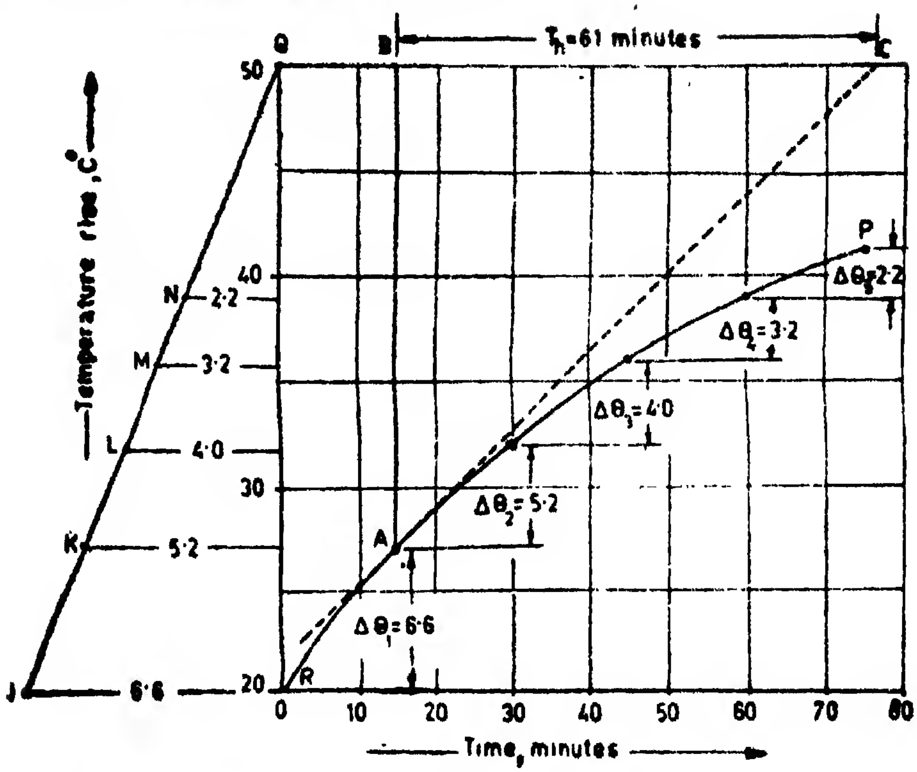


Fig. 3·39.

3·22. Rating of Machines. The rating of machines refers to the whole of the numerical values of electrical and mechanical quantities with their duration and sequences as signed to the machines by the manufactures and stated on the rating plate, the machine complying with the specified conditions. The duration of the sequence may be indicated by the qualifying term.

The assignment of the rating is to be made by the manufacturer to specify the capabilities of the machine. Irrespective of whether the machine carries an indication of the duty type, it shall carry a plate giving the values of quantities assigned to it by the machine manufacturer. In addition, since electrical machines have a time rate of temperature-rise and since the rise of temperature is limited according to standards, a qualifying term may be used with the term rating to give the duration for which the machine may be run at the assigned values while complying with the standards. Where a machine is manufactured for general purposes, it is capable of supplying its power rating indefinitely and the qualifying term signifies this. Where a machine is manufactured with the intention that it may be used to supply varying loads or loads including periods of no-load or where the machine may be in a state of rest and de-energized, the qualifying term will signify this.

3·23. Selection of Motor Power Ratings. The selection of power ratings of motors for electric drives is a matter of economic and operational interest in any industry. This is because the proper selection of motor has a predominant effect on both capital and running costs of electric drives. A motor of insufficient capacity does not operate the drive satisfactorily on account of low output and efficiency. Since the motor is overloaded, it has a shorter life span and also a possibility of burn out on account of execessive temperature rise.

On the other hand if a motor of higher power rating is used, the motor is underutilised and therefore the economic efficiency of the installation is reduced and the drive becomes expensive and has large energy losses. **Over motoring** (using a motor of higher rating than is required by load) leads to higher capital costs and increased losses because of lower efficiency at reduced load. In a.c. drives, motors working at reduced loads lead to poor power factor leading to uneconomic loading of supply circuits and apparatus.

In order to select the motor power rating properly, it is not only necessary to know the load under steady state conditions but also the loads that are met with under transient conditions. For this purpose use is made of **Load Diagrams (Time sequence graphs)** which show the variation of motor torque, power and load current as function of time.

The motors selected according to a given load diagram, should be fully loaded and in no case their temperature rise exceed the maximum permissible as specified by the manufacturer. In addition, the motors selected should operate properly during periods of overload and possess sufficient starting torque to accelerate the driven load up to the desired speed within the required time.

In most of the situations, motors are selected on the basis of their heating since temperature rise of the motor is the prime factor in determining its life. The capacity of the motor is then checked for overload capacity. Motor power capacity is selected in accordance with the work the motor is desired to perform simultaneously ensuring that the motor will operate with permissible limits of mechanical loading.

Since the motors operate under diverse operating (loading) conditions, it is pertinent here to define the **Rating** of a motor. The **Rating** of a motor is the power output or the designated operating power limit based upon certain definite conditions assigned to it by the manufacturer. An electrical motor is normally rated upon thermal basis of temperature rise *i.e.* maximum possible temperature at which the insulating materials may be operated without deterioration. The types of ratings are defined for electric motors depending upon the load and duration.

In majority of industrial and other applications, motors carry constant load continuously. However, in many cases the motor operation is often stopped before the machine reaches its final steady temperature rise, or the load is reduced and the temperature is lowered on account of reduced losses.

3·24. Types of Duties and Ratings.

The following are the types of duty as per IS : 4722-1968 "Specification for Rotating Electric Machinery."

(*i*) S_1 : continuous duty (*ii*) S_2 : short time duty

(*iii*) S_3 : Intermittent periodic duty

(*iv*) S_4 : Intermittent periodic duty with starting

(*v*) S_5 : Intermittent periodic duty with starting and braking

(*vi*) S_6 : Continuous duty with intermittent periodic loading

(*vii*) S_7 : Continuous duty with starting and braking

(*viii*) S_8 : Continuous duty with periodic speed changes.

3·24·1. Continuous Duty (Duty Type S_1). On this duty the duration of load is for a sufficiently long time such that all the parts of the motor attain thermal equilibrium *i.e.* the motor attains its maximum final steady temerature rise. Examples of drives with continuous duty are continuously running fans, pumps and other equipment which operate for several hours and even days at a time. The simplified load diagram for this duty is a horizontal straight line as shown in Fig. 3·40. The **continuous rating** of a motor may be defined as the load that may be carried by the machine for an indefinite time without the temperature rise of any part exceeding the maximum permissible value.

3·24·2. Short Time Duty (Duty Type S_2) : The motor operates at a constant load for some specified time which is then followed by a period of rest. The period for load is so short that the machine cannot reach its thermal equilibrium *i.e.* steady temperature rise while the period for rest is so long that the motor temperature drops to the ambient temperature. Railway turntable, navigation lock gates are some examples of the drives which operate on short time duty. The simplified diagram for short time duty is shown in Fig. 3·41.

The **short time rating** of a motor may be defined as its output at which it may be operated for a certain specified time without exceeding the maximum permissible value of temperature rise. The period of operation is so short that the temperature rise of the motor does not reach its final steady value and the period of rest is so long that the motor returns to cold conditions.

Standard short time ratings are : 10, 30, 60 and 90 minutes.

3·24·3. **Intermittent Periodic Duty (Duty type S_3)** : On intermittent duty the periods of constant load and rest with machine de-energized alternate. The load periods are too short to allow the motor to reach its final steady state value while periods of rest are also too small to allow the motor to cool down to the ambient temperature. This type of duty cycle is encountered in cranes, lifts and certain metal cutting machine tool drives. The simplified load for this type of duty is shown in Fig. 3·42.

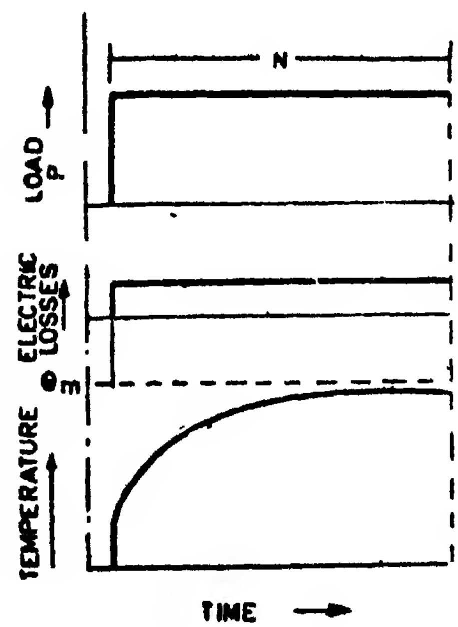
For the evaluation of intensity of heating due to intermittent period loads, use is made of duty factor. The **Duty factor** (also called **Load factor** or **Cyclic duration factor**) is generally defined as the ratio of the heating (working) period to the period of whole cycle.

$$\text{Duty factor } \varepsilon = \frac{t_h}{t_h + t_e} \qquad \ldots(3{\cdot}67)$$

where t_h=heating period and t_e=period of rest. The duty factor is determined on the basis of a cycle 10 minutes long.

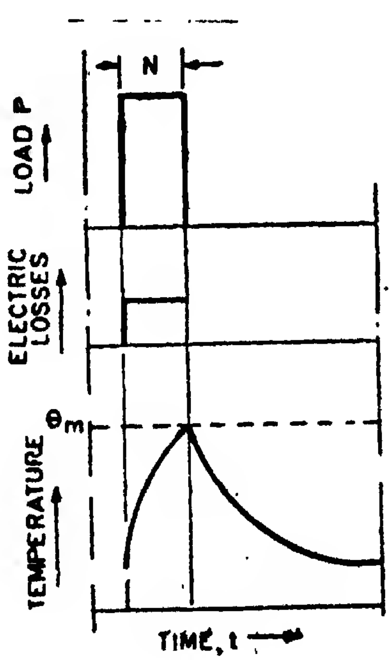
The **Intermittent Rating** of a motor applies to an operating condition during which short time load periods alternate with periods of rest or no load without the motor reaching the thermal equilibrium and without the maximum temperature rising above the maximum permissible value. In this duty the current does not significantly affect the temperature rise. The duty factor for this operation is :

$$\varepsilon = \frac{N}{N+R} \qquad \ldots(3{\cdot}68)$$



N=operation under rated conditions, s ;
θ_m=maximum temperature, °C.

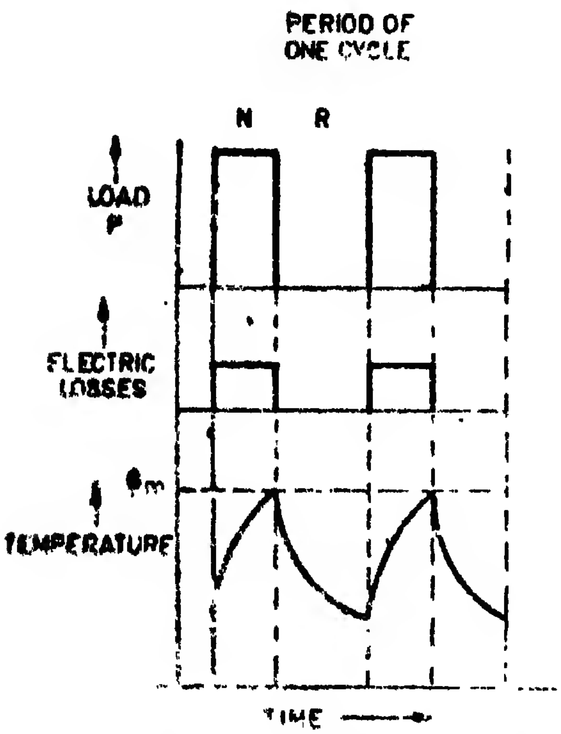
Fig. 3·40. Continuous duty. Duty type S_1



N=operation under rated conditions, s ;
θ_m=maximum temperature attained during duty cycle, °C.

Fig. 3·41. Short time duty. Duty type S_2.

3·24·4. Intermittent Periodic Duty with Starting (Duty Type S_4). This type of duty consists of a sequence of identical duty cycles each consisting of a period of starting, a period of operation at constant load and a rest period, the operating and rest periods are too short to obtain thermal equilibrium during one duty as shown in Fig. 3·43.

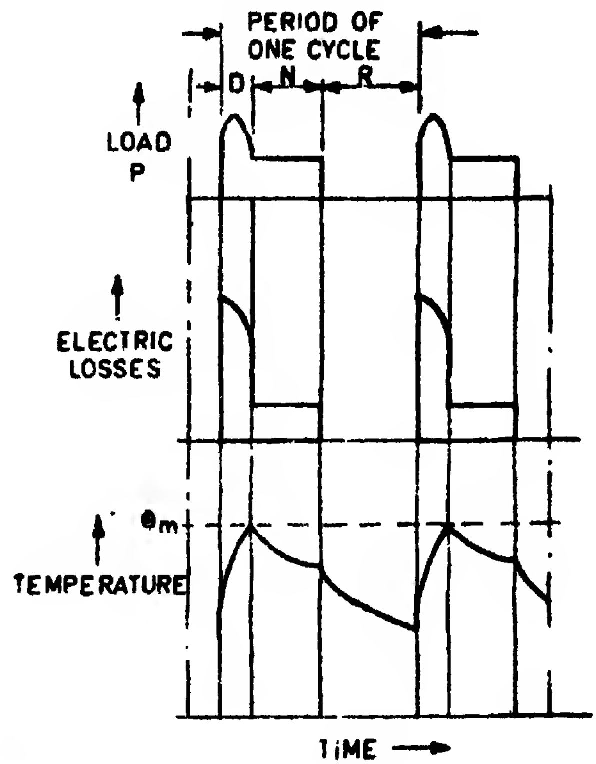


N=operation under rated conditions, s ;

R=operation at rest and de-energized, s, ;

θ_m=maximum temperature attained during the duty cycle, °C

Fig. 3·42 Intermittent periodic duty. Duty type S_3



D=starting period, s ;

N=operation under rated conditions, s ;

R=at rest and de-energized, s ;

θ_m=maximum temperature attained during the duty cycle, °C

Fig. 3·43. Intermittent periodic duty with starting. Duty type S_4.

In this duty the stopping of the motor is obtained either by natural decelaration after disconnection of the electric supply or by means of breaking such as mechanical brake which does not cause additional heating of windings.

The duty factor is given by :

$$\epsilon=\frac{D+N}{D+N+R} \quad \ldots(3\cdot69)$$

3·24·5. Intermittent Periodic Duty with Starting and Braking (Duty Type S_5). This type of duty consists of a sequence of identical duty cycles each consisting of a period of starting, a period of operation at constant load, a period of braking and a rest period. The operating and rest periods are too short to obtain thermal equilibrium during one duty cycle as shown in Fig. 3·44. In this duty braking is rapid and is carried out by electrical means.

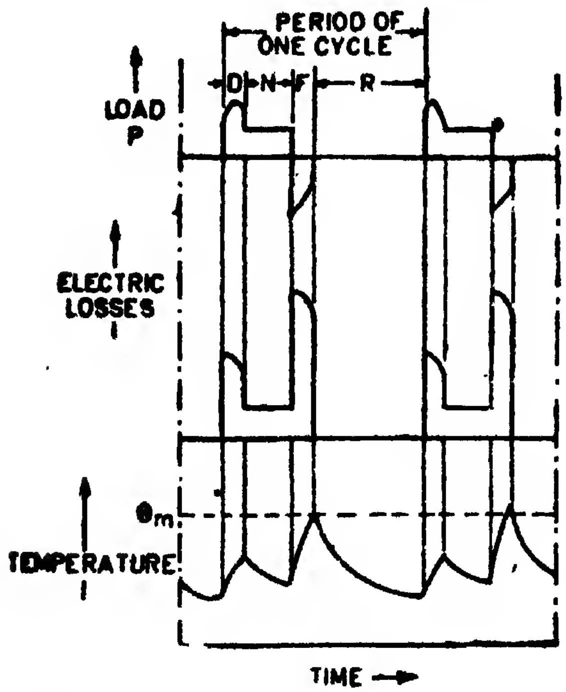
The duty factor is :

$$\varepsilon = \frac{D+N+F}{D+N+F+R} \quad \ldots(3.70)$$

3·24·6. Continuous Duty with Intermittent Periodic Duty (Duty Type S_6). This type of duty consists of a sequence of identical duty cycles each consisting of a period of operation at constant load and period of operation at no load. The machines with excited windings have normal no load voltage excitation during the load period. The operation and no load periods are too short to attain thermal equilibrium during one cycle as shown in Fig. 3·45.

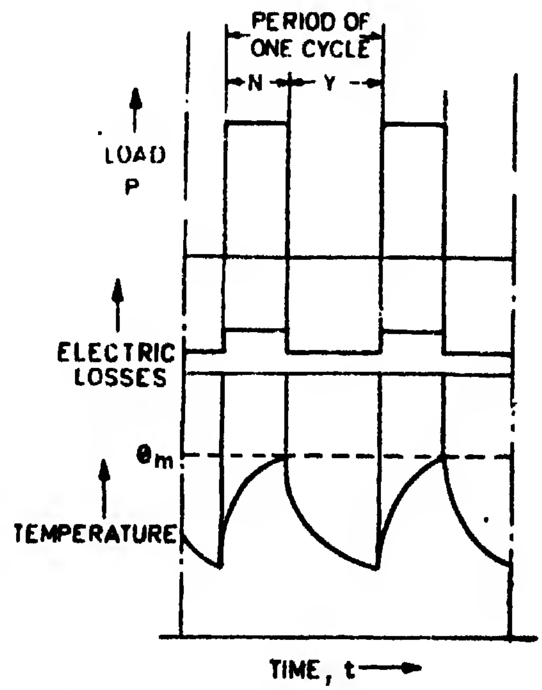
The duty factor is given by :

$$\varepsilon = \frac{N}{N+V} \quad \ldots(3.71)$$



D=Starting time, s ;
N=Operating under rated conditions, s ;
F=braking period, s ;
R=Rest and de-energized period, s ;
θ_m=maximum temperature attained during duty cycle, °C

Fig. 3·44. Intermittent periodic duty with starting and braking duty (Duty type S_5).



N=operation under rated conditions, s ;
V=operation on no load, s ;
θ_m=maximum temperature attained during the duty cycle

Fig. 3·45. Continuous duty with intermittent periodic duty (Duty type S_6)

Unless and otherwise specified the duty cycle is 10 minutes. The recommended duty factors are 15, 25, 40 and 60 percent.

3·24·7. Continuous Duty with Starting and Braking (Duty Type S_7). This type of duty consists of a sequence of identical duty cycles each having a period of starting, a period of operation at constant load and a period of electric braking. There is no rest or de-energized period. The load diagram is shown in Fig. 3·46.

The duty factor for this duty cycle is 1.

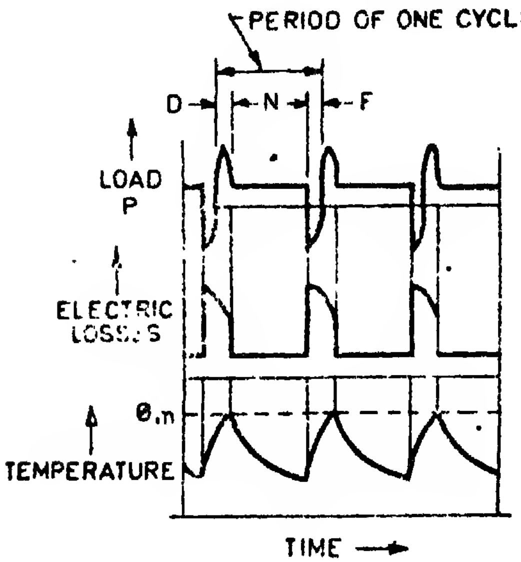
3·24·8. Continuous Duty with Periodic Speed Changes (Duty Type S_8). This type of duty consists of a sequence of identical duty cycles each consisting of a period of operation at constant load corresponding to a determined speed of rotation, followed immediately by a period of operation at another load corresponding to a different speed of operation. The operating period is too short to attain thermal equilibrium during one duty cycle there being no rest and de-energized period. This duty cycle is shown in Fig. 3·47.

The various duty factors are :

$$\varepsilon=\frac{D+N_1}{D+N_1+F_1+N_2+F_2+N_3} \quad \ldots(3\cdot72)$$

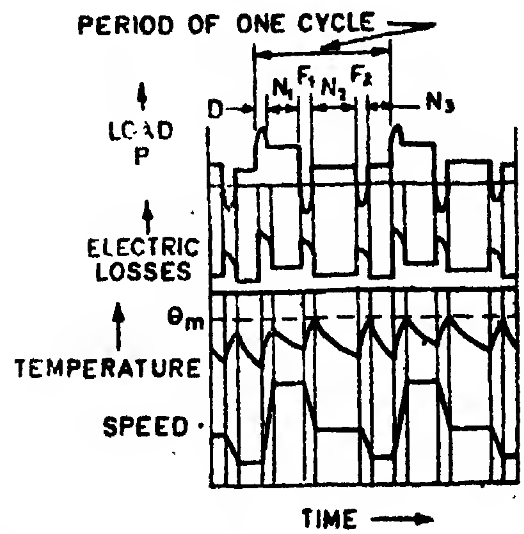
$$=\frac{F_1+N_2}{D+N_1+F_1+N_2+F_2+N_3} \quad \ldots(3\cdot73)$$

$$=\frac{F_2+N_3}{D+N_1+F_1+N_2+F_2+N_3} \quad \ldots(3\cdot74)$$



D=starting period, s ;
N=operation under normal conditions, s ;
F=period of electric braking s, ;

Fig 3·46. Continuous duty with starting and electric braking. (Duty type S_7)



F_1, F_2=braking periods, s ;
D=acceleration period, s ;
N_1, N_2, N_3=operation under rated conditions, s ;
θ_m=maximum temperature attained during duty cycle, °C

Fig. 3·47. Continuous duty with periodic speed changes (Duty type S_8)

3·25. Ambient Temperature and Ratings. The operating temperature of a motor should never exceed the maximum permissible temperature because it will result in deterioration and breakdown of insulation and will shorten the service life of motors.

In order to simplify the heating calculations, it is general practice to base the motor power ratings on a standard value of temperature say 35°C. Accordingly, the power given on the name plate of a motor corresponds to the power which the motor is capable of delivering without overheating at an ambient temperature of 35°C. At an ambient temperature considerably below 35°C, the motor can safely deliver a some what higher output than its rating because there is a greater difference between the ambient temperature and the maximum allowable temperature of insulation used in the motor. Similarly at ambient temperatures greater than 35°C, the load on the motor

should be smaller than that indicated on its name plate unless of course special measures are taken to improve the cooling to bring down the temperature rise.

The duty cycle is closely related to temperature and is generelly taken to include the enviromental factors also. A 100 kVA machine (intermittent rating) might be converted to a 200 kVA machine if continuously operated at North pole with an ambient temperature of —30°C. This is because all the heat generated would still be insufficient to make the temperature exceed the maximum allowable under such cold ambient conditions.

3·26. Overload Capacity of Motors. The rating of a machine can be determined from heating considerations. However the motor so selected should be checked for its overload capacity and starting torque. This is because the motor selected purely on the basis of heating may not be able to meet the mechanical requirements of the load to be driven by it. Table 3.5 lists the co-efficients used to determine the permissible instantaneous overload torque of different electric motors.

Table 3 5. Motor instantaneous torque overload co-efficients

S. No.	*Type of Motor*	*Torque overload co-efficient*
1	D C. motors	2 (for special types upto 3 or 4)
2	Squirrel cage induction motors	2—2·5
3	Double cage and deep bar squirrel cage motors	1.8—3
4	Synchronous motors	2—2.5 (For special types upto 3—4)
5	A.C. commutator motors	2—3

In the case of d.c. machines the maximum overload capacity is determined on the basis of commutation, while in the case of synchronous and induction motors it is determined by maximum electro-magnetic torque.

3·27. Selection of motor capacity for continuous rating. The majority of electric machines used in drives operate continuously at a constant or only slightly variable load. The selection of the motor capacity for these applications is fairly simple in case the approximate constant power input is known. For such applications there is no necessity to check the motor for the possibility of overheating or overloading during operation. Selection of the motor for a rating equal to the known required power input presents no difficulty because it is sure that this rating is the maximum permissible with respect to heating as the manufacturer designs and rates the motor so as to attain maximum utilization of the materials in it at rated output.

Although, the losses during the starting period of a motor are greater than those under rated load on account of increased value of current drawn during starting. They (losses) may be neglected during starting period because starting under continuous duty conditions is not so frequent and also the starting time is very short, and hence the starting has practically no influence on motor heating. However, in certain cases it becomes necessary to check the motor for sufficient starting torque when it is known that the machine concerned has a high breakaway torque or a high inertia.

For continuous duty at constant or slightly varying loads a motor is selected referring to the catalogue of standard rating motors. In case the rating listed in the catalogue is slightly less than the required one, motor of next higher rating is selected.

In many applications the power input required for a motor is not known beforehand and therefore certain difficulties arise in such cases. The power input required for normal

machines with continuous load such as pumps, fans and machine cutting tools, can be calculated theoretically or with the help of empirical formulae and co-efficients obtained from experimental results and data acquired over the years of using similar types of machines. For the determination of ratings of machines whose load characteristics have not been thoroughly studied, it becomes necessary to determine the load diagram (A **load diagram** portrays the variation of power output versus time). Sometimes, statistical methods are used to ascertain the input power requirement of a machine.

The power requirement for continuous duty can be calculated theoretically for only a relative small group of machines. Typical representatives of this group are pumps and fans.

The temperature of the motor changes continuously when the load is variable. On account of this, it becomes difficult to select the motor rating according to the heating. The analytical study of heating becomes highly complicated if the load diagram is irregular in shape or when it has a large number of steps. Therefore, it becomes extremely difficult to select the motor capacity through analysis of the load diagram due to lack of accuracy of this method. On the other hand it is not correct to select the motor according to the lowest or highest load because the motor would be overloaded in the first case and underloaded in the second case. It may look practicable to select the motor on the basis of average value of the load, say the average current or average power. This is also incorrect because the average ordinate of the load diagram does not take into account the dependence of variable losses upon the square of the motor current. The motor should be selected on the basis of average load only when the load fluctuations are comparatively small.

In case the fluctuations in the load are considerable, any motor selected on the basis of average power load will be of insufficient capacity. Therefore, the motor so selected is over loaded and consequently has a short service life or premature failure.

3·27. Methods used for determination of motor rating for variable load drives. It is clear from the above discussion that it is necessary to use suitable methods to calculate the proper rating of motors for variable load drives. The four commonly used methods are :

(*i*) method of average losses, (*ii*) equivalent current method,

(*iii*) equivalent torque method, and (*iv*) equivalent power method.

3·27·1. Method of average losses. The method consists of finding average losses Q_{av} in the motor when it operates according to the given load diagram. These losses are then compared with Q_{nom}, the losses corresponding to the continuous duty of the machine when operated at its nominal rating. The method of average losses presupposes that when $Q_{av}=Q_{nom}$, the motor will operate without temperature rise going above the maximum permissible for the particular class of insulation.

$$Q_{per}=\frac{Q_{nom}}{S\lambda} \qquad \text{...(3·75)}$$

and therefore

$$Q_{m}=Q_{av}/S\lambda \qquad \text{...(3·76)}$$

$$Q_{m}=Q_{per}=\frac{Q_{av}}{S\lambda}=\frac{Q_{nom}}{S\lambda} \qquad \text{...(3·77)}$$

Fig. 3·48 shows a simplified load diagram for a certain drive. The loss diagram (losses Q plotted versus time) of the electric motor is shown. The rating of the electric motor can be found from method of successive approximations. To start with a motor is selected in accordance with the required capacity calculated by multiplying the average power load

a factor of safety of 1·1 to 1·3. The losses of the motor are calculated for each portion of the load diagram by referring to the efficiency curve of the motor. The average losses are given by

$$Q_{av}=\frac{Q_1t_1+Q_2t_2+Q_3t_3+\ldots\ldots+Q_nt_n}{t_1+t_2+t_3+\ldots+t_n} \qquad \ldots(3\cdot78)$$

The average losses as found from Eqn. 3·78 are compared with the losses of selected motor at rated efficiency. In case, the two losses are equal or differ by a small amount the motor is selected. However, in case the losses differ considerably, another motor is selected and the calculations repeated till a motor having almost the same losses as the average losses, is found. It should be checked that the motor selected has a sufficient overload capacity and starting torque.

The method of average losses does not take into account the maximum temperature rise under variable load conditions. However, this method is accurate and reliable for determining the average temperature rise of the motor during one work cycle. No doubt the motor is subject to short time peaks in temperature rise, its practical life is unaffected. The disadvantage of the method of equal losses is that it is tedious to work with and also many a times the efficiency curve is not readily available and the efficiency has to be calculated by means of empirical formulae which may not be accurate to work with.

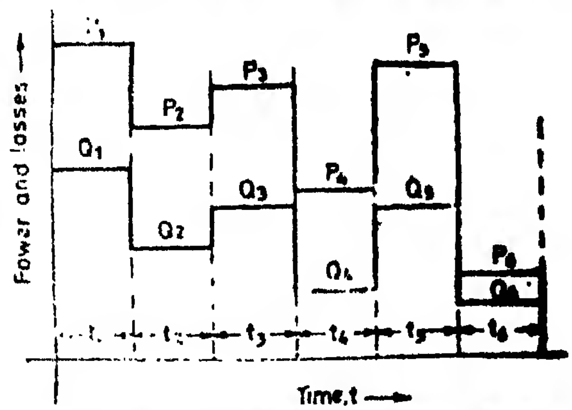


Fig. 3·48. A simplified power load diagram and loss diagram for variable load conditions.

3·28 2. **Equivalent current method**. The equivalent current method is based upon the assumption that the actual variable current may be replaced by an equivalent current I_{eq} which produces the same losses in the motor as the actual current.

$$\therefore \qquad I_{eq}=\sqrt{\frac{I_1^2t_1+I_2^2t_2+I_3^2t_3+\ldots+I_n^2t_n}{t_1+t_2+t_3+\ldots+t_n}} \qquad \ldots(3\cdot79)$$

The heating and cooling conditions in self ventilated machines depend upon its speed. At low speeds the cooling conditions are poorer than at normal speeds. Therefore, if the work cycle involves slow speed operation, it must be taken into consideration when using Eqn. 3·79

The equivalent current as found from Eqn. 3·79 should be compared with the rated current of the motor selected and the conditions $I_{eq}<I_{nom}$ should be met. (I_{nom} is the rated current of the machine).

The machine selected should also be checked for its overload capacity. For d.c. motors, *$I_{max}/I_{nom}<2$ to 2·5. In case of induction motors **$T_{max}/T_{nom}<1\cdot65$ to 2·75.

In case the overload capacity of the motor selected is not sufficient it becomes necessary to select a motor of higher power rating.

The equivalent current may not by easy to calculate especially in cases where the current load diagram is irregular as shown in Fig. 3·49. The equivalent current in such cases is calculated from the following expression.

$$I_{eq}=\sqrt{\frac{1}{\sum_1^n t}\int^{\Sigma t} i^2\,dt} \qquad \ldots(3\cdot80)$$

*I_{max}=maximum current during the work cycle.
**T_{max}=maximum load torque.
T_{nom}=torque of the torque at rated power and speed.

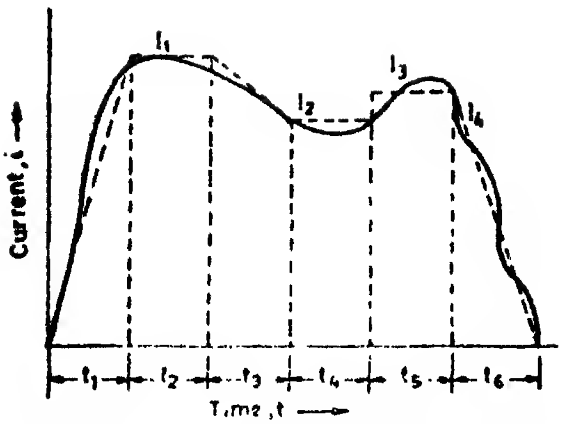


Fig. 3·49. Simplification of irregular current load diagram by short segments of straight lines.

The value of the integral may be found with the help of an integrator. An easier way is to break up the current load diagram into a series of straight line geometrical figures.

For a triangular shaped diagram :

$$I_{eq}=\sqrt{I_1/3} \qquad \ldots(3\cdot81)$$

For a trapezoidal diagram :

$$I_{eq}=\sqrt{\frac{I_1^2+I_1I_2+I_2^2}{3}} \qquad \ldots(3\cdot82)$$

The above method allows the equivalent current values to be calculated with accuracy sufficient for practical purposes.

3·28·3. Equivalent Torque and Equivalent Power methods. It often becomes necessary to use torque or power load diagrams for the selection of suitable capacity of motor. The equivalent torque or power is found in the same manner as the equivalent current.

The torque is directly proportional to current (assuming constant flux and constant power factor) and therefore the equivalent torque is :

$$T_{eq}=\sqrt{\frac{T_1^2t_1+T_2^2t_2+T_3^2t_3+\ldots+T_n^2t_n}{t_1+t_2+t_3+\ldots+t_n}} \qquad \ldots(3\cdot83)$$

The equation for equivalent torque follows directly from Eqn. 3·83 as power is directly proportional to the torque. At constant speed or where the changes in speed are small, the equivalent power is given by the following relationship :

$$P_{eq}=\sqrt{\frac{P_1^2t_1+P_2^2t_2+P_3^2t_3+\ldots+P_n^2t_n}{t_1+t_2+t_3+\ldots+t_n}} \qquad \ldots(3\cdot84)$$

The equivalent current method is the most accurate out of the three methods discussed above. This method may be used to determine the motor capacity for all uses except where it is necessary to take into account the changes in so called "constant losses" *i.e.* the iron and the mechanical losses.

The equivalent torque method cannot be used for cases where equivalent current method cannot be applied. The equivalent torque method cannot be used for selection of motor rating for cases in which the field flux does not remain constant like d.c. series motors and for squirrel cage induction motors under starting and braking conditions.

The disadvantage of the equivalent power method is that it cannot be used for motors whose speed varies considerably under load, especially when dealing with starting and braking conditions.

Example 3·28. *An induction motor has to perform the following duty cycle*

75 kW for 10 minutes,
No load for 5 minutes,
45 kW for 8 minutes,
No load for 4 minutes,

which is repeated indefinitely. Determine a suitable capacity of a continuously rated motor. Motors of standard (continuous) ratings of 45, 55, 75 kW are available. The ratio of maximum torque to nominal torque should be less than 1·8.

Solution. The capacity of continuously rated motor to perform the above duty cycle can be found by using equivalent power method.

From Eqn. 3·81, the capacity of motor is :

$$P=\left[\frac{P_1^2t_1+P_2^2t_2+P_3^2t_3+P_4^2t_4}{t_1+t_2+t_3+t_4}\right]^{1/2}$$

$$=\left[\frac{(75)^2\times 10+0\times 5+(45)^2\times 8+0\times 4}{10+5+8+4}\right]^{1/2}=51{\cdot}8 \text{ kW}.$$

A motor with a standard rating of 55 kW is selected. Since the induction motor is practically a constant speed motor, the ratio of maximum torque to nominal torque is equal to the ratio maximum power to nominal power.

$$\therefore \qquad \frac{T_{max}}{T_{nom}}=\frac{75}{55}=1{\cdot}36$$

which is less than the maximum allowable value of 1·8.

Example 3·29. *Determine the rated current of a transformer for the following duty cycle : 500 A for 3 minutes, a sharp increase 1000 A and constant at this value for 1 minute, gradually decreasing for 2 minutes to 200 A and constant at this value for 2 minutes, gradually increasing to 500 A during 2 minutes and the repetition of the cycle.*

Solution. The load diagram is plotted in Fig. 3·50.

The load diagram can be divided into rectangles and trapeziums (in this case) with the help of line segments. The equivalent current with sides I_1 and I_2 is :

$$I_{eq}=\left(\frac{I_1^2+I_1I_2+I_2^2}{3}\right)^{1/2}$$

$\therefore$ Equivalent current for the entire cycle is :

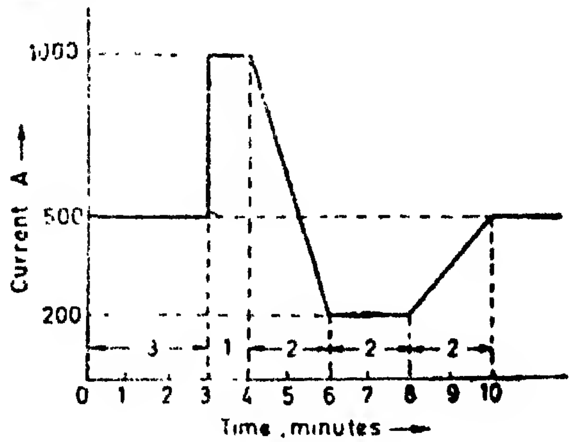


Fig. 3·50.

$$I_{eq}=\left[\frac{(500)^2\times 3+1000^2\times 1+(1/3)(1000^2+1000\times 200+200^2)\times 2}{3+1+2+2+2}\right.$$
$$\left.+\frac{200^2\times 2+1/3(200^2+200\times 500+500^2)\times 2}{3+1+2+2+2}\right]^{1/2}$$

$=537{\cdot}6$ A.

Example 3·30. *A motor driving a colliery winder has to deliver a load rising uniformly from zero to a maximum of 1500 kW in 20 s during the accelerating period, 750 kW for 40 s during the full speed period and during the deceleration period of 10 s, when the regenerative braking takes place the kW which is returned to the supply falls from an initial value of 250 to zero. The interval for decking (period of rest) before the next load cycle starts is 20 s. Estimate a suitable rating for the motor.*

Solution. The rating of a continuously rated motor to perform the given duty is found by using equivalent power method. The load diagram is shown in Fig. 3·51.

It is divided into two triangles and a rectangle.

From Eqn. 3·81, equivalent rating of motor

$$P=\left[\frac{1/3(1500)^2\times 20+(750)^2\times 40+1/3(250)^2\times 10}{20+40+10+20}\right]^{1/2}$$

$=648$ kW.

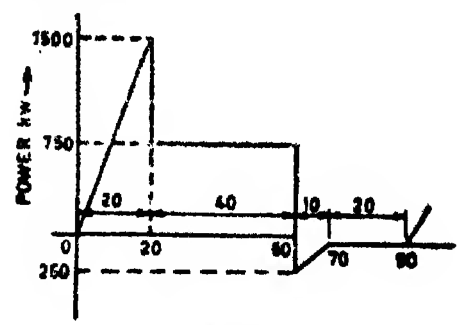


Fig. 3·51

Example 3·31. *The speed and torque required of a d.c. motor driving a mine winder vary during a cycle as shown in Fig. 3·52. Find the mean continuous rating of motor based upon (a) equivalent torque, (b) equivalent power.*

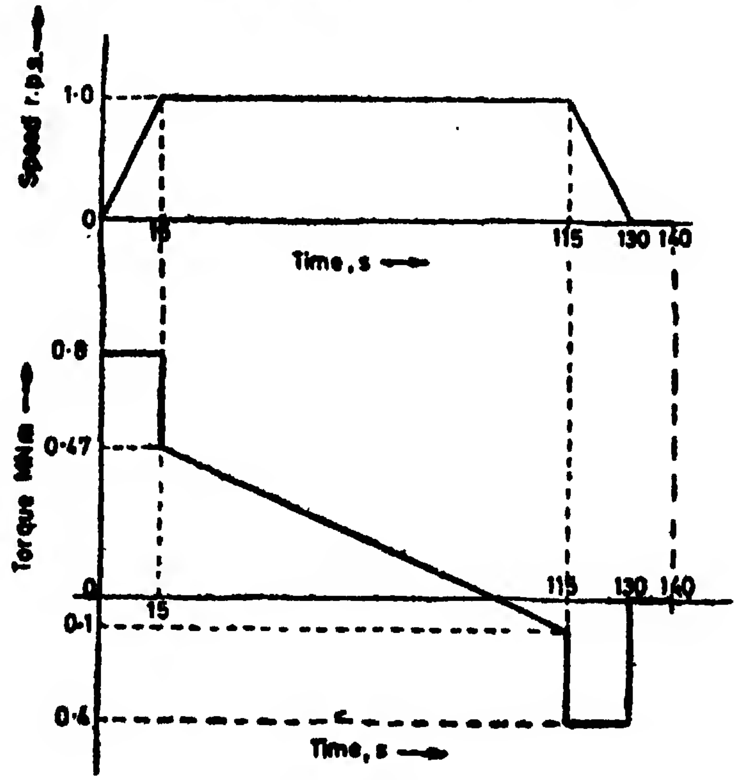


Fig. 3·52

Solution. (*a*) The load torque is divided into rectangles and a trapezium for finding the equivalent torque.

The equivalent torque is :

$$T_{eq}=\sqrt{\frac{1}{140}\left[0{\cdot}8\times15+\frac{(0{\cdot}47^2-0{\cdot}47\times0{\cdot}1+0{\cdot}1^2)\times100}{3}+(-0{\cdot}4)^2\times15\right]}$$

$=0{\cdot}36$ MNm $=0{\cdot}36\times10^6$ Nm

Power $P=T_{eq}\,\omega=0{\cdot}36\times10^6\times2\pi\times1$ W $=2260$ kW.

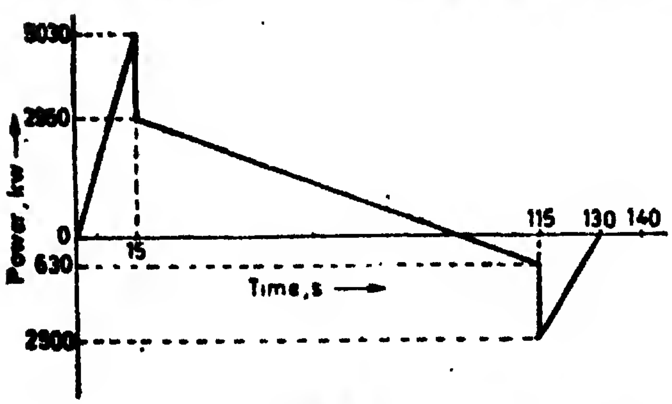


Fig. 3·53

(*b*) The power load diagram is drawn by calculating power from product of torque and angular speed at every instant.

For example power at $t=15$ s is

$0{\cdot}8\times10^6\times2\pi\times1$

$=5{\cdot}03\times10^6$ W $=5030$ kW.

The power load diagram is shown in Fig. 3·53.

The power load diagram is divided into two triangles and a trapezium.

Equivalent power $P_{eq}=\sqrt{\frac{1}{140}\left[\frac{(5030)^2\times15}{3}+\frac{(2950^2-2950\times630+630^2)\times100}{3}+\frac{(-2900)^2\times15}{3}\right]}$

$=1688$ kW.

3·29. Temperature Rise with Short Time Ratings. Suppose,

θ_m = final steady temperature rise with continuous rating. This may be assumed as maximum allowable temperature rise, θ_{per}.

T_h = heating time constant,

θ_m' = final steady temperature rise that would be attained if the machine were allowed to run for an indefinite time at its short time rating,

t_h = time for which the machine is run at its short time rating,

θ_{sh} = temperature rise with short time rating at $t=t_{sh}$.

From Eqn. 3·57, we can write

$$\theta_{sh}=\theta'_m(1-e^{-t_h/T_h}) \quad ...(3\cdot 85)$$

In order that the insulation of the machine is not operated beyond the specified temperature limit, the temperature rise at $t=t_h$ should not exceed the final steady temperature rise obtained for continuous operation of the machine
or θ_{sh} should not exceed θ_m.

Thus the maximum allowable temperature rise during short time rating is, $\theta_{sh}=\theta_m$.

Putting this value in Eqn. 3·85, we have :

$$\theta_m=\theta'_m(1-e^{-t_h/T_h})$$

or

$$\frac{\theta'_m}{\theta_m}=\frac{1}{1-e^{-t_h/T_h}}.$$

Obviously θ'_m is $\frac{1}{1-e^{-t_h/T_h}}$ times θ_m.

Thus the permissible losses in the machine during short time rating can also be as many times greater. Since $\frac{1}{1-e^{-t_h/T_h}}$ has a value greater than unity therefore with the given dimensions, the output of a machine with a short rating is larger than that of machine with continuous rating.

In other words, for a short time load t_h, a motor with a lower power rating should be selected so as to utilize it fully as regards its heating.

3·30. Selection of motor capacity for short time. The ratio of losses on short time duty to those corresponding to rated load on continuous duty, for the same motor temperature is called the **Heating Overload Ratio**.

Heating overload ratio $p_h=Q_{sh}/Q_{nom}$ ...(3·86)

where Q_{sh} = permissible losses on short time duty,

Q_{nom} = losses at rated load on continuous duty.

Since the steady state temperature rise is proportional to losses, the heating overload ratio may be written as :

$$p_h=\theta_m'/\theta_m \quad ...(3\cdot 87)$$

From Eqn. 3·85, $\theta_m'/\theta_m=1/1-e^{-t_hT_h}$ and therefore

$$p_h=\frac{1}{1-e^{-t_h/T_h}} \quad ...(3\cdot 88)$$

If heating overload ratio is known, it is possible to determine the time the motor may be allowed to carry the load. The permissible time for short time rating is

$$t_h=T_h \log_e \frac{p_h}{p_{h-1}} \quad ...(3\cdot 89)$$

The actual temperature rise-time curve differs from the theoretical relationship especially at the beginning of the heating period. Therefore in determining the capacity of a motor for short time duty, it becomes necessary to select the motor for a heating time constant whose value is smaller greater the overload of the motor during operation.

The **Mechanical Overload Ratio** is defined as :

$$p_m = P_{sh}/P_{nom} \qquad \ldots(3\cdot90)$$

where p_{sh}=permissible short time rating

and p_{nom}=continuous rating of the motor.

Suppose P_i are the constant losses and P_c are the variable losses for continuous duty at rated load. Therefore, we can write :

$$p_h = \frac{Q_{sh}}{Q_{nom}} = \frac{P_i + (P_{sh}/P_{nom})^2 P_c}{P_i + P_c}$$

$$= \frac{K + p_m^2}{K+1} \qquad \ldots(3\cdot91)$$

where $$K = \frac{\text{ratio of constant losses}}{\text{ratio of variable losses at rated load}} = \frac{P_i}{P_c} \qquad \ldots(3\cdot92)$$

$$\therefore \qquad p_m = \sqrt{(K+1)\,p_h - K} \qquad \ldots(3\cdot93)$$

If constant losses are assumed to be zero. K ...

$$\therefore \qquad p_m = \sqrt{p_h} \qquad \ldots(3\cdot94)$$

From Eqns 3·88 and 3·93, it can be written :

$$p_m = \left[\frac{K+1}{1-e^{-t_h/T_h}} - K\right]^{1/2} \qquad \ldots(3\cdot95)$$

Eqn. 3·95 makes it possible to find directly the mechanical overload ratio for a given loss factor K and per unit time value t_h/T_h. Knowing the mechanical overload ratio, the rating of machine for short time duty can be found.

Example 3·32. *An induction motor has a final steady temperature rise of 40°C when running at its rated output. Calculate its half hour rating for the same temperature rise if the copper losses at rated output are 1·25 times its constant losses. The heating time constant is 90 minutes.*

Solution Let P be the rated output and xP be the half hour rating. Also let P_c be the copper losses at full load and P_i be the other constant losses

$\therefore$ Total losses at full load$=P_c+P_i$

and total losses with short time rating$=x^2 P_c+P_i$.

Specific heat dissipation and surface remaining the same the temperature rise is proportional to the losses. Suppose θ_{msh} is the final steady temperature rise with short time rating.

$$\frac{\theta_m'}{\theta_m} = \frac{x^2 P_c + P_i}{P_c + P_i} = \frac{1\cdot25x^2 + 1}{2\cdot25} \quad \text{as } P_c = 1\cdot25\, P_i.$$

The temperature rise after ½ hour of operation at short time rating should not exceed $\theta_m = 40$°C. $\therefore$ From Eqn. 3·85

$$\theta_{sh} = \theta_m' \,(1 - e^{-t_h/T_h})$$

or $$40 = \theta_m\,[1 - e^{\frac{-1}{2\times 3/2}}] \quad \text{or} \quad \theta_m = 141\cdot1\text{°C}$$

$\therefore$ We have : $$\frac{1.25x^2+1}{2.25}=\frac{141.1}{40}$$

or $$x=2.36.$$

Thus ½ hour rating of the machine is 2·36 times its continuous rating.

Example 3·33. *During a temperature rise test at full load on a 100 kVA transformer, temperatures recorded were 60°C after one hour and 72°C after two hours. Find the time for which the transformer may safely be loaded to : 00 kVA. Ambient temperature is 40°C and full load copper loss (at 100 kVA) is twice the iron loss.*

Solution. Temperature rise after one hour$=60-40=20$°C.

Temperature rise after two hours$=72-40=32$°C.

We have $$\theta=\theta_m\ (1-e^{-t/T_h})$$

or $$20=\theta_m(1-e^{-1/T_h} \qquad \text{...(i)}$$

and $$32=\theta_m=(1-e^{-2/T_h}) \qquad \text{...(ii)}$$

From (*i*) and (*ii*) we get :

$$\theta_m=50°\text{C} \quad \text{and} \quad T_h=1.96 \text{ hours.}$$

Short time rating. The short time rating of the transformer is 2 times its continuous rating. Therefore, the full load copper loss with short time rating is 4 times the copper loss with continuous rating. Suppose θ_m' is the maximum steady temperature rise with short time rating.

$$\therefore \qquad \frac{\theta_m'}{\theta_m}=\frac{\text{loss with short time rating}}{\text{loss with continuous rating}}=\frac{4P_c+P_i}{P_c+P_i}=\frac{9P_i}{3P_i}=3$$

or $$\theta_m'=3\theta_m=3\times 50=150°\text{C}.$$

Now suppose the machine is allowed to run for t_{sh} time on its short time rating. This time t_{sh} is such that the temperature does not exceed $\theta_m=50$°C

We can thus write, $50=150(1-e^{-t_h/1.96})$

$\therefore$ Time for short time rating $t_h=0.795$ hour.

Example 3·34. *If one hour rating of a machine is $\sqrt{3}$ times the continuous rating and if steady temperature rise for one hour rating is twice that on normal load. Find the ratio of iron to copper loss at full load.*

Solution. Let one hour rating of transformer be x times its continuous rating.

Losses with continuous rating$=P_c+P_i$.

Losses with short time rating$=x^2P_c+P_i$.

Let θ_m and θ_m' be the final steady temperature rises with continuous and 1 hour rating respectively

$$\therefore \qquad \frac{\theta_m'}{\theta_m}=\frac{P_i+x^2P_c}{P_i+P_c}$$

$$\frac{\theta_m'}{\theta_m}=2 \text{ and } x=\sqrt{3}$$

$$\therefore \qquad 2=\frac{P_i+3P_c}{P_i+P_c} \text{ or } P_i=P_c.$$

Thus iron losses are equal to full load copper loss for continuous rating.

Example 3·35. *Half hour rating of a motor is 37 5 kW. The heating time constant is 90 minutes, find the heating and mechanical overload ratios and from there find the continuous rating of the motor. The maximum efficiency of the motor occurs at 70% full load.*

Solution. From Eqn. 3·88, heating overload ratio

$$p_h=\frac{1}{1-e^{-t_h/T_h}}=\frac{1}{1-e^{-30/90}}=3{\cdot}5.$$

Maximum efficien y occurs at 70% full load.

∴ Constant losses $P_i=(0{\cdot}7)^2P_c=0{\cdot}49\ P_c$

where P_c=copper loss at full load

Now, $K=P_i/P_c=0{\cdot}49$

From Eqn. 3·93 mechanical overload ratio

$$p_m=\sqrt{K+1)\,p_h-K}$$

$$=\sqrt{(0{\cdot}49+1)\times 3{\cdot}53-0{\cdot}49}=218$$

∴ Continuous rating of motor

$$P_{nom}=\frac{P_{sh}}{p_m}=\frac{37{\cdot}5}{2{\cdot}18}=17{\cdot}2\ \text{kW}.$$

Example 3·36. *A 500 kVA transformer has a total loss of 7·5 kW at full load. The rate of heat dissipation from tank walls is 300 W/C° rise and the heat energy required to raise its temperature by 1°C is 0·45 kWh, calculate (i) the final steady temperature rise and thermal time constant of transformer, (ii) half hour rating of the transformer to give the same temperature rise as in (i) if the copper loss at full load (500 kVA) is twice the iron loss.*

Solution. It is given that

$$S\lambda=300\ \text{W/°C}\ ;\quad Gh=450\ \text{Wh and } Q=7500\ \text{W}.$$

(*i*) Final steady temperature rise

$$\theta_m=\frac{Q}{S\lambda}=\frac{7500}{300}=25\text{°C}.$$

Heating time constant

$$T_h=\frac{Gh}{S\lambda}=\frac{450}{300}=1{\cdot}5\ \text{hour}.$$

(*ii*) Final steady temperature rise with short time rating

$$\theta_m'=\frac{1}{\theta_m[1-e^{-t_h/T_h}}=\frac{1}{25[1-e^{-1/(2\times 1\cdot 5)}]}=88{\cdot}1\text{°C}$$

Let the ½ hour rating be x times continuous rating.

$$\therefore\quad \frac{\text{Loss with half hour rating}}{\text{Loss with continuous rating}}=\frac{\theta_m'}{\theta_m}$$

or

$$\frac{x^2P_c+P_i}{P_c+P_i}=\frac{88{\cdot}2}{25}$$

But $P_c=2P_i$

$$\therefore\quad \frac{2x^2+1}{3}=\frac{88{\cdot}3}{25}\ \text{ or } x=2{\cdot}19.$$

∴ Half hour rating of transformer=2·19×500=1095 kVA.

3·31. Temperature Rise of Machines with Intermittent Short Time Ratings. Suppose a machine starts its intermittent short time operation from cold state (temperature rise=0). Let the operating time of the machine (when it is being heated) be t_h and the

pause time (when it is being cooled) be t_c. The temperature rise of the machine during the first operation will be given by Eqn. 3·57. When the pause period starts, the machine cools and the cooling takes place according to the Eqn. 3·58. During the second and subsequent

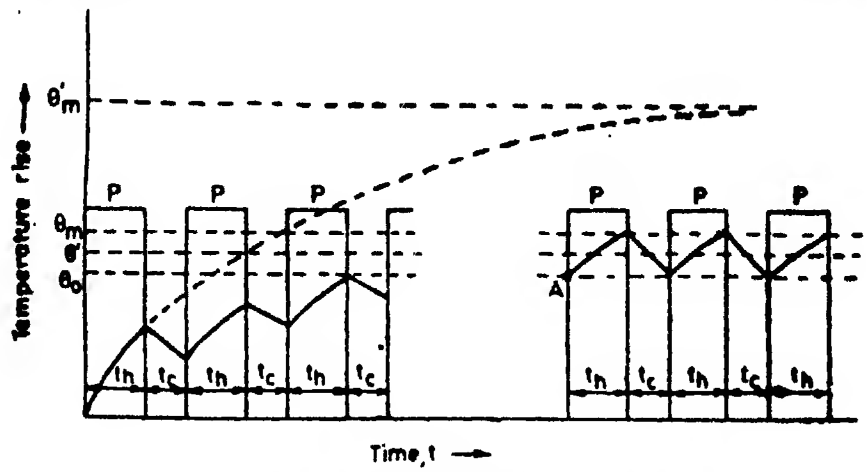


Fig. 3·54. Intermittent short time rating.

cycles, the heating takes place according to Eqn. 3·56 and the cooling, according to Eqn. 3·58. Proceeding as above, we get a curve for the intermittent rating. After some time, the above duty becomes practically uniform, and temperature rise of the machine varies within the range θ_m and θ_o as shown in Fig. 3·54. This shows that after an indefinite number of operations, the rise in temperature after heating becomes equal to fall in temperature after cooling.

t_h=heating period, t_c=cooling period,

T_h=heating time constant, T_c=cooling time constant,

θ_m'=final steady temperature rise if the machine were allowed to heat indefinitely

θ_n'=final steady temperature rise if the machine were allowed to cooling indefinitely

θ_m=upper temperature rise limit,

θ_o=lower temperature rise limit.

Considering any point A, (Fig. 3·54) where the duty becomes uniform. From Eqn. 3·55,

Heating time between θ_o and θ_m,

$$t_h = -T_h \log_e \frac{\theta_m' - \theta_m}{\theta_m' - \theta_o}$$

Similarly, cooling time between θ_m and θ_o,

$$t_c = -T_c \log_e \frac{\theta_o - \theta_n'}{\theta_m - \theta_n'}$$

When t_c and t_h are small as compared with cooling and heating time constants, the slopes of rising and falling curves may be considered as uniform over the interval $\theta_1 - \theta_2 = d\theta$ or in other words the heating and cooling curves may be taken as rectilinear over this period.

Let θ'=mean temperature rise$=(\theta_m+\theta_o)/2$

The slope of heating curve at θ', (by differentiating Eqn. 3·56 and putting $t=0$ and $\theta_i=\theta'$)

$$\frac{d\theta}{dt}=\frac{\theta_m'-\theta'}{T_h}$$

$\therefore$ Temperature difference produced in time t_h,

$$\theta_m-\theta_n=t_h\frac{d\theta}{dt}=t_h\frac{\theta_m'-\theta'}{T_h} \qquad ...(3·96)$$

Similarly the temperature difference produced in time t_c,

$$\theta_m-\theta_n=t_c\frac{d\theta}{dt}=t_c\frac{(\theta'-\theta_n)}{T_c} \qquad ...(3·97)$$

From Eqns. 3·96 and 3·97

$$t_h\frac{\theta_m-\theta'}{T_h}=t_c\frac{\theta'-\theta_n}{T_c}$$

or mean temperature rise, $\theta'=\dfrac{\theta_m\, t_h/T_h+\theta_n\, t_c/T_c}{t_h/T_h+t_c/T_c}$ ...(3·98)

Eqn. 3·96 should be used only when the heating and cooling periods are quite small as compared with the corresponding thermal time constants.

Let us consider the case of ratings of motors working intermittent periodic duty. The curves for temperature rise with this duty are shown in Fig. 3·46. The load diagram is seen to consist of regularly repeated intervals of constant load P having a constant duration.

It is wrong to select a motor of continuous rating P because it will be under-utilized. If the capacity of the motor is correctly selected, the maximum temperature rise does not reach θ_m' (greater than the permissible) but tends to approach a value θ_m (which is equal to maximum permissible value of temperature) after a large number of cycles. It is precisely on this basis that the capacity of the motor meant for intermittent periodic duty is selected. The temperature rise fluctuates between the limits θ_m and θ_n.

In order to find the rating of intermittent periodic duty, the following analysis is done. This analysis assumes that the heating time constant T_h equals the cooling time constant T_c. For the heating period :

$$\theta_m=\theta_m'\,(1-e^{-t_h/T_h})+\theta_n\, e^{-t_h/T_h}$$

and for cooling period :

$$\theta_n=\theta_m\, e^{-t_c/T_h} \qquad (\text{as } T_c=T_h \text{ as assumed})$$

From above,

$$\theta_m=\theta_m'\,(1-e^{-t_h/T_h})+\theta_m\, e^{-(t_h+t_c)/T_h}$$

Dividing by θ_m and putting $p_h=\theta_m'/\theta_m$

we have :
$$1=p_h\,(1-e^{-t_h/T_h})+e^{-(t_c+t_h)/T_h}$$

Thus we get, heating overhead ratio

$$p_h=\frac{1-e^{-(t_h+t_c)/T_h}}{1-e^{-t_h/T_h}}=\frac{1-e^{\frac{-t_h}{\epsilon T_h}}}{1-e^{-t_h/T_h}} \qquad ...(3·99)$$

If $t_c=\infty$, which corresponds to a short time duty, Eqn. 3·99 reduces to :

$$p_h=\frac{1}{1-e^{-t_h/T_h}}.$$

Fig. 3·55 shows the variation p_h with duty factor for various values of t_h/T_h. The points where the curves end the p_h axis, and where $\varepsilon=0$, correspond to short time duty. A characteristic point is that with coordinates $\varepsilon=1$ and $p_h=1$. Irrespective of the value of t_h/T_h all the curves converge on this point because this point represents the continuous operation. It is clear from Fig. 3·55 that when $\varepsilon>0.6$, the value of p_h is quite small and mechanical overload ratio is even less and therefore when the duty factor $\varepsilon>0.6$, it is necessary to select the motor for a capacity rating equivalent to continuous duty.

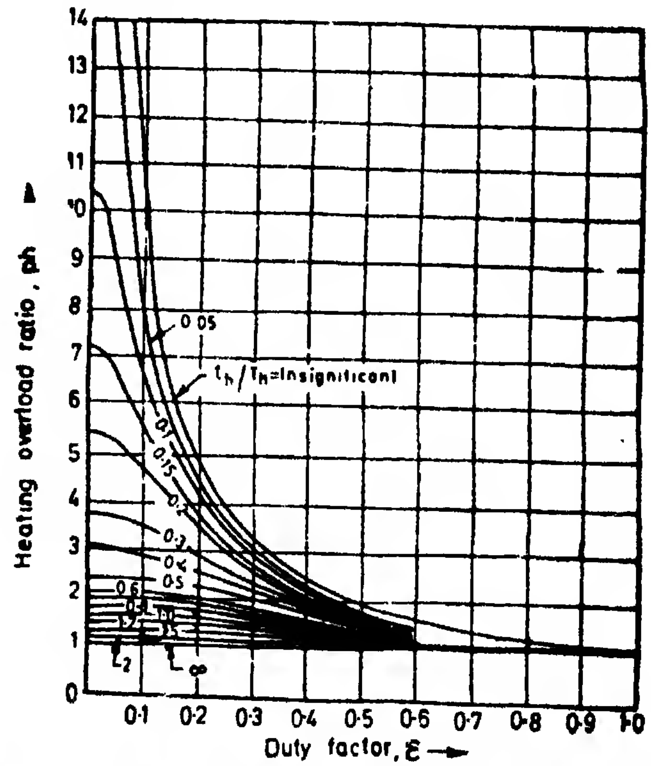


Fig. 3.55. Variation of heating overload ratio with duty factor for different values of t_h/T_h.

The duty factor for motors for intermittent duty are 0·15, 0·25, 0·4 and 0·6.

3·32. Selection of Motor Capacity for Short Time Ratings. It has been explained earlier that a motor operated on a short time duty can be overloaded more than when operated on continuous duty. The value of overload depends upon the values of heating and mechanical overload ratios as calculated from Eqns. 3·88 and 3·95 or can be found by referring to Fig. 3·55. These ratios are valid for a simple single-step load diagram. However, in practice, the load diagram is complicated because it may involve many steps and also include periods of starting and braking. Equivalent torque method may be used for calculation of ratings. The rating selected may be checked for mechanical overload. Equivalent power method may be used when there are no wide variations of speed. It is not rational to use general purpose motors meant for continuous duty for short time duty. The reasons for this are explained below :

(*i*) The general purpose motors are able to utilize their heating capacity fully when used for short time duty except in those rare cases where t_h/T_h has a relatively high value. Their heating capacity cannot be fully utilized because of limitations imposed on heating and mechanical overload ratios.

(*ii*) All the parts of an electric motor do not have the same heating time constant and this aspect needs careful examination For example, the commutator and field windings of a d.c. series wound motor may have a much smaller time constant as compared with that of

armature winding when the machine is used for continuous duty, the difference in time constants is of no significance at all. However, when the machine is used for short time duty the commutator and the field windings heat up much quicker and therefore limit the over loading thereby preventing the much higher heating capacity of the armature being utilized. Thus it is evident that machines used for short time duty be designed differently than for continuous duty. The commutators and field windings should be designed for higher heating time constants by making them stronger and heavier in construction.

Short time duty motors are designed for standard operating periods of 15, 30, or 60 minutes.

3·33. Selection of Motor Capacity for Intermittent Periodic Duty. The capacity of general purposes motors when used for Intermittent Periodic Duty can be calculated from Eqn. 3·99 or from Fig 3·55. If the load diagram consists of many steps, the equivalent load for the working period should be determined first. The motor is then chosen from the value of equivalent load and from permissible overload ratio.

Special motors are designed for intermittent periodic duty due to economic reasons. These special motors have greater starting torque and a higher overload capacity as compared to general purpose continuous duty motors and name plate indicates the duty factor for which the motor has been designed.

The standard duty factors for intermittent periodic duty motors are 0·15, 0·25, 0·4 and 0·6.

Fig. 3·56 shows an intermittent duty load diagram with four steps. The equivalent torque is given by :

$$T_{eq}=\sqrt{\frac{T_1^2t_1+T_2^2t_2+T_3^2t_3+T_4^2t_4}{t_1+t_2+t_3+t_4}} \quad \ldots(3{\cdot}100)$$

The period of rest t_r is not induced in Eqn. 3·100 because it is taken care of by the duty factor of the motor. If the design duty factor

$$\epsilon=\frac{t_h}{t_h+t_r}=\frac{t_1+t_2+t_3+t_4}{t_1+t_2+t_3+t_4+t_r} \quad \ldots(3{\cdot}101)$$

differs from the standard value, it is necessary to select a motor having the higher duty factor rating and calculate its power capacity for the specified duty conditions.

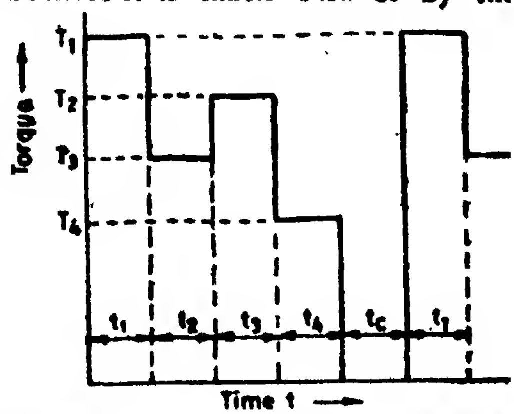


Fig. 3·56. Four step intermittent periodic duty load diagram.

In self ventilated motors, the cooling is poorer under rest, starting and braking conditions as compared with those at normal operating speed. The duty factor is then calculated as under

$$\epsilon=\frac{t_{st}+t_{ss}+t_b}{t_{ss}+\alpha t_r+\beta(t_{st}+t_b)} \quad \ldots(3{\cdot}102)$$

where t_{st}, t_{ss} and t_b = period of starting, steady speed operation and braking respectively,

α and β = factors which take into account the poorer cooling condition during rest and starting and braking.

When the duty factor is greater than 0·6, motors designed for continuous duty are used while if duty factor is less than 0·1, motors designed for short time duty are used.

Many a times, a situation arise where it becomes necessary to use a motor designed for a particular duty factor to be used at a different duty factor. The equivalent power for

which the motor is selected should remain the same under both the conditions. Suppose a motor of rating P_1 and duty factor ε_r is to be used at a duty factor ε_2. The new power rating P_2 for duty factor ε_2 is found by equating the equivalent powers in both the cases.

$$P_{eq}\left[\frac{P_1^2 t_{h_1}}{t_{h_1}+t_{c_1}}\right]^{1/2}=\left[\frac{P_2^2 t_{h_2}}{t_{h_2}+t_{c_2}}\right]^{1/2} \quad \text{...(3·103)}$$

but $\varepsilon_1=\dfrac{t_{h_1}}{t_{h_1}+t_{c_1}}$ and $\varepsilon_2=\dfrac{t_{h_2}}{t_{h_2}+t_{c_2}}$

$\therefore$ $P_1^2\varepsilon_1=P_2^2\varepsilon_2$

or $P_2=P_1\sqrt{\varepsilon_1/\varepsilon_2}$ ...(3·104)

A more precise formula that takes into account the constant losses is used. This relationship is :

$$P_2=P_1\sqrt{(K+1)\frac{\varepsilon_1}{\varepsilon_2}-K} \quad \text{...(3·105)}$$

where K=ratio of constant losses to variable losses.

Example 3·37 *An induction motor works at full load for 20 minutes and at no load for 15 minutes. The heating time constant is 2 hours and the cooling time constant is 3 hours. If the final steady temperature rise on the continuous full load is 50°C, find the approximate mean temperature rise when the machine operates indefinitely on its load cycle. Given : copper loss at full load=500 W, no load loss=300 W.*

Solution.

Total loss at full load =500+300=800 W.

Total loss at no load=300 W.

As the final steady temperature rise is proportional to loss to be dissipated, therefore final steady temperature rise when running on no load is,

$$\theta_n=\frac{300}{800}\times 50=18·7°\text{C}.$$

Since the heating and cooling periods are quite small as compared to the respective thermal constants and therefore Eqn. 3·98 can be used to calculate mean temperature rise.

Mean temperature rise,

$$\theta'=\frac{\theta_m' t_h/T_h+\theta_n' t_c/T_c}{t_h/T_h+t_c/T_c}=\frac{(50\times 20/120)+(18·75\times 15\times 180)}{20/120+15/180}$$

$$=39·58°.$$

Example 3·38 *A transformer has a final steady temperature rise of 75°C at full load and a heating time constant of 3 hour. The copper loss at full load is twice the iron loss. Calculate the temperature rise of the transformer at the end of the following load cycle after startin from cold conditions :*

full load—2 hours

no load—1 hour

20 percent overload—1 hour.

Solution. As the final steady temperature rise is proportional to the total loss, final steady temperature rise at no load,

$$=\frac{\text{loss at no load}}{\text{loss at full load}}\times\text{temperature rise at full load}$$

$$=\frac{1}{2+1}\times 75=25°\text{C}.$$

Final steady temperature rise at 20 per cent overload

$$=\frac{\text{loss at 20 per cent overload}}{\text{loss at full load}}\times\text{temperature rise at full load}$$

$$=\frac{2(1\cdot2)^2+1}{2+1}\times75=97°C.$$

2 hours of full load

We have, $\theta_m=75°C$, $\theta_i=0°C$, $t=2$ hour and $T_h=3$ hour.

From Eqn. 3·57, temperature rise after this period,

$$\theta=\theta_m(1-e^{-t/T_h})=75(1-e^{-2/3})$$
$$=36\cdot5°C$$

1 hour of no load

We have, $\theta_n=25°C$, $\theta_i=36\cdot5°C$,

$t=1$ hour and

$T_c=3$ hours.

From Eqn. 3·58 temperature rise after 1 hour of no load,

$$\theta=\theta_n(1-e^{-t/T_c})+\theta_i e^{-t/T_c}$$
$$=25(1-e^{-1/3})+36\cdot5\times e^{-1/3}=33\cdot2°C$$

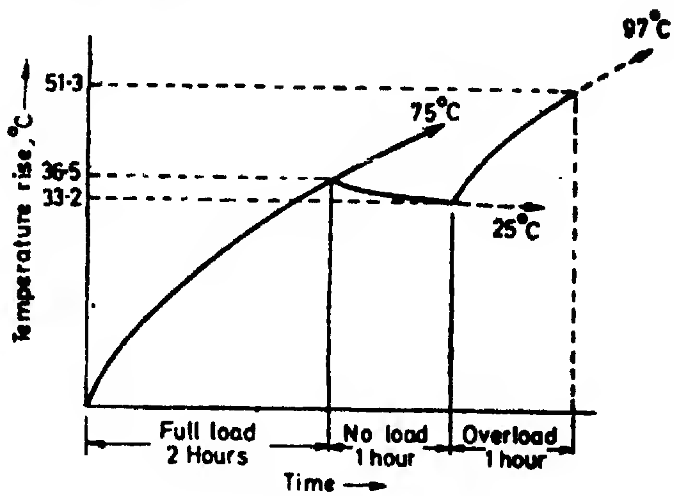


Fig. 3·57

1 hour of overload

$$\theta_m=97°C,\ \theta_i=33\cdot2°C,\ t=1 \text{ hour and } T_h=3 \text{ hour}$$

From Eqn. 3·56, temperature rise after 1 hour of overload,

$$\theta=\theta_m(1-e^{-t/T_h})+\theta_i e^{-t/T_h}$$
$$=97(1-e^{-1/3})+33\cdot2\times e^{-1/3}=51\cdot3°C$$

Temperature at the end of load cycle is 51·3°C.

Example 3·39. *A transformer has a thermal time constant of 4 hours. The final steady temperature rise at full load is 60°C and the copper loss at full load is 2 times the iron loss. Find out the maximum and minimum temperature rises of the transformer on a repeated cycle of 3 hours on full load and one hour at no load.*

Solution. The cycle of full load and no load is repeated indefinitely and therefore after some time the temperature rises vary between limits i.e. θ_m (maximum) and θ_0 (minimum) as shown in Fig. 3·54

When heating :

The transformer runs at full load and therefore its maximum steady temperature rise $\theta_m'=60°C$ and since the heating starts with a temperature rise $\theta_s=\theta_2$ and ends at θ_m we have

$$\theta_m=\theta_m'(1-e^{-t/T_h})+\theta_0 e^{-t/T_h} \text{ (see Eqn. 3·56)}$$

or $$\theta_m=60(1-e^{-3/4})+\theta_0 e^{-3/4}$$

or $$\theta_m=31\cdot66+0\cdot472\,\theta_0 \quad ...(i)$$

When cooling :

Losses at full load $=P_c+P_i=2P_i+P_i=3P_i$

Losses at no load $=P_i$.

∴ Final steady temperature rise when cooling

$$\theta_n'=\frac{p_i}{3p_i}\times\theta_m'=\frac{1}{3}\times60=20°C.$$

The cooling starts with an initial temperature rise $\theta_i=\theta_m$ and finishes at $\theta=\theta_0$ (with a final steady temperature rise $\theta_n'=20°C$.)

$$\therefore \qquad \theta_0=\theta_n'(1-e^{-t/T_c})+\theta_m e^{-t/T_c} \qquad \text{(See Eqn. 3·58)}$$

or
$$\theta_0=20(1-e^{-1/4})+\theta_m e^{-1/4}$$

or
$$\theta_0=4{\cdot}42+0{\cdot}779\ \theta_m.$$

From (*i*) and (*ii*),

$$\theta_m=53{\cdot}4°C \text{ and } \theta_0=46°C.$$

Example 3·40. *A 250-V, d.c. shunt motor has armature and field resistances of 0·24 Ω and 250 Ω respectively, and takes 4 A at no load. On full load the total input current is 70 A and the final steady temperature rise is 55°C. Estimate the overload that can be permitted for 1 hour period, if any such overload period is preceded by a period of atleast 2 hours during which the motor is shut down. The temperature rise must never exceed 55°C. Take heating time constant as 1 hour and cooling time constant 1·2 hours.*

Solution. Shunt field current $=\frac{250}{250}=1$ A.

Armature current at no load $=4-1=3$ A.

Armature copper loss at no load $=(3)^2\times0{\cdot}25=2{\cdot}25$ W.

Total input at no load $=250\times4=1000$ W.

Constant loss $=1000-2{\cdot}25=997{\cdot}75$ W.
$\simeq 1000$ W.

Total input at full load $=250\times70=17500$ W.

Armature current at full load $=70-1=69$ A.

Armature copper loss at full load $=(69)^2\times0{\cdot}25=1190$ W.

Total loss at full load $=1190+1000=2190$ W.

Motor output at full load $=17500-2190=15310$ W.

The cycle of overload for 1 hour and shut down for 2 hours is repeated indefinitely and, therefore, after some time the temperature rise varies between two fixed limits *i.e.* θ_m (maximum) and θ_0 (minimum) as shown in Fig. 3·54.

It is given that maximum temperature rise is not to exceed 55°C.

or
$$\theta_m=55°C.$$

Suppose the maximum steady temperature rise with overload is θ_m'

$$\therefore \qquad \theta_m=\theta_m'(1-e^{-t/T_h})+\theta_0 e^{-t/T_h} \qquad \text{(See Eqn. 35·6)}$$

or
$$55=\theta_m'(1-e^{-1/1})+\theta_0 e^{-1/1}$$

or
$$0{\cdot}632\ \theta_m+0{\cdot}368\ \theta_0=55. \qquad ...(i)$$

Since the machine is shut down and, therefore, final steady temperature rise with cooling is 0°C.

Hence
$$\theta_0=\theta_m e^{-t/T_c}$$

or
$$\theta_0=55\times e^{-2/1{\cdot}2}=10{\cdot}4°C.$$

$\therefore$ From (*i*), $\theta_m'=81°C$.

Hence allowable loss at overload $=\frac{\theta_m'}{\theta_m}\times$ loss at full load

$$=\frac{81}{55}\times2190=3230\text{ W}.$$

Allowable copper loss at overload $=3230-1000=2230$ W.

Allowable armature current at overload$=\sqrt{2230/0{\cdot}25}=94{\cdot}4$ A.

Motor input current at overload$=94{\cdot}4+1=95{\cdot}4$ A.

Total motor input at overload$=250\times95{\cdot}4=23850$ W.

Motor output at overload $=23850-2230-1000=20620$ W.

Percentage output at overload $=\dfrac{20620}{15310}\times100=135$.

Therefore, the machine can be overloaded by 35 percent.

Example 3·41 *An induction motor is to be used on short time duty for a load having a two stepped load diagram. The load is 30 kW for 15 minutes and 20 kW for 30 minutes. Calculate the rating of continuous duty motor suitable for this drive if the variable losses at full load are twice the constant losses and the heating time is 90 minutes. Select a motor of standard rating with an overload capacity of 2·5.*

Standard ratings available are : 7·5, 11, 15, 18·5, 22 kW.

Solution. Equivalent power $P_{eq}=\left[\dfrac{P_1^2t_1+P_2^2t_2}{t_1+t_2}\right]^{\frac{1}{2}}$

$$=\left[\frac{(30)^2\times15+(20)^2\times30}{15+30}\right]^{\frac{1}{2}}=23{\cdot}8\text{ kW.}$$

Heating power ratio p_h $=\dfrac{1}{1-e^{-t_h/T_h}}=\dfrac{1}{1-e^{-45/90}}=2{\cdot}54$

$$K=\frac{\text{constant loss}}{\text{variable losses}}=0{\cdot}5.$$

∴ Mechanical overload ratio

$$p_m=\sqrt{(K+1)\,p_h-K}=\sqrt{(0{\cdot}5+1)\times2{\cdot}54-0{\cdot}5}=1{\cdot}82$$

Hence, continuous rating of motor $=\dfrac{23{\cdot}8}{1{\cdot}82}=13$ kW.

The nearest standard higher rating is 15 kW.

The maximum load that the motor can carry is $2{\cdot}5\times15=37{\cdot}5$ kW.

The maximum load is 30 kW and therefore the 15 kW motor is suitable as regards heating as well as mechanical considerations.

Example 3·42. *An intermittent periodic duty motor has to work on a three stepped load curve : 100 kW for 2 minutes, 70 kW for 5 minutes, and 80 kW for 3 minutes followed by a rest period of 20 minutes. Calculate the capacity of intermittent periodic duty machine with standard duty cycle. The fixed losses are equal to the variable losses at full load. The cooling during rest is 70% as effective as during running operation. The standard duty cycles are 0·15, 0·25, 0·4 and 0·6.*

Standard ratings available are : 37, 45, 55, 75 and 90 kW.

Solution. Equivalent power

$$P_{eq}=\sqrt{\frac{(100)^2\times2+(70)^2\times5+(80)^2\times3}{2+5+3}}=79{\cdot}8\text{ kW.}$$

From Eqn. 3·102,

$$\text{Duty factor }\varepsilon_1=\frac{t_1+t_2+t_3}{t_1+t_2+t_3+\alpha t_c}=\frac{2+5+3}{2+5+3+0{\cdot}7\times20}=0{\cdot}417$$

The next higher duty cycle is 0·6.

$\therefore$ Rating of a 0·6 duty cycle with intermittent duty motor

$$P_2 = P_1 \sqrt{(K+1)\frac{\varepsilon_1}{\varepsilon_2} - K} \qquad \text{(see Eqn. 3·105)}$$

$$= 79.8\sqrt{(1+1)\times\frac{0.417}{0.6} - 1} = 49.8 \text{ kW.}$$

Therefore a motor with 55 kW and 0·6 duty cycle is choosen.

3·34. Rapid Heating of Conductors. Under faulty conditions which last for a very short duration the conductors may heat very rapidly. As the fault is cleared after a very small time, we can assume that no heat is dissipated to the surroundings. This means that all the heat which is generated is stored in the conductors.

Let

l = length of conductor, m;
a_s = area of conductor, m²,
ρ = resistivity, Ω m ;
I_s = current carried by conductor, A;
t = time for which current is carried, s ;
h = specific heat of conductor material, J/kg—°C ;
g = density of conductor material, kg/m³ ;
θ = temperature rise, °C.

$$\text{Heat generated} \quad = I_s^2 \rho \frac{l}{a_s} \cdot t \qquad \ldots(i)$$

$$\text{Weight of conductor} \quad G = g l a_s \text{ kg.}$$

$$\text{Heat stored} \quad = G h \theta = g l a_s \times h \theta \qquad \ldots(ii)$$

Equating (i) and (ii) for rapid heating, we have :

$$\text{Temperature rise} \quad \theta = \frac{I_s^2 \rho \dfrac{l}{a_s} t}{g l a_s h} = \frac{I_s^2 \rho t}{a_s^2 g h} \text{ °C} \qquad \ldots(3·106)$$

Putting $\delta = I/a_s$ = current density, A/m², we have :

$$\theta = \frac{\delta^2 \rho t}{g h} \text{ °C} \qquad \ldots(3·107)$$

Example 3·43. *A copper conductor of sectional area 30 mm² carries a current of 20 kA for 50 ms. The conductor data is resistivity = 0 021 × 10⁻⁶ Ω m, specific heat = 418 J/kg—°C, and density = 8900 kg/m³.*

Calculate the temperature rise.

Solution. From Eqn. 3·106, temperature rise $\theta = \dfrac{I_s^2 \rho t}{g a_s^2 h}$

$$= \frac{(20)^2 \times (10^3)^2 \times 0.021 \times 10^{-6} \times 50 \times 10^{-3}}{8900 \times (30)^2 \times 10^{-6} \times 418} = 125\text{°C.}$$

3·35. Calculation of Hot Spot Temperature. The internal flow of heat from the parts in which it is actually generated to the cooling surfaces from which it is transferred to the cooling medium is important in determining the hot spot temperature (which is normally the temperature of conductor). This is because the insulating materials are subjected to the hot spot temperature.

The hot spot temperature rise above the ambient temperature is made up of three components.

(i) Temperature gradient across the slot insulation, θ_{ins},

(ii) Temperature rise of cooling surface above that of cooling medium, θ_s.

(iii) Temperature rise of cooling medium over ambient temperature, θ_c.

∴ Hot spot temperature rise over ambient temperature is :

$$\theta_w=\theta_{ins}+\theta_s+\theta_c \qquad \ldots(3\cdot108)$$

Thus, in order to calculate the value of θ_w we have to calculate its three components θ_{ins}, θ_s and θ_c. Because of the complex nature of heat flow in a machine it is very difficult to predetermine the values of temperature gradient across the slot insulation. However we, can estimate temperature rise of the surface with sufficient accuracy by using the relation

$$\theta_s=Qc/s \qquad \text{(see Eqn. 3·52)}$$

The surface temperature rise θ_s is of little importance, the temperature of importance is the inside temperature of windings, θ_w. However, by keeping the surface temperatures below limit, the internal temperatures are also kept low. This can be illustrated by an example. Let us consider the case of a machine using class B insulating materials which can withstand a hot spot temperature of 120°C. Suppose the maximum ambient temperature is 40°C. Therefore, the maximum allowable temperature rise of hot spot is 120−40=80°C. From our experience of designing similar machines, we can know that the temperature gradient across the slot insulation is around 16°C and the temperature rise of cooling medium is about 14°C. Therefore the maximum allowable temperature rise of surface over cooling medium is

$$\theta_s=80-16-14=50°\text{C}.$$

Thus by calculations (using Eqn. 3·52) we find the value of θ_s. If it is below 50°C, the maximum allowable in this case, we can be sure that the insulating materials are not being subjected to a temperature beyond their maximum allowable. This means that the durability of insulation, on which the life of the machine depends, will be ensured. The designer must, however, see that sufficient ventilation is provided to allow the heat to be carried away without requiring a great difference in temperature of internal portion where the losses occur and the surfaces which dissipate heat to the cooling medium.

Assuming that proper ventilation is provided so that the internal temperatures are not greatly in excess of the surface temperatures, it is merely necessary to see that the cooling surface is sufficient to dissipate losses or in other words if the temperature decrement across the insulation is kept low, the heating conditions in the machine can be assessed by merely considering the surface temperature rise θ_s.

3·36. Calculation of Surface Temperature Rise. Using relation 3·52 we find steady temperature rise of surface, $Q_s=Qc/s$

Thus the maximum steady temperature rise can be calculated if we know the surface area and the cooling co-efficient c. The value of c depends upon the part to be cooled, the speed of the cooling medium and the configuration of the surface.

In view of the complexity of the aerodynamic phenomenon in the machine, the velocity of cooling medium over different surfaces can only be determined approximately. Also it is difficult to estimate directions and magnitude of heat flow. Therefore, the value of cooling co-efficient can be known with sufficient accuracy only if experimental data is available Thus the application of Eqn. 3·52 should only be made with safety and the results should be confirmed with experimental data.

The values of cooling co-efficient for various types of surfaces can be taken from Table 3·6. V is the peripheral speed in metre per second while S is measured in m².

Table 3·6. Value of Cooling Coefficient c

Part	*c*	*V*	*Remarks*
Cylindrical surface of stator and rotor	$\frac{0.03 \text{ to } 0.05}{1+0.1\,V}$	Relative peripheral speed	Use low values for forced cooling
Back of stator core	0·025 to 0·04	zero	
Cylindrical surface of d.c. armatures	$\frac{0.015 \text{ to } 0.035}{1+0.1\,V}$	Armature peripheral speed	Use lower values for large open machines
Stationary field coils	$\frac{0.14 \text{ to } 0.16}{1+0.1\,V}$	Armature peripheral speed	Based on total coil surface
	$\frac{0.0 \text{ to } 0.08}{1+0.1\,V}$		Based on exposed coil surface
Rotating field coil	$\frac{0.08 \text{ to } 0.12}{1+0.1\,V}$	Armature peripheral speed	Based on total coil surface
	$\frac{0.06 \text{ to } 0.08}{1+0.1\,V}$		Based on exposed coil surface
Ventilating ducts	$\frac{0.03 \text{ to } 0.2}{V}$	Air velocity in ducts	Velocity in ducts is 10 per cent of peripheral speed of core
Commutator	$\frac{0.015 \text{ to } 0.025}{1+0.1\,V}$	Commutator peripheral speed	

3·36·1. Calculation of Temperature Rise of Armatures. The temperature rise of armatures can be determined by Eqn. 3·52. In order to apply this, a knowledge of total heat lost, cooling surface and cooling coefficient is necessary. It is assumed that the current density in the conductors has been so chosen that the end connections are not appreciably hotter than the armature as a whole. Under these conditions, the heat to be dissipated by cooling surfaces of the armature core would consist of (*i*) the hysteresis and eddy current losses in armature core and teeth. (*ii*) the I^2R losses in the active portion of the armature winding *i.e.*, in the part of the winding embedded in the slots. These losses are equal to $(2L/L_{mt})P_c$.

where P_c = total copper losses in the armature winding,

L = gross length of machine core,

L_{mt} = length of mean turn of an armature coil.

The total armature cooling surface may be composed of the following:

(*i*) the outside cylindrical surface of armature,

(*ii*) the inside cylindrical surfaces of armature,

(*iii*) the two end surfaces of armature,

(*iv*) one surface of each duct.

Referring to Fig. 3·58.

Outside cylindrical surface of armature $S_o = \pi D_o L$ ...(3·108)

and let its cooling co-efficient be c_o

Loss dissipated/°C rise of temperature$=S/c$ watt (for $\theta=1$)

or loss dissipated/°C from outside cylindrical surface of armature$=S_o/c_o$.

Similarly loss dissipated/°C from incide cylindrical surface of armature$=S_i/c_i$

where S_i=inside cylindrical surface of armature$=\pi D_i L$

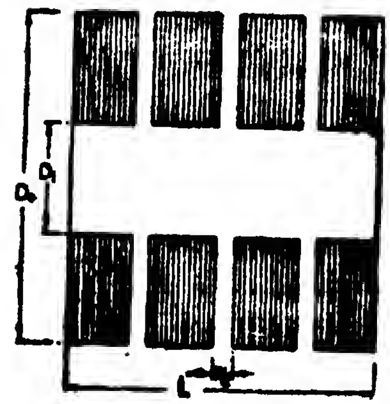


Fig. 3·58. Armature with radial ventilating ducts.

c_i=cooling co-efficient for inside cylindrical surface.

The cooling surface of two ends of armature core and radial ventilating ducts is (considering one surface of each radial duct)

$$S_d = (\pi/4) \times (D_o^2 - D_i^2)(2 + n_d) \quad \text{...(3·109)}$$

where n_d=number of ventilating ducts.

∴ Loss dissipated/°C from end surfaces and radial ventilating ducts

$$= S_d/c_d \quad \text{...(3·110)}$$

where c_d=cooling co-efficient for ducts.

From above, total loss dissipated/°C rise

$$= S_o/c_o + S_i/c_i + S_d/c_d \quad \text{...(3·111)}$$

∴ Temperature rise of armature

$$\theta = \frac{\text{total loss to be dissipated}}{\text{loss dissipated/°C rise of temperature}}$$

$$= \frac{Q}{S_o/c_o + S_i/c_i + S_d/c_d} \quad \text{...(3·112)}$$

The values of various cooling co-efficients can be taken from Table 3·6.

3·36·2. Calculation of Temperature Rise of Field Coils. The problem of calculation of temperature rise of field coils is complicated by the fact that the fanning action of rotor has an important effect on the capability of machine to dissipate heat. Moreover, the spacing between the poles and the presence of interpoles affects the cooling conditions of main poles. It is, therefore, clear that the heat dissipated from surfaces of field coils will depend upon the type of construction of machine, nature and area of surface, or whether stationary as in d.c. machines or revolving as in synchronous machines, and the heat absorbing properties of surrounding medium. Therefore, the pre-determination of temperature rise of field coils is based upon empirical co-efficients derived from experience and experimental investigations.

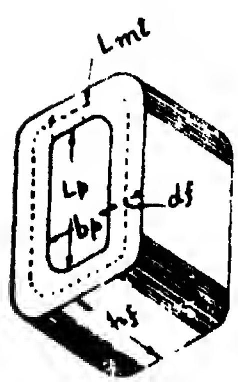


Fig. 3·59. Field Coil.

The losses to be dissipated from surfaces of field coils are the copper losses in the windings. There are no iron losses in the cores of field coils of d.c. and synchronous machines as they are subjected to non-varying magnetizations.

The calculation of temperature rise may be based upon total coil surface. Considering Fig. 3·59, the total exposed coil surface,

$$S=2L_{mt}\,d_f+2\,L_{mt}\,h_f=2L_{mt}\,(d_f+h_f) \quad ...(3\cdot113)$$

where, L_{mt}=length of mean turn of field coil

$$=2(b_p+d_f+L_p+d_f)=2\,(L_p+b_p)+4d_f.$$

For coil of Fig. 3·34, we have :

d_f=radial depth of field coil,

h_f=axial height of field coil,

L_p=length of pole core,

b_p=width of pole core.

Temperature rise of field coils can be calculated by applying Eqn. 3·5 . The cooling co-efficient for field coils is taken from Table 3·6.

3·36·3. Calculation of Temperature Rise of Commutators. The temperature rise of a commutator depends upon many factors which cannot be incorporated in a formula. The peripheral speed of the surface will definitely have effect on the value of cooling co-efficient but the influence of high speeds on rotating cylindrical surfaces is not so great as may be expected. The fanning action of risers connecting the bars to the armature coils largely influence the temperature rise in the case of small commutators but this factor is of less importance where the axial length of commutators is large.

The probable cooling co-efficient of a commutator in which no special cooling devices are used but for which risers have a good fanning action, is

$$c_c=\frac{0\cdot012}{1+0\cdot1\,V_c} \quad ...(3\cdot114)$$

where V_c=peripheral speed of commutator, m/s.

This value of cooling co-efficient takes into consideration only the cylindrical surface of the commutators. The cooling co-efficients based upon some portion of riser surface in addition to commutator surface are given in Table 3·6.

3·37. Measurement of Temperature Rise. The calculation of temperature rise of a machine is not simple due to complex heat flow through materials having different conductivities and heat transfer co-efficients. Experience is the only guide in designs. In order to ascertain whether sufficient provision has been made for cooling, heat runs are done during which temperature rises are recorded against time elapsed.

3·38. Methods of measurement of Temperature Rise*. Three methods of determining the temperature rise of windings and other parts are recognized :

(*a*) Thermometer method.

(*b*) Resistance method.

(*c*) Embedded temperature detector method.

3·38·1. Thermometer Method. In this method the temperature is determined by thermometers applied to the accessible surfaces of completed machine. **Thermometer** is usually applied to the surface of a machine part. Therefore, it indicates the temperature of the **surface** at one point only. The term thermometer also includes non-embedded thermocouples and resistance thermometers provided they are applied to the points accessible to the usual bulb thermometers. When bulb thermometers are employed in places where there is any varying or moving magnetic field, alcohol thermometers should be used in perference to mercury thermometers as the latter are unreliable under these conditions.

3·38·2. Resistance Method. In this method the temperature of winding is determined by increase in resistance of the winding. Therefore, this method involves the measurement of the resistance, both cold and hot, and estimating the average temperature rise by use

*Reference IS : 4722—1968.

of the resistance temperature co-efficient. This method is used for windings only. The temperature rise $\theta=\theta_2-\theta_a$ may be obtained from the ratio of resistance by the formula :

$$\frac{R_2}{R_1}=\frac{\theta_2+235^*}{\theta_1+235} \quad \ldots(3\cdot115)$$

where R_2=resistance of the winding at the end of the test,

R_1=initial resistance of winding (cold),

θ_2=temperature (°C) of the winding at the end of the test,

θ_1=temperature (°C) of the winding (cold) at the moment of initial resistance measurement,

θ_a=temperature (°C) of cooling medium at the end of the test.

For practical purposes the following alternative formula may be found convenient.

$$\text{Temperature rise} \quad \theta=\theta_2-\theta_a=\frac{R_2-R_1}{R_1}(235+\theta_1)+\theta_1-\theta_a \quad \ldots(3\cdot116)$$

In the case of a.c. machines, resistance measurements may be made without interruption the test by **method of superposition** which consists of applying a small d.c. measuring current superposed upon the load current.

3·38·3. Embedded Temperature Detectors (E.T.D.). Embedded temperature detectors are resistance thermometers or thermocouples built into the machine during construction at points which are inaccessible when the machine is completed.

At least six detectors are built into the machine, suitably distributed around the circumference and placed in positions along the length of the core at which highest temperatures are likely to occur.

The embedded detectors are protected from contact with cooling medium when the machine has two coil sides per slot, the detectors are located between the insulated coil sides within the slot. Where there are more than two coil sides per slot, the detectors are located between insulated coil sides where the highest temperature is likely to occur. The embedded temperature detectors give the temperature of one **internal point**.

The three methods described above are used for different parts or places in a machine and therefore do not refer to the same thing and hence do not give the same estimates of the **hot spot temperatures**. In case, an embedded temperature detector can be placed, during the course of construction, at a point which by experience shows the highest temperature it will give an indication of **hot spot temperature**. However, in practice it becomes virtually impossible to place the detector at this point on account of problems of insulation as the detector cannot be placed too close to the conductors (which are at a high potential).

Through, methods for measurement of resistance of a winding in operation, it is more customary to make resistance (or thermometric) measurements immediately after shut down. Extrapolation techniques are then used for the period between actual shut down and start of temperature measurement. The temperature rise-time curve during cooling period is plotted and extrapolated backward to the instant of shut down by means of simple application of exponential curve.

3·39. Measurement of winding temperature. Resistance or embedded temperature detector methods are used for measurement of temperature of a.c. stator windings of :

(*i*) turbine type machines having a rated output of 5000 kW or kVA or more,

(*ii*) salient pole machines and induction machines having a rated output of 5000 kW or kVA or more,

(*iii*) machines having a core length of one metre or more.

*This is applicable if the windings are of copper. In case windings are made of material other than copper, reciprocal of resistance temperature co-efficient at 0°C should be used in place of 235.

Embedded temperature detectors are preferred.

For field windings and stator windings of machines other than the large machines listed above or machines having one coil side per slot, the increase of resistance of windings is used for measurement of temperature.

The thermometer method is used where neither the embedded temperature detector method nor the resistance method is applicable.

The thermometer method is most suited for measurement of temperature of low resistance windings like interpole and compensating windings. It is also used for temperature measurement of single layer windings.

Example 3·38. *The resistance of shunt field of a d c. motor is 252 ohm at 20°C. The resistance of the winding rises to 303 ohm after a heat run on the machine. Calculate the temperature rise of the field coil if the ambient temperature is 20°C if the conductors are made of copper.*

Solution. From Eqn. 3·115

$$\frac{R_2}{R_1}=\frac{\theta_2+235}{\theta_1+235} \text{ or } \frac{303}{252}=\frac{\theta_2+235}{20+235}$$

Temperature of field coil after heat run $\theta_2=72°$C.

Temperature rise $\theta=72-20=52°$C.

UNSOLVED PROBLEMS

1. The copper loss in a winding is 0·5 W. This loss is dissipated through a layer of paper having a thermal resistivity of 6 Ω m. Find the temperature difference between the two sides of the paper if the thickness of paper is 0·5 mm and its surface area is 250 mm². [Ans. 6°C]

2. The thickness of the slot insulation around the conductors in the slots of a d.c. generator is 1.5 mm. The outside dimensions of the two insulated coil sides (in contact with iron of the slot) are 12·5 mm wide by 25 mm deep. The total cross section of the copper in the slot is 200 mm² and the current density is 5 A/mm². Calculate the temperature difference between copper conductors and the iron armature stampings. Assume thermal resistivity of insulation=8 Ω m (thermal) and electrical resistivity of copper as $0·02\times10^{-6}$ Ω m. [Ans. 16°C]

3. The thermal conductivity of assembled armature stampings is 40 times as great in the direction of laminations as in the direction perpendicular to laminations. Calculate the loss that will be conducted across the laminations in a stack 0·1 m thick and 20×10^{-3} m² in cross section with a difference of temperature of 15°C, given that a difference of temperature of 5°C will cause 64 W to be conducted through a stack 6×10^{-3} in area and 25 mm thick measured along the laminations. [Ans. 4 W]

4. A cylindrical conductor 0·1 m in diameter is surrounded by a coating of micanite 60 mm thick. The thermal resistivity of micanite is 4 Ω m. If the conductor has a loss of 43 watt per metre length, calculate (*a*) temperature of conductor (*b*) the temperature of outer surface of micanite. Take the dissipation from outer micanite surface=12·4 W/m²–°C and ambient temperature=20°C. [Ans. 50·5°C, 25°C]

5. The winding of an oil immersed transformer is 20 mm thick radially. It is wound on a former of negligible thermal conductivity. The power loss in the winding in 25 W/kg, the space factor is 0·75 and the thermal resistivity of the insulating materials is 7·5 Ω m. Find the temperature of hot spot above that of winding surface. Copper weighs 8900 kg/m³. [Ans. 34°C]

6. A field coil has a length of mean turn of 0·8 m, a space factor 0·545, a current of 2·55 A at a terminal voltage of 55 V, and an overall cross-section of 105×60 mm². Estimate the hot spot temperature above that of the surface of the coil. Thermal resistivity of insulating materials=8 Ω m (thermal). [Ans. 29·5°C]

7. A 15 MVA alternator has a full load efficiency of 0·96 with bearing friction losses being 10 per cent of total losses. The cooling air enters the machine at a temperature of 22°C, with a pressure of 760 mm of mercury and issues at 40°C. Calculate the volume of the inlet air circulating through the machine. If the inlet air temperature is reduced to 20°C and pressure raised to 765 mm of mercury, what will be change in the volume of inlet air, the outlet temperature remaining the same. [Ans. 26·5 m³/s ; 23·5 m³/s]

[**Hint.** Subtract the friction losses from the total losses as these losses occur outside the machine.]

8. In a closed air circuit for cooling a 60 MW alternator having an efficiency of 97%, 85 per cent of the losses are carried away by the cooling air. Calculate the weight of air required in kg per minute for a temperature rise of 30°C when the alternator is on full load. The specific heat of air at constant pressure is 1000 J/kg–°C. [Ans. 3150 kg.]

9. The full load efficiency of a 100 MW hydrogen cooled turbo-alternator is 99 per cent. The hydrogen enters the machine with a temperature of 25°C and leaves the machine with a temperature of 55°C. Determine the volume of hydrogen required if the hydrogen pressure is 1500 mm above a gauge pressure of 760 mm of mercury. Specific heat of hydrogen is 12600 J/kg—°C. Volume of 1 kg of hydrogen at N.T.P. is 11·2 m^3.
[Ans. 11·2 m^3/s]

10. The full load losses in a 16 MVA transformer are : iron loss, 82 kW; copper loss, 134 kW. The tank is 3·6 m high × 2·9 m long × 1·35 m wide and contains a cooling coil through which 200 litres of water per minute are passed. Assuming that the tank sides dissipate 7·5 W/m^2—°C and the average temperature rise of tank walls is 30°C, estimate the temperature rise of the cooling water. [Ans. 15°C]

11. An induction motor is heated to a temperature of 60°C and is then shut down. Calculate its temperature at a time 20 minutes after the shut down if the cooling time constant is 60 minutes. The ambient temperature is 30°C. [Ans. 51·5°C]

12. During a test on a transformer the load was kept constant and the rates of change of temperature rise obtained were 0·075°C per minute when temperature rise was 19°C and 0·055°C per minute when the temperature rise was 27°C. Estimate the thermal time constant and the final steady temperature rise of transformer.
[Ans. 400 minute ; 49°C]

13. A 150 kW, 250 V d.c. generator has an armature capable of dissipating 60 W/m^2—°C. Calculate the temperature rise if the armature resistance is 0·009 Ω ; the hysteresis loss 1760 W, and the eddy current loss 215 W. The radiating surface of generator is 1·5 m^2. [Ans. 58°C]

14. A machine run on steady full load showed the following temperature rises at the end of the specified time intervals.

Time, hours	0·25	0·5	1·0	1·5	2·0	2·5	3·0
Temperature rise, °C	9·5	17·0	29·2	38·0	44·2	48·7	52

Find graphically the final temperature rise and the heating time constant. If under the same ventilation conditions the machine were to be so loaded that the final steady temperature rise should not exceed 45°C, what temperature rise would be obtained at the end of one hour at this loading, starting from cold.
[Ans. 60°C, 1·5 hour ; 21·9°C]

15. A transformer gave a temperature rise of 20°C after one hour and 32°C after 2 hours on full load. What is the final steady temperature rise at full load ? If the transformer is worked on 50 per cent overload, how long will it take to attain the same temperature rises ? The copper losses on full load are twice the iron losses. [Ans. 50°C ; 1·5 hours]

16. Compare the 1-hour ratings of a transformer with a time constant of 4 hours under the following conditions :

(*i*) iron losses are equal to copper losses at full load,

(*ii*) iron losses are half of copper losses at full load.

[Ans. 2·83, 2·51 times continuous full load rating]

17. The power loss in a naturally cooled transformer is 20 kW on full load, and its rate of dissipation of heat is 0·4 kW/°C rise of temperature. The heat energy required to raise temperature by 1°C is 0·8 kWh.

Find the rise of temperature (*a*) two hours after switching on if the current is constant over this period at half full load value, and (*b*) after a further hour on full load. At half load, the copper loss is equal to the iron loss. [Ans. 12·64°C, 27·3°C]

18. A 100 kVA transformer has an efficiency of 95 per cent at 75 per cent of full load and 0·8 power factor, this being the maximum efficiency at this power factor. For the purposes of cooling, the transformer may be considered to be 1200 kg of homogeneous material having a specific heat of 700 J/kg—°C and a surface area of 10 m^2, the surface emitting heat at 12 W/m^2—°C. Find the thermal time constant and the full load temperature rise.

If the transformer is connected to the supply but is then on load for two hours, followed immediately by a 25 per cent over load, at what time must this load be disconnected so that the temperature rise does not exceed the full load value ? [Ans. 1·94 hours, 86·9°C ; 2·24 hours]

19. A motor driving some haulage equipment for a mine has to deliver a load which follows the following load cycle :

37 kW f0r 10 minutes,

No load for 4 minutes,

18·5 kW for 10 minutes,

No load for 6 minutes,

the cycle being repeated indefinitely. Estimate a suitable size of continuously rated motor for the purpose.

[Ans. 24 kW]

20. A motor has a heating time constant of 70 minutes and a cooling time constant of 90 minutes when stationary. When run continuously on full load of 18·5 kW, the final temperature rise is 40°C. What load could the motor deliver for 10 minutes if this is followed by a shut down period long enough for it to cool without the temperature rise exceeding 40°C ? If it is on an intermittent load of 10 minutes on load and followed by 15 minutes shut down, what is the maximum value of load which it could supply during the on load period, without maximum temperature rise exceeding 40°C ? Assume losses to be proportional to square of load. [Ans. 52·4 kW, 26·6 kW]

21. An oil-cooled transformer equipped with fans and radiators attains, on full load, a temperature of 36·5°C after 2 hour and a final steady temperature of 70°C. If the transformer operates on full load for 30 minutes at regular intervals, what time must be allowed between each load period if the maximum temperature rise is not to exceed 30°C ? Air temperature, is 20°C. The transformer is disconnected from supply in between the load periods. The fans are not in operation during the interval the transformer is disconnected and the cooling time constant is 400 minutes under these conditions. [Ans. 29 minutes]

22. The maximum allowable hot spot temperature in an alternator is 120°C. Find the maximum allowable ambient temperature in which this machine can be safely operated if the temperature gradient across the slot insulation is 18°C, the temperature rise of iron above the temperature of coolant is 38°C and the temperature rise of the coolant above the ambient medium is 24°C [Ans. 40°C]

4
Magnetic Circuits

BASIC PRINCIPLES OF MAGNETIC CIRCUITS

4·1. Basic Formulae. The path of the magnetic flux is called a magnetic circuit. A magnetic circuit is analogous to an electric circuit. A review of laws of magnetic circuits is given below.

Let Φ=magnetic flux, Wb ;
A=area of the magnetic path, m^2 ;
l=length af magnetic path, m ;
B=flux density, Wb/m^2 ; H='at'=magnetising force, A/m ;
AT=total mmf, A ;
$\mu=\mu_r\,\mu_0$=absolute permeability of the magnetic material, H/m ;
μ_0=permeability of free space=$4\pi\times10^{-7}$ H/m ;
μ_r=relative permeability ; S=reluctance, A/Wb ;
$\Lambda=\dfrac{1}{S}$=premeance, Wb/A.

In an electric circuit Ohm's law expresses a relationship between current, emf, and resistance ; while in a magnetic circuits a similar relation exists relating flux, mmf and reluctance. This relation is :

$$\Phi=\frac{AT}{S} \qquad ...(4·1)$$

or

$$\Phi=AT\times\Lambda \qquad ...(4·2)$$

Reluctance

$$S=\frac{\text{length}}{\text{area}}\times\frac{1}{\text{permeability}}=\frac{l}{A\mu} \qquad ...(4·3)$$

H='at'=mmf per unit length=flux×reluctance per unit length

$$=\Phi\times\frac{1}{l}\cdot\frac{l}{A\mu}=\frac{\Phi}{A}\cdot\frac{1}{\mu}=\frac{B}{\mu} \qquad ...(4·4)$$

For the case of a material of length l, and carrying a uniform flux, the total mmf AT is :

$$AT=H\times l=\text{'}at\text{'}\times l \qquad ...(\ ·5)$$

In a series magnetic circuit, the total reluctance is the sum of reluctances of individual parts or

$$S=S_1+S_2+S_3+...... \qquad ...(4·6)$$

where S=total reluctance

and $S_1, S_2, S_3......$=reluctances of individual parts.

The total mmf acting around a complete magnetic circuit is :

The mmf $$AT=\Phi S=\Phi[S_1+S_2+S_3+......]$$

$$=AT_1+AT_2+AT_3......$$

$$=at_1l_1+at_2l_2+at_3l_3+... \qquad ...(4·7)$$

or $$AT=\oint \text{'}at\text{'} \,.\, l. \qquad ...(4·8)$$

Eqns. 4·7 and 4·8 represent the circuital law for magnetic circuits where at_1, at_2, at_3, etc., are the mmfs per metre for individual part and l_1, l_2, l_3...... etc. are lengths of parts connected in series.

In parallel circuits, the same mmf is applied to each of the parallel paths and the total flux divides between the paths in inverse proportion to their reluctances, as in corresponding electric circuits or

$$\Phi=\Phi_1+\Phi_2+_3+...... \qquad ...(4·9)$$

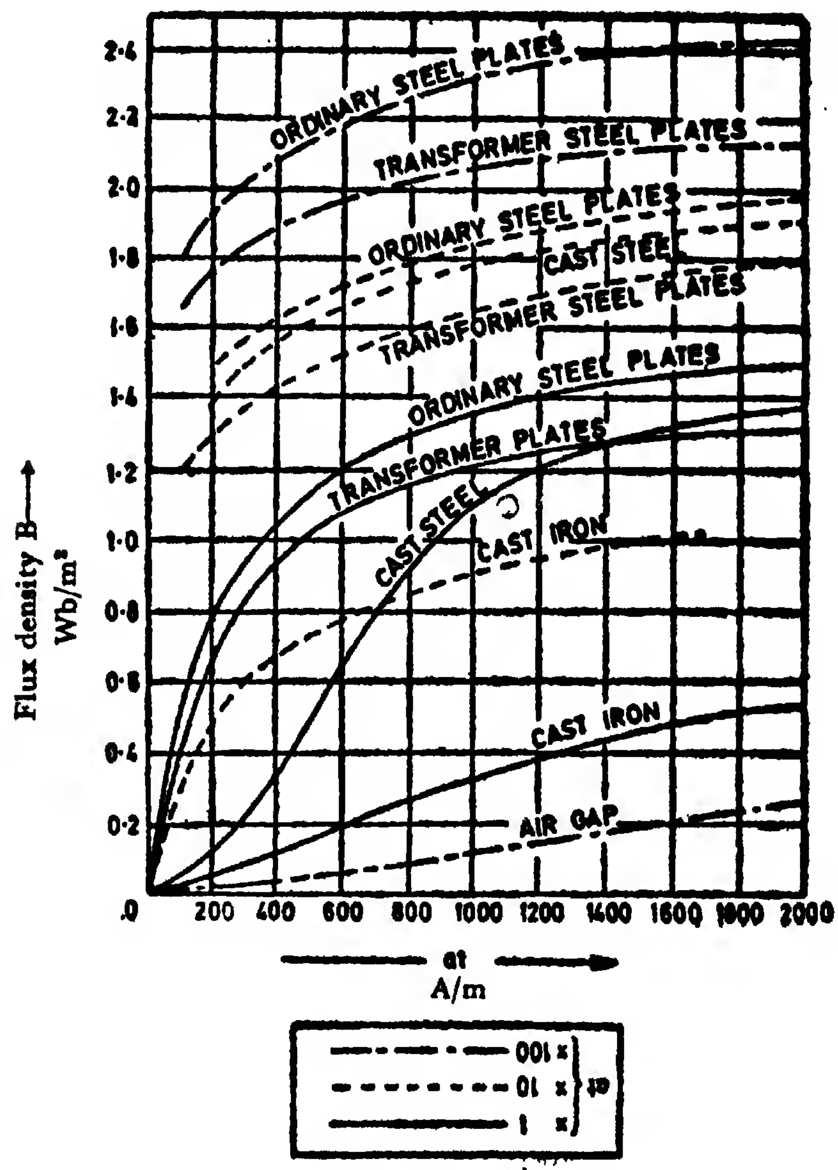


Fig. 4·1. B—'at' curves.

Dividing by AT, the applied mmf

or $$\frac{\Phi}{AT}=\frac{\Phi_1}{AT}+\frac{\Phi_2}{AT}+\frac{\Phi_3}{AT}+\ldots\ldots$$

or $$\frac{1}{S}=\frac{1}{S_1}+\frac{1}{S_2}+\frac{1}{S_3}+\ldots\ldots$$

$$\Lambda=\Lambda_1+\Lambda_2+\Lambda_3+\ldots\ldots \quad \ldots (4\cdot10)$$

where S = total reluctance of magnetic circuit and

S_1, S_2, S_3.............etc. = reluctance of individual parts

Λ = total permeances of magnetic circuit = $1/S$

while Λ_1, Λ_2, Λ_3.......etc. are permeances of individual parts.

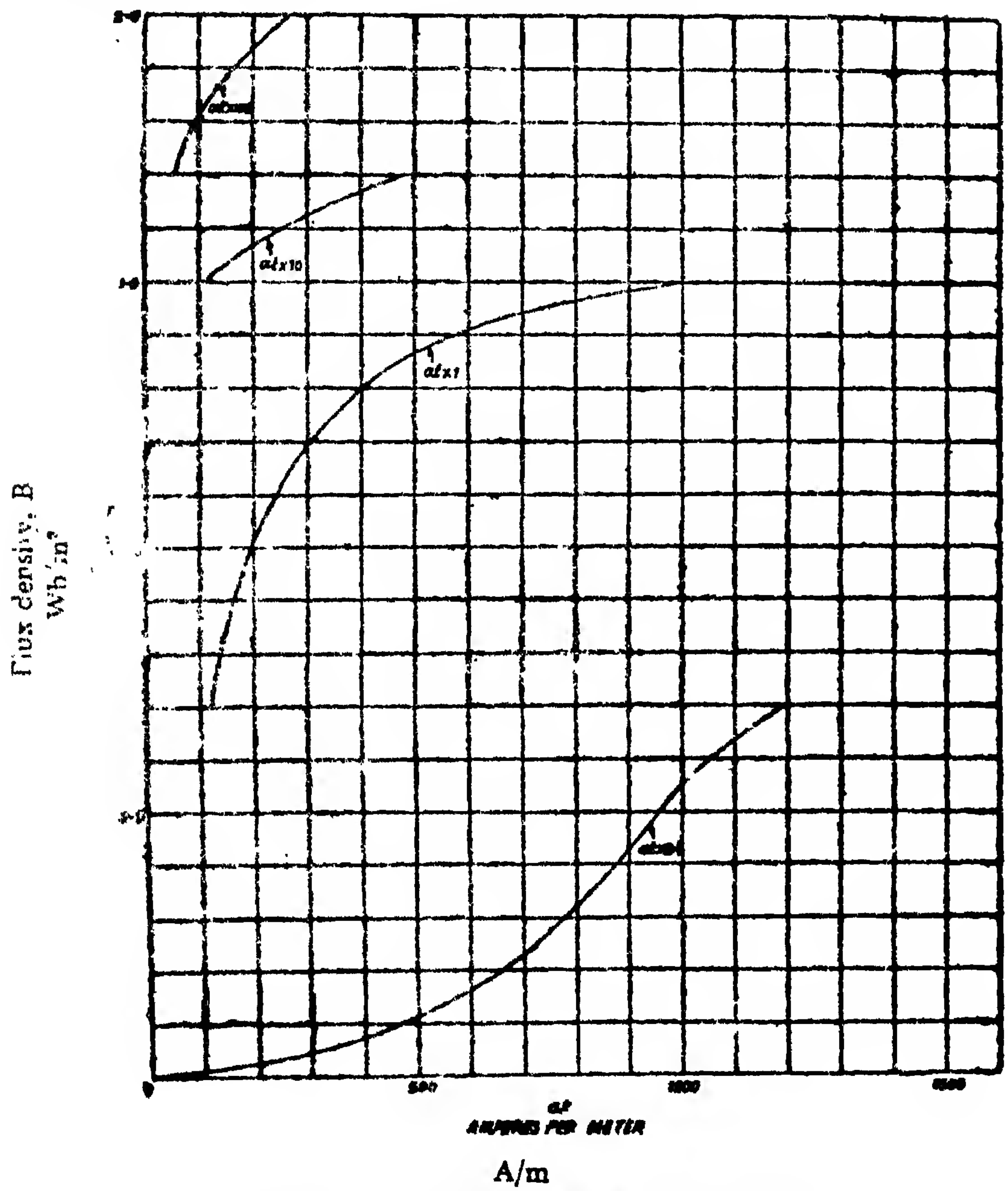


Fig. 4·2. B—H curve for Electrical Steel (non-oriented) Lohys stampings (dynamo grade).

4·2. Magnetization Curves (B—H or B—'at' Curves). The analogies between electric and magnetic circuits have certainly a value as aids to understanding of magnetic circuits but there are some essential differences between them :

1. The electric current is a true flow of electrons but there is no flow of magnetic flux and so the term flux is misleading in this context.

2. In an electric circuit, energy is consumed so long as the current flows while in a magnetic circuit, energy is expended in creating flux and not in maintaining it.

3. The most important difference between electric and magnetic circuits arises from the fact that whereas the resistance of an electric circuit is not directly dependent upon the value of current flowing and may be regarded as constant for ordinary purposes, the value of reluctance of a magnetic material is dependent upon the value of flux passing through it. The relative permeability of ferromagnetic materials may vary between a few hundred and about five hundred thousand. Consequently the mmf per metre length '*at*' has a value that lies between 2 B to 2000 B.

For this reason (No. 3 above) the actual values of permeability and reluctance are hardly used in magnetic circuit calculations. In actual practice magnetization (*B—H* or *B—'at'*) curves of magnetic materials relating flux density B to mmf per unit length, H or *at* are used for the rapid determination of necessary excitation.

4·2·1. B-'at' Curves. Fig. 4·1 shows the B - H curves of commonly used magnetic materials. Strictly, each specimen of a material has a unique *B-'at'* relation, but it is a normal practice to use average curves as given in Figs. 4·1—4·4. Figs. 4·2, 4·3 and 4·4 show the *B—H* curves of dynamo grade steel, transformer grade steel and cold rolled grain oriented steel respectively as supplied by Sankey Pressings Division of M/s Guest Keen Williams Ltd..

For work on digital computers, the analytic relations between B and H prove more convenient. Two of the most used mathetical relationships are given below :

$$B=\frac{aH}{1+bH} \qquad \ldots(4\cdot11)$$

and

$$B=\frac{a_0+a_1H+a_2H^2+\ldots\ldots}{1+b_1H+b_2H^2+\ldots\ldots} \qquad \ldots(4\cdot12)$$

where a, b, a_0, a_1, a_2, b_1, b_2 are constants, Eqn. 4·11 gives reasonable and approximate values while Eqn. 4·12 is better overall fit. If alternating magnetization is used, only odd powers of H must be used in Eqn. 4·12.

Non-magnetic materials (like air etc.) have a constant value of permeability and so the *B-'at'* curve for them is a straight line passing through the origin.

For air or any other non-magnetic material, mmf per metre

$$H=\frac{B}{\mu}=\frac{B}{4\pi\times10^{-7}}=800{,}000\ B \qquad \ldots(4\cdot13)$$

where B is in Wb/m².

4·3. Magnetic Leakage. It is impossible to confine all the magnetic flux to a given path (there being no magnetic insulator), and therefore the designer's problem becomes that of providing a path of low reluctance so that comparatively little flux leaks away from the path and then supplying a somewhat larger mmf to compensate for the flux which leaks away. This flux which strays away completes its circuit by paths which prevent its utilization in the functioning of the apparatus or machinery.

For the operation of electric machinery, some air gaps are necessary in the magnetic paths but these air gaps should be kept to a minimum of length and maximum of cross-section so as to reduce their reluctance. A long air gap of small cross-section would

require a large mmf resulting in large coils of many turns and would also result in a tendency for the flux to wander away from its main path. This flux which strays away from the main path is called the **leakage flux**. The leakage flux does not contribute to either transfer or conversion of energy.

However, the leakage flux affects the **performance** of rotating machines and transformers. The leakage flux affects the excitaton demands of salient pole machines, the leakage reactance of windings on which the performance of the a.c. machines is primarily based, the forces between windings, especially under short conditions, voltage regulation of a.c. generators and transformers, commutation conditions in d.c. machines, stray load losses, circulating currents in transformer tank walls and several other performance indices of importance.

For magnetic circuit calculations, a term **'leakage co-efficient'** is introduced in order to take into account the leakage flux. The value of this leakage co-efficient is defined as

Leakage co-efficient,

$$C_l=\frac{\text{useful flux}+\text{leakage flux}}{\text{useful flux}}$$

$$=\frac{\text{total flux}}{\text{useful flux}} \qquad \ldots(4\cdot14)$$

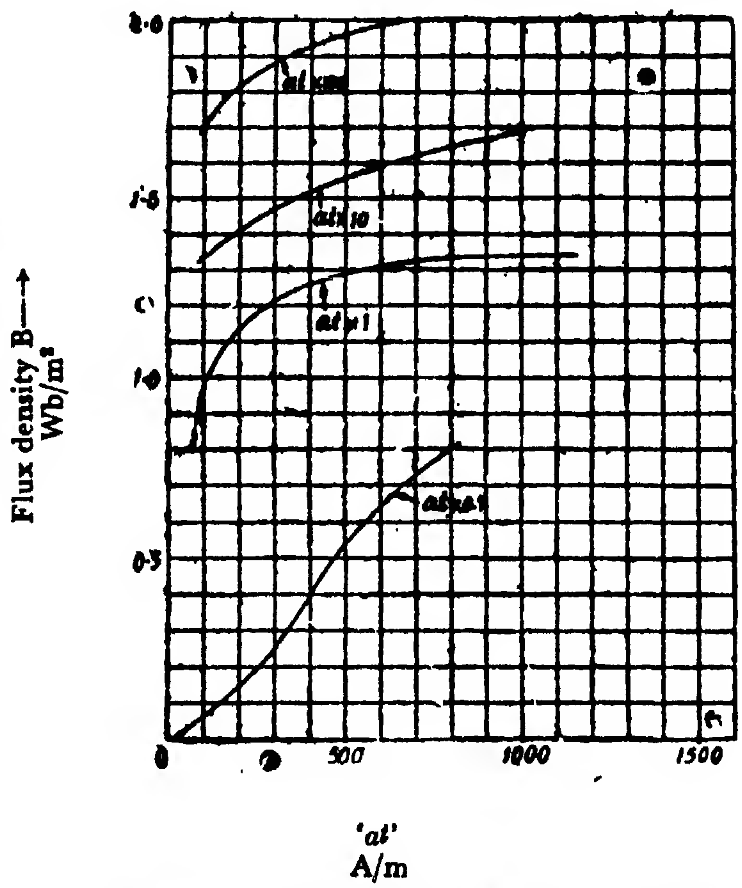


Fig. 4·3. *B—H* curve for electrical steel (non oriented) Grades 203 and 190, Grades 92 and 86 (cold rolled non oriented transformer steel) of M/s G.K.W.

MAGNETIC CIRCUIT CALCULATIONS

4·4. Calculation of Total Mmf. The calculation of total mmf required to establish the requisite flux in a magnetic circuit involves the knowledge of dimensions and configuration of the magnetic circuit. The magnetic circuit is split up into convenient parts which may be connected in series or parallel. The flux density is calculated in every part and mmf per unit length, *'at'* is found by consulting *'B-'at'* curves. The summation of mmfs in series gives the total mmf.

The method looks quite simple but there are some parts in the magnetic circuits, like air gap and tapered teeth which present complex magnetic problems. These problems are solved with special techniques outlined below.

4·4·1. Mmf for Air Gap.

Let L=length of core, l_g=gap length,

y_s=slot pitch, W_s=width of slot,

W_t=width of tooth, W_o=slot opening,

n_d=number of radial ducts, and W_d=width of each duct.

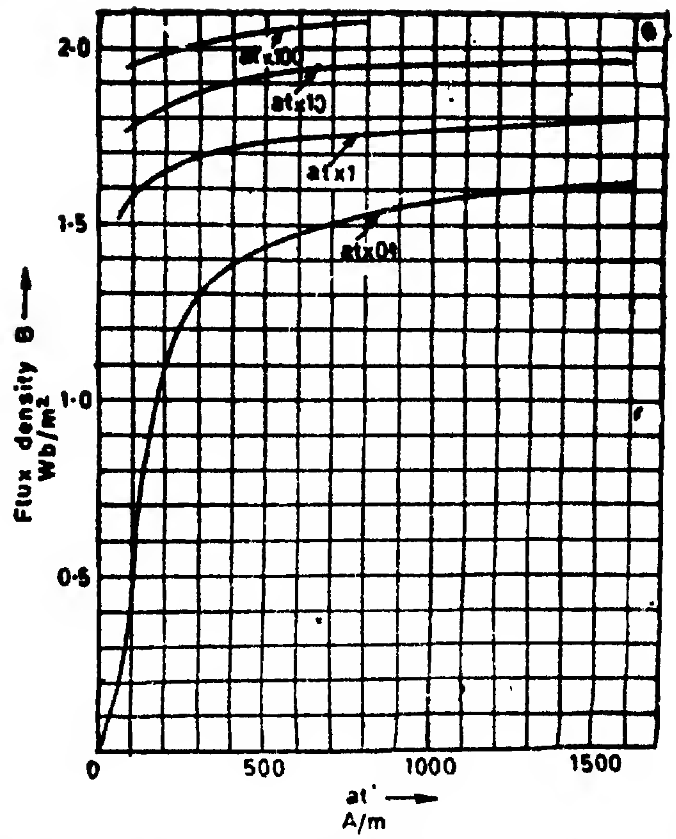


Fig. 4·4. B—H curve for Electrical steel (cold rolled grain oriented) Grade 123 (Grade 56 of M/s G.K.W.)

The iron surfaces around the air gap are not smooth and so the calculation of mmf for the air gap by ordinary methods gives wrong results. The problem is complicated by the fact that :

1. One or both of the iron surfaces around the air gap may be slotted so that the flux tends to concentrate on the teeth rather than distributing itself uniformly over the air gap.

2. There are radial ventilating ducts in the machine for cooling purposes which effect in a similar manner as above.

3. In salient pole machines, the gap dimensions are not constant over whole of the pole pitch.

Consider the iron surfaces on the two sides of the air gap to be smooth as shown in Fig. 4·5. The flux is uniformly spread over the entire slot pitch and goes straight across the air gap.

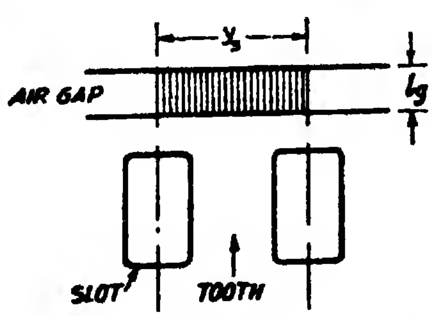


Fig. 4·5

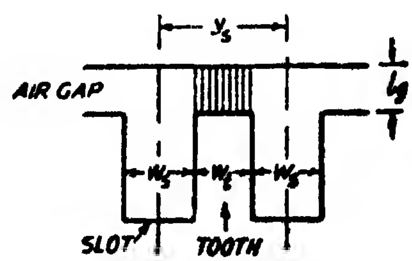


Fig. 4·6

If we confine our attention to only one slot pitch, the reluctance of air gap

$$S_g = \frac{l}{\mu_0 A} = \frac{l_g}{\mu_0 L y_s} \qquad \text{...(4·15)}$$

In a slotted armature, however, the effective area of flux path is substantially decreased resulting in an increase in reluctance of air gap. Consider the case of a slotted armature with a very small gap length as shown in Fig. 4·6. The flux in this case is only confined to the tooth width

$\therefore$ effective or contracted slot pitch

$$y_s' = W_t = y_s - W_s \qquad \ldots(4\cdot16)$$

Reluctance of air gap of a slotted armature

$$S_g = \frac{l_g}{\mu_0 y_s' L} = \frac{l_g}{\mu_0 L(y_s - W_s)} \qquad \ldots(4\cdot17)$$

There is, however, some fringing of flux around the teeth edges in a slotted armature. The flux penetrates down the slot as shown in Fig. 4·7. It is obvious that the reluctance of air gap in this case is more than that in the case of a smooth armature (Fig. 4·5) but lesser than that in the case where the whole flux is assumed to be confined over the tooth width (Fig. 4·6). A simple method to calculate reluctance in this case is to assume that the air gap flux is uniformly distributed over the whole of slot pitch except for a fraction of slot width as shown in Fig. 4·8. This fraction depends upon ratio of slot width to air gap length. Thus the flux of one slot pitch is distributed over $W_t + \delta W_s$.

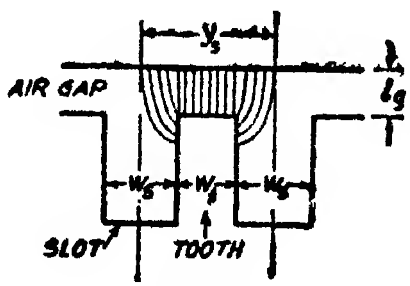


Fig. 4·7

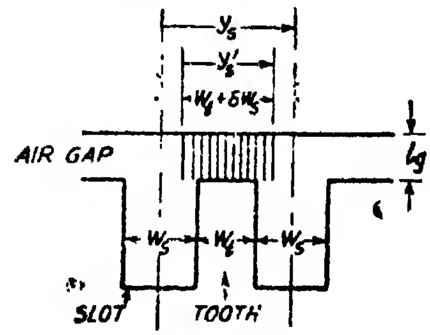


Fig. 4·8

$\therefore$ Effective or contracted slot pitch

$$\begin{aligned} y_s' &= W_t + \delta W_s = W_t + W_s + \delta W_s - W_s \\ &= y_s - (1-\delta)W_s \\ &= y_s - K_{cs} W_s \end{aligned} \qquad \ldots(4\cdot18)$$

where K_{cs} is the Carter's gap co-efficient which depends upon the ratio slot width/gap length. The value of Carter's co-efficient can be taken from Fig. 4·9. An empirical formula which gives the value of K_{cs} directly is :

$$K_{cs} = \frac{1}{1+5\, l_g/W_s} \qquad \ldots(4\cdot19)$$

Another useful relationship which can be used for calculation of Carter's co-efficient for parallel sided open slots is :

$$K_{cs} = \frac{2}{\pi}\left[\tan^{-1} y - \frac{1}{\pi} \log\sqrt{(1+y^2)}\right] \qquad \ldots(4\cdot20)$$

where $y = W_s/2l_g$

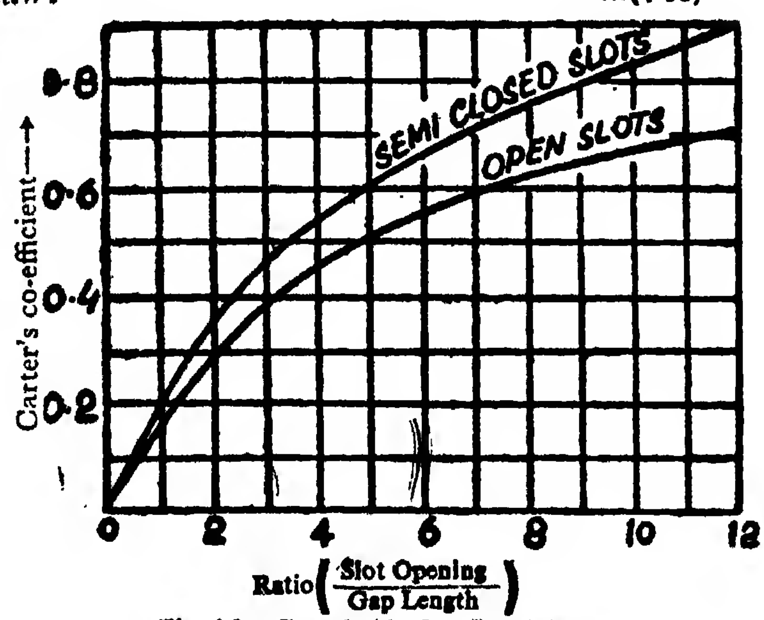


Fig. 4·9. Carter's Air Gap Co-efficient.

Reluctance of air gap with slotted armature

$$S_s = \frac{l_g}{\mu_0 y_s' L} = \frac{l_g}{\mu_0 L(y_s - K_{cs} W_s)} \quad \text{...(4·21)}$$

Let ratio of reluctance of air gap of slotted armature to reluctance of air gap of smooth armature be K_{gs}. Therefore, from Eqns. 4·15 and 4·21,

$$K_{gs} = \frac{y_s}{y_s - K_{cs} W_s} \quad \text{...(4·22)}$$

where K_{gs} is called the **gap contraction factor for slots**. Therefore, the reluctance of air gap with slotted armature is K_{gs} times that with smooth armature and K_{gs} has a value greater than unity.

The provision of radial ventilating ducts results in contraction of flux in the axial direction as shown in Fig. 4·10. It is clear that the effective axial length of the machine is reduced owing to presence of ducts and this resuls in an increase in the reluctance of air gap. We can derive a similar expression for ventilating ducts by treating stacks of laminations as teeth and the ducts as slots.

∴ Contracted or effective axial length

$$L' = L - K_{cd}\, n_d\, W_d \quad \text{...(4·23)}$$

where K_{cd} is the Carter's co-efficient for ducts.

Values of K_{cd}, can be taken from Fig. 4·9 by using ratio (duct width/gap length) in place of ratio (slot width/gap length).

Let the ratio of reluctance of air gap with ducts to reluctance of air gap without ducts be K_{gd}.

$$\therefore \quad K_{gd} = \frac{L}{L - K_{cd} n_d W_d} \quad \text{...(4·24)}$$

K_{gd} is called **gap contraction factor for ducts.**

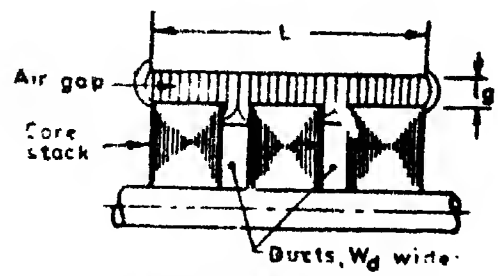


Fig. 4·10. **Effect of radial ventilating ducts.**

When two ventilating ducts, one on stator and the other on rotor are exactly opposite to each other, the Carter's co-efficient must now be based upon ratio $\frac{\text{duct width}}{\frac{1}{2}\text{ gap length}}$, as to an approximation we can ascribe only one half gap to each duct.

The effect of both slotting and ventilating ducts can be allowed for in a single expression. Considering one slot pitch.

Reluctance of air gap of a smooth armature without ducts

$$= \frac{l_g}{\mu_0 L y_s}.$$

Reluctance of air gap of a slotted armature with ducts

$$= \frac{l_g}{\mu_0 L' y_s'}$$

$$\text{Ratio } \frac{\text{reluctance of slotted armature with ducts}}{\text{reluctance of smooth armature without ducts}}$$

$$K_g = \frac{y_s}{y_s'} \cdot \frac{L}{L'} \quad \text{(4·25)}$$

$$= \frac{y_s}{y_s - K_{cs} W_s} \times \frac{L}{L - K_{cd} n_d W_d}$$

$$K_g = K_{gs} \times K_{gd} \quad \text{...(4·26)}$$

where K_g is the ratio of reluctance of air gap of a slotted armature with ducts to reluctance of air gap of a smooth armature without ducts and is called **total gap contraction factor for slots and ducts.**

For induction motors, with slots on both sides of air gap, it is customary to calculate gap contraction factors for both rotor and stator slots.

K_{gss} = gap contraction factor for stator slots,

K_{gsr} = gap contraction factor for rotor slots,

and K_{gs} = total gap contraction factor for slots,

$$=K_{gss}\times K_{gsr}. \qquad ...(4.27)$$

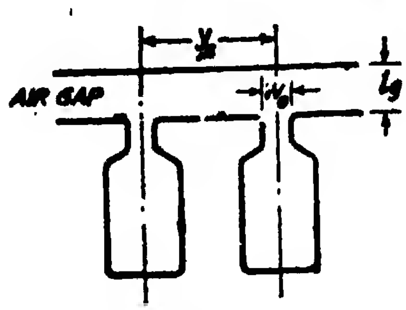


Fig. 4·11

The expressions involving K_{cs} used above apply only to open parallel sided slots of great depth. When slots are shallow, K_{cs} is decreased. Also when semi-enclosed slots are used K_{cs} is increased since now it depends on ratio (slot opening/gap length) rather than on ratio (slot width/gap length). (See Fig. 4·11)

Fig. 4·9 also shows values of Carter's co-efficient for semi-enclosed slots. Eqns. 4·18 and 4·22 are modified for semi-enclosed slot as :

Contracted or effective slot pitch $y_s'=y_s-K_{cs}W_o$ ...(4·28)

Gap contraction factor $K_{gs}=\dfrac{y_s}{y_s-K_{cs}W_o}$ ...(4·29)

The reluctance of air gap with slotted armatures is higher than with smooth armatures. The ratio of the two reluctances is equal to K_g, the gap contraction factor. In other words, the mmf required for the gap with slotted armatures is K_g times the mmf required for gap with smooth armatures. From Eqn. 4·11,

Mmf per metre for air gap = 800,000 B

∴ Mmf required for air gap having a length l_g metre, with smooth armatures,

$$AT_g=\text{mmf/m}\times\text{length}=800{,}000\ B\ l_g \qquad ...(4.30)$$

∴ Mmf required for air gap, having a length l_g, with slotted armatures,

$$AT_g=800{,}000\ K_g\ B\ l_g \qquad ...(4.31)$$

Eqn. 4·31 can be written as :

$$AT_g=800{,}000\ K_g\frac{\Phi}{A_g}l_g$$

where Φ = flux per pole, Wb ;

A_g = actual area of air gap per pole, m².

Now $$AT_g=800{,}000\ \frac{\Phi}{A_g/K_g}l_g$$

$$=800{,}000\ \frac{\Phi}{A_g'}l_g \qquad ...(4.32)$$

where $$A_g'=A_g/K_g \qquad ...(4.33)$$

This means that there is a contraction in the air gap area and the gap area has contracted to a value $A_g'=A_g/K_g$.

Area of gap per pole

$$A_g = \text{slot per pole} \times \text{slots pitch} \times \text{core length}$$

$$= \frac{S}{p} \times y_s \times L \qquad \ldots(4{\cdot}34)$$

where S = total number of slots and p = number of poles.

Contracted or effective gap area per pole

$$A_g' = \frac{A_g}{K_g} = \frac{(S/p) \times y_s \times L}{Ly_s/L'y_s'} \text{ as } K_g = \frac{Ly_s}{L'y_s'}$$

$$= \frac{S}{p} \times y_s' \times L' \qquad \ldots(4{\cdot}33)$$

The mmf required for a smooth armature is :

$$AT_g = 800{,}000\ B\ l_g$$

while for a slotted armature the air gap mmf is :

$$AT_g = 800{,}000\ K_g B l_g = 800{,}000\ B\ (K_g l_g)$$

The above relation may be interpreted as that the length of air gap is increased K_g times (instead of saying that area is reduced to $1/K_g$ times as earlier) due to the provision of slots and ducts. Thus effective gap length

$$l_{ge} = K_g l_g \qquad \ldots(4{\cdot}36)$$

and therefore, K_g in this case is called **"gap expansion factor"**.

Effect of Saliency. In the case of salient pole machines, the length of air gap is not constant over the whole pole pitch. This gives rise to different values of air gap density over the pole pitch. Thus to know the value of reluctance of the air gap, it is necessary to know the distribution of magnetic field in air gap. The methods for the determination of flux density distribution are given later in this chapter.

Fig. 4·12 shows a typical flux distribution curve for a salient pole machine.

Fig. 4·12 (a) shows the flux tubes passing from field to armature. The equivalent electric circuit for this magnetic circuit is a number of resistances connected in parallel with each resistance representing a flux tube. In parallel electric circuits, it is sufficient to know the value of only one resistance and the current flowing through it in order that the voltage across the circuit be known.

Similarly by analogy, we have only to know the reluctance of one flux tube and the flux flowing through it in order to find the mmf required for air gap.

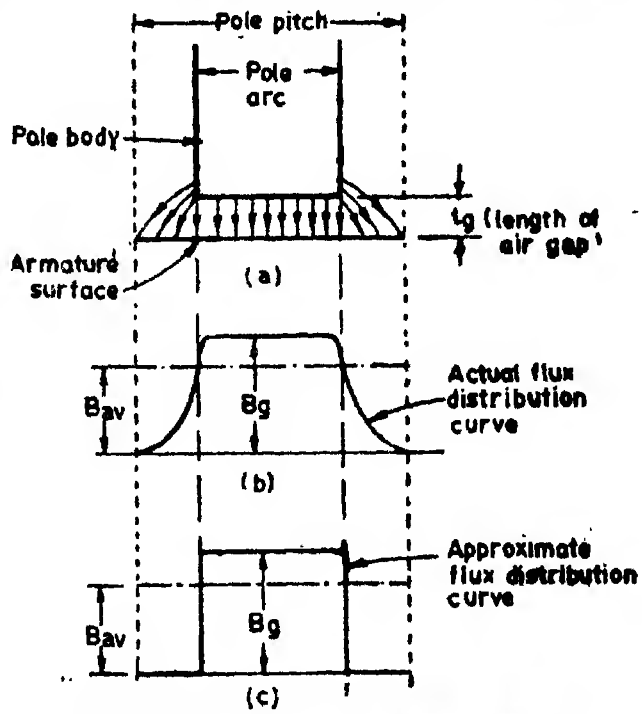


Fig. 4·12. Flux density distribution curve in salient pole machines.

$\therefore$ Mmf required for air gap of a salient pole machine

$$AT_g = \text{flux in a flux tube} \times \text{reluctance of flux tube.}$$

Let us consider a flux tube at the centre of the pole.

Flux in the flux tube at the centre

$$=\text{flux density}\times\text{area of flux tube}$$

$$=B_g\times\text{area of flux tube.}$$

Reluctance of flux tube at the centre

$$=\frac{\text{effective length of air gap at the centre}}{\mu_0\times\text{area of flux tube at the centre}}$$

$$=\frac{K_g\, l_g}{4\pi\times10^{-7}\times\text{area of flux tube at the centre}}$$

$\therefore$ $$AT_g=B_g\times\text{area of flux tube at the centre}\times\frac{K_g\, l_g}{4\pi\times10^{-7}\times\text{area of flux tube at the centre}}$$

$$=800{,}000\ B_gK_gl_g \qquad \ldots(4\cdot37)$$

where K_g=gap contraction factor for a gap length at the centre of the pole.

The flux tube at the centre of pole is chosen because its actual length is known. The length of the flux tube at the centre of the pole is exactly equal to the length of air gap there. Therefore, the value of K_g is based upon the air gap length l_g.

The **field form factor** K_f is defined as :

$$K_f=\frac{\text{average gap density over the pole pitch}}{\text{maximum flux density in the gap}}=\frac{B_{av}}{B_g}\cdot \qquad \ldots(4\cdot38)$$

We have $$B_{av}=\frac{\text{flux per pole}}{\text{area per pole}}=\frac{\Phi}{\tau L}=\frac{\Phi}{(\pi D/p)L}$$

where τ=pole pitch$=\pi D/p$

The value of B_g can be calculated after determining the value of field form factor K_f. The methods determining K_f are given later in this chapter. Fig. 4·12 (c) shows, an approximate flux distribution curve for a salient pole machine. From this,

field form factor $$K_f=\frac{\text{average flux density}}{\text{maximum flux density}}$$

$$\simeq\frac{\text{pole arc}}{\text{pole pitch}}=\psi \qquad \ldots(4\cdot39)$$

The assumption $K_f=\psi$ is fairly correct for machines with normal proportions and a fair degree of saturation. With long air gaps the pole arc may be considered as extended by say, 4 l_g to take account of fringing.

Example 4·1. *The stator of a machine has a smooth surface but its rotor has open type of slots with slot width W_s=tooth width, W_t=12 mm. and the length of air gap l_g=2 mm. Find the effective length of air gap if the Carter's co-efficient=$\frac{1}{1+5\ l_g/W_s}$. There are no radial ducts.*

Solution. Carter's co-efficient for slots

$$K_{cs}=\frac{1}{1+5\times2/12}=0\cdot545.$$

Slot pitch $y_s=W_s+W_t=24$ mm.

From Eqn. 4·12, gap contraction for slots

$$K_{gs}=\frac{y_s}{y_s-K_{cs}W_s}=\frac{24}{24-0\cdot545\times12}=1\cdot37.$$

Since there are no ducts, gap contraction factor for ducts, K_{gd}=1.

From Eqn. 4·26, total gap contraction factor $K_g = K_{gs} \times K_{gd} = 1{\cdot}37 \times 1 = 1{\cdot}37$.

$\therefore$ Effective gap length (see Eqn. 4·36) $l_{gs} = K_g l_g = 1{\cdot}37 \times 2 = 2{\cdot}74$ mm.

Example 4·2. *Calculate the mmf required for the air gap of a machine having core length=0·32 m including 4 ducts of 10 mm each. pole arc=0·19 m ; slot pitch=65·4 mm ; slot opening=5 mm ; air gap length=5 mm ; flux per pole=52 mWb. Given Carter's co-efficient is 0·18 for opening/gap=1, and is 0·28 opening/gap=2.*

Solution. Ratio $\dfrac{\text{slot opening}}{\text{gap length}} = \dfrac{0{\cdot}5}{0{\cdot}5} = 1$

$\therefore$ Carter's co-efficient for slots $K_{cs} = 0{\cdot}18$.

This is a salient pole machine with semi-enclosed slots.

Gap contraction factor for slots $K_{gs} = \dfrac{y_s}{y_s - K_{cs} W_0}$ (see Eqn. 4·29)

$$= \frac{65{\cdot}4}{65{\cdot}4 - 0{\cdot}18 \times 5} = 1{\cdot}014$$

Ratio $\dfrac{\text{duct width}}{\text{gap length}} = \dfrac{1}{0{\cdot}5} = 2.$

$\therefore$ Carter's co-efficient for ducts $K_{cd} = 0{\cdot}28$.

Gap contraction factor for ducts

$$K_{gd} = \frac{L}{L - K_{cd} n_d W_d} \text{ (See Eqn. 4·24)}$$

$$= \frac{0{\cdot}32}{0{\cdot}32 - 0{\cdot}28 \times 4 \times 1 \times 10^{-3}} = 1{\cdot}036$$

Total gap contraction factor $K_g = 1{\cdot}014 \times 1{\cdot}036 = 1{\cdot}05$.

Flux density at the centre of pole $B_g = \dfrac{\text{flux/pole}}{\text{pole arc} \times \text{core length}}$

$$= \frac{52 \times 10^{-3}}{0{\cdot}19 \times 0{\cdot}32} = 0{\cdot}854 \text{ Wb/m}^2.$$

$\therefore$ Mmf required for air gap $AT_g = 800{,}000\ K_g\ B_g\ l_g$

$$= 800{,}000 \times 1{\cdot}05 \times 0{\cdot}854 \times 5 \times 10^{-3} = 3587 \text{ A.}$$

Example 4·3. *Estimate the effective gap area per pole of a 10 pole, slip ring induction motor with following data :*

stator bore =0·65 m, core length=0·25 m, No. of stator slots=90
stator slot opening=3 mm, rotor slots=120
rotor slot opening=3mm, air gap length=0·95 mm
Carter's co-efficient for ducts=0·68, Carter's co-efficient for slots=0·46
number of ventilating ducts=3 each on rotor and stator,
width of each ventilating duct=10 mm.

Solution. Stator slot pitch $= \dfrac{\pi \times 650}{90} = 22{\cdot}7$ mm.

Gap contraction factor for stator slots

$$K_{gss} = \frac{y_s}{y_s - W_0 K_{cs}} = \frac{22{\cdot}7}{22{\cdot}7 - 3 \times 0{\cdot}46} = 1{\cdot}065.$$

Rotor diameter $=0.65-2\times0.95=0.6481$ m

Rotor slot pitch $=\dfrac{\pi\times0.6481}{120}=0.01697$ m

Gap contraction factor for rotor slots $K_{gsr}=\dfrac{16.97}{16.97-3\times0.46}=1.089$

Gap contraction factor for slots $K_{gs}=K_{gss}\times K_{gsr}=1.065\times1.089=1.16.$

Gap contraction factor for ducts $K_{gd}=\dfrac{L}{L-K_{cd}n_dW_d}$

$$=\frac{250}{250-0.68\times3\times10}=1.089.$$

Total gap contraction factor $K_g=K_{gs}\times K_{gd}=1.16\times1.089=1.26$

Actual area of air gap per pole $A_g=\dfrac{\pi\times0.65}{10}\times0.25=51.05\times10^{-3}$ m².

From Eqn. 4.33, effective air gap area per pole

$$A_g'=A_g/K_g=51.05\times10^{-3}/1.26=40.52\times10^{-3}\text{ m}^2.$$

Example 4.4. *A 175 MVA, 20 pole water wheel generator has a core of length 1.72 m and a diameter of 6.5 m. The stator slots (open) have a width of 22 mm, the slot pitch being 64 mm and the air gap length at the centre of the pole is 30 mm. There are 41 radial ventilating ducts each 6 mm wide. The total mmf per pole is 27000 A. The mmf required for the air gap is 87% of the total mmf per pole. Estimate the average flux density in the air gap if the field form factor is 0.7.*

The Carter's co-efficient can be calculated from the following relationship

$$K_c=\frac{2}{\pi}\left[\tan^{-1}y-\frac{1}{y}\log\sqrt{1+y^2}\right]$$

where $y=W_s/l_g$ for slots

$=W_d/2\,l_g$ for ducts

W_s, W_d are widths of slot and duct respectively and l_g is length of air gap.

Solution.

$$\text{Ratio }\frac{\text{slot width}}{2(\text{gap length})}=y=\frac{22}{2\times30}=0.367.$$

Carter's co-efficient for slots

$$K_{cs}=\frac{2}{\pi}\left[\tan^{-1}y-\frac{1}{y}\log\sqrt{1+y^2}\right]$$

$$=\frac{2}{\pi}\left[\tan^{-1}0.367-\frac{1}{0.367}\log\sqrt{1+0.367^2}\right]=0.176$$

$\therefore$ Gap contraction factor of slots $K_{gs}=\dfrac{y_s}{y_s-K_{cs}W_s}=\dfrac{64}{64-0.176\times22}=1.1064$

$$\text{Ratio }\frac{\text{duct width}}{2(\text{gap length})}=y=\frac{6}{2\times30}=0.1$$

$\therefore$ Carter's co-efficient for ducts

$$K_{cd}=\frac{2}{\pi}\left[\tan^{-1}0.1-\frac{1}{0.1}\sqrt{1+0.1^2}\right]=0.05.$$

$\therefore$ Gap contraction factor for ducts

$$K_{gd}=\frac{L}{L-K_{cd}\,n_d\,W_d}=\frac{1720}{1720-0.05\times41\times6}=1.007$$

Total gap expansion factor $K_g=1.1064\times1.007=1.114.$

Mmf required for air gap $AT_g=0.87\times27000=23490$ A,

But $AT_g=800{,}000\,K_g\,B_g\,l_g$.

Hence maximum flux density in air gap

$$B_g=\frac{AT_g}{800{,}000\,K_g\,l_g}=\frac{23490}{800{,}000\times1{\cdot}114\times30\times10^{-3}}=0{\cdot}8786\ \text{Wb/m}^2$$

Average flux density in the air gap

$$B_{av}=K_f\,B_g=0{\cdot}7\times0{\cdot}8786=0{\cdot}615\ \text{Wb/m}^2.$$

4·4·2. Net Length of Iron. The cores of magnetic circuits are built up with laminated steel plates wherever required. These laminations or stampings are insulated from each other by paper, stuck to one side of the lamination, Kaolin clay or enamel. Moreover in order to have an effective cooling of the machine, the length of the core is divided into packets of about 40 to 80 mm width separated by vent spacers. These vent spacers form ventilating ducts through which air is circulated. These ducts are radial as shown in Fig 4·10 and their width normally varies from 8 to 10 mm.

From above it is clear that whole of the length is not occupied by iron ; some part of the length is taken up by ventilating ducts and some part by insulation between steel laminations and air spaces created by irregularities in thickness of laminations. It is usual to define iron space factor, called **stacking factor**, as the ratio of actual length of iron in a stack of assembled core plates to total axial length of stack.

Gross iron length L_i=length of slot portion conductor

=core length−length of ventilating ducts

$$=L-n_dW_d \qquad \ldots(4{\cdot}40)$$

Net iron length $$L_i=K_i(L-n_dW_d) \qquad \ldots(4{\cdot}41)$$

where K_i=stacking factor for iron which largely depends upon thickness of plates and the type of insulating material employed. The manufacturers specify the stacking factor for a single lamination. The stacking factor depends upon the thickness of core and the thickness and the type of insulation used for laminations. The stacking factor for built up cores is smaller and an average value of 0·9 may be assumed for all practical purposes.

4·4·3. Mmf for Teeth. The calculation of mmf necessary to maintain the flux in the teeth is difficult owing to the following complex problems :

1. The teeth are wedge-shaped or tapered when parallel sided slots are used. This means that the area presented to the path of flux is not constant and this gives different values of flux density over the length of teeth.

2. The slot provides another parallel path for the flux, shunting the tooth. The teeth are normally worked in the saturation region and therefore their permeability is low, and as a result an appreciable portion of the flux goes down the depth of the slots. The presence of two parallel paths, the reluctance of one part depending upon the degree of saturation in the other, makes the problem intricate.

Tapered Teeth. The mmf required for teeth can be easily calculated whatever may be their shape, if the flux going down the slot is neglected. The correction, to take slot flux into account, can be incorporated later on.

Following are the methods usually employed for the calculation of mmf required for tapered teeth.

(a) *Graphical Method.* The mmf per metre for the whole length of tooth is not uniform as the flux density is not the same everywhere. Therefore, to obtain correctly the value of total mmf, it is necessary to construct a graph showing the manner in which 'at' varies over the length of the tooth. The mean ordinate of this graph gives the equivalent 'at' for the whole of the tooth. The total mmf for the teeth is given by $\int H\,dl$, the integration being carried out for the complete height of tooth.

Therefore, total mmf required for the tooth,

AT_t = mean ordinate × height of tooth

$= at_{mean} \times l_t = at_{mean} \times d_s$ ...(4·42)

The height of tooth l_t is equal to d_s, the depth of slot.

To determine this at_{mean}, it is necessary to construct first a graph showing the manner in which the flux density varies. From the known value of flux per tooth, the flux density is evaluated for a number of sections along the length of the tooth from tip to root (Fig.4.13). The corresponding values of 'at' are found from the B—'at' curve of the material and are plotted. The value of at_{mean} is obtained from the graph, as shown.

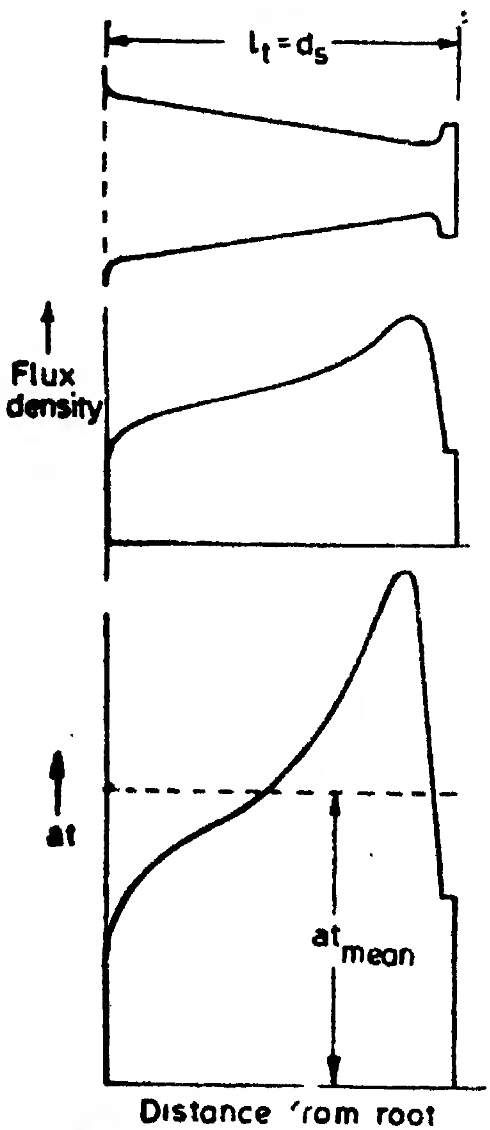


Fig. 4·13. B-at curve of tapered tooth and calculation of at_{mean}

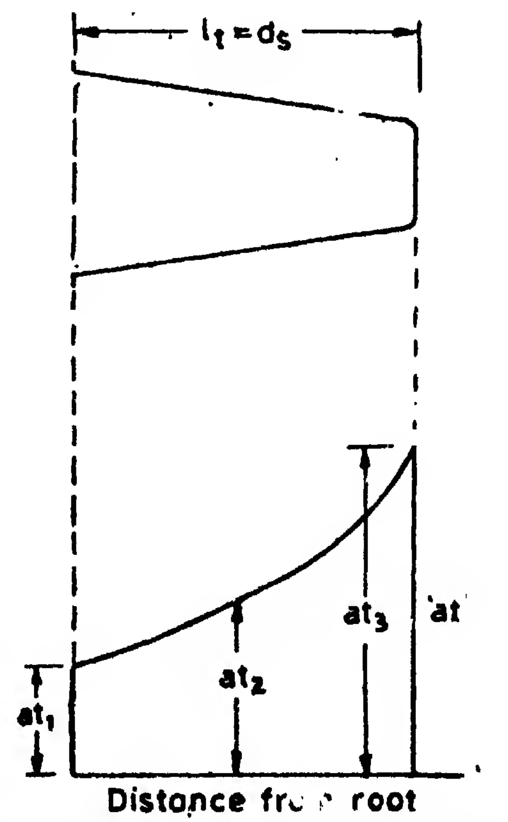


Fig. 4·14. Calculation of at_{mean} for tapered tooth using Simpson's rule

(*b*) *Three ordinate Method* (*Simpson's rule*). This method can be applied to teeth of very simple form and of a small taper and is based upon the assumption, that the curve relating 'at' with flux density, is a parabola. In this method, values of 'at' are obtained at three equidistant points, the ends of the tooth and its centre,

The mean value of 'at' is given by :

$$at_{mean}=\frac{at_1+4at_2+at_3}{6} \qquad \ldots(4.43)$$

where at_1, at_2, at_3 are the values of 'at' for the three sections shown in Fig 4.14.

(c) $B_{t1/3}$ ***Method.*** This method is applied to teeth of small taper and is based upon the assumption that value of 'at' obtained for flux density at a section 1/3 of tooth height from the narrow end is the mean of 'at' for whole of the tooth. This method is the most simple of all the methods and results are sufficiently accurate if the teeth are worked at low saturation.

Let $B_{t1/3}$=flux density at 1/3 height from narrow end,

$at_{1/3}$=value of mmf per metre for $B_{t1/3}$ as obtained from $B-$'at' curve

$\therefore$ Total mmf for teeth $AT_t=at_{1/3}\times l_t=at_{1/3}\times d_t \qquad \ldots(4.44)$

4·4·4. Real and Apparent Flux Densities. It has already been stated above that the slot provides an alternative path for the flux to pass, although the flux entering an armature from the air gap follows paths principally in iron. If the teeth density is high, the mmf acting across the teeth is very large and as the slots are in parallel (Fig. 4.15) with the teeth, this mmf acts across the slots also. Thus at saturation densities, the flux passing through the slots becomes large cannot be neglected, and any calculation based upon 'no slot flux' leads to wrong results. This means that the real flux passing through the teeth is always less than the total or **apparent flux**. As a result, the **'real flux density'** in the teeth is always less than the **'apparent flux density'**.

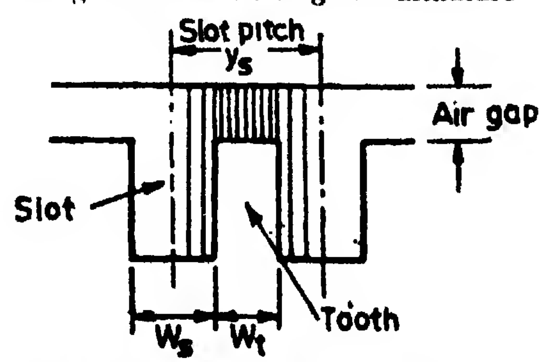


Fig. 4.15. Slot flux due to saturation in teeth.

The apparent flux density is defined as :

$$B_{app}=\frac{\text{total flux in a slot pitch}}{\text{tooth area}}$$

and the real flux density as :

$$B_{real}=\frac{\text{actual flux in a tooth}}{\text{tooth area}}.$$

In an actual machine, taking the flux over one slot pitch, there are two parallel paths,

1. **Iron path.**

Area of iron path A_t=tooth width$\times$net iron length$=W_t\times L_i$

2. **Air path.**

Area of air path A_a=total area$-$iron area

$=$(slot pitch$\times$core length)$-$(tooth width$\times$net iron length)$=y_s\times L-W_t\times L_i$.

If Φ_s is the flux over one slot pitch, we have :

$$\Phi_s=\Phi_t+\Phi_a$$

where Φ_t=flux passing through iron over a slot pitch,

Φ_a=flux passing through air over a slot pitch.

$$\therefore \quad B_{app}=\frac{\text{total flux over a slot pitch}}{\text{iron area over a slot pitch}}=\frac{\Phi_s}{A_t}$$

$$=\frac{\Phi_t}{A_t}+\frac{\Phi_a}{A_t}$$

$$=B_{real}+\frac{\Phi_a}{A_a}\cdot\frac{A_a}{A_t}$$

$$=B_{real}+B_aK \qquad (4\cdot45)$$

B_a=flux density in air$=\mu_0 H=4\pi\times10^{-7}$ at_{real}

where at_{real}=mmf per metre across the tooth for tooth density B_{real}.

$$\text{and } K=\text{ratio } \frac{A_a}{A_t}=\frac{\text{air area}}{\text{iron area}}=\frac{Ly_s-L_iW_t}{L_iW_t} \qquad ...(4\cdot46)$$

$$\therefore \quad B_{real}=B_{app}-4\pi\times10^{-7}\,'at'_{real}\,K \qquad ...(4\cdot47)$$

$$=B_{app}-4\pi\times10^{-7}\,'at'\,(K_s-1) \qquad ...(4\cdot48)$$

$$\text{where } K_s=1+K=\frac{\text{total area}}{\text{iron area}}=\frac{Ly_s}{L_iW_t} \qquad ...(4\cdot49)$$

(It should be borne in mind that the value $'at'_{real}$ used above is corresponding to the real flux density B_{real}). The slot and the tooth form a parallel magnetic circuit and therefore the mmf across them is the same. Since the length of flux path through slot and tooth is the same and hence mmf per metre length for both is the same. The mmf per metre in tooth is $'at'_{real}$ corresponding to real flux density B_{real} the mmf per metre across the slot is also $'at'_{real}$).

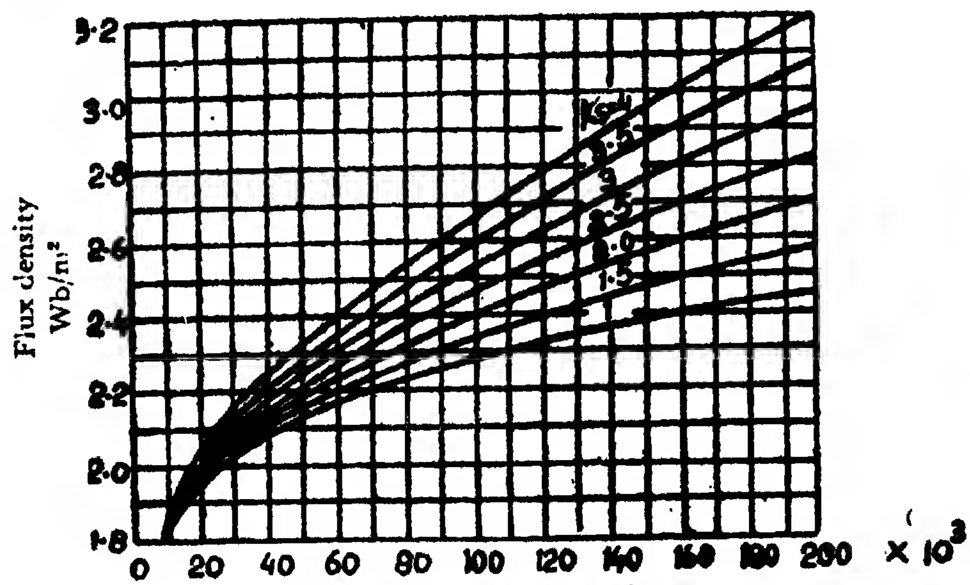


Fig. 4·16. Mmf per metre 'at' A/m

Eqn. 4·48. cannot be easily solved as there are two unknowns, B_{real} and $'at'_{real}$, the latter depends upon the value of former. In order to solve this equation for machine with any dimensions it is desirable to have data available which immediately gives the values of $'at'_{real}$ and B_{real} for various values of B_{app} and K_s. Fig. 4·16 shows a family of curves from which B_{real} and 'at' can readily be obtained. These curves correspond to $B-H$ curve of ordinary steel laminations given in Fig. 4·1.

There is another method available for obtaining the values of B_{real} and 'at'. This method is comparatively lengthy but useful for individual cases.

Eqn. 4·48. involves two unknowns and so for solution, another equation is needed.

The second relation is defined by the B—'at' curve of magnetic material used for the teeth. B—'at' curve is drawn and the straight line $B_{app} = B_{real} + 4\pi \times 10^{-7}$ 'at'$_{real}$ $(K_s - 1)$ is laid out as shown in Fig. 4·17. When 'at'$_{real} = 0$, $B = B_{app}$. This corresponds to point A. When $B_{real} = 0$, 'at'$_{real} = \dfrac{B_{app}}{4\pi \times 10^{-7}(K_s - 1)}$. This corresponds to point A'.

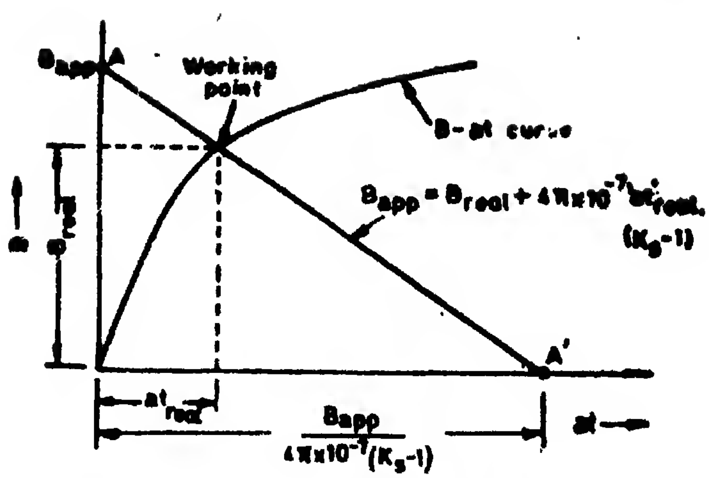


Fig. 4·17. Determination of B_{real}

A straight line is drawn connecting points A and A'. The intersection of straight line AB with B—'at' curve gives the value of B_{real} and the corresponding value of 'at'$_{real}$. The intersection of the two curves gives the value of B_{real} and the corresponding value of 'at'$_{real}$.

Example 4·5. *A laminated tooth of armature steel in an electrical machine is 30 mm long and has a taper such that maximum width is 1·4 times the minimum. Estimate the mmf required for a mean flux density of 1·9 Wb/m² in this tooth. Use Simpson's rule. The B-'at' curve for the material of tooth is :*

B Wb/m²	1·6	1·8	1·9	2·0	2·1	2·2	2·3
'at' A/m	3700	10,000	17,000	27,000	41,000	70,000	109,000

Solution. Let W_{t1}, W_{t2} and W_{t3} be respectively the maximum, mean and minimum widths of tooth.

$$W_{t1} = 1{\cdot}4\ W_{t3} \text{ (given)}; \quad W_{t2} = \frac{W_{t1} + W_{t3}}{2} = 1{\cdot}2\ W_{t3}$$

Flux density at any section of tooth $B_t = \dfrac{\text{flux/tooth}}{\text{area of section}}$

$$= \frac{\text{flux/tooth}}{\text{net iron length} \times \text{width of tooth}}$$

∴ The flux density at any section of the tooth is inversely proportional to the tooth width.

Let B_{t1}, B_{t2} and B_{t3} be respectively the minimum, mean and maximum flux densities.

∴ $B_{t1} = B_{t2} W_{t2} / W_{t1} = 1{\cdot}9 \times 1{\cdot}2 / 1{\cdot}4 = 1{\cdot}63$ Wb/m² ; $B_{t2} = 1{\cdot}9$ Wb/m²

and $B_{t3} = B_{t2} W_{t2} / W_{t3} = 1{\cdot}9 \times 1{\cdot}2 = 2{\cdot}28$ Wb/m².

The B-at curve is plotted in Fig. 4·18. From this curve the values of mmf per metre for flux densities $B_{t1} = 1{\cdot}63$ Wb/m², $B_{t2} = 1{\cdot}9$ Wb/m² and $B_{t3} = 2{\cdot}28$ Wb/m² are respectively :

$at_1 = 4500$ A, $at_2 = 17000$ A, $at_3 = 100{,}500$ A

Applying simpson's rule, mean value of mmf per metre

$$at_{mean} = \frac{at_1 + 4at_2 + at_3}{6} \qquad \text{(See Eqn. 4·43)}$$

$$= \frac{4500 + 4 \times 17000 + 100{,}500}{6} = 28833 \text{ A/m.}$$

∴ **Total mmf required** $= at_{mean} \times l_t = 28833 \times 30 \times 10^{-3} = 865$ A

Example 4·6. *The armature of a d.c. machine has a diameter of 0·2 m, 29 parallel slots each 6 mm wide and 25 mm deep ; the core is 0·15 m long with 1 duct 10 mm wide, and the air gap 5 mm long. Insulation on the stampings is 10% of the thickness.*

If the maximum flux density under the pole is 0·9 Wb/m², determine the mmf required to overcome the reluctance of gap and teeth. Carter's co-efficient for the slots is 0·275 and for the slots is 0·275 and for the duct 0·39. The magnetization curve for the iron is as follows :

B Wb/m²	1·4	1·6	1·8	2·0	2·2	2·3
H A/m	1800	3000	6500	19400	63000	112000

Solution. Slot pitch at the gap surface $y_s = \frac{\pi D}{S} = \frac{\pi \times 0 \cdot 2}{29}$ m $= 21 \cdot 67$ mm.

Since the insulation on laminations is 10% of the thickness of the laminations, the stacking factor K_i is 0·9.

$$\text{Gap contraction factor for slots } K_{gs} = \frac{21 \cdot 67}{21 \cdot 67 - 0 \cdot 275 \times 6} = 1 \cdot 082$$

$$\text{Gap contraction factor for ducts } K_{gd} = \frac{150}{150 - 0 \cdot 39 \times 1 \times 10} = 1 \cdot 027$$

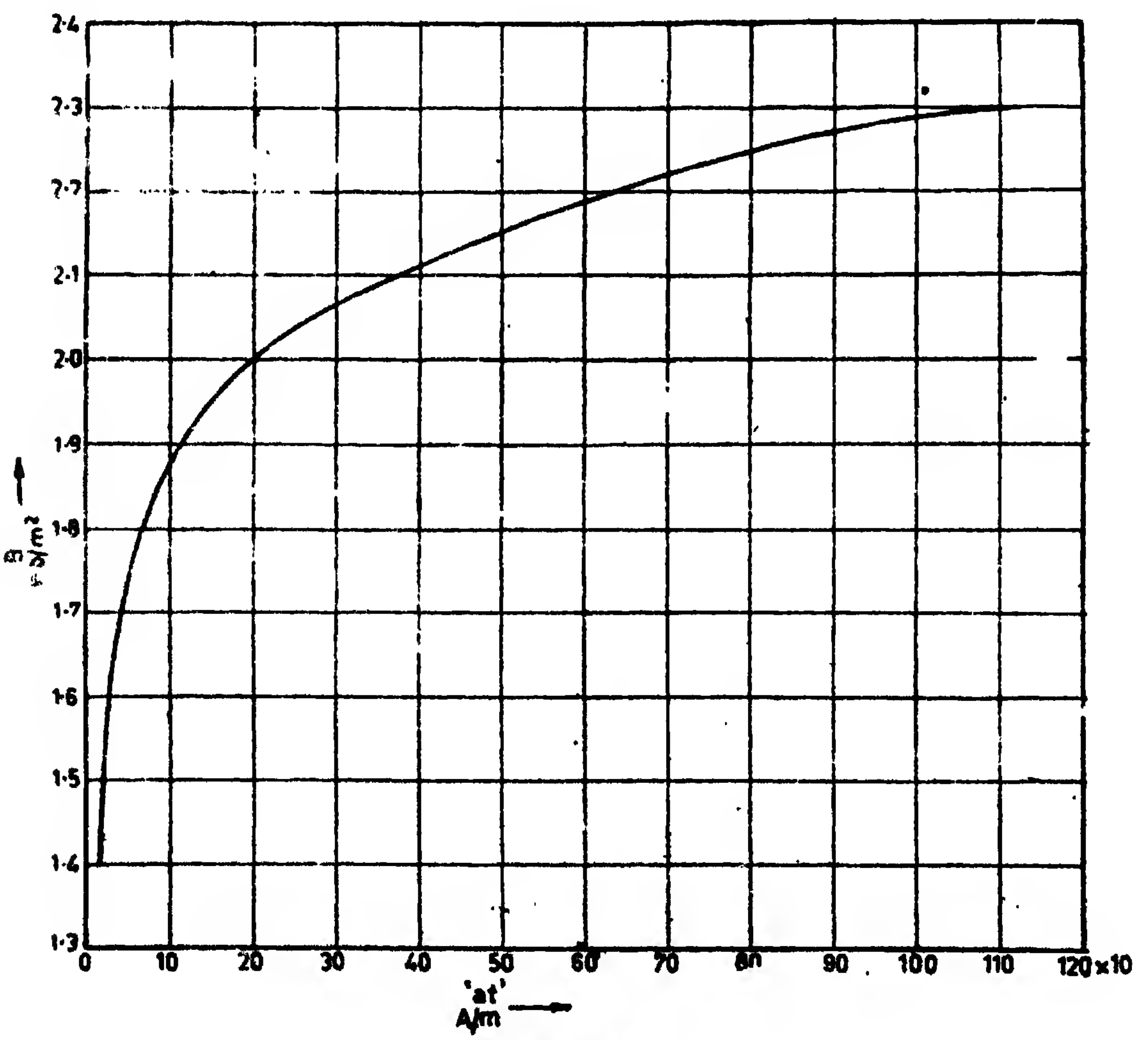


Fig. 4·18

Gap contraction factor $K_g=1{\cdot}082\times1{\cdot}027=1{\cdot}111$

$\therefore$ Mmf required for air gap $AT_g=800{,}000\,K_g\,B_g\,l_g$

$=800{,}000\times1{\cdot}111\times0{\cdot}9\times3\times10^{-3}=2400$ A.

The B-'at' curve for the ferromagnetic material used is plotted in Fig. 4·18. The mmf required for the teeth is calculated by using simpson's rule and $B_{t\frac{1}{3}}$ method.

The flux over one slot pitch passes through one tooth.

Flux over a slot pitch $\Phi_t=B_g\,y_s\,L=0{\cdot}9\times21{\cdot}67\times10^{-3}\times0{\cdot}15=2{\cdot}925\times10^{-3}$ Wb.

Simpson's Rule Top of Tooth. Slot pitch $y_s=21{\cdot}67$ mm.

Tooth width $W_{t_1}=21{\cdot}67-6=15{\cdot}67$ mm.

Net iron length $L_i=K_i(L-n_dW_d)=0{\cdot}09(0{\cdot}15-1\times1\times10^{-2})=0{\cdot}1341$ m.

$$\text{Flux density } B_{t_1}=\frac{\phi_t}{L_iW_{t_1}}=\frac{2{\cdot}925\times10^{-3}}{0{\cdot}1341\times15{\cdot}67\times10^{-3}}=1{\cdot}39\text{ Wb/m}^2$$

From Fig. 4·18 $at_1=1790$ A/m

Middle of Tooth. Diameter $=0{\cdot}2-0{\cdot}025=0{\cdot}175$ m.

Slot pitch $y_{s2}=\dfrac{\pi\times0{\cdot}175}{29}$ m $=18{\cdot}96$ mm.

Tooth width $W_{t2}=18{\cdot}96-6=12{\cdot}96$ mm.

Flux density $B_{t2}=B_{t_1}\times W_{t_1}/W_{t2}=1{\cdot}39\times\dfrac{15{\cdot}67}{12{\cdot}96}$

$=1{\cdot}681$ Wb/m²

From Fig. 4·18 at_2 $=4000$ A/m

Root of Tooth. Diameter $=0{\cdot}2-2\times0{\cdot}025=0{\cdot}15$ m.

Slot pitch $y_{s3}=\dfrac{\pi\times0{\cdot}15}{29}$ m $=16{\cdot}25$ mm.

Tooth width $W_{t3}=16{\cdot}25-6=10{\cdot}25$ mm.

Flux density $B_{t3}=B_{t_1}\times W_{t_1}/W_{t3}=1{\cdot}39\times15{\cdot}67/10{\cdot}25=2{\cdot}125$ Wb/m².

From Fig. 4·18 $at_3=42500$

By simpson's rule, $at_{mean}=\dfrac{at_1+4at_2+at_3}{6}=\dfrac{1790+4\times4000+42500}{6}$

$=10050$ A/m.

$\therefore$ Mmf required by teeth $AT_t=at_{mean}\times d_s=10500\times25\times10^{-3}=263$ A.

$\therefore$ Total mmf required for gap and teeth $=AT_g+AT_t=2400+263=2663$ A.

Diameter at $\frac{1}{3}$ height from root of tooth $=D-2\times2/3\,d_s$

$=0{\cdot}2-2\times2/3\times0{\cdot}025=0{\cdot}1667$ m.

Slot pitch $=\dfrac{\pi\times0{\cdot}1667}{29}$ m $=18{\cdot}06$ mm.

Tooth width $W_{t_1/3}=18{\cdot}06-6=12{\cdot}06$ mm.

Flux density $B_{t_1/3}=B_{t_1}\times W_{t_1}/W_{t_1/3}=1{\cdot}39\times15{\cdot}67/12{\cdot}06$

$=1{\cdot}809$ Wb/m²

From Fig. 4·18 $at_{1/3}=11000$ A/m

$\therefore$ Mmf required for teeth $AT_t=11000\times25\times10^{-3}=275$ A

$\therefore$ Total mmf required for gap and teeth $=2400+275=2675$ A.

Example 4·7. *Calculate the apparent flux density at a particular section of a tooth from the following data :*

Tooth width=12 mm ; slot width=10 mm ; gross core length=0·32 m ; number of ventilating ducts=4, each 10 mm wide ; real flux density=2·2 Wb/m² ; permeability of teeth corresponding to real flux density=31·4×10⁻⁶ H/m (henry per metre) ; stacking factor=0·9.

Solution. We have magnetizing force or mmf per metre, $H=B/\mu$

$\therefore$ Mmf per metre corresponding to real flux density $B_{real}=2{\cdot}2$ Wb/m² and permeability $\mu=31{\cdot}4\times10^{-6}$ is :

$$\text{'}at\text{'}=\frac{2{\cdot}2}{31{\cdot}4\times10^{-6}}=70{,}063 \text{ A/m.}$$

Net iron length $L_i=0{\cdot}9(0{\cdot}32-4\times10\times10^{-3})=0{\cdot}252$ m.

Slot pitch $\quad y_s=W_t+W_s=12+10=22$ mm

From Eq. 4 48, $\quad K_s=\dfrac{Ly_s}{L_iW_t}=\dfrac{0{\cdot}32\times22}{0{\cdot}252\times12}=2{\cdot}328.$

From Eqn. 4.48, apparent flux density

$$B_{app}=B_{real}+4\pi\times10^{-7}\,at\,(K_s-1)$$
$$=2{\cdot}2+4\pi\times10^{-7}\times70{,}068(2{\cdot}328-1)$$
$$=2{\cdot}317 \text{ Wb/m}^2.$$

Example 4·8. *Determine the apparent flux density in the teeth of a d c. machine when the real flux density is 2·15 Wb/m² ; slot pitch 28 mm ; slot width 10 mm and the gross core length 0·35 m. The number of ventilating ducts is 4, each 10 mm wide. The magnetising force for a flux density of 2·15 Wb/m² is 55000 A/m. The iron stacking factor is 0·9.*

Solution. Net iron length $L_i=k_i(L-n_dW_d)=0{\cdot}9(0{\cdot}35-4\times10\times10^{-3})$

$$=0{\cdot}279 \text{ m}$$

$\therefore$ Tooth width $\quad W_t=y_s-W_s=28-10=18$ mm

From Eqn. 4·49, $\quad K_s=\dfrac{Ly_s}{L_iW_t}=\dfrac{0{\cdot}35\times28}{0{\cdot}279\times18}=1{\cdot}95$

Corresponding to $B_{real}=2{\cdot}15$ Wb/m², the value of 'at'$=55000$ A/m,

Apparent flux density

$$B_{app}=B_{real}+4\pi\times10^{-7}\,\text{'}at\text{'}\,(K_s-1)$$
$$=2{\cdot}15+4\pi\times10^{-7}\times55000(1{\cdot}95-1)=2{\cdot}2156 \text{ Wb/m}^2.$$

Example 4·9. *The armature core of a d.c. machine has a gross length of 0·33 m including 3 ducts each 10 mm wide, and the iron space factor is 0·9. If the slot pitch at a particular section is 25 mm and the slot width 14 mm, estimate the true flux density and the mmf per metre for teeth at this section corresponding to an apparent flux density of 2·3 Wb/m². The magnetization curve data for armature stampings is :*

B Wb/m²	*1·6*	*1·8*	*1·9*	*2·0*	*2·1*	*2·2*	*2·3*
'*at*' A/m	*3700*	*10,000*	*17,000*	*27,000*	*41,000*	*70,000*	*109,000*

Solution. The B 'at' curve is plotted in Fig. 4·19.

Net iron length $\quad L_i=0{\cdot}9(0{\cdot}33-3\times10\times10^{-3})=0{\cdot}27$ m.

Tooth width $\quad W_t=y_s-W_s=25-14=11$ mm

$\therefore \quad K_s=\dfrac{Ly_s}{L_iW_t}=\dfrac{0{\cdot}33\times25}{0{\cdot}27\times11}=2{\cdot}78.$

and real flux density $\quad B_{real}=B_{app}-4\pi\times10^{-7}\,\text{'}at\text{'}\,(K_s-1)$

$$=2{\cdot}3-4\pi\times10^{-7}\,\text{'}at\text{'}\,(2{\cdot}78-1)$$
$$=2{\cdot}3-2{\cdot}237\times10^{-6}\,\text{'}at\text{'}.$$

This is the equation of a straight line. The line can be drawn by locating two points on it. The two points are located as under.

When $'at'=0$, $B_{real}=2{\cdot}3$ Wb/m². This is represented by point A.

When $'at'=70000$, $B_{real}=2{\cdot}3-2{\cdot}237\times10^{-6}\times70{,}000=2{\cdot}143$ Wb/m²

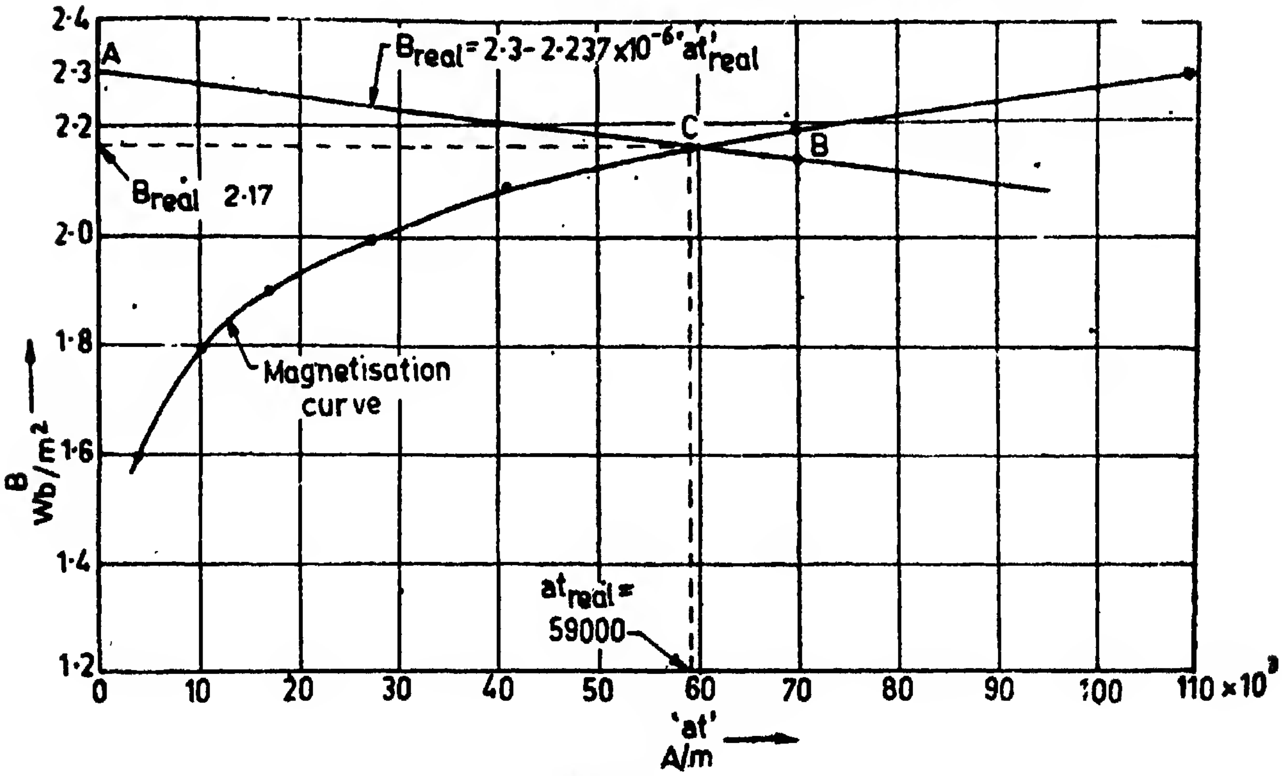


Fig. 4·19

This is represented by point B. A straight line is drawn through A and B cutting the magnetisation curve at C. Point C is the operating point. From point C,

$$B_{real}=2{\cdot}17 \text{ Wb/m}^2 \text{ and } 'at'=59000 \text{ A/m}.$$

Example 4·10. *A d.c. machine has the following data :*

pole arc $=0{\cdot}3$ m, length of machine $=0{\cdot}36$ m, length of air gap $=8$ mm, slot pitch at the air gap surface $=25$ mm, slot pitch at the bottom of slots $=25$ mm, depth of slot $=60$ mm, width of slot $=12$ mm.

number of ventilating ducts in armature	$=5$
width of each ventilating duct	$=10$ mm
flux per pole	$=79\times10^{-3}$ Wb.

Calculate :

(a) the mmf required for air gap,

(b) the mmf required for teeth neglecting the slot flux,

(c) the mmf required for teeth considering the slot flux.

Use B—'at' curve for ordinary steel plates as shown in Fig. 4·1. For the case of saturation consult Fig. 4·16 for finding 'at'. Fig. 4·9 should be referred to for finding the Carter's co-efficient.

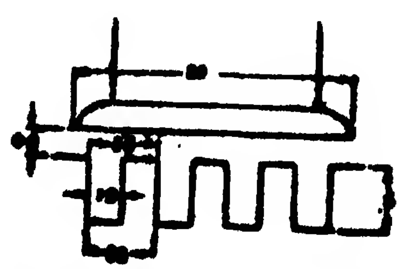


Fig. 4·20. Details of d.c. machine of Example 4·10 (All dimensions in cm)

Solution.

Fig. 4·20 gives the details of the machine.

Gap contraction factor for slots :

$$\text{Ratio } \frac{\text{slot width}}{\text{gap length}}=\frac{1·2}{0·8}=1·5.$$

Carter's co-efficient, from Fig. 4·9 corresponding to 1·5, for open slots, $K_{cs}=0·21$.

K_{gs}=gap contraction factor for slots

$$=\frac{y_s}{y_s-K_{cs}W_s}=\frac{25}{25-0·21\times12}=1·112.$$

Gap contraction factor for ducts :

Duct width=01 mm and number of ducts=5.

$\therefore$ $$\text{Ratio } \frac{\text{duct width}}{\text{gap length}}=\frac{10}{8}=1·25$$

From Fig. 4·9 corresponding to 1·25, for open slots, K_{cd} is 0·18.

Gap contraction factor for ducts

$$K_{gd}=\frac{L}{L-K_{cd}\,n_dW_d}=\frac{0·36}{0·36-0·18\times5\times10\times10^{-3}}=1·026$$

Total gap contraction factor $K_g=K_{gs}\times K_{gd}=1·112\times1·026=1·141$

(*a*) ***Mmf required for gap***

Assuming a rectangular distribution of flux over pole arc.

$\therefore$ Form factor $K_f=\psi$.

$$\text{Gap density at the centre of pole } B_g=\frac{\text{flux per pole}}{\text{pole arc}\times\text{length of core}}$$

$$=\frac{72\times10^{-3}}{0·3\times0·36}=0·667 \text{ Wb/m}^2$$

$\therefore$ Mmf required for the air gap=800,000 $K_gB_gl_g$

$$=800,000\times1·141\times0·667\times8\times10^{-3}=4870 \text{ A}.$$

(*b*) ***Mmf required for teeth neglecting saturation***

Net iron length $=K_i(L-n_dW_d)=0·9\,(0·36-5\times10\times10^{-3})=0·279$ m.

$$\text{No. of teeth in the pole arc}=\frac{\text{pole arc}}{\text{tooth pitch at the gap surface}}=\frac{0·3}{25\times10^{-3}}=12$$

Tooth width at the top of slot $=25-12=13$ mm.

Tooth width at the bottom of slot $=22-12=10$ mm.

Flux density in the tooth is maximum at the bottom, as the section there is minimum, calling this as $B_{t(max)}$.

$\therefore$ $$B_{t(max)}=\frac{\text{flux per pole}}{\text{number of teeth in a pole arc}\times L_i\times\text{width of tooth at the bottom}}$$

$$=\frac{72\times10^{-3}}{12\times0·279\times10\times10^{-3}}=2·15 \text{ Wb/m}^2.$$

The value of flux density at any height x from narrow end

$$B_t(x)=\frac{B_{t(max)}}{1+ax}$$

where $$a=\frac{\text{width at the top}-\text{width at the bottom}}{\text{height of teeth}}=\frac{13-10}{60}=0·05.$$

The flux density in the teeth is calculated at various sections and corresponding values of 'at' are taken from B—'at' curves (Fig. 4·1). The results are tabulated as under :

Distance from the narrow end of tooth cm	*Width of tooth* cm	B_{app} Wb/m²	'at' A/m
0	1·0	2·15	55000
1·0	1·05	2·05	35000
2·0	1·10	1·955	20000
3·0	1·15	1·87	13500
4·0	1·20	1·79	9500
5·0	1·25	1·72	7000
6·0	1·30	1·65	4750

(*i*) *Graphical method.*

Now a graph is drawn between distance from narrow end and 'at'. From the graph (Fig. 4·21),

average value of mmf per metre

at_{mean}=18700 A/m.

∴ Total mmf required for teeth

$$=18700\times\frac{6}{100}=1122\text{ A}.$$

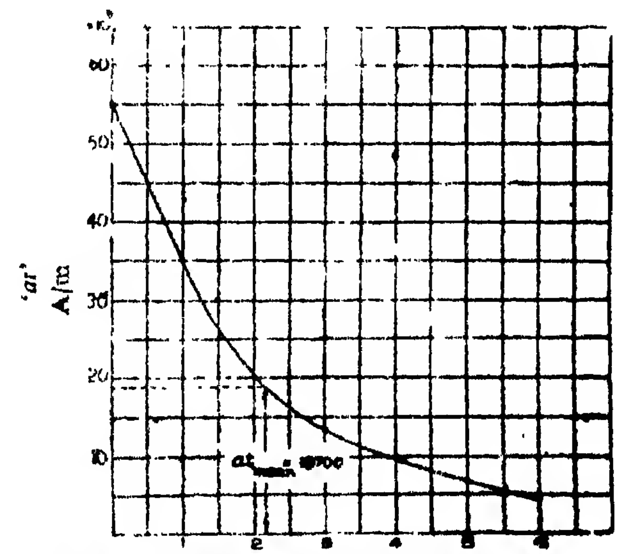


Fig. 4·21. Distance from narrow end in cm.

(*ii*) *Simpson's rule*

Take the three equidistant sections at the top, middle and bottom of teeth

at_1=55000, at_2=13500,

at_3=4750.

$$\therefore\quad 'at'_{mean}=\frac{at_1+4at_2+at_3}{6}=\frac{55000+4\times13500+4750}{6}$$

$$=18{,}960\text{ A/m}.$$

Total mmf required for teeth $=189.0\times\frac{6}{100}=1137$ A.

(*iii*) **$Bt_{1/3}$ method.**

Flux density at 1/3 length (2 cm) from narrow end = 1·96 Wb/m².

$at_{1/3}$ from graph = 20,000 A/m.

This value is higher than the value of 'at' got from graphical method. This is because this method does not give accurate results for teeth working at saturation.

Total mmf required for teeth $=20{,}000\times\frac{6}{100}=1200$ A.

(c) *Mmf for teeth considering saturation.* The areas and apparent flux densities at the six sections are known, the mmf per metre corresponding to various values of B_{app} and K_s are found from Fig. 4·16 and tabulated below. There is no use of evaluating K_s for sections where flux density does not exceed 1·9 Wb/m^2 as the difference in real and apparent flux densities is perceptible in only when there is saturation. Use Fig. 4·1 for '*at*' when B_{app} is less than 1·9 Wb/m^2.

Distance from the narrow end cm	*Tooth area* $A_i = W_t \times L_i$ cm^2	*Total area* $= A_a + A_t$ $= y_s \times L$ cm^2	$K_s = \frac{y_s L}{W_t L_i}$	B_{app} Wb/m^2	'*at*' A/m
0	27·9	79·2	2·84	2·15	37000
1·0	29·3	81·0	2·77	2·05	26000
2·0	30·7	82·8	2·7	1·955	17000
3 0	32·1	84·6	2·64	1·87	13000
4·0	33·5	86·4	2·58	1·79	9500
5·0	34·9	88·2	—	1·72	7000
6·0	36·3	90·0	—	1·65	4750

$$'at'_{mean} \text{ from Simpson's rule} = \frac{37000+4\times13000+4750}{6} = 15625 \text{ A/m.}$$

$$\text{Total mmf required for teeth} = 15625 \times \frac{6}{100} = 937 \text{ A.}$$

IRON LOSSES

4·5. Types of Iron Losses. When ferromagnetic materials are subjected to a flux in a fixed direction in space and having a magnitude varying in time, losses are produced in the material. These losses are called **Iron Losses** or **Core Losses.** The cores of transformers and reactors and the armatures of d.c. and a.c. machines, which carry alternating flux are subject to iron losses. The iron or core losses have two components :

(*i*) hysteresis loss, and (*ii*) eddy current loss.

4·5·1. Hysteresis Losses. Cyclic (alternating) magnetization of a material occurs with a certain loss of energy. The hysteresis loss is due to a form of inter-molecular friction. The loss per cycle is proportional to the area of the hysteresis loop and depends upon the quality of the material. The loss due to hysteresis for a material operating in an alternating magnetic field is usually expressed in terms of loss per unit volume or per unit mass. The hysteresis loss may be written as :

Hysteresis loss $p_h = K$ (area of loop) f W/m^3 or W/kg ...(4·50)

where K is a constant which takes into account the quantities involved.

Steinmetz developed an empirical relationship to express this loss in following terms

$$p_h = K_h f B_m^k \qquad ...(4·51)$$

In this expression

p_h = hysteresis loss, W/m^3 or W/kg, B_m = maximum flux density, Wb/m^2

K_h = hysteresis co-efficient, k = Steinmetz co-efficient

f = frequency of magnetization, Hz.

The value of Steinmetz co-efficient k approximately varies between 1·5 to 2·5 and is 2 for all modern magnetic materials used for electrical machine.

4·5·2. Eddy Currents and Eddy Current Losses. The term "eddy currents" is applied to those electric currents which circulate within a mass of conducting material when the latter is situated in a varying magnetic field. The conducting material may be considered as consisting of large number of closed conducting paths, each of which behaves like a short circuited winding. The varying magnetic field induces eddy emfs in these closed elemental paths giving rise to eddy currents. These eddy currents produce loss in power resulting in heating of materials. This loss is of considerable importance as it effects the efficiency and heating of electrical machines.

The magnetic materials used for varying magnetic fields are laminated (made up of thin sheets insulated from each other) so as to reduce eddy currents and associated losses, as by laminating, the area of paths of eddy currents is reduced giving rise to a large value of resistance.

4·5·3. Eddy current loss in thin sheets. Fig 4·22 shows a thin plate of thickness t and width b, the thickness being considerably smaller than the width. The height of laminations is h. The eddy currents produce a magnetic field of their own which opposes the main magnetic field. The result is non uniform distribution of magnetic field over the cross section of laminations. However, for the purposes of this analysis, a uniform flux distribution is assumed. Suppose this sheet carries a field with a flux density $B=B_m \sin \omega t$, the fieldrunning parallel to the axis of the sheet. Eddy currents would flow in the sheet in the elemental paths as shown.

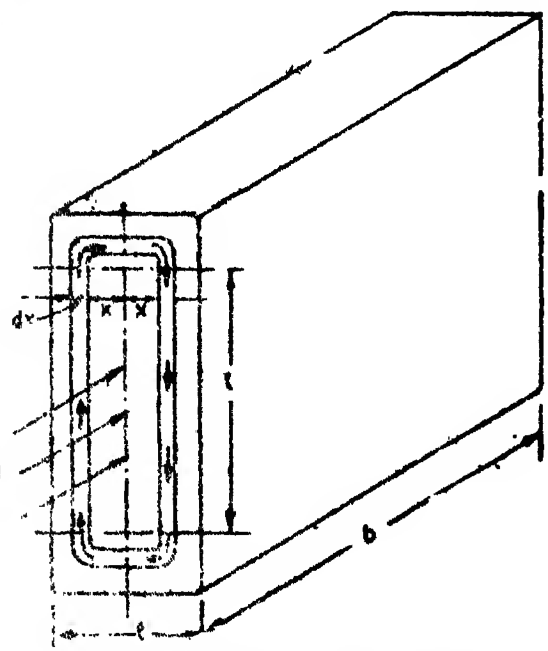


Fig. 4·22 Eddy currents in Laminations.

Take an elemental path of thickness dx at a distance x from the axis

Flux enclosed by the path

$$=2x \times l \times B = 2x\,l\,B$$
$$=2xl\,B_m \sin \omega t.$$

Instantaneous value of eddy emf induced in this elemental path is,

$$e_{ex}=\frac{d}{dt}\,(2x\,l\,B_m \sin \omega t)$$
$$=2x\,l\,\omega\,B_m \cos \omega t.$$

Rms value of eddy emf in the elemental path is

$$E_{ex}=\sqrt{2}\,x\,l\,\omega\,B_m$$
$$=2\sqrt{2}\,x\,\pi\,f\,B_m.$$

Since the thickness of laminations is very small and therefore the resistance is almost entirely due to such vertical paths within the section of iron as shown.

Let ρ be the resistivity of material in Ωm.

∴ Resistance of each elemental path is,

$$R_{ex}=\frac{2l\rho}{b\,dx}=\frac{2l\rho}{b\,dx}$$

$\therefore$ Eddy current in the elemental path is,

$$I_{ue}=\frac{E_{ue}}{R_{ue}}=\frac{2\sqrt{2}xl\pi fB_m}{2l\rho/bdx}=\frac{\sqrt{2}\pi fB_m lxdx}{\rho}$$

Eddy current loss in this path with 1 metre length and 1 metre width is

$$P_{ue}=I_{ue}^2\,R_{ue}$$

$$=\frac{2\pi^2 f^2 B_m^2}{\rho^2}\,b^2x^2(dx)^2\;\frac{2l\rho}{bdx}=\frac{4\pi^2 f^2 B_m^2 lbx^2dx}{\rho}$$

$\therefore$ Total eddy current loss

$$P_e=\int_{x=0}^{x=t/2}\frac{4\pi^2 f^2 B_m^2}{\rho}\,lb\,x^2dx$$

$$=\frac{\pi^2 f^2 B_m^2}{6\rho}\,lb\,t^3\ \mathrm{W} \qquad \ldots(4\text{·}52)$$

$\therefore$ Eddy current loss per unit volume

$$p_e=\frac{\pi^2 f^2 B_m^2\,lb\,t^3}{6\rho}\times\frac{1}{lbt}=\frac{\pi^2 f^2 B_m^2 t^2}{6\rho}\ \mathrm{W/m^3} \qquad \ldots(4\text{·}53)$$

$$=K_e f^2 B_m^2\ \mathrm{W/m^3} \qquad \ldots(4\text{·}54)$$

where

$$K_e=\frac{\pi^2 t^2}{6\rho} \qquad \ldots(4\text{·}55)$$

The thickness t of laminated material is of considerable importance as regards the eddy current loss. It is clear from Eqn. 4·53 that the eddy current loss is proportional to t^2 (square of thickness). This is precisely the reason why iron cores subjected to alternating magnetization are liminated. Thus it is economical at 50 Hz, to go to the trouble and expense of dividing iron into laminations about 0·4 mm thick and insulating them from one another. It is also clear from Eqn. 4·53, that the eddy current loss is inversely proportional to the resistivity of the material. Therefore, high resistivity materials which have a high silicon content are used for the cores in order to reduce eddy current loss. However, the use of high resistivity material, results in higher magnetizing mmf and consequently higher magnetizing current.

4·5·4. Total Iron Loss. Total iron loss is the sum of hysteresis and eddy current loss and is expressed by the formula

$$p_i=p_h+p_e$$

$$=K_h f B_m^k+K_e f^2 B_m^2\ \mathrm{W/m^3 \text{ or } W/kg} \qquad \ldots(4\text{·}56)$$

Since $k=2$ for most of the modern ferro-magnetic materials used in electrical machines, we have :

$$p_i=(K_h f+K_e f^2)B_m^2\ \mathrm{W/m^3 \text{ or } W/kg} \qquad \ldots(4\text{·}57)$$

Theoretically, the above relationships may be used for the calculation of total iron loss. However, the iron losses in actual machines are higher than those given by the above expressions. The reasons for this difference are explained below :

(i) There are additional **anomalous losses** in machines. These anomalous losses are possibly due to movements of the boundaries between microscopic magnetic regions. The extramental investigations show that the anomalous losses are proportional to $f^2B_m^2/(a+f^2)$ where a is a constant.

(*ii*) The relationships (Eqn. 4·51 and Eqn. 4·53) are based upon the assumption that the field varies sinusoidally with time and acts along a single axis which is parallel to the plane of laminations. In practice, the flux density at a point in the core neither varies sinusoidally with time nor alternates along a single axis. The non-sinusoidal nature of flux variations alter both the eddy current as well as the hysteresis losses. The eddy current loss for sinusoidal waveform is given by :

$$p_e=\frac{\pi^2 f^2 B_m^2}{6\rho}\,t^2=\frac{(1\cdot11)^2 f^2 B_m^2}{6\rho}\,t^2 \text{ W/kg} \qquad \ldots(4\cdot58)$$

The form factor for a purely sinusoidal waveform is 1·11. Therefore, the eddy current loss for any non-sinusoidal waveform with a form factor K_f is given by :

$$p_e=\frac{K_f^2 f^2 B_m^2}{6\rho}\,t^2 \text{ W/kg} \qquad \ldots(4\cdot59)$$

The non-sinusoidally varying flux also changes the hysteresis loss. This is clear from Fig. 4·24. The re-entrant part of the hysteresis loop gives an additional loss which makes the eddy current loss greater than is indicated by value of B_m used in Eqn. 4 51.

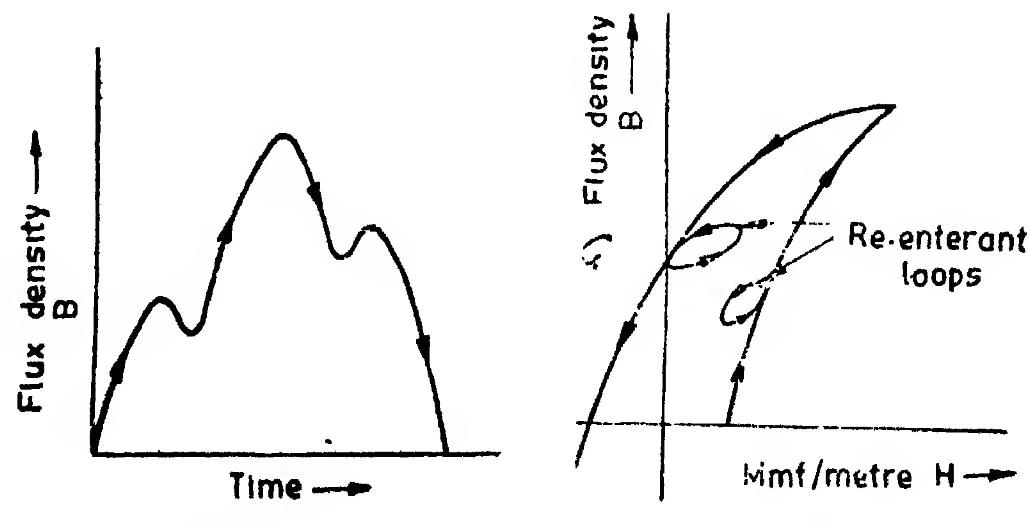


Fig. 4·24. Re-entrant Hysteresis loops.

Eqns. 4·51 and 4·53 give fairly good results in the case of static machines like transformers, reactors, and a.c. electromagnets in which the flux oscillates along a path that is practically fixed. However, the rotors of rotating electromagnetic machines are subjected to fluxes that change their direction on account of rotation. This gives rise to a phenomenon called **"rotating hysteresis"**. Fig. 4·25 shows a typical case of the variations of hysteresis loss at different values of flux density for rotating electrical machinery. The rotational hysteresis loss is greater at low magnetization than the corresponding hysteresis loss due to alterneting magnetic field, while at high flux densities the rotating hysteresis loss actually decreases becoming quite low at very high values of flux densities.

There are additional eddy current losses produced in the rotors of rotating electric machinery on account of the fact that the flux cannot everywhere be confined to a direction parallel to the plane of laminations. Some flux enters the rotor at the two ends and also the sides of the teeth through the spaces provided for ventilation. Since this flux enters the iron in a direction normal to the plane of laminations, it gives rise to an appreciable iron loss. It is pertinent to point out that in the case of teeth of armatures of d.c. machines, the variation of flux density is a combination of *linear and rotational magnetization, and therefore the loss under these conditions is uncertain.

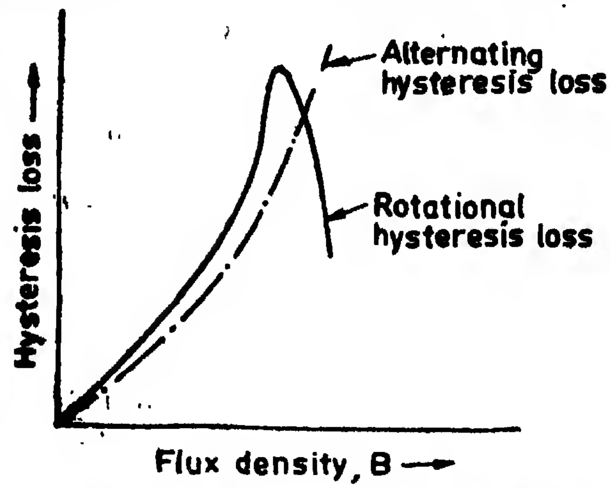
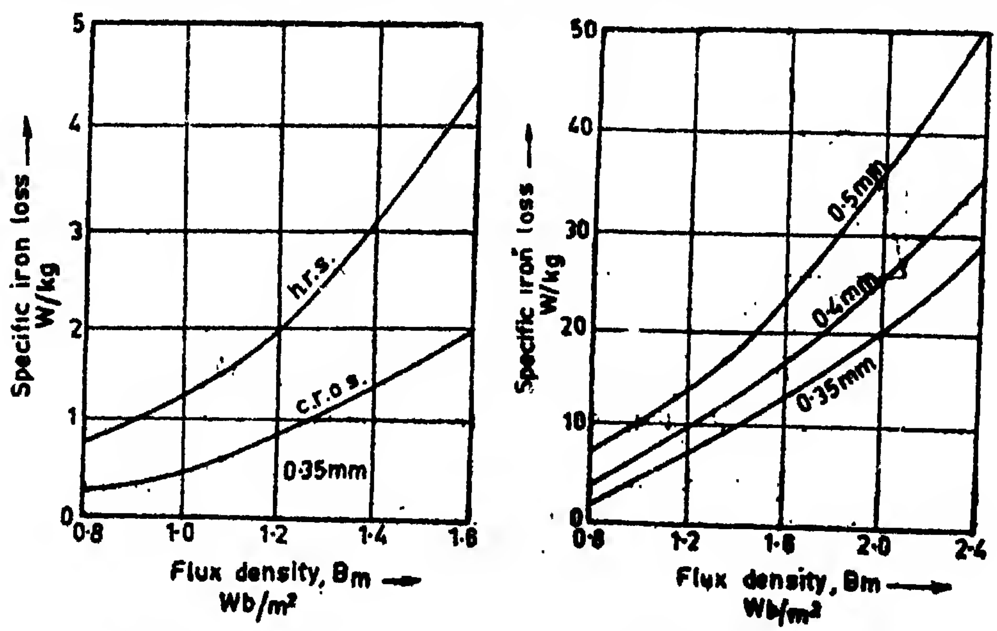


Fig. 4·25. Rotational alternating hysteresis loss.

(*i*) There are some practical factors which lead to increased iron loss in built up cores. The laminations may be shorted by burns on the edges due to worn dies, by grinding, or by faulty insulation on rivets, clamping bolts or the laminations themselves, thus giving increased eddy current loss.

Mechanical strains set up by cold working or assembly of core affect the nature of the material and thus increase the iron loss, sometimes as much as a factor of 2·5.

There are additional iron losses owing to distortion of flux produced by armature reaction on load. This is because the iron losses are proportional to maximum value of flux



(*a*) Transformers (*b*) Rotating machines

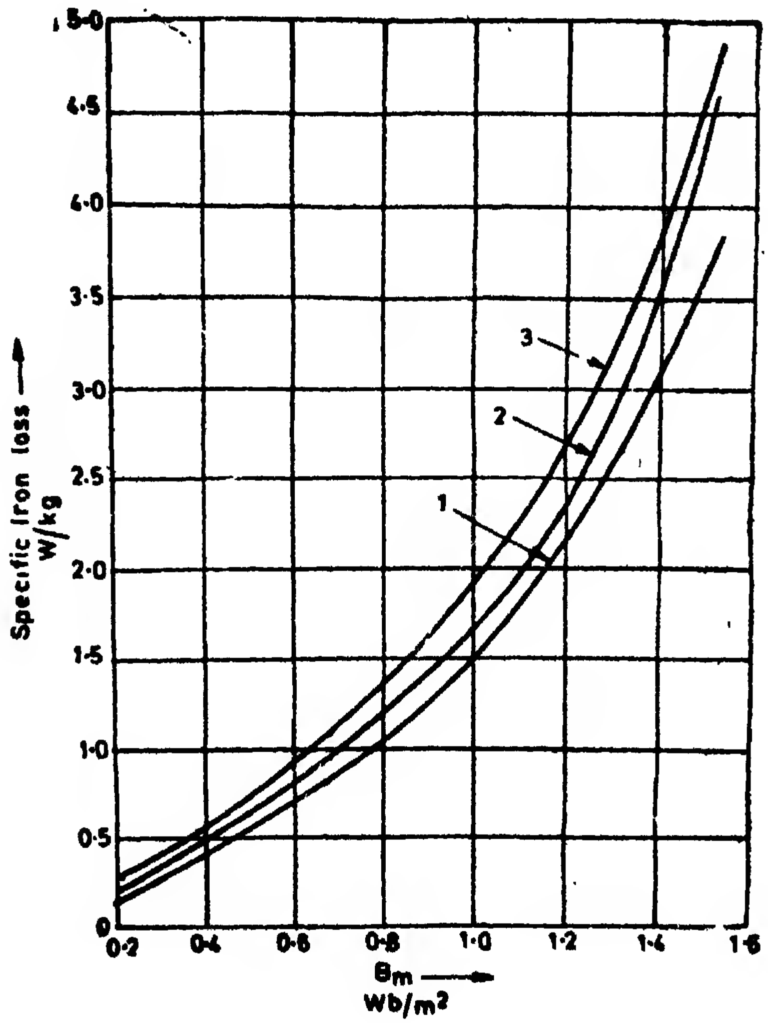
Fig. 4·26. Typical loss curves for transformers and rotating machines.

*The iron loss may be regarded as a mass effect distinguished only by the manner in which the flux variations occur. The magnetization may be (*i*) linear or (*ii*) rotational. Linear magnetization means a flux that varies sinusoidally on the same axis, while rotational magnetization means a roughly constant field whose axis varies cyclically or rotates.

density and maximum value of flux density increases with load on account of distortion of field form giving rise to increased losses. The iron losses also increase on account of ageing effect.

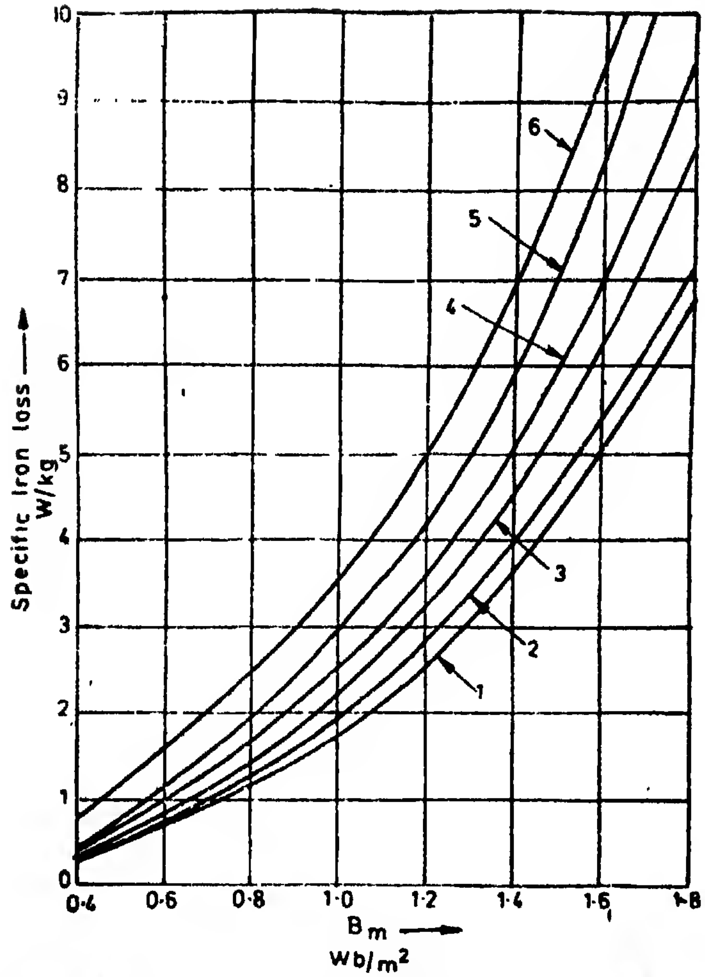
Due to the above mentioned reasons, it is impossible to calculate the iron losses in built up cores. In practice, loss curves which represent the specific iron loss as a function of maximum flux density are used. The loss curves are obtained by measurements done on built up cores and thus take into account some of the factors listed above. Typical loss curves for transformers and rotating machines are shown in Figs. 4·26 (*a*) and (*b*).

Loss curves for different grades of stampings manufactured by Sankey Pressings Division of M/s Guest Keen Williams is given in Figs. 4·27 to 4·30.



1—48 Quality 2—41 Quality 3—42 Quality

Fig. 4·27. Loss curves of Electrical sheet steel (Non oriented) 0·35 mm thick.

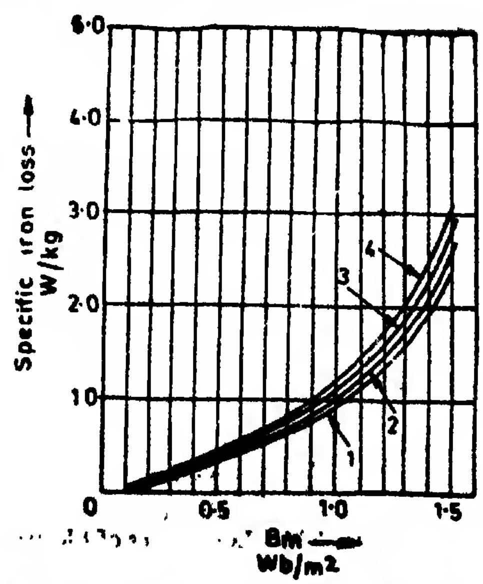


1—48 Quality 2—41 Quality 3—42 Quality
4—Medium Resistance 5—Special Lohys 6—Lohys

Fig. 4·28. Loss curves of Electrical sheet steel (Non oriented) 0·5 mm thick.

The loss curves are prepared by the manufacturers of stampings on the basis of laboratory tests done on prepared specimens.

In fact, the results obtained in the laboratory, by testing prepared specimens, cannot be relied upon as the losses in the built up cores of actual machines are invariably more than the losses obtained by laboratory tests. In order to take additional iron losses into account the losses as obtained from iron loss curves must be multiplied by a factor which is 1·4 to 1·6 for d.c. machines, 1· to 1·4 for induction motors, 1·4 to 1·6 for salient pole synchronous machines and 1·15 to 1·25 for turbo-alternators.



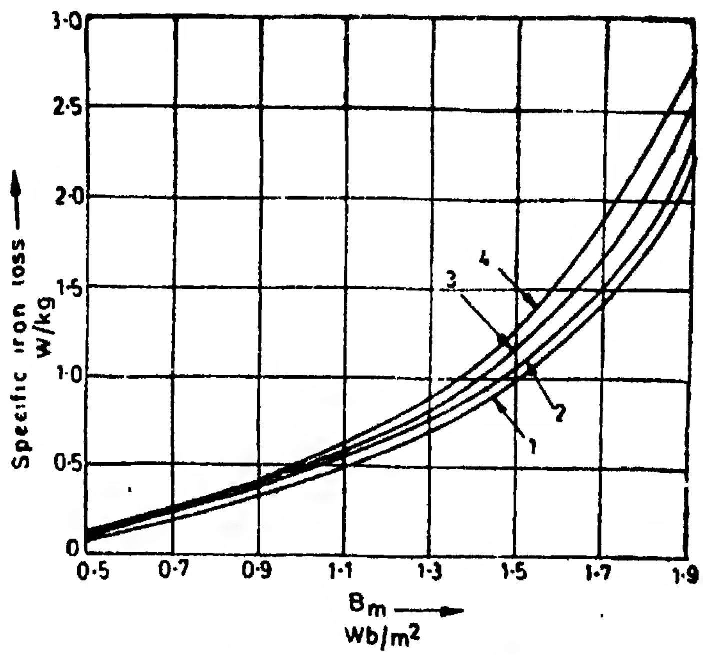
1—Grade 74 2—Grade 80 3—Grade 86 4—Grade 92

Fig. 4·29. Loss curves of Electrical sheet steel (non oriented) 0·35 mm thick.

As pointed out earlier the two losses *i.e.* hyteresis and eddy current losses may

be taken as proportional to B_m^2. (See Eqn. 4·57). But even for same flux density the specific iron loss greater for teeth than core. At 50 Hz the specific iron loss may be written as

$$p_i = aB_m^2 \text{ W/kg} \qquad \ldots(4\cdot58)$$



1—Grade 48 2—Grade 51 3—Grade 56 4—Grade 62

Fig. 4·30. Loss curves of Electrical sheet steel (oriented) 0·33 mm thick

when 'a' is a constant whose values are given below :

Machine	*Part*	*a*
A.C.	Core Teeth	4·7 6·5
D.C.	Core Teeth	6·5 2·3

At flux densities above 1·6 Wb/m² the losses increase rapidly above the values calculated by Eqn. 4·58.

4·6. Pulsation Losses. In a rotating electrical machine, the armatures are slotted and as a result when the rotor moves there are rapid changes of local gap reluctance. This change in reluctance gives rise to flux pulsations *i.e.* changes in the air gap flux a condition which produces additional losses called **pulsation losses** in the teeth and pole faces. This effect is considerably aggravated if the length of air gap is small as compared with slot openings. Slotting also produces harmonic fields which cause high frequency losses near the gap surface. These losses may sometimes be considerable (as in the case of induction motors) although they are difficult to calculate.

Example 4·11. *Calculate the specific iron loss in a specimen of alloy steel for a maximum flux density on 3·2 Wb/m² and a frequency of 50 Hz, using 0·5 mm thick sheets. The resistivity of alloy steel is 0·3×10⁻⁶ Ωm. The density is 7·8×10³ kg/m³. Hysteresis loss in each cycle is 400 J/m³.*

Solution. From Eqn. 4·53, eddy current loss

$$p_e=\frac{\pi^2 f^2 B_m^2 t^2}{6\rho}\ \text{kg/m}^3 = \frac{\pi^2 f^2 B_m^2 t^2}{6\rho\times 7\cdot 8\times 10^3}\ \text{W/kg}$$

$$=\frac{\pi^2\times 50^2\times 1\cdot 2^2\times(0\cdot 5\times 10^{-3})^2}{6\times 0\cdot 3\times 10^{-6}\times 7\cdot 8\times 10^3}=0\cdot 633\ \text{W/kg}$$

Hysteresis loss $p_h=\dfrac{400\times 50}{7\cdot 8\times 10^3}=2\cdot 564$ W/kg at 50 Hz.

Tot. l iron loss $P_i=2\cdot 564+0\cdot 633=3\cdot 2$ W/kg.

Example 4·12. *A specimen of cold rolled grain oriented 0·3 mm thick stampings has a resistivity of 0·5×10⁻⁶ Ωm. The published hysteresis loop is essentially rectangular in form, with a co-ercive force of 1_ A/m for all values of peak flux densities upto 1·6 Wb/m². The manufacturer quotes the iron loss in the material as 1·2 W/kg, with a sinusoidal flux density 1·0 Wb/m² (peak) at 100 Hz. Calculate the loss in the material from its properties and compare it with the quoted value. The mass density is 7650 kg/m³.*

Solution. The material is subjected to a peak flux density of 1Wb/m² and therefore the value of co-ercive force is 12 A/m for all values of flux densities to which the material is subjected. The hysteresis loop is thus a rectangle with sides 2×12=24 A/m and 2×1 =2Wb/m².

Loss per cycle=area of hysteresis loop=24×2=48 W-s/m².

Hysteresis loss at 100 Hz,

$$p_h=\frac{48\times 100}{7650}=0\cdot 627\ \text{W/kg}$$

From Eqn. 4·53 eddy current loss,

$$p_e=\frac{\pi^2 f^2 B_m^2 t^2}{6\rho}\ \text{W/m}^3=\frac{\pi^2 f^2 B_m^2 t^2}{6\rho\times 7650}\ \text{W/kg}$$

$$=\frac{\pi^2\times(100)^2\times(1\cdot 0)^2\times(0\cdot 3\times 10^{-3})^2}{6\times\cdot 5\times 10^{-6}\times 7650}=0\cdot 387\ \text{W/kg}$$

Total specific iron loss $p_i=0\cdot 627+0\cdot 387=1\cdot 014$ W/kg.

The calculated iron loss is smaller than the quoted.

Example 4·13. *The hysteresis loss in a sample of iron was found to be 4·9 W/kg at a frequency of f=50 Hz and at a maximum flux density of B_m=1Wb/m². (a) Calculate therefrom the co-efficient η in the expression, loss/cycle=η $B_m^{1\cdot 7}$ J/m³ : the specific gravity of iron is 7·5, (b) calculate the loss per kg at f=25 Hz and a flux density B_m=1·8 Wb/m².*

Solution. Specific gravity of iron=7·5,

∴ Density of iron=7500 kg/m³

Energy loss due to hysteresis $=\eta\, B_m^{1.7}$ J/m³.

$\therefore$ Hysteresis iron loss (power)/kg $p_h=\eta f B_m^{1.7}\times 7500$ ...(*i*)

It is given that the hysteresis iron loss p_e at $f=50$ Hz and $B_m=1$ Wb/m² is 4·9 W/kg.

$$\therefore \qquad 4.9=\eta\times(50)\times(1)^{1.7}\times 7500$$

or $$\eta=1307\times 10^{-6}.$$

Hence, hyteresis loss per kg at 25 Hz and a flux density of 1·8 Wb/m² is :

$$p_h=\eta f B_m^{1.7}\times 7500=1307\times 10^{-6}\times 25\times(1.8)^{1.7}\times 7500$$
$$=6.66 \text{ W/kg}.$$

Example 4·14. *A laminated iron cylinder is rotated in a magnetic field. The iron loss is 250 W at 600 r.p.m. and 312 W at 712 r.p.m. Find the loss if the laminations were twice as thick, the induction density increased by 20% and the speed were 720 r.p.m. Take Steinmetz co-efficient as 1·6.*

Solution. The frequency of pulsations is proportional to rotational speed.

Let N be the speed in r.p.m. and V_i be the volume of iron.

From Eqn. 4.53; eddy current loss $P_e=\dfrac{\pi^2 f^2 B_m^2 t^2}{6\rho}\, V_i$

$$=\frac{\pi^2 A^2 N^2 B_m^2 t^2}{6\rho}\, V_i$$

$=K_e'N^2$ where $f=AN$ and A is a constant since the frequency of flux pulsations is directly proportional to the speed of rotation.

From Eqn. 4·51, hysteresis loss

$$P_h=K_h f\, B_m^{1.6}V_i=K_h AN\, B_m^{1.6}V_i=K_h'N$$

where K_e' and K_h' are constants if B_m and t remain constant.

$\therefore$ Total iron loss $P_i=K_e'N^2+K_h'N$

Substituting the values of P_i and N in the above expression

$$500=K_e'(480)^2+K_h'(480) \qquad \text{...(i)}$$
$$312=K_e'(600)^2+K_h'(600) \qquad \text{...(ii)}$$

From relations (*i*) and (*ii*)

$$K_e'=0.00104,\ K_h'=0\ 5425$$

The hysteresis loss increases in direct proportion frequency (which is proportional to speed in this case), flux density raised to power 1·6 and is independent of the thickness of laminations. The eddy current loss on the other hand increases in proportion to flux density squared, frequency squared and square of the thickness of laminations.

$\therefore$ Total loss B_m increased by 20%, thickness increased to twice the original value and $N=720$ r.p.m.

$$P_i=0.00104\,(720)^2\times(1.2)^2\times 2^2+0.5425\times(720)\times(1.2)^{1.6}=3622 \text{ W}.$$

MAGNETIC LEAKAGE CALCULATIONS

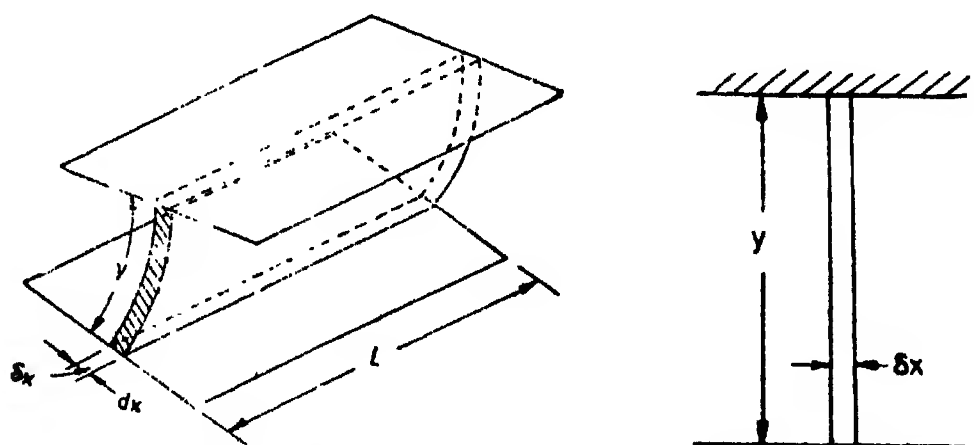
4·7. Effects of Leakage Flux. It has already been stated that it is impossible to confine all the flux to useful paths ; there being always a leakge flux.

If the leakage flux alternates, it will induce voltage in any winding with which it links. This is known as the leakage reactance voltage. The reactance corresponding to this voltage plays an important part in the performance of a.c. machines.

In d.c. machines, the leakage flux passing in non-useful paths affects the field excitation of the machines. The excitation has to be increased to compensate for loss of flux. Although the leakage flux is constant with time but during commutation it is reversed when the coil currents are reversed, giving rise to reactance voltage. This reactance voltage opposes the change in current and makes the commutation difficult.

The estimation of leakage flux is difficult owing to the complex geometry leakage paths. It is impossible to get very accurate results as no amount of mathematics can define complexity of leakage flux. Normally the results obtained are checked against experimental data.

4·8. Specific Permeance. The leakage flux can be assumed to consist of flux tubes of length y and a constant width δx along the effective depth or length L of the field as shown in Fig. 4·31 (*a*).



(*a*) in three dimensions (*b*) in two dimensions

Fig. 4·31. **Leakage flux.**

Permeance of a tube $\delta\Lambda=\mu_0\dfrac{\text{area of tube}}{\text{length of tube}}=\mu_0\dfrac{L\delta x}{y}$

(In this case only reluctance of the path through air has been considered and the reluctance of flux path through iron path has been neglected).

$\therefore$ Permeance of whole field $\Lambda=\mu_0\displaystyle\int L\frac{dx}{y}=L\mu_0\int\frac{dx}{y}$ .. (4·59)

Specific permeance is defined as permeance per unit length or depth of field.

$\therefore$ From Eqn. 4·59,

Specific permeance $\lambda=\dfrac{\Lambda}{L}=\mu_0\displaystyle\int\frac{dx}{y}$ ...(4·60)

If the length of tubes is constant over a height h as in Fig. 4·31 (*b*)

Specific permeance $\lambda=\mu_0\displaystyle\int_0^h\frac{dx}{y}=\mu_0\frac{h}{y}$ ...(4·61)

Eqn. 4·61 is based upon the assumption that the mmf is constant over all flux tubes as the integration is carried out.

The above expressions for specific permeance have to be modified if whole of the leakage flux does not link with all the turns. The analysis for this case is done below :

Let T=total number of turns,

I_z=current in each conductor,

Φ=effective flux ; this flux is a hypothetical flux which is assumed to link with all the T turns producing same number of flux linkages as are produced by actual arrangement of winding and field.

Now
$$\Lambda=\text{effective permeance}=\frac{\text{effective flux}}{\text{total mmf}}=\frac{\Phi}{TI_z}$$
$$=\frac{T\Phi}{(T)(TI_z)}=\frac{\text{total flux linkages}}{\text{total turns}\times\text{total mmf}} \qquad \ldots(4\cdot62)$$

The flux linkages of the actual arrangement can be found by dividing the flux into infitesimal parts $d\Phi_x$, each represented by a line or tube of force linking with T_x turns.

Flux linkages of T_x turns$=T_x\,d\Phi_x$

$\therefore$ Total flux linkages of winding$=\int T_x d\Phi_x$

but, $d\Phi_x$=mmf producing this flux$\times$permeance of infinitesimal part
$$=I_zT_x\times\mu_0L\frac{dx}{y}\;.$$

$\therefore$ Total flux linkages $=\int T_x\times I_zT_x\times\mu_0L\dfrac{dx}{y}\;.=\mu_0LI_z\int T_x^2\dfrac{dx}{y}\;.$

By denfiition of effective permeance (Eqn. 4·62)
$$\Lambda=\frac{\text{total flux linkages}}{\text{total turns}\times\text{total mmf}}=\frac{\mu_0LI_z\int T_x^2\dfrac{dx}{y}}{T(TI_z)}$$
$$=\mu_0L\int\left(\frac{T_x}{T}\right)^2\frac{dx}{y} \qquad \ldots(4\cdot63)$$

$$\therefore\ \text{Effective specific permeance}\ \lambda=\frac{\Lambda}{L}=\mu_0\int\left(\frac{T_x}{T}\right)^2\frac{dx}{y} \qquad \ldots(4\cdot64)$$

Hence, specific permeance of a differential path of depth dx and length y,
$$d\lambda x=\mu_0\left(\frac{T_x}{T}\right)^2\frac{dx}{y} \qquad \ldots(4\cdot65)$$

4·9. Leakage Reactance. When leakage flux is associated with a winding carrying "steady state" alternating current a reactive voltage is produced. The magnetic reluctance offered to leakage flux is predominantly due to the air path under normal conditions and so leakage reactance is directly proportional to current, giving a constant inductance. Therefore the reactive voltage may be regarded as due to a voltage drop in a constant **leakage reactance.**

Let us consider a coil of T turns each carrying a current of I_z, the effective permeance of flux paths being Λ. Inductance is defined as flux linkages per unit current.

$\therefore$ Inductance $= \dfrac{\text{flux linkages of the winding}}{\text{current}} = \dfrac{T\Phi}{I_z}$

but $\Phi = \text{mmf} \times \text{permeance} = TI_z\Lambda$.

$\therefore$ Inductance $= \dfrac{T \times TI_z\Lambda}{I_z} = T^2\Lambda$ ...(4·66)

If the current in the coil pulsates with a frequency f, it has a reactance,

$$X = 2\pi f \times \text{inductance} = 2\pi f T^2\Lambda = 2\pi f T^2 L\lambda \quad ...(4·67)$$

where $\lambda = \mu_0 \displaystyle\int \left(\frac{T_x}{T}\right)^2 \frac{dx}{y}$.

If λ is the value of the specific permeance for the leakage paths, Eqn. 4·67 gives the value of the leakage reactance of the winding.

The difficulty in applying Eqn. 4·67 lies in the estimation of λ. It is customary to assume that the mmf required for the iron parts of the leakage flux path is negligible. It is then obvious to concentrate on non-magnetic paths of the leakage flux and an attempt is made to fit geometry into these paths. The magnetic circuit is split into series and parallel paths so that the calculation of specific permeance λ is made possible. The problem of calculation of leakage flux or leakage reactance now reduces down to estimation of specific permeance.

When the leakage paths are complicated they are often found to be capable of division into parallel strips, each strip being associated with the length of conductor in part of the turn. The resultant permeance is :

$$\Lambda = \mu_0(A_1/l_1 + A_2/l_2 + A_3/l_3 + \ldots) \quad ...(4·68)$$

where l_1, l_2, l_3......are the lengths and A_1, A_2, A_3,......are the areas of the flux paths. The areas A_1, A_2, A_3,......etc., are usually products of widths w_1, w_2, w_3,......etc. and lengths L_1, L_2, L_3,......etc. of that part of the winding. Thus

$$\Lambda = \mu_0(L_1 w_1/l_1 + L_2 w_2/l_2 + L_3 w_3/l_3 + \ldots) \quad ..(4·69)$$

$$= L_1\lambda_1 + L_2\lambda_2 + L_3\lambda_3 + \ldots \quad ...(4·70)$$

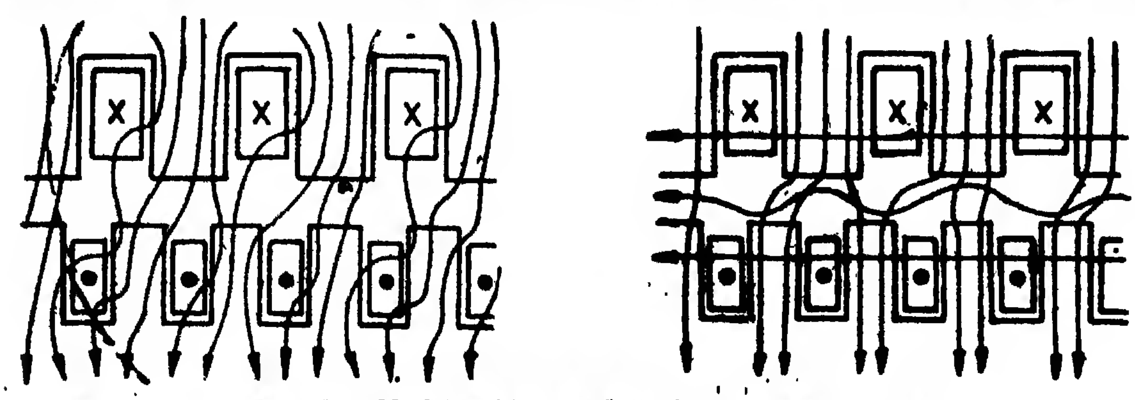
where λ_1, λ_2, λ_3,...are the specific permeances of the portion of the coil and their values are given by :

$$\lambda_1 = \mu_0 w_1/l_1,\ \lambda_2 = \mu_0 w_2/l_2,\ \lambda_3 = \mu_0 w_3/l_3 \quad ...(4·71)$$

4·10. Armature Leakage. The principle components of armature leakage flux are :

1. **Slot Leakage flux.** Slot leakage flux is shown in Fig. 4·37. It crosses the slot from one tooth to the next, linking with that portion of the conductors below it by returning through the iron.

The term **"Armature"** means the member which carries distributed d.c. or a.c. windings. The leakage flux in armature of rotating machines is superimposed upon the mutual (useful)



(a) Fig 4·32. Useful and leakage fluxes in armature (b)

flux in the air gap region. The distribution of the air gap flux is modified on account of existence of leakage flux. The distribution of total flux in air gap region is shown in Fig. 4·32 (*a*). However, the leakage flux in the overhang has a separate identity. Fig. 4·32 (*b*) shows its arbitrary two components, the mutual (useful) flux and the leakage flux.

As stated earlier, the useful flux links with both the windings while the leakage flux either links with one winding or is so directed that it does not contribute to transfer of useful energy.

2. **Tooth top leakage flux.** This leakage flux passes from top of one tooth to the top of another tooth. This leakage flux is quite important in machines having large gap lengths like d.c. machines and synchronous machines while in induction machines, it is normally negligible. Fig. 4·33 shows the tooth top leakage flux.

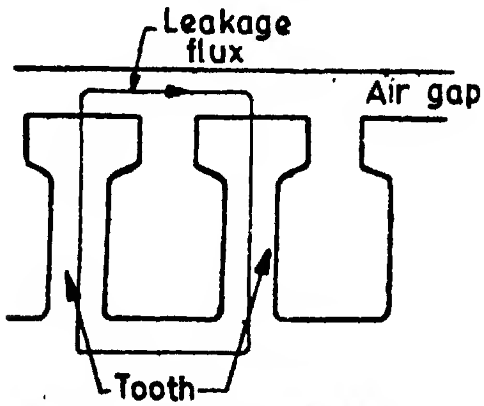


Fig. 4·33. Tooth top leakage flux.

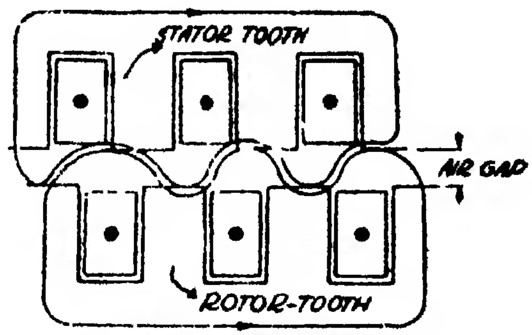


Fig. 4·34. Zigzag leakage flux.

3. **Zigzag leakage flux.** This flux passes from one tooth to another in a zigzag fashion across the air gap (Fig. 4·34). The magnitude of this flux depends upon the length of air gap and the relative positions of tips of teeth.

4. **Overhang leakage flux.** The overhang portion of armature windings produces a separate leakage flux. Its magnitude depends upon the arrangement of overhang and the proximity metal masses, such as core stiffners and end covers having conducting and magnetic properties. Fig. 4·35 shows the overhang leakage flux. It is clear, that this leakage flux has distinct separate identity and does not modify the value and distribution of the total flux.

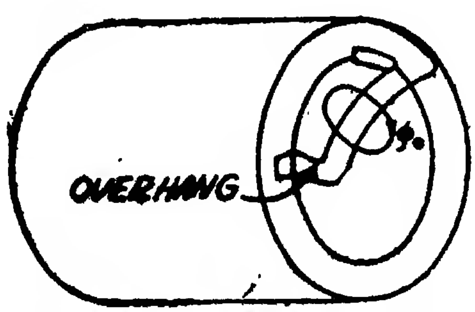
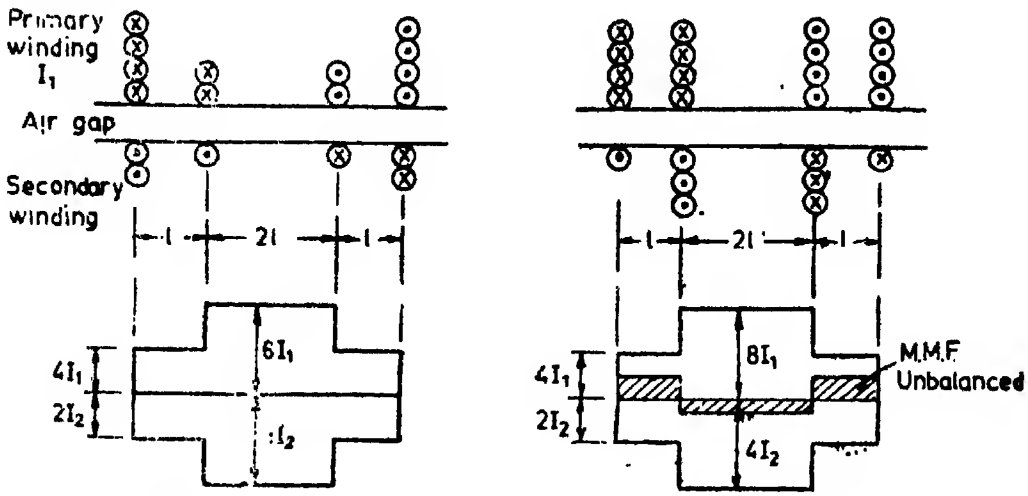


Fig 4·35. Overhang leakage flux.

The leakage fluxes given above account for most of the leakage reactance, but there are some other components, given below, which are normally small in magnitude except in exceptional cases.

5. **Harmonic or Differential leakage flux.** This is also called **belt leakage flux** and is due to the fact that the primary and secondary mmf distributions are not, in general, similar. Any unbalanced components will cause harmonic fluxes, each of which rotates at its own synchronous speed causing a fundamental frequency reactive voltage drop

in the primary. Fig. 4·36 shows the result of difference of mmf distribution between stator and rotor mmf across the air gap of a machine. Suppose the primary to secondary winding turns ratio is 2/1. Fig. 4·36 (*a*) shows the two windings, primary and the secondary on the opposite sides of the air gap. In order that the two mmfs balance each other both in magnitude and distribution, the secondary winding current should be two times he primary winding current everywhere *i.e.* $I_2=2I_1$



(*a*) **Similar mmf distribution.** (*b*) **Dissimilar mmf distribution.**

Fig. 4·36. Differential or Harmonic leakage.

When the mmfs of the windings are balanced both in magnitude and also in distribution, there is no differential or harmonic leakage flux. Let us now consider a case wherein the turns ratio is the same but there is dissimilar distribution. This is shown in Fig. 4·36 (*b*) The condition for zero secondary winding linkage is that :

$$72\, lI_1 - 34\, lI_2 = 0$$

or $$I_2 = \frac{72}{34} I_1 = 2{\cdot}12$$

Therefore, the ratio of currents is 2·12 which differs from the turns ratio of 2. The primary winding linkage, however, is

$$160\, l\, I_1 - 72\, l\, I_2 = 7{\cdot}5\, l\, I_1$$

Therefore, the primary and the secondary winding mmfs cannot exactly balance each other. Thus there exists a net mmf which causes a leakage flux which is called the **differential** or **harmonic leakage flux.**

The **harmonic leakage flux** is on account of the fact that if the spatial distribution of mmfs of the primary and the secondary windings is not the same, the difference in the harmonic contents of the two mmfs causes harmonic leakage fluxes. The differential or harmonic leakage flux is mainly a result of higher space harmonics and can be neglected for normal windings. In squirrel cage induction motors, the currents in the rotor (acting as secondary winding) can balance the stator (primary) current at every point without restraints and therefore there is a no harmonic leakage flux in these machines.

6. **Skew leakage flux.** This is only present when the slots are skewed. Skewing is generally done in squirrel cage induction motors to eliminate harmonic torques and noise. If the rotor slots are skewed, the voltage in rotor conductors is reduced. This results in apparent decrease in mutual flux creating a large difference between total flux and mutual flux. This is the same as the effect of increase in stator leakage flux and leakage reactance.

7. **Peripheral leakage flux.** This flux exists circumferentially round the air gap without linking with any of the windings. It is negligible for most of the machines.

4·11. Slot leakage. A parallel sided slot accomodating one or more conductors is shown in Fig. 4·37. (a) When the conductors carry current, they produce a slot leakage flux pattern. Flux patterns under steady state can be obtained by applying the condition that the magnetic vector potential within the slot must satisfy the Poisson's equation within the section of the conductors, and Laplace's equation in other parts. In case there is large space between the bottom of the conductor, and its width is comparative smaller than that of the slot, a flux pattern as shown in Fig. 4·37 (a) is obtained. For this set-up, the flux does not go straight across the slot. However, in all low voltage machines, the proportions of the conductors and slot are as shown in Fig. 4·37 (b). For these cases, it is justifiable to assume that an elemental leakage flux path is completed around the bottom of the slot as a result of the assumption that the ferromagnetic meterial has an infinite permeability. The flux path can be assumed to be straight across the slot for steady and power frequency currents. The slot leakage can be calculated by making the following assumptions :

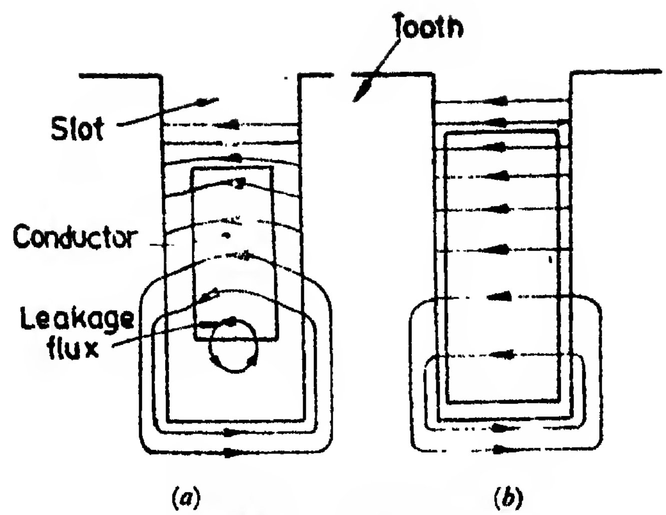


Fig. 4·37 Slot Leakage Flux

1. The current in the slot conductors is uniformly distributed over their cross-section.
2. The leakage path is straight across the slot and a round the iron at the bottom.
3. The permeance of air paths is only considered. The reluctance of iron paths is assumed as zero.

The slot leakage permeance depends upon the shape of the slot and the arrangement of the windings in the slot. We will first consider single layer windings.

4·11·1. Leakage permeance of parallel sided slot. Consider the slot shown in Fig. 4·38. The area occupied by conductors is shaded. The leakage paths over each of portions h_1, h_2, h_3, h_4 are indicated.

Let Z_s=conductors per slot,

I_z=current carried by each conductor,

and L=length of the conductor portion of slot.

Conductor Portion. Consider a strip of height dx at a distance x from the bottom of the conductors.

Permeance of the strip $=\mu_0\times\dfrac{\text{area of the flux path}}{\text{length of flux path}}=\mu_0\dfrac{Ldx}{W_s}$

Conductors producing flux in the strip$=\dfrac{x}{h_1}Z_s$

Mmf producing flux in this strip $=\dfrac{x}{h_1}Z_sI_s$

Flux in the strip,

$d\Phi_s$=mmf producing the flux×permeance of the strip

$$=\frac{x}{h_1}Z_sI_s\times\mu_0 L\frac{dx}{W_s}=\mu_0 L\frac{Z_sI_s}{W_s}\cdot\frac{x}{h_1}dx.$$

Portion of conductors linking with flux in the strip$=Z_s\dfrac{x}{h_1}$.

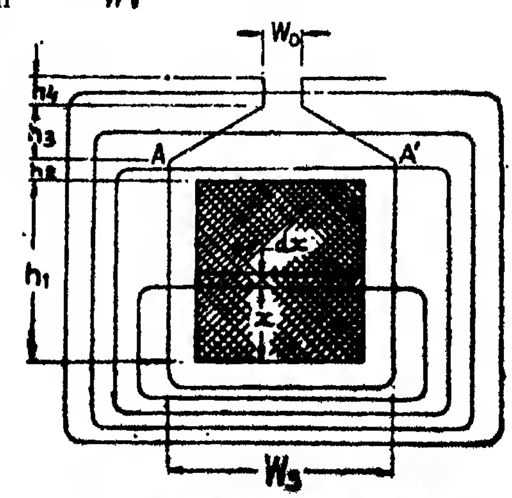


Fig. 4·38. Parallel sided slot.

$\therefore$ Flux linkages of the flux in the strip. $d\psi_x$=flux in the strip × no. of conductors linked

$$=\mu_0 L\frac{Z_s I_s}{W_s}\frac{x}{h_1}dx\times Z_s\frac{x}{h_1}=\mu_0\frac{L}{W_s}Z_s^2 I_s\left(\frac{x}{h_1}\right)^2 dx$$

$\therefore$ Total flux linkages of the conductor portion of the flux,

$$\psi=\int_0^{h_1}\mu_0\frac{L}{W_s}Z_s^2\left(\frac{x}{h_1}\right)^2 dx=\mu_0\frac{L}{W_s}Z_s^2 I_s\frac{h_1}{3}.$$

From Eqn. 4·62, effective permeance of conductor portion

$$\Lambda_1=\frac{\text{total flux linkages}}{\text{total turns}\times\text{total mmf}}$$

$$=\frac{\mu_0\frac{L}{W_s}Z_s^2 I_s\frac{h_1}{3}}{Z_s\times I_s Z_s}=\mu_0 L\frac{h_1}{3W_s}.$$

$\therefore$ Specific permeance for the conductor portion $\lambda_1=\frac{\Lambda_1}{L}=\mu_0\frac{h_1}{3W_s}$.

Same results are obtained by the application of Eqn. 4·64. A tube of force at a distance x from the bottom of conductors links with x/h_1 of the total conductors.

$$\therefore \qquad T'_x=\frac{x}{h_1}T'=\frac{x}{h_1}Z_s \text{ and } y=W_s.$$

Hence from Eqn. 4·64, specific permeance of conductor portion,

$$\lambda_1=\mu_0\int_0^{h_1}\left(\frac{T_x}{T}\right)^2\frac{dx}{y}=\mu_0\int_0^{h_1}\frac{x^2}{h_1^2}\frac{dx}{W_s}=\mu_0\frac{h_1}{3W_s}$$

Non-conductor portions. The flux tubes in the non-conductor portions link with all the conductors. Also these tubes of force are produced by all the conductors in the slot. Therefore, Eqn. 4·60 can be applied for the calculation of specific permeances of non-conductor portions.

Height h_2 : Length of flux path, $y=W_s$.

Specific permeance $\lambda_2=\mu_0\int_0^{h_2}\frac{dx}{y}=\mu_0\int_0^{h_2}\frac{dx}{W_s}=\mu_0\frac{h_2}{W_s}$

Height h_3 :

$$y=(W_o-W_s)\,x/h_3+W_s \text{ taking } x \text{ from } AA'$$

$\therefore$ Specific permeance

$$\lambda_3=\mu_0\int_0^{h_3}\frac{dx}{y}=\mu_0\int_0^{h_3}\frac{dx}{(W_o-W_s)\,x/h_3+W_s}$$

$$=\mu_0\frac{h_3}{W_o-W_s}\log_e\frac{W_o}{W_s}$$

$$=\mu_0\frac{h_3}{W_s-W_o}\log_e\frac{W_s}{W_o}\simeq\mu_0\frac{2h_3}{W_s+W_o}$$

The specific permeance can be determined by taking the length of the flux path 'y' as the mean of the widths W_s and W_o.

Thus, $$y=\frac{W_s+W_o}{2}$$

$\therefore$ Specific permeance

$$\lambda=\mu_0\int_0^{h_3}\frac{dx}{y}=\mu_0\int_0^{h_3}\frac{dx}{(W_s+W_o)/2}=\mu_0\,\frac{2h_3}{W_s+W_o}$$

Height h_4 :

Length of flux path $y=W_o$

Specific permeance $$\lambda_4=\mu_0\int_0^{h_4}\frac{dx}{y}=\mu_0\int_0^{h_4}\frac{dx}{W_o}=\mu_0\,\frac{h_4}{W_o}.$$

As all the leakage flux paths are in parallel the total specific slot permeance λ_s is the sum of the individual specific permeances.

Total specific slot permeance for parallel sides slots

$$\lambda_s=\lambda_1+\lambda_2+\lambda_3+\lambda_4$$

$$=\mu_0\left[\frac{h_1}{3W_s}+\frac{h_2}{W_s}+\frac{2h_3}{W_s+W_o}+\frac{h_4}{W_o}\right] \qquad ...(4\cdot72)$$

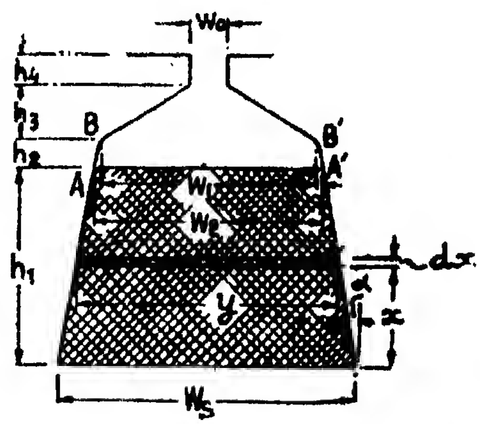


Fig. 4·39. Tapered slot.

4·11·2. Specific permeance of tapered slot. Considering the tapered slot shown in Fig. 4·39.

Conductor portion h_1. Length of strip at a distance x from the bottom of the conductors.

$$y=W_s-2x\tan\alpha \text{ where } \tan\alpha=\frac{W_s-W_2}{2h_1}.$$

Portion of conductors producing flux in the strip

$$T_x=\frac{(y+W_s)x}{(W_2+W_s)h_1}Z_s, \text{ as } T=Z_s.$$

$\therefore$ Specific permeance of the conductor portion

$$\lambda_1=\mu_0\int_0^{h_1}\left(\frac{T_x}{T}\right)^2\frac{dx}{y} \quad \text{(Eqn' 4·64)}$$

$$=\mu_0\int_0^{h_1}\frac{\left(\dfrac{y+W_s}{W_2+W_s}\right)^2\dfrac{x^2}{h_1^2}Z_s^2}{Z_s^2}\,\frac{dx}{y}.$$

$$=\mu_0\,\frac{h_1}{W_s}\,\frac{K^2-\dfrac{K^4}{4}-\log K-\dfrac{3}{4}}{(1-K)(1+K^2)^2} \text{ where } K=\frac{W_2}{W_s}$$

The appropriate value of $\lambda_1=\mu_0\frac{2h_1}{3(W_s+W_2)}$.

This is based upon the fact that the average length of the flux paths is $y=(W_s+W_2)/2$.

Non conductor portion. The flux tubes in the non-conductor portions are produced by all the conductors in the slot and they link with all the conductors. Therefore, Eqn. 4·60 can be applied for calculation of specific permeance of non-conductor portions

Height h_2 : $\lambda_2\cong\mu_0\frac{2h_2}{W_2+W_1}$.

Height h_3 : $\lambda_3\cong\mu_0\frac{2h_3}{W_1+W_o}$.

Height h_4 : $\lambda_4=\mu_0\frac{h_4}{W_o}$.

Total specific slot permeance for tapered slots

$$\lambda_s=\lambda_1+\lambda_2+\lambda_3+\lambda_4$$

$$=\mu_0\left[\frac{2h_1}{3(W_s+W_2)}+\frac{2h_2}{W_1+W_2}+\frac{h_3}{W_1+W_o}+\frac{h_4}{W_o}\right] \quad ..(4\cdot73)$$

4·11·3. Specific permeance of circular slots.

Conductor portion. The length of a infinitesimal strip at a distance x from the bottom slot (Fig. 4·40) is,

$$y=2r\sin\alpha$$

where $\cos\alpha=\frac{r-x}{r}$

or $dx=r\sin\alpha\, d\alpha$.

Area of segment of height x

$$=\frac{r^2}{2}(2\alpha-\sin 2\alpha).$$

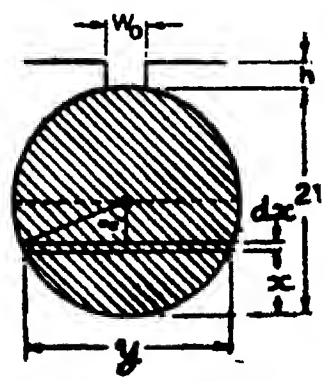


Fig. 4·40. Circular Slot.

∴ Portion of conductors producing flux in the strip

$$=\frac{\frac{r^2}{2}.(2\alpha-\sin 2\alpha)}{\pi r^2}Z_s=\frac{\alpha-\frac{1}{2}\sin 2\alpha}{\pi}Z_s.$$

Flux produced in the strip.

$$d\Phi_s=\text{mmf producing flux}\times\text{permeance of strip}$$

$$=\left(\frac{\alpha-\frac{1}{2}\sin 2\alpha}{\pi}Z_s I_z\right)\times\left(\mu_0\frac{Ldx}{y}\right)$$

$$=\left(\frac{\alpha-\frac{1}{2}\sin 2\alpha}{\pi}Z_s I_z\right)\left(\mu_0 L\frac{r\sin\alpha\, d\alpha}{2r\sin\alpha}\right)$$

$$=\left(\frac{\alpha-\frac{1}{2}\sin 2\alpha}{\pi}Z_s I_z\right)\left(\mu_0\frac{L}{2}d\alpha\right)$$

$$=\frac{\mu_0 Z_s I_z L}{2\pi}(\alpha-\tfrac{1}{2}\sin 2\alpha)\, d\alpha$$

Turns linking with flux $d\Phi_s$ are $\frac{\alpha-\frac{1}{2}\sin 2\alpha}{\pi}Z_s$.

Flux linkages with flux in the strip

$$d\psi_s=\frac{\mu_0 Z_s\, I_s\, L}{2\pi}(\alpha-\tfrac{1}{2}\sin 2\alpha)\, d\alpha.\left(\frac{\alpha-\frac{1}{2}\sin 2\alpha}{\pi}\right)Z_s$$

$$=\frac{\mu_0 Z_s^2\, I_s\, L}{2\pi^2}(\alpha-\tfrac{1}{2}\sin 2\alpha)^2\, d\alpha.$$

Hence, total flux linkages of flux in the conductor portion of the slot

$$\psi=\frac{\mu_0 Z_s^2 I_s\, L}{2\pi^2}\int_0^{\pi}(\alpha-\tfrac{1}{2}\sin 2\alpha)^2\, d\alpha=0.623\ \mu_0\ Z_s^2\ I_s\ L$$

$\therefore$ Effective permeance of the conductor portion (Eqn. 4·62)

$$\Lambda_c=\frac{\text{flux linkages}}{I_s\, Z_s^2}=0.623\ \mu_0 L$$

Specific permeance of conductor portion $\lambda_c=\frac{\Lambda_c}{L}=0.623\ \mu_0.$

The value of specific permeance is raised to 0·66 μ_0 to compensate for the actual flux pattern.

Thus, $\lambda_c=0.66\ \mu_0.$

Specific permeance of slot opening

$$\lambda_1=\int_0^h \mu_0\frac{dx}{y}=\mu_0\ \frac{h}{W_o}$$

Hence total specific slot permeance for circular slots

$$\lambda_s=\lambda_c+\lambda_1=\mu_0\left[0.66+\frac{h}{W_o}\right] \qquad \text{...(4·74)}$$

4·11·4. Specific permeance of semicircular bottom portion. Fig. 4·41 shows a slot with a semicircular bottom. The permeance calculations are based upon the assumption that the semicircular bottom portion does not contain any conductors. The portion is shown in enlarged form in Fig. 4·42.

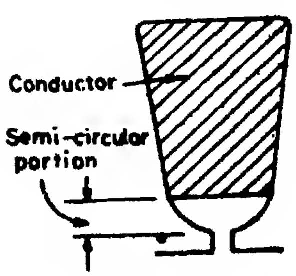


Fig. 4·41. Slot with semi-circular bottom.

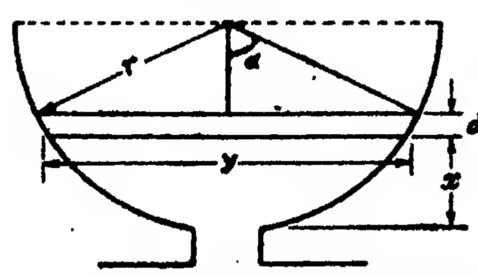


Fig. 4·42. Semi-circular bottom portion.

The length of infinitesimal strip at a distance x from the bottom of slot $y=2r\sin\alpha$.

Also $x=r-r\cos\alpha \quad \therefore\ dx=r\sin\alpha\, d\alpha.$

Conductors producing flux in the strip = conductors per slot $=Z_s$ and the flux in the strip links with all the conductors in the slot.

$\therefore$ We can apply Eqn. 4·60 for the calculation of specific permeance of semi-circular portion.

$$\lambda=\mu_0\int\frac{dx}{y}=\mu_0\int_0^{\pi/2}\frac{r\sin\alpha\,d\alpha}{2r\sin\alpha}=\frac{\pi}{4}\mu_0 \qquad \ldots (4\cdot75)$$

4·11·5. Specific permeance of closed type of slots. Closed type of slots are often used for small induction motors to increase the leakage reactance which in turn decreases the starting current. The low starting current permits the use of direct on line (DOL) starters which are the cheapest of the induction motor starters.

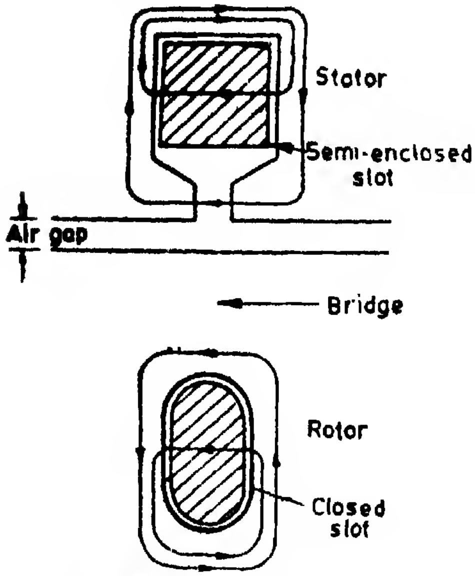


Fig. 4·43. Closed slot.

Small induction motors use rotors with die cast aluminium cage. The 'bridge' *i.e.*, iron part above the slot mechanically retains the molten metal during casting. Fig 4·43 shows a closed type of rotor slots wherein there is no slot opening. The leakage flux path on top of the slot is through iron, which is called 'bridge'. Therefore, the specific permeance of this path depends upon the permeance of iron which in turn depends upon the degree of saturation and hence upon the relative permeability of iron. The specific permeance of a closed slot can be determined by multiplying the last term in Eqn. 4·72 by μ_r, the effective relative permeability of bridge. The value of μ_r depends upon the degree of saturation in the bridge and therefore it is very difficult to assess. The value of μ_r varies from 25 at low conductor currents down 3 or 4 when the current is large, (a condition which is obtained when starting induction motors).

4·11·6. Specific slot permeances for complicated shaped slots. The relations derived above for specific slot permeance are very useful for slots of simple shapes. When the shape of the slot is less simple, the calculation of specific slot permeance becomes more diffi-

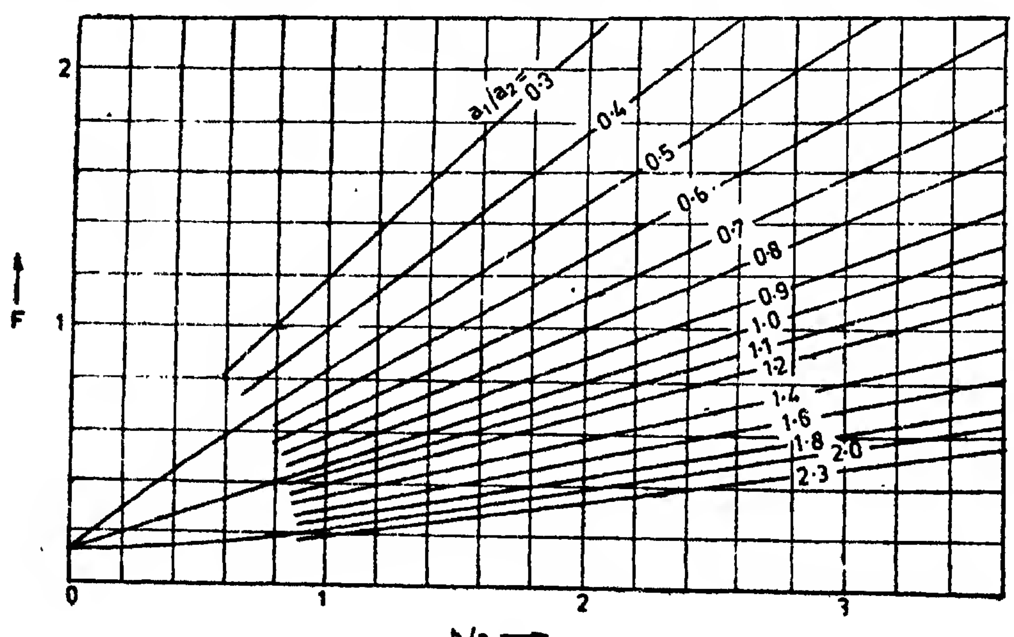


Fig. 4·44

cult. However evaluation of specific slot permeance, for the most commonly used slots, can be made from Figs. 4·44 and 4·45.

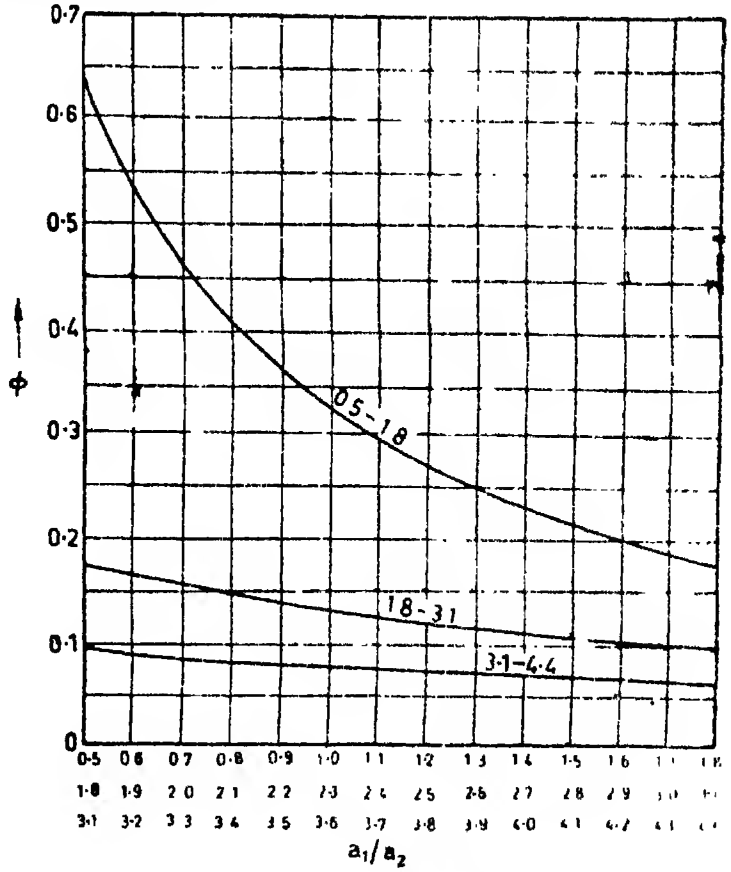


Fig. 4·45

These types of solts are commonly used for single phase and small three phase induction motors.

1. For slot of Fig. 4·46.

Specific slot permeance $\lambda_s = \mu_0\left[F + \frac{d}{c} + \frac{2e}{e+a_1}\right]$ ...(4·76)

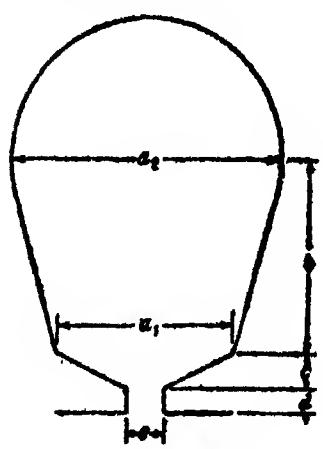


Fig. 4·46

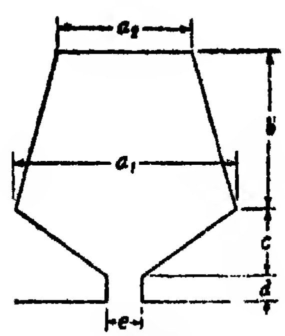


Fig. 4·47

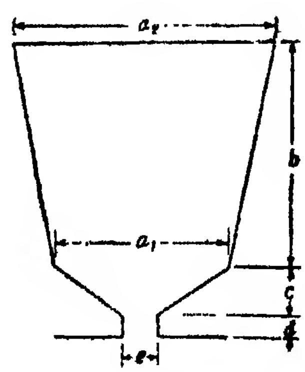


Fig. 4·48

The value of F is taken from Fig. 4·44.

2. For slots of Fig. 4·47 and 4·48.

Specific slot permeance $\lambda_s = \mu_0 \left[\Phi \frac{b}{a_2} + \frac{d}{e} + \frac{2c}{e+a_1} \right]$ ...(4·77)

The value of Φ is taken from Fig. 4·45,

3. For slot of Fig. 4·49.

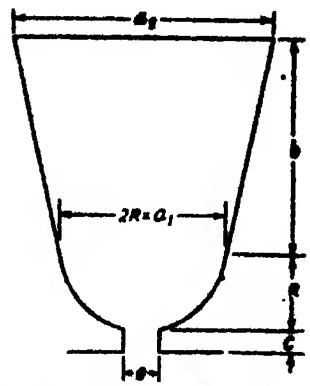


Fig. 4·49

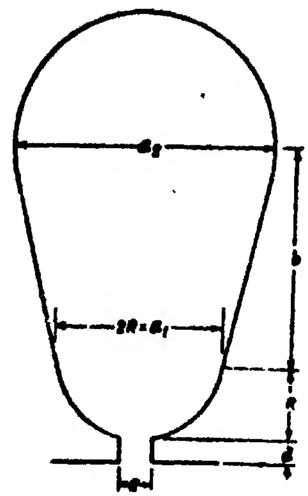


Fig. 4·50

Specific slot permeance

$$\lambda_s = \mu_0 \left[\Phi \frac{b}{a_2} + \frac{d}{e} + \frac{\pi}{4} \frac{\cos^{-1}(e/2R)}{90^\circ} \right] \quad ...(4·78)$$

The value of Φ is taken from Fig. 4·45.

4. For slot of Fig. 4·50,

Specific slot permeance $\lambda_s = \mu_0 \left[F + \frac{d}{e} + \frac{\pi}{4} \frac{\cos^{-1}(e/2R)}{90^\circ} \right]$ .. (4·79)

The value of F is taken from Fig. 4·44.

4·11·7. Specific permeance of slots with double layer windings. In a double layer winding, there are two layers per slot. Normally there are two coil sides per slot. One coil side is at the top of the slot while the other is at the bottom. Fig. 4·51 shows A as the top coil side and B as the bottom coil side. The total reactance of the coils will be made of the reactances of the top and bottom coil sides plus twice the mutual reactance between the top and the bottom coil sides.

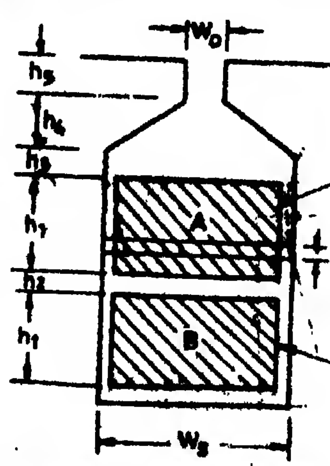


Fig. 4·51. Slot with double layer winding.

From Eqn. 4·72, for parallel sided slots,

specific slot permeance of the top coil side

$$\lambda_A = \mu_0 \left[\frac{h_1}{3W_s} + \frac{h_2}{W_s} + \frac{2h_4}{W_s + W_0} + \frac{h_5}{W_0} \right] \quad ...(4·80)$$

and similarly,

Specific slot permeance of the bottom coil side

$$\lambda_B=\mu_0\left[\frac{h_1}{3\,W_s}+\frac{h_1+h_2+h_3}{W_s}+\frac{2h_4}{W_s+W_o}+\frac{h_5}{W_o}\right] \quad \ldots(4\cdot81)$$

The presence of two coils sides results in mutual inductance between them. The specific slot permeance corresponding to the mutual flux is now to be calculated. Let us consider the effect of the flux produced by the top coil side linking with the bottom coil side.

Conductor portion. Consider an infinitesimal strip of height dx at a distance x from the bottom of top coil side.

$$\text{Mmf producing this flux}=I_z\,Z_L\cdot\frac{x}{h_1}$$

where Z_L=conductors per layer.

$$\text{Permeance of the strip}=\mu_0\frac{L\,dx}{W_s}$$

$\therefore$ Flux in the strip $d\Phi_x=I_zZ_L\dfrac{x}{h_1}\times\mu_0\dfrac{Ldx}{W_s}=\mu_0\,I_z\,Z_L\dfrac{L}{W_s}\,\dfrac{x}{h_1}dx.$

The flux links with all the conductors in the bottom coil side (we are considering only the mutual flux).

$\therefore$ Flux linkages due to flux in the strip

$$d\psi_x=\mu_0\,I_z\,Z_L\frac{L}{W_s}\,\frac{x}{h_1}dx\,Z_L=\mu_0\,I_z\,Z_L^2\,\frac{L}{W_s}\,\frac{x}{h_1}dx$$

Total flux linkages corresponding to the mutual flux

$$=\mu_0\,I_z\,Z_L^2\,\frac{L}{W_s}\int_0^{h_1}\frac{x}{h_1}dx=\mu_0\,I_z\,Z_L^2L\frac{h_1}{2W_s}$$

Effective permeance for the mutual flux

$$\Lambda_{1m}=\frac{\mu_0\,I_z\,Z_L^2\,L\dfrac{h_1}{2W_s}}{I_z\,Z_L^2}=\mu_0\,L\frac{h_1}{2W_s}$$

$\therefore$ Specific permeance of the conductor portion for the mutual flux

$$\lambda_{1m}=\frac{\Lambda_{1m}}{L}=\mu_0\,\frac{h_1}{2W_s}$$

Non conductor portion. In the non-conductor portion, the flux is produced by all the conductors in the top coil side and the flux so produced links with all the conductors in the bottom coil side. Therefore, Eqn. 4·60 can be applied for the non-conductor portion.

Using, Eqn. 4·60, the specific permeances are :

$$\lambda_{3m}=\mu_0\,\frac{h_3}{W_s},\ \lambda_{4m}=\mu_0\,\frac{2h_4}{W_s+W_o},\ \lambda_{5m}=\mu_0\frac{h_5}{W_o}$$

$\therefore$ Total specific slot permeance corresponding to mutual flux

$$\lambda_{sm}=\lambda_{1m}+\lambda_{3m}++\lambda_{4m}+\lambda_{5m}$$

$$=\mu_0\left[\frac{h_1}{2W_s}+\frac{h_3}{W_s}+\frac{2h_4}{W_s+W_o}+\frac{h_5}{W_o}\right] \quad \ldots(4\cdot82)$$

Specific permeance of bottom coil side for its own flux is λ_B and is given by Eqn. 4·81. Specific permeance corresponding to flux produced by top coil side and linking with bottom coil side is $\lambda_{AB}=\lambda_{sm}$ and is given by Eqn. 4·82.

$\therefore$ Effective specific permeance for bottom coil side $\lambda_b=\dfrac{\lambda_B+\lambda_{AB}}{2}$

Similarly effective specific permeance for top coil side $\lambda_t=\dfrac{\lambda_A+\lambda_{BA}}{2}$

Since each coil has one coil side in the bottom and the other in the top of the slot,

Specific slot permeance for double layer windings,

$$\lambda_s=\frac{\lambda_t+\lambda_b}{2}=\frac{\lambda_A+\lambda_B+\lambda_{AB}+\lambda_{BA}}{4}$$

$$=\frac{\lambda_A+\lambda_B+2\lambda_{AB}}{4} \qquad \text{.(4·83)}$$

where $\lambda_{AB}=\lambda_{BA}=\lambda_{sm}$

Substituting the values of λ_A, λ_B and λ_{AB} in Eqn. 4·83, we have specific slot permeance for double layer windings

$$\lambda_s=\frac{\lambda_A+\lambda_B+2\lambda_{AB}}{4}$$

$$=\mu_0\left[\frac{2h_1}{3W_s}+\frac{h_2}{4W_s}+\frac{h_3}{W_s}+\frac{2h_4}{W_o+W_s}+\frac{h_5}{W_o}\right] \qquad \text{...(4·84)}$$

If we neglect the insulation between two coil sides *i.e.* $h_2=0$, we have

$$\lambda_s=\mu_0\left[\frac{2h_1}{3W_s}+\frac{h_3}{W_s}+\frac{2h_4}{W_o+W_s}+\frac{h_5}{W_o}\right] \qquad \text{...(4·85)}$$

If we compare Eqn. 4·85 (for a double layer winding) and Eqn. 4·72 (for a single layer winding), we find that the two are identical (Comparing Fig. 4·38 with Fig. 4·51 we find that the height of conductor portion in single layer winding is h_1 while in two layer winding it is $2h_1$). **Thus Eqn. 4·72 can be used for double layer windings as well.**

4·11·8. Specific permeance of slots of special purpose Induction Motors. Deep bars and *T* bars are used for the rotors of squirrel cage induction motors to give a high torque at starting and a good efficiency while running.

The frequency of rotor currents of squirrel cage induction motors at starting near about 50 Hz during starting and below 5 Hz when running.

'T' Bar Rotors. *A 'T'* bar rotor conductor is shown in Fig. 4·52. The upper portion of the conductor has an area 'a_1' where the lower portion of the conductor has an area a_2.

At low frequencies (about 5 Hz), the current distribution throughout the conductor cross-section a_1 and a_2 is uniform.

The specific slot permeance for this case is given by :

$$\lambda_s=\frac{\mu_0}{(a_2+a_1)^2}\left[\frac{h_1}{W_s}\cdot\frac{a_1^2}{3}+\frac{h_2}{W_s}\left\{a_1^2+a_1a_2+\frac{a_2^2}{3}\right\}\right]+\mu_0\frac{h_3}{W_s} \qquad \text{...(4·86)}$$

The total conductor cross-section is (a_1+a_2) and the two parts share the current proportional to their area.

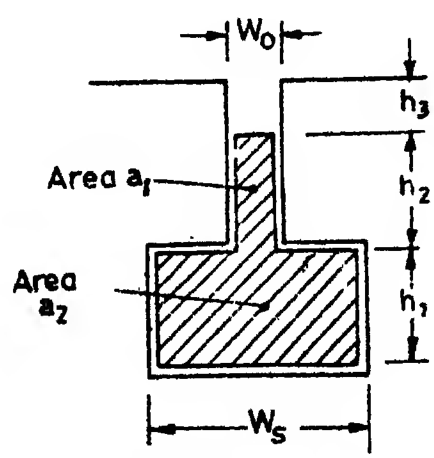


Fig. 4·52. *T* bar rotor.

At starting, the frequency of the rotor currents is almost equal to the supply frequency *i.e.* 50 Hz and therefore the leakage reactance of the lower portion of the conductor a_2 is quite high. This forces almost the entire current to flow through the upper part of the conductor which has an area of 'a_1'. Therefore, the specific slot permeance under starting conditions is :

$$\lambda_s = \mu_0 \left[\frac{h_1}{2W_s} + \frac{h_2}{3W_s} + \frac{h_3}{W_s} \right] \qquad \ldots(4{\cdot}87)$$

The values of specific slot permeance fall in between during the period of starting and speed changes and these values are of signifiance.

4·11·9. Specific permeance of deep bar rotor slots. Deep bar rotor slots as used in squirrel cage induction motors are shown in Fig. 4·53 (*a*) and (*b*). Deep bar rotors are used for increasing the starting torque of the induction motors. The increased starting torque is obtained by increasing the effective resistance of the stator at starting. The principle of operation of deep bar rotors is the same as that of *T* bar rotors. Consider first a squirrel cage rotor having deep and narrow bars as shown in Fig. 4·53. The general configuration of the slot leakage field produced by the current in the bar within the slot is shown in the diagram. Assuming that the iron used has an infinite permeability, all the leakage flux lines would close in the paths below the slots as shown. Consider that a bar consists of a number of layers of differential depth ; one at the top and the other at the bottom as shown in Fig. 4·54. The leakage inductance of the bottom layer is considerably larger than that of the layer because the bottom layer links with a large leakage flux. Thus the leakage reactance of the bottom layer is much more than that of the top layer. At starting the frequency of rotor emf is equal to the supply frequency and therefore

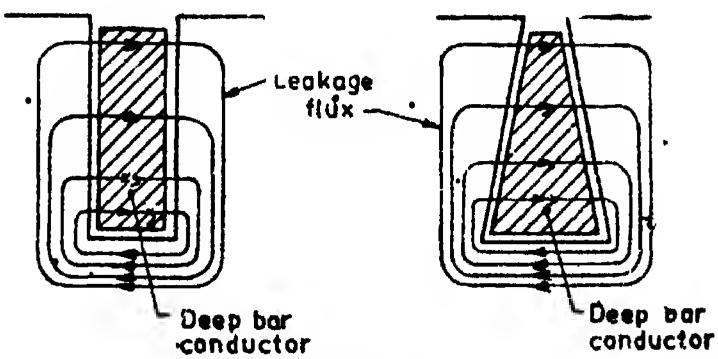


Fig. 4·53. Deep bar rotors.

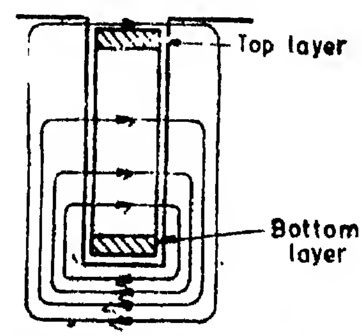


Fig. 4·54 Deep bar rotor represented by the top and a bottom layer.

it is leakage reactance which is the determining parameter for the distribution of rotor current between the top and bottom layers (since the two layers are connected in parallel). Consequently, at starting the current in the top layer is much greater than that in the bottom layer. Therefore, the concentration of current is mainly in the top layer. This results in large increase in the effective resistance of the rotor and consequently increase in starting torque. Since the distortion in rotor current distribution depends upon the inductive effect, the effective resistance is a function of the frequency. The effective resistance is also a function of the depth of the bar. A squirrel carge rotor with deep bars can be readily designed to have an effective resistance at stator frequency (stand still) several times greater than its d.c. resistance. As the rotor accelerates, the frequency of rotor currents drops down and therefore the leakage reactance gradually loses its effect to distribute current, the current distribution is then mainly determined by the resistance of top and bottom portions. Under running conditions, the resistance of the rotor is its d.c. resistance and the current distribution is uniform.

The values of specific slot permeance for deep bar rotors can be calculated from Eqns. 4·86 and 4·87 under actual operating conditions and using suitable modifications.

4·12. Tooth top leakage flux. Tooth top leakage depends primarily on the length of air gap. It is negligible in the case of machines with very small air gaps like induction motors while it is appreciable in d.c. machines where the air gap is larger. The presence of interpoles in the case of d.c. machines increases the tooth top leakage flux.

The calculation of this flux depends upon the assumption of shape of its path. In machines with small air gaps, its path can be assumed to be straight lines circumferentially along the air gap. Eqn. 4·60 is applied to such a case giving,

length of flux path y=slot pitch y_s.

height of flux path h=length of air gap l_g.

∴ Specific permeance for tooth top leakage

$$\lambda_t=\mu_0\int_0^h \frac{dx}{y}=\mu_0\int_0^{l_g}\frac{dx}{y_s}=\mu_0\,\frac{l_g}{y_s} \qquad \ldots(4·88)$$

For large gaps without interpoles, the leakage paths can be taken as sum of two quadrants and a straight portion (Fig. 4·55).

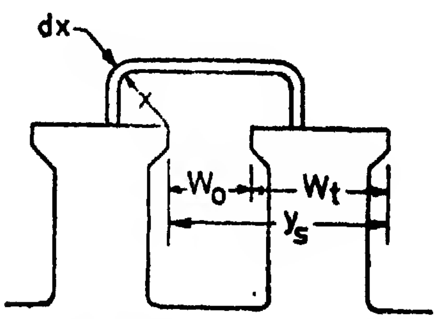


Fig. 4·55. Tooth top leakage.

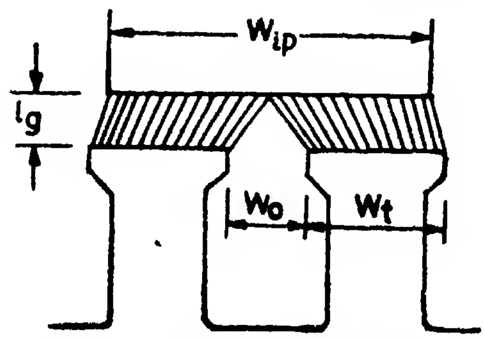


Fig. 4·56. Tooth top leakage with interpoles.

The length of path $y=W_o+\pi x$.

Using Eqn. 4·60,

$$\lambda_t=\mu_0\int_0^{W_t/2}\frac{dx}{W_o+\pi x}=\frac{\mu_0}{\pi}\log_e\left(1+\frac{\pi W_t}{2W_o}\right) \qquad \ldots(4·89)$$

The field distribution in the case of interpolar machines varies according to the position of slot with respect to interpole. Accurate results can only be obtained by taking mean of permeances for various positions.

An inspection of Fig. 4·56 will indicate that an approximation to the permeance of the path of the flux crossing from one tooth tip to next via interpole shoe is, when the slot is centrally placed under the interpole.

$$\text{Tooth top permeance, } \Lambda_t=\mu_0\frac{\text{area of flux path}}{\text{length of flux path}}$$

$$=\mu_0\,\frac{\left(\frac{W_{ip}}{2}\right)L}{2l_{gi}}=\mu_0\,\frac{W_{ip}\,L}{4\,l_{gi}}$$

where W_{ip}=width of interpole

l_{gi}=length of air gap under the interpole.

$$\text{Tooth top specific permeance } \lambda_t=\mu_0\,\frac{W_{ip}}{4l_{gi}} \qquad \ldots(4·90)$$

*Eqn. 4·90 holds if the length of interpole is equal to the length of the machine. But normally the length of the interpole is between half to two-thirds that of the machine. For accuracy it would then be necessary to calculate the permeance under the interpole and that outside. However, an empirical formula given below holds good for average condition,

$$\lambda_i = \mu_o \frac{W_{ip}}{6 l_{gi}} \quad \text{.. (4·91)}$$

4·13. Zigzag Leakage

Let AT_s = mmf per slot, y_{ss} = stator slot pitch,
W_{ts} = width of stator tooth, y_{sr} = rotor slot pitch,
W_{tr} = width of rotor tooth,
H = magnetic potential gradient acting circumferentially around the air gap.

If x be the distance between centre lines of a rotor and a stator tooth as shown in Fig. 4·57, then,

Mmf between them, $AT_x = Hx = \dfrac{AT_s\, x}{y_{ss}}$.

This mmf causes the flux to pass from one tooth to another. The permeance for the path of zigzag leakage flux will depend upon the relative positions of rotor and stator and the simplest way to find average effect is to sum the stored magnetic energies in the various positions.

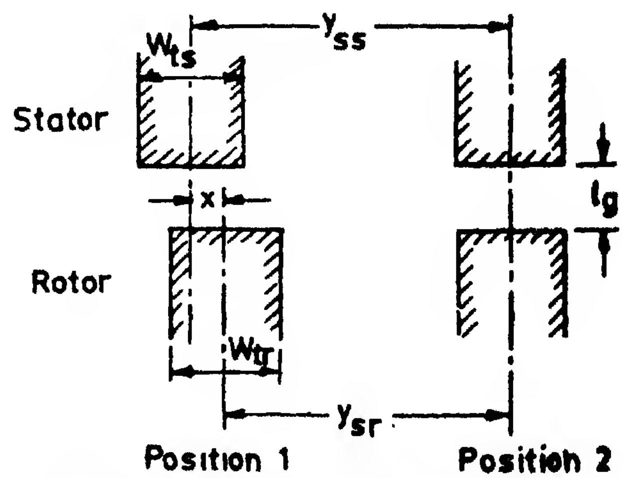


Fig. 4·57. Determination of zigzag permeance.

Stored energy at any position.

$$= \tfrac{1}{2}^* AT_x^2 \Lambda$$

where Λ = permeance in a particular position

For a tooth in position (1),

$$\Lambda_1 = \frac{\mu_o L}{l_g}\left[\frac{W_{ts}+W_{tr}}{2} - x\right]$$

for $\left(\dfrac{W_{ts}-W_{tr}}{2} < x < \dfrac{W_{ts}+W_{tr}}{2}\right)$

∴ Stored energy in position 1

$$W_1 = \tfrac{1}{2}\mu_o \frac{AT_s^2\, x^2\, L}{l_g y_{ss}^2}\left(\frac{W_{ts}+W_{tr}}{2} - x\right)$$

For a tooth in position (2),

$$\Lambda_2 = \frac{\mu_o}{l_g} L W_{tr} \text{ for } \left(0 < x < \frac{W_{ts}-W_{tr}}{2}\right)$$

Hence stored energy in position 2,

$$W_2 = \tfrac{1}{2}\mu_o \frac{AT_s^2 L}{l_g} \frac{x^2}{y_{ss}^2} . W_{tr}.$$

---

*Stored energy $= \frac{1}{2} LI^2$

But inductance $L = \dfrac{1}{2}\dfrac{T^2}{S}$

∴ Stored energy $= \dfrac{1}{2}\dfrac{T^2 I^2}{S} = \dfrac{1}{2}(TI)^2$

Adding these energies and finding their average over the rotor slot pitch y_{sr},

$$W_{av}=\frac{1}{2y_{sr}}\left[\int_{(W_{ts}-W_{tr})/2}^{(W_{ts}+W_{tr})/2} W'_1 dx + \int_{0}^{(W_{ts}-W_{tr})/2} W_2 dx\right]$$

$$=\tfrac{1}{2}\mu_0\, AT_z^2 L\ \frac{W_{ts}\, W_{tr}(W_{ts}^2+W_{tr}^2)}{12 l_g\, y_{ss}\, y_{sr}}.$$

Zigzag permeance $\Lambda_z=\dfrac{W_{av}}{\frac{1}{2}AT_z^2}$

$\therefore$ Zigzag specific permeance

$$\lambda_z=\frac{\Lambda_z}{L}=\frac{W_{av}}{\frac{1}{2}LAT_z^2}$$

$$=\mu_0\ \frac{W_{ts}\, W_{tr}\,(W_{ts}^2+W_{tr}^2)}{12\, l_g\, y_{ss}^2\, y_{sr}} \qquad \ldots(4.92)$$

The following relationship may also be used for calculation of zigzag specific permeance

$$\lambda_z=\mu_0\ \frac{(W_{ts}+W_{tr})^4}{96\, l_g\, y_{ss}^2\, y_{sr}} \qquad \ldots(4.93)$$

4.14. Overhang Leakage. The exact calculation of overhang leakage is very difficult. This is because the overhang leakage reactance depends upon the length of overhang and its shape (whose geometry is quite complicated, the spacing between stator and rotor overhangs, and the proximity and configuration of the neighbouring magnetic and conducting parts like cores, clamping and stiffening plates, machine frame and retaining rings etc.). The leakage reactance also depends upon the degree of saturation in the ferromagnetic parts.

Therefore, it is impossible to evolve a simple relationship which incorporates the above factors. Calculations based upon ideal shapes of coils yield empirical relationships which are unreliable. One of the relationships is :

Specific overhang permeance

$$\lambda_o=\frac{\mu_0 K_s \tau^2}{L_o \pi y_s} \qquad \ldots(4.94)$$

where K_s=slot leakage factor which is taken from Fig. 4.58.

τ=pole pitch,

L_o=length of conductor in overhang,

y_s=slot pitch

The simple and direct method of calculation of leakage reactance

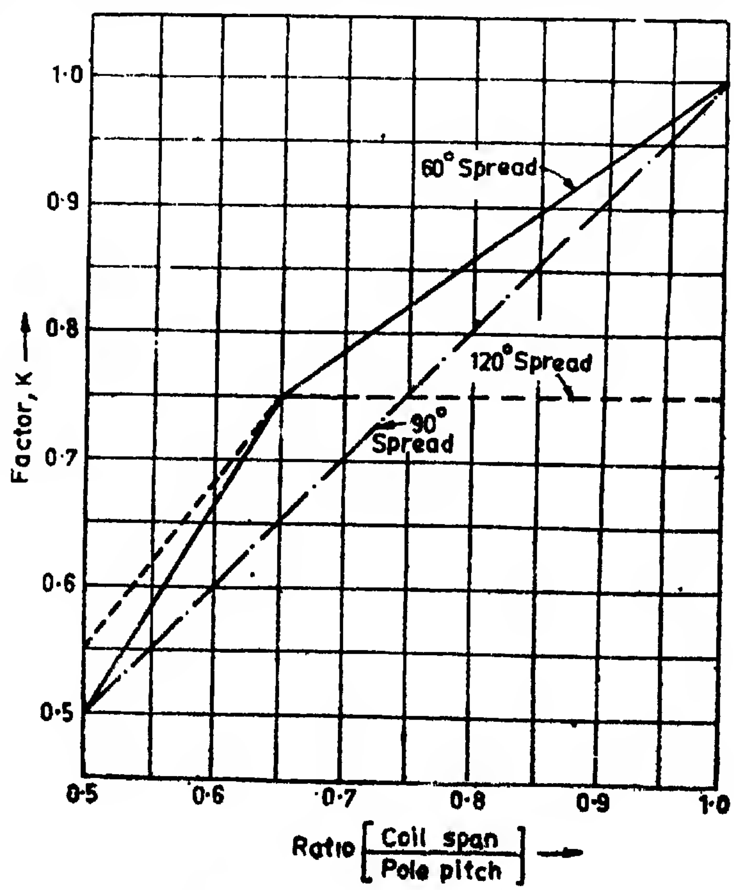


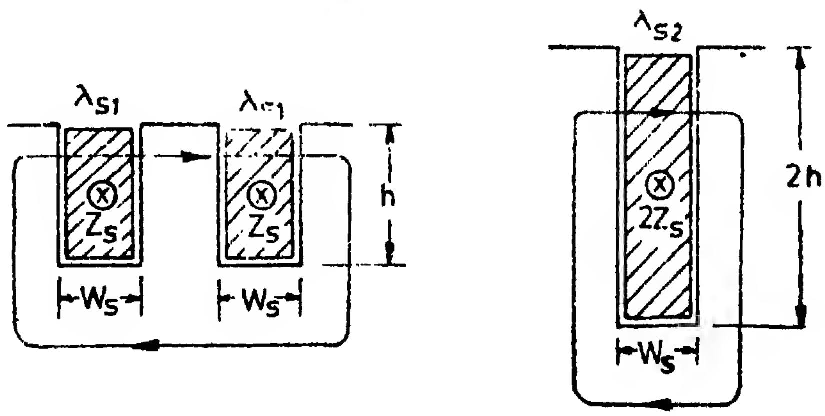
Fig. 4.58. Slot leakage factor.

as given by Eqn. 4·94 fails to give a correct assessment of the overhang leakage permeance for turbo-machines especially turbo-alternators.

4·15. Leakage Reactance calculations of polyphase machines

Let Z_s=conductors per slot, q=slots per pole per phase :
p=number of poles, T_{ph}=turns per phase

Let us first consider only the slot leakage flux and the leakage reactance produced due to it. Fig. 4·59 (*a*) shows an arrangement where there are 2 slots of each phase under one



(*a*) $q=2$ (*b*) $q=1$

Fig. 4·59. Effect of number of slots per pole per phase on leakage flux.

pole. The width of each slot is W_s, the depth of the conductor portion is h and each slot contains Z_s conductors, there being $2Z_s$ conductors of each phase under one pole.

Let us consider the specific permeance of the conductor portion only. The specific permeance of each slot is

$$\lambda_{s1}=\mu_o\ \frac{h}{3W_s}$$

Now the leakage flux travels through width of 2 slots. The magnetic circuit consists of reluctance of two slots connected in series and therefore the resultant specific permeance is,

$$\lambda_1=\frac{\lambda_{s1}}{2}=\mu_o\ \frac{h}{6W_s}.$$

Consider now that each phase has one slot under a pole (half the value used earlier). The $2Z_s$ conductors are now accommodated in one slot instead of two earlier. Therefore the area required for each slot is twice that of the first case. The width of the slots cannot be increased as this will result into a small cross-section for the teeth, resulting in a very high value of flux density in them—a high flux density in teeth is not desirable as it leads to large magnetizing current and also, the small cross-section of teeth may make them mechanically weak. Therefore, the depth of the slot is made two times the depth of the slot in the first case. This is shown in Fig. 4·59 (*b*). The height of conductor is now $2h$. The specific slot permeance is,

$$\lambda_{s2}=\mu_o\ \frac{2h}{3W_s}.$$

The leakage flux (for each phase) has now to cross only one slot width, therefore the resultant specific permeance for the second case is

$$\lambda_2=\lambda_{s2}=\mu_o\ \frac{2h}{3W_s}.$$

The ratio of specific slot permeance with 1 slot per pole per phase to specific permeance with 2 slots per pole per phase is,

$$\frac{\lambda_1}{\lambda_2}=\frac{\mu_0 \cdot 2h/3W_s}{\mu_0 \cdot h/6W_s}=4.$$

The leakage reactance is directly proportional to specific slot leakage permeance. Therefore, the slot leakage reactance with 1 slot per pole per phase is 4 times the slot leakage reactance with 2 slots per pole per phase. Thus, the leakage reactance is inversely proportional to the square of the number of slots per pole per phase. It has been explained in subsequent chapters, that the rotating electrical machines—especially induction motors—should not have a small number of slots per pole per phase as otherwise they will have a large leakage reactance.

Let us consider a general case now.

Fig. 4·59 shows the slots of a phase under one pole. There are q slots per phase under one pole and they carry the same current. The conductors in slots of a phase under a pole produce leakage flux and this leakage flux has to traverse the slots of a phase under one pole (q slots). Thus if λ_s is the specific slot permeance, the effective specific slot permeance for slots of a phase under one pole is λ_s/q, as these permeances are in series as shown in Fig. 4·59 (a). The conductors producing this leakage flux are :

$$=\text{conductors per slot}\times\text{slots per pole per phase}=Z_s q.$$

From Eqn. 4·67, reactance $X=2\pi f T^2 L\lambda$

Now for conductors of a phase under one pole $T=Z_s\,q$, also $\lambda=\lambda_s/q$.

∴ Slot leakage reactance of conductors of a phase under one pole

$$=2\pi f(Z_s q)^2\,L\frac{\lambda_s}{q}=2\pi f Z_s^2 q L\lambda_s.$$

If the conductors of a phase under all the poles are connected in series, the slot leakage reactance per phase,

$$x_s=p\times 2\pi f Z_s^2 q L\lambda_s=2\pi f p q Z_s^2 L\lambda_s$$

But $\quad Z_s=2T_{ph}/qp$

$$x_s=2\pi f p q\left(\frac{2\,T_{ph}}{qp}\right)^2 L\lambda_s$$

$$=8\pi f T_{ph}^2 L\,(\lambda_s/qp) \qquad \ldots(4·95)$$

Similarly,

Overhang leakage reactance per phase, $x_o=8\pi f T_{ph}^2 L_o\,(\lambda_o/qp)$ ...(4·96)

Zigzag leakage reactances per phase $x_z=8\pi f\,T_{ph}^2 L\,(\lambda_z/qp)$ ...(4·97)

Tooth top leakage reactance per phase $x_t=8\pi f T_{ph}^2 L(\lambda_t/qp)$ ...(4·98)

Harmonic leakage reactance per phase $\lambda_h=8\pi f T_{ph}^2 L(\lambda_h/qp)$ ...(4·99)

4·16. Leakage with fractional pitch windings. If the winding in a machine is chorded the two coil sides in any given slot will carry currents differing in phase by an angle ϵ, the angle by which the winding is short pitched.

Specific slot permeance of bottom coil side, for its own flux, λ_B is given by Eqn. 4·81. Specific slot permeance, corresponding to flux produced by top coil side and linking with bottom coil side, $\lambda_{AB}=\lambda_m$, is given by Eqn. 4·82.

As the winding is chorded by an angle α, the current in coil side A differs in phase from that in coil side B by an angle α (Fig. 4·51). The component of current in coil side A in phase with current in coil side B is proportional to $\cos \alpha$.

$\therefore$ Specific permeance for bottom coil side, $\lambda_b = \dfrac{\lambda_B + \lambda_{AB} \cos \alpha}{2}$

and similarly specific permeance for top coil side

$$\lambda_t = \frac{\lambda_A + \lambda_{BA} \cos \alpha}{2}$$

where λ_A is taken from Eqn. 4·80 and $\lambda_{BA} = \lambda_{AB} = \lambda_m$ is taken from Eqn. 4·82. Since each coil has one coil side in the bottom and the other in the top of a slot, specific slot permeance with chorded winding is :

$$\lambda_s = \frac{\lambda_t + \lambda_b}{2} = \frac{\lambda_A + \lambda_B + (\lambda_{AB} + \lambda_{BA}) \cos \alpha}{4} = \frac{\lambda_A + \lambda_B + 2\lambda_{AB} \cos \alpha}{4}$$

Specific slot permeance of a full pitched winding (from Eqn. 4·83).

$$\lambda_s = \frac{\lambda_A + \lambda_B + 2\lambda_{AB}}{4}$$

$\therefore$ The slot pitch factor.

$$K_s = \frac{\lambda_A + \lambda_B + 2\lambda_{AB} \cos \alpha}{\lambda_A + \lambda_B + 2\lambda_{AB}} \quad \ldots(4·100)$$

This is the factor by which the permeance of a chorded winding is reduced. This factor should be introduced when finding out the slot, tooth top and zigzag leakage reactances *i.e.* this is the factor by which Eqns. 4·95, 4·97, 4·98 and 4·99 should be multiplied to obtain leakage reactance of chorded windings. The expression for finding out over hang leakage reactance (Eqn. 4·96) already includes the effect of chording. The value of K_s can be directly obtained from Fig. 4·58.

Example 4·12. *A 3 phase, 50 Hz, 6 pole induction motor has 3 slots per pole per phase. The stator core length is 0·12 cm and there are 225 turns per phase in stator. Two alternative sizes of almost equal area [Figs. 4·60 (a) and (b)] are available for stator slots. Calculate the stator slot leakage reactance per phase in each case and comment on the result. The machine has a single layer winding.*

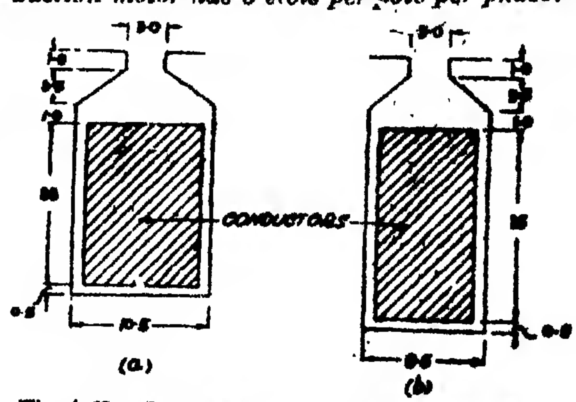


Fig. 4·60. Stator slots (All dimensions in mm).

Solution.

For Slot of Fig. 4·60 (*a*).

Specific slot permeance, (See Eqn. 4·72 for parallel sided slots)

$$\lambda_s = \mu_0 \left[\frac{h_1}{3 W_s} + \frac{h_2}{W_s} + \frac{2h_3}{W_s + W_0} + \frac{h_4}{W_0} \right]$$

$$= 4\pi \times 10^{-7} \left[\frac{28}{3 \times 10·5} + \frac{1}{10·5} + \frac{2 \times 3·5}{10·5 + 3} + \frac{1}{3} \right] = 23·1 \times 10^{-7}$$

From Eqn. 4·95, stator slot leakage reactance per phase

$$x_s = 8\pi f T^2_{ph} L \left(\frac{\lambda_s}{pq} \right) = 8\pi \times 50 \times (225)^2 \times 0·12 \times \frac{23·1 \times 10^{-7}}{6 \times 3} = 0·98\ \Omega.$$

For Fig. 4·60 (*b*) :

$$\lambda_s=4\pi\times10^{-7}\left[\frac{35}{3\times8\cdot5}+\frac{1\cdot0}{8\cdot5}+\frac{2\times3\cdot5}{8\cdot5+3}+\frac{1}{3}\right]=30\cdot5\times10^{-7}$$

$\therefore$ Stator slot leakage reactance per phase

$$x_s=8\pi\times50\times(225^2)^2\times0\cdot12\left(\frac{30\cdot5\times10^{-7}}{6\times3}\right)=1\cdot29\ \Omega.$$

The leakage reactance in the case of (*b*) is higher. Thus a deep and narrow slot gives a high value of leakage reactance as compared with a shallow and wide slot.

4·17. Leakage in salient pole machines. The leakage flux from salient poles can be determined accurately only by the method of flux plotting. However, the problem can be solved approximately in an idealized arrangement where pole axes are assumed parallel. This approximation is true only in the case of machines with large diameter and large number of poles.

Fig. 4·61 shows an idealized arrangement for non-interpole machines. There are four principal leakage paths.

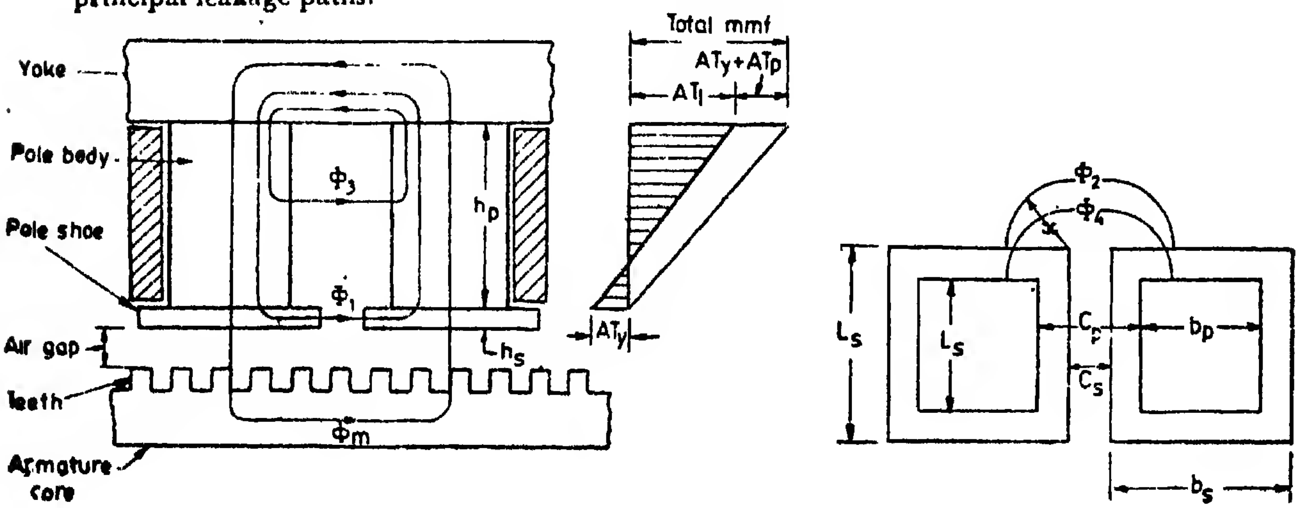


Fig. 4·61. Leakage in salient pole machines.

Φ_1, the leakage flux between the pole shoe inner surfaces,

Φ_2, the leakage flux between the pole shoe end surfaces,

Φ_3, the leakage flux between the pole body inner surfaces,

Φ_4, the leakage flux between the pole body end surfaces.

The total leakage flux is the sum of all the above leakage fluxes. Owing to the leakage flux, the pole and yoke carry larger flux. This results in larger pole and yoke sections or alternatively a larger mmf for the windings.

In the analysis given below the assumptions made are :

(*i*) the reluctance of iron parts is negligible and (*ii*) the leakage flux passing to the yoke directly is negligible.

Let AT=total mmf of each field coil, AT_g=mmf required for each air gap,

AT_t=mmf required for teeth, AT_c=mmf required for core,

AT_p=mmf required for pole, AT_y=mmf required for yoke.

$$\therefore\qquad AT=AT_g+AT_t+AT_c+AT_p+AT_y.$$

The mmf causing leakage fluxes Φ_1 and Φ_2, $=$mmf acting between points A and B

$=2$(total mmf/pole)-2 (mmf required for pole and yoke)

$=2[AT_g+AT_t+AT_c+AT_p+AT_y]-2[AT_p+AT_y]$

$=2[AT_g+AT_t+AT_c]=2AT_l$

where, $\quad AT_l=AT_g+AT_t+AT_c$

Then, $\Phi_1=$mmf$\times$permeance of flux path$=2\ AT_l\times\mu_0\ \dfrac{L_sh_s}{C_s}\times 2$

$$=4\mu_0AT_l\ \frac{L_sh_s}{C_s} \qquad \ldots(4{\cdot}101)$$

The factor 2 is used as there are two paths per pole such as path 1.

For Φ_2 there are four paths per pole such as path 2 on each pole. Assuming the path of flux Φ_2 to be made up of two quadrants and a straight line in the middle.

$$\Phi_2=4\times 2AT_l\mu_0\int_0^{b_s/2}\frac{h_s dx}{C_s+\pi x}$$

$$=8\mu_0\ AT_l\ \frac{h_s}{\pi}\log_e\left(1+\frac{\pi b_s}{2C_s}\right) \qquad \ldots(4{\cdot}102)$$

The mmf causing fluxes Φ_3 and Φ_4 is almost zero at the yoke increasing steadily to $2AT_l$ at the pole shoe.

$\therefore$ The value of mmf acting between M and $N=$ average of mmfs at the two extremes

$=AT_l$.

Therefore, with two paths such as 3 for Φ_3 and four paths such as 4 for Φ_4,

$$\Phi_3=2\mu_0AT_l\ \frac{L_ph_p}{C_p} \qquad \ldots(4{\cdot}103)$$

and

$$\Phi_4=4\mu_0AT_l\int_0^{b_p/2}\frac{h_p dx}{C_p+\pi x}$$

$$=4\mu_0AT_l\ \frac{h_p}{\pi}\ \log_e\left(1+\frac{\pi b_p}{2C_p}\right) \qquad \ldots(4{\cdot}104)$$

Hence, total leakage flux from pole shoes $\Phi_{sl}=\Phi_1+\Phi_2$

$$=4\ \mu_0AT_l\left[\frac{L_sh_s}{C_s}+\frac{2h_s}{\pi}\log_e\left(1+\frac{\pi b_s}{2C_s}\right)\right]$$

$$=4\ \mu_0AT_l\left[\frac{L_sh_s}{C_s}+1{\cdot}47\ h_s\log_{10}\left(1+\frac{\pi b_s}{2C_s}\right)\right] \qquad \ldots(4{\cdot}105)$$

Total leakage flux from pole body $\Phi_{pl}=\Phi_3+\Phi_4$

$$=2\ \mu_0AT_l\left[\frac{L_ph_p}{C_p}+\frac{2h_p}{\pi}\log_e\left(1+\frac{\pi b_p}{2C_p}\right)\right]$$

$$=2\ \mu_0AT_l\left[\frac{L_ph_p}{C_p}+1{\cdot}47\ h_p\log_{10}\left(1+\frac{\pi b_p}{2C_p}\right)\right] \qquad \ldots(4{\cdot}106)$$

Hence, total leakage flux $\Phi_l=\Phi_{sl}+\Phi_{pl}$

$$=2\ \mu_0AT_l\left[\left(\frac{L_ph_p}{C_p}+\frac{2L_sh_s}{C_s}\right)+1{\cdot}47\ \left\{h_p\log_{10}\left(1+\frac{\pi b_p}{2C_p}\right)\right.\right.$$

$$\left.\left.+2h_s\ \log_{10}\left(1+\frac{\pi b_s}{2C_s}\right)\right\}\right] \qquad \ldots(4{\cdot}107)$$

In machines, the length of pole is normally equal to length of shoe or $L_p = L_s$,

$$\therefore \quad \Phi_l = 2\mu_0 AT_l \left[L_p \left(\frac{h_p}{C_p} + \frac{2h_s}{C_s} \right) + 1{\cdot}47 \left\{ h_p \log_{10}\left(1 + \frac{\pi b_p}{2C_p}\right) + 2h_s \log_{10}\left(1 + \frac{b_s \pi}{2C_s}\right) \right\} \right] \quad ..(4{\cdot}108)$$

The value of leakage flux obtained from the above expression is less than the actual value as the method of calculation assumes a much restricted path of leakage flux. Actually in view of the assumptions made there is considerable discrepancy between the calculated and actual values of leakage flux. However, the treatment above gives a reasonable first approximation. The following relationships give approximate values of leakage fluxes.

$$\Phi_{sl} = 40AT \times 10^{-8} \quad ...(4{\cdot}109)$$

and

$$\Phi_{pl} = 160AT \times 10^{-8} \quad ...(4{\cdot}110)$$

If the value of useful flux is Φ_m, the flux at the back of the pole shoes $= (\Phi_m + \Phi_{sl})$, and the value of flux at the root or poles near the yoke $= (\Phi_m + \Phi_{sl} + \Phi_{pl})$

Leakage co-efficient at the root of poles,

$$C_l = \frac{\text{total flux}}{\text{useful flux}} = \frac{\Phi_m + \Phi_{sl} + \Phi_{pl}}{\Phi_m} = \left(\frac{\Phi_{sl} + \Phi_{pl}}{\Phi_m} \right) \quad ...(4{\cdot}111)$$

The leakage co-efficient normally varies between 1·1 to 1·2, the larger values applying to smaller machines.

4·17·1. Effect of Saturation and load on Leakage Co-efficient. The path of the leakage flux is mainly through air and to a close approximation the leakage increases in direct proportion to total mmf absorbed by gap, teeth and core. The reluctance of teeth and core is not constant but varies according to degree of saturation in the teeth. With saturation the mmf required by teeth increases considerably and so the value of leakage flux and hence leakage co-efficient increases rapidly with saturation.

Let AT_l = mmf for gap, teeth and core at normal voltage V,

AT_{l_1} = mmf for gap, teeth and core at voltage V_1,

and C_l = leakage co-efficient at voltage V.

Leakage co-efficient voltage V_1 is :

$$C_{l_1} = \frac{AT_{l_1}}{AT_l} \times \frac{V}{V_1} (C_l - 1) + 1. \quad ...(4{\cdot}112)$$

When a machine is loaded, the armature reaction is increased which decreases the value of useful flux. In order to compensate for the loss of useful flux, the excitation is increased. This increase in excitation results in larger values of leakage co-efficient. Hence the leakage co-efficient in a loaded machine has a larger value as compared with that in an unloaded machine.

MAGNETIZING CURRENT

4·18. Calculation of Magnetizing Current. The total mmf to be provided by the exciting winding is the sum of mmf for the several series parts of the magnetic circuit. The value of exciting or magnetizing current depends upon the total mmf required, the number of turns in the exciting winding and upon the way in which the winding is distributed.

4·18·1. Magnetizing current for concentrated windings. In a concentrated winding, it is permissible to assume that whole of the flux links with all the turns. For this case if AT is the total mmf and T is the number of turns of the magnetizing winding, the magnetizing current is :

$$I_m = \frac{AT}{T} \quad ...(4{\cdot}113)$$

This is the situation in transformers and in salient pole machines. In transformers, AT is always calculated on the basis of maximum flux density, and hence Eqn. 4·113 will give maximum magnetizing current $I_{m(max)}$. The r.m.s. value of magnetizing current is found from the following relationship.

$$I_m = \frac{I_{m(max)}}{K_{pk}} \qquad \ldots(4·114)$$

where K_{pk} = peak factor.

Since the current wave shape is not sinusoidal in general, K_{pk} differs from $\sqrt{2}$ and its value increases with the saturation of iron and can be obtained from maker's test data.

4·18·2. Magnetizing current for distributed windings. In a distributed magnetizing winding, the flux does not link with all the turns. A relationship can be established if the flux and the mmf are distributed in a known simple configuration.

(*i*) **Sinusoidal flux distribution.** If the flux is steady and is sinusoidally distributed in space, the value of magnetizing current is :

$$I_m = \frac{AT_m}{T} \qquad \ldots(4·115)$$

where, AT_m = mmf for the peak flux density B_m. If the flux pulsates sinusoidally in both time and space with a maximum flux density B_m, the r.m.s. value of magnetizing current is :

$$I_m = \frac{AT_m}{\sqrt{2}\,T} \qquad \ldots(4·116)$$

A three-phase winding uniformly distributed in space with 60° spread and excited with balanced sine wave currents, gives the amplitude of fundamental of magnetizing mmf

$$AT_{m1} = \frac{2·7\, I_m T_{ph} K_{w1}}{p} \qquad \ldots(4·117)$$

(This relationship is derived in Chapter 6)

where, I_m = magnetising current per phase,

T_{ph} = number of turns per phase,

K_{w1} = winding factor for fundamental, and p = number of poles.

The value of magnetizing current per phase (Eqn. 4·117)

$$I_m = \frac{0·37\, p\, AT_{m1}}{K_{w1} T_{ph}} \qquad \ldots(4·118)$$

(*ii*) **Non sinusoidal flux distribution.** The mmf distribution in the air gap can be taken as sinusoidal to a close approximation. If there is no saturation in the iron parts of the magnetic circuit, the flux density distribution in space is also sinusoidal. But owing to saturation in iron parts, the mmf produces a flat topped flux wave, which may be represented by :

$$B_\theta = B_{m_1} \sin\theta + B_{m_3} \sin 3\theta + B_{m_5} \sin 5\theta + \ldots\ldots$$

where, B_θ = flux density at an angle θ from neutral axis.

B_{m_1}, B_{m_3}, B_{m_5}, etc. are the maximum values of fundamental, 3rd harmonic and 5th har-

monic flux densities respectively. Generally, the harmonics above the third are small in magnitude and so the flux density wave shape can be considered to consist of a fundamental sine wave with a super-imposed third harmonic. This case has a special significance in the case of induction motors. Fig. 4·62 shows the flux distribution in the air gap of an induction motor with moderate saturation.

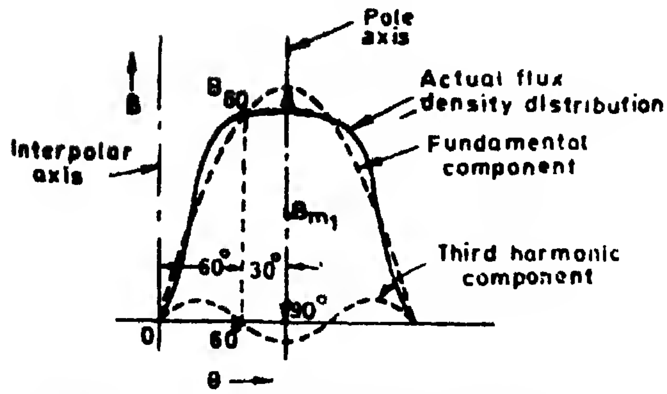


Fig 4·62. Flux-density distribution in space with its fundamental and third-harmonic components.

To calculate the required mmf any closed path can be chosen provided the flux density along that path is known. If we choose a path along the pole centres, the value of mmf cannot be calculated as the flux density along this path is not known, because the knowledge of this flux density involves the knowledge of third harmonic component.

Suppose we take two points which are displaced 60° from the inter polar axis (*i.e.* 30° from the pole axis).

Flux density at $\theta=60°$,

$$B_{60}=B_{m_1}\sin 60°+B_{m_3}\sin 3\times 60°=B_{m_1}\sin 60°$$

$$=\frac{\sqrt{3}}{2}B_{m_1} \qquad \ldots(4{\cdot}119)$$

It is clear from the above expression that B_{60} is the same whether third harmonic component is present or not. And this is the reason for preferring this path for calculation of magnetizing mmf in an induction motor.

Now B_{m_1} is sinusoidally distributed,

$$B_{m_1}=\frac{\pi}{2}B_{av_1}=\frac{\pi}{2}\frac{\Phi_1}{A_1}$$

where, Φ_1=fundamental flux per pole, A_1=effective area of each fundamental pole,

B_{av_1}=average flux density in air gap$=\frac{\Phi_1}{A_1}$ (fundamental component)

We have, $$B_{60}=\frac{\sqrt{3}}{2}B_{m_1}=\frac{\sqrt{3}}{2}\cdot\frac{\pi}{2}B_{av_1}$$

$$=1{\cdot}36\,B_{av_1}. \qquad \ldots(4{\cdot}120)$$

The calculations for the remainder of the magnetic circuit, which is affected by the non-sinusoidal flux distribution, can be worked out in a similar manner.

The mmf acting at $\theta=60°$,

$$AT_{60}=AT_{m_1}\sin 60°=\frac{\sqrt{3}}{2}AT_{m_1}=\frac{\sqrt{3}}{2}\times 2{\cdot}7\frac{I_{ph}T_{ph}K_{w_1}}{p}$$

$$\therefore I_{ph}=\frac{0{\cdot}427\,p\,AT_{60}}{K_{w_1}T_{ph}} \qquad \ldots(4{\cdot}121)$$

UNBALANCED MAGNETIC PULL

4·19. Magnetic Pull (Force). Consider an electromagnet arranged as shown in Fig. 4·63. Let us find the magnetic force between the two poles.

F=force between the two poles, N ; B=flux density in air gap, Wb/m² ;

A=area of each pole, m² ;

If one of the poles is moved by a distance dx, Work done $=F dx$

This work done is equal to the change of energy stored in magnetic field.

Change in energy stored in magnetic field

$$= \text{energy density} \times \text{change in volume}$$

$$= \frac{1}{2} \frac{B^2}{\mu_0} \times A\, dx = \frac{1}{2} \frac{B^2}{\mu_0} A\, dx$$

$$\therefore \quad F\, dx = \frac{1}{2} \frac{B^2}{\mu_0} A\, dx$$

or $$F = \frac{1}{2} \frac{B^2}{\mu_0} A \text{ N} = 0{\cdot}051 \frac{B^2}{\mu_0} A \text{ kg} \quad \ldots(4{\cdot}122)$$

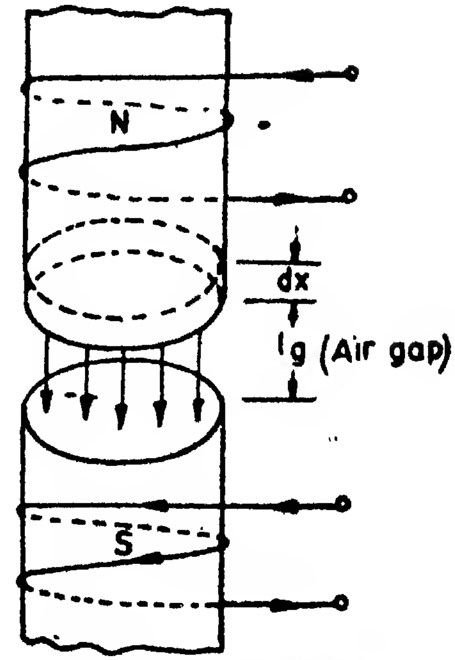


Fig. 4·63. Magnetic force between two poles.

Hence from above,

pull or force per unit area

$$P_m = \frac{1}{2} \frac{B^2}{\mu_0} \text{ N/m}^2 = 0{\cdot}051 \frac{B^2}{\mu_0} \text{ kg/m}^2 \quad \ldots(4{\cdot}123)$$

The flux density in the air gap, B, depends upon the mmf of the exciting winding. Let AT be the total mmf of the exciting winding. A portion of this mmf is required for the air gap and the rest for the iron parts of the magnetic circuit. Let AT_g be the mmf required for the air gap and AT_i for the iron parts.

$$\therefore \quad AT = AT_g + AT_i$$

Now, $$B = \frac{\mu_0 AT_g}{l_g}, \text{ where } l_g = \text{length of air gap.}$$

$\therefore$ From Eqn. 4·122, force

$$F = \frac{1}{2} \left(\frac{\mu_0 AT_g}{l_g} \right)^2 \frac{A}{\mu_0} = \frac{1}{2} \mu_0 \left(\frac{AT_g}{l_g} \right)^2 A \text{ N} \quad \ldots(4{\cdot}124)$$

If there is no saturation in the iron parts, the mmf required for them is small and therefore $AT = AT_g$.

This gives .

$$F = \frac{1}{2} \mu_0 \left(\frac{AT}{l_g} \right)^2 A \text{ N} = 0{\cdot}051 \left(\frac{AT}{l_g} \right)^2 A \text{ kg} \quad \ldots(4{\cdot}125)$$

4·20 Radial magnetic forces in rotating machines. Consider an ideal 2 pole machine as shown in Fig. 4·64. The rotor is set symmetrically within the stator bore. The rotor is, therefore concentric with the stator and the gap surfaces of both rotor and stator are purely cylindrical and thus length of air gap is uniform everywhere. The rotation of the rotor is on account of the formation of poles of opposite polarity on stator and rotor which exert a tangential force on the rotor. However, a much stronger magnetic force of attraction takes place between the stator and the rotor poles acting along a direction perpendicular to the rotor shaft axis. These forces therefore act radially. Suppose flux density B is uniform in the airgap.

The force of attraction between top stator and rotor poles is :

$$F_1 = \frac{1}{2} \frac{B^2}{\mu_0} A = \frac{1}{2} \mu_0 (AT_g/l_g)^2 A$$

where B=flux density in air gap, Wb/m^2 ; l_g=length of air gap, m ;
AT_g=mmf per pole for air gap, A ; A=area per pole, m^2 ; $=\pi DL/2$
D=stator bore, m; L=axial length, m.

If mmf required for the iron parts is negligible, the total mmf is consumed by the air gap.

Now, AT=total mmf per pole, A. For no saturation $AT_g=AT$

$$\therefore \quad F_1=\tfrac{1}{2}\,\mu_0\,(AT/l_g)^2\,A$$

In a symmetrical machine the mmf per pole and the area per pole are the same for all the poles. Since the length of air gap is uniform, the flux density in the air gap is uniform.

$\therefore$ Force of attraction between the bottom stator and rotor poles

$$F_2=\frac{1}{2}\,\frac{B^2}{\mu_0}\,A=\tfrac{1}{2}\mu_0\,(AT/l_g)^2\,A.$$

Forces F_1 and F_2 are equal and act in the opposite direction and hence their resultant is equal to zero. Therefore, in a symmetrical machine there is no resultant radial magnetic pull on the rotor.

In the analysis given above, a uniform flux density distribution has been assumed in the air gap. However, in alternating current machines the flux density distribution is not uniform but is sinusoidal.

Let us consider an elemental angle $d\theta$ at an angle θ from the x axis.

Flux density in the elemental angle, $B=B_m\sin\theta$

$\therefore$ Radial force acting on the elemental strip

$$=\frac{1}{2\mu_0}(B_m\sin\theta)^2\,\frac{DL}{2}\,d\theta=\frac{1}{4\mu_0}\,B_m\,DL\sin^2\theta\,d\theta.$$

Vertical component of the above force is :

$$=\frac{1}{4\mu_0}\,B_m^2\,DL\sin^2\theta\,d\theta.\sin\theta \quad =\frac{1}{4\mu_0}\,B_m^2\,DL\sin^3\theta\,d\theta.$$

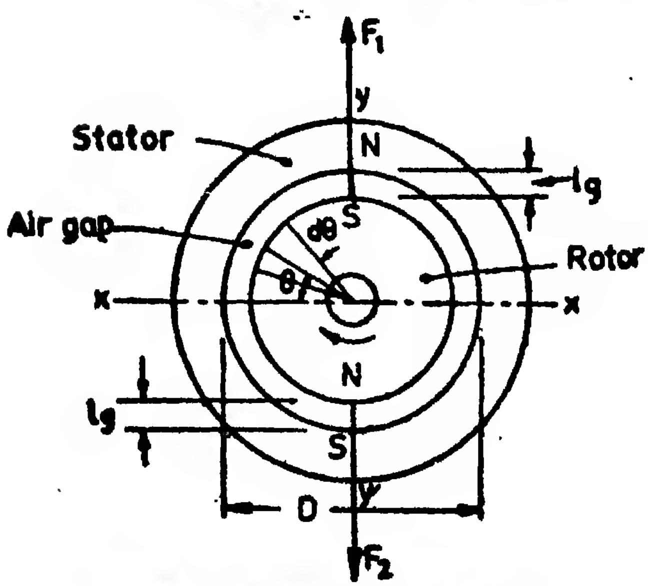


Fig. 4·64 Radial magnetic forces in symmetrical machine

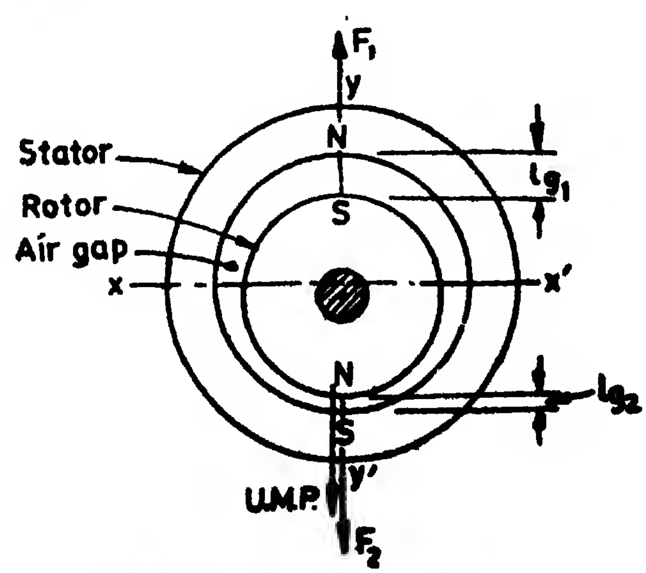


Fig. 4.65 Unbalanced magnetic pull

$\therefore$ Resultant magnetic pull per pole acting vertically

$$=\frac{1}{4\mu_0}B_m^2\ DL\int_0^{\pi}\sin^3\theta\ d\theta.$$

$$=\frac{1}{4\mu_0}B_m^2\ DL\times\ \frac{4}{3}=\frac{B_m^2}{3\mu_0}DL\ \ \text{N} \qquad \ldots(4{\cdot}126)$$

If the integration is carried over the whole periphery *i.e.* $0<\theta<2\pi$ the resultant pull is :

$$\frac{1}{4}\mu_0\ B_m^2\ DL\int_0^{2\pi}\sin^3\theta\ d\theta=0.$$

The resultant pull is zero, this is because the pull $(B_m^2/3\mu_0)\ DL$ of one pole is balanced by an equal and opposite force on the other pole.

It is necessary to allow dimensional tolerances during the process of manufacture of electrical machines. These tolerances are allowed on core-plate stampings, core assemblies, frames, end covers and bearings. The manufacturing costs are lowered in case wide tolerances are given. However, wide tolerances result in variations in the length of air gap which give rise to unbalanced magnetic pull (U.M.P.) as explained below.

Let us now consider a machine whose rotor is not concentric with the stator as shown in Fig. 4·65. The length of air gap at the top is l_{g1} while the length of air gap at the bottom is l_{g2}. The eccentricity may be caused by many reasons, the foremost being manufacture tolerances, poor quality of machining and wear of bearings. The wear of bearings causes the shaft to sink resulting in the air gap at the top (l_{g1}) becoming longer than the air gap, (l_{g2}) at the bottom.

The force of attraction between the top stator and rotor poles,

$$F_1=\tfrac{1}{2}\frac{B_1^2}{\mu_0}A=\tfrac{1}{2}\mu_0\ (AT/l_{g1})^2A$$

where B_1=flux density in the air gap at the top,

$$=\mu_0\frac{AT}{l_{g1}}\ \text{if saturation in iron parts is neglected.}$$

Similarly, force of attraction between bottom stator and rotor,

$$F_2=\tfrac{1}{2}\frac{B_2^2}{\mu_0}A=\tfrac{1}{2}\mu_0\ (AT/l_{g2})^2\ A$$

$$=\mu_0\frac{AT}{l_{g2}}\text{if saturation in iron parts is neglected.}$$

where B_2=flux density in the air gap at the bottom.

It is evident from above that force F_2 is greater than F_1 since $l_{g2}<l_{g1}$. Since $F_2>F_1$ and hence a resultant radial force (pull) acts on the rotor in the downward direction. This force or pull is called the **Unbalanced Magnetic Pull** (U.M.P.)

It should be borne in mind that it is not only the irregularity of the air gap that causes unbalanced magnetic pull but also any other assymmetry in the magnetic circuit or windings would cause U.M.P.

4·21. Calculation of unbalanced magnetic pull. When the rotor of an electric machine is concentric with the stator, the radial pull of all the poles on one side of the diameter is exactly balanced by the pull of poles on the opposite side. If on account of

any defect like wear of bearings or improper assembly of any part, the rotor is not concentric with the stator, the air gap is not uniform over the periphery. This would give rise to an unbalanced magnetic pull which tends to draw the rotor over to the side where the air gap between rotor and stator is smallest. Eqn. 4·122 is self explanatory in this respect. This equation shows that the pull is inversely proportional to square of length of air gap in case the mmf required for the iron parts of the magnetic circuit is neglected.

Let P_m=pull per unit area with normal air gap l_g,

and δ=length of air gap as modified by displacement of rotor centre.

$\therefore$ Pull per unit area with displaced rotor$=P_m\,(l_g/\delta)^2$.

The pull is exerted radially along the centre of area considered.

Consider the case of a rotor where it is displaced vertically downwards as shown in Fig. 4·66 (*a*).

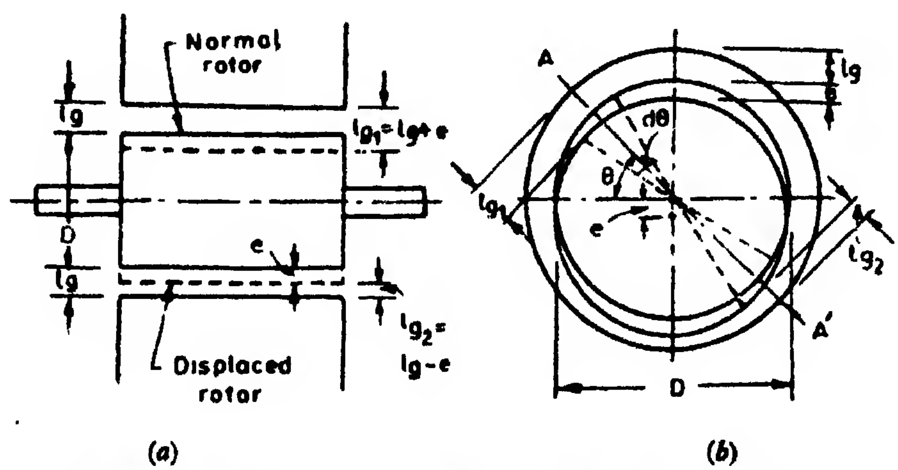


Fig. 4·66. Machine with rotor displaced vertically downwards.

The end view of rotor is shown in Fig. 4·66 (*b*).

Let e=displacement of rotor along downward direction,

and D_r=diameter of rotor, L=length of machine core.

Modified length of air gap at the top along vertical axis$=l_g+d$.

Modified length of air gap at the bottom along vertical axis$=l_g-d$.

Referring to Fig. 4·66 (*b*).

Consider a differential strip at an angle θ from horizontal axis, the axis of this strip being AA'.

Modified length of air gap at the top along AA', $l_{g1}=l_g+d\sin\theta$.

Modified length of air gap at the bottom along AA', $l_{g2}=l_g-d\sin\theta$.

There are two forces acting in the opposite direction on the rotor along axis AA'.

$\therefore$ Pull per unit area on the rotor along AA'

$$=P_m\left[\left(\frac{l_g}{l_g-e\sin\theta}\right)^2-\left(\frac{l_g}{l_g+e\sin\theta}\right)^2\right]$$

The expression within brackets is approximately equal to 4 $(e/l_g)\sin\theta$ as e is comparatively small.

$\therefore$ Pull per unit area on the rotor along AA' due to a pair of poles

$$=4\,P_m\,\frac{e}{l_g}\sin\theta \qquad ...(4·127)$$

Vertical component of the pull per unit area $=(4\,P_m\frac{e}{l_g}\sin\theta)\times\sin\theta$

$$=4P_m\frac{e}{l_g}\sin^2\theta \qquad ...(4\cdot128)$$

$$=2\,\frac{B^2}{\mu_0}\cdot\frac{e}{l_g}\sin^2\theta \qquad ...(4\cdot129)$$

as $P_m=\frac{1}{2}\,\frac{B^2}{\mu_0}$, Now area of strip $=\frac{DL}{2}\,d\,\theta$.

Total vertical pull acting on the strip $=4\,P_m\frac{e}{l_g}\sin^2\theta\,\frac{DL}{2}d\,\theta$

$$=2\,P_m\,DL\;\frac{e}{l_g}\sin^2\theta\,d\theta.$$

$\therefore$ Total pull acting on the rotor in the downward direction due to a pair of poles (unbalanced magnetic pull U.M.P.)

$$P_p=2\,P_m\,DL\,(e/l_g)\int_0^\pi\sin^2\theta\,d\theta$$

$$=\pi\,D\,LP_m\frac{e}{l_g} \qquad ...(4\cdot130)$$

$$=\frac{1}{2}\,\frac{\pi DL}{\mu_0}B^2\frac{e}{l_g} \qquad ...(4\cdot131)$$

Now, area per pole $A=\pi D\,l/2$ (since we are considering a pair of poles)

$$\therefore \qquad \text{U.M.P.}=2\,A\,P_m\,e/l_g \qquad ...(4\cdot132)$$

The formulae developed above for the calculation of unbalanced magnetic pull for a pair of symmetrically placed poles may be easily modified to render them applicable to machines without salient poles, such as induction motors or to machines having a large number of salient poles.

Unbalanced magnetic pull due to p poles, U.M.P.$=$pole pairs$\times P_p$

$$=\frac{p}{2}\times2\,A\,P_m\,e/l_g \;=pA\,P_m\,e/l_g \qquad ...(4\cdot133)$$

The above analysis has been done assuming a uniform value of flux density in the air gap. In general the gap flux density is not uniform. The values of flux density vary considerably over the gap surface under the pole face as, for instance, in a.c. machines under all conditions of loading, and d.c. machines under heavy load, when the flux distribution is distorted by the armature reaction. Under such conditions it becomes necessary to calculate the pull for several small areas over which the flux density is approximately constant and add these together to obtain total unbalanced magnetic pull per pair of poles.

In the case of machines, where the flux density distribution is sinusoidal, it is possible to obtain mathematically an expression for U.M.P. based upon r.m.s. value of flux density.

$\therefore$ For sinusoidal flux density distribution the unbalanced magnetic pull per pair of poles :

$$P_p=\frac{1}{2}\,\frac{\pi DL}{\mu_0}B^2\frac{e}{l_g} \quad \text{(See Eqn. 4·131)}$$

$$=\frac{1}{2}\,\frac{\pi DL}{\mu_0}\left(B_m/\sqrt{2}\right)^2\frac{e}{l_g}=\frac{1}{4}\,\frac{\pi DL}{\mu_0}\,B_m^2\,\frac{e}{l_g} \qquad ...(4\cdot134)$$

The simplified analysis given above assumes that stator and rotor axes, though displaced, remain parallel, which is not true in many cases such as when the alignments of

the two bearings differ. When calculating unbalanced magnetic pull in machines with sinusoidal flux density distribution it has been assumed that the peak value of the flux density remains the same irrespective of the eccentricity which is not correct. The unbalanced magnetic pull has been calculated for the worst possible condition i.e. the U.M.P. has been calculated for the case where the eccentricity is coincident with the axis of maximum flux density. However in practice the two are not coincident always but the position changes with load and speed.

Example 4·13. *The following data refers to a 55 kW, 4 pole d.c. generator :*
normal length of air gap = 5 mm ; area under each pole face = 45×10^{-3} m^2 ; flux density under pole centre with normal air gap = 0·75 Wb/m^2 ; vertical displacement of rotor = 0·8 mm.

Calculate :

(a) *the unbalanced magnetic pull acting downwards if the poles are centred 45° with the horizontal axis,*

(b) *the unbalanced magnetic pull acting downwards if there were only two poles placed on the vertical axis.*

Solution. (a) Magnetic pull per unit area with normal flux density

$$P_m=\frac{1}{2}\ \frac{B^2}{\mu_0}=\frac{1}{2}\times\ \frac{(0.75)^2}{4\pi\times10^{-7}}=223.8\times10^3\ \text{N/m}^2.$$

Unbalanced magnetic pull per unit area acting downwards due to a pair of poles

$$=4\ P_m\ \frac{e}{l_g}\sin^2\theta\quad\text{(See Eqn. 4·128)}$$

$$=4\times223.8\times10^3\times\ \frac{0.8}{5}\times(0.707)^2=71.6\times10^3\ \text{N/m}^2.$$

Unbalanced magnetic pull acting downwards per pair of poles

$$=71.6\times10^3\times45\times10^{-3}=3223\ \text{N}$$

There are four poles or 2 pairs of poles.

∴ Total unbalanced magnetic pull acting downwards $=2\times3222=6444$ N.

(b) The pull with 2 poles centred on the vertical axis can be calculated from Eqn. 4·128 by putting $\theta=90°$.

Unbalanced magnetic pull per unit area acting downwards for 2 poles on vertical axis,

$$=4\times223.8\times10^3\times\ \frac{0.8}{5}\times(1)^2=143.2\times10^3\ \text{N/m}^2.$$

Total unbalanced magnetic pull acting downwards with 2 poles on vertical axis

$$=143.2\times10^3\times45\times10^{-3}=6444\ \text{N}.$$

This unbalanced magnetic pull is the same as with 4 poles centred 45° with horizontal axis. From above, the poles should not be kept along the vertical axis as this arrangement results in high U.M.P.

(The problem has been solved by assuming that the flux density is 0.75 Wb/m^2 every where under the pole face. However, this is not true as the flux density falls down near the pole faces).

Example 4·14. *Find the maximum out-of-balance magnetic pull (unbalanced magnetic pull) for a 4 pole, 45 kW d.c. motor for an eccentricity of 1 mm in a normal air gap length of 5·5 mm. Assume a rectangular flux distribution under the pole arc with a flux density of 0·893 Wb/m^2. The pole arc subtends an angle of 60°. The area under the pole arc is 32×10^{-3} m^2.*

Solution. The arrangements of poles which give rise to maximum magnetic pull for a 4 pole machine are :

(*i*) 2 poles on the horizontal axis and 2 poles on vertical axis ;

and (*ii*) all the 4 poles centred at 45° with vertical axis.

These two arrangements give the same value of unbalanced magnetic pull as has been shown in Example 4·14.

Let us consider the first arrangement wherein 2 poles are on the vertical axis and while the other 2 are on horizontal axis. The arrangement is shown in Fig. 4.67 The pair of poles which are arranged vertically only contribute towards the unbalanced magnetic pull while the poles arranged horizontally do not contribute to unbalanced magnetic pull. Let us consider an elementary strip at an angle θ from vertical axis.

The radial flux density B under the pole arc is uniform and has a value of 0·893 Wb/m².

The vertical component of the air gap flux density $= B \cos\theta$.

The length of air gap at angle θ from the vertical axis $= l_g + e\cos\theta$.

Eqn. 4·127 can be modified to take into account both the variations in vertical gap density and also the change in reference axis from horizontal to vertical.

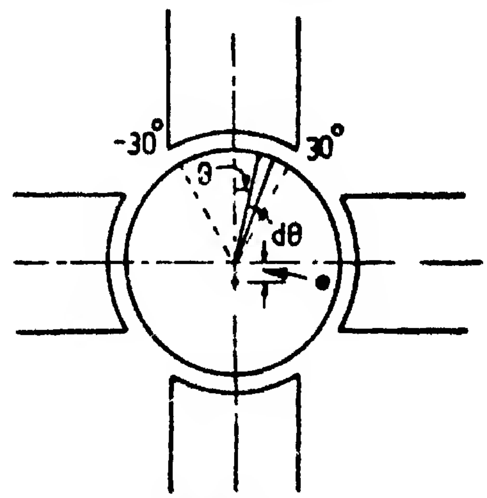


Fig. 4·67

Pull per unit area on rotor along an axis θ from vertical axis

$$=4\times\frac{1}{2\mu_0}(B\cos\theta)^2\times\frac{e}{l_g}\cos\theta$$

$$=2\,\frac{B^2}{\mu_0}\;\frac{e}{l_g}\cos^3\theta.\quad \text{N/m}^2.$$

If the area subtended by angle $d\theta$ over the corresponding length be $A\,d\theta$, the total unbalanced magnetic pull acting downwards is

$$\text{U.M.P.}=2\,\frac{B^2A}{\mu_0}\;\frac{e}{l_g}\int_{-30^\circ}^{+30^\circ}\cos^3\theta\,d\theta$$

$$=2\times\frac{(0{\cdot}893)^2}{4\pi\times10^{-7}}\,\frac{3}{\pi}\times32\times10^{-3}\times\frac{1}{5{\cdot}5}\times\frac{11}{12}=5860\quad\text{N}.$$

as $\frac{\pi}{3}A = 32\times10^{-3}$ and the integral is 11/12.

Example 4·15. *A 75 kW, 50 Hz, 2 pole machine with sinusoidally distributed flux has the following design data :*

Axial length of core=0·2 m : Stator bore=0·5 m ; length of air gap=5 mm ; peak magnetising mmf per pole=4500 A. Calculate

(*i*) *magnetic pull per pole when the rotor is symmetrically centred,*

(*ii*) *unbalanced magnetic pull if the rotor axis is displaced by 1 mm,*

(*iii*) *ratio of unbalanced magnetic pull to useful force. Neglected saturation.*

Solution. Since there is no saturation the entire magnetizing mmf per pole is used for the air gap.

Peak value of flux density $B_m = \frac{\mu_0 AT_m}{l_g} = \frac{4\pi \times 10^{-7} \times 4500}{5 \times 10^{-3}} = 1{\cdot}13$ Wb/m².

(i) The flux distribution is sinusoidal,

$\therefore$ From Eqn. 4·126 magnetic pull per pole $= \frac{B_m^2}{3\mu_0} DL$

$$= \frac{(1{\cdot}13)^2 \times 0{\cdot}5 \times 0{\cdot}2}{3 \times 4\pi \times 10^{-7}} = 33900 \text{ N} = 33{\cdot}9 \text{ kN}$$

(ii) Unbalanced magnetic pull per pair of poles :

$$P_p = \frac{\pi DL}{4\mu_0} B_m^2 \frac{e}{l_g} \quad \text{(See Eqn. 4·134)}$$

$$= \frac{\pi \times 0{\cdot}5 \times 0{\cdot}2}{4 \times 4\pi \times 10^{-7}} \times (1{\cdot}13)^2 \times \frac{1}{5} = 16000 \text{ N} = 16 \text{ kN.}$$

(iii) Speed $= \frac{2f}{p} = \frac{2 \times 50}{2} = 50$ r.p.s.

Useful torque $T = \frac{75000}{2\pi \times 50} = 238{\cdot}7$ Nm.

Useful force $F = \frac{T}{D/2} = \frac{238{\cdot}7}{0{\cdot}5/2} = 954{\cdot}8 \text{ N} \simeq 0{\cdot}95$ kN

Ratio of unbalanced magnetic pull to useful force $= \frac{16}{0{\cdot}95} = 16{\cdot}8$.

4·22. Effect of saturation on unbalanced magnetic pull. The relationships derived in Art. 4·20 and 4·21 are based upon the assumption that the total mmf is consumed by the air gap. But with the reduction of air gap, the gap reluctance decreases giving rise to saturation in iron parts. Thus the assumption that the flux density is inversely proportional to the length of air gap is not valid on account of saturation. However, with displacement of rotor, there is moderate saturation only and results obtained from relationships derived earlier are fairly correct. Moreover, the results obtained from those relationships are on the higher side.

In induction motors where the length of air gap is very small and therefore even a slight eccentricity may cause saturation. It has been found experimentally that the unbalanced magnetic pull is proportional to the eccentricity ϵ where $\epsilon = e/l_g$ for eccentricities not greater than 0·1. The increase in U.M.P. for ϵ between 0·1 to 0·3, the increase is small on account of saturation since saturation limits the value of peak flux density. Leakage flux, which increases with load, saturates tooth tips and therefore U.M.P. is greater under light or no load conditions.

The saturation in the iron parts of the magnetic circuit can be taken into account by using an **equivalent length** of air gap instead of the actual length of air gap in the expressions already derived. The equivalent length of air gap determined by a graphical method using magnetization curve of machine. This method is explained in Example 4·16

Example 4·16. *Calculate the unbalanced magnetic pull in a machine with the following data :*

Normal length of air gap = 1 mm ; normal gap flux density = 0·65 Wb/m² ; stator core length = 0·33 m ; pole pitch = 0·28 m ; number of poles = 8 ; height of stator and rotor teeth = 40 mm each ; flux density in stator and rotor teeth = 1·6 Wb/m².

The *B-'at'* for the material used for stator and rotor teeth is given below :

Flux density Wb/m²	1·2	1·6	1·8	2·0	2·2
'at' A/m²	600	3,600	10,000	24,000	70,000

The vertical displacement of rotor is 0·25 mm.

Solution. The calculations for magnetization curve with normal air length are given below :

Flux density in teeth B_t Wb/m²	1·2	1·6	1·8	2·0	2·2
'*at*' for teeth A/m	600	3,600	10,000	24,000	70,000
Mmf for stator and rotor teeth '*at*' × 0·08 A	48	288	800	1,920	5,600
Air gap flux density $B_g=(B_t/1{\cdot}6)\times 0{\cdot}65$ Wb/m²	0·488	0·650	0·731	0·813	0·894
Mmf required for air gap $=800{,}000\ Bl_g$ A	390	520	585	650	715
Total mmf A	438	808	1,385	2,750	6,315

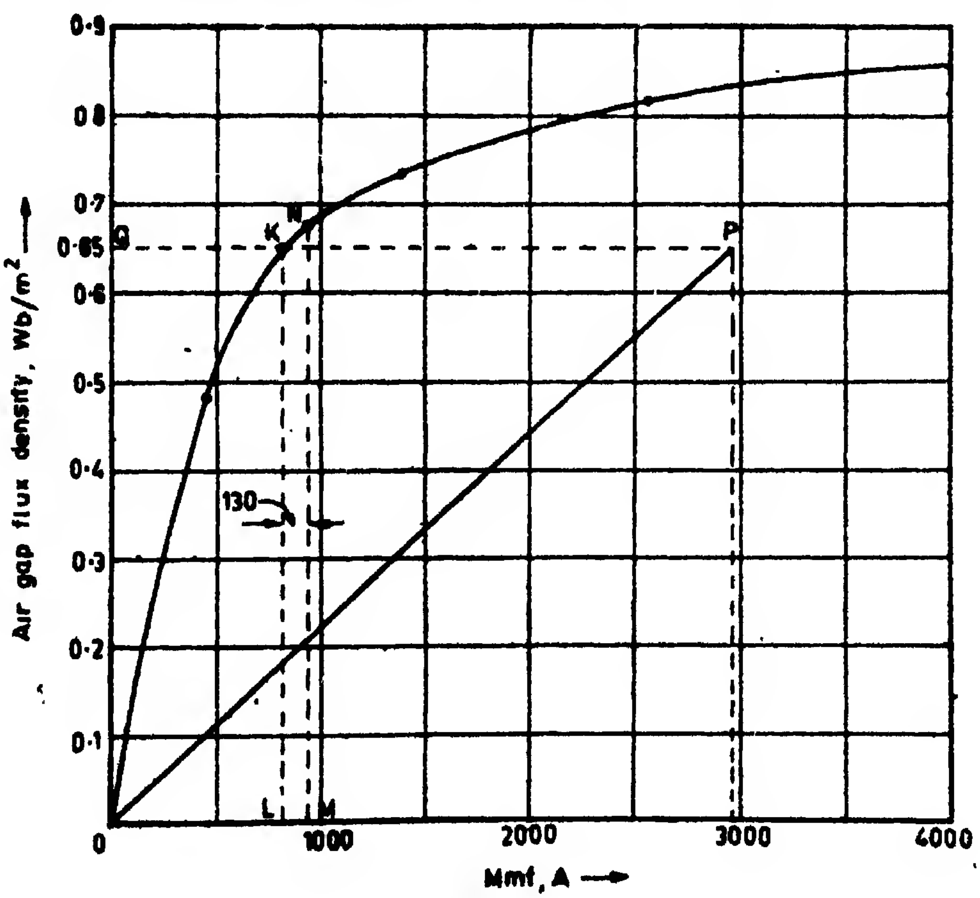


Fig. 4·68

The magnetization curve is plotted between total mmf and the flux density in the air gap as shown in Fig. 4 68. Draw a horizontal line QP at the normal working density, 0·65 Wb/m^2 in this case. Mmf required for an air gap length e for normal working flux density,

$$=800{,}000\ B_{g}e=800{,}000\times 0\cdot 65\times 0\cdot 25\times 10^{-3}=130\ \text{A}.$$

Referring to Fig. 4·68, choose a point K on the magnetization curve near the normal working flux density (0·65 Wb/m^2). Draw a vertical line KL and lay LM=80,000 Be=130 A (in the case). At M draw a vertical line cutting the magnetization curve at N. Join K and N. Draw a line OP' from the origin parallel to chord KN and cutting the horizontal line QP at P'

The equivalent length of air gap which takes into account is given by :

$$l_{ge}=\left(\frac{QP}{LM}\right)e=\left(\frac{2960}{130}\right)\times 0\cdot 25=5\cdot 69\ \text{mm}.$$

as $QP=2960.$

Area per pole A=pole pitch × axial length of core=0·28×0·33=0·0924 m^2.

Pull per unit area with normal air gap

$$P_m=\frac{1}{2}\frac{B^2}{\mu_0}=\frac{1}{2}\times\frac{(0\cdot 65)^2}{4\pi\times 10^{-7}}=168\cdot 1\times 10^3\ \text{N}$$

Total unbalanced magnetic pull

$$\text{U.M.P.}=pAP_m\frac{e}{l_{ge}}\quad \text{(See Eqn. 4·133)}$$

$$=8\times 0\cdot 0924\times 168\cdot 1\times 10^3\times\frac{0\cdot 25}{5\cdot 69}=5460\ \text{N}.$$

4·22·1. Practical aspects of unbalanced magnetic pull. Some of the practical aspects of unbalanced magnetic pull that must be taken into account while designing electrical machines are given below :

(*i*) Unbalanced magnetic pull is very large especially in induction motors. This is because even a small eccentricity can cause a fairly large unbalanced magnetic pull as the length of air gap in induction motors is very small ; in many cases it is less than a mm. In induction motors, the length of the air gap must be made as small as possible in order to obtain best operating conditions, the foremost being the power factor of the machine. Every care must be taken to keep this air gap uniform around the rotor periphery. In order to maintain the uniformity of the air gap in induction motors, the following aspects are incorporated in their design and manufacture.

(*a*) Ball bearings are used so that a good alignment can be maintained over a long period. The use of ball bearings prevent wear and also reduce noise.

(*b*) Stator windings of induction motors are designed with parallel paths having equalizer connections. These windings are so connected that they provide automatically a mitigating influence to overcome the effects of the unbalance.

(*c*) Induction motors are designed such that they have a rigid shaft of small length and also they are provided with bearings of bigger size. A stiff and short shaft gives negligible eccentricity and therefore the U.M.P. is small. The use of bigger bearings gives an almost perfect concentricity for rotor with stator.

(*ii*) Certain slot combinations of rotor and stator slots have a strong tendency to produce noise and vibrations. The slots on both stator and rotor produce harmonic fields. If two harmonic fields, with number of poles differing by 2, co-exist in the air gap of an induction motor, they produce unbalanced radial magnetic pull, and conesquent radial vibration of rotor as a whole. Also, symmetrical radial forces of high frequency are produced by the super-imposition of rotating magnetic fields due to different pole numbers. These phenomena create stator vibrations and magnetic noise.

Noise production is greatly accentuated by U.M.P. especially if its variations correspond to natural frequency of the stator structure. During the starting period of induction

motors, the U.M.P. has an alternating component at the rotor slot frequency and sets up vibrations at speeds given by the natural frequency divided by the number and rotor slots, and thereby possibly producing noise.

(*iii*) There is considerable homopolar flux provided in 2 pole machines due to assymetry in the air gap. The flux path is completed through shaft and frame. There is modulation of the air gap length which induces an emf in the bars of squirrel cage windings of induction motors. This effect may be counteracted by use of parallel paths and equalizer connections in the stator windings.

FIELD FORM

4·23. Introduction. The flux, in passing from poles into the armature, does not confine itself over the pole arc, but spreads out over the entire pole pitch. The flux will distribute itself in the air gap in such a way that the total reluctance is minimum. It is sometimes advantageous and necessary to determine the distribution of flux or field form in the air gap. For example, in d.c. machines, it is absolutely necessary to know the field form, in order to have knowledge of commutation conditions. In a.c. machines the flux distribution curve determines the waveshape of the voltage generated in the armature windings.

The field form in salient pole machines can be plotted by using any of the following techniques :

(*i*) Carter's fringe curves,
(*ii*) Flux plotting by method of curvilinear squares.

4·23·1. Carter's fringe curves. There is no difficulty in determining the flux density at any portion of the armature lying under the pole shoe if we neglect saturation ; this makes the flux density vary inversely as the length of the air gap.

Suppose B_g=maximum flux density in the air gap (at the centre of the pole),
l_g=length of air gap at the centre of the pole,
B_x=flux density in the gap at a distance x from the centre of the pole,
l_{gx}=length of air gap at a distance x from the centre of the pole.

∴ Neglecting saturation in iron parts

$$B_x=\frac{l_g}{l_{gx}}B_g \qquad \ldots(4\cdot135)$$

This equation is applied for portions of armature under the pole shoe but it is difficult to determine the flux distribution in the interpolar region as no such simple calculation is possible there. F.W. Carter worked on this problem and his results are given here in a graphical form. The actual shape of the fringing portion (portion in the interpolar space) of the flux distribution curve depends upon the dimensions of machine. Fringe curves have been drawn for various values of ratio C_n/l_{gt} as shown in Fig. 4·69.

where C_n=distance of pole tip from neutral axis

and l_{gt}=length of air gap at the pole tip

The values of relative flux densities can be read off directly. The curves for ratio C_n/l_{gt} not given in Fig. 4·69 can be interpolated.

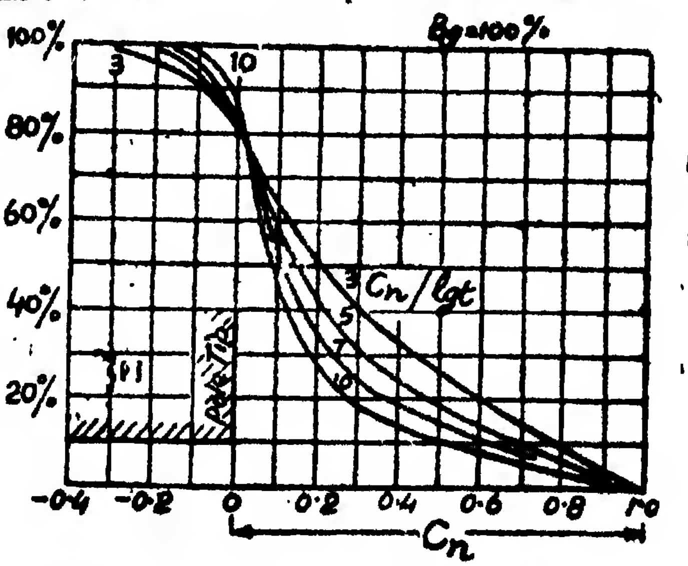


Fig. 4.69. Carter's Fringe Curves.

4·23·2. Flux Plotting (by method of curvilinear squares).

The flux path in the air gap under the pole may be divided into tubes of flux as shown in Fig. 4·70. The depth of each tube being unity in a direction parallel to shaft.

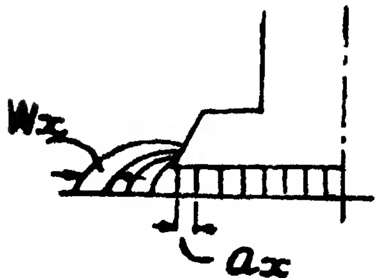


Fig. 4·70. Flux Plotting.

Let W_x = mean width of a flux tube,

l_{gx} = mean length of the flux tube.

Permeance of this flux tube considering unit depth

$$\Lambda_x = \mu_0 \frac{W_x \times 1}{l_{gx}}$$

$$= \mu_0 \frac{W_x}{l_{gx}}$$

∴ Flux in the tube Φ_x = mmf × permeance $= AT_f \, \mu_0 \, W_x/l_{gx}$

where AT_f = field mmf per pole.

∴ Flux density at armature surface where width of tube is a_x,

$$B_x = \frac{\Phi_x}{a_x \times 1} = AT_f \, \mu_0 \frac{W_x}{a_x \, l_{gx}} \qquad ...(4·136)$$

Now at the centre of the pole : $a_x = W_x$ and $l_g = l_{gx}$

∴ Flux density at the centre of pole $B_g = AT_f \, . \, \mu_0/l_g$. ...(4·137)

From Eqns. 4·136 and 4·137, we have :

$$B_x = \frac{W_x l_g}{a_x \, l_{gx}} B_g. \qquad ...(4·138)$$

The following rules should be observed for flux plotting by method of curvilinear squares :

(*a*) the flux lines leave and enter from surfaces, bounding the gap, at right angles if it is assumed that iron has infinite permeability as compared with air,

(*b*) the flux and equipotential lines intersect at right angles,

(*c*) the flux and equipotential lines are so drawn that each flux tube is divided into equal number of curvilinear squares. In such a case the flux density at any point will be proportional to the length of side of the square at the centre of the pole, to the length of the side of a square at the point where the flux density is desired to be evaluated. If larger number of squares are used over the top tips, the ratio of side of the squares must be multiplied by ratio of number of squares in order to get the flux density. The flux distribution curve is obtained by plotting points on the armature surface as abscissas and corresponding value of flux density as ordinates.

Method. A simplified method of flux plotting which yields sufficiently accurate results is given below.

The armature surface and the pole arc are laid out ; it is necessary to show only half of the pole pitch. The armature surface is divided into any number of equal parts and then the centre lines of tubes of flux are drawn in such a way that they leave and enter the iron surfaces at right angles. If it is assumed that the average width of tubes is equal to their maximum width, we have : $W_x = a_x$

or from Eqn. 4·138, $B_x = (l_g/l_{gx})B_g$ ...(4·139)

This means that the flux density is inversely proportional to the length of flux tube. Fig. 4·73 shows the flux plot of a synchronous machine. For this, the outline of half of pole

pitch is laid out to a large scale and the armature surface is divided into six parts with 0 at the interpolar axis and 6 on the pole centre. With the aid of rules for flux plotting by curvilinear squares, flux lines and equipotentials are laid out. It is then assumed that the flux density at the pole centre is 100. From Eqn. 4·139, flux density, at a place where length of flux line is l_{gx}, is $B_x=(l_g/l_{gx})B_g$.

The flux distribution curve is drawn (shown in Fig. 4·74) from the results obtained from Eqn. 4·139 applied at the different points. The flux distribution curve shows a positive value at position 0 (at the interpolar axis). This should be zero, so a straight line is drawn from the ordinates at position 0 drawn to zero, at the point where the pole level begins. The pole level begins at position 4 for the case shown in Fig. 4·72. The actual flux density is taken as the ordinate between the straight line and the original flux distribution curve. This means that the true flux distribution curve is obtained by subtracting the ordinates corresponding to straight line from the original flux distribution curve. The true flux distribution curve is indicated by $b_0, b_1, b_2, b_3, b_4, b_5, b_6$.

4·24. Air gap flux distribution factor (Field form factor). The air gap flux distribution factor K_f can be defined as the ratio of area under the actual flux distribution curve to the area of a rectangle having the same base and an ordinate equal to the maximum flux density.

or
$$K_f=\frac{\text{area under the flux distribution curve}}{\text{maximum ordinate}\times\text{pole pitch}}$$
$$=\frac{\text{average ordinate}}{\text{maximum ordinate}}=\frac{\text{average flux density}}{\text{maximum flux density}}=\frac{B_{av}}{B_g} \qquad \ldots(4·140)$$

Thus the flux distribution factor K_f can be defined as the ratio of average flux density over the pole pitch to the maximum flux density. K_f is also known as field form factor. The area under the flux distribution curve can be measured either by a planimeter or by method of mid ordinates.

4·25. Harmonic analysis of flux distribution curve. The flux distribution in rotating machines is of varied form depending upon the shape of the pole, the distribution of field windings and the load conditions. Ideally, it should be sinusoidal in the case of a.c. machines and rectangular under the pole arc in the case of d.c. machines.

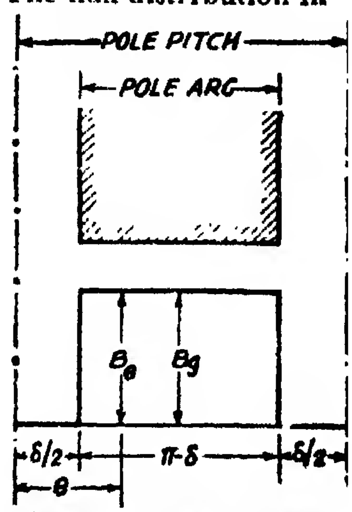


Fig. 4·71. Rectangular field form.

4·25·1. Rectangular flux-distribution curve. The field form of a salient pole machine is rectangular as shown in Fig. 4·71 if the air gap under the pole arc is uniform and if fringing effects are neglected. The field form can be analysed for its harmonic contents with the help of Fourier series

The pole arc spans an angle $(\pi-\delta)$ and the maximum gap density is B_g. Now the field form is symmetrical about the pole axis and also the north and south poles of a machine are similar, and thus there are no even harmonics, no cosine terms and the constant term is zero. Therefore, the flux density at any angle θ from the interpolar axis can be written as

$$B_\theta=\sum_{n=1}^{n=\infty} B_{mn}\sin n\theta$$

where n is an odd integer.

$$\therefore \qquad B_\theta=B_{m_1}\sin\theta+B_{m_3}\sin 3\theta=B_{m_5}\sin 5\theta+\ldots\ldots \qquad \ldots(4·141)$$

where B_{mn}=amplitude of nth harmonic

$$=\frac{2}{\pi}\int_0^{\pi} f(\theta)\sin n\theta\, d\theta.$$

Now
$$f(\theta)=0 \quad \text{for } 0<\theta<\frac{\delta}{2}$$
$$=B_g \quad \text{for } \frac{\delta}{2}<\theta<\pi-\frac{\delta}{2}$$
$$=0 \quad \text{for } \pi-\frac{\delta}{2}<\theta<\pi.$$

∴ Amplitude of nth harmonic

$$B_{mn}=\frac{2}{\pi}\left[\int_{\delta/2}^{\pi-\delta/2} B_g \sin\theta\, d\theta\right]$$

$$=\frac{4B_g}{n\pi}\cos\frac{n\delta}{2} \qquad \ldots(4{\cdot}142)$$

∴ The Fourier expression for field form is

$$B_\theta=\frac{4}{\pi}B_g\left[\cos\frac{\delta}{2}\sin\theta+\cos\frac{3\delta}{2}\sin 3\theta+\cos\frac{5\delta}{2}\sin 5\theta+\ldots\ldots\right] \qquad \ldots(4{\cdot}143)$$

Amplitude of fundamental is :

$$B_{m_1}=\frac{4}{\pi}B_g\cos\frac{\delta}{2}. \qquad \ldots(4{\cdot}144)$$

4·23·2. Actual flux distribution curve. The actual flux distribution curve is far from rectangular as shown in Fig. 4·74. A method which gives estimate of harmonics upto the seventh is described below :

The pole pitch is divided into 6 equal parts as described earlier, the coefficients of harmonics and fundamental are :

$$B_{m_1}=0{\cdot}086\, b_1+0{\cdot}167\, b_2+0{\cdot}236\, b_3+0{\cdot}289\, b_4+0{\cdot}323\, b_5+0{\cdot}167\, b_6$$
$$B_{m_3}=0{\cdot}236\, b_1+0{\cdot}333\, b_2+0{\cdot}236\, b_3+0 \quad -0{\cdot}236\, b_5-0{\cdot}167\, b_6$$
$$B_{m_5}=0{\cdot}323\, b_1+0{\cdot}167\, b_2-0{\cdot}236\, b_3-0{\cdot}289\, b_4+0{\cdot}086\, b_5+0{\cdot}167\, b_6$$
$$B_{m_7}=0{\cdot}323\, b_1-0{\cdot}167\, b_2-0{\cdot}236\, b_3+0{\cdot}289\, b_4+0{\cdot}086\, b_5-0{\cdot}167\, b_6$$

The equation of the flux distribution curve is

$$B_\theta=B_{m_1}\sin\theta-B_{m_3}\sin 3\theta+B_{m_5}\sin 5\theta+B_{m_7}\sin 7\theta+\ldots$$

The average flux dentity

$$B_{av}=\frac{2}{\pi}\left(B_{m_1}+\frac{1}{3}B_{m_3}+\frac{1}{5}B_{m_5}+\frac{1}{7}B_{m_7}\ldots\ldots\ldots\right) \ldots(4{\cdot}145)$$

and the r.m.s. flux density

$$B=\sqrt{\tfrac{1}{2}(B_{m_1}^2+B_{m_3}^2+B_{m_5}^2+B_{m_7}^2+\ldots\ldots)} \qquad \ldots(4{\cdot}146)$$

Example 4·17. *The pole profile of a 3000 kVA, 32 pole, salient pole, synchronous generator is shown in Fig. 4·72. The inner diameter of stator is 3·2 m and the pole pitch τ=0·314 m. The ratio of pole arc to pole pitch is 0·74. The length of air gap at the pole centre is 5·5 mm and this gap length is maintained over one third of pole pitch as indicated. The length of air gap is 10 mm at the pole tips.*

Draw the flux distribution curve of the machine by method of flux plotting. Determine the amplitudes of different harmonics and also the field form factor.

Solution. The stator bore and the pole surfaces shown in Fig 4·73 are drawn by techniques given for pole drawing in chapter on Synchronous Machines. The flux plot is drawn by using method of curvilinear squares.

Half of the pole pitch is divided into 6 parts and the flux plot is drawn by method of curvilinear squares. The flux distribution curve drawn with results obtained from the flux plot is shown in Fig. 4·74.

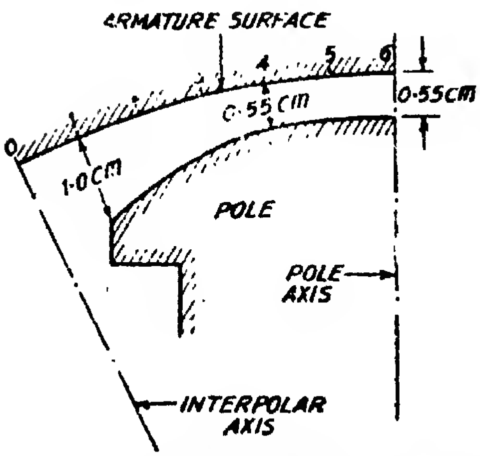


Fig. 4·72. Pole profile of synchronous machine. of Example 4·17

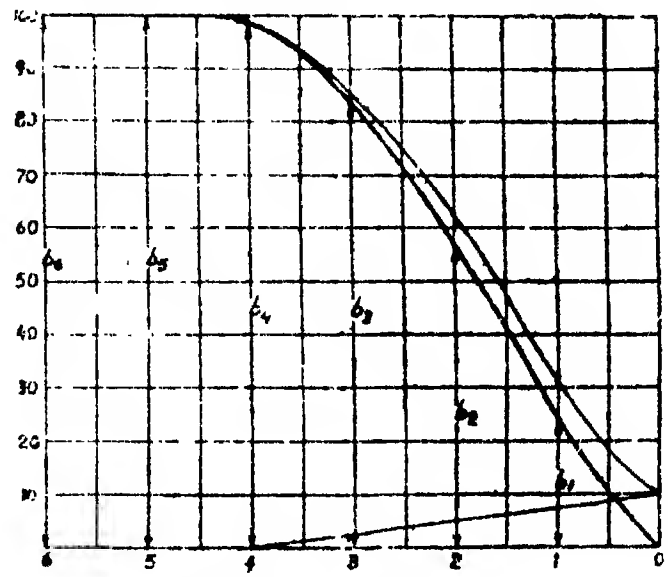


Fig. 4·74. Flux distribution curve.

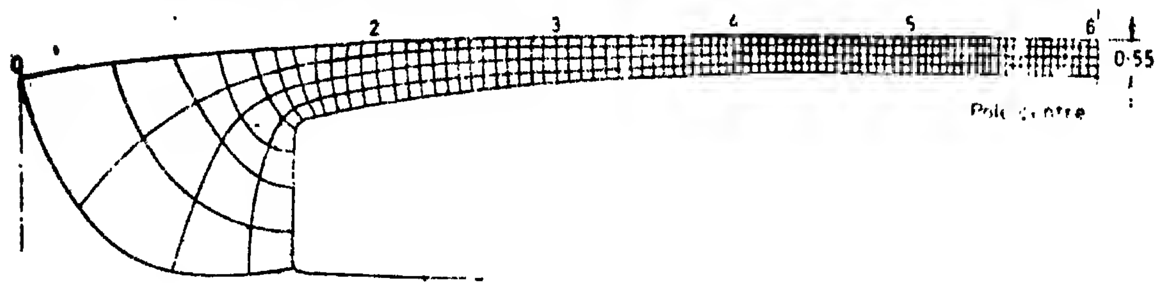


Fig. 4·73

From Fig. 4·73,

b_1=24, b_2=55, b_3=82, b_4=98, b_5=100 and b_6=100.

The amplitudes of fundamental and harmonics of the gap density are :

$B_{m1}=0{\cdot}086\ b_1+0{\cdot}167\ b_2+0{\cdot}236\ b_3+0{\cdot}289\ b_4+0{\cdot}323\ b_5+0{\cdot}167\ b_6=108{\cdot}06,$

$B_{m3}=0{\cdot}236\ b_1+0{\cdot}333\ b_2+0{\cdot}236\ b_3+0-0{\cdot}236\ b_5-0{\cdot}167\ b_6=3{\cdot}36$

$B_{m5}=0{\cdot}323\ b_1+0{\cdot}167\ b_2-0{\cdot}236\ b_3-0{\cdot}289\ b_4+0{\cdot}086\ b_5+0{\cdot}167\ b_6=-5{\cdot}25$

$B_{m7}=0{\cdot}323\ b_1-0{\cdot}167\ b_2-0{\cdot}236\ b_3+0{\cdot}289\ b_4+0{\cdot}086\ b_5-0{\cdot}167\ b_6=-0{\cdot}75.$

$\therefore$ The flux distribution curve can be defined as :

$B_\theta=B_{m1}\sin\theta+B_{m3}\sin 3\theta+B_{m5}\sin 5\theta+B_{m7}\sin 7\theta+\ldots\ldots$

$\quad=108{\cdot}06\sin\theta+3{\cdot}36\sin 3\theta-5{\cdot}25\sin 5\theta-0{\cdot}75\sin 7\theta+\ldots\ldots$

Average value of flux density

$$B_{av}=\frac{2}{\pi}\left(B_{m1}+\frac{B_{m3}}{3}+\frac{B_{m5}}{5}+\frac{B_{m7}}{7}\right)=\frac{2}{\pi}\left(108{\cdot}06+\frac{3{\cdot}36}{3}-\frac{5{\cdot}25}{5}-\frac{0{\cdot}75}{7}\right)=70{\cdot}2.$$

$\therefore$ Field form factor $=\dfrac{B_{av}}{B_g}=\dfrac{70{\cdot}2}{100}=0{\cdot}702.$

UNSOLVED PROBLEMS

1. A 15 kW 230-V, 4 pole d.c. machine has the following data: armature diameter$=0·25$ m; armature core length$=0·125$ m; length of air gap at pole centre$=2·5$ mm; flux per pole$=11·7\times10^{-3}$ Wb, ratio $\dfrac{\text{pole arc}}{\text{pole pitch}}=0·66$.

Calculate the mmf required for air gap (*i*) if the armature surface is treated as smooth (*ii*) if the armature is slotted and the gap contraction factor is 1·18. [Ans 1443 A : 1700 A]

2. Determine the air gap length of a d.c. machine from the following particulars: gross length of core$=0·12$ m; number of ducts=one and is 10 mm wide; slot pitch$=25$ mm; slot width$=10$ mm: Carter's co-efficient for slots and ducts$=0·32$; gap density at pole centre$=0·7$ Wb/m²; field mmf per pole$=3900$ A; mmf required for iron parts of magnetic circuit$=800$ A. [Ans. 4·7 mm]

3. Find the effective value of flux density at the pole centre of a d c. machine if; actual flux density at pole centre$=0·82$ Wb/m²; length of air gap$=6$ mm; width of tooth$=18$ mm; width of slot$=13$ mm; width of packets$=50$ mm, width of duct$=10$ mm. Take Carter's co-efficient for ducts and slots from Fig. 4·9. Also calculate the mmf for air gap.

[Solution $W_s/l_g=13/6=2·17$; $K_{cs}=0·3$; $K_{gs}=\dfrac{31}{31-0·3\times13}=1·145$

$W_d/l_g=10/6=1·66$; $K_{cd}=0·25$;

For calculation of contraction factor of ducts the stacks are treated as teeth and ducts as slots

$$K_{gd}=\frac{50+10}{(50+10)-0·25\times10}=1·045\;;\quad K_g=1·145\times1·045=1·195.$$

Effective gap density$=K_gB_g=1·195\times0·82=0·982$ Wb/m²

$AT_g=800,000\times0·982\times6\times10^{-3}=4710$ A]

4. An armature tooth 50 mm in height has a width of 16 mm at the top and 12 mm at the root. The real flux density at the root is 2·15 Wb/m². Calculate the mmf required to magnetize this tooth, by (*i*) graphical method (*ii*) Simpson's rule (*iii*) B_t 1/3 method. The following *B*-'*at*' curve may be used.

B, Wb/m²	1·5	1·6	1·7	1·8	1·9	2·0	2·1	2·2
at, A/m	2,000	3,000	5,000	8,000	14,500	24,000	40,000	80,000

[Ans. 950 A for graphical method]

5. Calculate the apparent flux density at a section of the teeth of an armature of a d.c machine from the following data at that section:

Slot pitch$=24$ mm; slot width$=$tooth width$=12$ mm; length of armature core including 5 ducts 10 mm each$=0·38$ m, iron stacking factor$=0·92$.

True flux density in teeth at that section is 2·2 Wb/m² for which the mmf is 70,000 A/m. [Ans. 2·332 Wb/m²]

5. The apparent flux density in the net iron section of an armature tooth is 2·1 Wb/m² at a section of slot pitch 27 mm and width 15·5 mm. The armature is 0·25 cm long and has 3 ducts each 10 mm wide. Estimate the true flux density and mmf per metre for the tooth at the given section. The *B*. '*at*' curve is:

B, Wb/m²	1·7	1·8	1·9	2·0	2·1
'*at*' A/m	10,000	17,500	30,000	90,000	160,000

[Ans. 1·96 Wb/m²; 59000 A]

7. A laminated iron cylinder is rotated in a given magnetic field. The iron loss is 250 W at 600 r.p.m. and 312 W at 720 r.p.m. Find approximately the loss if the laminations were twice as thick, the flux density increased by 25% and the speed were 800 r.p.m. [Ans. 917W]

8. The hysteresis loss in a sample of iron is found to be 4·9 W/kg at a frequency of 50 Hz and a maximum flux B_m of 1 Wb/m². (*a*) Calculate the co-efficient η in the expression for: hysteresis loss in J/m³ per cycle$=\eta\,B_m^{1·7}$. The specific gravity of iron is 7·5. (*b*) Calculate the hysteresis loss in W/kg at 25Hz and $B_m=1·8$ Wb/m². [Ans. (*a*) 735 (*b*) 6·67 W/kg]

9. Determine the leakage permeance per metre length of a rectangular semi-enclosed slot having the following dimensions (all in mm):

Slot width$=10$; slot opening$=4·5$; height of conductor portion$=26$; height above conductor and below wedge$=1$; wedge height$=3·5$; lip height$=1·5$ [Ans. $1·78\mu_0$]

10. A 2 pole d.c. machine has an air gap flux density of 0·7 Wb/m² and a gap length of 5 mm. The area of each pole is 20×10^{-2} m² and the poles are vertically mounted. Calculate the unbalanced magnetic pull for 10% eccentricity in the air gap. [Ans. 780 N]

5
Electromagnets

5·1. Introduction. Electromagnets are used for the purposes of pulling, lifting and holding. The large size electromagnets are used for lifting heavy loads while small electromagnets are used for holding of armatures of relays and valves. The general principles of operation and design are the same for all types of electromagnets irrespective of their application. However, problems of heat dissipation and insulation, and also constructional details become more complicated as the size of the electromagnet increases.

5·2. Types of Electromagnets. Electromagnets are classified into two general categories :

(*i*) tractive type, (*ii*) portative type.

Tractive Electromagnets. These electromagnets are usually known as solenoids and are designed to produce mechanical movement by means of an armature, which is attached to the mechanical part required to do work. Tractive magnets are operated either from a.c. or d.c. supply. Tractive magnets are used in electric bells and buzzers, telegraph apparatus, electric clocks, time stamps, electric pendulums, current limiting magnetic switches and relays, telephone relays magnetic brakes, circuit breakers, track switches and electric valves.

Portative Electromagnets. These electromagnets generally function as holding magnets. They are usually operated from d.c. supply.

Portative magnets are used as lifting magnets, magnetic chucks, magnetic clutches, magnetic couplings, magnetic separators and low voltage releases.

There are many types of electromagnets but for the sake of brevity, only three commonly used types of electromagnets are described here. These are :

(*i*) Flat-faced armature type,

(*ii*) Horse shoe type, and

(*iii*) Flat-faced plunger type.

5·2·1. Flat-faced Armature type. Fig. 5·1 shows a cross-section of an ordinary circular lifting magnet of flat-faced armature type. This type of magnet is used primrily to produce large force through a relatively small distance. As generally constructed in larger sizes, it is made of cast steel and is used for lifting scrap iron, sheet iron and iron castings. It has magnetic circuit of extremely short length and great cross-rectional area with two working air gaps. These gaps, magnetically in series, are mechanically in parallel, and hence produce a holding surface of large effective area.

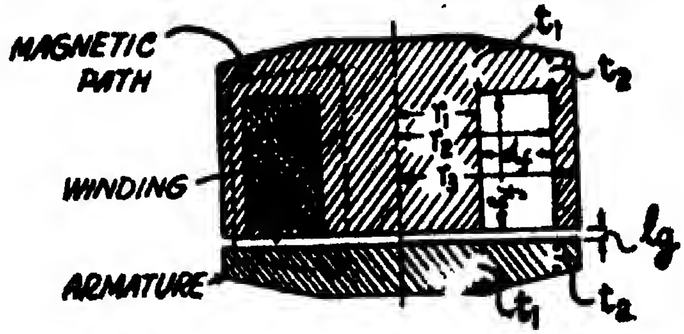


Fig. 5·1. Flat faced armature type electromagnet.

It is clear from Eqn. 4·125 that the force in this type of magnet varies inversely as the square of the air gap length. This condition exists under ideal conditions wherein the effects of saturation and magnetic leakage are negligible. However, in practice the actual force-stroke (force-air gap length) curve departs from the ideal inverse square law relationship as saturation and magnetic leakage effects exist in an actual magnet. Fig. 5·2 shows ideal force-stroke curve (which follows the inverse square law) and the actual force-stroke curve. There is a wide discrepancy between the actual and the ideal curves for short strokes *i.e.* when the length of air gap is small. This is because with short air gaps the mmf required for the air gap is small and hence a large proportion of the mmf forces the flux through the iron parts resulting in high values of flux densities in them. Thus with small air gaps, saturation sets in the iron parts and therefore the inverse squre law no longer holds good. Since with saturation, the permeability of iron parts decreases and therefore the mmf required for iron parts becomes larger in the air gap resulting in lower values of flux density (as compared to the gap density under ideal conditions). Thus for short strokes the saturation in iron parts, which prevents the gap flux density to rise in proportion to decrease in length of air gap, reduces the force. This effect of saturation to decrease the force is somewhat compensated for by increase in the force brought about by increase in value of air gap flux owing to decrease in the value of leakage flux with decrease in air gap length

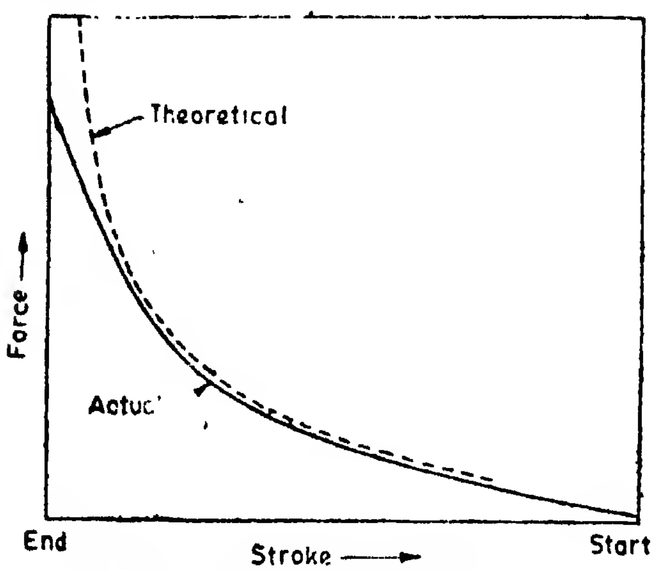


Fig. 5·2. Force-stroke curves of flat faced Armature type magnet.

5·2·2. Horse shoe type. Fig. 5·3 shows a cross-section of a horse shoe or a bipolar type of electromagnet. This is also flat-faced armature type but is built in much smaller sizes as compared with circular magnets as, in general, it is not so economical. Horse shoe

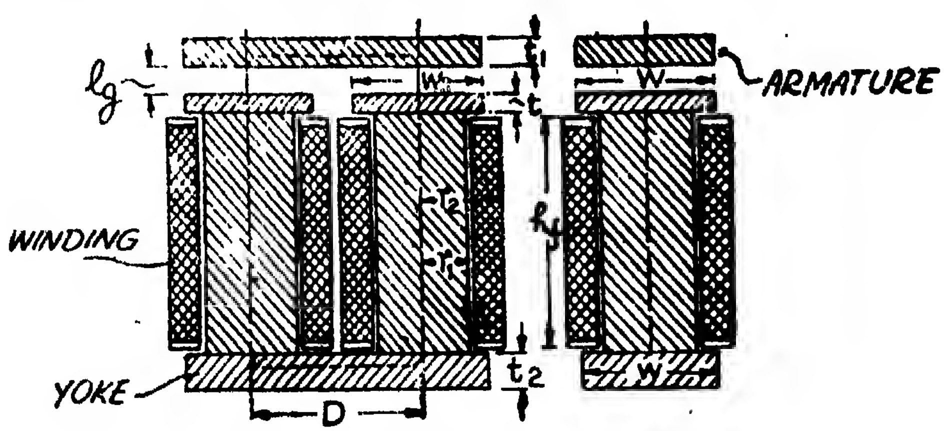


Fig. 5·3. Horse shoe type electromagnet.

type of construction is usually employed for small magnets because of its mechanical adaptability and the ease with which it can be constructed. This construction results in economy in weight over a large range of force-stroke values as with the help of its polar enlargements, the values of flux density in various parts of the magnetic circuit can be adjusted to optimum. In fact it is a versatile type of magnet which can be used as

*Stroke is defined as the linear displacement of armature.

alternative to plunger types of magnets where sliding plunger or fixed cylindrical gap sometimes become undesirable. The force-stroke curves for horse shoe magnets are similar to those of flat faced armature type (Fig. 5·2).

5·2·3. Flat-faced Plunger type. Fig. 5·4 shows a cross-sectional view of cylindrical flat faced plunger type of magnet. Its magnetic circuit is usually short and heavy and has only one air gap. This is adapted to produce a relatively small force through a longer distance (than the flat faced type). This is because for the same cross sectional area of iron as the flat faced armature type or horse shoe type, its air gap reluctance decreases half as fast with decrease in air gap length (as it has only one gap while the flat faced armature type and the horse shoe type have two gaps) and therefore, develops half the force. However, since the mmf is effective across one air gap, instead of two in series, it can develop this smaller force through twice the stroke.

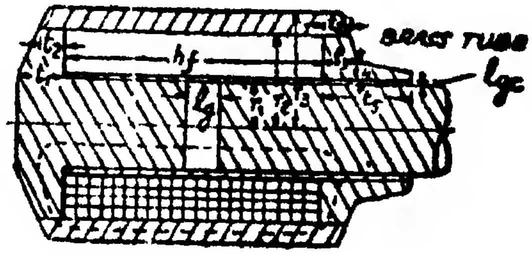


Fig. 5·4. Flat faced plunger type electromagnet.

The force-stroke characteristics of flat-faced plunger type magnets are similar to those of flat faced armature and horse shoe type of magnets.

The force-stroke characteristics of magnets described above show a large difference in force at the beginning and the end of the stroke, the force stroke characteristics following an almost inverse square law. A more uniform force-stroke characteristic may be obtained by shaping the faces of plunger and core as shown in Fig. 5·5. Conical type plunger magnet as shown in Fig. 5·5 (*a*) gives a more flat curve because the reluctance of the working gap decreases less rapidly with motion. The force-stroke curves can be flattened still more by using stepped cylindrical faced plunger (*b*) and taper plunger (*c*) type magnets.

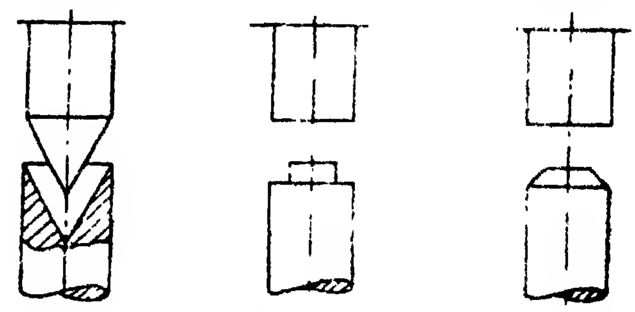


Fig. 5·5 Shapes of plunger and core

5·3. Construction of Electromagnets. Electromagnets consist of a ferromagnetic core which carries the flux and a winding which produces a flux when excited by an external source.

5·3·1. Electromagnet core materials. Soft magnetic materials are used for construction of core of the electromagnets. Most of these materials contain the ferromagnetic elements like iron, nickel and cobalt in various combinations. Sometimes some non ferromagnetic elements like silicon, molybdenum, and chromium are added to obtain desirable from materials.

5·3·2. Electromagnets coils. Coils are used in electromagnets as an exciting source for production of magnetic field. A coil, usually, consists of wire wound like a helical thread to form a layer, there being one or more layers to the coil. Insulation, such as paper is sometimes placed between layers. The usual material for the conductors is copper. In some cases aluminium is used. The cross-section of coil is generally rectangular and the cross-section of conductors is usually round except in coils made of heavy wire where a square, or a rectangular section with rounded corners is used.

The coil insulation is of the sheet form. It is usually made of cloth or paper treated with varnish, glass, synthetic resin bonded paper and synthetic resin impregnated compressed laminations of wood.

5·4. Design of Magnet coils. The coils for electromagnets are usually circular in shape and rectangular in cross-section. The various terms connected with circular coils (Fig. 5·6) are explained below.

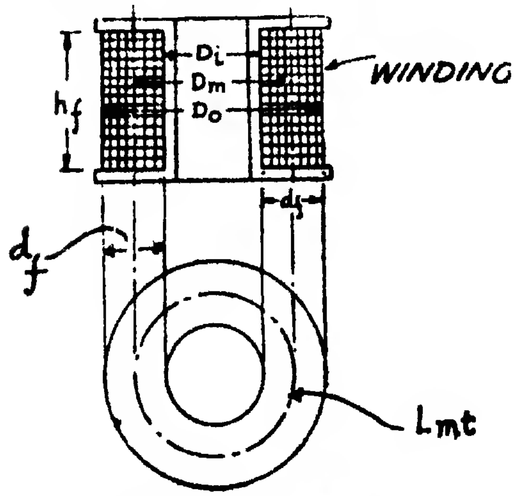


Fig. 5·6. Eelectromagnet coil.

D_i =inside diameter of coil,

D_o =outside diameter of coil,

D_m =mean diameter of coil,

d_f =radial depth of coil,

h_f =axial length of coil,

L_{mt}=length of mean turn.

Mean diameter $D_m=\frac{D_i+D_o}{2}=D_i+d_f$.

Outside diameter $D_o=D_i+2d_f=D_m+d_f$.

Depth of winding

$$d_f=\frac{D_o-D_i}{2}=D_o-D_m=D_m-D_i.$$

Cross winding area $A_w=h_f\times d_f$.

Length of mean turn, $L_{mt}=\pi D_m=\pi\left(\frac{D_i+D_o}{2}\right)=\pi(D_o-d_f)=\pi(D_i+d_f)$

Total heat dissipating surface$=2\ L_{mt}\ (d_f+h_f)$.

Outer cylindrical heat dissipating surface$=\pi\ D_oh_f$

Outer and inner cylindrical heat dissipating surface$=2\ L_{mt}\ h_f$.

Coming to the design of electromagnet coils,

Let AT=mmf per coil, A ; V=terminal voltage, V ;

ρ=resistivity of conductor, Ω/m/mm² ; δ=current density, A/mm² ;

L_{mt}=length of mean turn, m ; I=current, A ;

T=number of turns ; R=resistance, Ω ;

a=area of conductor, mm² ; S_f=space factor ;

A_w=total winding section, m², ; S=cooling surface of coil, m² ;

c=cooling co-efficient, C°-m²/W

θ_m=temperature rise of coil surface over ambient medium, °C.

The ambient temperature is usually assumed as 20°C.

We have,

coil resistance $R=\frac{\text{voltage across the coil}}{\text{current in the coil}}=\frac{V}{I}$.

Resistance of coil number of turns × resistace of each turn $= T\frac{\rho L_{mt}}{a}$

$$\therefore \qquad \frac{V}{I} = T\frac{\rho L_{mt}}{a}$$

or area of conductor $a = IT\frac{\rho L_{mt}}{V} = AT\frac{\rho L_{mt}}{V}$ ...(5·1)

The value of 'a' obtained from Eqn 5·1 may be rounded off to the necessary standard size as given in conductor tables in chapter 17.

The value of resistivity used in Eqn. 5·1 should be corresponding to working temperature. The resistivity and resistance temperature coefficient of copper at 20°C are 0·01734 $\Omega/m/mm^2$ and 0·00393 /°C respectively

The resistivity of copper at working temperature is :

=0·01734[1 + 0·00393 (temperature rise above 20°C)].

Current. The actual value of current is not known until the conditions of cooling are known. A preliminary value of current is obtained by assuming a suitable current density δ.

$$\therefore \qquad I = \delta a \qquad ...(5·2)$$

Turns. The number of turns are obtained by dividing the total mmf by value of current obtained above

$$T = \frac{AT}{I} \qquad ...(5·3)$$

Winding Section. The total area occupied by conductors is equal to the product of number of turns and the area of each conductor.

Total winding area

$$A_w = \frac{\text{number of turns} \times \text{area of each conductor}}{\text{space factor}}$$

$$= \frac{T\,a}{S_f}\,\text{mm}^2 = \frac{T\,a}{S_f} \times 10^{-6}\ \text{m}^2 \qquad ...(5·4)$$

But A_w = height of winding × depth of winding

$= h_f \times d_f \quad m^2$ ...(5·5)

if both h_f and d_f are expressed in m.

From Eqns. 5·4 and 5·5.

$$\left.\begin{aligned} a &= \frac{S_f A_w}{T} \times 10^6\ \text{mm}^2 \\ &= \frac{S_f h_f d_f}{T} \times 10^6\ \text{mm}^2 \end{aligned}\right\} \qquad ...(5·6)$$

Temperature Rise. Total heat to be dissipated $P = I^2 R$

Temperature rise $\theta_m = (P\,c/S)$ where c = cooling co-efficient and S = dissipating surface.

The dissipating surface of the coil considering only the outer and inner cylindrial surfaces is $S = 2\,L_{mt}\,h_f$

Therefore, $\theta_m = \frac{I^2 R\,c}{2 L_{mt} h_f}$ But $R = \frac{T\rho L_{mt}}{a}$

Substituting the value of a from Eqn. 5·6.

$$R=\frac{T^2\,\rho\,L_{mt}}{S_f\,h_f\,d_f}\,10^{-6}$$

$$\therefore\ \theta_m=\frac{I^2\,T^2\,\rho\,L_{mt}\,c}{2\,L_{mt}\,h_f\times S_f\,h_f\,d_f}\,10^{-6}=\frac{\rho\,c}{2\,S_f d_f}\left(\frac{AT}{h_f}\right)^2\times10^{-6}\ {}^\circ\mathrm{C} \qquad \ldots(5{\cdot}7$$

Permissible temperature rise is determined by the class of insulation used. Table 5·1 shows the various kinds of insulations used for magnet coils with permissible temperature rises based on 40°C as the maximum ambient temperature.

Eqn. 5·7 is used when the coil is continuously operating. Continuous ratings are based on the maximum permissible temperature rises shown in Table 5·1.

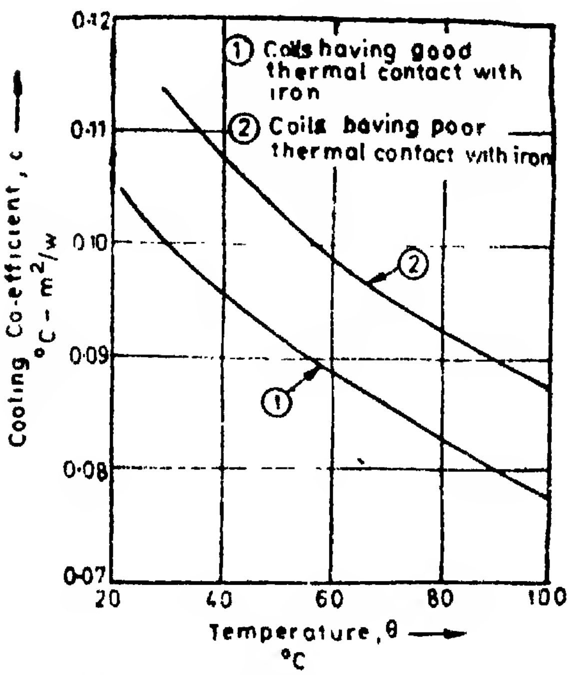


Fig. 5·7. Coolin co-efficient temperature curves of magnet coil.

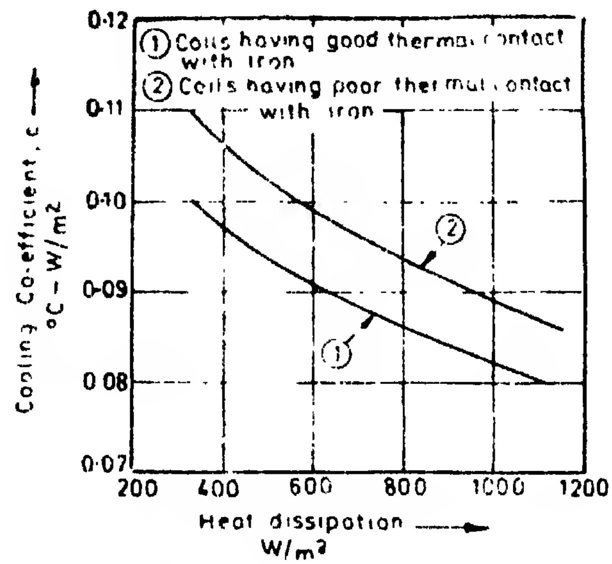


Fig. 5·8. Cooling co-efficienct—watt per m² curves of magnet coils

Since most magnet coils, however, are not energised continuously, consideration must be given to the duty cycle. If the coil is energised for a fraction of time equal to ϵ, Eqn. 5·7 is modified as

$$\theta_m=\frac{\epsilon\rho c}{2S_f d_f}\left(\frac{AT}{h_f}\right)^2\times10^{-6}\ {}^\circ\mathrm{C} \qquad \ldots(5{\cdot}8)$$

Eqn. 5·8 can be used if the heating period is small, not greater than quarter as compared with the thermal time constant.

The value of cooling co-efficient 'c' to be used in Eqns. 5·7 and 5·8 may be taken from Fig. 5·7 and 5·8.

Space Factor. Space factor is defined as the ratio of active conductor section to total conduction section From Fig. 5·9 if d is the diameter of bare conductor and d_1 is the diameter with insulation, the space factor with this arrangement :

$$S_f=\frac{(\pi/4)\,d^2}{d_1^{\,2}}=0{\cdot}78\left(\frac{d}{d_1}\right)^2 \qquad \ldots(5{\cdot}9)$$

Table 5·1. Limiting temperature rise in °C for various coil insulations

Types of coils	Class A insulation organic impregnated (cotton, silk, paper, enamel)		Class B insulation inorganic impregnated (mica, asbestos, glass)	
	*T	*R	*T	*R
1. Wire wound coil	65	85	85	105
2. Single layer series coils with exposed uninsulated surfaces	90	—	105	—

*T—By thermometer *R—By resistance measurement

When the conductors bed as shown in Fig 5 10

$$S_f = \frac{(\pi/4)\, d^2}{d_1^2} \cdot \frac{2}{\sqrt{3}} = 0{\cdot}9 \left(\frac{d}{d_1}\right)^2 \qquad \ldots(5\ 10)$$

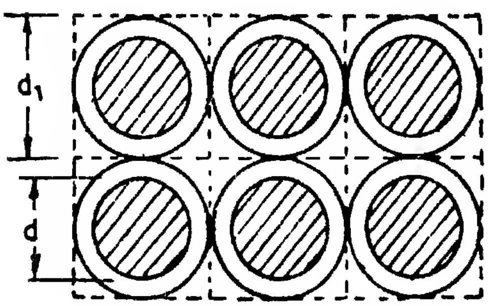


Fig. 5·9. Space factor when conductors do not bed.

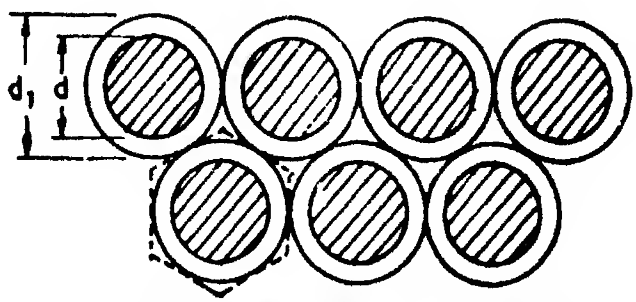


Fig. 5·10. Space factor when conductors bed.

An average value of $S_f = 0{\cdot}8\,(d/d_1)^2$ is usually used for round conductors. The space factor for conductor sections other than circular is estimated from experience.

It should be understood that the space factor calculated from Eqns. 5·9 and 5·10 is for conductors only and not for the whole winding. The space factor for the whole winding will have a lower value as it (winding) contains other insulation in addition to insulation on the conductor.

Example 5·1. *A plunger type magnet has to lift a mass of 200 kg from a distance of 5 mm. The area of pole face is 5×10^{-3} m². Find the current required if the exciting coil has 3000 turns. Assume that the mmf required for iron parts = 10 percent of air gap mmf. Neglect fringing.*

Solution. The plunger type magnet exerts force through one pole face only. Therefore, force exerted

$$F = 0{\cdot}051\ \frac{B^2 A}{\mu_0}\ \text{kg (see Eqn. 4·122).}$$

or

$$200 = \frac{0{\cdot}051\, B^2 \times 5\times10^{-3}}{4\pi\times10^{-7}}$$

or flux density in air gap $B = 0{\cdot}993$ Wb/m²

Mmf required for air $= 800{,}000\ Bl_g = 800{,}000\times0{\cdot}993\times5\times10^{-3} = 3972$ A.

Mmf required for iron parts $= 0{\cdot}1\times3972 = 397$ A.

Total mmf AT=mmf for air gap+mmf for iron parts

$=3972+397=4369$ A.

Current in exciting coil $I=\frac{AT}{T}=\frac{4369}{3000}=1.456$ A.

Example 5.2. *The area of each of two poles of a cast steel circular magnet is 1.25×10^{-3} m². The mean length of flux path through iron is 0.6 m. The magnet lifts steel plates of negligible reluctance with an effective gap of 6 mm at each pole. Find (a) the force exerted and (b) the flux density in air gap, when the excitation is 12000 A. The B—H curve for cast steel is*

B Wb/m²	0·7	0·9	1·0	1·1	1·2
H A/m	630	780	900	1100	1400

Solution. The mmf required for iron parts is not a linear function of the flux density and since the flux density under the operating conditions is not known, the problem has to be solved by plotting the magnetization curve of the magnet *i.e.* by finding the total mmf required for various values of flux density. The total mmf is the sum to mmf required for two gaps and the mmf required for iron parts. The results are :

Flux density B Wb/m²	0·7	0·9	1·0	1·0	1·2
Mmf for two gaps $AT_g=2\times800{,}000\,Bl_g$ A	6720	8640	9500	10560	11520
Mmf per metre for iron at_i A/m	630	780	900	1100	1400
Mmf for iron parts $AT_i=at_i\times l_i$ A	378	464	540	660	840
Total mmf $AT=AT_g+AT_i$ A	7098	9108	10140	11220	12360

The magnetization curve is shown in Fig. 5·11. From this flux density corresponding to an mmf of 12000 A is 1·17 Wb/m².

Since force is exerted through two pole faces in a circular magnet, therefore force

$$F=2\times\frac{B^2A}{2\mu_0\times9.81}\text{ kg}$$

$$=\frac{2\times(1.17)^2\times1250\times10^{-4}}{2\times4\pi\times10^{-7}\times9.81}=13850\text{ kg.}$$

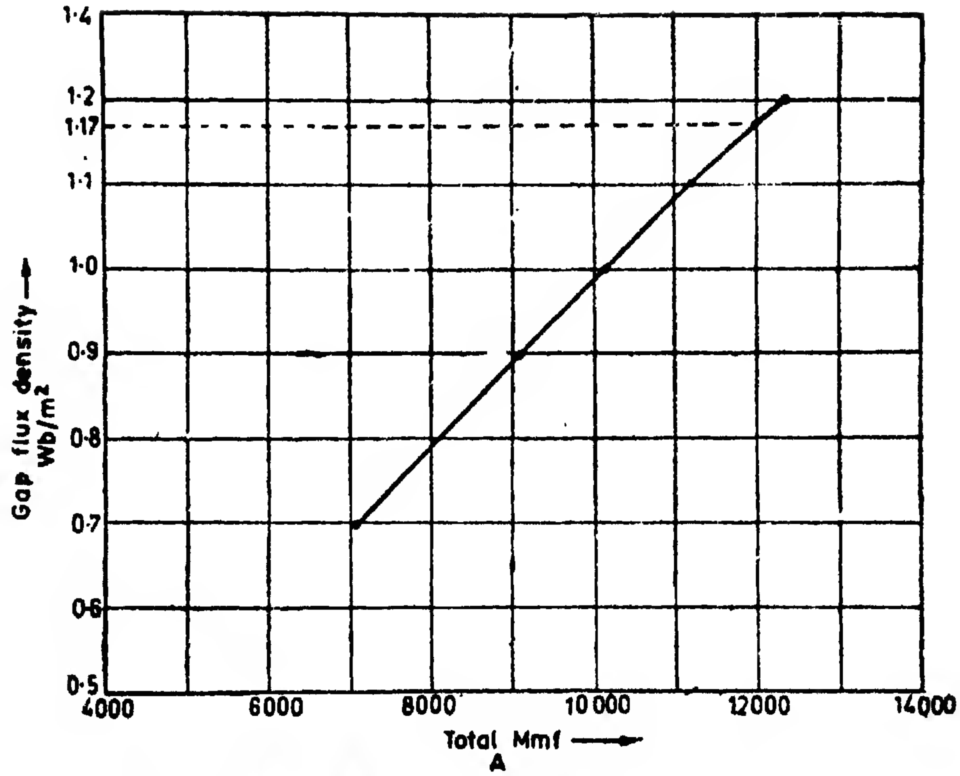


Fig. 5·11. Magnetization curve pertaining to Example 5·2.

Example 5·3. *A plunger type magnet exerts a pull of 200 kg through a 25 mm gap, the reluctance of the iron portions of the circuit being such that only 75 percent of total mmf of the coil is available to send the flux across the gap. Calculate the pull of the magnet when the gap is reduced to 12·5 mm and the total mmf is reduced by 10 percent, given that the mmf to overcome the reluctance of iron portions of the magnetic circuit in the second instance is twice as great in the first instance.*

Solution. Let the mmf required for the air gap in the first case be AT_{g1}.

Total mmf in the first case $AT_1 = \frac{AT_{g1}}{0.75} = 1.33\ AT_{g1}$

Mmf required for iron parts in first case $AT_{i1} = AT_1 - AT_{g1} = 0.33\ AT_{g1}$.

The mmf required for iron parts in second case is twice as great as in first case (given). Therefore mmf required for iron parts in second case :

$$AT_{i2} = 2AT_{i1} = 0.66\ AT_{g1}$$

Total mmf in second case :

$$AT_2 = AT_{g2} + AT_{i2} = AT_{g2} + 0.66\ AT_{g1}.$$

But $$AT_2 = 0.9\ AT_1 = 0.9 \times 1.33\ AT_{g1} = 1.197\ AT_{g1}.$$

$\therefore$ Mmf required for air gap in second case :

$$AT_{g2} = AT_2 - 0.66\ AT_{g1} = 1.197\ AT_{g1} - 0.66\ AT_{g1} = 0.537\ AT_{g1}$$

Now, force $$F = \frac{\mu_0}{2}\left(\frac{AT_g}{l_g}\right)^2 A \qquad \text{(see Eqn. 5·4.)}$$

$\therefore$ Force in the first case $F_1 = \frac{AT_{g1}^2}{2l_{g1}^2} A.$

Force in second case $F_2=\dfrac{\mu_0 AT g_2^2}{2l_{g2}^2}A$.

Hence from above, force in second case

$$F_2=\left(\frac{AT_{g2}}{AT_{g1}}\right)^2\left(\frac{l_{g1}}{l_{g2}}\right)^2 F_1$$

$$=\left(\frac{0{\cdot}537\ AT_{g1}}{AT_{g1}}\right)^2\left(\frac{25}{12{\cdot}5}\right)^2\times 200=231\ \text{kg}.$$

Example 5·4. *A contactor coil is wound on a former 80 mm in between flanges, 75 mm in flange diameter, and 30 mm in gross diameter tube. (a) Calculate the winding depth, winding space and length of mean turn. (b) Calculate the space factor and the number of turns (i) when the conductors bed and (ii) when they do not bed if the coil is wound with 35 SWG S.C.C. copper wire having an area of 0·0357 mm², a bare diameter of 0·213 mm with 0·05 mm thick cotton covering.*

Solution.

(*a*) Winding depth $\quad d_f=\dfrac{D_o-D_i}{2}$ (See Fig. 5·6)

$$=\frac{75-30}{2}=225\ \text{mm}.$$

Winding space $\quad A_w=h_f\times d_f=(80\times10^{-3})\times(22{\cdot}5\times10^{-3})=1{\cdot}8\times10^{-3}\ \text{m}^2.$

(*b*) Diameter of bare conductor $d=0{\cdot}213$ mm

Diameter of insulated conductor $d_1=0{\cdot}213+2\times0{\cdot}05=0{\cdot}313$ mm.

(*i*) From Eqn. 5·9 space factor when the conductors do not bed

$$S_f=0{\cdot}78\left(\frac{d}{d_1}\right)^2=0{\cdot}78\left(\frac{0{\cdot}213}{0{\cdot}313}\right)^2=0{\cdot}361.$$

From Eqn. 5·10, number of turns

$$T=\frac{S_f A_w\times10^6}{a}=\frac{0{\cdot}36\times1{\cdot}8\times10^{-3}}{0{\cdot}0357}\times10^6=18200.$$

(*ii*) From Eqn. 5·10, space factor for conductors when they bed

$$S_f=\left(\frac{d}{d_1}\right)^2=0{\cdot}9\left(\frac{0{\cdot}213}{0{\cdot}313}\right)^2=0{\cdot}417$$

Number of turns $\quad T=\dfrac{0{\cdot}417\times1{\cdot}8\times10^{-3}}{0{\cdot}0357}\times10^6=21{,}025.$

Example 5·5. *A cylindrical magnet has 0·1 m inside diameter and 0·18 m outside diameter while the height is 0·15 m. Find what percentage of total turns must be unwound from outside so that the resistance of the coil is reduced to 3/7 of its original value.*

Solution. Resistance of coil winding

$$R=\frac{T\rho L_{mt}}{a}$$

But $\quad Ta=S_f h_f d_f$

and $\quad d_f=\dfrac{D_o-D_i}{2}$

$\therefore \quad T=\dfrac{S_f h_f d_f}{a}=S_f\, h_f\,\dfrac{(D_o-D_i)}{2}$

Also $\quad L_{mt}=\pi D_m=\dfrac{\pi(D_o+D_i)}{2}$

$$\therefore \qquad R = \frac{S_f h_f}{a}\left(\frac{D_o - D_i}{2}\right)\rho \cdot \frac{\pi(D_o + D_i)}{2} \cdot \frac{1}{a}$$

$$= \frac{\pi \rho S_f h_f}{4a^2} \cdot (D_o^2 - D_i^2)$$

Let suffixes 1 and 2 respectively refer to the original winding and the changed winding. Therefore we can write :

resistance of original winding $R_1 = \frac{\pi\rho S_{f_1} h_{f_1}}{4a_1^2}(D_{o_1}^2 - D_{i_1}^2)$ ...(*i*)

and resistance of changed winding $R_2 = \frac{\pi\rho S_{f_2} h_{f_2}}{4a_2^2}(D_{o_2}^2 - D_{i_2}^2)$ ...(*ii*)

But when we unwind the turns to reduce the resistance, there is a change only in the outer diameter of the coil, while the other terms remain the same.

This means $S_{f_2} = S_{f_1}$, $h_{f_2} = h_{f_1}$, $D_{i_2} = D_{i_1}$ and $a_2 = a_1$

$$\therefore \qquad R_2 = \frac{\pi\rho S_{f_1} h_{f_1}}{4a_1^2}(D_{o_2}^2 - D_{i_1}^2) \qquad ...(iii)$$

Hence from (*i*) and (*iii*) : $\frac{R_2}{R_1} = \frac{(D_{o_2}^2 - D_{i_1}^2)}{(D_{o_1}^2 - D_{i_1}^2)}$.

But $\qquad \frac{R_2}{R_1} = \frac{3}{7}$

$$\therefore \qquad \frac{D_{o_2}^2 - D_{i_1}^2}{D_{o_1}^2 - D_{i_1}^2} = \frac{3}{7} \qquad \text{or} \qquad \frac{D_{o_2}^2 - (0{\cdot}1)^2}{(0{\cdot}18)^2 - (0{\cdot}1)^2} = \frac{3}{7}$$

or outer diameter of winding in second case $D_{o_2} = 0{\cdot}14$ m.

Depth of winding in second case $d_{f_2} = \frac{0{\cdot}14 - 0{\cdot}1}{2} = 0{\cdot}07$ m.

Depth of winding in original case $d_{f_1} = \frac{0{\cdot}018 - 0{\cdot}10}{2} = 0{\cdot}04$ m.

No. of turns in original case $T_1 = \frac{S_{f_1} h_{f_1} d_{f_1}}{a_1}$

No. of turns in second case $T_2 = \frac{S_{f_2} h_{f_2} d_{f_2}}{a_2} = \frac{S_{f_1} h_{f_1} d_{f_2}}{a_1}$.

$$\therefore \qquad T_2 = \frac{d_{f_2}}{d_{f_1}} \cdot T_1 = \frac{0{\cdot}02}{0{\cdot}04} T_1 = 0{\cdot}5\ T_1$$

Turns to be unwound $= T_1 - 0{\cdot}5\ T_1 = 0{\cdot}5\ T_1$.

Therefore 50 percent of original turns have to be unwound to make the resistance 3/7th of its original value.

Example 5·6. (*a*) *Prove that the maximum mmf that can be produced by an exciting coil, with given overall dimensions, temperature rise, cooling co-efficient and space factor is independent of the exciting voltage.*

(*b*) *The exciting coil of an electromagnet is to dissipate 4000 W. The length mean turn is 1 6 m, winding area 35×10^{-3} m² and space factor 0 65. Determine the maximum mmf that can be produced by the coil. Take specific resistance of copper as 0·02 Ω/m/mm².*

Solution (*a*) Area of conductor

$$a = \frac{AT\ \rho\ L_{mt}}{V} \text{ (see Eqn. 5·1)}$$

$$= \frac{(AT)\ L_{mt}}{VI} \cdot I = \frac{(AT)\ \rho\ L_{mt}\ I}{P}.$$

where $\quad P$ = total power dissipated.

$$\therefore \quad \text{Mmf } AT = \frac{P\,a}{\rho\, L_{mt}\, I}$$

Now $$a = \frac{S_f\, A_w}{T} \times 10^6 \qquad \text{(with '}a\text{' in mm}^2 \text{ and } A_w \text{ in m}^2\text{)}$$

From (*i*), $$AT = \frac{P}{\rho\, L_{mt}\, I} \cdot \frac{S_f\, A_w \times 10^6}{T} = \frac{P\, S_f\, A_w \times 10^6}{\rho\, L_{mt}\, (AT)}$$

or $$(AT)^2 = \frac{P\, S_f\, A_w \times 10_6}{\rho\, L_{mt}}$$

or $$\text{Mmf } AT = \sqrt{\frac{P\, S_f\, A_w \times 10^6}{\rho\, L_{mt}}} \qquad \ldots(ii)$$

Now power loss $$P = \frac{S\,\theta_m}{c} \qquad \ldots((iii)$$

where $\quad S$ = heat dissipating surface,

θ_m = maximum permissible temperature rise,

and $\quad c$ = cooling co-efficient.

$\therefore$ From (*ii*) and (*iii*),

$$\text{Mmf } AT = \sqrt{\frac{S\,\theta_m\, S_f\, A_w \times 10^6}{c\,\rho\, L_{mt}}} \qquad \ldots(iv)$$

The above equation does not involve the voltage and all the terms are constant given). Therefore the maximum mmf developed by a coil with given overall limensions, temperature rise, cooling coefficient and space factor is independent of exciting voltage.

(*b*) Using relation (*ii*) derived above

$$\text{Mmf } AT = \sqrt{\frac{P\, S_f\, A_w \times 10^6}{\rho\, L_{mt}}}$$

$$= \sqrt{\frac{4000 \times 0{\cdot}65 \times 35 \times 10^{-3} \times 10^6}{0{\cdot}02 \times 1{\cdot}6}} = 53{,}300 \text{ A.}$$

Example 5·7. *A d.c. crane magnet is built to dissipate about 5 kW at a supply voltage of 100 V. The winding space is 0·2×0·2 m². Taking a winding space factor of 0·6 and a specific resistance of 0·022 Ω/m/mm², calculate the number of turns in the magnet, the cross-section of the conductor used, and the total mmf. The length of mean turn is 2 m.*

Solution.

$\therefore$ Current $$I = \frac{P}{V} = \frac{5000}{100} = 50 \text{ A.}$$

Resistance $$R = \frac{V}{I} = \frac{100}{50} = 2\ \Omega.$$

Also resistance $$R = \frac{T\rho L_{mt}}{a}$$

or area of conductor $$a = \frac{T\rho L_{mt}}{R} = \frac{T \times 0{\cdot}022 \times 2}{2} = 0{\cdot}022\ T \text{ mm}^2.$$

Now, $$a = \frac{S_f A_w}{T} \cdot 10^6 = \frac{0{\cdot}6 \times 0{\cdot}2 \times 0{\cdot}2 \times 10^6}{T} \text{mm}^2.$$

$$\therefore \qquad 0.022\,T = \frac{0.6\times0.2\times0.2\times10^6}{T}$$

or number of turns $\qquad T=1044$

and area of conductor $\qquad a=0.022\times1044=23\ \text{mm}^2.$

Mmf $\qquad =50\times1044=52200$ A.

Example 5·8. *Design the winding of a lifting magnet to obtain greatest mmf with a power dissipation of 6 kW with a supply voltage of 100 V. The gross winding space is $0.27\times0.23\ \text{m}^2$ and the coil has a 15 mm asbestos wrapping. The insulation between turns is 1 mm and between layers is 2·5 mm. The length of mean turn is 2 m The resistivity of wire is $0.023\ \Omega/\text{m}/\text{mm}^2$. The approximate value of space factor is 0·55.*

Solution.

Current $\qquad I=\dfrac{6000}{100}=60$ A and resistance $R=\dfrac{100}{60}=1.67\ \Omega.$

Coil height without wrapping $h_f=0.23-2\times0.015=0.20$ m.

Coil depth without wrapping $d_f=0.27-2\times0.015=0.24$ m.

$\therefore$ Winding area $A_w=0.20\times0.24=0.048\ \text{m}^2.$

Area of conductor, $a=T\dfrac{\rho L_{mt}}{R}=\dfrac{S_f A_w}{T}\ .\ 10^6\ \text{mm}^2$

$$T^2=\frac{RS_fA_w}{\rho L_{mt}}\ .\ 10^6=\frac{1.67\times0.55\times0.048\times10^6}{0.023\times2}=0.958\times10^6$$

$\therefore$ Number of turns $T=979.$

Conductor section 'a'$=\dfrac{T\rho L_{mt}}{R}=\dfrac{979\times0.023\times2}{1.67}=27.1\ \text{mm}^2.$

Using a conductor of 14×2 mm section.

Area of conductor 'a'$=14\times2=28\ \text{mm}^2.$

The insulation between layers is 2·5 mm thick.

$\therefore$ Number of layers height wise$=\dfrac{200}{14+2.5}=12.$

The insulation between turns is 1 mm. thick

$\therefore$ Number of turns depth wise$=\dfrac{240}{2+1}=80.$

$\therefore$ Number of turns used $T=12\times80=960.$

Total copper area$=960\times28\ \text{mm}^2=0.0269\ \text{m}^2.$

$\therefore$ Actual space factor $S_f=\dfrac{0.0269}{0.048}=0.56.$

Actual resistance $\quad R=\dfrac{960\times0.023\times2}{28}=1.58\ \Omega.$

Actual current $\qquad =\dfrac{100}{1.58}=63.4$ A.

Actual power dissipation$=100\times63.4$ W$=6.34$ kW.

Actual mmf provided $\qquad =960\times63.4=60860$ A.

Example 5·9. *A cylindrical magnet coil provides an mmf of 5500 A with 55 V across its terminals. The heat dissipation from its external cylindrical surface is 1000 W/m². The*

inside diameter of the coil is 0·10 m and the length of mean turn is 0·43 m. If the copper space factor is 0·6, calculate :

(i) the cross section of the conductor (ii) the height of the coil (iii) the number of turns in the coil.

Assume the resistivity of wire as 0·02 Ω/m/mm².

Solution. Area of conductor $a=AT\dfrac{\rho L_{mt}}{V}=\dfrac{5500\times0{\cdot}02\times0{\cdot}43}{55}=0{\cdot}86$ mm²

∴ Length of mean turn $L_{mt}=\pi(0{\cdot}1+d_f)=0{\cdot}43$ m. (given)

∴ Depth of winding $d_f=0{\cdot}0375$ m $=37{\cdot}5$ mm.

Outer diameter of coil $=0{\cdot}1+2\times0{\cdot}0375=0{\cdot}175$ m.

Heat dissipating surface = outer cylindrical surface $=\pi(0{\cdot}175)\,h_f=0{\cdot}55\,h_f$.

∴ Heat dissipated $=1000\times0{\cdot}55\,h_f=550\,h_f$

Current $I=\dfrac{\text{power}}{\text{voltage}}=\dfrac{550\,h_f}{55}=10\,h_f$.

∴ Mmf $=IT=10\,h_f\,T$.

Turns $T=\dfrac{S_fA_w}{a}\times10^6=\dfrac{h_fd_fS_f}{a}\times10^6=\dfrac{h_f\times0{\cdot}0375\times0{\cdot}6}{0{\cdot}86}\times10^6$.

Substituting the value of T obtained above,

mmf $=10\,h_f\times\dfrac{h_f\times0{\cdot}0375\times0{\cdot}6}{0{\cdot}86}\times10^6=0{\cdot}2616\times10^6h_f^2=5500$ (given).

∴ Height of coil $h_f=0{\cdot}145$ m

Current $=10\,h_f=10\times0{\cdot}145=1{\cdot}45$ A and turns $T=\dfrac{5500}{1{\cdot}45}=3800$.

Example 5·10. *It is required to establish a flux density of 1·2 Wb/m² in an air gap 10 mm long and 0·12×0·05 m² in cross-section with the help of an electromagnet shown in Fig. 5·12. Design the coil of the electromagnet giving its height and area of cross-section of conductors and number of turns in the winding. Also calculate the width of core winding. The coil is operated from a 110 V supply. Assume space factor = 0·6, resistivity of copper = 0·02 Ω/m/mm² and current density = 2 A/mm². The heat dissipated from outside coil surface should not exceed 2000 W/m². Mmf required for iron parts is negligible.*

Assume that the height of coil is approximately 5 times its width.

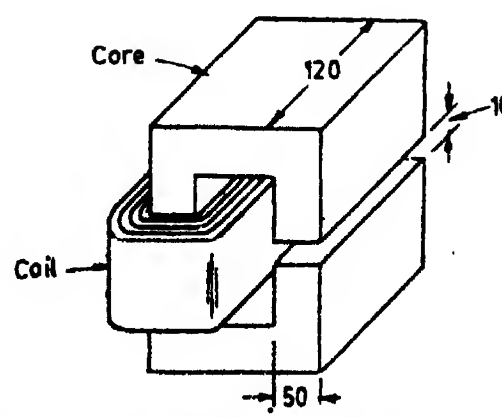


Fig. 5·12. Electromagnet of Example 5·10. All dimensions in mm.

Solution. Mmf required for gap,

$AT_g=800{,}000\,Bl_g=800{,}000\times1{\cdot}2\times10\times10^{-3}=9600$ A.

Total mmf $AT=9600$ A as the mmf required for iron parts is negligible.

Winding area $A_w=\dfrac{Ta}{S_f}10^{-6}=\dfrac{(AT)a}{I\,S_f}10^{-6}=\dfrac{AT}{\delta S_f}10^{-6}$

$$=\frac{9600}{2\times0{\cdot}6}\times10^{-6}=8000\times10^{-6}\ \text{m}^2$$

where δ = current density $= I/a$, A/mm²

Now $A_w = h_f \times d_f$, or $h_f \times d_f = 8000 \times 10^{-6}$

$h_f = 5\, d_f$, $\therefore$ $5d_f^2 = 8000 \times 10^{-6}$

or depth of winding $d_f = 0{\cdot}64$ m and height of winding $h_f = 0{\cdot}20$ m.

The plan of core and winding is shown in Fig. 5·13. Length of mean turn

$L_{mt} = 2(0{\cdot}12 + 0{\cdot}04) + 2(0{\cdot}05 + 0{\cdot}04)$
$= 0{\cdot}50$ m.

Length of outer turn $L_o = 2(0{\cdot}12 + 2 \times 0{\cdot}04) + 2(0{\cdot}05 + 2 \times 0{\cdot}04) = 0{\cdot}66$ m.

Area of each conductor

$$a = \frac{(AT)\,\rho\, L_{mt}}{V}$$

$$= \frac{9600 \times 0{\cdot}02 \times 0{\cdot}5}{110} = 0{\cdot}873 \text{ mm}^2.$$

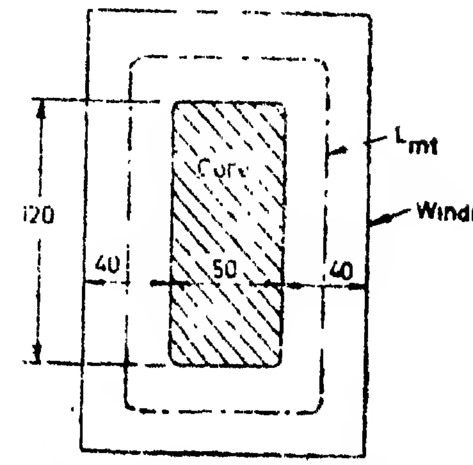


Fig. 5·13. Plan of core and winding. All dimensions in mm.

Exciting current $= \delta a = 2 \times 0{\cdot}873$; $I = 1{\cdot}746$ A.

Number of turns in the winding

$$T = \frac{AT}{I} = \frac{9600}{1{\cdot}746} = 5500.$$

Power dissipation $P = VI = 110 \times 1{\cdot}746 = 192$ W.

Outside coil surface $S = 2\, L_{mt}\, d_f + h_f\, L_o = 2 \times 0{\cdot}5 \times 0{\cdot}04 + 0{\cdot}2 \times 0{\cdot}66 = 0{\cdot}172$ m²

Power dissipated from outer surface of coil

$$= \frac{P}{S} = \frac{192}{0{\cdot}172} = 1116 \text{ W/m}^2$$

This is within the maximum permissible value of 2000 W/m²

Example 5·11. *The channel and coil of a magneto-hydro-dynamic machine are shown in Fig. 5·14. A field of B Wb/m² is to be established in the channel. The mmf required for iron parts is negligible.*

(a) *Show that the depth of coil is* $d_f = B/\mu_0 S_f \delta$.

(b) *Show that the power loss in the coil is given approximately by :*
$P = 2\,\rho\,\delta\, B l_c\, h_f/\mu_0$
Assume channel length l_c *to be very large as compared to width* w_c.

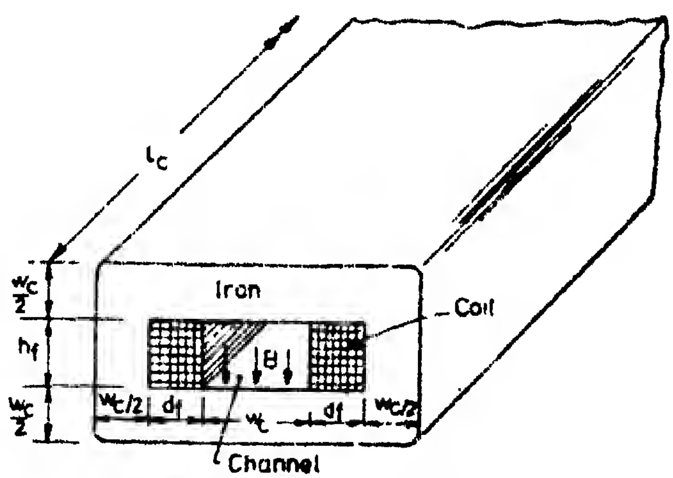


Fig. 5·14. M.H.D. Machine

(c) *Show that the magnetic field energy stored in the channel per watt of power dissipation is approximately* $Bw_c/4\rho\,\delta$.

where S_f = *space factor,* δ = *current density and* ρ = *resistivity.*

(d) *A magneto hydrodynamic generator requires a channel with* $l_c = 25$ m, $w_c = 4$ m *and* $h_f = 0{\cdot}8$ m. *The flux density in the channel is 1·5 Wb/m². The water cooled coil is operated at a current density of* 10×10^6 A/m² (10 A/mm²). *The space factor is 0·4 and the operating*

temperature is 40 °C. Estimate the power dissipated by coil. Calculate also the weight of iron and copper used in the generator. Density of copper is 8900 kg/m³ and that of iron 7800 kg/m³. Resistivity of copper at 40°C is $1 \cdot 87 \times 10^{-8}$ Ωm.

Solution. (*a*) The flux density B is to be established across an air gap of length h_f.

$\therefore$ Mmf required for air gap$=\dfrac{B}{\mu_0}h_f$

Total mmf $AT=AT_g=\dfrac{B}{\mu_0}h_f$, since the mmf required for iron parts is negligible

Now $AT=TI=T\,\delta a$ But $Ta=S_f A_w=S_f h_f d_f$

$\therefore$ $AT=\delta S_f h_f d_f$ (with a expressed in m² and δ in A/m²)

or $\dfrac{B}{\mu_0}h_f=\delta S_f h_f d_f$

$\therefore$ Width of winding $d_f=B/\mu_0 S_f \delta$

(*b*) Length of mean turn $L_{mt}=2l_c$.

Power dissipated $P=VI=TI\,\dfrac{\rho L_{mt}}{a}\,I=(TI)\rho L_{mt}\left(\dfrac{I}{a}\right)$

But $IT=\text{total mmf}=\dfrac{B}{\mu_0}\,h_f$ and $\dfrac{I}{a}=\delta$

$\therefore$ $P=\dfrac{Bh_f}{\mu_0}\cdot\rho\, L_{mt}\,\delta$

Now $L_{mt}=2\,l_c$

$\therefore$ Power dissipated $P=\dfrac{Bh_f}{\mu_0}\rho.\,2l_c\,\delta=\dfrac{2\rho\delta Bl_c h_f}{\mu_0}$.

(*c*) Total magnetic energy stored in the channel

$W=$energy density × volume of channel $=\frac{1}{2}\dfrac{B^2}{\mu_0}\times l_c w_c h_f$

Magnetic energy stored per unit power dissipation

$$=\frac{W}{P}=\frac{\frac{1}{2}B^2 l_c w_c h_f/2\mu_0}{2P\delta Bl_c h_f/\mu_0}=\frac{Bw_c}{4\rho\delta}$$

Power dissipated $P=\dfrac{2\rho\delta B\,l_c h_f}{\mu_0}$

$$=\frac{2\times1\cdot87\times10^{-8}\times10\times10^6\times1\cdot5\times25\times0\cdot8}{4\pi\times10^{-7}}\ \text{W}=8\cdot93\ \text{MW}$$

Depth of coil $d_f=\dfrac{B}{\mu_0 S_f\delta}=\dfrac{1\cdot5}{4\pi\times10^{-7}\times0\cdot4\times10\times10^6}=0\cdot3$ m.

Volume of copper $=S_f\times2l_c h_f d_f=0\cdot4\times2\times25\times0\cdot8\times0\cdot3=4\cdot8$ m³.

Weight of copper $=8900\times4\cdot8=42\ 7\times10^3$ kg

Volume of iron $=l_c\times\dfrac{w_c}{2}\times2\left(h_f+\dfrac{w_c}{2}+w_c+2d_f+\dfrac{w_c}{2}\right)$

$=l_c w_c(h_f+2d_f+2w_c)=25\times4(0\cdot8+2\times0\cdot3+2\times4)=940$ m³.

Weight of iron $=7800\times940=7\cdot33\times10^6$ kg

5·6. Index Number of electromagnets. The shape of a magnet is dependent upon its force and stroke. In general a large force magnet is characterized by large diameter while a small force magnet is small in diameter. A long stroke magnet is long in length and a short stroke magnet short if they are to be economically designed. The cross-sectional area of working gap of a magnet is directly proportional to the force and hence the diameter is proportional to the square root of force. The length is proportional to the stroke. The ratio square root of force and the stroke is an index of the shape of the magnet and hence is called **index number**. Therefore

$$\text{Index number} = \sqrt{F}/s \qquad \ldots(5·11)$$

where F=force of magnet, kg ; and s=stroke of magnet, m.

The index number forms a logical basis for determining the type of magnet to be used for a particular application. Table 5·2 shows the range of index numbers economically covered by each magnet.

Table 5·2. Economical range of Index Numbers

Type of Magnet	*Economical Index Number range*
Flat faced armature	Above 7500
Horse shoe	200 to 20000
Flat faced plunger	1400 to 7500
Conical faced plunger (45°)	30 to 1400
Tapered plunger	17 to 30
Leakage flux	Below 17

5·7. Design of Flat-Faced armature type circular magnet. The design of electromagnets is based upon four fundamental equations. (For dimensions refer to Fig. 5·1).

(i) Force Equation. The force exerted by an electromagnet is :

$$F=\frac{B^2A}{2\mu_0}\ \text{N}=0·051\ \frac{B^2A}{\mu_0}\ \text{kg}.$$

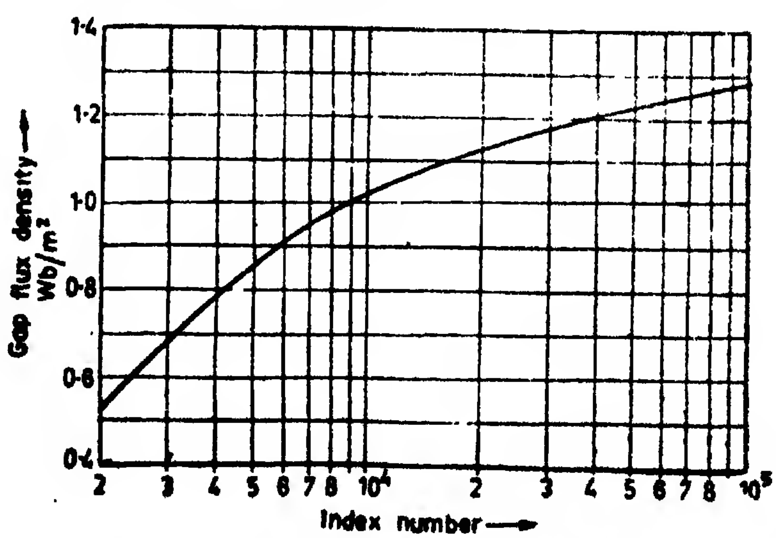


Fig. 5·15. Gap flux density-Index number curve of flat faced electro magnet.

The circular magnet has two working faces and therefore the force exerted by this magnet is :

$$F=2\times 0{\cdot}51\ \frac{B^2A}{\mu_0}=0{\cdot}102\ \frac{B^2A}{\mu_0}\ \text{kg.} \quad ...(5{\cdot}12)$$

The value of flux density to be used is taken from Fig. 5·15.

(*ii*) **Magnetic circuit equation.** Some mmf is consumed in the iron parts whose dimensions at present are unknown. However this mmf may be estimated as 15 per cent of total mmf.

∴ Total mmf required

AT = mmf for two air gaps + mmf for iron parts. $=2\times 800{,}000\ Bl_g+0{\cdot}15\ AT$

$$=160{,}0000\ Bl_g+0{\cdot}15\ AT \quad ...(5{\cdot}13)$$

(*iii*) **Heating Equation.** From Eqn. 5·8, Temperature rise, Maximum temperature rise

$$\theta_m=\frac{q\rho c}{2S_f d_f}\left(\frac{AT}{h_f}\right)^2\times 10^{-6}\ {}^\circ\text{C}$$

The ratio of height to depth of winding is :

$h_f/d_f=3$ to 4 for index numbers upto 40,000

$=2\text{-}3$ to 4 for index numbers 40,000 to 80,000

$=2$ to 4 for index numbers 80,000 and above.

Now depth of winding $d_f=r_2-r_1$ (See Fig. 5·1)

$$\therefore \quad \theta_m=\frac{\epsilon\rho c}{2S_f(r_2-r_1)}\left(\frac{AT}{h_f}\right)^2\times 10^{-6} \quad ...(5{\cdot}14)$$

(*iv*) **Voltage Equation.** Voltage $V=IR=IT\ \frac{\rho L_{mt}}{a}$

$$=(AT)\ \frac{\rho\pi(r_1+r_2)}{a} \quad ...(5{\cdot}15)$$

5·?·1. Design of a small circular magnet.

Problem. *Design a flat faced armature type circular magnet with the following data :*
stroke $=0{\cdot}625$ *mm* ; *force* $=200$ *kg* ; *voltage* $=6$ *volt* ;
temperature rise $=70\ {}^\circ C$ *above an ambient temperature of* $20\ {}^\circ C$.
The magnet is continuously rated.

Solution. For dimensions refer to Fig. 5·1.

$$\text{Index number}=\frac{\sqrt{F}}{s}=\frac{\sqrt{200}}{0{\cdot}625\times 10^{-3}}=22600$$

Referring to Fig. 5·15, flux density corresponding to index number 22600 is 1·17 Wb/m².

Taking a flux density of 1·1 Wb/m² in the air gap.

From Eqn. 5·12, for a flat faced magnet, $F=\frac{0{\cdot}102\ B^2A}{\mu_0}$

or area of inner pole $A=\frac{F\mu_0}{0{\cdot}102\ B^2}=\frac{200\times 4\pi\times 10^{-7}}{0{\cdot}102\times(1{\cdot}1)^2}=2{\cdot}04\times 10^{-3}\ \text{m}^2$

Area $A=\pi r_1^2$ where r_1=radius of inner pole

$\therefore$ $r_1=(2.04\times10^{-3}/\pi)^{1/2}$ m$=25.5$ mm.

Now $AT=1600{,}000\ Bl_g+0.15\ AT$

or $0.85\ AT=1600{,}000\times1.1\times0.625\times10^{-3}$ $\therefore$ $AT=1300$ A.

Resistivity at a temperature rise 70°C above an ambient temperature of 20°C,

$$\rho=0.01734\ (1+0.00393\times70)=0.022\ \Omega/\text{m, mm}^2.$$

Assume $c=0.085$ °C$-$m²/W. (Refer to Fig 5.8)

In this type of magnet, the coil is generally wound on a brass bobbin, for the smaller sizes, which fits snugly into the coil space. Thus the coil has a good thermal contact with iron parts. Therefore a lower value of 'c' is being taken and this value is based upon outer cylindrical surface only.

$S_f=0.5$. The space factor of bobbin wound coils varies from 0.6 for large low voltage coil to 0.4 for small high voltage coils. As this coil is a small low voltage coil, a value of 0.5 is assumed.

$$\text{and}\quad \frac{h_f}{d_f}=\frac{h_f}{r_2-r_1}=4.$$

From Eqn. 5.14, temperature rise

$$\theta_m=\frac{\varepsilon\,\rho\,c}{2\,S_f(r_2-r_1)}\left(\frac{AT}{h_f}\right)\times10^{-6}\ \text{°C}.$$

As the coil is continuously rated, $\varepsilon=1$.

$$\therefore\quad 70=\frac{1\times0.022\times0.085}{2\times0.5\ h_f/4}\left(\frac{1300}{h_f}\right)^2\times10^{-6}$$

or height of winding $h_f=56.5\times10^{-3}$m$=56.5$ mm.

From above, depth of winding $r_2-r_1=\dfrac{h_f}{4}=\dfrac{56.5}{4}=14.1$ mm.

$$r_2=14.1+r_1=14.1+25.5=39.6\text{ mm}.$$

The rest of the dimensions of the iron path are now computed on the basis of maintaining the iron cross section at all points equal to that of inner pole core, *i.e.* πr_1^2.

Calculation of t_1. The lines of force are shown in Fig. 5.1.

Area of flux path$=2\ \pi r_1\ t_1$.

This area should be equal to the area of the inner pole core.

$\therefore$ $2\ \pi\ r_1\ t_1=\pi\ r_1^2$. or $t_1=r_1/2=12.75$ mm.

Area of Conductor. From Eqn. 5.15, $V=\dfrac{(AT)\ \rho\pi\ (r_1+r_2)}{a}$

or area of conductor $a=\dfrac{(AT)\ \rho\ \pi\ (r_1+r_2)}{V}$

$$=\frac{1300\times0.022\times\pi\times(25.5+39.6)\times10^{-3}}{6}$$

$$=0.975\text{ mm}^2.$$

Diameter of conductor $d=(0.975\times4/\pi)^{1/2}=1.11$ mm.

A standard synthetic enamel covered conductor of diameter 1.12 mm is available. (See Table 17.7.) Area of conductor used $a=(\pi/4)\times(1.12)^2=0.985$ mm².

The outer diameter of enamelled conductor,

$$d_1 = 1\cdot215 \text{ mm (using medium covering).}$$

Coil Design. The gross winding depth (r_2-r_1) includes the following thicknesses besides the thickness of the winding itself.

(*a*) thickness of bobbin tube$=0\cdot8$ mm,

(*b*) thickness of insulation between bobbin and coil. This consists of oiled linen or mica about 0·4 mm thick,

(*c*) coil insulation consisting of mica about 0·4 mm thick. Press board washers about 0·2 mm thick are also inserted at the two ends,

(*d*) allowances of about 0·1 mm between insulated coil and iron to cover up irregularities.

$\therefore$ Total allowances$=0\cdot8+0\cdot4+0\cdot8+0\cdot1=2\cdot1$ mm.

Net winding depth$=14\cdot1-2\cdot1=12\cdot0$ mm.

$$\therefore \quad \text{Number of layers depth wise} = \frac{12\cdot0}{1\cdot215} \simeq 10$$

Besides the insulated wire, the height of winding h_f includes :

(*a*) thickness of two flanges of the bobbin each equal to thickness of tube.

Total thickness$=2\times0\cdot8=1\cdot6$ mm.

(*b*) mica about 0·6 mm thick and two press board washers 0·2 mm thick.

Total thickness$=0\cdot6+2+0\cdot2=1.0$ mm.

(*c*) allowance for irregularities about 0·2 mm.

Total allowances$=1\cdot6+1\cdot0+0\cdot2=2\cdot8$ mm.

$\therefore$ Net winding height$=56\cdot5-2\cdot8=53\cdot7$ mm.

$$\text{Number of layers height wise} = \frac{53\cdot7}{1\cdot215} = 44.$$

$\therefore$ Total number of turns $=44\times10=440$.

Total cross section of bare copper$=440\times0\cdot985=433\cdot4$ mm^2.

$$\therefore \quad \text{Actual space factor } S_f = \frac{433\cdot4}{56\cdot5\times14\cdot1} = 0\cdot544$$

Length of mean turn $L_{mt}=\pi(r_1+r_2)=\pi(25\cdot5+39\cdot6)\text{mm}=0\cdot2045$ m.

$$\therefore \quad \text{Resistance of winding } R = \frac{440\times0\cdot022\times0\cdot2045}{0\cdot985} = 2\cdot01\ \Omega.$$

$$\text{Current } I = \frac{6}{2\cdot01} = 2\cdot985 \text{ A.}$$

Actual mmf $AT=440\times2\cdot985=1313$ A.

Temperature rise from Eqn. 5·14,

$$\theta_m = \frac{1\times0\cdot022\times0\cdot085}{2\times0\cdot544\times14\cdot1\times10^{-3}} = \left(\frac{1313}{56\cdot5\times10^{-3}}\right)^2 \times10^{-6} = 65\cdot5\,°\text{C}.$$

Calculation of r_3. For equal iron area, $\pi(r_3^2-r_2^2)=\pi r_1^2$.

or $$r_3=\sqrt{r_1^2+r_2^2}=\sqrt{25\cdot5^2+39\cdot6^2}=47\cdot1 \text{ mm.}$$

Calculation of t_2. For equal iron areas, $2\pi r_2\, t_2=\pi r_1^2$

or $$t_2=\frac{r_1^2}{2r_2}=\frac{(25\cdot5)^2}{2\times39\cdot6}=8\cdot2 \text{ mm.}$$

2. Design of a Large Circular Magnet

Problem. *Design a 110 V d.c. circular magnet to lift ingots of weight 20,000 kg from a distance of 5 mm. The following data may be assumed :*

flux density in the air gap = 0·8 Wb/m², flux density in the iron parts = 1·2 Wb/m²,
mmf required for the iron parts = 10 percent of that of gaps
leakage factor = 1·25, current density = 2 A/mm², space factor = 0·5
depth of winding = 2 × height of winding.

Solution. Force exerted by two poles $= \dfrac{0.102\, B^2 A}{\mu_0}$ kg.

$$= \frac{0.102 \times (0.8)^2 A}{4\pi \times 10^{-7}} = 20{,}000 \text{ (given)}$$

$\therefore$ Area of pole core $A = \dfrac{20{,}000 \times 4\pi \times 10^{-7}}{0.102 \times (0.8)^2} = 0.385$ m².

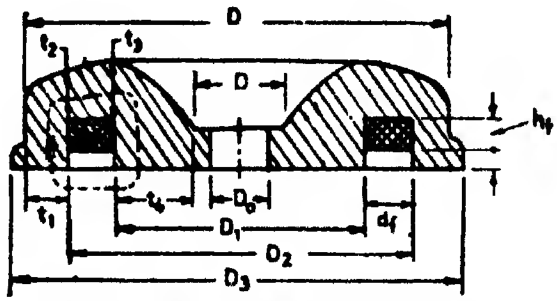


Fig. 5·16. Large flat faced circular magnet.

A hole of diameter $D_0 = 0.15$ m is provided at the centre of the inner pole (Refer to Fig. 5·16).

Area of the inner pole $= (\pi/4)\,(D_1^2 - 0.15^2) = 0.385$ m².

or $D_1 = 0.716$ m. $\simeq 0.72$ m.

Mmf required for the two air gaps $= 2 \times 800{,}000\; B\, l_g$

$= 2 \times 800{,}000 \times 0.8 \times 5 \times 10^{-3} = 6400$ A.

Mmf required for the iron parts $= \dfrac{1}{10} \times 6400 = 640$ A.

$\therefore$ Total mmf required $= 6400 + 640 = 7040$ A.

Total winding area $A_w = \dfrac{Ta}{S_f} \times 10^{-6} = \dfrac{TI}{S_f\,\delta} \times 10^{-6}$

$$= \frac{7040}{0.5 \times 2} \times 10^{-6} = 7.04 \times 10^{-3} \text{ m}^2.$$

Also, $A_w = d_f \times h_f = 2h_f^2$ (as $h_f = 2d_f$) or $2h_f^2 = 7.04 \times 10^{-3}$.

Height of winding $h_f = 0.0593$ m $\simeq 0.06$ m $= 60$ mm.

Depth of winding $d_f = 2\,h_f = 2 \times 0.06 = 0.12$ m $= 120$ mm.

$\therefore$ Inside diameter of outer pole $D_2 = D_1 + 2\,d_f = 0.72 + 2 \times 0.12 = 0.96$ m.

The outer diameter of outer pole D_3, can be found by equating the area of the outer pole with that of the inner pole.

or $(\pi/4)\,(D_3^2 - D_2^2) = 0.385$ or $(\pi/4)\,(D_3^2 - 0.96^2) = 0.385$

$\therefore$ $D_3 = 1.19$ m

Air gap flux per pole $=0{\cdot}8\times0{\cdot}385=0{\cdot}308$ Wb.

Flux in iron = leakage factor × air gap flux $=1{\cdot}25\times0{\cdot}308=0{\cdot}385$ Wb.

Area of iron in the path of flux

$$=\frac{\text{flux per pole}}{\text{flux density in iron}}=\frac{0{\cdot}385}{1{\cdot}2}=0{\cdot}321\ \text{m}^2.$$

Outer diameter of the magnet shell is given by :

$$(\pi/4)\,(D_4^2-D_3^2)=0{\cdot}321 \quad \text{or} \quad (\pi/4)\,(D_4^2-0{\cdot}96^2)=0{\cdot}321$$

or $$D_4=1{\cdot}15\ \text{m}.$$

Thickness of metal in outer shell

$$t_1=\frac{D_4-D_3}{2}=\frac{1{\cdot}15-0{\cdot}96}{2}=0{\cdot}095\ \text{m}.$$

Area of iron above the exciting winding at diameter $D_2=\pi D_2 t_2$

or $$t_2=\frac{0{\cdot}321}{\pi\times0{\cdot}96}=0{\cdot}106\ \text{m}.$$

Area of iron above the exciting winding at diameter $D_1=\pi D_1 t_3$

or $$t_3=\frac{0{\cdot}321}{\pi\times0{\cdot}72}=0{\cdot}142\ \text{m}.$$

Diameter of hole in the casting above the inner pole is given by :

$$(\pi/4)\,(D_1^2-D^2)=0{\cdot}321 \quad \text{or} \quad D=0{\cdot}33\ \text{m}.$$

Thickness of metal $$t_4=\frac{D_1-D}{2}=\frac{0{\cdot}72-0{\cdot}33}{2}=0{\cdot}195\ \text{m}.$$

Design of Coil. Leaving 10 mm each, as allowance for insulation and former etc both in depth and height,

net winding depth $=0{\cdot}12-0{\cdot}01=0{\cdot}11$ m $=110$ mm.

and net winding height $=0{\cdot}06-0{\cdot}01=0{\cdot}05$ m $=50$ mm.

Length of mean turn $L_{mt}=\pi(D_1+d_f)=\pi(0{\cdot}72+0{\cdot}12)=2{\cdot}64$ m.

Area of conductor $$a=AT\frac{\rho L_{mt}}{V}=\frac{7040\times0{\cdot}023\times2{\cdot}64}{110}=3{\cdot}83\ \text{mm}^2.$$

(The resistivity of copper is taken as $0{\cdot}023\ \Omega/\text{m}/\text{mm}^2$)

∴ Diameter of conductor $d=2{\cdot}21$ mm.

Using round copper conductors with fine d c.c.

From table 17·5,

diameter (bare) of standard conductor $=2{\cdot}24$ mm.

∴ **Area of conductor used, $a=(\pi/4)\times2{\cdot}24^2=3{\cdot}94$ mm².**

Diameter of insulated conductor $=2{\cdot}515$ mm.

∴ Number of layers depth wise $=\dfrac{110}{2{\cdot}515}\simeq43.$

Number of layers height wise $=\dfrac{50}{2{\cdot}515}\simeq20.$

∴ Total turns $T=43\times20=860.$

Actual resistance of winding $=T\dfrac{\rho L_{mt}}{a}=\dfrac{860\times0{\cdot}023\times2{\cdot}64}{3{\cdot}94}=13{\cdot}25\ \Omega.$

Actual current $I=\frac{110}{13.25}=8.3$ A Actual mmf$=860\times8.3=7138$ A.

Power $=110\times8.3=913$ W.

Total dissipating area of coil considering all the surfaces,

$$S=2\ L_{mt}\ (d_f+h_f)=2\times2.64\ (0.12+0.06)=0.95\ \text{m}^2.$$

Power dissipation from coil surface$=\frac{913}{0.95}=961$ W/m²

Corresponding to 961 W/m², cooling coefficient $c=0.088$°C$-$m²/W.

(See Fig. 5.8 for coils with good thermal contact with iron).

Temperature rise $\theta_m=\frac{913\times0.088}{0.95}=84.5$°C.

The temperature rise of coils is, however, much smaller than 84.5°C as the coil is not continuously in the circuit and is put in the circuit intermittently.

5.8. Design of Horse shoe-type magnet. For dimension refer to Fig. 5.3.

The four fundamental design equations for a horse shoe magnet are :

(i) Force Equation. There are two working gaps and therefore the force exerted is

$$F=0.102\ \frac{B^2\ A}{\mu_0}\ \text{kg.} \qquad ...(5.16)$$

The value of flux density is taken from Fig. 5.17

(ii) Magnetic Circuit Equation. The mmf required for iron parts may be taken as between 10 to 20 percent of total mmf. As there are two gaps, the mmf is :

$$AT=1600{,}000\ Bl_g+(0.1\text{ to }0.2)\ AT \qquad ...(5.17)$$

(iii) Heating Equation. Temperature rise

$$\theta_m=\frac{r\rho c}{2S_f\ (r_2-r_1)}\left(\frac{AT}{h_f}\right)^2\ \times10^{-6}\ \text{°C}. \qquad ...(5.18)$$

The depth winding $d_f=r_2-r_1$.

Ratio h_f/d_f is usually 6 to 8 for horse shoe magnets.

(iv) Voltage Equation. Voltage $V=AT\frac{\rho\pi(r_1+r_2)}{a}$ ...(5.19)

Problem. *Design a horse-shoe magnet with the following data :*

force$=$5 kg ; stroke$=$2.5 mm ; voltage$=$120 V ;

temperature rise $=$70°C above an ambient temperature of 20°C.

The magnet is continuously rated.

Solution. For dimensions refer to Fig. 5.3.

Index number $=\frac{\sqrt{5}}{2.5\times10^{-3}}=894.$

Corresponding to index number 894, the air pap density is nearly 0.3 Wb/m². (See Fig. 5.17).

The value of flux density used is 0.4 Wb/m².

Area of each pole face $A=\frac{F\mu_0}{0.102\ B^2}=\frac{5\times4\pi\times10^{-7}}{0.102\times(0.4)^2}=0.385\times10^{-3}$ m².

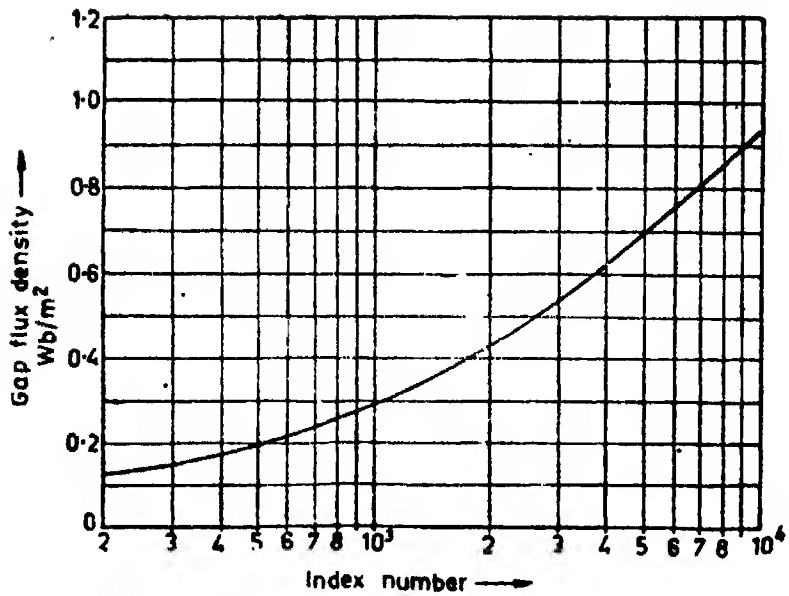


Fig. 5·17. Gap flux density-index number curves of horse shoe electro-magnet

Using square pole faces, we have,

Width of pole face $W=\sqrt{0.385\times10^{-3}}$ m$=0.0196$ m$\simeq20$ mm.

$\therefore$ Actual area of pole face$=20\times10^{-3}\times20\times10^{-3}=0.4\times10^{-3}$ m^2

Flux in the air gap $=0.4\times0.4\times10^{-3}=0.16\times10^{-3}$ Wb.

Taking a leakage factor 1·8 for the yoke.

Flux at the base of poles $\Phi_y=1.8\times0.16\times10^{-3}=0.288\times10^{-3}$ Wb.

Taking a flux density of 1·5 Wb/m^2 in the pole core.

Area of pole core $=\dfrac{0.288\times10^{-3}}{1.5}=0.192\times10^{-3}$ m^2

$\therefore$ Radius of pole core $r_1=\sqrt{(1/\pi)\times0.192\times10^{-3}}$ m$\simeq8$ mm.

Taking the mmf for iron to be 20 percent of total mmf.

From Eqn. 5·17, $AT=1600{,}000\times0.4\times2.5\times10^{-3}+0.2\,AT$

$\therefore$ Total mmf $AT=2000$ A.

We have $\theta_m=70°$C and $\varepsilon=1$

$\rho=0.022\ \Omega/\text{m}/\text{mm}^2$

$c=0.095°$C—m^2/W (See Fig. 5·7. The coil is in poor thermal contact with iron.)

$S_f=0.4$ as it is a small, high voltage magnet.

and $\dfrac{h_f}{d_f}=\dfrac{h_f}{r_2-r_1}=7$

$\therefore$ From Eqn. 5·18,

Temperature rise $\theta_m=\dfrac{\varepsilon\rho c}{2S_f(r_2-r_1)}\left(\dfrac{AT}{h_f}\right)^2\times10^{-6}$

or $70=\dfrac{1\times0.022\times0.095}{2\times0.4\times h_f/7}\cdot\left(\dfrac{2000}{h_f}\right)^2\times10^{-6}$

or height of winding $h_f=0{\cdot}1$ m and depth of winding $d_f=r_2-r_1=\frac{0{\cdot}1}{7}$

$$=0{\cdot}014 \text{ m}=14 \text{ mm}.$$

$\therefore$ $r_2=14+8=22$ mm.

From Eqn. 5·19. $V=\dfrac{AT\ \rho\pi(r_1+r_2)}{a}$

or $$a=\frac{2000\times0{\cdot}022\times\pi(8+22)\times10^{-3}}{120}=0{\cdot}0346 \text{ mm}^2.$$

Diameter of bare conductor $d=0{\cdot}21$ mm.

From Table 17·7, the nearest standard conductor has a diameter $d=0{\cdot}212$ mm.

Actual area 'a' $=\dfrac{\pi}{4}\times(0{\cdot}212)^2=0{\cdot}0353$ mm^2.

Taking a clearance of 3 mm between adjacent coils, the distance between pole centres.

$$D=2r_2+3=2\times22+3=47 \text{ mm}.$$

Taking width of yoke equal to width of pole face and the area of yoke equal to area of pole core,

$$t_2=\frac{\pi r_1^2}{W}=\frac{\pi\times8^2}{2}\simeq10 \text{ mm}.$$

The leakage factor for armature can be taken as 1·3.

Flux in armature Φ_a=leakage factor$\times$air gap flux$=1{\cdot}3\times0{\cdot}1\times10^{-3}=0{\cdot}278\times10^{-3}$ Wb.

Taking a flux density of 1·5 Wb/m^2.

Area of armature $=\dfrac{0{\cdot}278\times10^{-3}}{1{\cdot}5}=0{\cdot}185\times10^{-3}$ m^2.

Taking width of armature equal to width of pole face, the thickness of armature.

$$t_1=\frac{0{\cdot}185\times10^{-3}}{20\times10^{-3}}=9{\cdot}25 \text{ mm},$$

t can be taken equal to 5 mm.

Coil Design. The outside diameter of enamelled wire from Table 17·7 is 0·258 mm.

The gross winding depth r_2-r_1 will include besides the thickness of winding :

(*a*) coil tube of kraft paper 0·8 mm thick

(*b*) coil cover 0·1 mm thick

$\therefore$ Net winding depth$=14-0{\cdot}8-0{\cdot}1=13{\cdot}1$ mm.

Number of layers depth wise$=\dfrac{13{\cdot}1}{0{\cdot}258}\simeq50.$

Besides winding, the height of coil includes two paper margins of 0·4 mm each.
Net winding height$=100-2\times0{\cdot}4=99{\cdot}2$ mm.

Number of layers height wise$=\dfrac{99{\cdot}2}{0{\cdot}258}=384.$

$\therefore$ Total turns$=384\times50=19200.$

Actual space factor $=\dfrac{1\ 200\times0{\cdot}0353}{100\times14}=0{\cdot}484.$

Length of mean turn $=\pi(r_1+r_2)=\pi\times30\times10^{-3}=0.0942$ m.

Resistance of winding $=\dfrac{19200\times0.022\times0.0942}{0.0353}=1127\ \Omega.$

Actual current $=\dfrac{120}{1127}=0.1065$ A.

Actual mmf $=19200\times0.1065=2045$ A.

Actual temperature rise $\theta_m=\dfrac{1\times0.022\times0.095}{0.484\times14\times10^{-3}}\times\left(\dfrac{2045}{0.1}\right)^2\times10^{-6}=64.5°$C.

5·9. Design of a Plunger type magnet. The four fundamental design equations of a plunger type magnet shown in Fig. 5·4 are :

(*i*) **Force Equation.** As there is only one working gap, the force exerted is

$$F=0.051\ \frac{B^2A}{\mu_0}\ \text{kg}. \qquad ...(5.20)$$

The value of air gap flux density is taken from Fig. 5·18.

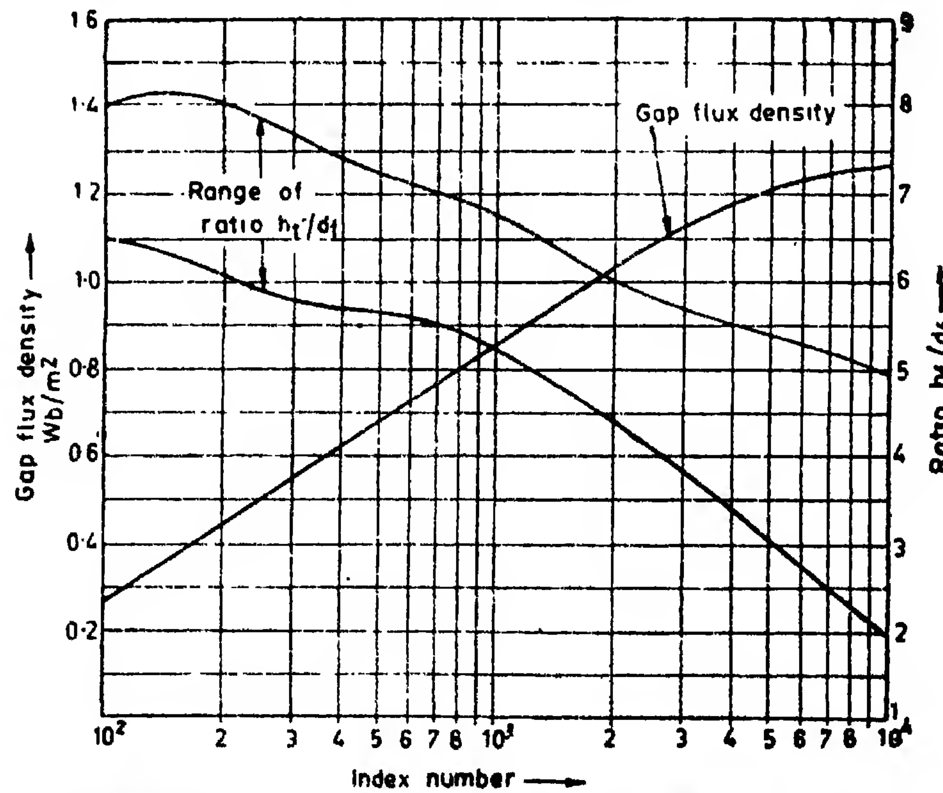


Fig. 5·18. Gap flux density, h_f/d_f -index number curves of flat faced plunger type magnets.

(*ii*) **Magnetic Circuit Equation.** Referring to Fig. 5·4, it can be seen that the magnetic circuit of this type of magnet includes, besides the working air gap and iron, a fixed cylindrical air gap of length l_{gc} and having an axial width t_g. This gap must be included in the calculations. The mmf allocated to this gap is entirely arbitrary. Where it is desired to reduce the size, a large percentage of total mmf should be allocated to this gap. This mmf ranges from 5 to 15 percent of total coil mmf.

Taking mmf required for iron parts to be 20 per cent and that required for cylindrical gap as 10 per cent of total coil mmf, the total mmf is :

$$AT=800{,}000\ Bl_g+0.3\ AT \qquad ...(5.21)$$

(iii) Heating Equation. Temperature rise.

$$\theta_m = \frac{\varepsilon\rho c}{2S_f(r_2-r_1)}\left(\frac{AT}{h_f}\right)^2 10^{-6}\ °C. \qquad ...(5\cdot22)$$

The depth of winding $d_f=(r_2-r_1)$. Fig. 5·18 gives the range of ratio h_f/d_f for different values of index number.

(iv) Voltage Equation. Voltage $V=(AT)\dfrac{\rho\pi(r_2+r_1)}{q}$ ...(5·23)

Problem. *Design a plunger type electromagnet with following specifications :*
stroke=3 mm ; force=50 kg ; voltage=120 V ;
temperature rise=70 °C above an ambient temperature of 20°C.
The magnet has an intermittent rating with heating time = 0·1.

Solution. For dimensions refer to Fig. 5·4

Index number $= \dfrac{\sqrt{50}}{3\times10^{-3}} = 235\cdot7$

Referring to Fig 5·18, a flux density of 1 Wb/m² is used in the air gap.

From Eqn. 5·20, $F=0\cdot051\,\dfrac{B^2A}{\mu_0}$.

or area of plunger $A=\dfrac{F\mu_0}{0\cdot051\,B^2}=\dfrac{50\times4\pi\times10^{-7}}{0\cdot051\times(1)^2}=1\cdot23\times10^{-3}\ \text{m}^2.$

∴ Radius of plunger $r_1=\sqrt{(1/\pi)\times1\cdot23\times10^{-3}}\ \text{m}\simeq20$ mm.

Now, for a plunger type magnet

$$AT=800{,}000\ Bl_g+0\cdot3\ AT \text{ (See Eqn. 5·21)}$$

or

$$AT=\frac{800{,}000\ Bl_g}{0\cdot7}=\frac{800{,}000\times1\times3\times10^{-3}}{0\cdot7}=3430\ \text{A}.$$

Assume $\rho=0\cdot022\ \Omega/\text{m/mm}^2$.

$c=0\cdot085$°C—m²/W. (Refer to Fig. 5·7. The coil is in good thermal contact with iron.)

$S_f=0\cdot45$ as the coil is small and the voltage is relatively high.

$\dfrac{h_f}{d_f}=\dfrac{h_f}{r_2-r_1}=5$ (See Fig. 5·18).

We have, $\theta_m=70$°C and $q=0\cdot1$.

From Eqn. 5·22,

Temperature rise $\theta_m=\dfrac{\varepsilon\rho c}{2S_f(r_2-r_1)}\left(\dfrac{AT}{h_f}\right)^2\times10^{-6}$

or $70=\dfrac{0\cdot1\times0\cdot022\times0\cdot085}{2\times0\cdot45\times h_f/5}\left(\dfrac{3430}{h_f}\right)^2\times10^{-6}$

or height of winding $h_f=56\times10^{-3}$ m$=56$ mm.

∴ depth of winding $d_f\ r_2-r_1=\dfrac{5\cdot6}{5}=11\cdot2$ mm.

or $r_2=11\cdot2+r_1=11\cdot2+20\cdot0=31\cdot2$ mm.

The shell and the end pieces are made of an inferior magnetic material compared with plunger, and hence they should be operated at a lower maximum flux density. Therefore, the area of shell and end pieces is increased to 125 per cent of plunger area. Computing rest of the dimensions, for the outer shell :

$$\pi(r_3^2-r_2^2)=1\cdot25\ \pi r_1^2$$

or $$r_3=\sqrt{1\cdot25\,r_1^2+r_2^2}=\sqrt{1\cdot25\times(20)^2+(31\cdot2)^2}=38\cdot4 \text{ mm}.$$

We have, $2\pi r_1 t_1=1\cdot25\,\pi r_1^2$. $\therefore$ $t_1=0\cdot625\,r_1=12\cdot5$ mm

Also $2\pi r_2 t_2=1\cdot25\,r_1^2$. $\therefore$ $t_2=0\cdot625\,r_1^2/r_2=8$ mm.

Flux through the fixed cylindrical gap $\Phi_0=$leakage factor$\times\pi r_1^2 B$

$$=1\cdot45\,\pi(20\times10^{-3})\times1=1\cdot82\times10^{-3}\text{ Wb}.$$

(Assuming leakage factor$=1\cdot45$)

The thickness of brass tube is 0·5 mm and the clearance between plunger and brass tube is 0·05 mm.

Length of cylindrical gap $l_{gc}=0\cdot5+0\cdot05=0\cdot55$ mm.

Area of flux path through cylindrical gap

$$=\pi(2r_1+l_{gc})t_5=\pi(2\times20+0\cdot55)10^{-3}\times t_5=0\cdot127\,t_5.$$

Flux density in cylindrical air gap

$$B_{gc}=\frac{1\cdot82\times10^{-3}}{0\cdot127\,t_5}=\frac{0\cdot0143}{t_5}\text{ Wb/m}^2.$$

Mmf for cylindrical air gap $AT_{gc}=0\cdot1\,AT=343$ A.

Therefore, we have $AT_{gc}=8000,00\quad B_{gc}\,l_{gc}$

or $$800{,}000\times\frac{0\cdot0143}{t_5}\times0\cdot55\times10^{-3}=343$$

or $$t_5=0\cdot018\text{ m}=18\text{ mm}$$

Coil Design. Conductor area $a=\dfrac{(AT)\,\rho\pi(r_1+r_2)}{V}$

$$=\frac{3430\times0\cdot022\times\pi\times(20+31\cdot2)\times10^{-3}}{120}=0\cdot101\text{ mm}^2$$

Diameter of bare conductor $d=0\cdot359$ mm.

Synthetic enamelled conductors are used. Referring to Table 17·7,

a standard conductor of 0·355 mm bare diameter is selected.

Actual area of conductor $a=(\pi/4)\times(0\cdot355)^2=0\cdot099$ mm^2.

The gross depth of winding includes the following allowances :

(*i*) 0·5 mm thick brass tube and 0·05 mm clearance,

(*ii*) 0·4 mm insulation between brass tube and coil,

(*iii*) 0·6 mm insulation outside the coil,

(*iv*) 0·4 mm allowance for irregularities.

Total allowance$=0\cdot55+0\cdot4+0\cdot6+0\cdot4=1\cdot95$ mm.

Net winding depth$=1\cdot12-1\cdot95=9\cdot25$ mm.

Outside diameter of insulated conductor$=0\cdot415$ mm (see Table 17·7).

Number of layers depth wise$=\dfrac{925}{0\cdot415}=22$.

Making an allowance of 0·5 mm for mica and 0·75 mm of fibre at both ends,

net winding height$=56-0\cdot5-2\times0\cdot15=54$ mm

Number of layers height wise$=\dfrac{54}{0\cdot415}=130$.

Total turns $T=130\times 22=2860$

Space factor $=\dfrac{2860\times 0.099}{56\times 11.2}=0.45$

Length of mean turn$=\pi(r_1+r_2)=\pi(20+31.2)\times 10^{-3}=0.161$ m.

Coil resistance $=2860\times\dfrac{0.022\times 0.161}{0.099}=102.3\ \Omega$

Current $=\dfrac{120}{102.3}=1.175$ A.

Total mmf $=1.173\times 2860=3350$ A.

Temperature rise $\theta_m=\dfrac{0.1\times 0.022\times 0.085}{2\times 0.45\times 11.2\times 10^{-3}}\left(\dfrac{3350}{56\times 10^{-3}}\right)^2\times 10^{-6}.=66.4°C$

5·10. Magnetic Clutches. There are two types of magnetic clutches.

(i) Friction clutches. They are annular type and are provided with friction rings. The connections to the electromagnet of the clutch is made through slip rings. Fig. 5·19 shows a friction type magnetic clutch. The type of clutch is designed for brief periods of slippage when connecting the load (such as when connecting a 200 kW generator to an engine running at 150 r.p.m. without stopping) but not at maximum load when connected.

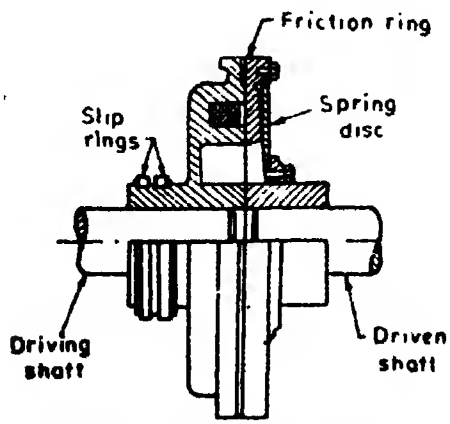


Fig. 5·19. Magnetic clutch.

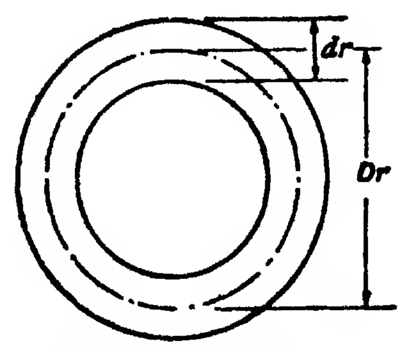


Fig. 5·20. Friction ring.

(ii) Magnetic Couplings. Magnetic clutches without friction rings are known as magnetic couplings.

5·11. Design of Friction Clutch. Let

D_r=mean diameter of friction ring, m ;
A_r=area of contact of friction surface, m^2 ;
p=pressure over friction surface, N/m^2 ;
n=speed, r.p.s.
μ=co-efficient of friction.

Therefore, force on friction surfaces$=pA_r$.

Tangential force which produces slipping$=\mu pA_r$.

Power transmitted kW$=T\omega\times 10^{-3}$

$$=(\mu pA_r)\frac{D_r}{2}(2\pi n)\times 10^{-3}$$

$$=(\pi\mu pA_rD_rn)\times 10^{-3}$$

$$\therefore \qquad A_r \times D_r = \frac{\text{kW} \times 10^3}{\pi \mu p n}$$

A factor of safety must be allowed, partly for the reason that the peak power transmitted is generally considerably in excess of the average full load power and it is to guard against slipping at such times.

Let F.S. be the factor of safety, which will generally have a value between 2 to 3. Introducing this factor, we have

$$A_r \times D_r = \frac{\text{F.S.} \times \text{kW} \times 10^3}{\pi \mu p n} \qquad \ldots(5\cdot24)$$

The value of co-efficient of friction μ for pressures between 350 to 700 kN/m² is about 0·15 for iron on iron and 0·35 to 0·4 for special asbestos friction lining on iron.

Area of contact A_r = mean periphery of friction ring × radial depth of ring

$$= \pi D_r \times d_r$$

$$\therefore \qquad \pi D_r^2 d_r = \frac{\text{F.S.} \times \text{kW} \times 10^3}{\pi \mu p n}$$

or

$$D_r^2 d_r = \frac{\text{F.S.} \times \text{kW} \times 10^3}{\pi^2 \mu p n} \qquad \ldots(5\cdot25)$$

The mean diameter D_r and radial depth d_r of friction ring should be so proportioned as to allow room for the electromagnet between the shaft and the friction ring.

Now, force to be transmitted $= pA_r = \pi p D_r d_r$

To this we must add the force exerted by the spring (which keeps the two parts of the clutch apart when the electric circuit is open).

Total force to be transmitted $F = pA_r +$ force of spring.

The problem now reduces to designing a flat faced armature type circular electromagnet to exert a force F as given above through a distance of about 1·5 mm with the outer diameter of the outer pole equal to inner diameter $(D_r - d_r)$ of friction ring.

The flux density in the air gap may be as high as 1·0 to 1·4 Wb/m².

Example 5·12. *What pressure would be necessary between contact surfaces each 60×10^{-3} m² in area in a magnetic clutch of average diameter 1 metre running at 200 r.p.m., the co-efficient of friction being 0·2 and the power to be transmitted being 50 kW when the clutch is just on the point of slipping?*

Solution. From Eqn. 5·24, we have :

$$A_r \times D_r = \frac{\text{F.S.} \times \text{kW} \times 10^3}{\pi \mu p n}$$

or

$$\text{pressure } p = \frac{\text{F.S.} \times \text{kW} \times 10^3}{\pi \mu n \, A_r \, D_r}$$

We have factor of safety F.S. = 1 as the clutch is about to slip

kW = 50, $\mu = 0\cdot2$, $n = 200/60$

$A_r = 0\cdot06$ m², and $D = 1$ m.

$$\therefore \qquad p = \frac{1 \times 50 \times 10^3}{\pi \times 0\cdot2 \times (200/60) \times 0\cdot06 \times 1} = 40 \times 10^3 \text{ N/m}^2.$$

Design Problem. *Design the field magnet of an electromagnetic clutch to transmit 45 kW h.p. at 375 r.p.m. with a factor of safety of 3. Assume co-efficient of friction = 0·4, pressure on friction ring = 450 kN/m², mean diameter of friction ring = 0·5, coil voltage = 110 volt d.c.*

Solution. Speed $=\frac{375}{60}=6{\cdot}25$ r.p.s.

From Eqn. 5·24,

$$A_r \times D_r = \frac{\text{F.S.}\times \text{kW}\times 10^3}{\pi \mu p n} = \frac{3\times 45\times 10^3}{\pi \times 0{\cdot}4\times 450\times 10^3\times 6{\cdot}25}$$
$$=0{\cdot}0382 \text{ m}^2.$$

∴ Area of contact of friction ring $A_r=0{\cdot}0382/0{\cdot}5=0{\cdot}0764$ m^2 (as $D_r=0{\cdot}5$ m)

But $A_r=\pi D_r d_r$

∴ Radial depth of friction ring $d_r=\frac{A_r}{\pi D_r}=\frac{0{\cdot}0764}{\pi\times 0{\cdot}5}=0{\cdot}0486$ m.

Taking $d_r=0{\cdot}05$ m $=50$ mm, $A_r=\pi\times 0{\cdot}5\times 0{\cdot}05=0{\cdot}0785$ m^2.

Force to be transmitted $=pA_r=450\times 10^3\times 0{\cdot}0785=35325$ N.

Considering in addition the pressure of spring, force to be transmitted

$F=40{,}000$ N.

The clearance between two sides of clutch is assumed as 1·5 mm.

The problem now reduces to designing a circular magnet to exert a force of 40,000 N through a distance of 1·5 mm.

The design details as regards magnetic circuit and winding can be worked out in a manner similar to the one used for design of large circular magnets.

UNSOLVED PROBLEMS

1. A horse-shoe magnet with a uniform cross-section of 0·07 m^2 is used to lift mild steel plates of negligible reluctance. The length of flux path through iron is 2·5 m. Assuming the flux leakage to be negligible, calculate the current which must be passed through its two coils having 800 turns each to support a weight of 6000 kg. The B—H curve for magnet material is :

B Wb/m^2	0·6	0·8	1·0	1·1	1·2
H A/m	600	720	900	1030	1250

[Ans. 1·45 A]

2. The following particulars relate to horse-shoe type magnet, total pull=30 kg ; total mmf of both coils=7000 A ; air gap flux density=0·78 Wb/m^2 ; length of each air gap=5 mm. Calculate the pull when the mmf is increased to 8700 A, which has the effect of increasing the mmf required for iron parts by 50 percent. [Ans. 44 kg]

3. A contactor coil is wound on a former 0·10 m between flanges, 0·075 m in flange diameter and 0·025 m in gross diameter of tube. Calculate the number of turns which can be accommodated in the winding if it is wound with S.C.C. copper conductor having bare diameter=0·122 mm and covered diameter=0·222 mm. Assume that the conductors do not bed. Also calculate the resistance of winding at 20°C if area of cross-section of each conductor is 0·0117 mm^2. [Ans. 50,000, 11,400 Ω]

4. A large oil switch is closed by means of an electromagnet having 800 turns and taking 176 A at normal voltage and temperature. It is desired to unwind sufficient number of turns to enable a current of 200 A to pass. Find how many turns should be removed from outside. The dimensions of the coil are . Outside diameter=0·1 m ; Inside diameter=0·225 m. [Ans. 73]

5. A coil of 4000 turns has an outside radius of 50 mm and an inside radius 20 mm. Estimate the number of turns to be removed from outside to reduce the resistance by 25 per cent, [Ans. 750]

6. A circular lifting magnet has a gap length of 12·5 mm, flux density in air gap=0·5 Wb/m², mmf for air gap=0·9 of total mmf, length of mean turn=0·5 m, coil voltage=55 volt. Calculate the area of cross-section of conductor of the coil winding. Assume an average temperature rise of 40°C over an ambient temperature of 20°C. [Ans. 2 mm²]

7. Design the exciting coil of a 110 volt lifting magnet to obtain the greatest mmf with a power dissipation of 6 kW. The gross winding space is 0·26×0·22 m² and length of mean turn=2 m Assume space factor=0·55 and resistivity of copper at working temperature=0·02 Ω/m/mm².
[Ans. turns=1260, area of conductor=25 mm², mmf=68600 A]

8. A coil of a solenoid operated switch requires 10,000 A. The inside diameter is 0·06 m and the length 0·2 m. The heat dissipation is 1500 watt per m² of outside cylindrical surface at working temperature. The operating voltage=100 volt. Find the number of turns, the conductor cross-section and the current. Take resistivity=0·02 Ω/m/mm² and space factor=0·6.
[Ans. turns=7600, conductor cross-section=0·625 mm², current=1·32 A]

9. A magnetic actuator shown in Fig. 5·21 is used to raise a load through a distance. The coil has 10000 turns and carries a current of 1 A. The magnetic material is not saturated upto a flux density of 1·5 Wb/m².

(a) Determine the maximum air gap for which a flux density of 1·5 Wb/m² can be established with a current of 1 A.
(b) Calculate the force exerted with value of air gap determined in (a).
(c) The density of material is 7800 kg/m³. Determine the approximate value of mass of the load which can be lifted against the force of gravity.
(d) What is the value of current required to lift the unloaded actuator from a distance of 4·17 mm.
[Ans. 4·17 mm ; 584 kg ; 578 kg ; 0·101 A]

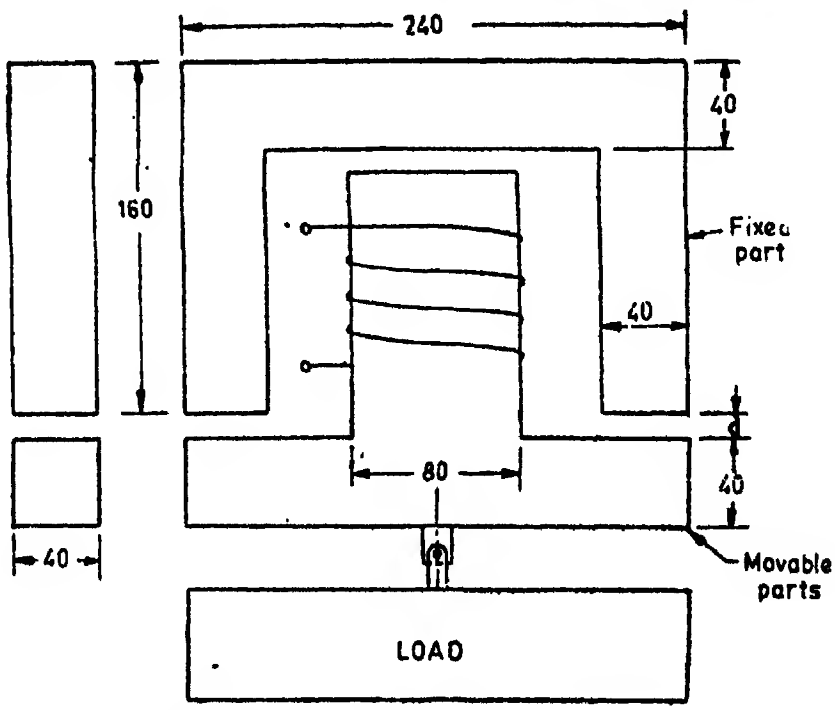


Fig. 5·21. Magnetic Actuator.
(All Dimensions in mm)

10. An electromagnet coil has an internal diameter of 0·3 m and external diameter of 0·4 m. Its height is 0·2 m. The outside cylindrical surface of the coil can dissipate 1000 W/m². Calculate the total mmf of the coil if voltage applied across the coil in 50 V. Assume space factor=0·6 and resistivity=0·02 Ω/m/m².
[Ans. 8500 A]

11. Show that the ohmic loss per unit mass in a coil is proportional to the square of the current density. Find the ohmic loss/kg in a coil worked at a current density of 1·5 A/mm² at the working temperature. Resistivity of copper=0·02 Ω/m/mm². The coil is made of copper which weighs 8900 kg/m³. [Ans. 5·05 W/kg]

12. A circular lifting magnet is to lift a weight of 15 Mg from a distance of 6 mm. The diameter of inner pole is 0·4 m and the area of outer annular pole is equal to the area of inner solid pole. Design the

exciting winding of square cross-section for this magnet giving the height and width of winding, number of turns, area of each conductor and the winding current. Assume the following data : current density = 2·4 A/mm^2 ; space factor = 0·5 ; length of mean turn = ·5 m, resistivity = 0·022 Ω/m/mm^2, inner diameter of exciting winding = diameter of inner pole (*i.e.* 0·4 m) : outer diameter of exciting winding = inner diameter of outer annular pole. The excitation voltage is 220 V. The mmf required for iron parts is 20 percent of total mmf.

[Ans. Turns = 2780, I = 5·22 A, a = 2·175 mm^2, $h_f = d_f$ = 0·11 m ; outer diameter = 0·738 m]

13. The following particulars relate to a magnetic clutch. Calculate the area of contact of friction ring.

Data : kW = 15 ; r.p.m = 400 ; factor of safety = 3 ; coefficient of friction = 0·17 ; pressure at contact surface = 250 kN/m^2 ; mean diameter of ring = 0·5 m. [Ans. 0·1 m^2]

14. The plunger of a flat faced cylindrical plunger type electromagnet has a cross-sectional area of 150 × 10^{-6} m^2. The coil of the magnet has 3000 turns and a resistance of 8 Ω. It is connected to a 12 V d.c. source. The magnetic material may be assumed perfect upto its saturation flux density 1·6 Wb/m^2.

(*a*) Determine the force as a function of gap length

(*b*) Over what range of gap length will the force on plunger be essentially constant because the saturation density has been reached ?

(*c*) If the plunger moves slowly from a gap of 10 mm to fully closed position, calculate the mechanical energy produced.

$$\left[\text{Ans.}\quad F = \frac{19 \times 10^{-6}}{l_g^2}\ \text{N} ;\ l_g = 3{\cdot}53\ \text{mm} ;\ 8{\cdot}9\ \text{J}\right]$$

6

Armature Windings

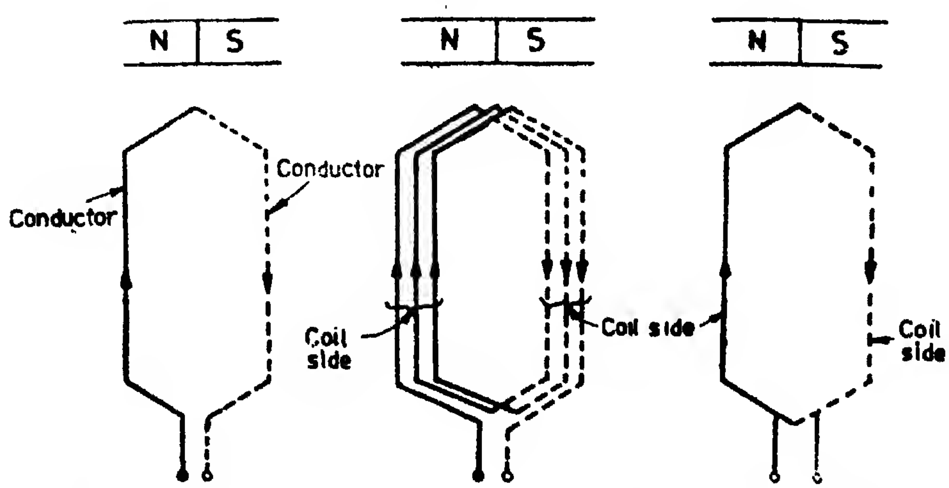
6·1. Introduction. Armature winding of a machine is defined as an arrangement of conductors designed to produce emfs by relative motion in a heteropolar magnetic field. The action of rotating electromagnetic machines is dependent upon the conversion of power that takes place through the medium of magnetic field ; an emf being induced in the winding that experiences a change of flux linkages.

Electrical machines employ groups of conductors distributed in slots over the periphery of the armature. The groups of conductors are connected in various types of series—parallel combinations to form **armature winding.** The conductors are connected in series so as to increase the voltage rating while they are connected in parallel to increase the current rating. Some of the commonly used terms associated with windings are exlpained below :

1. Conductor. The active length of wire or strip in the slot.

2. Turn. A turn consists of two conductors separated from each other by a pole pitch or nearly so, and connected in series as shown in Fig. 6·1 (*a*). The conductors forming a turn are kept a pole pitch apart in order that the emfs in two are additive to produce maximum resultant emf.

3. Coil. A coil may consist of a single turn or may consist of many turns, placed in almost similar magnetic position, connected in series. In the former case, the coil is called a **single turn coil** while in the latter case, it is known as **multi-turn coil.** A single turn coil is shown in Fig. 6·1 (*a*). Fig 6·1 (*b*) shows a 3-turn coil. However, any number of turns could be included in a coil before bringing out the coil ends.



(*a*) Single turn coil (*b*) 3 turn coil (*c*) Representation of multi-turn (or single turn coil)

Fig. 6·1. Single and Multi-turn coils.

4. **Coil side.** A coil consists of two coil sides which are placed in two different slots which are almost a pole pitch apart. The group of conductors on one side of the coil form one coil side while the conductors on the other side of the coil situated a pole pitch (or approximately a pole pitch apart) form the second coil side. Fig. 6·1 (*c*) shows the representation of a coil irrespective of whether it is a single turn or a multiturn coil.

The connections joining the conductors form the **end connectors** or in the mass, the **overhang** or **end-winding**. When the two coil sides forming a coil are spaced exactly one pole pitch apart they are said to be of **full pitch**. However, the coil span may be less than a pole pitch, in which case the coil is described as **short pitched** or **chorded**.

6·2. Single layer and two layer windings. There are two basic physical types for the windings. They deal differently with the mechanical problem of arranging coils in sequence around the armature. The two types are : (*i*) Single layer winding, and (*ii*) Double layer winding.

Single layer winding. Fig. 6·2 (*a*) shows an arrangement for a single layer winding. In this type of winding arrangement, the whole of the slot is occupied by one coil side of a coil. Single layer windnigs are not used for machines having commutators. However, some advantages of these windings are :

(*i*) Single layer windings allow the use of semi-enclosed and closed types of slots.

(*ii*) With single layer windings, coils preformed at one end can be used. These coils are pushed through the slots from one end of the core and are connected up during the winding process at the other end which happens to be free end. The use of preformed coils wherein the insulation can be properly applied and consolidated is a great advantage especially in large output high voltage machines.

High voltage single layer windings use small groups of concentrically placed coils. These coils are interlinked in such a way as to minimize both space taken up outside the slot and in the overhang connections. A coil group employs coils of different coil spans (and hence of different shapes), with average span of coil group being equal to pole pitch. Small rating low voltage induction machines use mush type single layer winding.

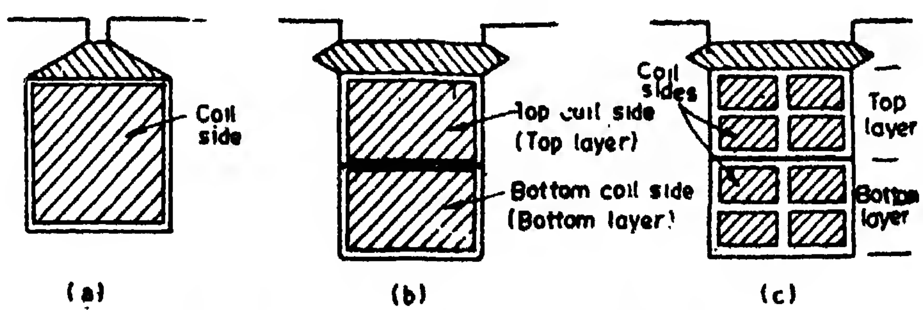


Fig. 6·2. Single and Double layer windings.

Double layer winding. The two layer windings have identical coils with one coil side of each coil lying in top half of the slot and the other coil side in bottom half of another slot exactly or approximately one pole pitch away. Fig. 6·2 (*b*) shows a double layer winding with 2 coil sides per slot. Each layer may contain more than one coil side in case large number of coils are required. Fig. 6·2 (*c*) shows an arrangement wherein there are 8 coil sides per slot. Open slots are frequently used to house double layer windings.

The advantages of double layer windings are :

1. Double layer windings give a neat arrangement with all coils being identical.

2. Double layer windings give greater flexibility in design because of the ease with which the coil span (or coil pitch) can be chosen.

6·3. Closed and open windings. Armature windings are classified into two categories :

(*i*) Closed type and (*ii*) Open type.

Closed windings. In this winding there is a closed path around the armature. Thus if starting at any point, the winding is followed through all its turns, the starting point is reached again. The current flowing into a closed winding is through brushes placed on a commutator whose segments are connected to different armature coils. The armature current divides itself into parallel paths which retain their spatial orientation irrespective of the position of the rotating winding. Even though the coils carrying this current are continuously changing, the view of the winding from the brush always remains the same and thus the brushes maintain the same polarity. This is possible only through the use of a commutator. The closed windings are used in d.c. and a.c. commutator machines because in these machines they are a must in order that the commutator may accomplish its purpose as a switching device. Closed windings are always double layer type.

Open windings. Alternating current machines where commutator is not used, it is not necessary to use a closed winding. These machines use open winding. In open windings, the armature may be left open at one or more points. The ends of each section of the winding can be brought out to terminals and then any desired inter-connection can be made externally. Where an open winding is possible, it is universally preferred to a closed winding because of the flexibility in design and the freedom of connections.

Open windings may be either single layer or double layer type and are used in a.c. machines like induction and synchronous machines.

Closed windings which are adapted for connection to a commutator are usually called "**D.C. Armature Windings**" while open windings which are exclusively used for a.c. machines are known as **A.C. Armature Windings.** These two types of windings are discussed in details in this chapter.

D.C. ARMATURE WINDINGS

6·4. Coils and Coil Sides. D.C. armature windings are closed and double layer type with each coil, consisting of an upper coil side at the top of one slot and a lower coil side situated at the bottom of another slot approximately a pole pitch away. The coils are diamond shaped made in special forming machines. The back and front bends are constructed so that one coil side is on a circumferentially higher level than the other. This construction makes it possible to place the higher coil side in the top of a slot, and the lower coil side in the bottom of another slot (Fig. 6·3).

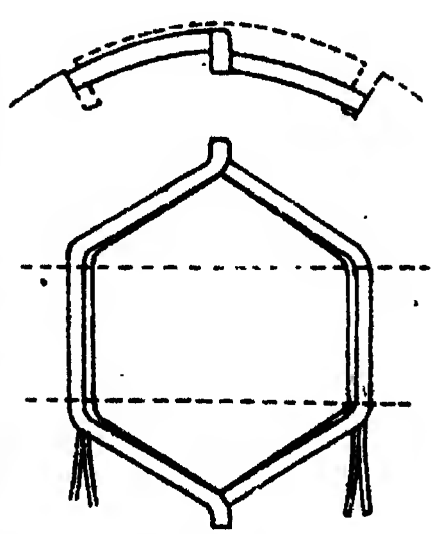


Fig. 6·3. Diamond shaped coil

The number of slots required in a machine is equal to the number of coils if the slot contains two coil sides, one in each layer. But in high voltage and low speed machines, the number of coils is large and it therefore, becomes, necessary to have 4, 6, 8 or even more coil sides per slot (2, 3, 4 or more coil sides per layer) as it, is not always possible to increase the number of slots. Fig. 6·4 shows a winding having 6 coil sides per slot.

6·5 Numbering Scheme. All the top coil sides are numbered odd while all the bottom coil sides are num-

bered even as shown in Fig. 6·4. It is to be noted that coil side 2 is under coil side 1, while coil side 4 is under coil side 3. The numbers 1, 3, 5, 7 etc. at the top and 2, 4, 6, 8 etc. at the bottom progress clockwise.

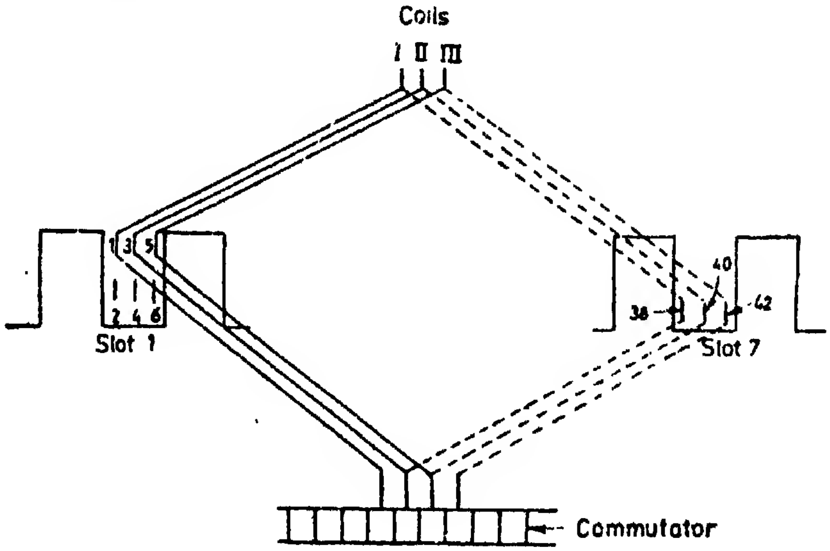


Fig. 6·4. Winding with 6 coil sides per slot.

6·6. Coil Span. The number of teeth embraced by the two coil sides of a coil is known as its coil span. It is usual that the coil span of each armature coil be made equal to the pole pitch (in terms of number of teeth embraced) although the coils are deliberately chorded in special cases, while with certain winding arrangements, it is not possible to have the coil span exactly equal to the pole pitch *i.e.* where the number of slots is not divisible by the number of poles. Consider a winding with 37 slots and 4 poles ; a pole pitch covers $37/4=9\frac{1}{4}$ slots. Thus the coil span can either be 9 or 10 slots (or teeth). It is usual to take the nearest whole number, 9 in this case. If the coil span is chosen as 9 slots and if the top coil side of a coil lies in slot 1, the bottom coil side will lie in $(1+9)=10$th slot.

6·7. Types of D.C. Windings. Modern d.c. machines employ two general types of windings :

(*a*) Lap windings (*b*) Wave windings.

These two types of windings differ from each other in two general ways : (*i*) the number of circuits between positive and the negative brushes, and (*ii*) the manner in which the coil ends are connected to the commutator segments.

The coils in both lap and wave windings are identically formed.

The most commoly used windings are the simplex lap and wave windings. In the **simplex lap winding** (Fig. 6·5), the finish F_1 of coil I is connected to start S_2 of coil II starting under the same pole as start S_1 of coil I. The connector connecting F_1 and S_2 is connected to commutator segment 2 adjacent to segments 1 to which start S_1 of the coil I and the finish of last coil are connected.

In the **simplex wave winding** (Fig. 6·6), the finish F_1 of coil I is connected to start S_{11} of coil XI (for this particular case) which is located under a similar pole to one under which coil I started. This means that the starts of two consecutive coils (S_1 and S_{11}) are two pole pitches apart in wave winding.

6·8. Winding pitches.

6·8·1. Back Pitch. The distance between top and bottom coil sides of a coil, measured around the back of the armature (away from the commutator) is called the back

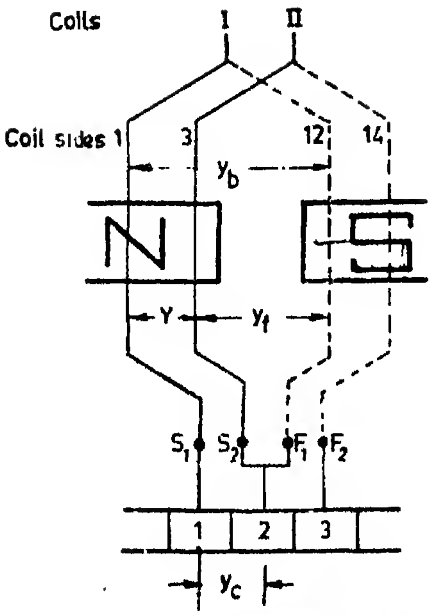


Fig. 6·5. Lap winding.

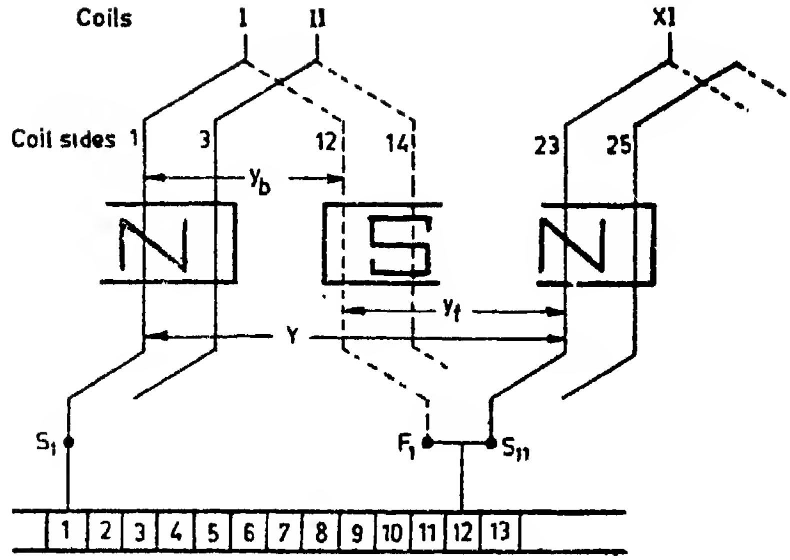


Fig. 6·6. Wave winding.

pitch and is designated as y_b. The back pitch is measured in terms of coil sides. Since y_b is the difference between an even and an odd number and must, therefore, always be an odd number. In Fig. 6·5 and Fig. 6·6, the two coil sides of coil No. I are numbered 1 and 12, therefore $y_b=12-1=11$ in these cases. The back pitch of a coil determines the size of the coil and is nearly equal to coil sides per pole.

6·8·2. Front Pitch. The distance between two coil sides connected to the same commutator segment is called the **front pitch** and is designated as y_f. The front pitch for the lap winding shown in Fig. 6·5 is $y_f=(12-3)=9$ and that for the wave winding shown in Fig. 6·6 is $y_f=(23-12)=11$. It is noted from above that the front pitch for both lap and wave windings is odd. If we trace the connection for lap winding the movement is backwards (from coil side No. 12 to coil side No. 3) while for a wave winding the movement is in the forward direction (coil side No. 12 to No. 23). Thus, the front pitch determines the type of winding only and it does not affect the size of the coils.

6·8·3. Winding Pitch. The distance between the starts of two consecutive coils measured in terms of coil sides is called **winding pitch** and is designated as Y.

$$Y=y_b-y_f \text{ for lap winding} \qquad \ldots(6\cdot1)$$

$$Y=y_b+y_f \text{ for wave winding} \qquad \ldots(6\cdot2)$$

For the simplex lap winding shown in Fig. 6·5; the winding pitch, $Y=11-9=2$.

While the back pitch for the simplex wave winding shown in Fig. 6·6 is

$$Y=11+11=22.$$

6·8·4. Commutator Pitch. The distance between the two commutator segments to which the two ends (starts and finish) of a coil are connected is called the **commutator pitch**. It is measured in terms of commutator segments and is designated as y_c. Thus from Figs. 6·5 and 6·6,

$y_c=2-1=1$ in the case of simplex lap winding for all cases ; and

$y_c=12-1=11$ in the case of simplex wave winding of Fig. 6·6.

On tracing through the simplex lap winding in Fig. 6·5, a segment adjacent to the first one is reached after traversing through one coil, while for simplex wave winding (Fig. 6·6), the adjacent segment is reached after traversing through $p/2$ coils.

6·7. Simplex Lap Winding. As discussed earlier the back pitch y_b is an odd integer and is approximately equal to coil sides per pole. Therefore :

$$y_b=\frac{2C}{p}\pm K \qquad \text{...(6·3)}$$

where,

C=number of coils in the armature, p=number of poles, and

K=a number (integer or fraction) to make y_b an odd integer nearly equal to $2C/p$ *i.e.* coil sides per pole.

There are two ways of connecting the armature coil sides to the commutator (*i*) progressive, (*ii*) retrogressive.

In the progressive winding (Fig. 6·7), the jointing to the commutator progress around the commutator in the same direction as the coils progress around the armature. In this case, finish of coil I is connected to the start of coil II lying to the right. Thus for progressive lap winding :

$$\left.\begin{aligned} Y&=y_b-y_f=+2 \\ y_c&=Y/2=+1. \end{aligned}\right\} \begin{matrix}(a)\\(b)\end{matrix} \qquad \text{..(6·4)}$$

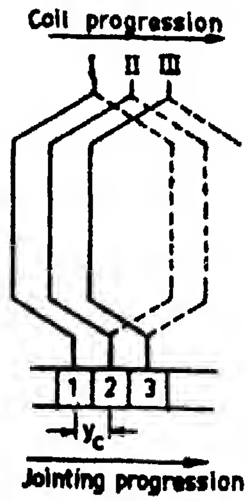


Fig. 6·7. Progressive lap winding.

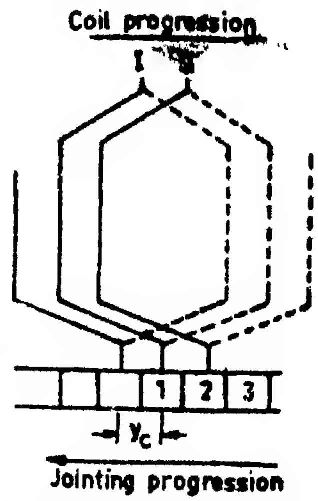


Fig. 6·8. Retrogressive lap winding.

In the retrogressive winding (Fig. 6·8), the jointing to the commutator progresses around the commutator in the opposite direction to the progress of coils around the armature. The finish of coil II is connected to start of coil I lying to the left.

Thus, for retrogressive lap winding :

$$\left.\begin{aligned} Y&=y_b-y_f=-2 \\ \text{and} \quad y_c&=Y/2=-1 \end{aligned}\right\} \begin{matrix}(a)\\(b)\end{matrix} \qquad \text{...(6·5)}$$

Thus, for a simplex lap winding :

$$\left.\begin{aligned} Y&=\pm 2 \\ \text{and} \quad y_c&=\pm 1 \end{aligned}\right\} \begin{matrix}(a)\\(b)\end{matrix} \qquad \text{...(6·6)}$$

Progressive type lap windings are commonly used while use of retrogressive windings is rare.

Example 6·1. *Draw the winding diagram in radial form for a 4 pole, 12 slot simplex lap connected d.c. generator with commutator having 12 segments. Indicate the position of brushes.*

Solution. Slots per pole=12/4=3.

Number of coil C=number of commutator segments=12.

Number of coil sides=$2C$=24.

∴ Coil sides/slot=24/12=2

From Eqn. 6·3, back pitch $y_b=\frac{2C}{p}\pm K=\frac{24}{4}\pm 1=7,\ 5$

$y_b=7$ is used since this permits the use of progressive winding.

From Eqn. 6·4 (*a*), for a progressive lap winding

front pitch $y_f=y_b-2=7-2=5$.

(In case $y_b=5$ is used, y_f is either 7 or 3. If $y_f=7$ is used, it results in a retrogressive winding which should be avoided. On the other and $y_f=3$ cannot be used since the winding pitches should be nearly equal to coils sides per pole in order to get maximum emf)

For drawing the winding diagram in radial form, the armature with 12 slots each having two coil sides and the commutator with 12 segments are drawn as shown in Fig. 6·9. In order to facilitate the connections, a winding table is drawn which is given below.

Winding Table*

$$
\begin{array}{l}
1 \leftarrow 8 \rightarrow 3 \leftarrow 10 \leftarrow 5 \leftarrow 12 \\
\rightarrow 7 \leftarrow 14 \rightarrow 9 \leftarrow 16 \rightarrow 11 \leftarrow 18 \\
\rightarrow 13 \leftarrow 20 \rightarrow 15 \leftarrow 22 \rightarrow 17 \leftarrow 24 \\
\rightarrow 19 \leftarrow 2 \rightarrow 21 \leftarrow 4 \rightarrow 23 \rightarrow 6 \\
\rightarrow 1
\end{array}
$$

Starting with coil side 1 at the front it is connected to commutator segment 1. Coil side 1 is connected to coil 8 at the back. Coil side 8 is connected to coil side 3 at the front. The junction of coil side 8 and coil side 3 is connected to segment 2. Proceeding in similar way, all the coil sides are connected. Also the connections to the commutator segments are made.

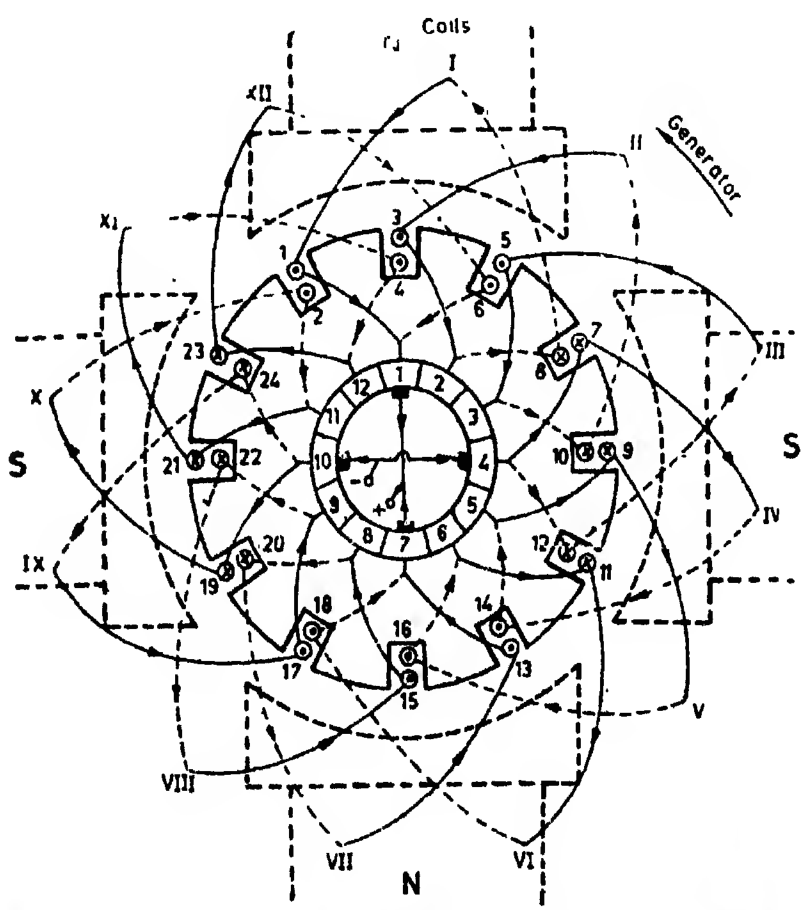


Fig. 6·9. Simplex lap winding of a 4 pole, 12 slot, 12 segment d c. generator.

The polarity of four poles and the direction of rotation are arbitrarily chosen. The slots containing coil sides 1 to 6 and 13 to 18 are under the influence of N poles while slots

*The arrow from right to left (←—) indicates connection at the back (with $y_b=7$) and the arrow from left to right (—→) denotes connection at the front (with $y_f=5$).

containing coil sides 7 to 12 and 19 to 24 are under S poles. The direction of the emfs in coil sides is marked by using Fleming's right hand rule. The direction of current flowing is the same as that of the emf. The directions of current flowing is marked for the coil sides and the end connections. Current flows from commutator into cross marked coil sides while current flows from dot marked coil sides into the commutator. The commutator segments are connected to the junctions of top and bottom coil sides at the front end.

The brushes are placed on those commutator segments, the two coil sides of which carry current in the same direction. A careful inspection of the winding diagram of Fig 6·9 shows that the current enters the commutator at segments 1 and 7 and leaves the commutator at segments 4 and 10. Thus four brushes are placed at segments 1, 4, 7 and 10. Brushes at segment 1 and 7 have the same polarity. They are joined together and the terminal is marked —ve (as the terminal where the current enters a generator is—ve). Similarly, brushes 4 and 10 are joined together and the terminal marked as +ve.

Starting at commutator segment 1 and progressing round the winding in clockwise direction, a derived diagram can be drawn as shown in Fig. 6·10. This diagram includes all coil sides and shows the directions of emf.

There are four (equal to number of poles) reversals of emf. Though the winding is short circuited on itself, there is no circulating current as the resultant emf across the circuit is zero. However, if connection is made to the winding at four equidistant point as shown, it is seen that four equal paths are formed from which an external circuit can be supplied. Each parallel path consists of all top coil sides under one pole together with the bottom coil sides of adjacent pole. In general, for a simplex lap winding the number of parallel paths is equal to the number of poles.

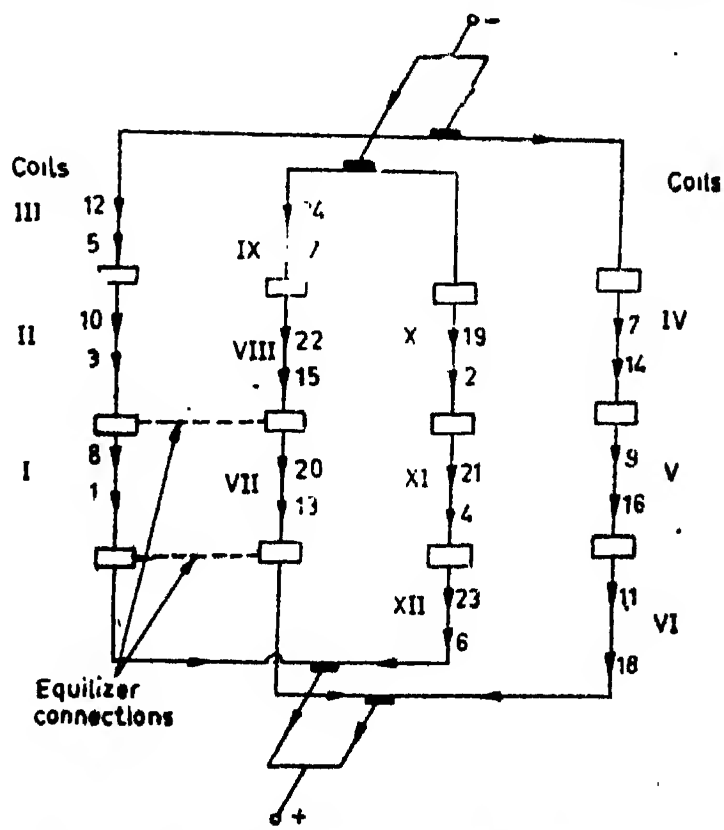


Fig. 6·10. Derived diagram for winding of Example 6·1.

Example 6·2. ***Draw the winding diagram in developed form for a simplex lap wound 24 slot, 4 pole d.c. armature with 24 commutator segments. Also draw the sequence diagram to show the position of brushes.***

Solution. Slots per pole$=24/4=6$

Number of coils $C=$number of commutator segments$=24$.

Number of coil sides$=2C=2\times24=48$.

$\therefore$ Coil sides per slot$=48/24=2$.

Back pitch $y_b = \frac{2C}{p} \pm K = \frac{48}{4} \pm 1 = 13$ or 11.

The winding table is given below.

Winding Table

1←—14—→ 3←—16—→ 5←—18—→ 7←—20—→ 9←—22—→11←—24
—→13←—26—→15←—28—→17←—30—→19←—32—→21←—34—→23←—36
—→25←—38—→27←—40—→29←—42—→31←—44—→33←—46—→35←—48
—→37←— 2—→39←— 4—→41←— 6—→43←— 8—→45←—10—→47←—12
—→1

The developed winding diagram is shown in Fig. 6·11. The direction of the emfs shown are for a generator with the poles on top of the winding.

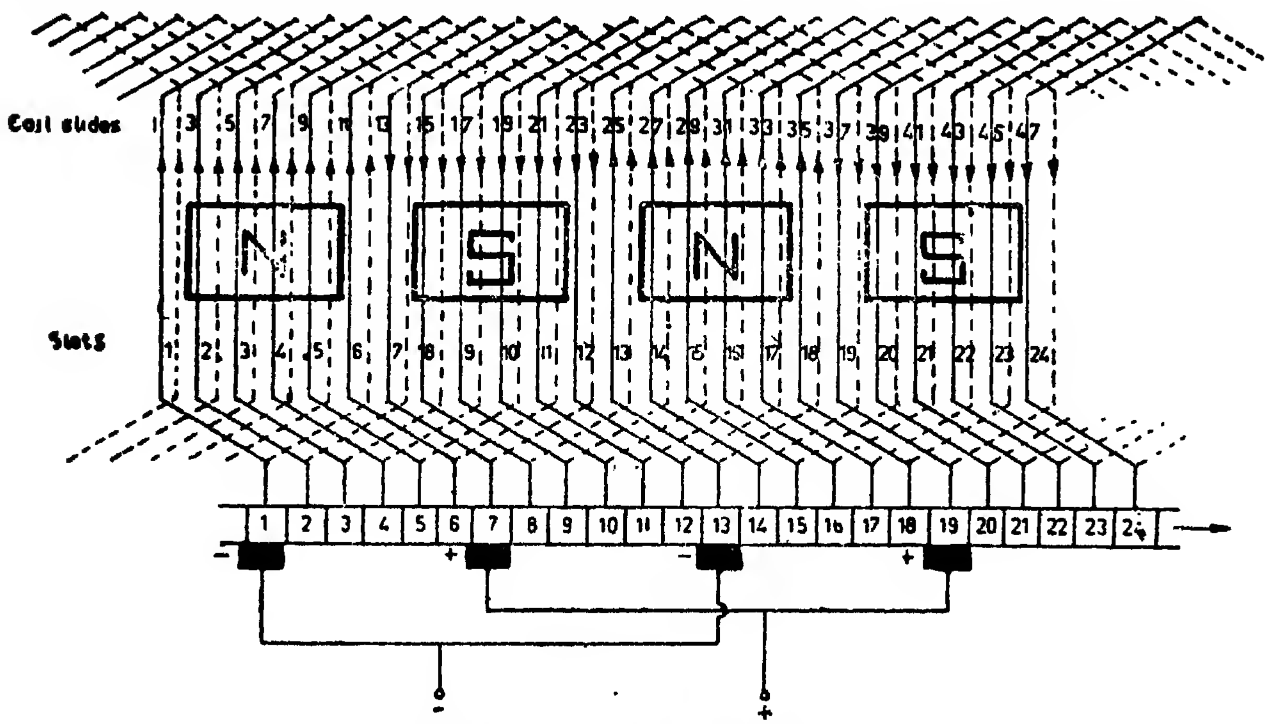


Fig. 6·11. Simplex lap winding.
Slots = 24 Poles = 4 coil sides/Slot = 2
$Y = +2$ $y_b = 13$ $y_f = 11$

A complete developed diagram of the winding is shown in Fig. 6·11 in which the top coil sides are shown as thick lines and the bottom coil sides are shown as dotted lines. The poles are at the top of conductors, each pole covering about 2/3 of a pole pitch. The directions of emfs induced in the conductors lying under each pole are indicated by arrow heads.

Sequence diagram. The sequence diagram for the above winding is shown in Fig. 6·12. This diagram is drawn to locate the position of the brushes. The brushes are placed on a commutator segment, the two coil sides connected to which carry current in the same direction. This diagram shows that a simplex lap winding has as many parallel paths as the number of poles (4 in this case as $p=4$) and the number of brush sets is equal to the number of poles.

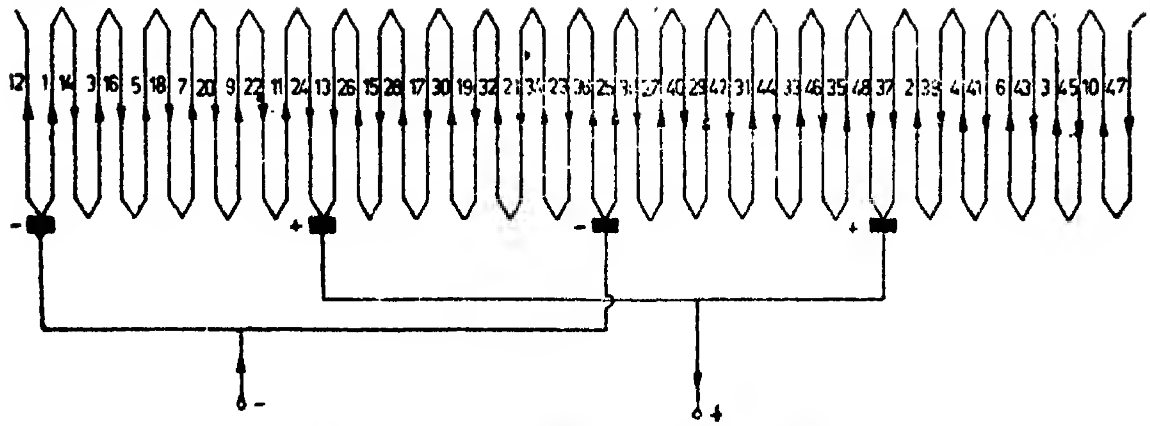


Fig. 6·12. Sequence diagram for winding of Fig. 6·11.

6·9. Simplex lap winding with more than 2 coil sides per slot. We have considered a lap winding with 2 coil sides per slot. Often there are 4 or more (even) coil sides per slot. In such cases the value of back pitch y_b should be so chosen that all the coils having coil sides in upper layer of one slot should also form the bottom layer of another slot. If the value of y_b is not properly chosen it may result in coil sides in the upper layer of one (same) slot being connected to bottom coil sides of two different slots. This requires the use of split coils. The coils with their top coil sides in one (same) slot but having their bottom coil sides in two different slots are called **split coils.** The use of split coils is not desirable from practical point of view. An example given below illustrates this.

Example 6·3. *A 4 pole d.c. armature having 40 slots and 120 coils is to be provided with a simplex lap winding. Work out a suitable winding arrangement so that split coils are not used.*

Solution.

Number of coils sides $=2\times120=240$.

Number of coil sides per slot $=240/40=6$

From Eqn. 6·3, back pitch $y_b=\dfrac{2C}{p}\pm K=\dfrac{240}{4}\pm1=59,\ 61.$

For $y_b=59$:

coil side 1 is connected to coil side $1+59=60$,

coil side 3 is connected to coil side $3+59=62$,

coil side 5 is connected to coil side $5+59=64$.

Considering Fig. 6·13, upper coil sides 1, 3, 5 are located in slot 1 while bottom coil side 60 is located in slot 10, and bottom coil sides 62 and 64 are located in slot 11. Thus it is clear that the top coil sides forming coils I, II, III are in one slot while their corresponding bottom coil sides are in two different slots. Thus the coils are not equal in length resulting in split coils which give considerable practical difficulties.

For $y_b=61$:

coil side 1 is connected to coil side $1+61=62$,

coil side 3 is connected to coil side $3+61=64$,

coil side 5 is connected to coil side $5+61=66$.

Considering Fig. 6·14, the top coil sides are in slot 1 while all the corresponding bottom coil sides are in slot 11. This winding has all the coils of equal length and permits the use of completely formed 3 element coils (a complete unit having 3 coils insulated enbloc), as shown in Fig 6·15.

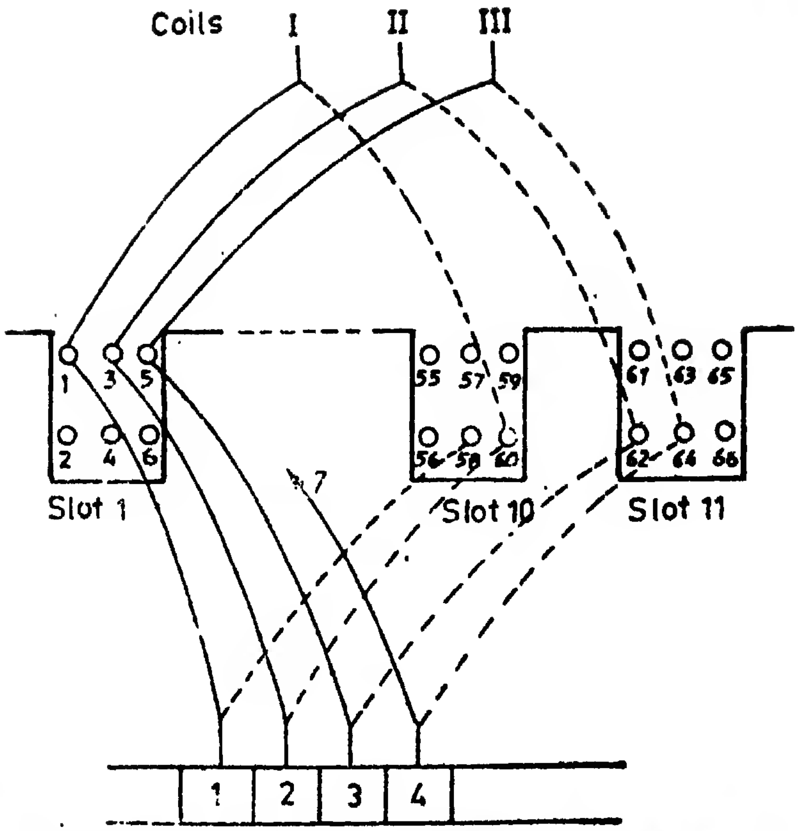


Fig. 6·13. Simplex lap winding with 6 coil sides/slot and having split coils.

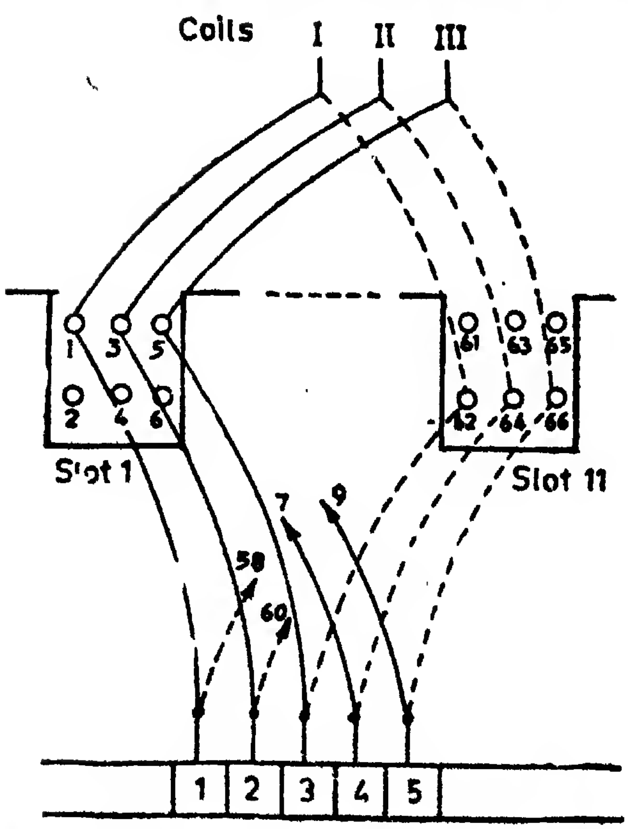


Fig. 6·14. Simplex lap winding with 6 coil sides/slot using unsplit coils.

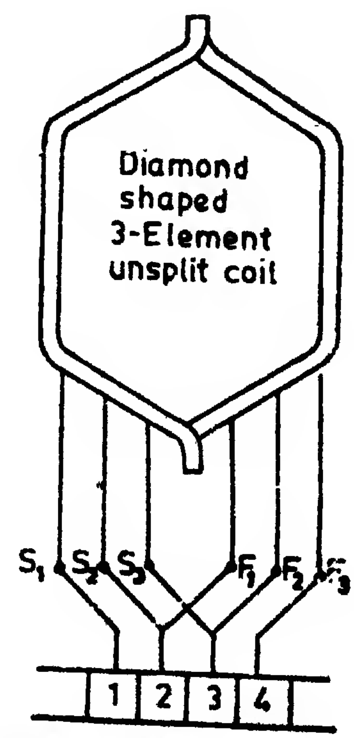


Fig. 6·15. 3-Element coil.

It should be borne in mind that at the front end (commutator end) coil sides belonging to different slots are no doubt connected together but these are merely connections and have nothing to do with practical formation of a coil.

In order to avoid split winding the value of y_b should be so chosen that $\dfrac{y_b-1}{\text{coil sides per slot}}$ =integer which is approximately equal to slots per pole.

6·10. Simplex Wave Winding. In a simplex wave winding as stated earlier, the back pitch y_b is an odd integer and has a value $y_b=(2C/p)\pm K$. In this winding, starting at one commutator segment and tracing the winding from coil to coil, after one trip around the armature (which is completed after $p/2$ coils), we arrive at a commutator segment next to the segment from which the start is made. The segment of arrival may be either ahead or behind the starting segment. The former is known as progressive winding while the latter as retrogressive.

This means that after passing over $p/2$ coils we come back to a coil side 2 coil sides ahead or behind the coil side where we started, giving :

$$\left.\begin{aligned} Y&=\frac{2C+2}{p/2} \text{ for progressive winding} \\ &=\frac{2C-2}{p/2} \text{ for retrogressive winding} \end{aligned}\right\} \quad \ldots(6{\cdot}8)$$

or, in general, for wave winding

$$Y=\frac{2C\pm 2}{p/2} \quad \ldots(6{\cdot}9)$$

The winding pitch Y is an even integer and is equal to (y_b+y_f). It is split up into two equal or approximately equal parts to obtain back and front pitches. The value of back pitch y_b is so selected that it does not give a split winding when the number of coil sides per slot is 4 or more.

In a wave winding, the two ends of a coil are connected to two segments which are approximately two pole pitches apart. Commutator pitch for wave winding is :

$$\left.\begin{aligned} y_c&=\frac{C+1}{p/2} \text{ for progressive winding} \\ &=\frac{C-1}{p/2} \text{ for retrogressive winding} \end{aligned}\right\} \quad \ldots(6{\cdot}10)$$

or in general $$y_c=\frac{C\pm 1}{p/2} \quad \ldots(6{\cdot}11)$$

As $Y_c=Y/2$ and the value of Y is always an even integer, the value of y_c should always be an integer.

Example 6·4. ***Draw the winding diagram in radial form for a 4 pole, 13 slot, simplex wave connected d.c. generator with a commutator having 13 segments. The number of coil sides per slot is 2. Indicate the position of brushes.***

Solution. Number of coils $C=\frac{1}{2}\times 2\times 13=13$

Number of coil sides $=2C=26$

From Eqn. 6·3, back pitch $y_b=\dfrac{2C}{p}\pm K=\dfrac{2\times 13}{4}\pm K=7.$

For progressive windings with commutator pitch

$$y_c=\frac{C\pm 1}{p/2}=\frac{13+1}{4/2}=7.$$

Total winding pitch $Y=2y_c=14$

∴ Back pitch $y_f=y_b-2=7$.

The 13 slots with two coil sides each and the commutator with 13 segments are drawn as shown in Fig. 6·16. The winding table is given below :

Winding Table

1← 8→15←22→ 3←10→17←24→ 5←12→19←26→ 7 →14←21← 2→ 9←16→23← 4→11←18→25← 6→13←20 → 1.

Now slots/pole$=13/4=3\frac{1}{4}$.

Therefore, there are 3 slots each under three poles and 4 slots under the fourth pole. The direction of emfs are then marked. The various connections are made as per the table and also the connections to the various commutator segments are drawn. On inspection it is found that the two coil sides connected to each of the segments 4 and 7 carry current in the same direction and therefore the brushes are placed on these two segments. Starting at

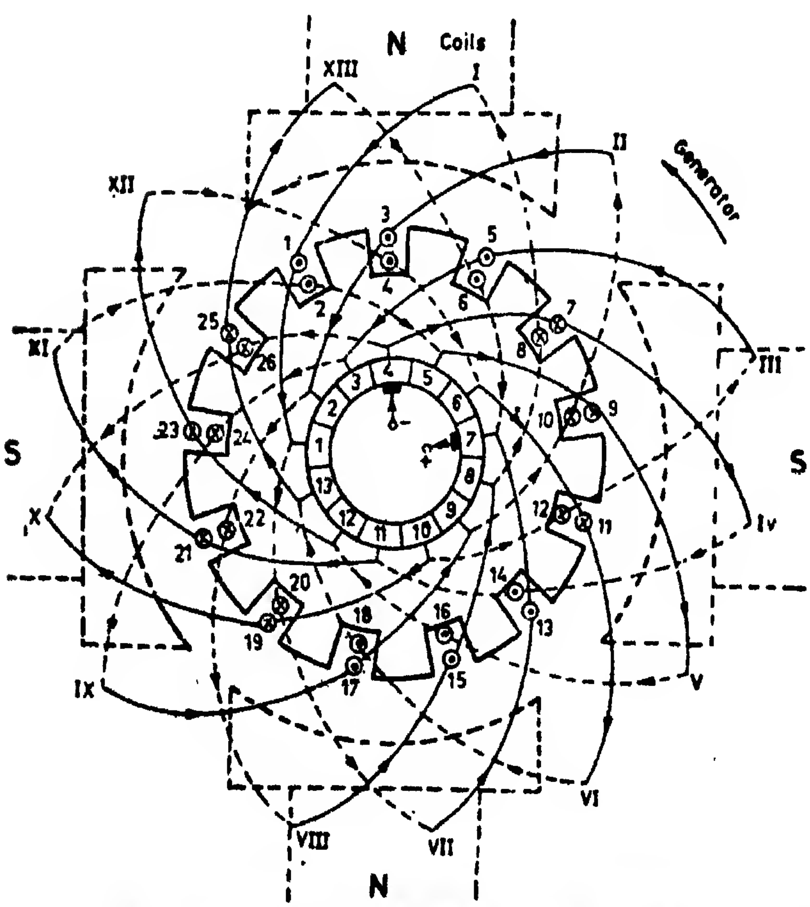


Fig. 6·16. Simplex wave winding of a 4 pole, 13 slot, 13 segment d.c. generator.

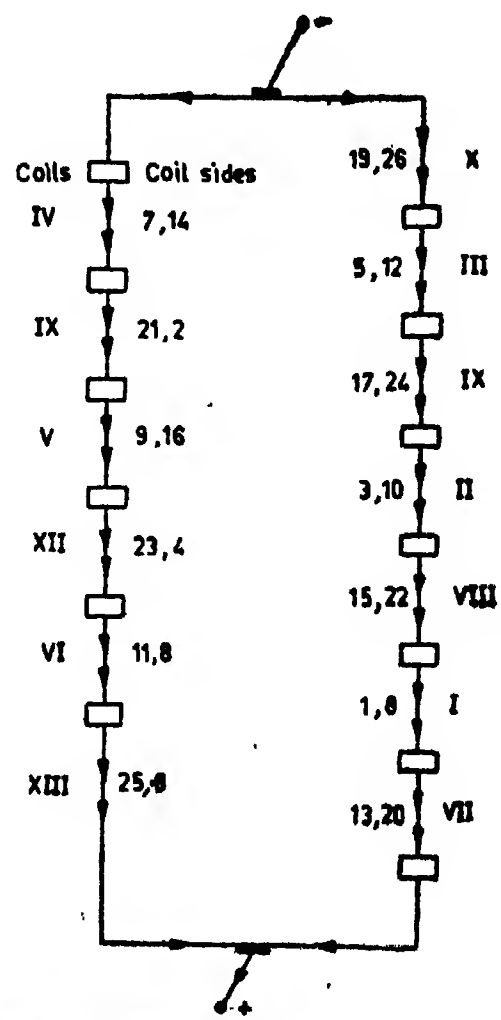


Fig. 6·17. Derived diagram for winding of Example 6·4.

commutator segment 4 and progressing round the winding in clockwise direction a derived diagram of the winding can be drawn as shown in Fig. 6·16. The diagram includes all the

coil sides. Going round the winding, it is found that there are only two reversals of emf, and therefore two parallel paths. This is always true whatever may be the number of poles. The derived diagram is shown in Fig. 6·17.

Example 6·5. *A 4 pole, simplex wave wound armature has 25 slots and 25 coils. The commutator has 25 segments. Work out its winding details.*

Solution. Slots per pole $=25/4=6\frac{1}{4}$. Number of coil sides $=2\times25=50$.

Coil sides per slot $=\dfrac{50}{25}=2$.

From Eqn. 6·9, winding pitch $Y=\dfrac{2C\pm2}{p/2}=\dfrac{2\times25\pm2}{4/2}=26, 24$

Take $Y=26$, Now $Y=y_b+y_f$

This gives $y_b=y_f=13$ (as both y_b and y_f have to be odd integers).

The choice of $y_b=13$, gives the coil span equal to 6 slots which is nearly equal to slots per pole.

Commutator pitch $y_c=\dfrac{25+1}{2}=13$ segments [See Eqn. 6·10 (*a*)]

A winding table is drawn in order to facilitate the winding connections.

Winding Table

$1\longrightarrow14\longrightarrow27\longleftarrow40\longrightarrow3\longleftarrow16\longrightarrow29\longleftarrow42\longrightarrow5\longleftarrow18\longrightarrow31\longleftarrow44$
$\longrightarrow7\longleftarrow20\longrightarrow33\longleftarrow46\longrightarrow9\longleftarrow22\longrightarrow35\longleftarrow48\longrightarrow11\longleftarrow24\longrightarrow37\longleftarrow50$
$\longrightarrow13\longleftarrow26\longrightarrow39\longleftarrow2\longrightarrow15\longleftarrow28\longrightarrow41\longleftarrow4\longrightarrow17\longleftarrow30\longrightarrow43\longleftarrow6$
$\longrightarrow19\longleftarrow32\longrightarrow45\longleftarrow8\longrightarrow21\longrightarrow34\longrightarrow47\longleftarrow10\longrightarrow23\longleftarrow36\longrightarrow49\longleftarrow12$
$\longrightarrow25\longleftarrow38\longrightarrow1.$

There are 6 slots each under 3 poles while there are 7 slots under the fourth pole. Therefore, slots 1—6 and 13—18 are under say north pole and slots 7—12 and 19—25 are under south pole. So coil sides 1—12 and 25—36 are under north pole while coil sides 13—24 and 37—50 are under south pole.

Fig. 6·18 shows the developed winding diagram for progressive wave winding.

Sequence diagram. Fig. 6·19 shows the sequence diagram of the wave winding from which we conclude that it has two parallel paths (irrespective of the number of poles) and has a minimum of two brush sets although it is possible to put as many brush sets as the number of poles.

6·11. Dummy Coils. Sometimes with a certain number of coils it is not possible to satisfy Eqn. 6·11. *i.e.*, $y_c=(2C\pm2)/p$ or $(C\pm1)/p/2$ where y_c should be an integer.

This may happen when a standard lamination has to be used and it becomes necessary to depart from exact symmetry and use a number of slots that does not confirm to the above condition. This involves the use of an extra coil which is not connected to the commutator but is put on the armature to get a mechanical balance. This coil is known as a **dummy coil** as it serves no electrical purpose. The addition of a dummy coil should only be done in relatively small machines for mechanical balance. Also dummy coil should not be used in large machines—its use adversely affects commutation conditions. The windings where dummy coils are used are sometimes called **forced windings.**

The following example illustrates the use of a dummy coil.

Example 6·6. *Work out the winding details of a 4 pole, simplex wave wound d.c. armature having 21 slots with 4 coil sides per slot.*

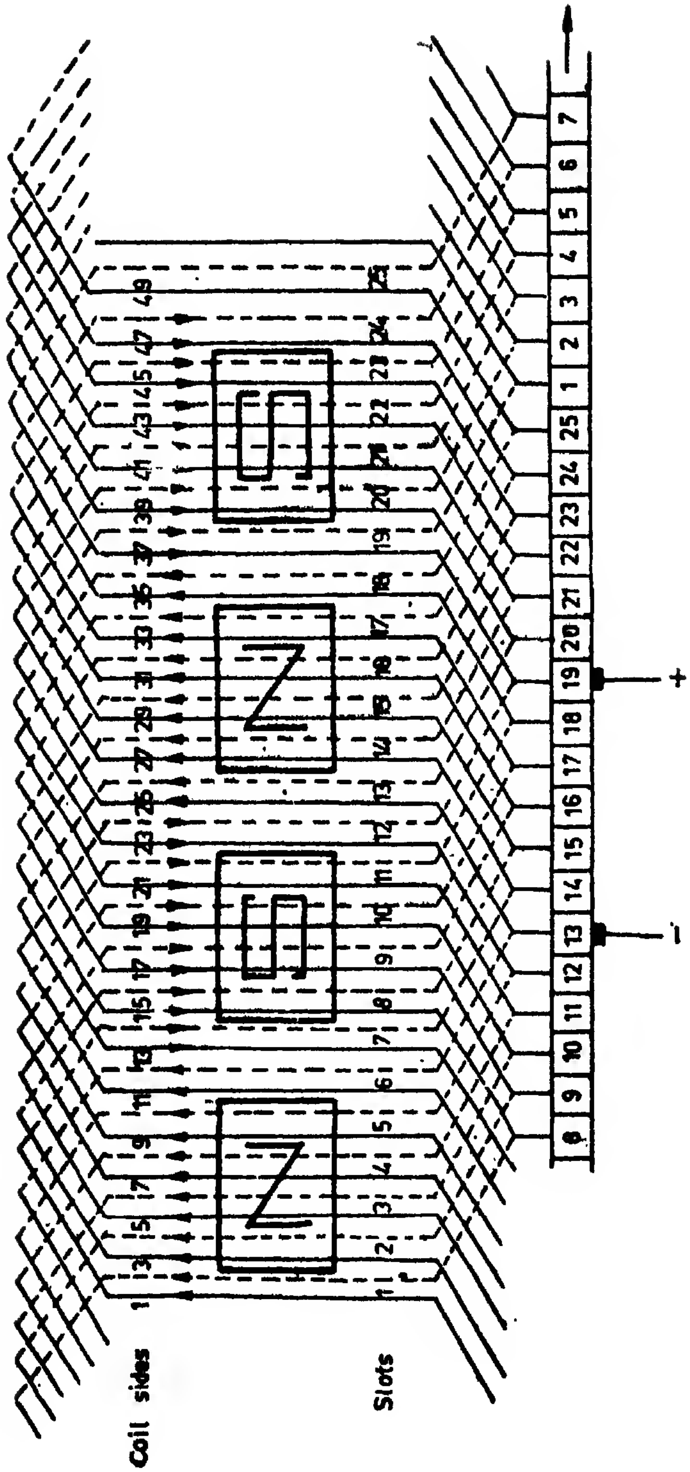


Fig. 6·18. Simplex, Wave Winding

Slot=25 Poles=4

Segments=25 Coil sides 1 slot=2

$Y=26$ $y_b=13$ $y_f=13$

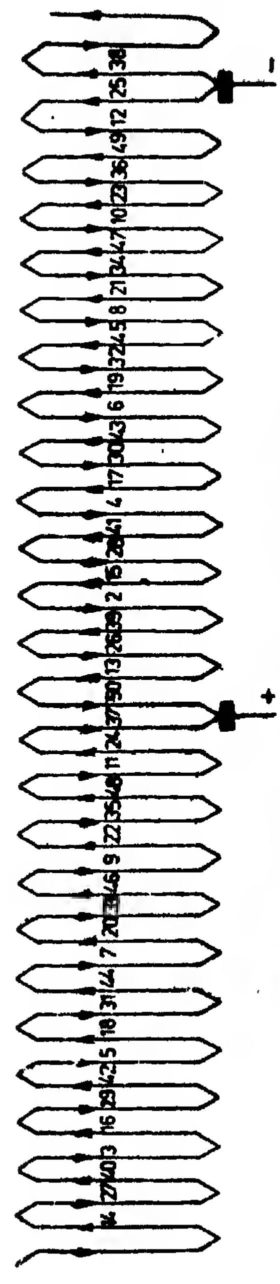


Fig. 6·19. Sequence diagram for winding of Fig. 6·18.

Solution. Total coil sides $=21\times4=84$. Total coils $C=84/2=42$

From Eqn. 6·11, $y_c=\dfrac{C\pm1}{p/2}=\dfrac{42\pm1}{4/2}=20\frac{1}{2}, 21\frac{1}{2}$.

But y_c should be an integer. Therefore it is not possible to use all the 42 coils as active coils in the machine.

The solution lies in making one coil inactive or dummy, therefore, with one coil as a dummy coil, number of active coils$=41$.

With $C=41$, we have $y_c=\dfrac{41\pm1}{4/2}=21$ (for progressive winding). $\therefore$ $Y=2, y_c=42$.

Taking $y_b=21$ and $y_f=21$.

We take $y_b=21$ as we have to satisfy Eqn 6·7.

$\left(i.e.\ \dfrac{y_b-1}{\text{coil sides per slot}}\text{ should be integer to avoid split coils}\right)$.

The winding is drawn by making one of the coils as dummy.

6·12. Equalizer Connections. A simplex lap winding has as many parallel paths as poles, and each parallel path is made up of coil sides which lie under two adjacent poles. If the flux under all poles is equal, the emf induced in all parallel paths will be equal. But in practice, it is possible that the emf induced in various parallel paths may not be equal owing to the following reasons :

(*i*) there may be a difference in the values of reluctance of corresponding iron parts of the magnetic circuit (which may be due to defects in castings and structural irregularities in core materials),

(*ii*) the length of air gap may not be the same under all the poles owing to some defects in machining or in assembling,

(*iii*) the poles may have different strengths owing to error in putting field windings.

The unequal values of emfs generated in different parallel paths give rise to resultant emfs which act across the armature windings. These emfs acting across the local armature circuits produce large circulating currents as the armature resistance is very small. These currents flowing through the brushes, result in considerable inequality of brush arm currents, give rise to I^2R losses in the winding on no load, add to the losses on full load both in the winding and brushes, and also introduce commutation difficulties, causing overheating and sparking. In order to rid the brushes of these circulating currents, **equalizer connections** are used. These equalizer connections or equalizers are low resistance copper conductors which connect those points in the winding which under ideal conditions have no difference of potential between them. If there are differences of potential between such points, as is normally the case, equalization of potential will result from the flow of current through these low resistance conductors which bypass the current. This bypassing of circulating currents relieves the brushes of excessive loading to which some of them would otherwise be subjected.

Consider the winding of Example 6·1 as shown in Fig. 6·9. Pairs of coils like I and VII, II and VIII etc occupy identical magnetic positions with respect to poles of like polarity. Therefore, their emfs rise, fall and reverse in time phase. Thus, they can be connected together in pairs as shown in Fig. 6·10. It is clear that the two points which are to be connected together must be two pole pitches apart. The equalizer connections may be in the form of rings in which case they are known as **equalizer rings**.

The perfect arrangement is to equalize the potential of all the coils but it is not possible in practice as the number of connections becomes very large. It is usual to have 10 to 20 rings, each having as many coils connected to it as the number of pair of poles.

The distance between coils of same potential

$$Y_{eq}=\frac{\text{total number of coils}}{\text{pair of poles}}=\frac{C}{p/2}=\frac{2C}{p}\ \text{coils} \qquad ...(6\cdot12)$$

Y_{eq} is called the equipotential pitch and is expressed in terms of coils.

Total number of tappings $=$ number of rings $\times$ pairs of poles

$$=m\times p/2=mp/2 \qquad ...(6\cdot13)$$

where $m=$ number of equalizer rings.

Distance between adjacent tappings $Y_{ph}=\dfrac{\text{total number of coils}}{\text{total number of taps}}$

$$=\frac{C}{m\times p/2}=\frac{2C}{mp}\ \text{coils.} \qquad ...(6\cdot14)$$

Y_{ph} is called the phase pitch and is expressed in terms of coils.

In order that equalizers may be used, the armature winding must be symmetrical. This requires that both the number of slots and the commutator segments be a multiple of pair of poles.

It is to be noted here that there are no circulating currents in a simplex wave winding even if the magnetic circuits under different poles are not uniform as in this winding the coil sides forming a parallel path are distributed over all the poles and hence all the parallel paths are affected equally by the asymmetry in the magnetic circuit. Thus there is no necessity of providing equalizer connections for a simplex wave winding.

Example 6·7. *Work out an arrangement for equalizer connections for 6 pole, 27 slot, simplex lap wound d.c. generator. There are 4 coil sides per slot. Use 9 equalizer rings. Show a part of the arrangement with help of a simple diagram.*

Solution. No. of coils $C=\frac{1}{2}\times4\times27=54$

Back pitch $y_b=\dfrac{2\times54}{6}\pm K=19$, Front pitch $y_f=19-2=17$

From Eqn. 6·12, equipotential pitch $y_{eq}=\dfrac{2C}{p}=\dfrac{2\times54}{6}=18$ coils

Total number of tappings $=\dfrac{mp}{2}=9\times\dfrac{6}{2}=27$

Distance between adjacent tappings $Y_{ph}=\dfrac{2C}{mp}=\dfrac{54}{27}=2$ coils.

Ring No. Y_{ph}	*I*	*II*	*III*	*IV*	*V*	*VI*	*VII*	*VIII*	*IX*
Coil No. Y_{eq} ↓	1	3	5	7	9	11	13	15	17
	19	21	23	25	27	29	31	33	35
	37	39	41	43	45	47	49	51	53

The winding arrangement is shown in Fig. 6·20. Thus coils 1, 19 and 37 are connected to equalizer I. Coil 1 is located in slots 1 and 5 is formed by coil sides. 1 and 18 while coil 19 is located in slots 10 and 14 and is formed by coil sides 37 and 54.

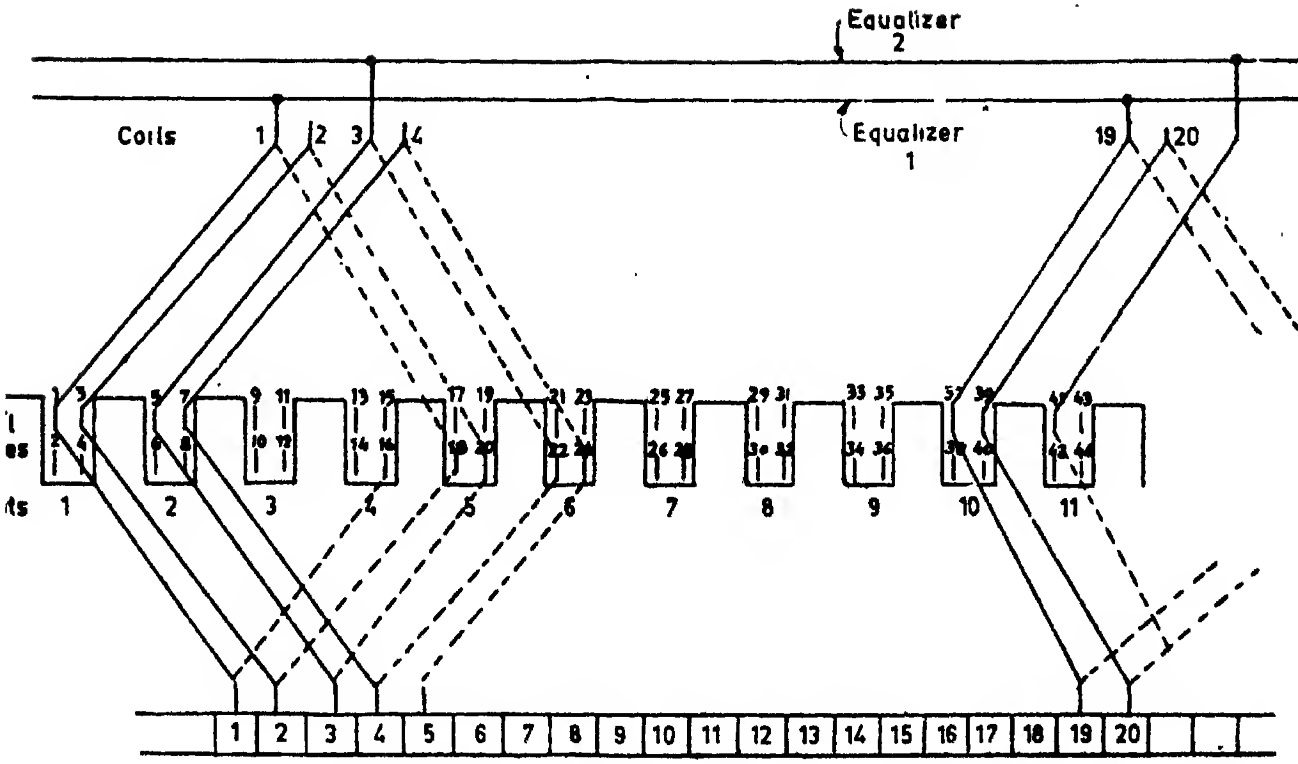


Fig. 6·20. Details of windings and equalizer connections of Example 6·7.

Example 6·8. *Work out an arrangement for equalizer connections for a 8 pole simplex lap wound d.c. machine having 200 coils. The number of equalizer rings is 10.*

Solution. Equipotential pitch, $Y_{es}=\frac{2C}{p}=2\times\frac{200}{8}=50$ coils.

Total number of tappings $=\frac{mp}{2}=10\times\frac{8}{2}=40.$

Distance between adjacent tappings

$$Y_{ph}=\frac{2C}{mp}=\frac{2\times200}{10\times8}=5 \text{ coils.}$$

The layout of tappings is :

Ring No. $Y_{ph}\rightarrow$	I	II	III	IV	V	VI	VII	VIII	IX	X
Coil No. $Y_{es}\downarrow$	1	6	11	16	21	26	31	36	41	46
	51	56	61	66	71	76	81	86	91	96
	101	106	111	116	121	126	131	136	141	146
	151	156	161	166	171	176	181	186	191	196

6·13. Multiplex Windings. The degree of multiplicity of a multiplex winding indicates the relative number of parallel paths with respect to the number of parallel paths in the corresponding simplex winding. For example, a duplex lap or wave winding is a lap or wave winding having twice as many parallel paths as a simplex lap or wave winding respectively. We can define the triplex or quadruplex windings in a similar manner.

6·14. Duplex Lap Windings. Fig 6·21 shows a duplex lap winding in which the commutator pitch y_c is 2. It is apparent that the coil sides in series between adjacent brush sets is half the coil sides per pole ; but if brushes of sufficient thickness to cover two segments are provided, the number of parallel paths becomes twice that in the case of simplex lap winding.

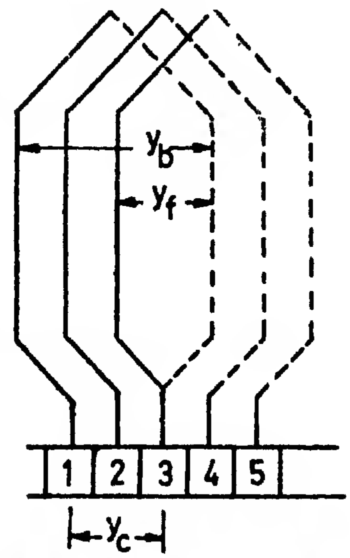


Fig. 6·21. Duplex lap winding.

This leads us to the following equations for duplex lap windings :

(*i*) Commutator pitch $y_c = \pm 2$ ...(6·15)

and winding pitch $Y = 2y_c = \pm 4$ ...(6·16)

(*ii*) $y_b - y_f = +4$ for progressive winding (*a*)...(6·17)

$= -4$ for retrogressive winding (*b*)

Duplex windings may be either singly or doubly re-entrant. The winding is **singly re-entrant**, if after tracing the entire winding the starting point is reached and **doubly re-entrant** if the starting point is reached after tracing half of the winding. With an even number of commutator segments, it is obvious that a closed circuit is formed after tracing half the coils and thus there will be two independent circuits giving a doubly re-entrant winding. With odd number of commutator segments, the winding would close only after tracing all the coils giving a singly re-entrant winding.

Example 6·9. *Give the winding details for a 4 pole, 36 slot, 72 segment d.c. armature with 8 parallel paths.*

Solution. As the number of parallel paths in 8 which is twice the number of poles, the winding is dupluex lap type.

Number of coils $C = 72$,

Coil sides/slot $= \dfrac{2 \times 72}{36} = 4$

Back pitch $y_b = \dfrac{2C}{p} \pm K = \dfrac{144}{4} \pm K = 37,\ 35$

We select $y_b = 37$ as this satisfies the relation

$$\frac{y_b - 1}{\text{coil sides per slot}} \text{ should be an integer.}$$

Now for a progressive duplex lap winding :

$y_c = +2$ and $Y = +4$ $\therefore$ Front pitch $y_f = y_b - 4 = 33$.

As the number of segments is even, the winding is doubly re-entrant.

6·15 Duplex Wave Windings. As in the case of lap windings, it is possible to wind the armature with two independent wave windings. If the commutator pitch y_c is made equal to $\dfrac{C \pm 2}{p/2}$ instead of $\dfrac{C \pm 1}{p/2}$ then, passing around the armature once, in tracing the winding, connection will not be made to a segment adjacent to the one started with, but to one 2 segments away from the starting point.

Wave windings may consist of a singly closed circuit (singly re-entrant) or two entirely separate closed circuits (doubly re-entrant). As in the case of a duplex lap winding, with an even number of commutator segments, there will be two independent windings if the commu-

tator pitch is even. The winding will be singly closed under all other conditions (like the number of commutator segments being odd or the commutator pitch being odd).

$$y_c=\frac{C\pm2}{p/2} \text{ for duplex wave winding.} \quad \ldots(6\cdot18)$$

Example 6·10. *Give the winding details for the following d.c. armatures :*

(*a*) *8 poles, 58 slots, 4 circuits with 58 segments,*

(*b*) *6 poles, 43 slots, 4 circuits with 43 segments.*

Solution. (*a*) The number of parallel paths is 4 while the number of poles is 8 and so the winding is duplex wave type.

Back pitch $y_b=\frac{2\times58}{8}\pm K=15$.

Commutator pitch $y_c=\frac{C\pm2}{p/2}=\frac{58\pm2}{8/2}=15, 14.$

Choosing $y_c=15,\quad Y=2y_c=30.$

$\therefore$ Front pitch $y_f=Y-y_b=30-15=15.$

As y_c is an odd integer, it gives a singly re-entrant duplex wave winding.

If $y_c=14$ is used we get doubly re-entrant winding since y_c is now even and also the number of segments is even.

(*b*) As the number of circuits is 4 while the number of poles is 6, it is a duplex wave winding.

$$y_b=\frac{2\times43}{6}\pm K=15, \qquad y_c=\frac{C\pm2}{p/2}=\frac{43+2}{6/2}=15.$$

$$Y=2y_c=30 \qquad \therefore \quad y_f=30-15=15.$$

As the number of segments is odd, the winding is singly re-entrant.

INTEGRATED APPROACH FOR WINDINGS

6·16. Relations between winding parameters. In the previous few pages, an attempt has been made to familiarize the reader with the basic ideas of d.c. armature windings. Simple approach coupled with examples has been given in order that the reader may grasp the fundamentals. In the next few pages, a general approach to all types of windings is being given. This general approach covers all the details worked out earlier.

In order to analyze the necessary relations between the winding parameters, let :

S=total number of slots on the armature ; p=number of poles ;

C=number of coils=number of segments ;

y_c=commutator pitch in terms of segments ;

C_s=coil span in terms of slots covered ; n=an even integer,

u=coil sides per slot=$2C/S$;

α_s=angle between adjacent slots, electrical radian,

ψ=phase angle between emfs of successive coils in the winding, electrical radian ;

Y_{eq}=equipotential pitch, in coils ; Y_{ph}=phase pitch, in coils.

In general, without regard to the winding, whether lap or wave, the coil span must nearly be equal to slots per pole or

Coil span $C_s \simeq S/p$ slots and is exactly equal to S/p for full pitch coils.

If C_s is less or more than S/p the coils are said to be **fractional pitch or chorded.**

Consider a winding with full pitch coils. If it is assumed that the flux distribution around the armature is sinusoidal, the emfs of individual coil sides are sinusoidal, and the amplitude of emfs can be represented by phasors. Consider that the winding has 2 coil sides per slot as shown in Fig. 6·22. (*a*). The angle between adjacent slots

$\alpha_s = \pi p/S$ electrical radian.

This is also the angle between the emfs generated in coil sides of adjacent slots. Fig. 6·22 (*b*) shows the slot emf stars of the winding, the firm lines represent the emfs generated in top coil sides while the dotted lines represent the emfs induced in bottom coil sides. It is clear that the emf stars for both the layers are identical and therefore it suffices to consider any layer. Fig. 6·22(*c*) represents the emf star of top coil sides only. Because we have eliminated bottom coil sides from the discussion, it becomes simpler to consider coils instead of coil sides. Fig. 6·22 (*d*) represents the emf star of coils of the windings, coil 1 lying in slot 1, coil 2 lying in slot 2 and so on [the numbers marked on the diagram 6·22 (*d*) correspond to coils or slots]. This case, when coil sides per slot $u=2$, is very important for analytical treatment since when $u=4$ (as in Fig. 6·23) all coil sides in the same slot have emfs identical in phase and identical in phase and magnitude and can be reduced in effect to $u=2$. This is on the assumption that the emf in a coil side is determined by the position of its slot in the magnetic field and not by its own position in the slot.

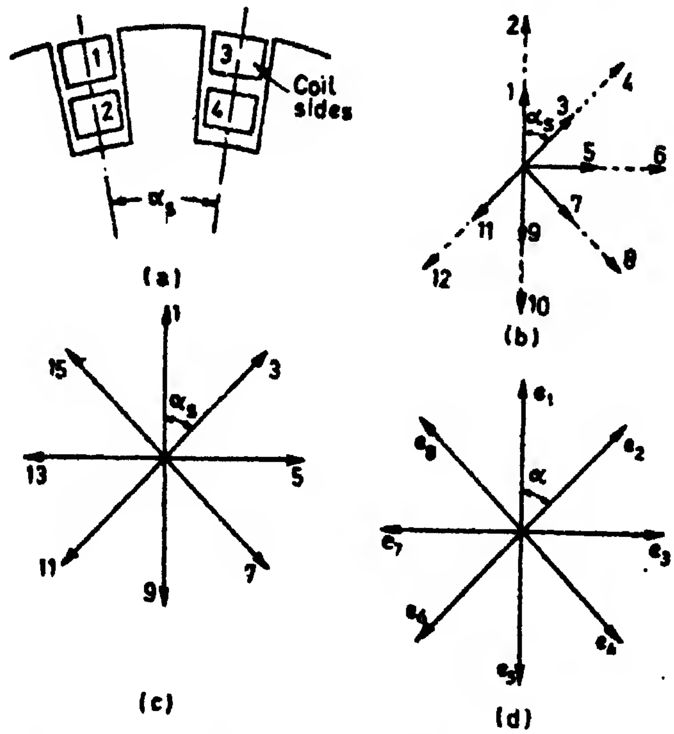


Fig. 6·22. Emf stars.

6·16·1. Conditions for a closed winding. If all the coils of a two layer winding with $u=2$ and $C=S$ are connected electrically in series, no circulating currents should flow if the winding is closed. This requires that there should not be any resultant emf acting around the armature circuit or the resultant of emfs of all the coils should be equal to zero.

Therefore, for a winding to be closed $\quad \Sigma\, e = 0 \quad$ ...(6·19)

Fig. 6·24 shows the emfs of C coils (in S slots, $C=S$), their resultant is zero only if

$$\sum_{1}^{C} \psi = C\psi = 2\pi,\ 4\pi,\ 6\pi \ldots = a\pi \qquad \text{...(6·20)}$$

where a is an even integer 2, 4, 6...etc. Thus for a closed winding with $u=2$ and $C=S$, we have :

$$S\,\alpha_s = \pi p \quad \text{and} \quad C\psi = \pi a.$$

$$\frac{\alpha_s}{\psi} = \frac{p}{a} \quad \text{or} \quad \psi = \frac{a}{p}.\alpha_s \qquad \text{.(6·24)}$$

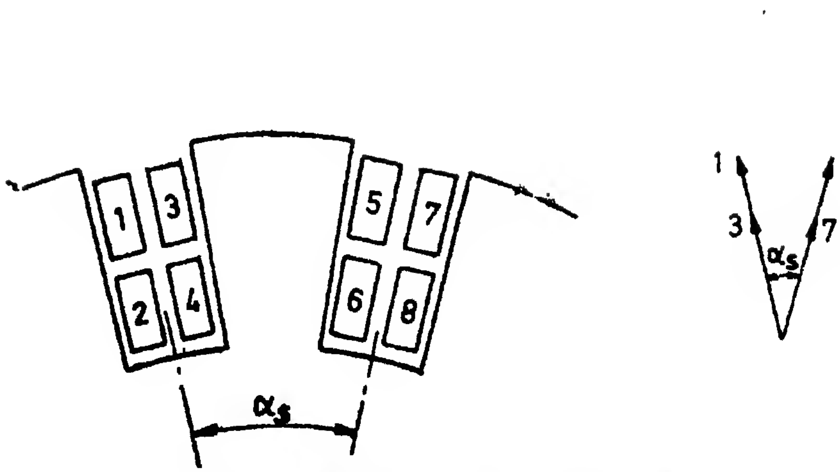


Fig. 6·23. Four coil sides/slot and their emf star.

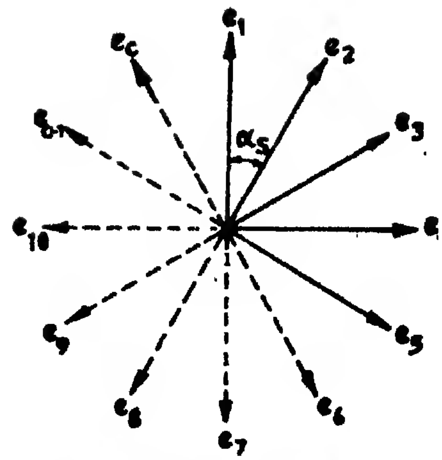


Fig. 6·24. Emf star for a closed winding.

6·16·2. Similar parts and number of parallel paths in a winding. Suppose for a winding, the summation $\sum_{1}^{C} \psi = C\psi = 2\pi$.

Thus $\sum_{1}^{C} e = 0$ as a phase shift of 2π is equivalent to 0.

Therefore, this winding can be closed. Now the brushes of opposite polarity are placed π radians apart and, therefore, this winding has 2 parallel paths between the brushes.

Suppose for a winding, the summation $\sum_{1}^{C} \psi = C\psi = 4\pi$.

This winding can be split up into 2 independent parts, each of which can be closed without allowing circulating currents of fundamental frequency to flow, as for this winding,

$$\sum_{1}^{C/2} \psi = \frac{C\psi}{2} = \frac{4\pi}{2} = 2\pi \text{ and, therefore } \sum_{1}^{C/2} e = 0.$$

Now, the brushes of opposite polarity are placed π radians apart and therefore each part consists of two parallel paths. Thus there are in all four possible parallel paths. This is true only when it is possible to divide the winding into two identical parts. These two parts, each consisting of two parallel paths, can be connected in parallel to increase the current rating of the machine.

The above results can be generalised for any winding having

$$\sum_{1}^{C} \psi = C\psi = a\pi \text{ (where } a \text{ is an integer).}$$

In this winding number of similar parts $= a/2$ and number of parallel paths $= a$.

This is upon the condition that the winding can be divided into $a/2$ identical parts each of which is capable of being closed.

Suppose 'a' is the h.c.f. of $2S$ and p such that $S' = \frac{2S}{a} = \frac{S}{a/2}$

and $$p' = \frac{p}{a} = \frac{p/2}{a/2}$$

are both integers, the winding can be divided into $a/2$ identical parts, having S' slots over p' pole pairs.

As $$S'\psi = \frac{S\psi}{a/2} = \frac{a\pi}{a/2} = 2\pi$$

and therefore $$\sum_{1}^{S'} e = 0$$

and $$S'\alpha_s = \frac{S\alpha_s}{a/2} = \frac{\pi p}{a/2} = 2\pi p' \qquad \ldots(6{\cdot}25)$$

Therefore if 'a' is the h.c.f. of $2S$ and p, the winding will have $a/2$ identical parts (each having two parallel paths) each being closed on itself without any flow of circulating currents of fundamental frequency. These '$a/2$' parts can be connected in series to increase the emf or they may be connected in parallel to increase the current rating of machines.

Example 6·11. *Find the number of identical parts and number of parallel paths in a 2 layer d.c. armature winding with 6 poles, 32 slots and 2 coil sides/slot.*

Solution. Number of slots $S=32$, number of coils $C=\frac{1}{2}\times 2\times 32=32$

Now h.c.f. of $2S$ and p is, $a=2$.

Therefore, the winding can have only $a/2=1$ part capable of being closed.

$\therefore$ Number of parallel paths $a=2$.

Example 6·12. *Find the number of parallel paths and identical parts in a two layer 24 slot, 4 pole d.c. armature with 2 coil sides/slot.*

Solution. Number of coils $C=24$, number of slots $S=24$.

Now h.c.f. of $2\,S$ and p is $a=4$.

Therefore there are $a/2=2$ identical parts each capable of being closed. Each part has 2 parallel paths and so the total number of parallel paths is $a=4$. This winding can be divided into two parts only when it is possible to have two identical parts.

The conditions for above are $S'=2S/a$ and $p'=p/a$ should both be integers:

For this winding, $S'=\frac{2\times 24}{4}=12$, $p'=\frac{4}{4}=1$ are both integers.

Thus it is possible to have 2 identical parts each having $S'=12$ slots in $p'=1$ pole pair.

6·16·3. Equipotential Pitch. In a winding having $a/2$ identical parts, it is possible to find $a/2$ points having the same potential at every instant, and therefore capable of being joined to common terminals by tappings to equalizer connections or commutator segments or slip rings. Such equipotential points are $S'=\frac{S}{a/2}=\frac{2S}{a}$ slots or coils apart (for $C=S$).

Therefore, equipotential pitch is

$$Y_{eq}=S=2S/a=2C/a \qquad \ldots(6{\cdot}26)$$

(Compare with Eqn. 6.12, for simplex lap winding, where $a=p$)

6·16·4. Phase Pitch. In order that the winding may be divided into symmetrical parts to be connected to m outside terminals (m equalizer rings in case of d.c. machines or m phases in a.c. machines) it is necessary to divide each of $a/2$ parts into m equal parts.

Therefore, number of tappings $= m\,a/2$ ...(6·27)

and phase pitch $Y_{ph} = \dfrac{\text{slots or coils}}{\text{total number of tappings}} = \dfrac{S}{ma/2} = \dfrac{2C}{ma}$ ...(6·28)

(Compare with Eqn. 6·14 for a simplex lap winding, where $a=p$)

Example 6·13. *A 4 pole, d.c. machine has 44 slots. It is to be provided with 2 equalizer rings. Find the number of taps and their arrangement. There are two coil sides per slot.*

Solution. As there are two coil sides per slot, number of coils $C=S=44$.
H.C.F. of $2S$ and p is $a=4$. Therefore, there are $a/2=2$ identical parts each having 2 parallel paths each.

As $S'=\dfrac{2S}{a}=22$ and $p'=\dfrac{p}{a}=1$ are both integers, it is possible to divide the winding into two identical parts.

From Eqn. 6·26, equipotential pitch $Y_{eq}=\dfrac{2\times 44}{4}=22$ coils.

Number of rings $m=2$ $\quad\therefore\quad$ Total number of tappings $=2\times 2=4$

$\therefore$ Phase pitch $Y_{ph}=\dfrac{2\times 44}{2\times 4}=11$ coils. (See Eqn. 6·28)

The arrangement of tappings is :

Ring No. Y_{ph} $\longrightarrow$	*I*	*II*
Coil No. $\downarrow$	1	12
Y_{eq}	23	34

6·16·5. Conditions of Electrical Symmetry. The conditions for symmetry for two layer closed windings are :

(1) If the winding is divided into $a/2$ identical parts :

(*i*) $2S/a$ should be an integer, and (*ii*) p/a should be an integer.

(2) If all the coils are electrically connected and there are no dummy coils $C=\frac{1}{2}uS$.

(3) If the winding is provided with tappings connected with m terminals, and in order that the winding between these m terminals be symmetrical, *i.e.* $Y_{ph}=2C/ma$ should be an integer.

This is the condition for m phase symmetry in a.c. machines.

6·16·6. Lap and Wave windings. We know that y_c is the commutator pitch in terms of commutator segments or number of coils between the starts of two electrically successive coils. If β (Fig.6·24) is the angle between two electrically successive coil, $\beta=y_c\,\alpha_s$

or $\quad y_c=\dfrac{\beta}{\alpha_s}$ ...(6·28)

Now $\quad \alpha_s=\dfrac{p\psi}{a}$ (from Eqn. 6·24)

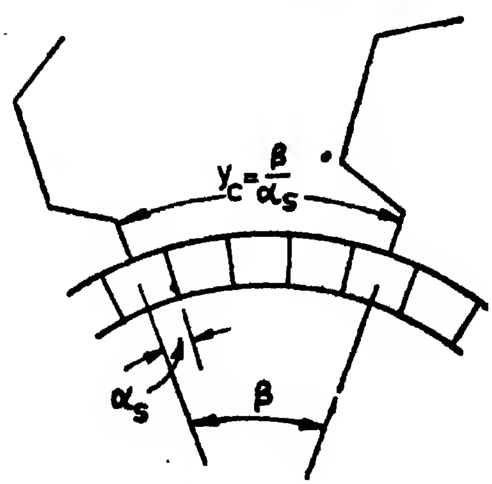


Fig. 6·24.

$$\therefore \quad y_c = \frac{\beta}{\psi} \cdot \frac{a}{p} \qquad \ldots(6\cdot29)$$

1. **Lap Windings.** Fig. 6·25 shows a lap winding.

For this winding, the angle between emfs of electrically successive coils is equal to physical displacement of the coils or $\psi=\beta$.

So, from Eqn. 6·28, $y_c=\dfrac{a}{p}$

$$\text{or in general } y_c=\pm\frac{a}{p} \qquad \ldots(6\cdot30)$$

The value of y_c has to be an integer and thus :

$$\frac{a}{p}=\text{integer}=1, 2, 3\ldots\ldots\text{etc.}$$

(*i*) For $y_c=1$, we get $a=p$

or number of parallel paths=number of poles.

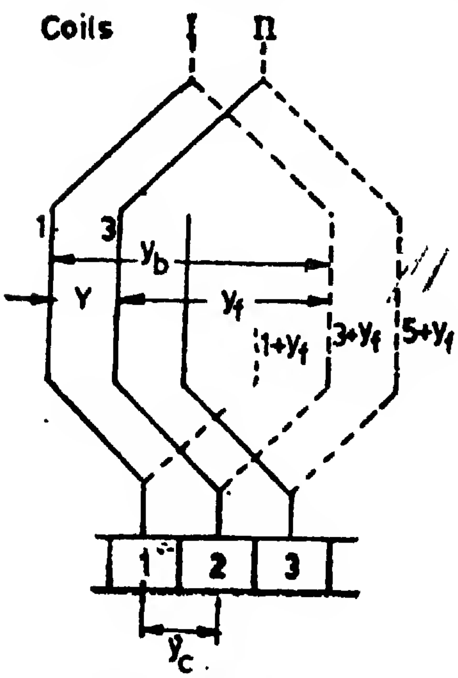


Fig. 6·25. Lap Winding.

This condition is obtained in the simplex lap winding as described earlier. Applying conditions of symmetry $p/a=1$, integer and if $2S/a$ is also an integer, the winding is symmetrical. As $a=p$, therefore for symmetry $2S/p$ should be an integer or the number of slots should be a multiple of pair of poles.

(*ii*) For $y_c=2$, we have, $a=2p$

or number of parallel paths=twice the number of poles.

This is the duplex lap winding.

Applying the conditions for symmetry $p/a=\frac{1}{2}$, which is not an integer.

Thus a duplex lap winding is never symmetrical.

Example 6·14. ***Find whether the following simplex lap windings are symmetrical or not :***

(*i*) ***4 poles, 26 slots, 26 coils.***

(*ii*) ***6 poles, 37 slots, 37 coils.***

Solution. (*a*) H.C.F. of $2S=52$ and $p=4$, is 4.

$\therefore$ $a=4$. Hence, there are four parallel paths.

Number of poles=number of parallel paths=4.

Applying conditions for symmetry

$$\frac{p}{a}=1 \text{ and } \frac{2S}{p}=13 \text{ are both integers.}$$

So the winding is symmetrical.

Thus if the number of slots is a multiple of pair of poles, we get a symmetrical simplex lap winding.

(*b*) H.C.F. of $2S=74$ and $p=6$, is 2. $\quad\therefore\quad a=2.$

It can give a symmetrical winding for $a=2$. But for a simplex lap winding $a=p$. The number of poles is 6 while symmetrical winding is possible with $a=2$.

Thus, it is not possible to have symmetrical simplex lap winding with this armature.

2. **Wave Windings.** The angle β between electrically successive coils cannot be exactly made equal to 2π because in that case the winding would close on itself, after connecting $p/2$ coils in series. Therefore, β must either be made slightly more or less than 2π (Fig. 6·26). A divergence of angle ψ from 2π will make successive emfs differ in phase by an angle ψ.

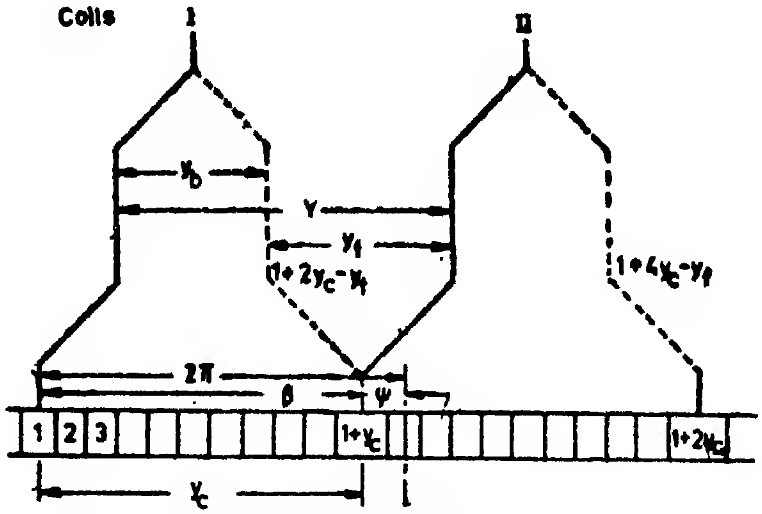


Fig. 6·26. Wave winding.

$\therefore \qquad \beta=2\pi\pm\psi.$

From Eqns. 6·28 and 6·29,

$$y_c=\frac{\beta}{\alpha_s}=\frac{\beta}{\psi}\cdot\frac{a}{p}=\frac{2\pi\pm\psi}{\psi}\cdot\frac{a}{p}=\frac{a}{p}\cdot\frac{2\pi}{\psi}\pm\frac{a}{p}.$$

Now $\psi=\frac{a}{p}\ \alpha_s$ and $\alpha_s=\frac{\pi p}{C}$ $\qquad\therefore\qquad \psi=\frac{\pi a}{C}$.

Hence $y_c=\frac{a}{p}\cdot\frac{2\pi}{\pi a}\pm\frac{a}{p}=\frac{2C\pm a}{p}$, where y_c is an integer.

Re-writing the above relationship as $a=py_c\pm 2C$.

Now p and y_c are both integers and p is always an even integer and therefore their product py_c is always an even integer. Also $2C$ is always an even integer (C being an integer). Hence 'a' $(=py_c\pm 2C)$ is always an even integer and its value depends upon the choice of y_c. From above, it is clear that we can have any even number of parallel paths irrespective of the number of poles.

(*i*) **Simplex wave winding.** As stated earlier, the value of parallel paths depends upon the choice of y_c. Thus, it is possible to have two parallel paths irrespective of number of poles. The winding with $a=2$, is a simplex wave winding. For such a winding :

$$y_c=\frac{2C\pm 2}{p}.$$

From the above expression it is clear that it is not possible to choose any number of coils as y_c has to be an integer. Thus, there is a fixed number of coils which can be used for a wave winding (depending, of course, on the value of p and y_c).

As the number of parallel paths is $a=2$ and thus there is only one part in the winding capable of being closed. Therefore, if a simplex wave winding satisfies the above relationship, it is always symmetrical as in this case, there will not be any dummy coil, also p/a is always an integer ('a' being equal to 2 and p being always an even integer).

(*ii*) **Duplex Wave Winding**. In this winding the number of parallel paths is equal to 4. In order that there may not be any dummy coil, it must satisfy the equation:

$$y_c=\frac{2C\pm4}{p}$$

where y_c is an integer.

The windings with no dummy coil will be symmetrical if the conditions

(*i*) $2S/a$=integer and (*ii*) p/a=integer are both satisfied.

Now, for duplex wave windings 'a' is always equal to 4 and, therefore, for symmetry the number of slots should always be a multiple of 2 and the number of poles should always be a multiple of 4. Thus, a duplex wave winding with odd number of pole pairs can never be symmetrical.

Example 6·15. *Find out whether the following windings are symmetrical or not:*

(*a*) *6 pole, 37 slot, 2 coil sides per slot, simplex wave winding.*

(*b*) *4 pole, 30 slot, 2 coil sides per slot, simplex wave winding.*

(*c*) *8 pole, 126 slot, 6 coil sides per slot, duplex wave winding.*

(*d*) *10 pole, 81 slot, 6 coil sides per slot, duplex wave winding.*

Solution. (*a*) Number of coils $C=\frac{1}{2}\,uS=\frac{1}{2}\times2\times37=37$.

$$y_c=\frac{2C\pm2}{p}=\frac{2C\times37\pm2}{6}=12\text{, an integer.}$$

Thus, there are no dummy coils. Also $p/a=3$ (integer) as $a=2$.

Therefore, it is a symmetrical winding.

(*b*) Number of coils $C=\frac{1}{2}\,uS=30$.

$$y_c=\frac{2\times30\pm2}{4}\text{, not an integer.}$$

So a simplex wave winding is not possible.

Making one coil as dummy, $C=29$. $\therefore\ y_c=\dfrac{2\times29\pm2}{4}=15,\ 14$.

Thus, it is possible to have a wave winding with 29 coils but it is unsymmetrical owing to the use of one dummy coil.

(*c*) Number of coils $C=\frac{1}{2}\times6\times126=378$.

For a duplex wave winding, $y_c=\dfrac{2\times378\pm4}{8}=95,\ 94$ (integers)

$\therefore$ No dummy coils are used.

For duplex wave winding $a=4$.

Applying conditions of symmetry, $2S/a=2\times126/4=53$ (integer)

and $p/a=8/4=2$ (integer).

Therefore, the winding is symmetrical.

(*d*) Number of coils $C=\frac{1}{2}\times6\times81=243$.

For duplex wave winding, $y_c=\dfrac{2\times243\pm4}{10}=49$ (integer).

Therefore, no dummy coils are used.

Applying conditions of symmetry : $p/a=10/4=2\frac{1}{2}$, not an integer.

Therefore, the winding is not symmetrical.

Example 6·16. (*a*) *If there are u coil sides per slot, show that the number of last coil side in nth slot is (un) and of the first coil side is (un—u+1).*

(*b*) *Show the position in the slot and the number of slots in which the following coil sides are located :*

(*i*) *coil side 53 when u=4,* (*ii*) *coil side 195 when u=10.*

Solution. Referring to Fig. 6·27. Consider slot No. 1 :

1st coil side No.=1. Last coil side No.=u.

Consider slot No. 2 :

1st coil side No.$=u+1=(2-1)u+1$. Last coil side No$=2u$.

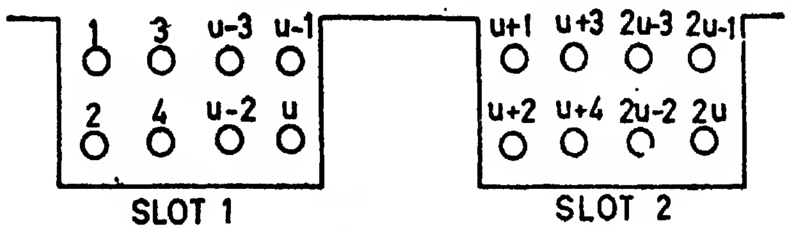


Fig. 6·27.

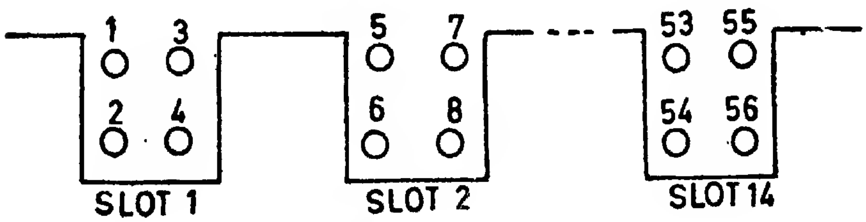


Fig. 6·28.

Similarly, for nth slot :

1st coil side No.$=(n-1)u+1=(un-u+1)$. Last coil side No$=(un)$

(*b*) Considering a coil side No. A,

Let $A=ux+y$

Then A is yth coil side in slot No. $(x+1)$

(*i*) $A=53$ and $u=4$. $\therefore$ $A=53=13\times4+1$

or coil side 53 lies in $(13+1)=14$th slot and it is the 1st coil side there. (Fig. 6·28).

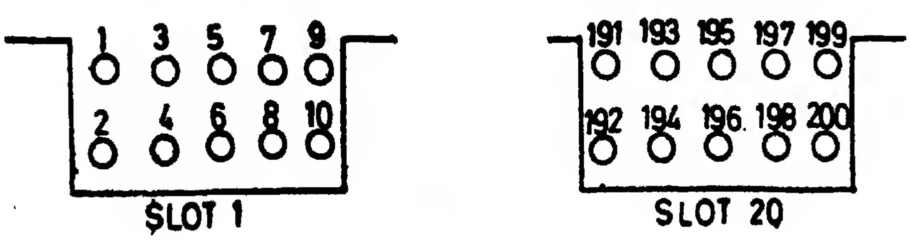


Fig. 6·29.

(ii) $A=195$, and $u=10$. $\therefore$ $A=195=19\times10+5$.

Thus, it lies in $(19+1)=20$th slot and is the fifth coil side there. (Fig. 6·29).

Example 6·17. *A 6 pole, 72 slot d.c. armature has 6 coils sides per slot. It is to be provided with a simplex lap winding having 8 equalizer rings. Find the number of commutator segments and the winding pitches for full pitch coils. Work out the arrangement for equalizers and find whether the winding is symmetrical or not.*

Solution. Number of slots $S=72$. Coil sides per slot $u=6$.

$\therefore$ Number of coils $C=\frac{1}{2}uS=\frac{1}{2}\times6\times72=216$.

Number of parallel paths $a=p=6$.

Number of commutator segments $=C=216$.

For full pitch coils, coil span $C_s=\frac{S}{p}=\frac{72}{6}=12$ slots.

Thus if the top coil side lies in slot 1, the bottom coil side should lie in slot $1+12=13$.

$$y_b=\frac{2C}{p}\pm K=\frac{2\times216}{6}\pm1=71,\ 73.$$

If we choose $y_b=71$, coil side 1 which lies in slot 1 is connected to coil side 72 which lies in slot 12 ; thus the coil span becomes 11 slots. This gives fractional pitch coils while if we choose $y_b=73$, coil side 1 lying in slot 1 is connected to coil side 74 lying in slot 13. This gives full pitch coils. Also with $y_b=73$, the relation $(y_b-1)/u=$integer is also satisfied and thus we do not get split winding.

Choosing progressive winding, $y_c=+1$ and $Y=+2$, $y_f=y_b-2=71$.

Equipotential pitch $Y_{eq}=\frac{2C}{p}=\frac{2\times216}{6}=72$ coils.

Total number of tappings $=\frac{mp}{2}=\frac{8\times6}{2}=24$.

Phase pitch $Y_{ph}=\frac{2C}{mp}=\frac{216}{24}=9$ coils.

Thus the arrangement of equalizer rings is :

Ring No.	$Y_{ph}\rightarrow$	*I*	*II*	*III*	*IV*	*V*	*VI*	*VII*	*VIII*
		1	10	19	28	37	46	55	64
Coil No.	$Y_{eq}\downarrow$	73	82	91	100	109	118	127	136
		145	154	163	172	181	190	199	208

Applying the conditions for symmetry,

(i) $\frac{2S}{a}=\frac{2\times72}{6}=24$, and (ii) $\frac{p}{a}=\frac{6}{6}=1$ are both integers. Also there are no dummy coils. Therefore, the winding is symmetrical.

Example 6·18. *The armature core of a 4 pole d.c. machine has 31 slots, each designed to accommodate 4 coil sides of a simplex wave winding. The winding has a total of 496 conductors. Find the total number of coils, turns per coil, commutator segments, and back, front and total pitches.*

Solution. Total number of coils $C=\frac{1}{2}\times4\times31=62$.

Total turns $=496/2=248$. Turns per coil$=248/62=4$.

Now $y_c=\dfrac{C\pm1}{p/2}=\dfrac{62\pm1}{4/2}$ is not an integer.

Thus it is not possible to connect all the 62 coils.

Use one dummy coil. $\therefore$ Number of active coils $C=61$

$\therefore$ $y_c=\dfrac{61\pm1}{4/2}=31,\ 30.$ Take $y_c=30$.

Back pitch $y_b=\dfrac{2C}{p}\pm K=\dfrac{2\times61}{4}\pm K=31,\ 29.$

If we choose $y_b=31$,

$\dfrac{y_b-1}{u}$ is not an integer and therefore this results in split coils.

Take $y_b=29$ as this does not give split coils.

Now $y_c=30$, and $Y=60$. $\therefore$ $y_f=Y-y_b=60-29=31$.

The winding is retrogressive.

Now coil side 1 in slot 1 is connected at the back to coil side 30 in slot 8 and therefore the coil span is 7 slots.

Number of commutator segments$=$number of active coils$=61$.

Example 6·19. *A 4 pole, 2 circuit armature is to develop an emf of 250 V at 550 r.p.m. with a flux of 0·04 Wb. There are 171 commutator segments and the armature has 57 slots. Find the number of armature conductors, turns per coil, coil sides per slot, winding pitches and coil span.*

Solution. This is a simplex wave winding.

Number of armature conductors $Z=\dfrac{Ea}{\Phi np}=\dfrac{250\times2}{0{\cdot}04\times(550/60)\times4}=341.$

Turns per coil $T_c=\dfrac{Z}{2C}=\dfrac{341}{2\times171}\simeq1.$

Actual number of armature conductors $Z=2\times1\times171=342$.

Coil sides $=2C=2\times171=342.$

Coil sides per slot $u=\dfrac{342}{57}=6.$

For a simplex wave winding, commutator pitch $y_c=\dfrac{C\pm1}{p/2}=\dfrac{171\pm1}{4/2}=86,\ 85.$

Taking $y_c=85$, $\therefore$ $Y=2y_c=170$, and $y_b=y_f=85$.

We take $y_b=85$ as this does not give split coils. With $y_b=85$, coil side 1 lying in slot 1 is connected to coil side 86 lying in slot 15. Therefore coil span is 14 slots.

Example 6·20. *The number of conductors in each circuit of a 8 pole simplex lap winding is to be within 5 per cent of 98, the number of slots per pole must be between 12 and 18. Give details of a suitable winding with about 8 equaliser rings.*

Solution. Assume number of equalizer rings $m=8$.

Number of tappings$=mp/2=8\times 8/2=32$.

Therefore the number of coils should be a multiple of 32 in order to have symmetrical equalizer connections. Also the number of slots and coil sides per slot, must be so chosen that the number of coils is a multiple of 32.

(*i*) Taking 16 slots per pole and 4 coil sides per slot.

Total number of slots$=16\times 8=128$.

Total number of coils$=\frac{1}{2}uS=\frac{1}{2}\times 4\times 128=256$.

This is a multiple of 32 (number of taps) and therefore symmetrical equalizer connections are possible.

Assuming number of conductors per parallel path to be 98, total number of conductors $Z=8\times 98=784$.

Using single turn coils.

Number of conductors$=2\times 256=512$.

This number is not within 5 per cent of 784. Therefore it is not possible to use single turn coils.

Using 2 turn coils.

Number of conductors$=2\times 2\times 256=1024$.

This is also not possible as 1024 is not within 5 per cent of 784.

Therefore it is not possible to use 2 turn coils.

Thus it is not possible to wind the machine with 16 slots per pole and 4 coil sides per slot.

(*ii*) Taking 16 slots per pole and 6 coil sides per slot.
Total number of slots $S=8\times 16=128$.
Total number of coils $C=\frac{1}{2}\times 6\times 128=384$.
This is a multiple of 32 and so symmetrical equalizer connections are possible.

Using single turns coils.

Total number of conductors$=2\times 384=768$.

This is within 5 per cent of 784 and thus it is possible to wind this machine with : slots$=128$, coils$=384$ and coil sides per slot$=6$.

Winding details are :

$$y_b=\frac{2C}{p}\pm K=\frac{2\times 384}{8}\pm K=97,\ 95.$$

Take $y_b=97$ as with this we do not get split coils. $\therefore\ y_f=97-2=95$.

Equipotential pitch $Y_{eq}=\frac{2\times 384}{8}=96$ coils and phase pitch $Y_{ph}=\frac{384}{32}=12$ coils.

Details of equalizer connections are :

Ring No. taps →	*I*	*II*	*III*	*IV*	*V*	*VI*	*VII*	*VIII*
Coil No. Y_{eq} ↓	1	13	25	37	49	61	73	85
	97	109	121	133	145	157	169	181
	193	205	217	229	241	253	265	277
	289	301	313	325	337	349	361	373

Example 6·21. *Find the possible windings for a 4 pole, 2 circuit armature. It is possible to punch any number of slots between 39 and 45. Commutators with segments between 122 and 130 are available.*

Solution. The winding to be used is a simplex wave winding.

The problem is solved by elimination of the unwanted winding arrangements.

1st Step. (*i*) Assuming 2 coil sides per slot.

Number of coils = number of segments. Therefore number of segments is between :

$$\tfrac{1}{2}\times2\times39=39 \text{ and } \tfrac{1}{2}\times2\times45=45.$$

(*ii*) With 4 coil sides per slot, number of segments is between :

$$\tfrac{1}{2}\times4\times39=78 \text{ and } \tfrac{1}{2}\times4\times45=90.$$

(*iii*) With 6 coil sides per slot, number of segments is between :

$$\tfrac{1}{2}\times6\times39=107 \text{ and } \tfrac{1}{2}\times6\times45=135.$$

(*iv*) With 8 coil sides per slot, number of segments is between :

$$\tfrac{1}{2}\times8\times39=156 \text{ and } \tfrac{1}{2}\times8\times45=180.$$

Therefore the number of segments given by 6 coil sides per slot are within the range of commutators given. Hence 6 coil sides per slot are used.

2nd Step. With 6 coil sides per slot,

39 slots give $\tfrac{1}{2}\times6\times39=117$ segments, 40 slots give $\tfrac{1}{2}\times6\times40=120$ segments,

41 slots give $\tfrac{1}{2}\times6\times41=123$ segments, 42 slots give $\tfrac{1}{2}\times6\times42=126$ segments,

43 slots give $\tfrac{1}{2}\times6\times43=129$ segments, 44 slots give $\tfrac{1}{2}\times6\times44=132$ segments,

45 slots give $\tfrac{1}{2}\times6\times45=135$ segments.

Now number of segments given are to be between 122 and 130. Therefore, slots 39, 40, 44 and 45 are eliminated. The remaining slots are 41, 42 and 43.

3rd Step. (*i*) With slots $S=41$ we have, coils $C=123$.

Commutator pitch $\quad y_c=\dfrac{2C\pm2}{p}=\dfrac{2\times123\pm2}{4}=62, 61$ (integers).

Therefore, 2 circuit winding is possible with slots $S=41$ and segments, $C=123$.

(*ii*) With slots $S=49$, we have, coils $C=126$.

Commutator pitch $\quad y_c=\dfrac{2\times126\pm2}{4}$ is not an integer.

Thus winding is not possible with $S=42$ and $C=126$.

(*iii*) With slots $S=43$ we have, coils $C=129$.

Commutator pitch $\quad y_c=\dfrac{129\times2\pm2}{4}=65, 64$ (integers).

Therefore winding is possible with slots $S=43$ and segments $C=129$.

Now the two possible arrangements are :

(*i*) slots = 41 and segments = 123. (*ii*) slots = 43 and segments = 129.

A.C. ARMATURE WINDINGS

6·17. Introduction. This book deals primarily with design of 3 phase induction and synchronous machines and as such three phase windings are extensively dealt with. The

emphasis is mainly on three phase windings though a brief reference to features of a general phase 'm' winding are also given at places. Windings for single phase machines are dealt with in chapter on single phase induction motors.

6·17·1. Number of Phases and Phase Spread. An a.c. winding, meant to be used for a 'm' phase system, should produce emfs of equal magnitude in all the phases. These emfs should have identical wave forms and equal frequency. Their displacement in time should be $\gamma=2\pi/m$ electrical radians. This is obtained by having similar pole phase groups (a pole phase group is defined as a group of coils of a phase under one pole) and arranging the groups to have an effective displacement of $\gamma=2\pi/m$ electrical radians in space.

Consider the case of a 12 slot armature having 2 poles and wound for three phases as shown in Fig. 6·30 (a). If the flux density wave shape is considered sinusoidal, the emfs of the conductors in the slots can be represented as phasors displaced from each other by an (electrical) angle, $\alpha_s=\pi p/S=\pi/6$ radian$=30°$ as shown in Fig. 6·30 (a).

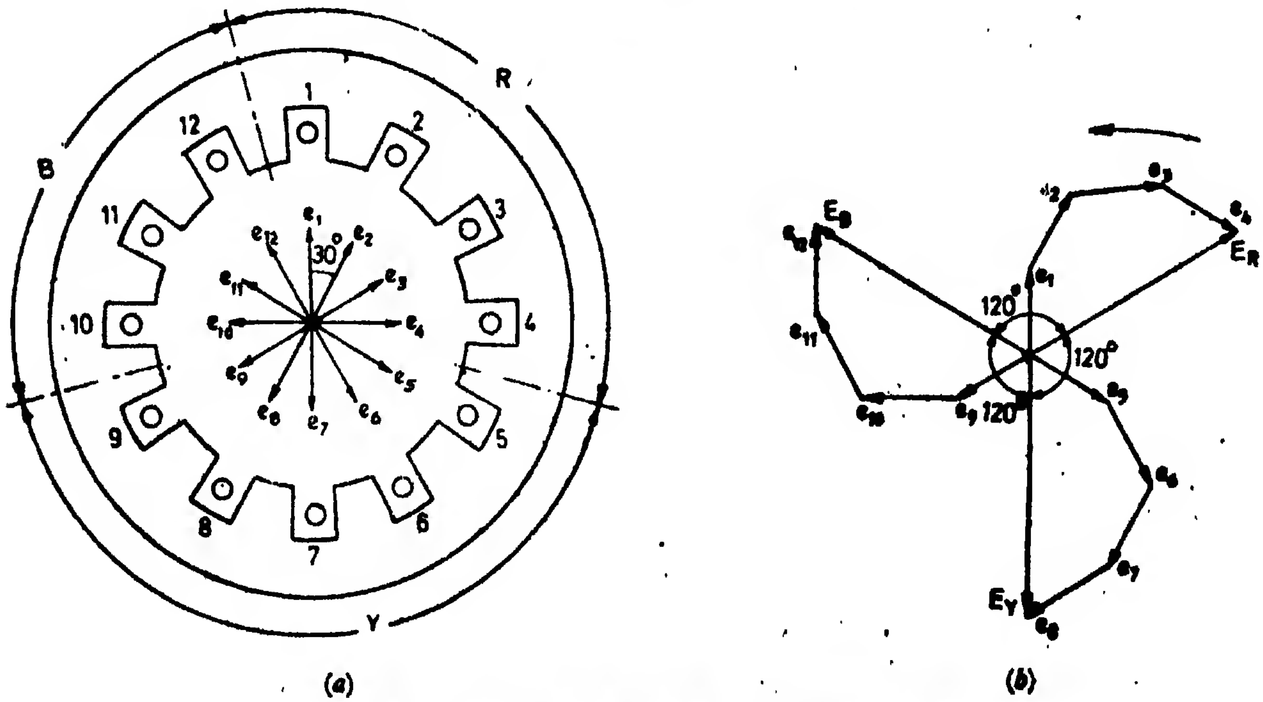


Fig. 6·30. Emfs of 3 phase winding with 120° phase spread.

If the winding is divided into three groups (one for each phase) spread over two pole pitches, the electrical displacement in space between the groups is $2\pi/3$ electrical radian or 120° electrical.

Each phase is located in four consecutive slots and so the phase spread is $4\times30°=120°$ electrical. If the conductors in the slots are connected as per the phasor diagram 6·30 (b), the summation of conductors emfs would give three emfs displaced 120° in time following a phase sequence of RYB in time. The space sequence is also RYB.

Let the winding be split up into six 60° phase groups spread over two pole pitches as shown in Fig. 6·31 (a).

Conductors of phase R are placed in slots, 1, 2 and 7, 8. Conductors of phase Y are placed in slots 5, 6 and 11, 12. Conductors of phase B are placed in slots 3, 4 and 9, 10. Conductors in slots 7, 8 are return conductors for conductors in slots 1, 2. Conductors in slots 11, 12 are return conductors for conductors in slots 5, 6. Conductors in slots 3, 4 are return conductors for conductors in slots 9, 10.

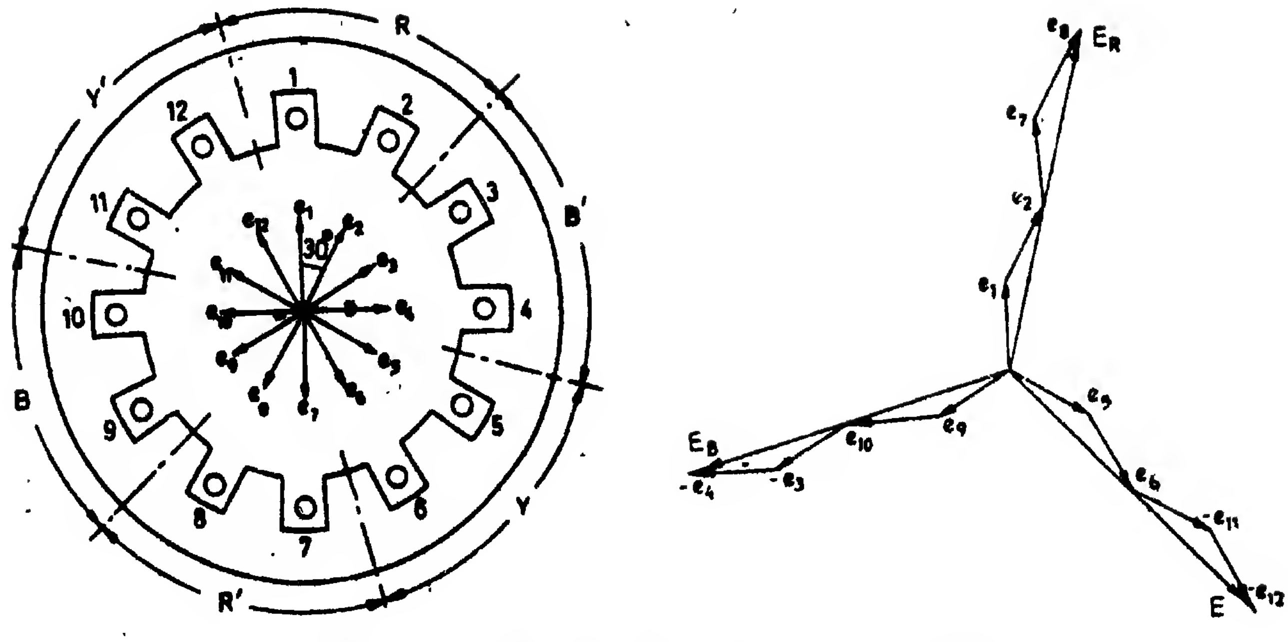


Fig. 6·31. Emfs of three phase winding with 60° phase spread.

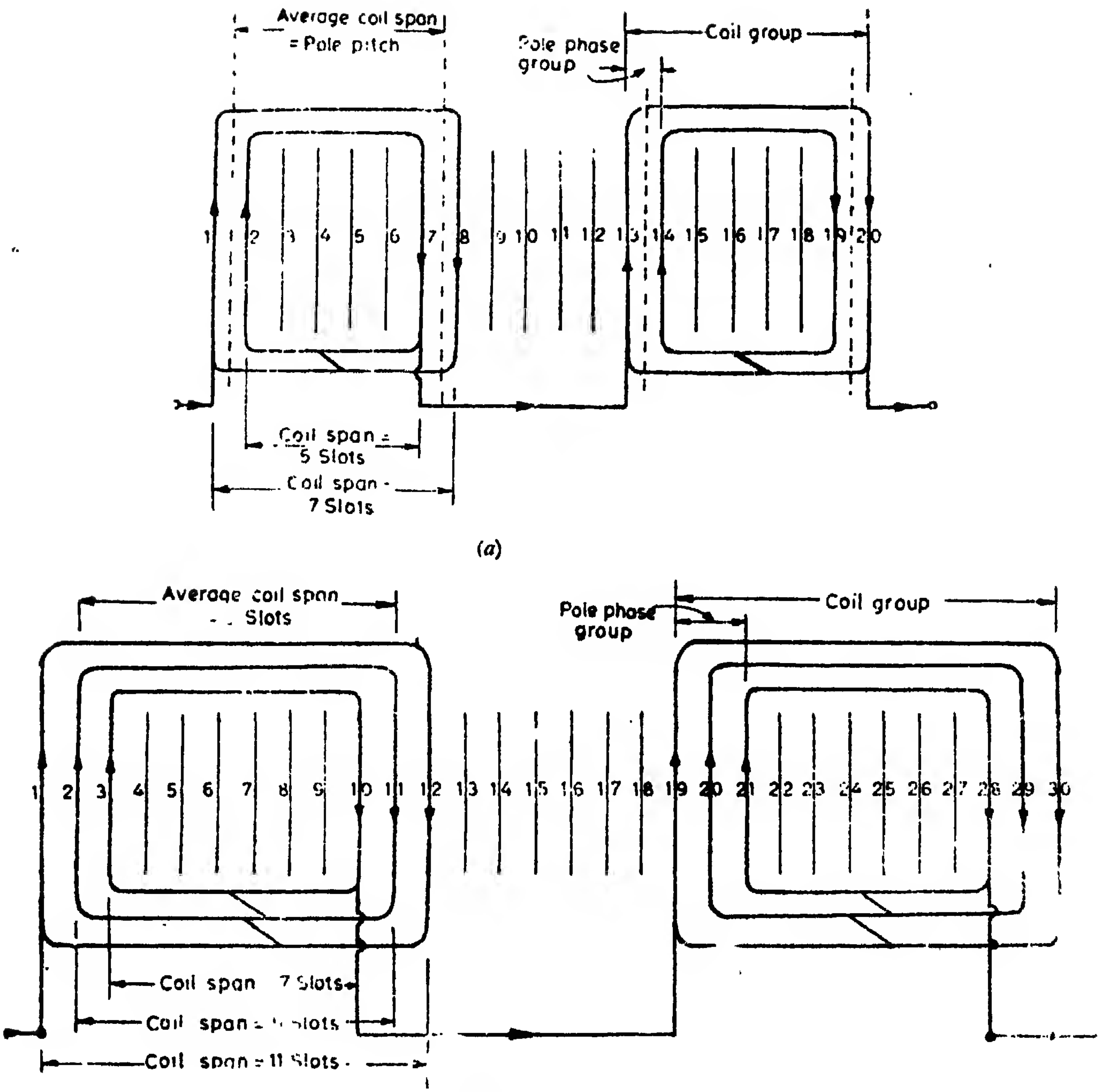
If the conductors are connected as represented by the phasor diagram 6·31 (*b*), we would still get three equal emfs displaced by 120° in time following a phase sequence *RYB*. The space sequence being *R B' Y R' B Y'*. Thus it is clear that with six 60° phase groups (three 60° groups per pole) spread over two pole pitches, it is possible to obtain three equal emfs displaced 120° in time. Generalizing for an m phase winding, it is possible to have m equal emfs displaced by $2\pi/m$ radian in time by having m phase groups per pole with a spread of π/m radian. The arrangement with a phase spread as $\sigma=\pi/m$ is normally used instead of $\sigma=2\pi/m$ as the latter is not adaptable for single layer windings.

6·18. Concentric Winding. Concentric windings are single layer windings which use concentic type of coils as described earlier. The coil span of the individual coils is different. The coil span of some coils is more than a pole pitch while the span of others is equal to or less than the pole pitch. These windings are so designed that the effective coil span of the winding is equal to that of a winding as a full pitch winding with some of the coils having a span greater than a pole pitch, some with less than a pole pitch but an effective span which makes the winding behave as if it had full pitched coils. They are of two types :

(*i*) hemitropic or half coil windings, (*ii*) whole coil windings. These windings are described below.

6·18·1. Hemitropic windings. These windings are also known as **unbifurcated windings**. In these windings the coils comprising a pair of pole phase groups under adjacent poles are concentric. These coils under two adjacent poles from one coil group and thus there is one coil group per pair of poles. Coil group and pole phase group are two different terms and these are distinguished as shown in Fig. 6·32 which shows a hemitropic winding.

There are 2 slots per pole phase or 6 plots per pole in hemitropic winding shown in Fig. 6·32 (*a*). The coil spans of the two coils forming a coil group are different. One has a coil span of 7 slots and the other 5 slots. With average or effective coil span being 5 slots (equal to slots per pole) and therefore the winding behaves as a full pitch winding. Similarly, in the hemitropic winding shown in Fig. 6·32 (*b*), the 3 coils in a coil group have an average (effective) coil span of 9 slots (which is equal to solts per pole).



(b)

Fig. 6·32. Hemitropic Winding.

It is clear that the overhang of a single layer concentric winding will have to be accommodated in more than one plane. Fig. 6·33 shows a Hemitropic winding having 3 slots per pole per phase with a 2 plane overhang. The same winding may have a 3 plane overhang as shown in Fig 6·34.

Number of coil groups = pole pairs × number of phases $= 3p/2$.

Considering a 3 phase winding with overhang arranged in two planes. In all cases when the number of pairs of poles is even, the number of coil groups is also even and therefore the overhang of half the coil groups lies in one plane while that of the other half lies in the second plane. But when the number of pole pairs is odd, the coil groups are also odd resulting in a cranked coil group whose ends partly lie in one plane and partly in

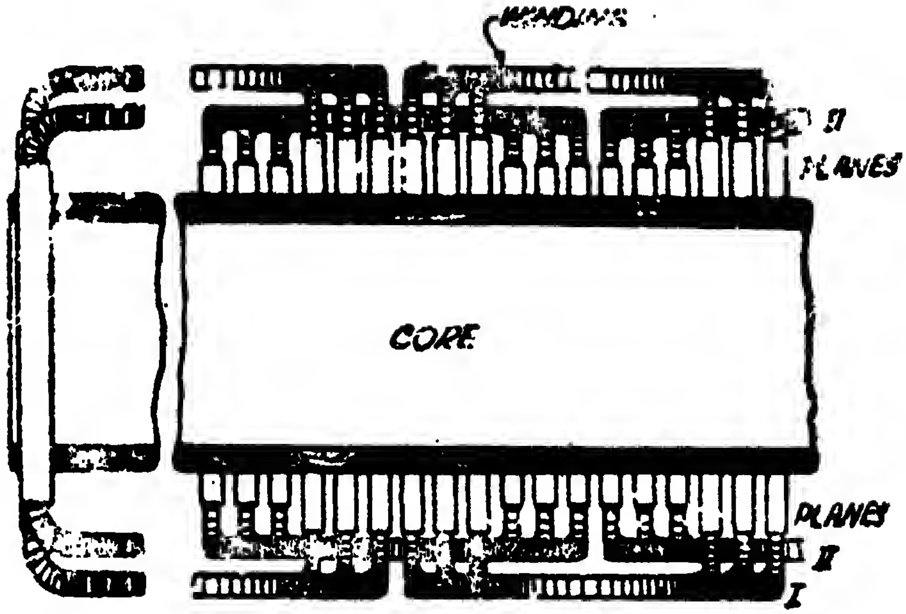


Fig. 6·33. Hemitropic winding with 2 plane overhang.

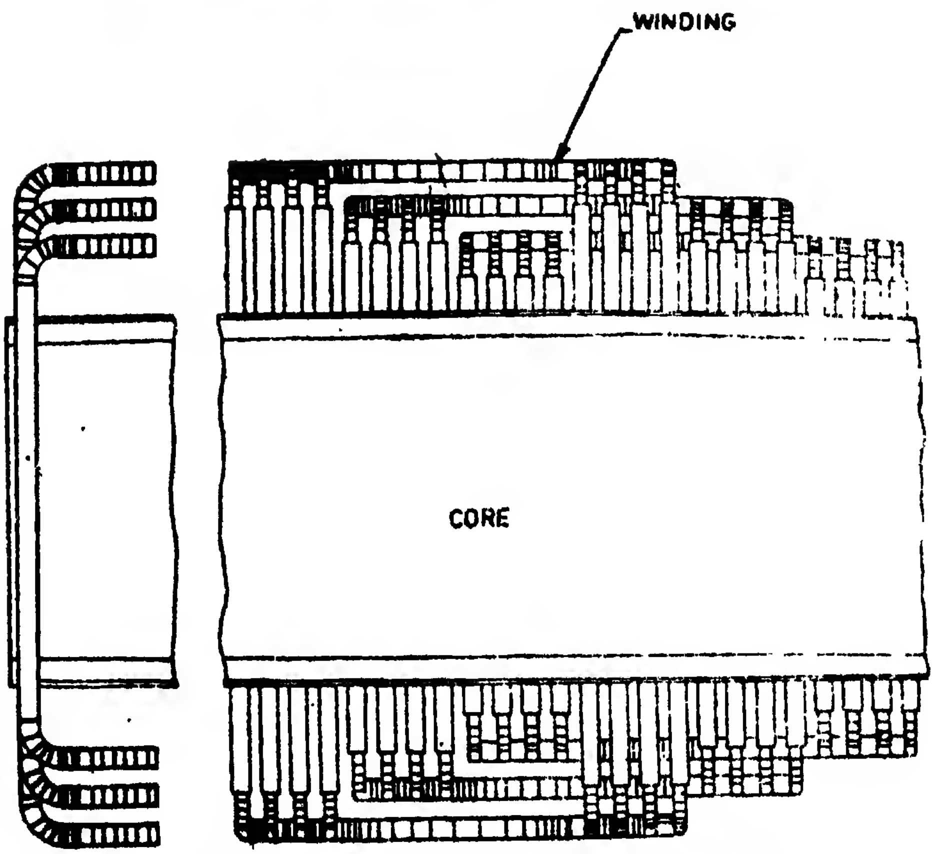


Fig. 6·34. Hemitropic winding with 3 plane overhang.

the other (Fig. 6·37). As the number of coil groups is a multiple of three in a 3-phase machine and therefore there is no question of a cranked coil group when a 3-plane overhang is used.

6·18·2. Whole coil windings. These are also known as **bifurcated windings**. In these windings, each pole phase group is split up into two sets of concentric coils, each set sharing its return coil sides with those of another pole phase group in the same phase. Thus we have one coil group per pole per phase. The number of coil groups is $3p$ and therefore the end connections in a 3-phase whole coil winding can be arranged to lie in either 2 or 3 planes. There are no cranked coil groups.

Fig. 6·35 shows a whole coil winding with 4 slots per pole per phase.

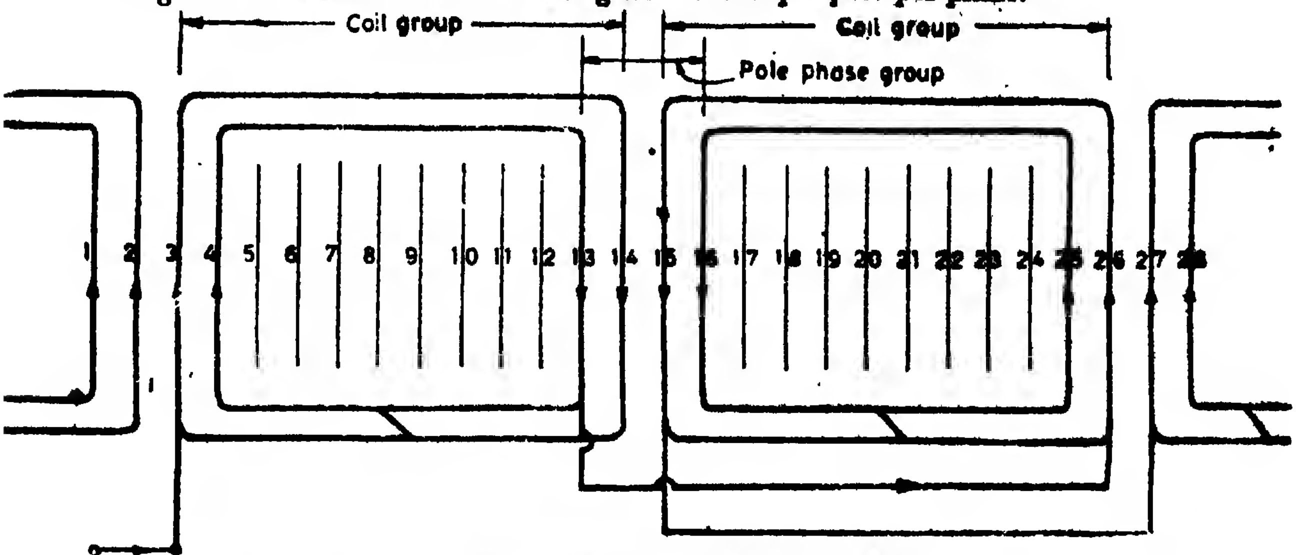


Fig. 6·35. Whole coil concentric winding.

Example 6·22. *Draw the winding diagrams of a 3-phase, 36 slot, 6 pole, a.c. armature for the following single layer concentric types of windings :*

(*i*) *Hemitropic—2 plane overhang,* (*ii*) *Hemitropic—3 plane overhang*

(*iii*) *Whole coil—3 plane overhang.*

Solution. Slots per pole per phase $=\dfrac{36}{3\times 6}=2.$

Slots per pole $=36/6=6$. $\therefore$ Angle between consecutive slots $\alpha_s=180/6=30°$.

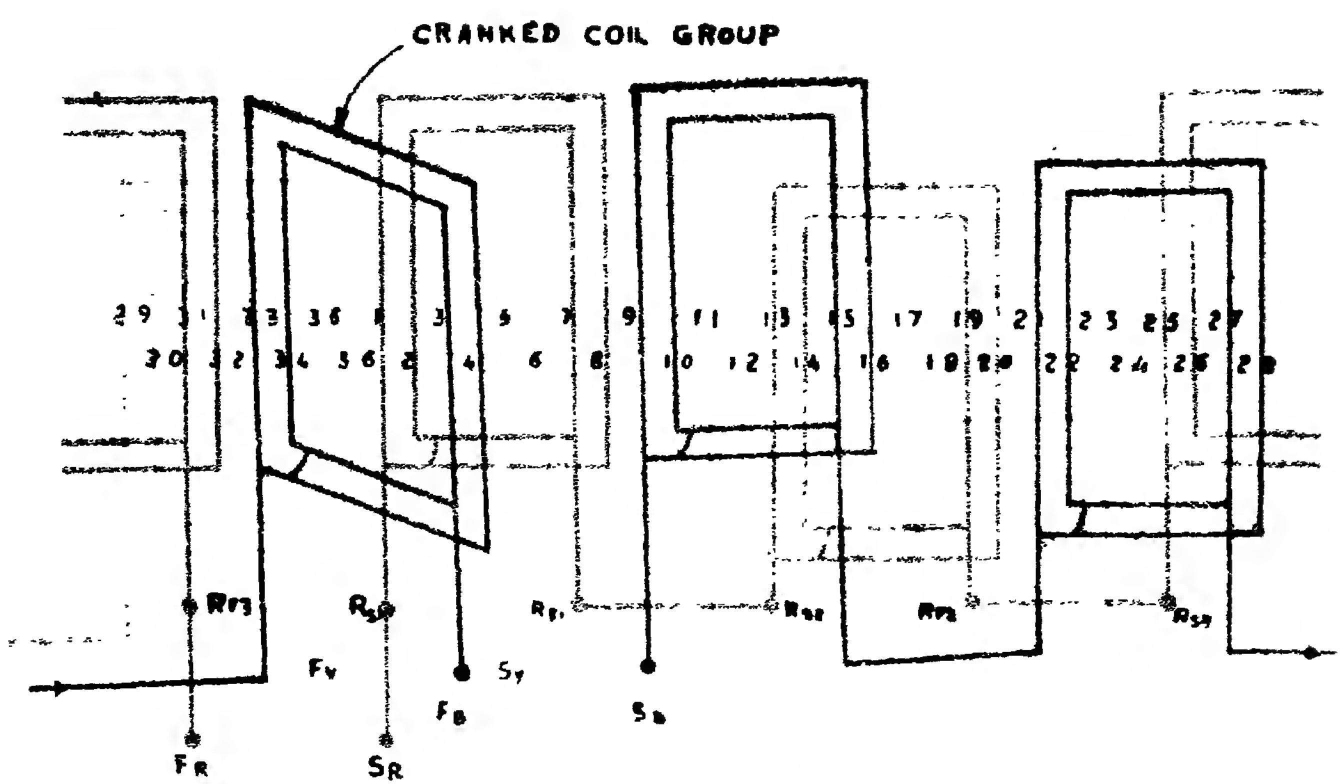
Phase spread $\sigma=60°$ is used with single layer windings and so there are 3 pole phase groups (one for each phase) under one pole. The allotment of slots to the three phases for a pole pair is given in Fig. 6·36 (*a*) and this allotment repeats itself for other pole pairs. The space sequence is kept as $RB'YR'BY'$ in order to get the phase sequence as RYB, Fig. 6·36 (*b*) shows the slot emf star.

Hemitropic. Starting phase R with the coil side in slot 1, the start of phase Y will be $120°/30°=4$ slots away *i.e.* in slot $(1+4)=5$ and the start of phase B will be in slot $(5+4)=9$.

The winding should be so connected that alternately we have North and South poels. Starting with coil side 1 in slot 1, it belongs to phase R. This should be connected to the last coil side of phase R under the next pole *i.e.*, 8 in this case. Coil side 2 should be connected to last but one coil side of the same phase under the next pole *i.e.* 7 in this case.

Now the two coils formed by coil sides 1, 8 and 2, 7 belong to the same phase and are concentric with each other. They form a coil group (while coil sides 1, 2 and 7, 8 form two separate pole phase groups). Now the coils formed by coil sides 1, 8 and 2, 7 are in the same coil group and they have to be connected. The rule for connecting together the coils in a coils group is that the end wire of one coil must be connected to the beginning of next coil in the group and so on. This is done in order that the emfs of the coils in a coil group, add.

The winding has one coil group per pair of poles for each phase and therefore there are in all 3 coil groups for each phase. These coil groups are similarly formed by the



3 Phase, 36 Slots, 6 Pole, Single Layer
HEMITROPIC CONCENTRIC WINDING
(2 Plane Overhang)

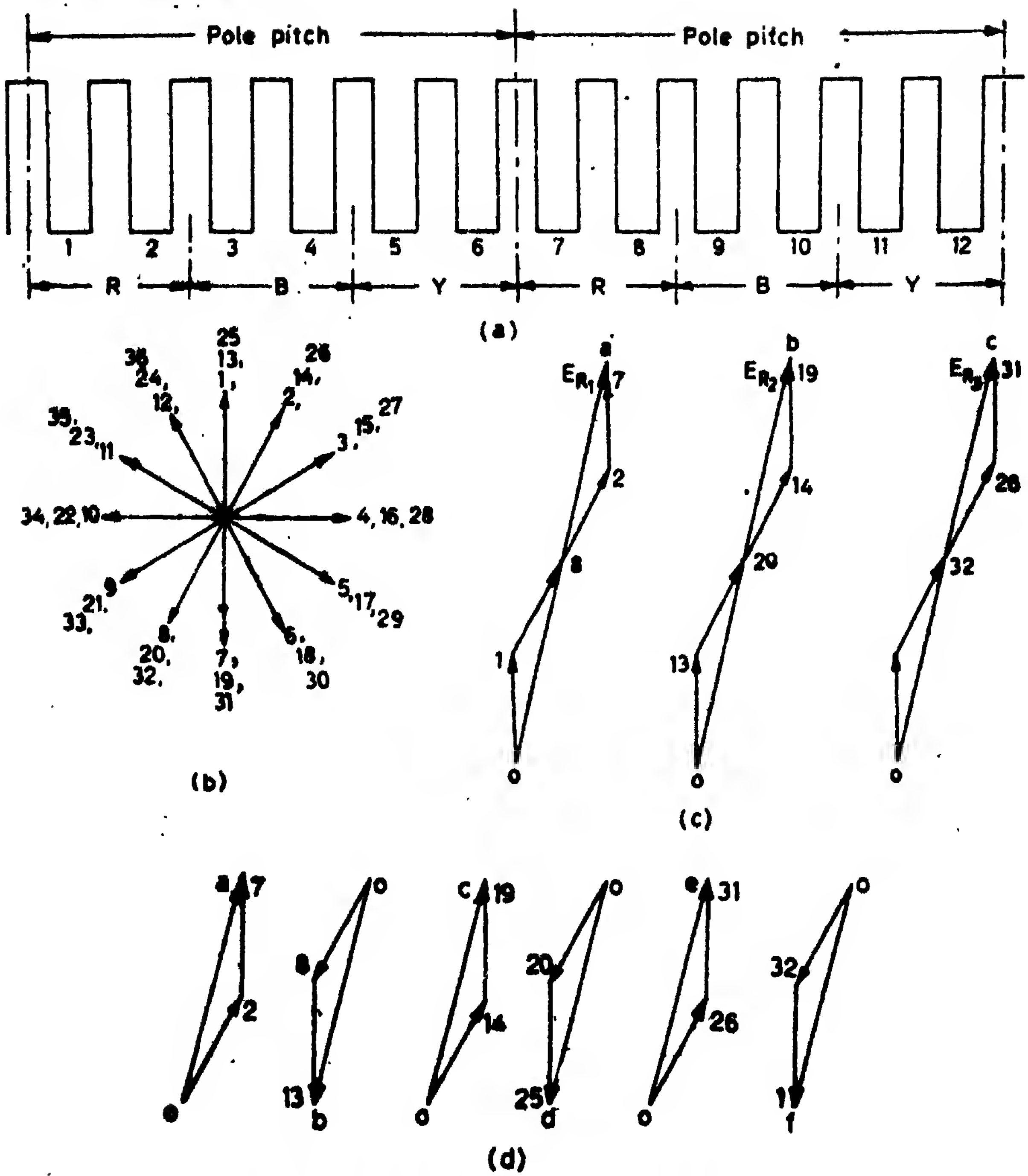


Fig. 6·36. Emfs of phase and coil groups of phase *R*.

interconnection of coils in the groups. For phase *R*, they are (Refer to Figs. 6·37 and 6·38).

Coil group	*Start*	*Finish*
I	Rs_1	Rf_1
II	Rs_2	Rf_2
III	Rs_3	Rf_3

Fig. 6·36 (*c*) shows the resultant emfs for three coil groups of phase *R*.

$oa = E_{R1}$ = emf of coil group *I*, $ob = E_{R2}$ = emf of coil group *II*,

and $oc = E_{R3}$ = emf of coil group *III*.

From this diagram it is clear that the three emfs are equal and are in phase with each other. The coil groups of each phase are connected together in such a way that the centre of every coil in a group has the same polarity either a North or a South. Thus if the first coil group of a phase forms a North pole so too must any succeeding groups of the same phase at the same instant form a North pole. Hence for series connection between coil groups the finish of one coil group is connected to start of next coil group.

Thus, R_{F1} is connected to R_{S2} R_{F2} is connected to R_{S3}

R_{S1} forms the start of phase R (designed as S_R).

R_{F3} forms the finish of phase R (designed as F_R)

The windings of other phases can be similarly laid and their terminals designated as :

S_Y, F_Y—start and finish of phase Y, respectively.

S_B, F_B— start and finish of phase B, respectively.

Fig. 6·37 shows a 2 tier (plane) hemitropic winding and as the number of pairs of poles is odd, there is a cranked coil group. Fig. 6·38 shows a 3 tier hemitropic winding. We have drawn the winding by connecting all the coil groups of a phase in series (or the number of parallel paths per phase is 1). It is clear that there are three (=pair of poles) similar coil groups per phase and therefore the maximum number of parallel paths can be 3 (=pair of poles) in this case.

Whole coil winding. The whole coil winding has one coil group per pole for each phase. The number of planes are normally 3 for three phase machines. In this example :

total number of coils$=36/2=18$, coils per phase$=18/3=6$,

number of coil of groups per phase$=6$. $\therefore$ Coils per group$=6/6=1$.

Thus in this example, a single coil is a coil group in itself.

Starting with coil in slot 2. The start of phase R lies in slot 2, that of Y in slot $(2+4)=6$ and of B lies in slot $(6+4)=10$.

Fig. 6·39 shows a complete winding diagram. This has 6 coil groups per phase and for phase R, they are :

Coil Group	*Start*	*Finish*
I	R_{S1}	R_{F1}
II	R_{S2}	R_{F2}
III	R_{S3}	R_{F3}
IV	R_{S4}	R_{F4}
V	R_{S5}	R_{F5}
VI	R_{S6}	R_{F6}

Fig. 6·36 (*d*) shows the emfs of the six coil groups of phase R with :

$Oa=E_{R1}=$emf across coil group *I* $Ob=E_{R2}=$emf across coil group *II*

$Oc=E_{R3}=$emf across coil group *III* $Od=E_{R4}=$emf across coil group *IV*

$Oe=E_{R5}=$emf across coil group *V* $Of=E_{R6}=$emf across coil group *VI*.

It is clear from Fig. 6·36 (*d*) that the emfs of all the coil groups are equal. But the emfs of the succeeding coil groups are 180° out of phase with each other. Therefore for a series connection the finish of first coil group of a phase is connected to the finish of 2nd coil group of the same phase. The start of second coil group is connected to the start of the third coil group and so on.

R_{S1} is the start of phase R (designated as S_R) R_{F1} is connected to R_{F2}

R_{S2} is connected to R_{S3} R_{F3} is connected to R_{F4}

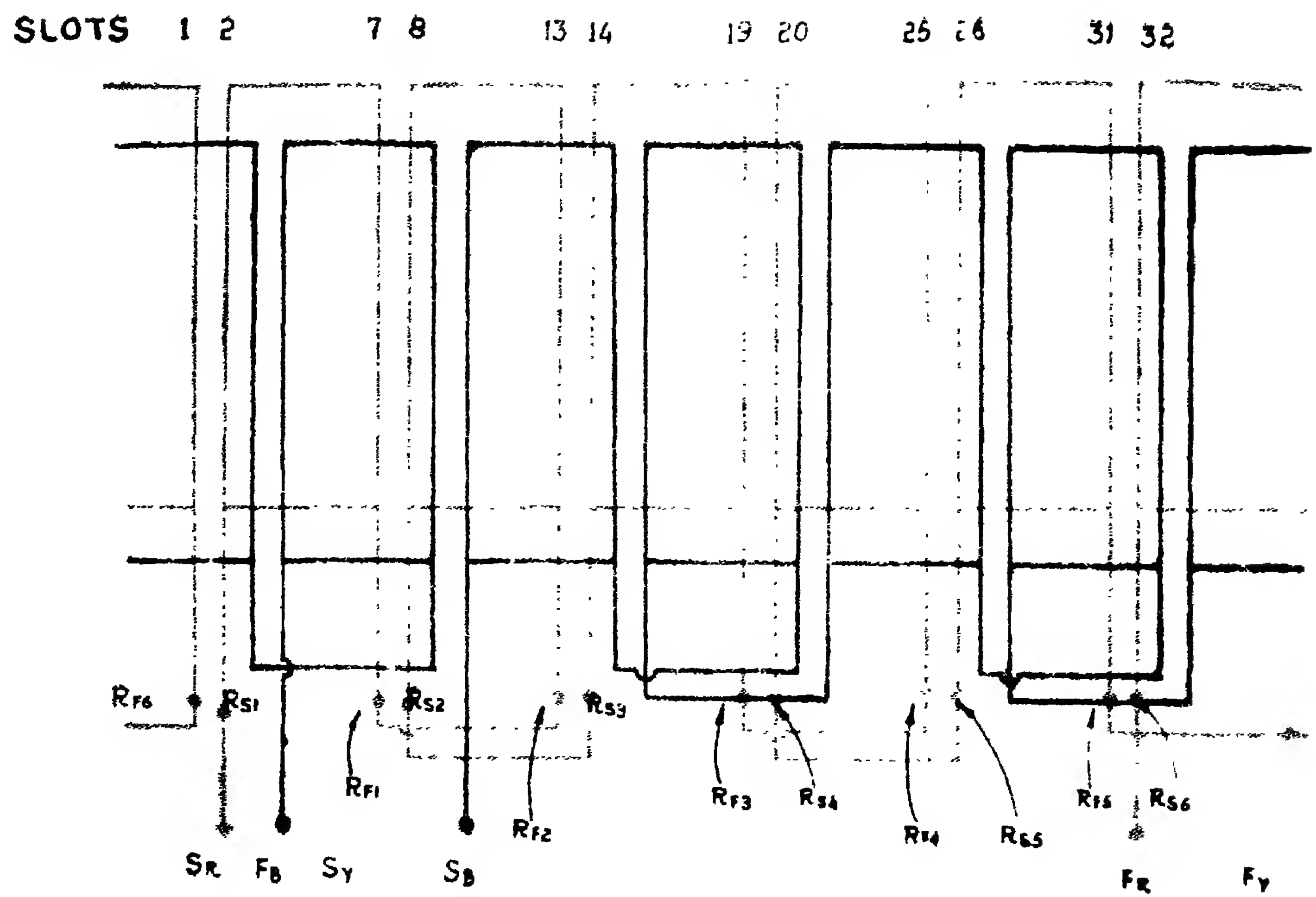
R_{S4} is connected to R_{S5} R_{F5} is connected to R_{F6}

R_{S6} is the finish of phase R (designated as F_R).

The winding for the other two phases can be similarly drawn. Their terminals are designated as

S_Y, F_Y—start and finish of phase Y respectively.

S_B, F_B—start and finish of phase B respectively.


SLOTS
1
2
7
8
13
14
19
20
25
26
31
32
R_{F6}
R_{S1}
R_{S2}
R_{F1}
R_{F2}
R_{S3}
R_{F3}
R_{S4}
R_{F4}
R_{S5}
R_{F5}
R_{S6}
S_R
F_B
S_Y
S_B
F_R
F_Y

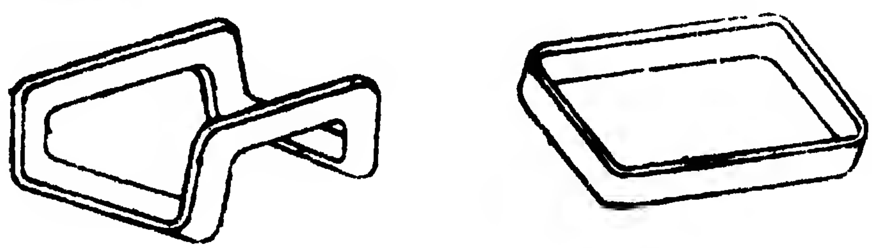
The winding has been drawn connecting all the coil groups of a phase in series (or there is one parallel circuit).

6·18·3. Mechanical details of concentric type coils. The types of coils to be used depends upon the shape of the slots. Except for very small machines, (which use round conductors placed in tapered slots to improve the shape of the tooth), all machines use parallel sided slots. These slots may be semi-enclosed or open. Different types of methods of coil formation and placing the winding on the armature are followed for open and semien-closed slots.

Coils used with open slots. Coils for windings to be placed in open slots are wound on formers or on flat plates and then the slot portions are bent up, to suit. The coils are preformed and fully insulated ready to be placed into position in slots.

The end connections of one tier (plane) must be bent in order to allow the second tier of windings to be placed in slots. Fig. 6·40 (*a*) shows bent coil to be used with semitropic winding. The coils in the second plane may be straight or may be bent. Fig. 6·40 (*b*) shows a straight coil.

Coils with semi-enclosed slots. Formed coils can be introduced from the top of the slots if open slots are used. But coils cannot be placed from the top of a semi-enclosed slot, the coils must be pushed through the slots. Therefore, the coils for semi enclosed slots must be made with one end open, *i.e.* one end is fully formed to size and shape and the other end is formed after the coil is placed in the slots. This type of coil, called a hair pin push through coil, is shown in Fig. 6·41. It is clear that only one end, the formed end, can be fully insulated before insertion into slots. The other end is bent and butt welded turn by turn after insertion. Hair pin coils are usually not preferred as jointing is both difficult and expensive.



(*a*) Bent coil. (*b*) Straight coil.

Fig. 6·40. Coils for concentric windings placed in open slots.

Fig. 6·41. Hair pin coil

6.19. Mush windings. This winding is very commonly used for small induction motors having circular conductors. This is a single layer winding where all the coils have same span (unlike the concentric winding where coils have different spans). Each coil is wound on a former, making one coil side shorter that the other. The winding is put on the core by dropping the conductors, one by one into previously insulated slots. The short coil sides are placed first and then the long coil sides. The long and short coil sides occupy alternate slots. It will also be observed that the ends of coil situated in adjacent slots cross each other *i.e.* proceed to left and right alternately. This is why sometimes it is known as a **basket winding**.

The following should be kept in view while designing a mush winding, that

(*i*) the coils have a constant span,

(*ii*) there is only one coil side per slot and therefore the number of coils is equal to number of slots,

(*iii*) there is only one coil group per phase per pole pair and therefore, the maximum number of parallel paths per phase is equal to pole pairs.

(*iv*) the coil span should be odd. Thus for a 4 pole 36 slot machine, coil span should be 36/4=9 slots while for a 4 pole, 24 slot machine, the coil span should not be 24/4=6 (*i.e.* even) ; it should be either 5 or 7 slots. This is because a coil consists of a long and a short coil side. The long and short coil sides are placed in alternate slots and hence one coil side will be in a even numbered slot and the other in a odd number slot giving a coil span which is a even integer.

Example 6·23. *Draw a winding diagram for a 4 pole, 36 slot, 3 phase mush connected armature.*

Solution. In setting out such a winding the slots are numbered from 1 to 36 and the long and short coil sides are alternately drawn.

Slots per pole per phase $=\frac{36}{3\times4}=3$. Thus a phase group has 3 slots.

Keeping a phase spread of $\sigma=60°$ and a phase sequence *RYB*, the slots allotted to the three phases are :

R :	1, 2, 3	10, 11, 12	19, 20, 21	28, 29, 30
B :	4, 5, 6	13, 14, 15	22, 23, 24	31, 32, 33
Y :	7, 8, 9	16, 17, 18	25, 26, 27	34, 35, 36.

For four poles the currents must flow in the conductors of phase *R* as shown in Fig. 6·42 (this is an arbitrary direction at any instant).

Thus in slots 1, 2, 3, the direction of current is marked as ↑ : in slots 10, 11, 12 as ↓ ; in slots 19, 20, 21 as ↑ and in slots 28, 29, 30 as ↓

We have coil span=36/4=9 slots (odd integer.)

Starting phase *R* in slot 1, coil side 1 is connected to coil side (1+9)=10. In mush winding we move to right and left alternately. We have moved to the right for connecting coil side 1, for connection of next coil side *i.e.* 2, we must move to the left. This coil side 2 is connected to coil side (2+36—9)=29. The whole winding for phase *R* can be completed in a similar manner. The coil groups are connected in such a manner that their emfs add.

Angle between slots $\alpha_s=4\times180/36=20°$

Thus the start of phase *Y* lies 120/20=6 slots away from that of phase *R* *i.e.*, in slot 7 and that of phase *B* lies in slot 13. The windings of these phases can also be similarly completed.

Example 6·24. *Draw a winding diagram for a 4 pole, 24 slot, 3 phase mush connected armature.*

Solution. Slots per pole per phase $=\frac{24}{3\times4}=2$.

Coil span=24/4=6 slots. This is an even number and hence winding is not possible with a coil span of 6 slots. A coil span of 5 slots is used.

The slots allotted to various phases are :

R	1, 2	7, 8	13, 14	19, 20
B	3, 4	9, 10	15, 16	21, 22
Y	5, 6	11, 12	17, 18	23, 24.

Fig. 6·43 shows the winding diagram for phase *R* only. The winding for other two phases may be similarly completed.

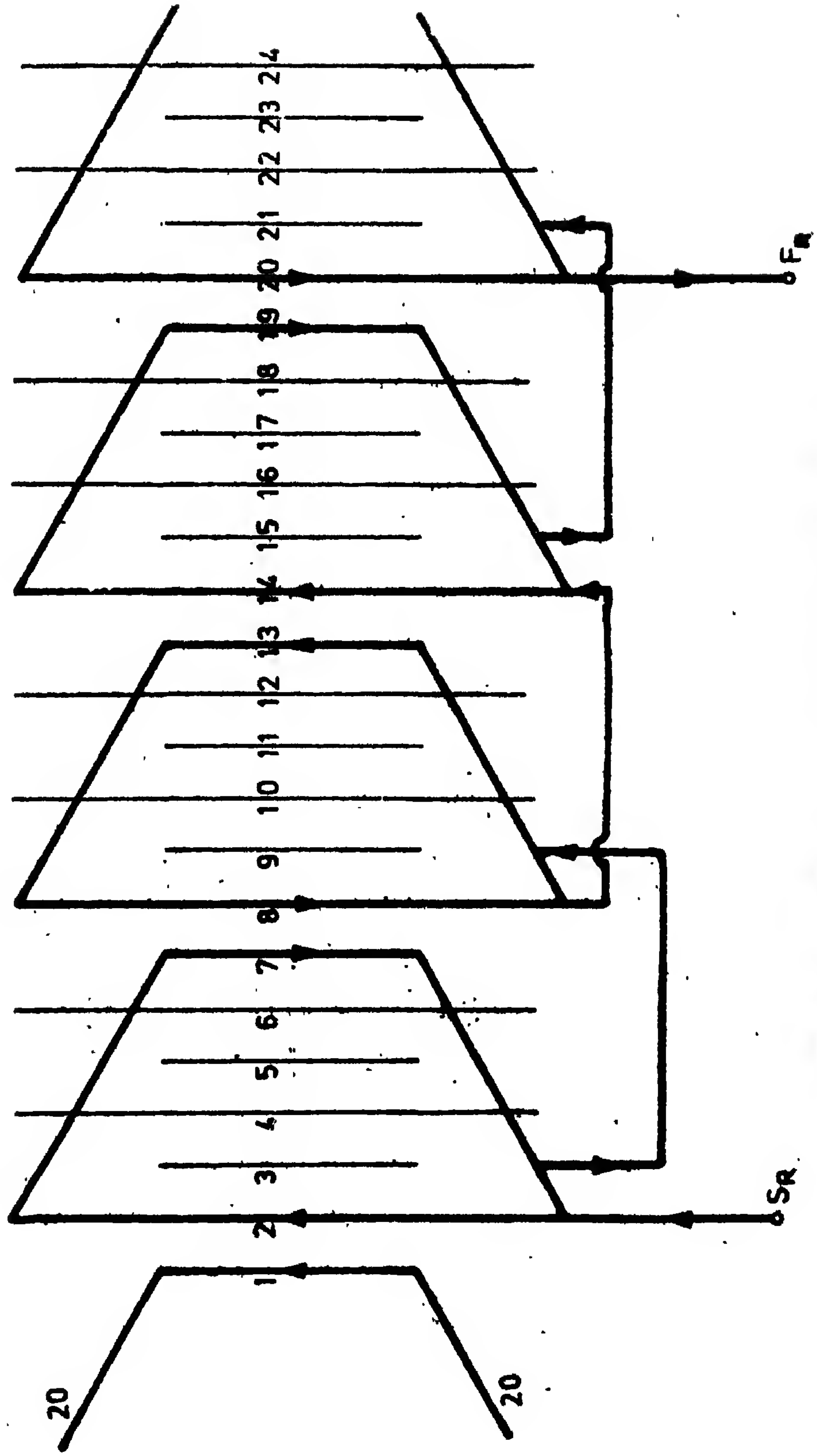


Fig. 6·43. 3 phase, 24 slot, 4 pole single layer mush winding.

6·20. Double layer windings. Double layer windings are universally used for armatures of synchronous generators and motors and most induction motors of large and medium sizes. They can be either lap or wave type. Lap windings have their widest field of application in the stators of high speed machines. Wave windings are used for wound rotors of medium and large size induction motors.

Double layer windings may be classified into two categories depending upon the method used to bring about the transition from the top layer to bottom layer. These are :

(*i*) lap type, and (*ii*) wave type.

These windings are the same as used in d.c. machines and give two different shapes to the end connections. The coils used in double layer windings are **diamond shaped** as in d.c. machines.

The number of coil sides per slot in double layer windings of a.c. machines is invariably 2. Therefore, number of coils is equal to number of slots or $C=S$.

The windings used for a.c. machines are usually chorded. The advantages of a chorded (short pitch or fractional pitch) winding are :

(*i*) the amount of copper used in the overhang is reduced, and

(*ii*) the magnitude of certain harmonics in the emf and also mmf is reduced.

Double layer windings can be either (*i*) integral slot, or (*ii*) fractional slot. When the number of slots per pole phase $q=S/mp$ is an integer, it is known as an **integral slot winding**. For example, consider a stator having 24 slots and wound for 4 poles and 3 phases. The number of slots pole per phase $q=24/(4\times3)=2$, which is an integer. The winding is thus an integral slot winding.

When the number of slots per pole and also the number of slots per pole per phase are not integers, the winding is known as **fractional slot winding**. The value of q is usually an improper fraction such as 8/5, 25/7 etc. Consider a stator having 78 slots to be wound for 3-phases and 8 poles, $q=78/(8\times3)=13/4=3\frac{1}{4}$. Thus q is not an integer and therefore it is a fractional slot winding.

6·21. Integral slot lap winding. The integral slot lap winding is explained with the help of following example.

Example 6·25. *Give the layout of a lap winding for the stator of a 3 phase ac machine having 4 poles and 24 slots. There are 2 coil sides/slot.*

Solution. Slots per pole phase $q=24/3\times4=2$

A phase spread of $\sigma=60°$ is used

∴ Number of pole phase groups/pole $=3$ (and total number of pole phase groups $=4\times3=12$)

Thus there are 3 phase groups per pole each comprising of $q=2$ slots. The distribution of slots, for phase sequence *RYB*, is :

R	1, 2	7, 8	13, 14	19, 20
B	3, 4	9, 10	15, 16	21, 22
Y	5, 6	11, 12	17, 18	23, 24

For full pitch coils, coil span $C_s=24/4=6$ slots. This means that the top coil side in slot 1 is to be connected to bottom coil side in slot $(6+1)=7$ or $y_b=13$ and $y_f=11$ in terms of coil sides.

Considering the coils at the end opposite to the connections, coil side 1 (top) is connected to coil side $(1+13)=14$ (bottom). Coil side 1 and 14 from a coil ; this coil may

be single turn or a multiturn coil. Coil side 14 is connected to coil side$(14-11)=3$, (top) at the front end. This connector C connects the last turn of the first coil of a pole phase group to the first turn of second coil of the same group.

Coil side 3 is connected to coil side $(3+13)=16$ (bottom) at the back. Coil sides 3 and 16 form the second and the last coil of this pole phase group. (If there were more coils per phase group, they would have been also connected in series).

Each phase has four ($p=4$) pole phase groups each containing $q=2$ coils. For phase R, the groups are

Group No.	Start	Finish
I	R_{S_1}	R_{F_1}
II	R_{S_2}	R_{F_2}
III	R_{S_3}	R_{F_3}
IV	R_{S_4}	R_{F_4}

These phase groups can either be connected in series or in parallel depending upon the requirements.

The starts of the phases must be displaced by 120° and so must the finishes. The angle between adjacent slots is $\alpha_s=4\times180/24=30°$.

So if the start of phase R lies in slot 1, the start of phase Y must be in slot $(1+120/30)=5$ and that of phase B in slot No $(5+120/30)=9$. This makes the phase sequence RYB. The starts of phases R, Y and B are represented by S_R, S_Y, S_B respectively.

Fig. 6·45 represents the emfs induced in various coil sides. e_1, e_2, e_3 ......etc. denote the emfs induced in coil sides 1, 2, 3......etc It is clear that in the case of full pitch coils, the emfs of two coil sides forming a coil add algebraically.

Now coil 1 comprises of coil sides 1 and 14 and its emf is represented by E_1 which is the algebraic sum of e_1 and e_{14} and the emf of coil 2 comprising of coil sides 3 and 16, is represented by E_2 which is the algebraic sum of e_3 and e_{16}. E_2 is displaced by 30° with respect to E_1.

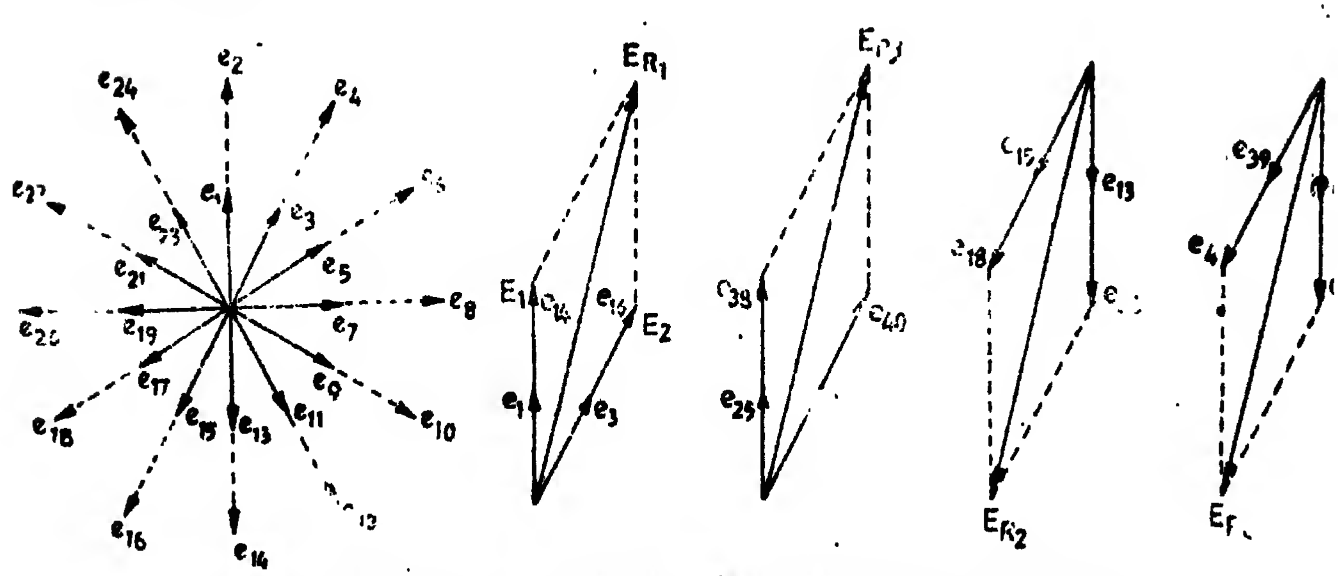


Fig. 6·45. Emfs of coils and pole phase groups of winding of Example 6·25.

The emf between R_{S1} and R_{F1} (I phase group) is the phasor summation of E_1 and E_2 and is represented by E_{R_1}. Similarly, the emf between R_{S2} and R_{F2} (II phase group) is represented by E_{R_2}.

E_{R_1}=emf between R_{S_1} and R_{F_1}

E_{R_2}=emf between R_{S_2} and R_{F_2}

E_{R_3}=emf between R_{S_3} and R_{F_3}

E_{R_4}=emf between R_{S_4} and R_{F_4}.

The emfs of all the phase groups are equal but the emfs of alternate phase groups are 180° out of phase with each other. (See Fig. 6·45). Therefore for series connection, finish of phase group I is connected to finish of phase group II of the same phase. Now in order that the emfs add, the start of phase group II is connected to start of phase group III and the finish of phase group III is connected to finish of phase group IV.

Thus, R_{S_1} forms the start of phase R and is designated as S_R.

R_{F_1} is connected to R_{F_2}

R_{S_2} is cennected to R_{S_3}

R_{F_3} is connected to R_{F_4}

R_{S_4} forms the finish of phase R and is designated as F_R.

It is noticed from the above example that the individual coils forming a pole phase group have to be connected in series while the various pole phase groups of a phase can either be connected in series or in parallel. As there are as many similar pole phase groups per phase as the number of poles the maximum number of parallel paths is equal to number of poles (4 in this case).

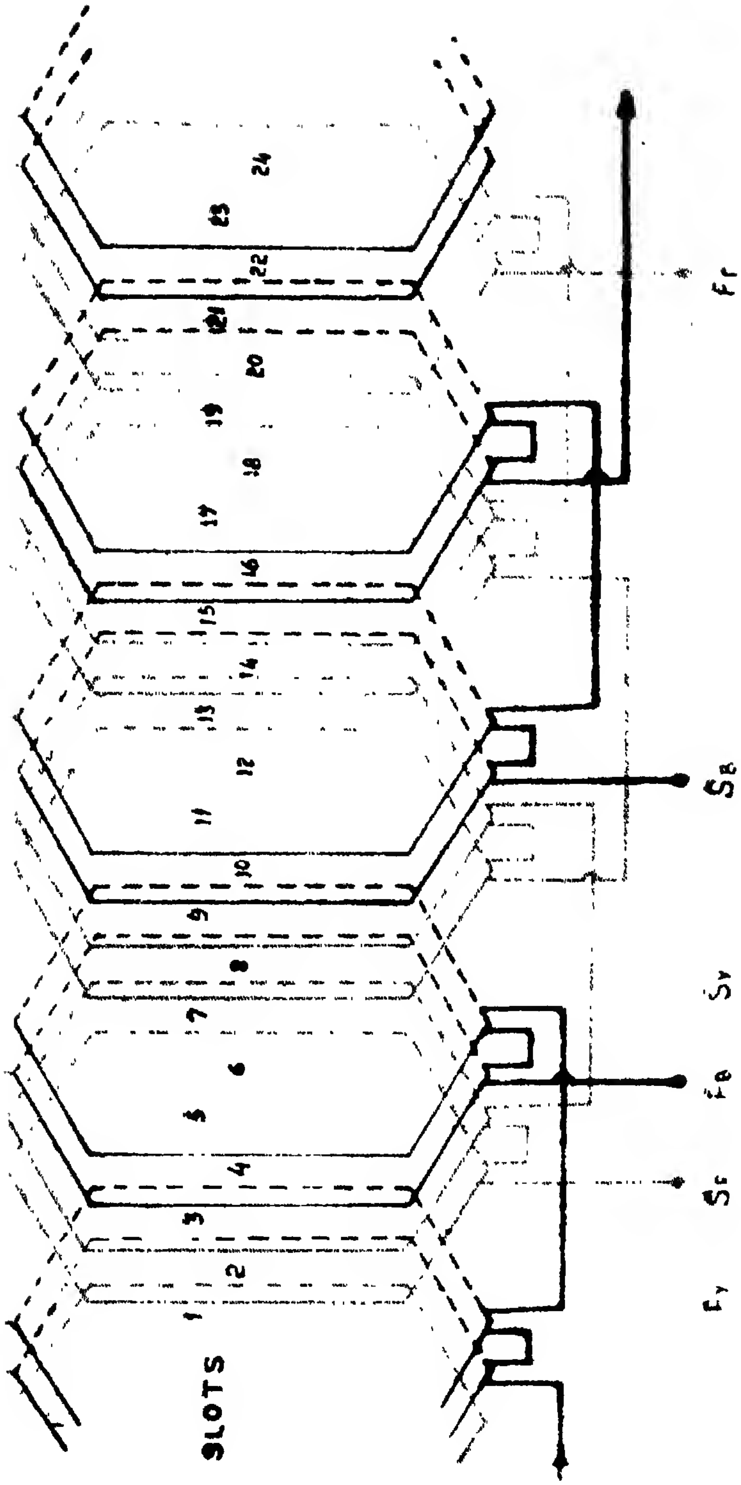
The winding arrangement can also be shown by a simple diagram called **clock diagram**. In the clock diagram, the arrangement of top and bottom coil sides is shown. The slot no. and the phases which occupy the top and bottom layers are indicated. Fig. 6·46 shows the clock diagram of the winding described above. (It should be kept in mind that since there are 4 poles 1° mechanical=2° electrical).

We have considered the case of full pitch coils in the above example. Let us now take up the same winding but with short pitch (chorded) coils. Suppose that the coils are chorded by one slot pitch *i.e.* coil span=slots per pole −1 or C_S=6−1=5 slots.

This means that if the top coil side of a coil lies in slot 1, the bottom coil side would lie in slot (1+5)=6. Thus y_b=11 and y_f=9.

The procedure for drawing this winding is the same as described earlier but initially we can assign slots to top coil sides only and the bottom coil sides go on occupying slots as the winding progresses. (In this winding, unlike the full pitch winding, the top and bottom coil sides in a slot may not belong to the same phase). Fig. 6·47 shows a double layer winding with chorded coils.

In fact, a clock diagram is very helpful in the layout of a winding with short pitch coils. The phases to which the top coil sides belong are first indicated. Now, this winding uses a coil span of 5 slots, therefore the top coil side in slot 1 belonging to phase R is connected to bottom coil side of slot 6. Thus R is written in the bottom coil side space of slot 6. The rest of the bottom spaces can now be allocated by proceeding serial wise.


SLOTS

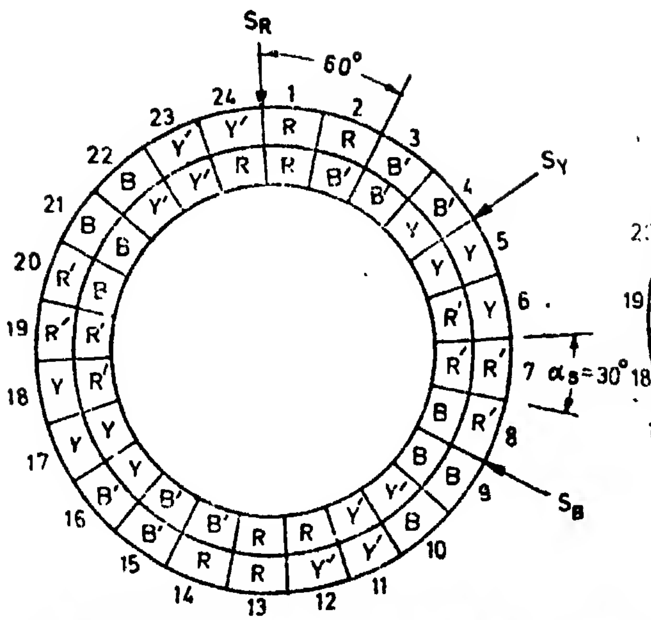


Fig. 6·48. Clock diagram of a 3 phase, 4 pole, 24 slot double layer winding with short pitch coils.

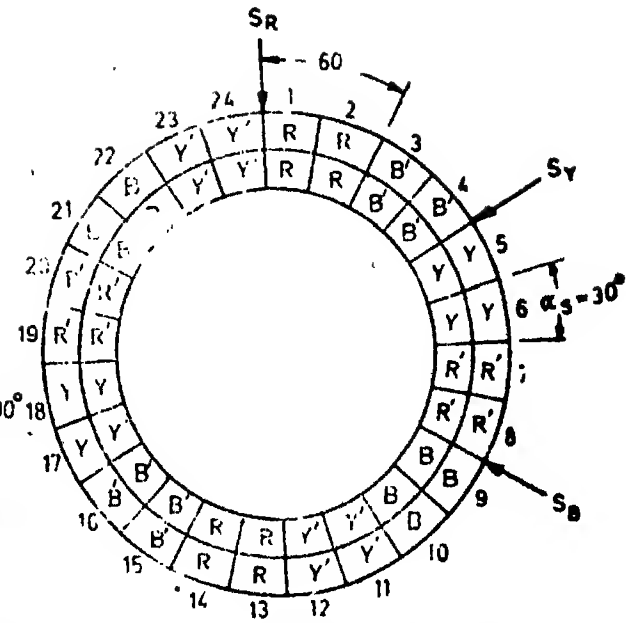


Fig. 6·46. Clock diagram of a 3 phase, 4 pole, 24 slot double layer winding with full pitch coil.

6·22. Integral slot wave windings. We have observed that special connectors are needed to connect individual coils and pole phase groups in series in the case of lap windings. The number of these connectors is very large especially in the case of multipolar (low speed) machines. This results in large additional expend ture of copper especially in the case of machines using heavy cross-section conductors. In such cases wave windings are used particularly when single turn coils or two coil sides per slot are adequate for the required voltage. With wave windings there is a great saving in end connections and jointing, as the coil groups are interconnected naturally due to the outward spread of coil ends.

The coil used in wave connected a.c. machine windings are identical with those employed in d.c. windings. The coil ends, in the case of d.c. machines are connected to the commutator while those in a.c. machines are joined together to form phases.

In a wave connected d.c. machine, the total winding pitch to $Y=\frac{4C\pm 4}{p}$ when Y is an even integer. However, if this relationship is used in a 3 phase a.c. machine the winding pitch,

$$Y=\frac{4C\pm 4}{p}=\frac{4\times 3pq\pm 4}{p}=\frac{12pq\pm 4}{p}.$$

Therefore, Y cannot be an even integer except in the case of $p=2$ (for which lap and wave windings are identical). Hence value of winding pitch used for 3 phase a.c. machines is $Y=12q$.

In an ordinary d.c. wave winding, one has coils of constant span and one proceeds with constant pitch of coil around the machine until one automatically comes to the next coil side from which one started and so on till the winding closes. But in a.c. wave windings, with integral number of slots per pole per phase, and $Y=12q$, the winding would close after completing one trip around the armature. Thus with a.c. windings one proceeds with a **normal step** round the machine, but to prevent closing of the winding after one tour, one has to take an **abnormal step**. After having completed half the winding, it is necessary to reverse the direction of travel around the machine.

The following example illustrates the procedure to lay a wave winding.

Example 6·26. *Give the layout of an a.c. wave winding for the rotor of a 3-phase, 4 pole induction motor having 24 slots. Each slot contains 2 coil sides.*

Solution.

Slots/pole$=24/4=6$. Slots per pole per phase $q=24/3\times4=2$.

Total number of coils$=2\times24/2=24$. Total number of coil sides$=2\times2$ $=48$.

Total winding pitch $Y=12q=12\times2=24$

Back pitch $y_b=13$ and front pitch $y_f=11$

This means that coil side 1 in slot 1 will be connected at the back to coil side 14 in slot 7.

$\therefore$ Coil span$=7-1=6$ slots and so full pitch coils are used. With full pitch coils, the slots can be assigned to both top and bottom coil sides at the very start. The distribution of slots to the 3 phases is :

Phase R	1, 2	7, 8	13, 14	19, 20
Phase B	3, 4	9, 10	15, 16	21, 22
Phase Y	5, 6	11, 12	17, 18	23, 24

All the coil sides are drawn and numbered as shown in Fig. 6·49.

Phase R starts with top coil side of slot 1 *i.e.*, coil side 1.

Proceeding clockwise. 1. Top coil side 1 in slot 1 is connected at the back side to bottom coil side $(1+13)=14$ in slot 7.

2. Bottom coil side 14 in slot 7 is connected at the front side to top coil side $(14+11)=25$ in slot 13.

3. Top coil side 25 in slot 13 is connected at the back side to bottom coil side $(25+13)=38$ in slot 19.

Bottom coil side 38 in slot 19 is connected at the front side to top coil side $(38+11-48)=1$ in slot 1. If we make this connection the winding closes and so coil side 38 is connected to coil side 3 in slot 2 instead of coil side 1 in slot 1. This is an abnormal step where $y_f=13$.

4. Bottom coil side 38 in slot 19 is connected at the front side to coil side 3 in slot 2.

5. Top coil side 4 in slot 2 is connected at the back side to bottom coil side $(3+13)=16$ in slot 8.

6. Bottom coil side 16 in slot 8 is connected at the front side to top coil side $(16+11)=27$ in slot 14.

7. Top coil side 27 in slot 14 is connected at the back side to bottom coil side $(27+13)=40$ in slot 20.

Bottom coil side 40 in slot 20 is connected at the front side to top coil side $(40+11-48)=3$ coil side in slot 2.

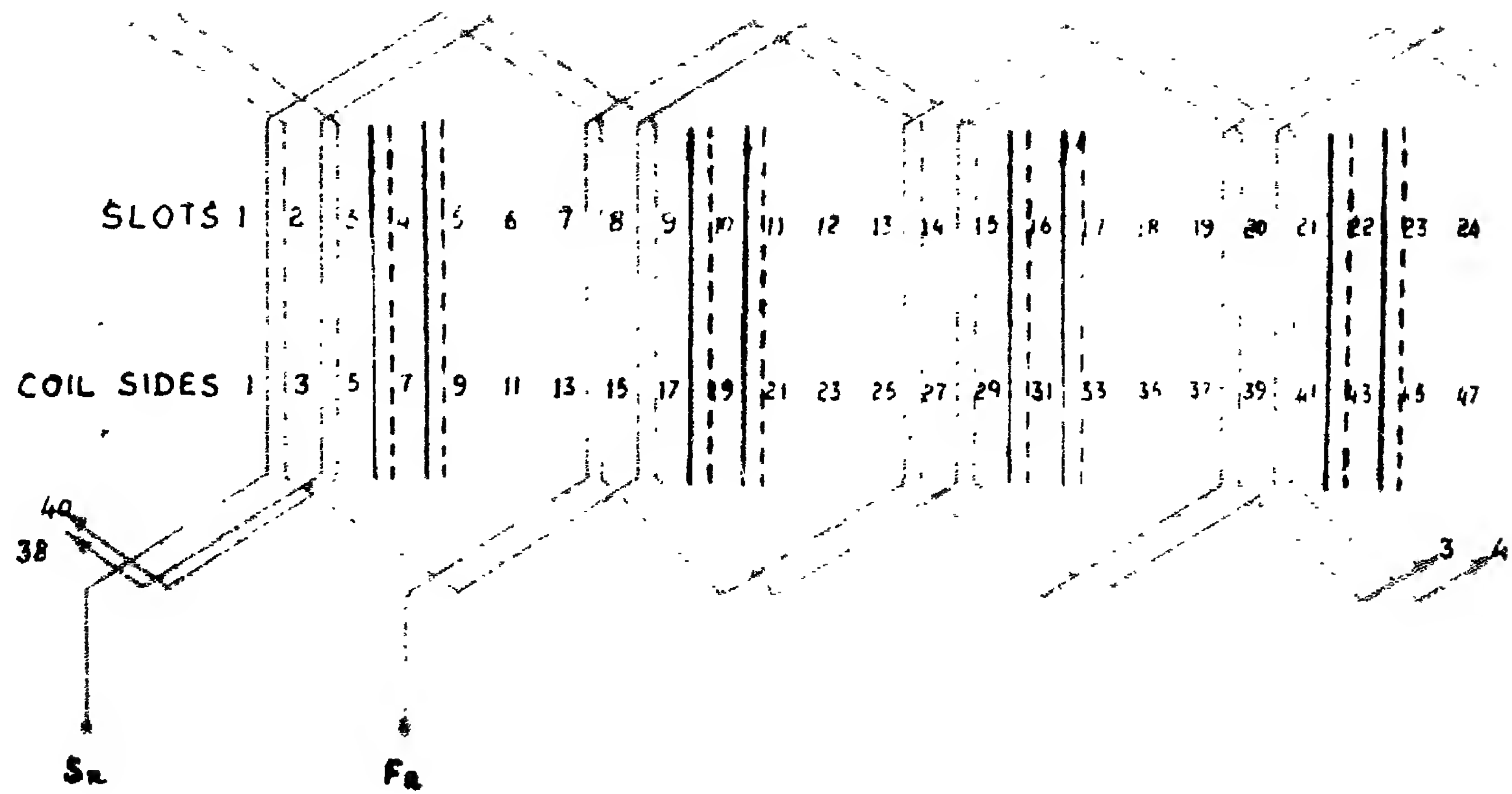
If we make this connection the winding would close. Also we cannot connect coil side 40 to coil side 5 as they belong to different phases. Therefore, coil side 40 is connected to coil side 4. This is another abnormal step.

8. Bottom coil side 40 in slot 20 is connected at the front side to bottom coil side 4 in slot 2.

Half of the winding of phase R is complete and therefore we must proceed anticlockwise.

Proceeding anticlockwise. 9. Bottom coil side 4 in slot 2 is connected at the back side to top coil side $(4+48-13)=39$ in slot 20.

10. Top coil side 39 in slot 20 is connected at the front side to bottom coil side $(39-11)=28$ in slot 14.


SLOTS 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24
COIL SIDES 1 3 5 7 9 11 13 15 17 19 21 23 25 27 29 31 33 35 37 39 41 43 45 47
40
38
3
4
S_R
F_R

11. Bottom coil side 28 in slot 14 is connected at the back side to top coil side 28−13)=15 in slot 8.

Top coil side 15 in slot 8 is connected at the front side to bottom coil side (15−11) =4 in slot 2. But coil side 4 is already connected up and so we connect coil side 15 to coil ide 2. This is an abnormal step.

12. Top coil side 15 in slot 8 is connected at the front side to bottom coil side 2 in lot 1.

13. Bottom coil side 2 in slot 1 is connected at the back side to top coil side 2+48−13)=37 in slot 19.

14. Top coil side 37 in slot 19 is connected at the front side to bottom coil side 37−11)=26 in slot 13.

15. Bottom coil side 26 in slot 13 is connected at the back side to top coil side 26−13)=13 in slot 7

The winding for phase *R* ends here as all the coil sides belonging to this phase have been connected. The winding for other two phases can be similarly drawn. It should be noted that in the case of rotor windings of induction motors (which are usually star connected), he three leads are brought out from parts of the winding which are displaced as nearly as possible 120 mechanical degrees apart, as well as being exactly 120 electrical degrees apart. Thus the start of phase *Y* is not in the same pole pair as the start of phase *R*, and also the tart of phase *B* is not in the same pole pair as the start of *Y*.

6·23. Fractional slot windings. The fractional slot windings where slots/pole/phase q, is not an integer, has many advantages. These advantages are :

1 In low speed a.c. machines the number of poles is large and so the number of slots per pole phase may become two or less. If a small integral number of slots per pole per phase ($q=1$, 2 or 3) are used, it would give rise to appreciate tooth harmonics in the emf induced. The large value of harmonic content in integral slot windings is due to the fact that the corresponding winding elements of each phase, under different poles, occupy similar positions with respect to pole to pole axis ; and so the harmonic emfs add algebraically. But if a winding is wound with a fractional number of slots per pole per phase, the corresponding winding elements of a phase under different poles, may occupy dissimilar positions with respect to pole axis. The displacement of these winding elements with respect to magnetic field is so selected that the higher harmonics are decreased without significantly affecting the fundamental. The use of fractional slot windings results in an effective distribution which is equivalent to infinite distribution even when the slots per pole per phase is small. This is explained by the fact that a winding with $q=2\frac{1}{4}=9/4$ behaves as if it has 9 slots per pole per phase resulting in infinite distribution. The infinite distribution, though not physical, results in considerable decrease of harmonic emfs.

2. Machine manufacturers generally have stocks of lamination notching gear with which stampings are made for a great variety of cores. Thus a particular number of slots, for which die and punch are available may be used for a range of machines running at different speeds as in synchronous machines, with consequent saving in cost of drawings and equipment.

3. The use of fractional slot windings results in reduction of mmf harmonics.

4. The leakage reactance of windings is reduced.

5. With fractional slot windings, chording is a must and, therefore, the cost of copper is reduced owing to shorter end connections.

In the integral slot windings every pole phase group has the same number of series connected coils. But in fractional slot windings all the pole phase groups do not have the same number of series connected coils as q is always a mixed number. Since fractional coils are impossible the only alternative left is not to make all the pole phase groups identical i.e, practical winding is only possible when one (or more) pole phase groups has one coil fewer than the other (or others).

When, however, it is found necessary to use a lamination that does not give an integral slot winding its number of slots must make it possible for all phases to have same number

of coils. This means that if a lamination is to be used for a 3 phase machine, the total number of slots should be divisible by 3 in order that each phase has the same number of coils.

Fractional slot windings tend to create non-uniform flux density distributions in the air gap, but the machine operation would be satisfactory if the phase groups are arranged around the stator core with a certain degree of uniformity.

Consider a 3-phase, 10 pole armature with 96 slots. The average number of slots per pole per phase is

$$q=\frac{96}{3\times10}=\frac{16}{5}=3\ \frac{1}{5}.$$

As each pole phase groups must have an integral number of coils, $q=\frac{16}{5}=3\ \frac{1}{5}$. can only be obtained if the 5 (denominator of q) phase groups under 5 poles have different number of coils totalling upto 16 coils (numerator of q). Now 16 coils for each phase lying under 5 poles can be obtained if we have,

4 pole phase group of 3 coils each and 1 pole phase group of 4 coils.

This gives the average value of $q=\frac{4\times3+1\times4}{5}=3\ \frac{1}{5}$

Five poles make one basic unit of this winding. As this winding has 10 poles, there are two units of 5 poles, each covering 16 slots of each phase. The $(4+1)=5$ pole phase groups of each phase in a unit must be connected in series and as there are $10/5=2$ such units, the maximum number of parallel paths is equal to 2 *i.e.* number of units.

Considering q in the form 16/5, where numerator and denominator have no common factor, we have,

(*i*) number of poles in a unit$=5$ (denominator of q),

(*ii*) number of slots per phase in each unit$=16$ (numerator of q),

(*iii*) number of units$=\dfrac{\text{total number of poles}}{\text{poles/unit}}=\dfrac{p}{\text{denominator of } q}$.

Considering the form $q=3\frac{1}{5}$, we observe that there are :

(*i*) $5-1=4$ groups of 3 coils each and (*ii*) 1 group of $3+1=4$ coils each.

The results of the above example can be expressed in general form .

$$q=\frac{S}{m\times p}=\frac{M}{d}=I+\frac{n}{d} \qquad \ldots(6\cdot31)$$

where M and d have no common divisor.

Thus, for a fractional slot winding :

(*i*) number of poles in a unit$=d$

(*ii*) number of slots per phase in each unit$=M=qd$

(*iii*) number of total slots in each unit$=mM$

(*iv*) number of units$=p/d$. (This is also the maximum number of parallel paths).

(*v*) Each phase, in a unit, contains :

$(d-n)$ groups of I coils each and n groups of $(I+1)$ coils each.

6·23·1. Layout of fractional slot windings. It is clear from above that in fractional slot windings, there are two types of pole phase groups consisting of different number of slots. In order to lay out the winding, the distribution of slots around the armature has to be worked out. This distribution of slots to various phases should be uniform in order that the machines give good performance.

The investigation of a fractional slot winding is carried out proceeding from a slot emf star.

Consider a 2 pole, 12 slot, 3-phase double layer winding. The two layers in the winding are entirely symmetrical.

Electrical angle between adjacent slots $\alpha_s=180\times2/12=30°$.

This is also the displacement between emfs induced in coil sides in adjacent slots. Fig. 6·50 is the slot emf star of this winding, the firm lines represent the emf induced in the

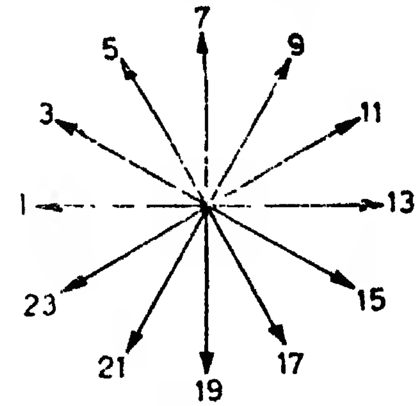


Fig. 6·51.

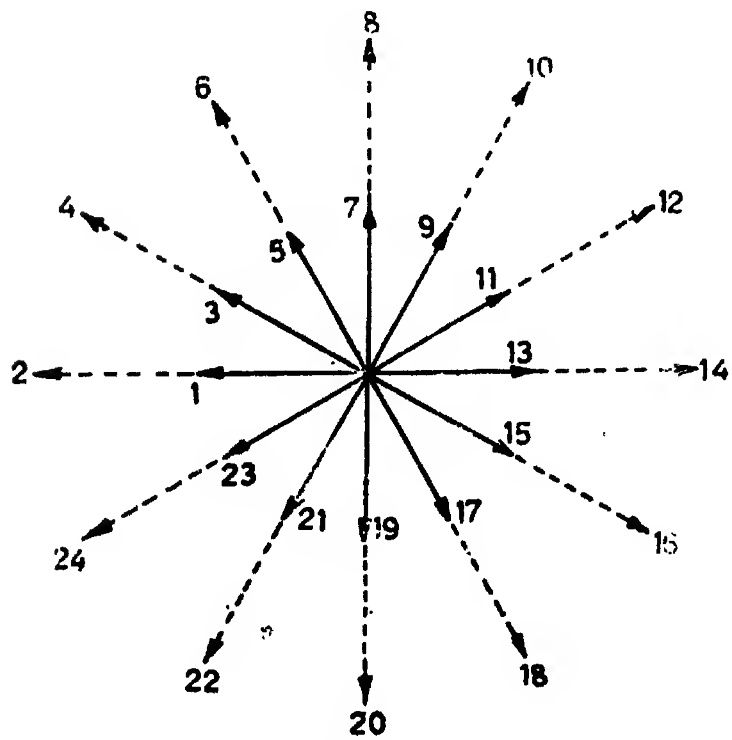


Fig 6·50.

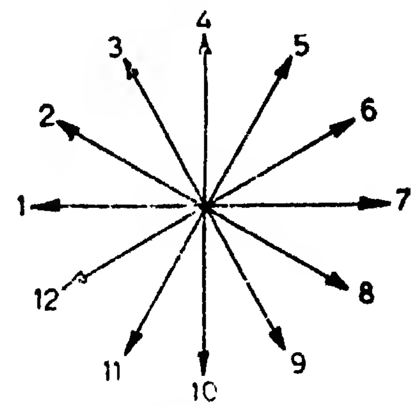


Fig. 6·52.

top coil sides. The dotted lines represent the emfs induced in the bottom coil sides. If full pitch coils are used, coil side 1 is conneted to coil side 14 ; the emfs of two are equal but are displaced through 180°. This is true of all the bottom coil sides and the corresponding top coil sides. If chorded coils are used, the emfs of corresponding bottom coil sides are shifted through an angle by which the coils are chorded. For example, if the coils are chorded by one slot pitch *i.e.* 30°, coil side 1 is connected to coil side 12. In this case the emfs of the two coil sides are equal but are displaced by $(180°-30°)=150°$. This means that the emfs of the corresponding coil sides are now shifted through 30° (the angle by which the coil is chorded) from the previous position.

From above, it is clear that the emf stars of both the layers are indentical but are shifted with respect to each other by the angle of chording. Therefore it suffices to consider any layer. Let us consider the top coil sides only (top layer). Fig. 6·51 shows the emf star of top coil sides. Because we have eliminated the bottom coil sides from the discussion, it becomes simpler to consider slots instead of coil sides. Fig. 6·52 shows the phase of emfs induced in the coil sides of various slots, this star of emfs is known as the **slot emf star** or **slot star**. The numbers marked represent the number of the slots (instead of number of the coil sides as taken earlier).

The distribution of slot is :

Phase	*Pole No.*	*Slot No.*
R	1	1, 2
	2	7, 8
B	1	3, 4
	2	9, 10
Y	1	5, 6
	2	11, 12

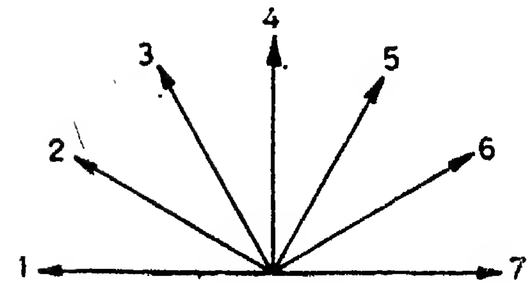


Fig. 6·53

The emf of phasor 1 (under pole No. 1) and 7 (under pole No. 2) are equal but are shifted through 180°. The connections between phasors 1 and 7 are so made that their emfs add. Therefore, this shift of 180° can be disregarded and phasor 7 can be taken as coincident with phasor 1. The same applies to other phasors in pairs *i.e.* phasors 2 and 8, 3 and 9, 4 and 10 and so on. Thus the star of a 2 pole integral slot winding can be represented by half of a circle. As poles form a basic unit of an integral slot winding, the whole of the winding can be represented by half of a circle. Fig. 6·53 shows the slot star represented by half of a circle.

Instead of representing slot star by graphical construction, the following description may be used.

Slot	1	2	3	4	5	6	7	8	9	10	11	12
Angle between slots	0	30	60	90	120	150	180	210	240	270	300	330

Ignoring the 180° phase shift.

Slots	1	2	3	4	5	6	7	8	9	10	11	12
Angle between slots	0	30	60	90	120	150	0	30	60	90	120	150

If we examine the sequence of phasors in the given integral slot winding it is clear from the above table that the sequence of phasors in the slot star (which is ultimately the sequence of slots in the magnetic field) is the same as the sequence of slots in the machine armature.

Considering a fractional slot winding, take the example of a 36 slot, 10 pole, 3 phase winding, we have,

$$\text{slots/pole/phase} \quad q=\frac{36}{10\times 3}=\frac{6}{5}=1\frac{1}{5}$$

Now $\quad M=6,\ d=5,\ I=1,\ n=1.$

Therefore in this winding, there are 2 basic units of $d=5$ poles each. Each unit of $d=5$ poles consists of $M=6$ slots of each phase.

Each phase has $d-n=4$ groups of $I=1$ coil and $n=1$ group of $I+1=2$ coils each.

Electrical angle between consecutive slots, $\alpha_s=180\times 10/36=50°$.

The basic unit in a fractional slot winding consists of d poles and so a slot star of d poles is considered. In this example, we have a slot star of $d=5$ poles.

Total number of slots in a unit $=mM=3\times 6=18$.

Thus the slot star of this winding will have 18 phasors.

Slots	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Angle between slots	0	50	100	150	200	250	300	350	400	450	500	550	600	650	700	750	800	850

Subtracting 180° or its multiplier :

Slots	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Angle between slots	0	50	100	150	20	70	120	170	40	90	140	10	60	110	160	30	80	130

Putting the slots in order of position they occupy in the magnetic field, the sequence is (as shown by slot star Fig. 6·54).

Slot.	1	12	5	16	9	2	13	6	17	10	3	14	7	18	11	4	15	8

and the angle between phasors is $\frac{180}{18}=10°$.

It is clear from above that unlike in integral slot windings, the sequence of phasors in the magnetic field is not the same as in the armature of the machine in the case of fractional slot windings. We have also seen that in fractional slot windings, the angle between the consecutive slots is not the same as the angle between phasors of the slot star.

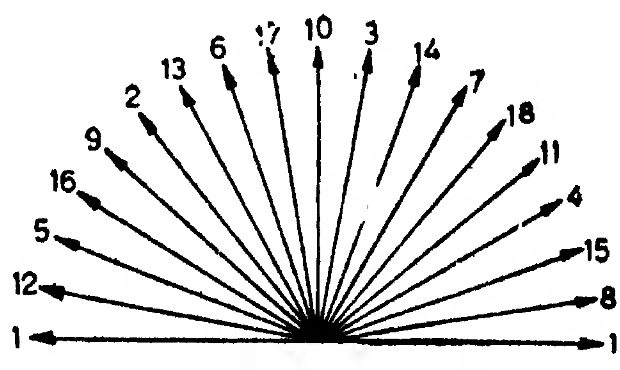


Fig. 6.54.

The angle between consecutive phasors in a slot star is known as the **magnetic field angle** and is equal to

$$\alpha_m=180/mM \qquad ...(6·32)$$

and angle between consecutive slot $\alpha_s=180p/S=180/mq$

and as $q=M/d$

$\therefore$ $\alpha_s=180\,d/mM$ ...(6·33)

From Eqns. 6·32 and 6·33 : $\alpha_s/\alpha_m=d$ ...(6·34)

in fractional slot windings. In the case of integral slot windings $\alpha_s=\alpha_m$.

In order to obtain high distribution factor, the phasors which lie closer should be assigned to a phase. Thus out of 18 phasors, the first $M=6$ phasors are assigned to phase R, middle 6 to phase B and the last 6 to phase Y (Fig. 6·54).

The distribution of slots in a basic unit of $d=5$ poles is :

Phase R	1	12	5	16	9	2
Phase B	13	6	17	10	3	14
Phase Y	7	18	11	4	15	8

In order to find a general procedure for distribution of slots, we should examine the above results. Starting with slot 1, the sequence of slots follows the series :

$$1,\ 1+11,\ 1+2\times11-mM,\ 1+3\times11-mM,...$$

As the total number of slots in a basic unit is mM, the value of mM (or its multiple) has to be subtracted from the terms which become larger than mM.

In general, the series is :

$1,\ 1+D,\ 1+2D\ 1+3D,\ 1+4D, \ldots\ldots\ldots [1+(mM-1)\ D-x\ mM]$

where D=difference between two slots which correspond to two consecutive phasors of slot star.

and x=any integer.

The electrical angle between two slots which correspond to two consecutive phasors of slot star$=\alpha_m+180\ P$

where P=number of pole pitches between the above slot.

This electrical angle should also be equal to $D\alpha_s$.

$$\therefore \quad D\alpha_s=\alpha_m+180P \quad \text{or} \quad D=\frac{\alpha_m}{\alpha_s}+\frac{180\ P}{\alpha_s}=\frac{1}{d}+\frac{mMP}{d}=\frac{1+mMP}{d} \qquad \ldots(6{\cdot}35)$$

as from Eqns. 6·33 and 6·34, $\alpha_m=180d/mM$ and $\alpha_m/\alpha_s=1/d$.

P is the smallest integer which makes D an integer.

Using Eqn. 6·35 for the case considered, $D=\dfrac{1+3\times 6P}{5}$.

The smallest value of P which makes D an integer, is 3.

Putting $P=3$ we get, $D=11$.

Example 6·27. *Give the layout of a double layer lap type winding for a 3 phase, 8 pole, 54 slot armature.*

Solution. We have $q=\dfrac{S}{mp}=\dfrac{54}{3\times 8}=\dfrac{9}{4}=2\dfrac{1}{4}$.

Thus $M=9, \quad d=4, \quad I=2, \quad n=1$.

The basic unit of this winding consists of $d=4$ poles. As there are 8 poles, so the winding has 2 units Each unit of 4 poles has $M=9$ slots of each phase or the total number of slots in a unit is $mM=27$.

Each phase in a unit has :

$d-n=3$ groups of $I=2$ coils each and $n=1$ group of $I+1=3$ coils each

Now $\quad D=\dfrac{1+mMP}{d}=\dfrac{1+3\times 9\times P}{4}$

The smallest value of P for D to be an integer is 1.

Putting $\quad P=1$ we have, $D=\dfrac{1+3\times 9\times 1}{4}=7$.

The $M=9$ slots assigned to phase R are :

$1,$	$1+D,$	$1+2D,$	$1+3D,$	$1+4D-mM,$	$1+5D-mM,$	$1+6D-mM,$	$1+7D-mM,$
1	8	15	22	2	9	16	23
$1+8D-2mM.$							
3							

These slots, when arranged according to their position in the machine, are :

1, 2, 3   8,9   15, 16   22, 23

The $M=9$ slots assigned to phase B are :

$1+9D-2mM,$	$1+10D-2mM,$	$1+11D-2mM,$	$1+12D-3mM,$	$1+13D-3mM,$
10	17	24	4	11
$1+14D-3mM,$	$1+15D-3mM,$	$1+16D-4mM,$	$1+17D-4mM.$	
18	25		12	

These slots, when arranged according to their position in the machine are :

4, 5 10, 11, 12 17, 18 24, 25

The remaining 9 slots out of a total of 27 in a unit, belong to phase Y. These, when arranged according to their sequence in the machine, are :

6, 7 13, 14 19, 20, 21 26, 27.

Considering the slots assigned to the 3 phases, the first pole phase group consists of slots 1, 2 and 3 and it belongs to phase R. The second pole phase group comprises of slots 4 and 5 and it belongs to phase B. The third pole phase group consists of slots 6 and 7 and it belongs to phase Y, and so on. Thus in a unit of 4 poles, the sequence of pole phase groups is :

Phase	R	B	Y	R	B	Y	R	B	Y	R	B	Y
No. of coils in a phase group	3	2	2	2	3	2	2	2	3	2	2	2

It is evident from above that each phase has, in a unit, 1 group of 3 coils each and 3 groups of 2 coils each. Splitting the above sequence in 3 parts.

Phase	$R\ B\ Y\ R$	$B\ Y\ R\ B$	$Y\ R\ B\ Y$
Coils in a phase group	3 2 2 2	3 2 2 2	3 2 2 2

We find that these three parts are similar. Thus it is only necessary to know the coil grouping of only a third of a basic unit.

Total number of units is p/d and the total number of phase groups for a 3 phase winding is $3p$.

Phase groups per unit $=\dfrac{3p}{p/d}=3d$ and $\frac{1}{3}$ of this is d.

Thus it is only necessary to determine the sequence of d phase groups in order to know the sequence of all the phase groups of the winding.

The connections and layout of phase groups is shown in Fig. 6·55.

In order to draw the winding diagram the top coil sides of all the phases are drawn. The bottom coil sides are drawn after deciding the coil span.

Coil span $C_s \simeq$ slots per pole $\simeq 27/4$

Taking a coil span of 6 slots. The top coil side in slot 1 is connected to bottom coil side in slot 1+6=7. Now the bottom coil sides can be drawn with

$C_s=6$ slots, and $y_b=13,\ y_f=11$.

Fig. 6·56 shows the complete winding diagram of a unit. The whole layout repeats itself after one unit.

Example 6·28. ***Give the winding layout of a 30 slot, 4 pole, 3-phase wave wound rotor of an induction motor.***

Solution. Slots/pole/phase $q=\dfrac{30}{3\times 4}=\dfrac{5}{2}=2\frac{1}{2}$.

Thus $M=5,\ d=2,\ I=2,\ n=1.$

Each unit covers 2 poles and there are two such units. Each unit contains 5 slots of each phase. Each phase in a unit has :

$d-n=1$ group of $I=2$ coils. and $n=1$ group of $I+1=3$ coils each.

From Eqn. 6·35, $D=\dfrac{1+3\times5\times P}{2}$.

The smallest value of P to make D an integer is 1. $\therefore$ $D=8$ with $P=1$.

The distribution of slots is :

Phase R	1, $1+D$,	$1+2D-mM$,	$1+3D-mM$,	$1+4D-2mM$.	
	1, 9	2	10	3	
Phase B	$1+5D-2mM$,	$1+6D-3mM$,	$1+7D-3mM$,		
	11	4	12		
	$1+8D-4mM$,	$1+9D-4mM$.			
	5	13			

Putting the slots in the sequence which they occupy in the machine

Phase R	1, 2, 3	9, 10
Phase B	4, 5	11, 12, 13,
Phase Y	6, 7, 8	14, 15.

All the top coil sides are drawn. The bottom coil sides can only be drawn if the coil span is known.

Coil span $C_s \simeq$ slots per pole $\simeq 30/4=7$ slots.

This means that top coil side in slot 1 is connected to bottom coil side in slot $1+7=8$, or $y_b=15$ coil sides. Winding pitch $Y=12\times2\frac{1}{2}=30$ $\therefore$ $y_f=30-15=15$.

Now the bottom coil sides can also be drawn. The winding can be completed in a similar manner as described for integral slot a.c wave windings in example 6·25.

Fig. 6·57 shows the winding diagram for phase R. The winding for phases B and Y can be similarly completed.

The distribution of slots to various phases can be done very easily by the following method :

1. Put down the fraction $\dfrac{\text{number of slots}}{\text{number of poles}}$ and reduce the numerator and the denominator to the lowest pair of whole numbers. The denominator is equal to d, the number of poles in a basic unit.

2. Make a table of squares having as many columns as the numerator of the reduced fraction and as many rows as the number of poles in a basic unit. The table is divided into three equal vertical parts (for three phases). The numerator should definitely be a multiple of number of phases *i e*. 3 in the case of 3 phase windings.

3. Start making crosses in the squares. Mark the first cross in the upper left square, proceed from left to right and put cross in every dth square where d is the denominator of above said fraction.

4. The number of coils in a phase group of a particular phase is equal to the number of crosses in a horizontal row within columns of that phase.

5. The sequence of pole phase groups follows from left to right and row by row starting from upper left to lower right.

The application of this method is illustrated by the following examples.

Example 6·29. *Determine the slot distribution and the pole phase group sequence for a 45 slot, 6 pole, 3 phase winding.*

Solution. 1. Write the fraction slots/poles=45/6 and reduce it to 15/2 where 15 and 2 are the lowest pair of whole numbers and the numerator (*i.e.* 15) is divisible by 3. Hence $d=2$.

Therefore there are 2 poles in each unit and so there are $p/d=3$ similar units.

2. Make a table having columns=numerator of the reduced fraction=15 and rows=denominator of the reduced fraction=2.

3. Divide the table in three vertical parts each having 15/3=5 columns and 2 rows. (See Table below).

Phase R					Phase B					Phase Y				
X		X		X		X		X		X		X		X
	X		X		X		X		X		X		X	

4. Mark cross in the upper left square and move from left towards right. Continue marking cross in every dth square. Here $d=2$ and so a cross is to be marked in every second square.

5. Count the number of crosses in each horizontal row in the columns of each phase. For example, in the first row, there are 3 crosses for phase R, 2 crosses for phase B and 3 crosses for phase Y. This means that first pole phase group of phase R has 3 coils, of phase B, 2 coils and that of phase Y has 3 coils.

6. Proceeding row by row from top left to right bottom, the sequence of coil groups is :

Phase	*R*	*B*	*Y*	*R*	*B*	*Y*
Coil in phase group	3	2	3	2	3	2

Thus in a unit, for each phase, there are : 1 group of 3 coils and 1 group of 2 coils.

Distribution of Slots. Starting with slot 1. The first phase group of phase R has 3 coils and thus slots 1, 2 and 3 are assigned to phase R. The first phase group of phase B has 2 slots and so slots 4 and 5 belong to phase B. The first phase group of Y has 3 slots so slots 6, 7 and 8 are assigned to it. The complete distribution of slots is :

Phase	*Slot No.*	
R	1, 2, 3	9, 10
B	4, 5	11, 12, 13
Y	6, 7, 8	14, 15

The winding repeats itself for the other two units.

Example 6·30. *Determine the pole phase group sequence and distribution of slots for a 3 phase, 48 slot, 10 pole winding.*

Solution. 1. Write the fraction slots/poles=48/10 and reduce it to 24/5 where 24 and 5 are the lowest pair of whole numbers.

Here $d=5$ and therefore are 5 poles per unit, and thus there are 2 such units.

2. Make a table having columns=numerator of reduced fraction=24

and rows=denominator of reduced fraction=5.

3. Divide the table into three vertical parts, each having 24/3=8 columns and 5 rows. (See table below).

Phase R								Phase B								Phase Y							
X					X					X					X					X			
	X					X					X					X					X		
		X					X					X					X					X	
			X					X					X					X					X
				X					X					X					X				

4. Mark cross in upper left square and move from left to right. Continue marking cross in every dth square. Here $d=5$ and so the cross is to be marked in every 5th square.

5. Count the crosses.

The sequence of coil groups is :

Phase	R	B	Y	R	B	Y	R	B	Y	R	B	Y	R	B	Y
No. of coils in phase group	2	2	1	2	1	2	2	1	2	1	2	2	1	2	1

and the distribution of slots in a unit is :

Phase	*Slot No.*				
R	1, 2	6, 7	11, 12	16	21
B	3, 4	8	13	17, 18	22, 23
Y	5	9, 10	14, 15	19, 20	24

6·23·2. Conditions for electrical balance or symmetry. We have laid out the fractional slot windings assuming that they are balanced. Let us examine whether they are balanced or not. Taking the example of a 36 slot, 10 pole, of 3 phase winding explained on page 278.

Consider the slot star of Fig. 6·54. All the phasors are equal in magnitude but are displaced with respect to each other by 10°. First six phasors are assigned to phase R. The next six phasors are assigned to phase B and the last six to phase Y. Refer to Fig. 6·58. If we find the resultant phasor of phasors of each phase, we see that the resultant phasor of phase B is shifted 60°, and the resultant phasor of phase Y is shifted 120°, from the resultant phasor of phase R. These three resultant phasors are equal in magnitude.

Reversing the connections of phase B, we get three equal phasors displaced 120° with respect to each other. Thus we get an absolutely balanced three phase winding.

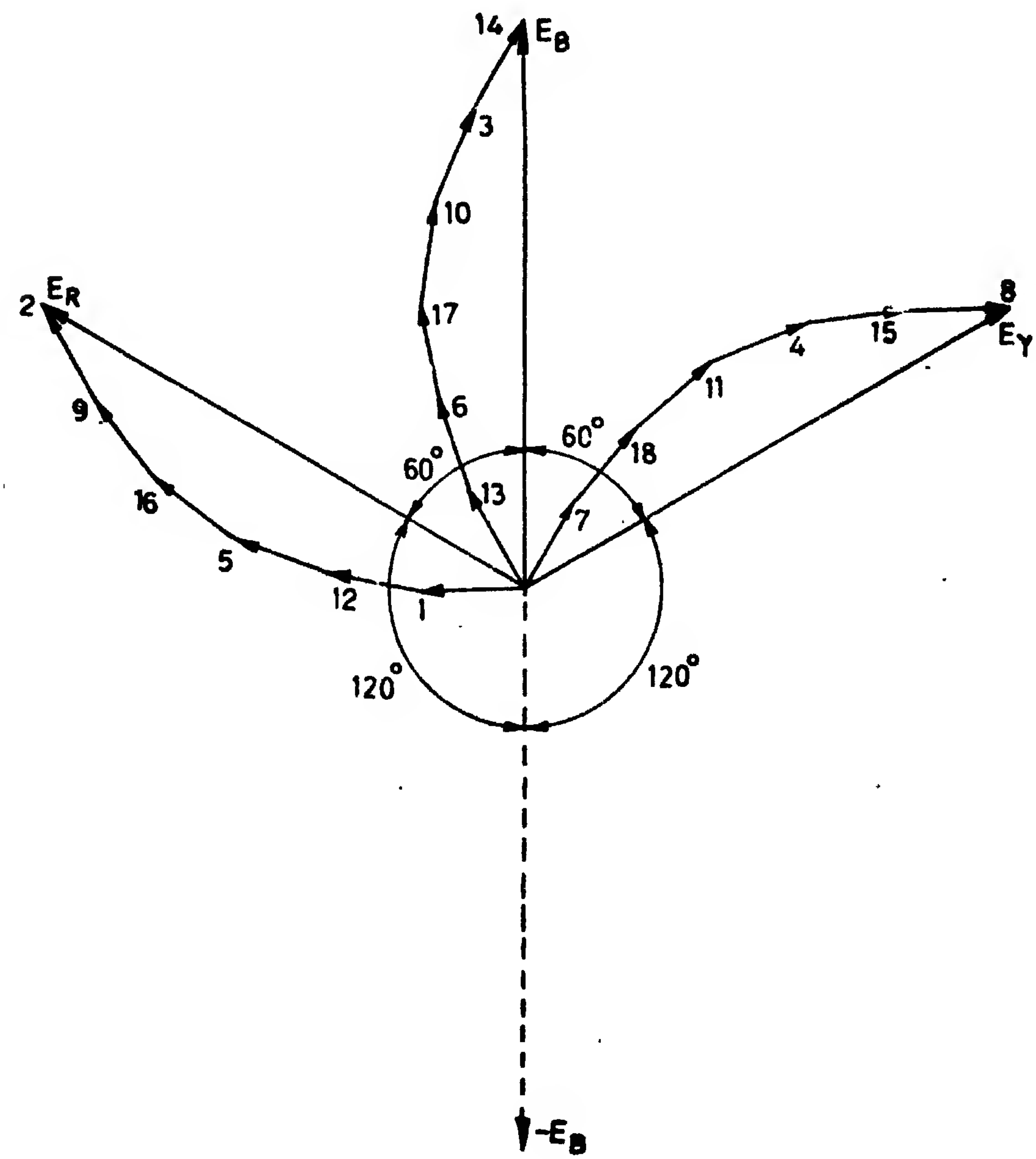


Fig. 6·58.

(i) Consider a 3 phase, 6 pole 48 slot winding.

We have $$q=\frac{42}{6\times 3}=\frac{8}{3}=2\frac{2}{3}$$

Here $M=8$ and $d=3$.

$\therefore$ $$D=\frac{mMP+1}{3}=\frac{3\times8\times P+1}{3}$$ (see Eqn. 6·35)

Thus there is no integral value P which makes D as integer. Therefore, the sequence of slots in the magnetic field is the same as the sequence of the slots in the machine (This can be seen by drawing the slot star. Fig. 6·59 shows the slot star with $\alpha_s=5\times180/48=22\cdot5°$). It clearly shows that the winding is unbalanced. Thus we can say that if the denominator of q is divisible by 3 in 3 phase windings and by m in m phase windings, the winding is not balanced.

$\therefore$ The first condition of balance is :

d/m should not be an integer.

Thus, for symmetry of the phases, phase pitch $Y_{ph}=2C/m$ should be an integer.

$$Y_{ph}=\frac{2C}{m}=\frac{2mpq}{m}.$$

In order that Y_{ph} be an integer the denominator in q should not be m or a multiple of m. Thus in a 3 phase machine, the denominator in q should not be 3 or multiple of 3.

(ii) Consider the case of a 3 phase, 4 pole, 28 slot winding, we have

$$q=\frac{28}{3\times4}=\frac{7}{3}=2\frac{1}{3}.$$

One unit comprises of $d=3$ poles. The number of poles is 4.

Thus the number of units$=(p/d)$ is not an integer and therefore the winding is not balanced.

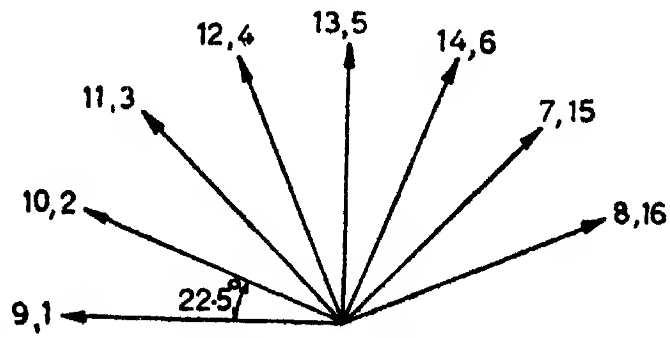


Fig. 6·59.

This is due to the number of slots not being a multiple of number of phases (28 is not a multiple of 3).

$\therefore$ The second condition of balance is that slots should be a multiple of number of phases or

S/m should be an integer.

6·24. Tappings and Openings. In order to provide phase-tappings or openings in commutator type windings (as in rotary convertors or a.c. windings with fractional slot windings), it is necessary to know the number and position of concerned coil, coils sides, and joints (or commutator segments).

The coil and coil side positions are found as below :

1. **Lap Windings.** (See Fig. 6·25).

Table 6·1

Coil No.	1	2	3	x
Joint or segment No.	1	2	3	x
Top coil side No.	1	3	5	$(2x-1)$
Bottom coil side No.	$1+y_f$	$3+y_f$	$5+y_f$	$(2x-1)+y_f$

2. **Wave Windings.** (See Fig. 6·26).

Table 6·2

Coil No.	1	2	3	x
Joint or segment No.	1	$1+y_c$	$1+2y_c$	$1+(x-1)y_c=b$
Top coil side No.	1	$1+2y_c$	$1+4y_c$	$1+2(x-1)y_c=2b-1$
Bottom coil side No.	$1-y_f$	$1+2y_c-y_f$	$1+4y_c-y_f$	$1+2(x-1)y_c-y_f=(2b-1)-y_f$

The positions of top and bottom coil sides in their respective slots for both lap and wave windings may be found by dividing the numerical values by u, the coil sides per slot.

Example 6·31. *A simplex wave winding has 8 poles, 105 slots and 105 coils. Find (a) tappings of a winding for a 3 phase load (b) openings for a symmetrical 3 phase winding with a phase spread of 60°.*

Solution.

(*a*) Phase pitch $Y_{ph}=\frac{2C}{ma}=\frac{2\times105}{3\times2}=35$ coils. Also $\frac{2S}{ma}=\frac{2\times105}{3}=70$.

As both Y_{ph} and $2S/m$ are integers, it is possible to obtain a 3 phase symmetrical winding.

Phase	R	Y	B
Joint	1	$1+Y_{ph}=36$	$1+2\ Y_{ph}=71$.

In the complexer polygon of Fig. 6·60 (*a*) it is seen that the positions of coils with these numbers are displaced by 120°.

(*b*) For a wave winding :

Commutator pitch $y_c=\frac{2C\pm a}{p}=\frac{2\times105\pm2}{8}=26$ coils.

Resultant pitch $Y=2y_c=52$ coil sides.

∴ Back pitch $y_b=27$ coils sides, front pitch $y_f=25$ coil sides.

Coils per phase in a 6 phase winding$=\frac{105}{6}=17\frac{1}{2}$.

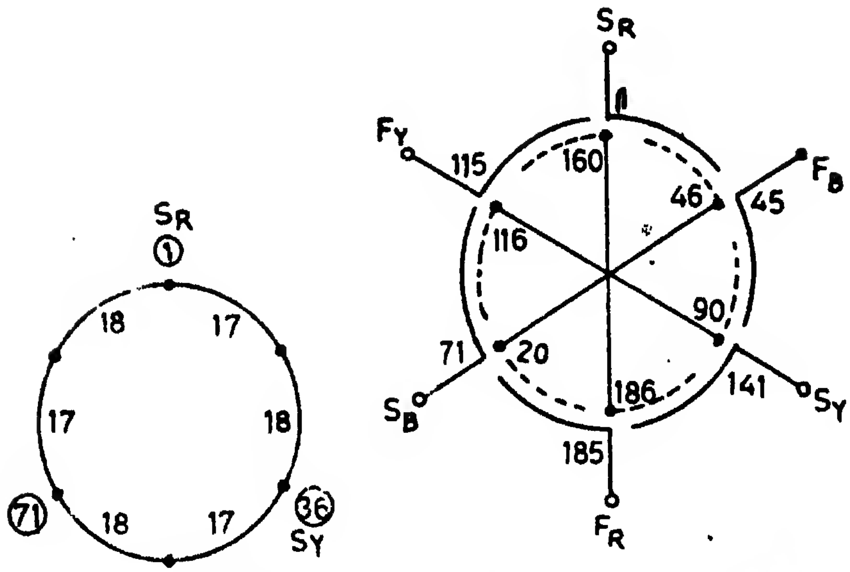
Each of the six portions, into which the winding is split, consists of 17 and 18 coils alternately.

Therefore the coils to be opened are :
(*i*) 1 (*ii*) $1+17=18$ (*iii*) $18+18=36$ (*iv*) $36+17=53$ (*v*) $53+18=71$ (*vi*) $71+17=88$.

Referring to Table 6·2.

Coil No. x	*Joint No.* $1+(x-1)\,y_c=b$	*Top Coil Side No.* $2b-1$	*Bottom coil Side No.* $(2b-1)-y_f$
S_R 1	1	1	186
F_B 18	23	45	20
S_Y 36	71	141	116
F_R 53	93	185	160
S_B 71	36	71	46
F_Y 88	58	115	90

Fig. 6·60 (*b*) shows connections for the three phases.



(*a*) Complexer polygon. (*b*) Connections for 3 phases.

Fig. 6.60

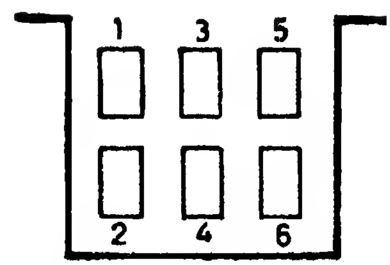


Fig. 6.61. Positions of coil sides in slots.

Example 6·32. *A simple wave connected armature is to be opened at 6 places. Given 8 poles, 57 slots and 6 coil sides per slot. Find :*

(*a*) *coil sides to which connections are made,*
(*b*) *positions of these coil sides in the slots.*

Solution. Number of coils $C=\frac{1}{2}uS=\frac{1}{2}\times6\times57=171$.

Commutator pitch $y_c=\dfrac{2\times171\pm2}{8}=43$ coils.

Resultant pitch $Y=2\times43=86$. Taking $y_b=43$ coil sides, $y_f=43$ coil sides.

Coils per phase in a 6 phase winding$=\dfrac{171}{6}=28\frac{1}{2}$.

Six portions, into which the winding is split, consists of 28 and 29 coils alternately.

Therefore the coils to be opened are :
(*i*) 1 (*ii*) 1+29=30 (*iii*) 30+28=58 (*vi*) 58+29=87 (*v*) 87+28=115 (*vi*) 115+29=144.

Referring to Table 6·2

Coil No. x	*Joint No.* $1+(x-1)\times y_c=b$	*Top Coil Side=2b—1* No.	*Top Coil Side=2b—1* Position	*Bottom Coil Side=2b—1—y_f* No.	*Bottom Coil Side=2b—1—y_f* Position
S_R 1	1	1	Slot 1, coil side 1	300	Slot 50, coil side 6
F_B 30	51	101	Slot 17, coil side 5	58	Slot 10, coil side 4
S_Y 58	58	115	Slot 20, coil side 1	72	Slot 12, coil side 6
F_R 87	108	215	Slot 36, coil side 5	172	Slot 29, coil side 4
S_B 115	115	229	Slot 39, coil side 1	186	Slot 31, coil side 6
F_Y 144	165	329	Slot 55, coil side 5	286	Slot 48, coil side 4

Connections. Connect coil side 6 in slot 50 to coil side 4 in slot 29. Connect coil side 6 in slot 12 to coil side 4 in slot 48. Connect coil side 6 in slot 31 to coil side 4 in slot 10. Refer to Fig. 6 61 for positions of coil sides in slots.

PRODUCTION OF EMF IN WINDINGS

6 25. **Choice of double layer windings**. It has been explained earlier that double layer windings are of two types: (*i*) lap and (*ii*) wave. The choice between lap and wave windings, in a.c. machine, is governed by many design considerations. However, it should be noted that parallel circuits do not arise (and hence do no restrict the choice) in the same way as for the closed commutator windings. It is, of course, possible to connect coils of open windings in parallel groups, provided that each phase has sufficient number of identical sections. Lap windings provide a greater number of such sections than the wave windings.

It is useful practice to use lap windings in a.c. machines. The coils forming the winding may be single turn or multiturn. The choice of coil depends upon the number of turns per phase required to generate emf, the current rating and the number of parallel paths in the winding.

Multiturn coils. An examination of the emf equation for a.c. machines reveals that the number of turns per phase varies directly as the voltage per phase and inversely as the flux per pole. Thus high voltage machines and machine with a large number of poles (having small value of flux per pole) have a relatively large number of turns per phase. Therefore, for such machines **multiturn coils** are used. When multiturn coils have to be used for a machine the choice lies between a double layer winding and a single layer winding. In double layer windings, requiring good slot insulation, completely insulated preformed coils are dropped into open slots. However, in small low voltage machines, when the requirements of insulation are not stringent, the conductors may be dropped one by one through the narrow openings of semi-enclosed slots insulated with a slot liner which serves as the main insulation.

Modern machine building practice favours the use of double layer windings except in cases where the large openings of open slot (used for double layer windings of high voltage machines) become large as compared with the length of air gap thereby resulting in large magnetizing mmf and poor power factor as in large high voltage induction motors.

Single turn coils. Machines like turbo-alternators have only two poles and therefore the flux per pole is large and hence the number of turns per phase is quite small. Also, low

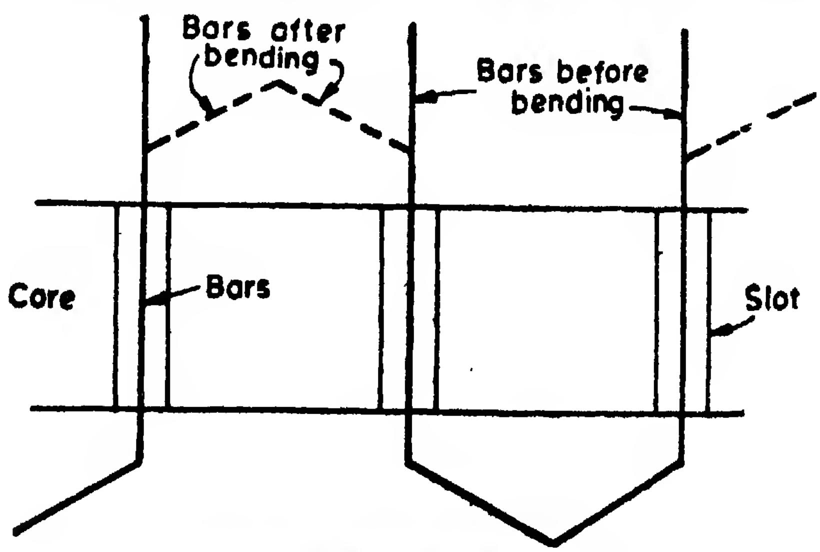


Fig. 6·62. Bar type wave winding.

voltage machines have a small number of turns per phase. Therefore, these machines require the use of single turn coils. **Single turn coils** require the use of **bar windings** which are commonly used for low voltage machines with large phase currents. There is usually one conductor per layer and the conductor is laminated and transposed along its length in order to reduce eddy current losses.

Bar windings are used in turbo-alternators and low voltage multipolar machines. For these machines double layer bar lap or wave winding may be used. In both the cases, bars are pushed through partially closed slots and are bent to shape at the other end as shown in Fig. 6·62. This practice is followed when the conductor section is moderate. However, in the case of turbo-alternators, the bars are quite heavy in cross-section and therefore they must be completely formed before being inserted into slots.

Bar type windings are also used for rotors of large slip ring induction motors where current handled is large. Bar wave windings are preferred in this case as lap windings gives a large number of connectors which is undesirable for rotors.

6·26 Construction of coils. Fig. 6·63 shows the various stages in the manufacture of multiturn diamond coils used in double layer windings. In case the coil section is not heavy, it is wound as a flat over two dowels. The slot portion of the coil is gripped in a machine and pulled apart to give the required coil span. Coils having heavy cross-section are built up on formers. The overhang portion is bent up to make angle with conductors in the slots forming an involute shape as shown in Fig. 6·63.

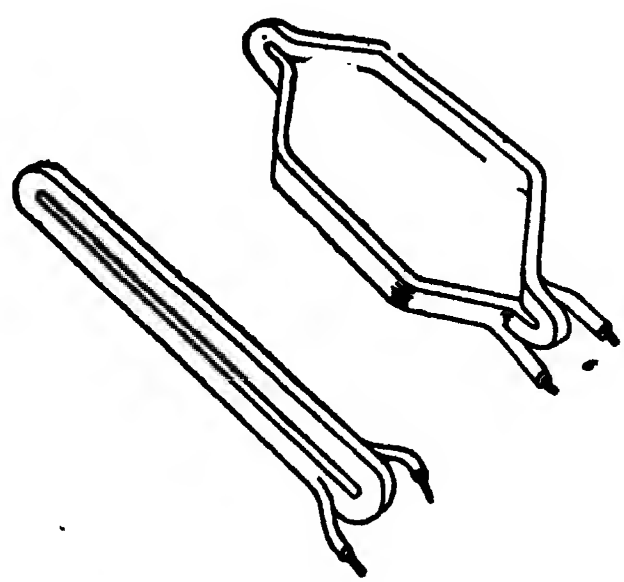


Fig. 6·63. Diamond type coil (Pulled type).

6·27. Emf Generated in a Conductor. The emf generated in a conductor on the armature of a rotating electrical machine is $e_0 = Blv$

where, B is the flux density ; l the length of conductor and v is the linear velocity of the conductor.

This emf is alternating in nature and the frequency of alternations is $f = pn/2$ where p is the number of poles and n is the speed in r.p.s.

The flux density distribution around the air gap of all properly designed electrical machines is symmetrical. The distributions are symmetrical in respect to abscissa and also in respect to polar axes. Thus they can be expressed by a Fourier series which does not contain any even harmonics.

Referring to Fig. 6·64,

flux density at any angle θ from interpolar axis

$$B\theta = B_{m1}\sin\theta + B_{m3}\sin 3\theta + \ldots\ldots + B_{mn}\sin n\theta + \ldots\ldots \quad \text{—(6·36)}$$

B_{m1}=amplitude of fundamental component of flux density,

B_{m3}= amplitude of 3rd harmonic component of flux density,

... .

B_{mn}=amplitude of nth (odd) harmonic component of flux density.

Therefore, emf generated in a conductor located at an angle θ from interpolar axis is :

$$e_c = [B_{m1}\sin\theta + B_{m3}\sin 3\theta + \ldots\ldots + B_{mn}\sin n\theta + \ldots\ldots]Lv$$

Now, linear velocity $v = \pi Dn = 2\pi Df/p$

where D=diameter of the armature at the air gap,

and L=active length of armature conductor.

Substituting the value of v in the above expression,

$$e_c = \left[B_{m1}\frac{\pi DL}{p}2f\sin\theta + B_{m3}\frac{\pi DL}{p}2f\sin 3\theta + \ldots\ldots\ldots + B_{mn}\frac{\pi DL}{p}2f\sin n\theta + \ldots\ldots \right]$$

There are p poles corresponding to the fundamental component of flux density.

$\therefore$ Area of each fundamental pole, $A_1 = \dfrac{\pi DL}{p}$.

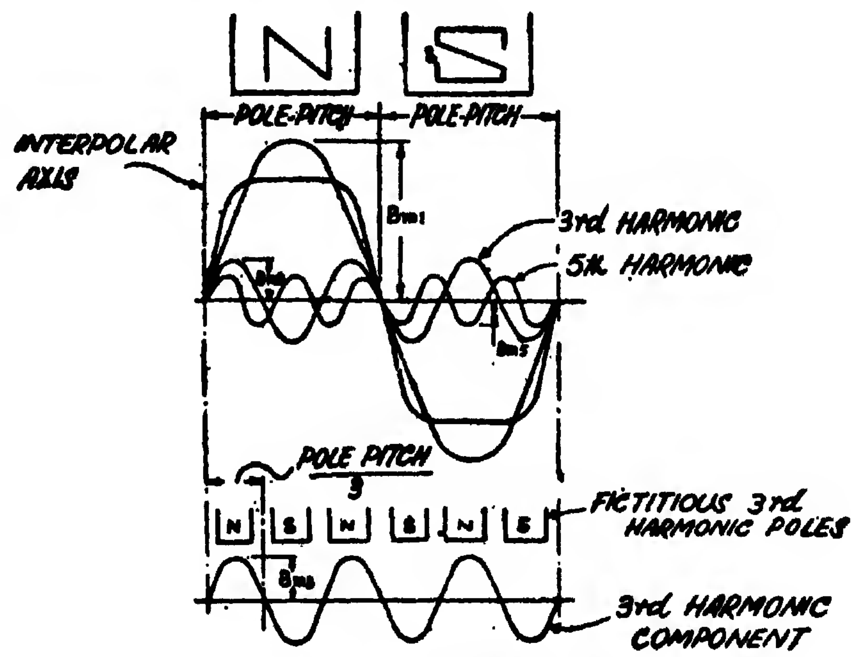


Fig. 6·64. Flux density distribution in space.

If there are p poles for the fundamental, there are np poles for the nth harmonic (Fig. 6.64).

Therefore, area of each nth harmonic pole $A_n = \pi DL/np = A_1/n$.

Hence,

$$e_c = [B_{m1}A_1 2f\sin\theta + B_{m3}A_1 2f\sin 3\theta + \ldots\ldots\ldots + B_{mn}A_1 2f\sin n\theta + \ldots\ldots\ldots]$$

$$= 2f[B_{m1}A_1\sin\theta + B_{m3}\,.\,3A_3\,.\,\sin 3\theta + \ldots\ldots\ldots + B_{mn}\,.\,nA_n\sin n\theta + \ldots\ldots\ldots]$$

Now each flux density harmonic determines a corresponding flux harmonic namely.

Fundamental component of flux per pole $\Phi_1 = A_1 \times \frac{2}{\pi} B_{m1} = \frac{2}{\pi} A_1 B_{m1}$ ...(6·37)

Similarly, nth harmonic flux per pole $\Phi_n = \frac{2}{\pi} A_n B_{mn}$ ...(6·38)

Substituting these values in expression for conductor emf,

$e_c = \pi f(\Phi_1 \sin\theta + 3\,\Phi_3 \sin 3\theta + \ldots + n\,\Phi_n \sin n\theta + \ldots\ldots)$ ...(6·39)

$\therefore$ Instantaneous value of fundamental frequency emf generated in a conductor

$$e_{c1} = \pi f\,\Phi_1 \sin\theta$$

and rms value of fundamental frequency emf generated in a conductor is :

$$E_{c1} = \frac{\pi}{\sqrt{2}} f\,\Phi_1 = 2{\cdot}22\,\Phi_1 f. \quad \ldots(6{\cdot}40)$$

and rms value of nth harmonic emf generated in a conductor is

$$E_{cn} = 2{\cdot}22\,\Phi_n \,.\, nf \quad \ldots(6{\cdot}41)$$

From Eqns. 6·39 and 6·40,

$$\frac{E_{cn}}{E_{c1}} = \frac{2{\cdot}22\,\Phi_n\,nf}{2{\cdot}22\,\Phi_1 f} = \frac{n\Phi_n}{\Phi_1}$$

but $A_1 = nA_n$ and $\Phi_1 = \frac{2}{\pi} A_1 B_{m1}$ and $\Phi_n = \frac{2}{\pi} A_n B_{mn}$

$$\therefore \quad \frac{E_{cn}}{E_{c1}} = \frac{n \times (2/\pi)\,A_n B_{mn}}{(2/\pi)\,A_1 B_{m1}} = \frac{nA_n B_{mn}}{A_1 B_{m1}} = \frac{B_{mn}}{B_{m1}} \quad \ldots(6{\cdot}42)$$

From Eqn. 6·42, it is clear that the relative magnitudes of harmonic emfs are proportional to their corresponding flux density components. Thus the emf generated in a conductor has the same waveform as that of that flux density distribution in space. This means that if the 3rd harmonic component of flux density is 10 per cent, the 3rd harmonic emf generated in the conductor would also be 10 per cent.

The rms value of resultant emf of a conductor is

$$\begin{aligned} E_c &= \sqrt{E^2_{c1} + E^2_{c3} + \ldots\ldots + E^2_{cn} + \ldots\ldots} \\ &= E_{c1}\sqrt{1 + (E_{c3}/E_{c1})^2 + \ldots\ldots + (E_{cn}/E_{c1})^2 + \ldots\ldots} \\ &= E_{c1}\sqrt{1 + (B_{m3}/B_{m1})^2 + \ldots\ldots + (B_{mn}/B_{m1})^2 + \ldots\ldots} \\ &= 2{\cdot}22\,\Phi_1 f\sqrt{1 + (B_{m3}/B_{m1})^2 + \ldots + (B_{mn}/B_{m1})^2 + \ldots\ldots} \end{aligned} \quad \ldots(6{\cdot}43)$$

6·28. Emf generated in full pitch coils. Fig. 6·65(a) shows a full pitch single turn coil. The two conductors forming a turn are 180 electrical degrees apart. The emf of such a turn is obtained as the phasor subtraction of the two individual conductor emfs E'_c and E''_c as shown in Fig. 6·65(b).

The rms value of a full pitch turn $E_t = 2E_c$.

In general, we may consider a full pitch coil having T_c turns connected in series, the emf generated in such a coil is :

$E_{coil} = T_c \times E_t = 2T_c\,E_t$

If a machine has p poles and like coils are placed one under each pole pair in the same position relative to the pole centres and if these $p/2$ coils are connected in series, there would form a concentrated winding. For this winding :

total turns T = turns per coil × number of coils $= T_c \,.\, p/2$.

Emf of a full pitch concentrated winding = number of coils in series × emf per coil

$$= (p/2)E_{coil} = 2T\,E_c$$

Substituting the value of E_c from Eqn. 6·43, emf of a concentrated full pitch winding

$$E = 2T \times 2{\cdot}22\ \Phi_1 f\ \sqrt{1+(B_{m3}/B_{m1})^2+\ldots\ldots\ldots\ldots+(B_{mn}/B_{m1})^2+\ldots\ldots}$$

$$= 4{\cdot}44\ T\ \Phi_1 f\ \sqrt{1+(B_{m3}/B_{m1})^2+\ldots\ldots\ldots\ldots+(B_{mn}/B_{m1})^2+\ldots\ldots} \quad \ldots(6{\cdot}44)$$

From above it is clear that the emf of a concentrated full pitch winding does not differ in waveform from the emf of a single conductor. Thus the waveform of emf of a full pitch concentrated winding is the same as that of the flux density.

From Eqn. 6·44, it follows that, the rms value of emf of fundamental frequency in a concentrated winding is,

$$E_1 = 4{\cdot}44\ T\ \Phi_1 f \quad \ldots(6{\cdot}45)$$

and the rms value of nth harmonic emf

$$E_n = 4{\cdot}44\ T\ \Phi_1 f \frac{B_{mn}}{B_{m1}} = 4{\cdot}44\ \Phi_1 f \frac{2/\pi\ A_n B_{mn}}{2/\pi\ A_n B_{m1}}$$

$$= 4{\cdot}44\ T\ \Phi_1\ nf \quad \ldots(6{\cdot}46)$$

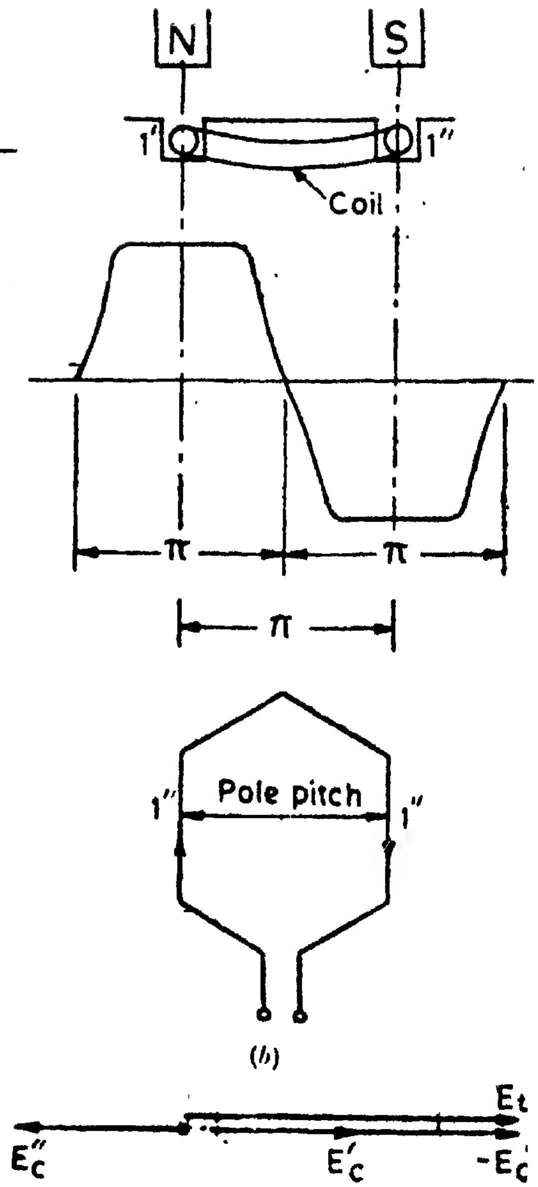


Fig. 6·65. Emf generated in full pitch coil.

6·29. Emf generated in a full pitch distributed winding. Armature windings of electrical machines are normally not concentrated but are distributed. The conductors are not concentrated in one slot per pole but are distributed in many slots.

Suppose a group of coils consists of N coils (A, B, C......N). The emfs of these coils, E_A, E_B, E_C,......E_N are all equal but are electrically displaced from each other by an angle ψ. The resultant emf of the coil group is the phasor summation of individual coil emfs. Considering Fig. 6·66, it is clear that the phasor sum is less than the arithmetic sum of coil emfs. Thus owing to distribution of conductors to more than one slot per pole, the resultant emf is less than the emf which would have been obtained had the winding been concentrated. In order to take into account the reduction in emf because of distribution of winding, a factor called **distribution factor** is introduced. Distribution factor is defined as :

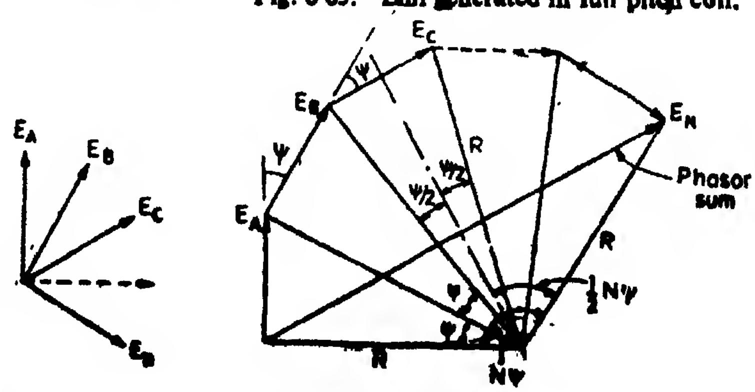


Fig. 6·66. Resultant emf

$$K_d = \frac{\text{actual voltage generated with distributed winding}}{\text{voltage generated in the winding considering it as concentrated}}$$

$$= \frac{\text{phasor sum of coil emfs}}{\text{arithmetic sum of coil emfs}}$$

From Fig. 6·66, $K_d=\dfrac{2R\sin N\psi/2}{N\times 2R\sin\psi/2}=\dfrac{\sin(N\psi/2)}{N\sin\psi/2}$ .. (6·47)

When the N coils, referred to above, from a pole phase group, we have

$N=q=$slots per pole per phase, and $N\psi=\sigma=$phase spread.

Therefore, distribution factor for fundamental

$$Kd_1=\frac{\sin\sigma/2}{q\sin\sigma/2q} \qquad ...(6·48)$$

The angle of displacement between two adjacent slots for a field harmonic of nth order is $n\psi$. The emf phasors of this harmonic are also displaced by the same angle *i e.* $n\psi$. Reasoning as before,

distribution factor for nth harmonic

$$K_{dn}=\frac{\sin n\sigma/2}{q\sin n\sigma/2q} \qquad ...(6·49)$$

From Eqns. 6·45 and 6·46 for a full pitch distributed winding, rms value of fundamental frequency emf $E_1=4.44\ T\Phi_1 f K_{d1}$ ...(6·50)

and rms value of nth harmonic emf $E_n=4·44\ T\Phi_n nf K_{dn}$ ...(6·51)

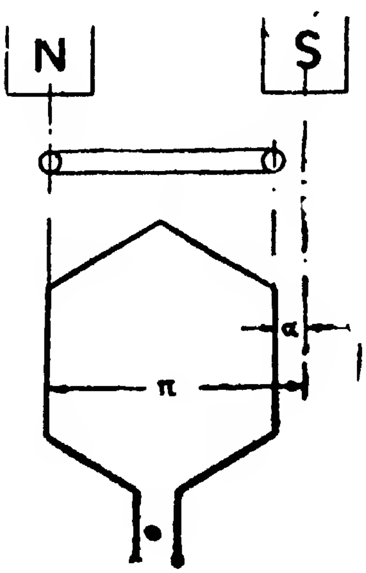
$$\therefore \quad \frac{E_n}{E_1}=\frac{n\Phi_n K_{dn}}{\Phi_1 Kd_1}=\frac{B_{mn} K_{dn}}{B_{m1} Kd_1}$$

Rms value of resultant emf of a distributed winding having T turns,

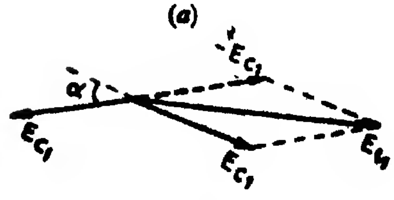
$$E=4·44\ T\ \Phi_1 f K_{d1}\sqrt{1+(B_{m3}Kd_3/B_{m1}Kd_1)^2+\ldots\ldots+(B_{mn}Kd_n/B_{m1}Kd_1)^2+\ldots\ldots} \qquad ...(6·52)$$

From Eqn. 6·52, it is clear that the emf generated in a full pitch distributed winding has a different waveform from that of the flux density. The magnitude of different harmonic emfs not only depends upon the amplitude of corresponding flux density harmonic but also on the distribution factors which may be different for different harmonics.

6·30. Emf generated in a fractional pitch concentrated winding. Alternating current windings are normally wound with fractional pitch (short pitch or chorded) coils *i.e.* the two coil side forming a coil are displaced by an angle less than 180°. Suppose α is the angle by which the coil span is short of 180° as shown in Fig. 6·67(*a*). The fundamental frequency emfs of the conductors forming a turn are equal in magnitude but are displaced from each other by an angle $(180-\alpha)$ as shown in Fig. 6·67(*b*).



(*a*)



(*b*)

Fig. 6·67. Emf of a chorded coil.

From the phasor diagram, the fundamental frequency emf of each turn is :

$$E_{t1}=\sqrt{E^2_{c1}+E^2_{c1}+2E^2_{c1}\cos\alpha}=2E_{c1}\cos\alpha/2.$$

If there are T_c turns per coil the fundamental frequency emf generated in a coil is,

$$E_{coil_1}=2T_c\ E_{c1}\cos\alpha/2.$$

If there are $p/2$ such coils under p poles, the fundamental frequency emf generated in a fractional pitch concentrated winding having T turns is,

$$E_1=(p/2)\times 2T_c\ E_{c1}\cos\alpha/2=2TE_{c1}\cos\alpha/2$$
$$=4·44\ T\Phi_1 f\cos\alpha/2 \qquad ...(6·53)$$

Comparing Eqn. 6·53 with Eqn. 6·45, we find that the emf reduces if we use fractional pitch coils. The reduction is proportional to $\cos\alpha/2$. We can write Eqn. 6·53 as :

$$E_1=4·44\ T\ \Phi_1 f K_{p1} \qquad ...(6·54)$$

where K_{p1} is called the **pitch factor** for the fundamental.

In general, considering the nth harmonic, the nth harmonic emfs generated in two coil sides of a fractional pitch coil would be displaced by an angle $n\,(180-\alpha)$ where n is an odd integer.

Therefore, pitch factor for nth harmonic $K_p = \cos n\alpha/2$ ...(6·55)

Thus the nth harmonic emf in a fractional pitch concentrated winding is :

$$E_n = 4{\cdot}44\ T\ \Phi_n\ n\ f\ K_{pn} \quad \text{...(6·56)}$$

and the total emf generated

$$E = 4{\cdot}44\ T\ \Phi_1\ f\ K_{p1} \sqrt{1+(B_{m3}\ K_{p3}/B_{m1}\ K_{p1})^2+\ldots\ldots+(B_{mn}\ K_{pn}/B_{m1}\ K_{p1})^2+\ldots\ldots} \quad \text{...(6·57)}$$

From Eqn. 6·57, it is clear that the waveshape of emf generated in a fractional pitch concentrated winding is different from that of flux density distribution. This is because the magnitude of different harmonic emfs depends upon their pitch factors which may be different for different harmonics.

6·31. Emf generated in fractional pitch distributed winding. The windings of most electrical machines are distributed two layer type with fractional pitch coils. Generalizing and including the effects of both distribution and chording, we have for a winding of T turns,

emf of fundamental frequency (Eqns. 6·50 and 6·54),

$$E_1 = 4{\cdot}44\ T\ \Phi_1\ f\ K_{p1}\ K_{d1} \quad \text{...(6·61)}$$

and nth harmonic emf (Eqns. 6·51 and 6·56)

$$E_n = 4{\cdot}44\ T\ \Phi_n\ nf\ K_{pn}\ K_{dn}$$

Total emf of the winding,

$$E = \sqrt{(E_1)^2+(E_3)^2+\ldots+(E_n)^2+\ldots} \quad \text{...(6·62)}$$

$$= 4{\cdot}44\ T\ \Phi_1\ f\ K_{p1}\ K_{d1} \sqrt{1+\left(\frac{B_{m3}\ K_{p3}\ K_{d3}}{B_{m1}\ K_{p1}\ K_{d1}}\right)^2+\ldots+\left(\frac{B_{mn}\ K_{pn}\ K_{dn}}{B_{m1}\ K_{p1}\ K_{d1}}\right)} \quad \text{...(6·63)}$$

Eqn. 6·63 shows that the waveform of the generated voltage is not the same as that of the flux density distribution. This is because the relative magnitudes of harmonic emfs would now depend upon the pitch and distribution factors in addition to the amplitudes of the flux density harmonics. The product of pitch factor and distribution factor is known as **winding factor.**

Winding factor for fundamental $K_{w1} = K_{p1}\ K_{d1}$ and for nth harmonic $K_{wn} = K_{pn}\ K_{dn}$.

Thus Eqn. 6·63 can be written as :

$$E = 4{\cdot}44\ T\ \Phi_1\ f\ K_{w1} \sqrt{1+(B_{m3}\ K_{w3}/B_{m1}\ K_{w1})^2+\ldots+(B_{mn}\ K_{wn}/B_{m1}\ K_{w1})^2+\ldots} \quad \text{...(6·64)}$$

6·32. Emf generated in a.c. machines. The effective value of voltage generated in each phase is given by Eqn. 6·64.

Voltage per phase,

$$E_{ph} = 4{\cdot}44\ T_{ph}\ \Phi_1\ f\ K_{w1} \sqrt{1+(B_{m3}\ K_{w3}/B_{m1}\ K_{w1})^2+\ldots+(B_{mn}\ K_{wn}/B_{m1}\ K_{w1})^2+\ldots} \quad \text{...(6·65)}$$

where T_{ph} = turns per phase.

Fundamental frequency emf per phase

$$E_{ph1} = 4{\cdot}44\ T_{ph}\ \Phi_1\ K_{p1}\ K_{d1} \quad \text{...(6·66)}$$

nth harmonic emf per phase $E_{phn} = 4{\cdot}44\ T_{ph}\ nf\ \Phi_n\ K_{pn}\ K_{dn}$ ...(6·67)

$$\frac{E_{phn}}{E_{ph1}} = \frac{4{\cdot}44\ T_{ph}\ nf\ \Phi_n\ K_{pn}\ K_{dn}}{4{\cdot}44\ T_{ph}\ f\ \Phi_1\ K_{p1}\ K_{d1}} = \frac{n\ \Phi_n\ K_{pn}\ K_{dn}}{\Phi_1\ K_{p1}\ K_{d1}} = \frac{n\ \Phi_n\ K_{wn}}{\Phi_1\ K_{w1}} = \frac{B_{mn}\ K_{wn}}{B_{m1}\ K_{w1}}$$

or
$$E_{phn} = E_{ph1}\ \frac{B_{mn}\ K_{wn}}{B_{m1}\ K_{w1}} \quad \text{...(6·68)}$$

6·33. Effect of distribution and chording in a.c. machines. From Eqn. 6·49, the distribution factor for nth harmonic $K_{dn}=\dfrac{\sin n\,\sigma/2}{q \sin n\sigma/2q}$

Where number of slots per pole per phase is large, the distribution is uniform giving the effect shown in Fig. 6·68. The arithmetic sum of the emfs is an arc while their phasor summation is the chord of that arc.

Therefore, distribution factor for fundamental

$$K_{d_1}=\frac{2R \sin \sigma/2}{R\sigma}=\frac{\sin \sigma/2}{\sigma/2} \quad \ldots(6·69)$$

Similarly, $K_{dn}=\dfrac{\sin n\,\sigma/2}{\sigma/2}$ ...(6·70)

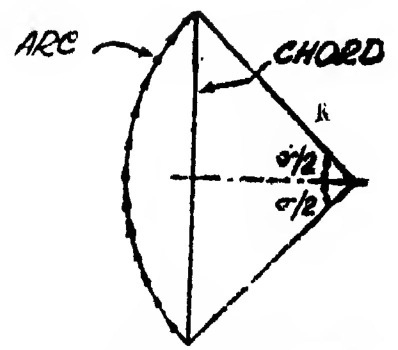


Fig. 6·68. Infinite Distribution.

The distribution under these conditions is known as **infinite distribution.** Actually when q is equal to or more than 5, the winding can be taken as infinitely distributed.

Considering an infinite distribution of winding. A phase spread of $\sigma=60°$ gives a distribution factor of 0·955 for the fundamental while with $\sigma=120°$, the distribution factor for fundamental is 0·827. As the distribution factor directly effects the output voltage, it is clear that the output of a machine would be 0·955/0·827=1·15 times as much with 60° phase spread as with phase spread=120°. However, if we consider the 3rd harmonic, we find that its distribution factor is zero with $\sigma=120°$. Thus, the voltage generated corresponding to 3rd harmonic and also all other harmonics whose order is a multiple of 3 *i.e.* **triplen harmonics**, would be equal to zero. Therefore, a phase spread of 120° eliminates the 3rd harmonic (and also other triplen harmonics).

Table 6·3

Distribution Factors

q=Slots/pole/phase→	2	3	4	∞
Order of harmonic				
$n=1$	0·966	0·96	0·957	0·955
$n=3$	0·707	0·666	0·653	0·636
$n=5$	0·259	0·218	0·205	0·191
$n=7$	0·259	0·178	0·158	0·136
$n=9$	0·707	0·333	0·271	0·212
$n=11$	0·966	0·178	0·126	0·087
$n=13$	0·966	0·218	0·126	0·074
$n=15$	0·707	0·666	0·271	0·127
$n=17$	0·259	0·96	0·158	0·056
$n=19$	0·259	0·96	0·205	0·050

Table 6 3 shows the distribution factor for the fundamental and harmonics of a 3 phase winding for 60° phase spread. From this table it is clear that the use of a large number of slots results in reduction of distribution factor for the harmonics while the fundamental remains almost unaffected. Thus, by using a large value for q, the harmonic emfs are reduced.

It has been discussed earlier that by chording a winding (using fractional pitch coils), its emf is reduced and the reduction factors are

$K_{p1}=\cos \alpha/2$ for the fundamental and $K_{pn}=\cos n\alpha/2$ for nth harmonic.

Suppose we chord the coils by 1/3rd of a pole pitch *i.e.* $\alpha=\pi/3=60°$.

Under this condition, $K_{p1}=\cos 60°/2=0{\cdot}866$, $K_{p3}=\cos\ 3\times60/2=\cos 90°=0$.

Thus 3rd and all triplen harmonics are eliminated from coil and phase voltages if the coils are chorded by $\pi/3$. In general the coils should be chorded by an angle $=\pi/n$ for elimination of nth harmonic.

The triplen harmonics, in fact, are not eliminated by chording the coils by 60° as given above because this would result in considerable reduction in emf of fundamental frequency (as $K_{p1}=0{\cdot}866$ for $\alpha=60°$). The triplen harmonics generated in a 3-phase machine are normally eliminated by star connection of phases. The line voltage of a star connected machine cannot contain any triplen harmonics unless a star point circuit is provided for them, which is quite unusual This is clear from Fig. 6·69. The fundamental emfs of a 3-phase windings are shown in Fig. 6·69 (*a*). These emfs E_{R1}, E_{Y1}, E_{B1} are displaced from each other by 120°. The fundamental line emfs are given by the phasor difference of phase emfs. Line emfs are :

$$\mathbf{E}_{RY1}=\mathbf{E}_{R1}-\mathbf{E}_{Y1},\ \mathbf{E}_{YB1}=\mathbf{E}_{Y1}-\mathbf{E}_{B1},\ \mathbf{E}_{BR1}=\mathbf{E}_{B1}-\mathbf{E}_{R1}$$

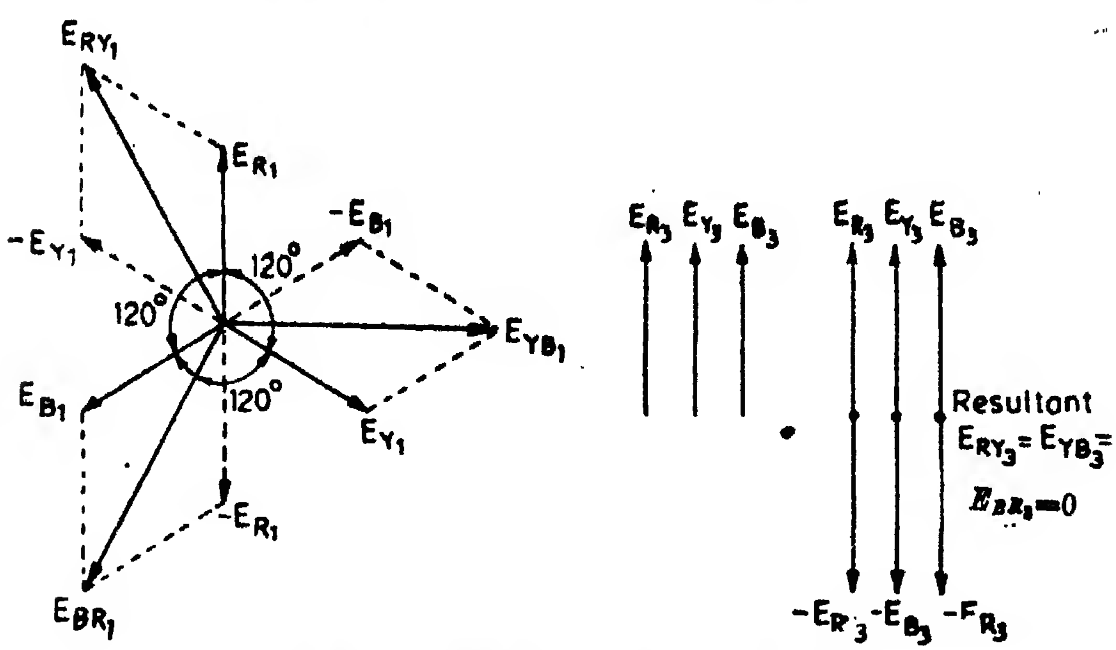


Fig. 6·69. Fundamental and 3rd harmonic emfs.

It is clear that the three line emfs are equal to $\sqrt{3}$ times the phase emfs. The third harmonic emfs are in the three phases are displaced from each other by an angle $3\times120°=360°$. Thus the 3rd harmonics (and other **triplens** *i.e.* harmonics whose order is a multiple of 3) are in phase with each other and hence are **cophasal** as show in Fig. 6·69 (*b*).

The 3rd harmonic emfs appearing across the three lines are phasor difference of 3rd harmonics emfs in two different phases. The difference of two equal emfs which are in phase with each other is zero *i.e.*,

$$\mathbf{E}_{RY_3}=\mathbf{E}_{R_3}-\mathbf{E}_{Y_3}=0,\ \mathbf{E}_{Y_3}=\mathbf{E}_{B_3}-\mathbf{E}_{BY_3}=0,\ \mathbf{E}_{B_3}-\mathbf{E}_{R_3}=-\mathbf{E}_{BY_3}=0$$

Hence the 3rd harmonic emf appearing across the lines of a star connected generator is zero. It is normally the practice to chord the winding to such an extent that 5th and 7th harmonics are reduced considerably. It is usual to chord the winding by $\alpha=30°$ as this gives,

$$K_{p1}=0{\cdot}966,\ K_{p3}=0{\cdot}707,\ K_{p5}=0{\cdot}259, \text{ and } K_{p7}=0{\cdot}259.$$

Thus the 3rd and the other triplen harmonics will not appear in the line voltage, 5th and 7th harmonics are considerably reduced, and the fundamental is not affected very much with chording by $\alpha=30°$.

Coming back to Eqn. 6.65, it is clear that the waveform of emfs generated will be quite different from that of the flux density waveform. This is due to the fact that the various harmonic emfs would depend not only upon their corresponding flux density components but also on their distribution and pitch factors. By properly designing the machine these factors can be kept low for the harmonics thereby reducing the harmonic voltages. Thus it is possible to have an approximately sinusoidal waveshape for the generated emf in a winding even though the flux density distribution is non-sinusoidal.

From above it is clear that, for a properly designed machine, the resultant emf per phase E_{ph} would be almost equal to its fundamental component E_{ph_1}. Thus Eqn. 6·66 for E_{ph_1} can be used for calculation of phase voltage E_{ph} for ordinary purposes. Further the value of Φ_1, fundamental component of total flux per pole differs slightly from that of Φ, total flux per pole. Thus either of them can be used in Eqn. 6·56 for the determination of E_{ph}.

6·33. Distribution Factor of Fractional Slot Windings. It has been explained that in a fractional slot winding, slots per pole per phase $q=S/mp=M/d$ (See Eqn. 6·31) or there are M slots per phase distribution over d poles. It has also been explained that the displacement between phasors of a fractional slots winding is α_m where, $\alpha_m=\pi/mM$ (See Eqn. 6·32).

Thus for a 3 phase machine $\alpha_m=\pi/3M$.

$\therefore$ There are M phasors of a phase displaced from each other by an angle $\alpha_m=\pi/3M$ and hence $N=M$ and $\psi=\alpha_m=\pi/3M$.

Distribution factor for fundamental

$$K_{d1}=\frac{\sin N\psi/2}{N\sin\psi/2}$$

$\therefore$ For a fractional slot winding

$$K_{d1}=\frac{\sin M\pi/6M}{M\sin\pi/6M}=\frac{\sin\pi/6}{M\sin\pi/6M} \quad \ldots(6{\cdot}71)$$

and

$$K_{dn}=\frac{\sin n\,\pi/6}{M\sin n\,\pi/6M} \quad \ldots(6{\cdot}72)$$

Thus a fractional slot winding as if it has M slots per pole per phase. Consider a winding with $q=1\frac{1}{4}=5/4$. This winding behaves as if it has 5 slots per pole per phase. Therefore, the winding gives a small value of distribution factors for higher order harmonics, thereby reducing their magnitude. This is inspite of the fact that the actual number of slots per pole per phase is nearly equal to 1

Example 6·33. *The phase of emf of a 3-phase 50 Hz alternator consists of a fundamental, a 20 per cent third harmonic and a 10 percent fifth harmonic. The amplitude of fundamental is 1000 V. Calculate the rms line voltage when the windings are connected in (a) star (b) delta. If the reactance per phase at 50 Hz is 10 Ω, calculate the circulating current when the machine is delta connected.*

Solution. Rms alue of fundamental frequency emf per phase

$$E_{ph1}=\frac{1000}{\sqrt{2}}=707 \text{ V}.$$

Rms value of 3rd harmonic component of phase voltage $E_{ph_3}=0{\cdot}2\times 707 \text{ V}=141{\cdot}4 \text{ V}$.

Rms value of 5th harmonic component of phase voltage $E_{ph_5}=0{\cdot}1\times 707{\cdot}1=70{\cdot}7 \text{ V}$.

(*a*) In star connection the third harmonic component will not appear across the lines and therefore the line voltage is equal to $\sqrt{3}$ times the phase voltage considering no third harmonic.

Phase voltage considering no third harmonic, $E_{ph}=\sqrt{E^2_{ph_1}+E^2_{ph_5}}=\sqrt{(707{\cdot}1)^2+(70{\cdot}7)^2}$
$=710{\cdot}6$ V.

$\therefore$ Line voltage$=\sqrt{3}\times 710{\cdot}6=1230{\cdot}8$ V.

(*b*) In delta connection, the third harmonic component of voltage acts around the delta. Third harmonic voltage appears as a voltage drop in the impedance of windings and therefore does not appear across the lines. Therefore, the line voltage is equal to phase voltage considering no third harmonic voltage,

$\therefore$ Line voltage$=\sqrt{E_{ph_1}{}^2+E_{ph_5}{}^2}=710{\cdot}6$ V.

The third harmonic component of the voltage circulates current around the delta. The third harmonic voltage components of three phases are cophasal and thus add algebraically around the delta.

$\therefore$ Voltage producing circulating current$=3E_{ph_3}$.

The reactance per phase for the fundamental frequency is 10 Ω and therefore the reactance per phase corresponding to 3rd harmonic is $3\times 10=30$ Ω.

$$\therefore \quad \text{Circulating current}=\frac{3\,E_{ph_3}}{3\times 30}=\frac{3\times 141{\cdot}4}{3\times 30}=4{\cdot}71 \text{ A}.$$

Example 6·34. *The flux distribution curve of a smooth core, 4 pole, 1500 r.p.m. synchronous generator is :*

$$B_\theta=1 \sin\theta+0{\cdot}25 \sin 3\theta+0{\cdot}2 \sin 5\theta+0{\cdot}1 \sin 7\theta \text{ Wb/m}^2$$

where θ is measured from interpolar axis. The pole pitch is 0·4 m and the core length is 0·3 m. Determine the equation of the induced emf per turn and its rms value if

(*i*) *the coils are full pitch.* (*ii*) *the coils are chorded by 4/5 of pole pitch*

Comment on the results obtained.

Solution.

Frequency $f=\frac{pn}{2}=\frac{4\times 1500}{2\times 60}=50$ Hz.

Area of each pole (fundamental) $A_1=$pole pitch $\times$ core length$=0{\cdot}4\times 0{\cdot}3=0{\cdot}12$ m².

Fundamental component of flux per pole$=\frac{2}{\pi}B_{m_1}A_1=\frac{2}{\pi}\times 1\times 0{\cdot}12=0{\cdot}0764$ Wb.

The distribution factor for fundamental and harmonics is 1 as we are considering only one turn.

$\therefore \quad K_{d_1}=K_{d_3}=K_{d_5}=K_{d_7}=1.$

(*i*) For full pitch coils, the pitch factor for fundamental and all the harmonics is 1

or $K_{p1}=K_{p3}=K_{p5}=K_{p7}=1$.

∴ Winding factor for the fundamental and all the harmonics is 1.

or $K_{w1}=K_{w3}=K_{w5}=K_{w7}=1$.

From Eqn. 6·66, Fundamental frequency emf is each turn

$E_{t_1}=4{\cdot}44\Phi_1 f K_{w1}=4{\cdot}44\times0{\cdot}0764\times50\times1=17$

From Eqn. 6·68,

*n*th harmonic induced emf per turn $E_{tn}=E_{t_1}\dfrac{B_{mn}\ K_{wn}}{B_{m1}\ K_{w1}}$

Therefore harmonic emfs are :

$$E_{t_3}=E_{t_1}\ \frac{B_{m3}\ K_{w3}}{B_{m1}\ K_{w1}}=1{\cdot}7\ \frac{0{\cdot}25\times1}{1\times1}=4{\cdot}25\text{ V.}$$

$$E_{t_5}=E_{t_1}\ \frac{B_{m5}\ K_{w5}}{B_{m1}\ K_{w5}}=1{\cdot}7\frac{0{\cdot}2\times1}{1\times1}=3{\cdot}40\text{ V.}$$

$$E_{t_7}=E_{t1}\ \frac{B_{m7}\ K_{w7}}{B_{m1}\ K_{w1}}=1{\cdot}7\frac{0{\cdot}1\times1}{1\times1}=1{\cdot}70\text{ V.}$$

The above values of emfs are rms values.

∴ The equation of the induced emf in a turn is

$\sqrt{2}(17\sin\omega t+4{\cdot}25\sin3\,\omega t+3{\cdot}4\ \sin5\,\omega t+1{\cdot}7\sin7\,\omega t)$

or $24\sin\omega t+6\sin3\,\omega t+4{\cdot}8\sin5\,\omega t+2{\cdot}4\sin7\,\omega t$

or $24(1\sin\omega t+0{\cdot}25\sin3\,\omega t+0{\cdot}1\sin5\,\omega t+0{\cdot}1\sin7\,\omega t)$ V.

Comparing the equation of induced emf with that of flux distribution, we find th they are identical. From this it is clear that in a full pitch, concentrated winding the rel tive magnitude of harmonics in the induced emf is the same as in the flux distribution.

Rms value of induced emf per turn $E_t=\sqrt{E_{t_1}^2+E_3^2+E_{t_5}^2+E_{t_7}^2}$

$$=\sqrt{(17)^2+(4{\cdot}25)^2+(3{\cdot}41)^2+(1{\cdot}7)^2}=17{\cdot}93\text{ V.}$$

(*ii*) Coil span$=4/5\times180°=144°$. ∴ Angle of chording $\alpha=180-144°=36°$

Pitch factor for fundamental $K_{p1}=\cos\alpha/2=\cos18°=0{\cdot}951$

Similarly pitch factors for the harmonics are :

$K_{p3}=\cos3\times18=0{\cdot}5878$, $K_{p5}=\cos5\times18=0$, $K_{p7}=\cos7\times18=-0{\cdot}5878$

∴ Winding factors for the fundamental and the harmonics are :

$K_{w1}=K_{d_1}\ K_{p1}=0{\cdot}951$, $K_{w3}=K_{d_3}\ K_{p3}=0{\cdot}5878$

$K_{w5}=K_{d_5}\ K_{p5}=0$, $K_{w7}=K_{d_7}\ K_{p7}=-0{\cdot}5878$.

Proceeding as in part (*i*)

$$E_{t_1}=4{\cdot}44\ \Phi_1\ f\ K_{w1}=4{\cdot}44\times0{\cdot}0764\times50\times0{\cdot}9511=16{\cdot}1\text{ V.}$$

$$E_{t_3}=E_{t_1}\ \frac{B_{m3}\ K_{w3}}{B_{m1}\ K_{w1}}=16{\cdot}1\ \frac{0{\cdot}25\times0{\cdot}5878}{1\times0{\cdot}9511}=2{\cdot}49\text{ V.}$$

$$E_{t_5}=E_{t_1}\ \frac{B_{m5}\ K_{w5}}{B_{m1}\ K_{w1}}=16{\cdot}1\ \frac{0{\cdot}2\times0}{1\times0{\cdot}9511}=0$$

$$E_{t_7}=E_{t_1}\ \frac{B_{m7}\ K_{w7}}{B_{m1}\ K_{w1}}=16{\cdot}1\ \frac{0{\cdot}1\times0{\cdot}5878}{1\times0{\cdot}951}=0{\cdot}995\text{ V }(-)$$

The equation of induced emf per turn is

$\sqrt{2}$ (16·1 sin ωt+2·49 sin $3\omega t$+0 sin $5\omega t$−0·995 sin $7\omega t$)

or 22·8 sin ωt+3·52 sin $3\omega t$−1·41 sin $7\omega t$

or 22·8(1 sin ωt+0·154 sin $3\omega t$−0·0618 sin 7 ωt)

Comparing the equation of induced emf per turn with that of flux distribution, we find that they are different. Thus we conclude that by chording the winding the wave shape of induced emf is different from the flux density distribution. We also find that the relative magnitudes of 3rd and 7th harmonics are considerably reduced while 5th harmonic is altogether absent. The absence of 5th harmonic is attributed to chording by α=36° (*i.e.*, 180/5).

Rms value of induced emf per turn $E_t=\sqrt{E_{t_1}^2+E_{t_3}^2+E_{t_5}^2+E_{t_7}^2}$

$=\sqrt{(16\cdot1)^2+(2\cdot49)^2+(0\cdot995)^2}=16\cdot35$ V.

Comparing the total emf per turn for full pitch and fractional pitch windings we find that the emf generated in the fraction pitch winding is smaller.

Example 6·35. *A 3-phase 4 pole, 50 Hz machine has an armature with a diameter of 0·5 m and length 0·3 m. The equation for flux density distribution is :*

$$B=0\cdot15\sin\theta+0\cdot03\sin3\theta+0\cdot02\sin5\theta \text{ Wb/m}^2.$$

Double layer winding is used for the armature which has 60 coils with 10 turns per coil. This phase spread is 60°.

(*a*) *Determine the voltage generated per coil and per phase for full pitch coils.*

(*b*) *The voltage generated per coil and per phase if coils have a span of 13/15 of a pole pitch.*

(*c*) *Determine the voltage between terminals for star connection for the above.*

Solution.

Area of each pole (fundamental) $A_1=\frac{\pi}{4}\times0\cdot25\times0\cdot3=0\cdot0589$ m².

Fundamental flux per pole $\Phi_1=(2/\pi)\,B_{m_1}A_1=(2/\pi)\times0\cdot15\times0\cdot0589=0\cdot00562$ Wb.

(*a*) Fundamental frequency emf per turn $E_{t_1}=4\cdot44\,\Phi_1 f K_{p_1} K_{d_1}$

Now, $K_{d_1}=K_{dn}=1$ when considering a turn or a coil and $K_{p_1}=K_{pn}=1$ for full pitch coils.

or $K_{w_1}=K_{wn}=1$

$\therefore$ $E_{t_1}=4\cdot44\times0\cdot00562\times50\times1\times1=1\cdot25$ V.

3rd harmonic emf in each turn $E_{t_3}=E_{t_1}\dfrac{B_{m_3}K_{w_3}}{B_{m_1}K_{w_1}}=1\cdot25\times\dfrac{0\cdot03}{0\cdot15}=0\cdot25$ V.

5th harmonic emf per turn $E_{t_5}=E_{t_1}\dfrac{B_{m_5}K_{w_5}}{B_{m_1}K_{w_1}}=1\cdot25\,\dfrac{0\cdot02}{0\cdot15}=0\cdot166$ V.

$\therefore$ **Total voltage per turn**$=\sqrt{1\cdot25^2+0\cdot25^2+0\cdot166^2}=1\cdot28$ **V.**

Voltage per coil=turns per coil×voltage per turn=10×1·28=12·8 V.

Number of coils per phase=60/3=20. For a double layer the number of slots is equal to the number of coils and hence the number of slots is 60.

$\therefore$ **Number of slots per pole phase** $q=\dfrac{60}{3\times4}=5$

Distribution factor for fundamental $K_{d_1}=\dfrac{\sin\sigma/2}{q\sin\sigma/2q}=\dfrac{\sin60/2}{5\sin60/2\times5}=0\cdot957.$

Distribution factors for 3rd and 5th harmonics are

$$K_{d3}=\frac{\sin(3\times 60/2)}{3\sin(3\times 60/2\times 5)}=0{\cdot}647,\quad K_{d5}=\frac{\sin(5\times 60/2)}{5\sin(5\times 60/2\times 5)}=0{\cdot}2$$

T_{ph}=coils per phase × turns per coil=20 × 10=200.

Fundamental frequency voltage per phase $E_{ph1}=4{\cdot}44\,f\,\Phi_1\,T_{ph}\,K_{d1}\,K_{p1}$

$$=4{\cdot}44\times 50\times 0{\cdot}00562\times 200\times 0{\cdot}957\times 1=239\text{ V.}$$

$$\therefore\quad E_{ph3}=E_{ph1}\frac{B_{m3}\,K_{w3}}{B_{m1}\,K_{w1}}=239\times\frac{0{\cdot}03\times 0{\cdot}647}{0{\cdot}15\times 0{\cdot}957}=32{\cdot}3\text{ V.}$$

and $E_{ph5}=E_{ph1}\dfrac{B_{m5}\,K_{w5}}{B_{m1}\,K_{w1}}=239\times\dfrac{0{\cdot}02\times 0{\cdot}2}{0{\cdot}15\times 0{\cdot}957}=6{\cdot}66$ V.

Voltage per phase $E_{ph}=\sqrt{E_{ph1}^2+E_{ph3}^2+E_{ph5}^2}=\sqrt{(239)^2+(32{\cdot}3)^2+(6{\cdot}66)^2}=241{\cdot}3$ V.

(*b*) Coil span (13/15) × 180=156°.

∴ Angle of chording α=180−156=24°.

$K_{p1}=\cos\dfrac{\alpha}{2}=\cos 12°=0{\cdot}978,\ K_{p3}=\cos 3\times 12=0{\cdot}809,\ K_{p5}=\cos 5\times 12=0{\cdot}5.$

Fundamental frequency emf in each turn

$E_{t1}=4{\cdot}44\,\Phi_1\,f\,K_{p1}$ (as $K_{dn}=1$ when we consider emf in one turn)

$=4{\cdot}44\times 0{\cdot}00562\times 50\times 0{\cdot}978=1{\cdot}22$ V.

3rd harmonic emf in each turn $E_{t3}=E_{t1}\dfrac{B_{m3}\,K_{w3}}{B_{m1}\,K_{w1}}=1{\cdot}22\dfrac{0{\cdot}03\times 1\times 0{\cdot}809}{0{\cdot}15\times 1\times 0{\cdot}978}=0{\cdot}202$ V.

5th harmonic emf in each turn $E_{t5}=1{\cdot}22\ \dfrac{0{\cdot}02\times 1\times 0{\cdot}5}{0{\cdot}15\times 1\times 0{\cdot}978}=0{\cdot}0832$ V.

∴ Total effective emf per turn $E_t=\sqrt{1{\cdot}22^2+0{\cdot}202^2+0{\cdot}0832^2}=1{\cdot}244$ V.

Voltage per coil=1·244 × 10=12·44 V.

Fundamental frequency voltage per phase with coil pitch of 13/15 pole pitch.

$$E_{ph1}=4{\cdot}44\,T_{ph}\,\Phi_1\,f\,K_{d1}\,K_{p1}=4{\cdot}44\times 200\times 0{\cdot}00562\times 50\times 0{\cdot}957\times 0{\cdot}978=233\text{ V.}$$

$$E_{ph3}=E_{ph1}\ \frac{B_{m3}\,K_{w3}}{B_{m1}\,K_{w1}}=233\times\frac{0{\cdot}03\times 0{\cdot}647\times 0{\cdot}809}{0{\cdot}15\times 0{\cdot}957\times 0{\cdot}978}=26{\cdot}1\text{ V.}$$

Similarly, $E_{ph5}=233\times\dfrac{0{\cdot}02\times 0{\cdot}2\times 0{\cdot}5}{0{\cdot}15\times 0{\cdot}975\times 0{\cdot}978}=3{\cdot}32$ V.

∴ Voltage per phase $E_{ph}=\sqrt{233^2+26{\cdot}1^2+3{\cdot}32^2}=234{\cdot}$ V.

(*c*) With star connection, the 3rd harmonic voltages do not appear across the line terminals even though they are present in the phase voltage.

With coil pitch =13/15 pole pitch, the phase voltage omitting the 3rd harmonic

$E_{ph}=\sqrt{E_{ph1}^2+E_{ph5}^2}=\sqrt{233^2+3{\cdot}32^2}=233$ V.

∴ Line voltage=$\sqrt{3}\times 233=404$ V.

Example 6·36. *The flux distribution in the air gap of a 50 Hz salient pole alternator may be taken as rectangular the base being two thirds of a pole pitch. (a) Calculate the rms value of the phase emf (considering harmonics upto 5th. Calculate the phase emf if the same total flux had a sinusoidal distribution.*

Given : turns per phase=120 ; phase spread 60° ; pole pitch=0·5 m ; stator core length 0·75 m ; maximam value of gap density=0·7 Wb/m². Full pitch coils are used.

Solution. The Fourier analysis of a simple rectangular field form has been done earlier. The flux density is given by :

$$B_\theta = \frac{4}{\pi} B_g \left[\cos \frac{\delta}{2} \sin \theta + \frac{1}{3} \cos \frac{3\delta}{2} \sin 3\theta + \frac{1}{5} \cos \frac{5\delta}{2} \sin 5\theta + \ldots\ldots \right]$$

where $\delta = \pi(1-\psi)$ where ψ is the ratio of pole arc to pole pitch.

In our case $\psi = 2/3$ and therefore $\delta = \pi/3$.

Amplitude of fundamental $B_{m1} = \frac{4}{\pi} B_g \cos \frac{\delta}{2} = \frac{4}{\pi} \times 0{\cdot}7 \times \cos \frac{\pi}{6} = 0{\cdot}772$ Wb/m².

Amplitude of 3rd harmonic $B_{m3} = \frac{4}{3\pi} B_g \cos \frac{3\delta}{2} = \frac{4}{3\pi} \times 0{\cdot}7 \times \cos \frac{\pi}{2} = 0.$

Amplitude of 5th harmonic $B_{m5} = \frac{4}{5\pi} B_g \cos \frac{5\delta}{2} = \frac{4}{5\pi} \times 0{\cdot}7 \cos \frac{5\pi}{6} = 0{\cdot}154$ Wb/m².

Since the coils are full pitch $K_{p1} = K_{p3} = K_{p5} = 1$.

Assuming infinite distribution, the distribution for the fundamental and the third harmonic are

$$K_{d1} = \frac{\sin \sigma/2}{\sigma/2} = \frac{\sin \pi/6}{\pi/6} = 0{\cdot}955$$

and $$K_{d5} = \frac{\sin 5\pi/6}{5\pi/6} = 0{\cdot}191.$$

The winding factors are $K_{w1} = 0{\cdot}955$ and $K_{w5} = 0{\cdot}191$.

Fundamental flux per pole $\Phi_1 = \frac{2}{\pi} B_{m1} A_1 = \frac{2}{\pi} \times 0{\cdot}0772 \times 0{\cdot}5 \times 0{\cdot}75 = 0{\cdot}184$ Wb.

Rms value of fundamental frequency emf $E_{ph1} = 4{\cdot}44 \times 50 \times 0{\cdot}184 \times 120 \times 0{\cdot}955$
$= 4660$ V.

The rms value of third harmonic emf is zero as the pitch factor is zero.

Rms value of 5th harmonic emf $E_{ph5} = E_{ph1} \frac{B_{m5} K_{w5}}{B_{m1} K_{w1}} = 4660 \times \frac{0{\cdot}514 \times 0{\cdot}191}{0{\cdot}772 \times 0.955} = 187$ V.

Rms value of E_{ph} phase emf $= \sqrt{(4660)^2 + (0)^2 + (187)^2} = 4670$ V.

(*b*) Total flux per pole $\Phi = 2/3 \times (0{\cdot}7)(0{\cdot}5 \times 0{\cdot}75)$
$= 0{\cdot}175$ Wb.

If this flux were sinusoidally distributed, rms value of phase emf

$E_{ph} = 4{\cdot}44 \times 50 \times 0{\cdot}175 \times 120 \times 0{\cdot}955 = 4450$ V.

6·34. Emf generated in d.c. machines. Fig. 6·70 shows a d.c. machine. The voltage appearing across the terminals of a d.c. machine is the summation of emfs generated in all the conductors, which at that instant of time are in one series path between the brushes.

If the armature conductors were so closely placed on the surface of the armature that they completely covered it and if the number of coils short-circuited by the brushes was the same at every instant, the number of conductors in one series path between the brushes would be the same at every instant. And with an infinitely distributed winding the net position of conductors would be the same with respect to the magnetic field at every instant. Under these conditions the voltage appearing across the brushes of a d.c. machine would be the same at all instants of time.

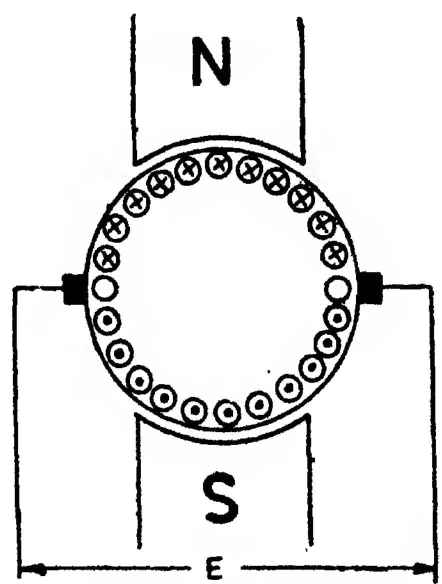


Fig. 6·70 Emf generated in d.c. machines.

Thus the emf generated in a d.c. machine

E = conductors in series per path × average voltage per conductor.

(This takes into account the distribution of winding).

From Eqn. 6·39, the instantaneous value of conductor emf

$e_c = \pi f \Phi_1 \sin \theta + 3\pi f \Phi_3 \sin 3\theta + \ldots\ldots + n\pi f \Phi_n \sin n\theta + \ldots\ldots$

Average value of conductor emf $E_{c(av)} = \dfrac{1}{\pi}\displaystyle\int_0^\pi e_{cond} d\theta$

$$= \frac{1}{\pi}\int_0^\pi (\pi f \Phi_1 \sin \theta + 3\pi f \Phi_3 \sin 3\theta + \ldots + n\pi f \Phi_n \sin n\theta + \ldots)\, d\theta$$

$$= 2f[\Phi_1 + \Phi_3 + \ldots\ldots + \Phi_n + \ldots.]$$

Now $[\Phi_1 + \Phi_3 + \ldots + \Phi_n + \ldots] = \Phi$, the total flux per pole.

$\therefore \qquad E_{c(av)} = 2f\,\Phi.$

Now if Z is the total number of armature conductors, the number of conductors per parallel path is Z/a if the number of parallel paths is a.

$\therefore$ Generated emf $\quad E = E_{c(av)} \times \dfrac{Z}{a} = 2f\,\Phi \dfrac{Z}{a} = \Phi Z n \dfrac{p}{a}$ ...(6·73)

Thus the emf generated in a d.c. machine would depend upon the total flux per pole and not on how it is distributed around the air gap.

6·35. Tooth Ripples (Slot Harmonics). The surface of the armature has been assumed as smooth while deriving the expressions for voltage generated in armature windings. However, the effect of slotting on the generated voltage cannot be ignored since certain harmonic emfs of particularly undesirable order may be produced because of the presence of slots in which the conductors are located. Consider Fig. 6·71, it is clear that the reluctance at any point in the air gap would depend upon whether a slot or a tooth is in the magnetic path. As there is a movement of the armature, the slots and teeth alternately occupy positions at this point and thus the reluctance varies. The ripples caused by the variation of reluctance from point to point in the air gap are of the form shown in Fig. 6·71. It is important to note that these ripples do not move with respect to conductors but glide over the flux distribution curve. This is because these ripples are due to slotting of armature and they are always opposite to the slots and teeth which cause them.

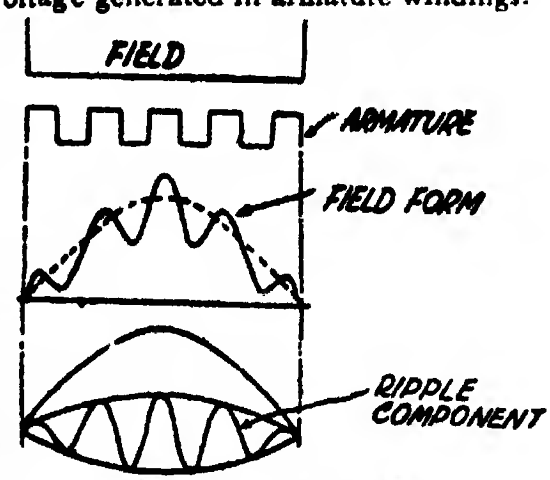


Fig. 6·71. Tooth Ripples.

Referring to Fig. 6·71, a complete cycle of this tooth ripple takes place in the time required for the pole to move through an angle equal to one slot pitch. Thus if there are q slots per pole phase and m phases there would be $2mq$ cycles of the tooth ripple for one complete cycle of fundamental frequency f or

frequency of tooth ripples

f_t = fundamental frequency × number of slots per pair of poles

$= 2mqf.$ ...(6·74)

The effect of the ripples may be considered as an additional flux Φ_t superposed on the main field. Considering the fundamental component Φ_1 of flux of the main field and ignoring the other field harmonics. Let the winding have T turns.

Flux linkages of armature winding $\psi=T(\Phi_1+\Phi_t \sin 2\pi f_t t) \cos 2\pi f t$

$$=T\Phi_1 \cos 2\pi f t+T\frac{\Phi_t}{2}\left[\sin 2\pi (f_t-f)t+\sin 2\pi (f_t+f) t\right]$$

$\therefore$ Generated emf $e=-\frac{d\psi}{dt}$.

$$=2\pi f T \Phi_1 \sin 2\pi f t-\frac{\Phi_t T}{2}\left[2\pi (f_t-f) \cos 2\pi (f_t-f)t+2\pi (f_t+f) \cos 2\pi (f_t+f)t\right] \quad ..(6\cdot75)$$

The first term on the right hand side of Eqn. 6·75 is the emf of fundamental frequency, while the remaining term is the emf due to tooth ripples. It is clear that the tooth ripple emf is made up of two frequencies (f_t-f) and (f_t+f). Thus it is evident that the slotting gives rise to emfs of two frequencies, which are

$$f_{t1}=f_t+f=f\,(2mq+1) \quad \text{and} \quad f_{t2}=f_t-f\,(2mq-1).$$

These harmonics in the generated emf are called **slot harmonics**.

In the above analysis only the fundamental of slot reluctance variation has been considered. Further ripples may occur due to harmonics. Thus a m phase winding with q slots per pole phase the slot harmonics of frequency $(2\,Amq\pm1)\,f$ are produced where $A=1, 2, 3$......etc. For a 3-phase winding, order of slot harmonics, $n=(6Aq\pm1)$.

Consider a common three phase winding with integral number of slots per pole per phase and a phase spread of 60°.

Let us evaluate the distribution factor slot harmonics.

$$K_{d(6Aq\pm1)}=\frac{\sin (6Aq\pm1)\,\pi/6}{q \sin (6Aq\pm1)\,\pi/6q}=\frac{\sin (Aq\pi\pm\pi/6)}{q \sin (A\pi\pm\pi/6q)}$$

Now A and q are both integers and therefore we can write,

$\sin (Aq\pi\pm\pi/6)=\sin \pm\pi/6$ and $\sin (A\pi\pm\pi/6q)=\pm\sin \pi/6q$.

Disregarding the sign $\quad K_{d(6Aq\pm1)}=\frac{\sin \pi/6}{q \sin \pi/6q}$

or $\quad K_{d(2Amq\pm1)}=\pm K_{d1}$

Therefore the distribution factors for the slot harmonics is equal to that of the fundamental. Hence if flux harmonics of these orders are present in the air gap, emf harmonics will appear in the phase voltage with the same percentage value.

Example 6·36. *A 4 pole, 50 Hz, 3-phase alternator has a single layer winding with 5 slots per pole per phase and 1 conductor per slot. The fundamental component of flux per pole is 1.8 Wb. The air gap flux contains a 10 per cent third harmonic, and a tooth ripple with a maximum amplitude of 5 per cent. Calculate the emf of each phase.*

Solution. Slots per pole per phase $q=5$.

Frequency of tooth ripples $f_t=2\,mqf=2\times3\times5f=30\,f$.

A ripple of frequency $2mqf$ is approximately equivalent to two combined harmonics of frequency $(2mq\pm1)\,f$, each with an amplitude half that of the ripple.

$\therefore$ A 5 per cent ripple of 30th order is approximately equivalent to 2·5 per cent harmonics of $(30\pm1)=31$st and 29th order.

Distribution factors for fundamental and third harmonic are :

$$K_{d_1}=\frac{\sin(60/2)}{5\sin(60/2\times5)}=0.957,\quad K_{d_3}=\frac{\sin 3(60/2)}{5\sin(5\times60/2\times5)}=0.645.$$

It has already been explained that the distribution factors for harmonics of order $(2mq\pm1)$ is the same as for the fundamental.

Thus $\quad K_{d(2mq\pm1)}=K_{d_1} \quad \therefore \quad K_{d_{31}}=K_{d_{29}}=K_{d_1}=0.957$

Turns per phase $=\dfrac{1\times5\times4}{2}=10.$

Full pitch coils are used $\quad \therefore \quad K_{p_1}=K_{p_3}=K_{p_n}=1$

Thus

$$K_{w_1}=K_{d_1}K_{p_1}=0.957,\ K_{w_3}=K_{d_3}K_{p_3}=0.645$$
$$K_{w_{29}}=K_{d_{29}}K_{p_{29}}=0.957,\ K_{w_{31}}=K_{d_{31}}K_{p_{31}}=0.957.$$

From Eqn. 6·66, fundamental frequency emf per phase

$$E_{ph_1}=4.44\,T_{ph}f\Phi_1K_{d_1}K_{p_1}=4.44\times10\times50\times1.8\times0.957\times1=3810\text{ V}.$$

Now, nth harmonic emf per phase

$$E_{ph_n}=\frac{B_{mn}}{B_{m_1}}\frac{K_{wn}}{K_{w_1}}\times E_{ph_1}\quad\text{[Eqn. 6·68].}$$

$\therefore$ 3rd harmonic emf per phase $E_{ph_3}=\dfrac{0.1\times0.645}{0.957}\times3810=257$ V.

Similarly 29th and 31st harmonic emfs are

$$E_{ph_{29}}=\frac{0.025\times0.957}{0.957}\times3810=95.2\text{ V},\quad E_{ph_{31}}=\frac{0.025\times0.957}{0.957}\times3810=95.2\text{ V}.$$

$\therefore$ Voltage per phase $E_{ph}=\sqrt{3810^2+257^2+2\times95.2^2}=3821$ V.

MMF DISTRIBUTION OF ARMATURE WINDINGS

6·36. Mmf distribution of concentrated coils. The armatures of all electrical machines (with the exception of a few) have distributed windings which are spread over a number of slots around the periphery of the air gap. The armature with full pitch coils whether for a d.c. machine or an a.c. machine has a current distribution such that one half of the armature conductors carry current in one direction and the other half in the opposite direction. The individual coils are so inter-connected that the resultant of field produced by the armature winding has the same number of poles as the field winding.

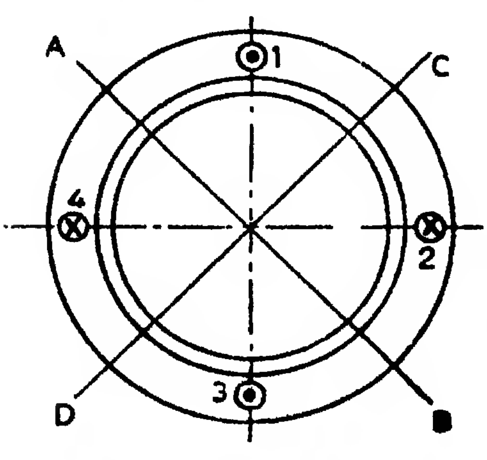


Fig. 6·72. Machine with two coils.

The magnetic fields produced by distributed winding can be studied by considering one of the most simple form of a single phase 4 pole winding. This winding consists of two coils with coil sides 1, 2, 3 and 4 as shown in Fig. 6·72. The dots and crosses indicate current towards and away from the reader respectively. A cylindrical concentric rotor is shown for the sake of simplicity. The same winding arrangement in the developed form is shown in Fig. 6·73. The general nature of the magnetic field produced by the current carrying conductors is shown by the dotted lines. Coil side 1, creates flux between OA and OC. Thus total mm.f produced by coil side is associated with flux lines in the portion of armature between OA and OC. Similarly, coil side 2 produces magnetic field between OC and OB, coil side 3 between OB and OD, and coil side 4 between OD and OA. Each

of these paths consists of reluctances of iron parts of stator, (armature in this case), two air gaps and iron parts of rotor (field, in this case). Since the permeability of the iron parts

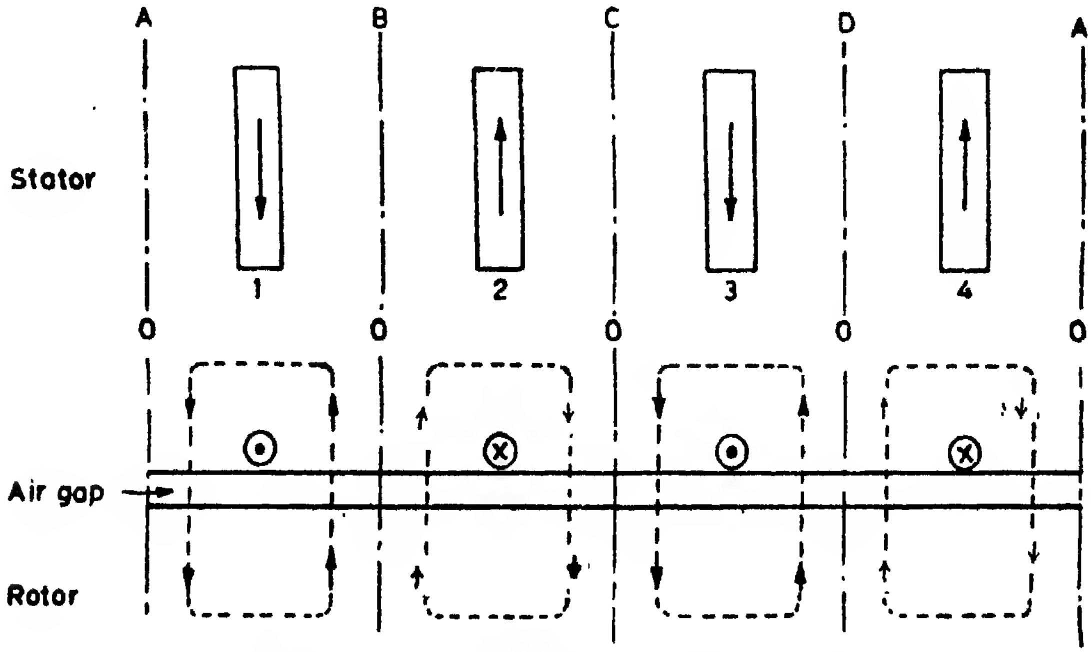


Fig. 6·73. Developed diagram of machine with two coils.

of both stator and rotor is much greater than that of air, it is sufficiently accurate to assume that all the reluctance of the magnetic circuit is in the two air gaps. Thus the mmf of the conductors is consumed by the air gaps. Since there are two air gaps in series in the magnetic field produced by each coil side, or half of the mmf of each coil side acts in the air gap to its left and other half acts in the air gap to its right.

Let T be the turns per coil and I be the current in each conductor. Therefore the mmf produced by each coil is TI. Around any closed of the closed paths shown the flux lines in Fig. 6·73, the mmf is TI considering coil side 1. Consider coil side 2. This coil side produces a magnetic field between OC and OB. Half of the mmf of this coil side *i.e.*, $TI/2$

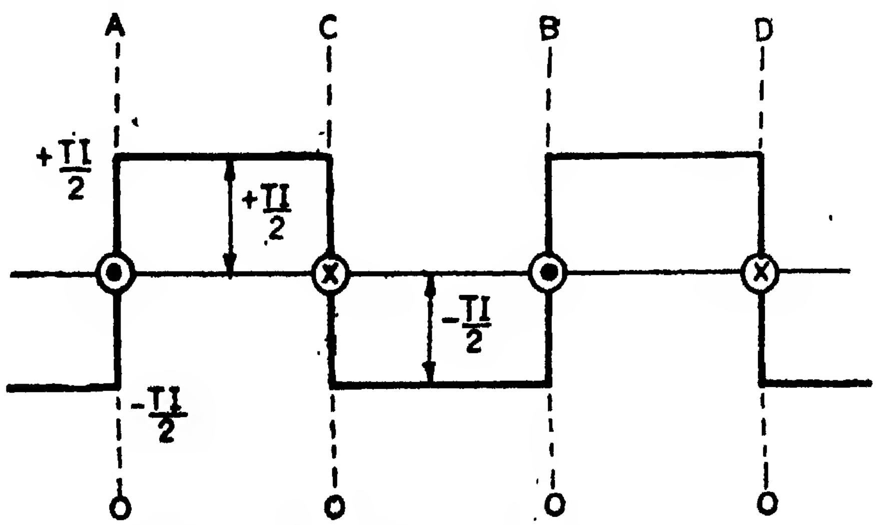


Fig. 6·74. Mmf distribution due to two coils

acts in the air gap to the left of the coil side forcing the flux downwards while the other half of the mmf acts in the air gap to the right of the coil side producing a flux acting upwards.

The mmf distribution of the two coils of Fig. 6·72 is shown in Fig. 6·74. The distribution is steplike with an amplitude of $TI/2$. Since the conductors are assumed to be concentrated at points *i.e.* the slots are assumed to be extremely narrow the mmf wave jumps abruptly by TI *i.e.* changes from $-TI/2$ to $+TI/2$ in crossing from one coil side to the other coil side.

6·37. Mmf distribution in d.c. machines. The armature and the field poles of a 2 pole d.c. machine are shown in Fig. 6·75. The armature has 6 slots per pole. The current distribution in the conductors will be as shown in the diagram provided full pitch coils are used and the brushes are placed on the interpolar axis.

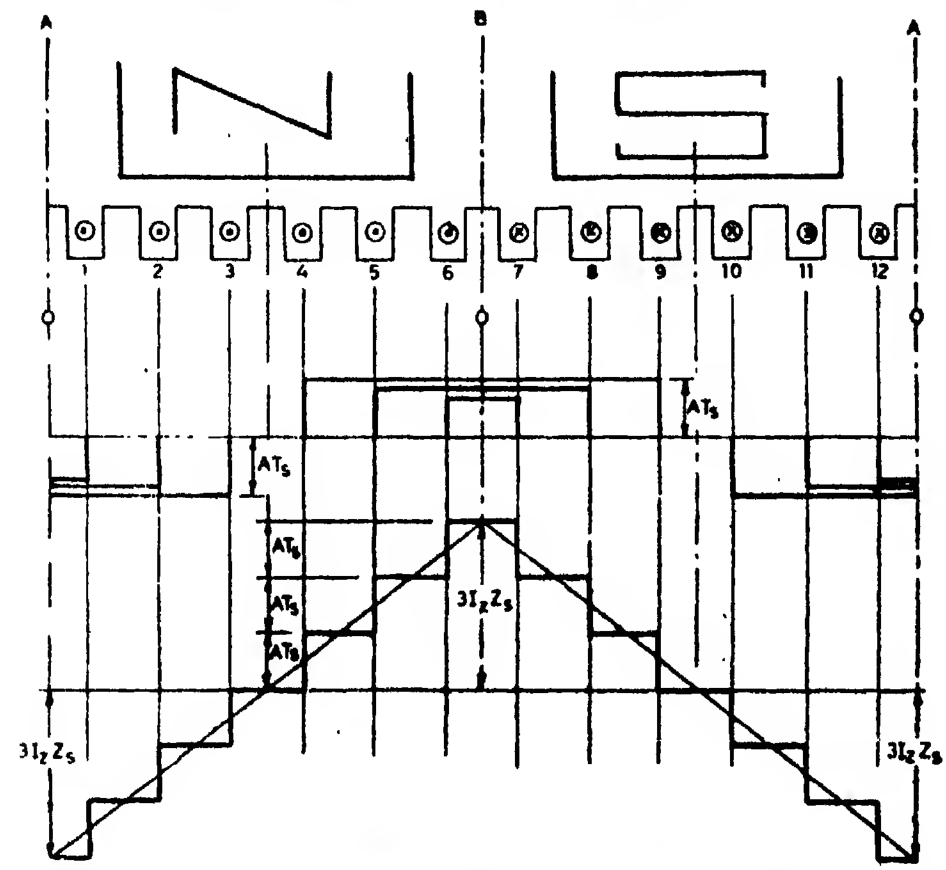


Fig. 6·75. Armature mmf distribution in a d.c. machine with 6 slots per pole.

It should be noted that the conductors in one half of the slots under each pole produce mmf in one direction while the conductors in other half of the slots under each pole produce mmf in the other direction. The conductors in each slot exert a constant mmf AT_s in the form of step over a pole pitch extending between the appropriate slot centres with

$AT_s = I_z . Z_s$ where I_z = current in each conductor.

The conductors in slots 4, 5 and 6 along with conductors in slots 7, 8 and 9 produce mmf along axis OB (with +ve sign) while conductors in slots 10, 11 and 12 together with conductors in slots 1, 2 and 3 produce mmf along axis OA (with −ve sign).

The conductors in slot 6 together with conductors in slot 7 form a unit to produce mmf along axis OA. The mmf of this unit is a step of height $AT_s = I_z Z_s$ which extends from

centre of slot 6 to centre of slot 7. Similarly, conductors in slot 5 and slot 8 form a unit, the mmf being a step of height $AT_s = I_s Z_s$ extending from centre of slot 5 to centre of slot 8. The third unit is formed by conductors in slot 4 and slot 9 with the step of height $AT_s = I_s Z_s$ extending from centre of slot 4 to centre of slot 9.

In a similar manner conductor in slots 1 and 12, 2 and 11 and 3 and 10 form three units. These units produce mmfs which are steps each of height AT_s. However, the mmfs produced are of opposite sign. The adjacent pole pitches have similar units but their mmfs are of opposite sign.

The resultant armature mmf distribution can be obtained by summing up of mmfs of various units. It is clear that the resultant armature mmf is a stepped wave. The peak value of the armature mmf is $3I_sZ_s$ and acts along the interpolar axis midway between the field poles. This stepped mmf wave can be represented approximately by a triangular wave as shown in Fig. 6·75. Now let us take the general case. Let

I_a=total armature current, a=number of parallel paths,
S=number of armature slots, p=number of poles,
Z=total number of armature conductors, Z_s=conductors per slot.

$\therefore$ Peak armature mmf AT_a=(number of units/pole) I_sZ_s

But number of units $=\frac{1}{2}\frac{S}{p}$

Also $I_s = I_a/a$ and $Z_s = Z/S$.

$$\therefore \quad AT_a = \left(\frac{1}{2}\frac{S}{p}\right)\times\frac{I_a}{a}\cdot\frac{Z}{S} = \left(\frac{I_a}{a}\right)\left(\frac{Z}{2p}\right) \qquad ...(6\cdot76)$$

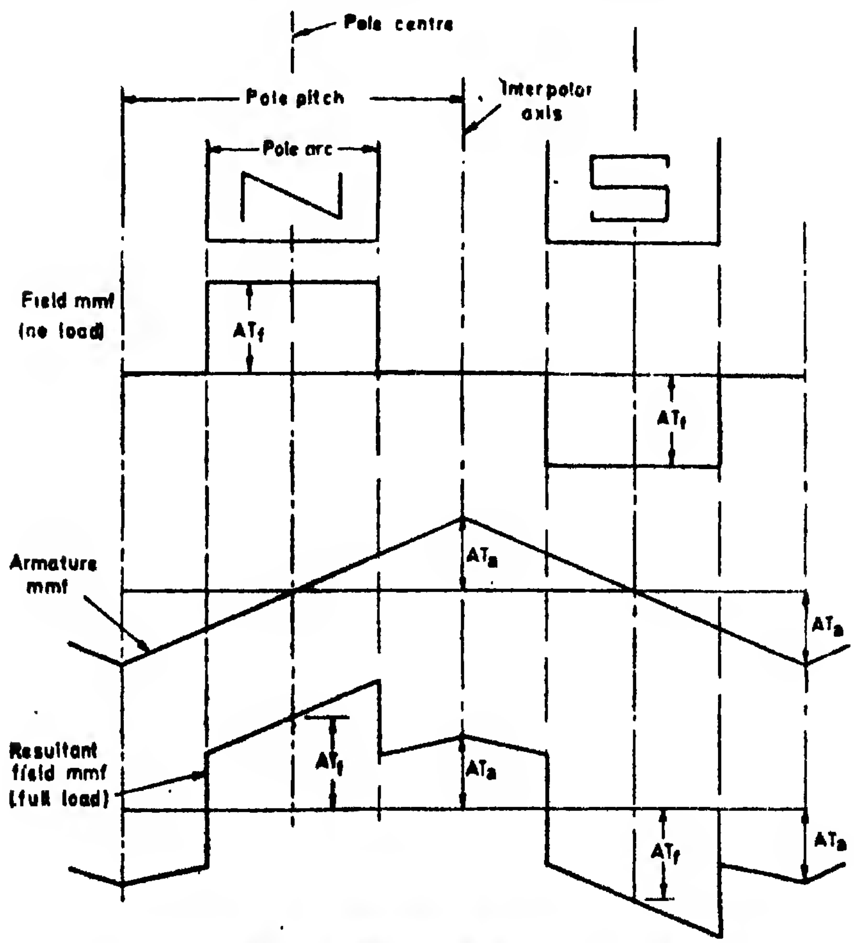


Fig. 6·76. Field mmf distribution at no load and full load.

Instead of neglecting the slot width and obtaining a stepped wave it may be assumed that the armature winding is uniformly distributed in which case the mmf wave becomes triangular in shape as shown in Fig. 6·76. This applies to any machine regardless of the number of slots per pole. Both forms are approximations but because of the relatively large number of coils generally used in d.c. machines, the assumption of uniform distribution is usually made and the triangular waveform is normally used. Thus for all d.c. machines the armature mmf approximates to a symmetrical triangular wave with an amplitude, $AT'_a=(I_a/a)\ (Z/2p)$.

The axis of the armature mmf wave is the interpolar axis so that the armature and field mmf are displaced by 90°. Armature reaction thus has a cross magnetization effect. Fig. 6·76 shows the resulting mmf distribution which is obtained by adding the armature and field mmfs.

Further details about armature reaction in d.c. machines are discussed in the chapter on d.c. machines.

Example 6·37. *The mmf developed by the field coils of a 4 pole d c. generator is 2000 ampere per pole. Its armature is wave connected and has 43 slots with 12 conductors per slot. The ratio of pole arc to pole pitch is 0·7 and the total armature current is 50 A. Calculate the armature mmf and the resultant mmf at*

(*i*) *the interpolar axis* (*ii*) *the pole tips and* (*iii*) *the pole centre.*

Also calculate the flux density at the two pole tips and the centre of the pole if the length of air gap is 4 mm. Neglect the mmf required for iron parts of magnetic circuit and also fringing effects.

Solution. The field mmf distribution is shown in Fig. 6·37 (*a*). It has a constant value of 2000 A over the pole arc and is zero in the interpolar region.

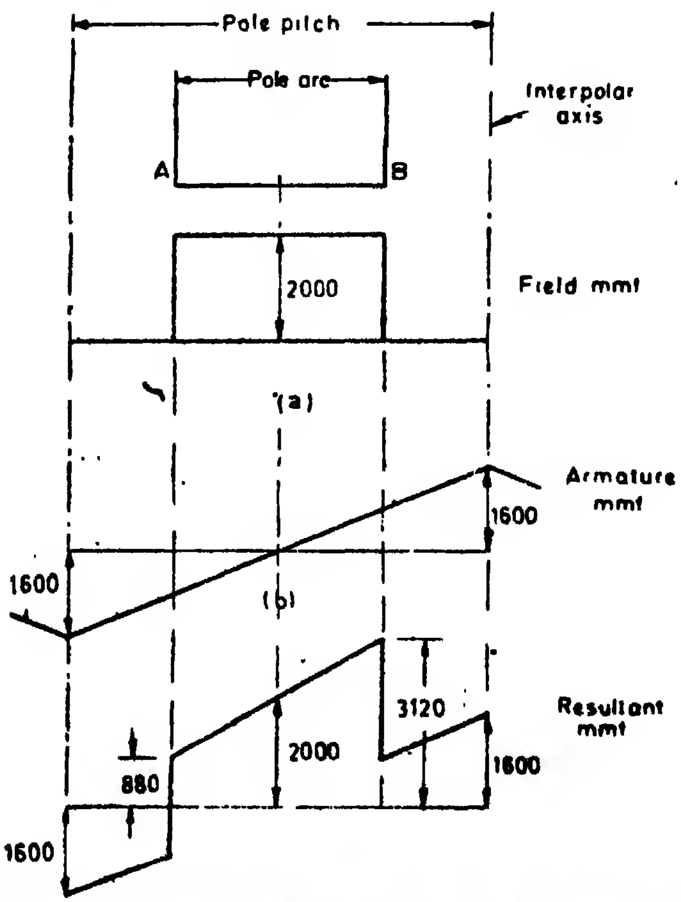


Fig. 6·77. Mmf distribution for d.c. machine of Example 6·37.

Amplitude of armature mmf per pole $(I_a/a)(Z/2p)=(50/2)(516/2\times4)=1600$ A.

The armature mmf is triangular in wave shape with an amplitude of 1600 A at the interpolar axis and zero at the pole centre.

The different mmfs at the interpolar axis are :

Armature mmf=1600 A, field mmf=0 and therefore resultant mmf=1600 A.

(*ii*) Armature mmf at pole tips$=\dfrac{\text{pole arc}}{\text{pole pitch}}\times AT_a=0{\cdot}7\times1600=1120$ A.

Field mmf at pole tips=2000 A.

At pole tip A the armature and field mmf are opposite in sign, therefore, resultant mmf at pole tip $A=2000-1120=880$ A.

At pole tip B, the armature and field mmfs are both of same sign, therefore, resultant mmf at pole tip $B=2000+1120=3120$ A.

(*iii*) The different mmfs at the pole centre are :

Armature mmf=0, field mmf=2000 A.

∴ Resultant mmf=2000 A.

Now, mmf required for the air gap=800,000 $B_g\, l_g$.

Flux density in the air gap under pole tip A

$$=\frac{\text{resultant mmf at } A}{800{,}000\ l_g}=\frac{880}{800{,}000\times0{\cdot}004}=0{\cdot}275\ \text{Wb/m}^2.$$

Similarly,

flux density in the air gap under pole tip $B=\dfrac{3120}{800{,}000\times0{\cdot}004}=0{\cdot}975$ Wb/m²

Flux density in the air gap at the pole centre$=\dfrac{2000}{800{,}000\times0{\cdot}004}=0{\cdot}625$ Wb/m².

6·38. Mmf distribution in 1 phase a.c. machines. Fig. 6·78 shows the mmf distribution of one phase. The trapizoidal wave shape is obtained if we assume a uniformly distributed winding (neglecting the steps produced because of the presence of slots).

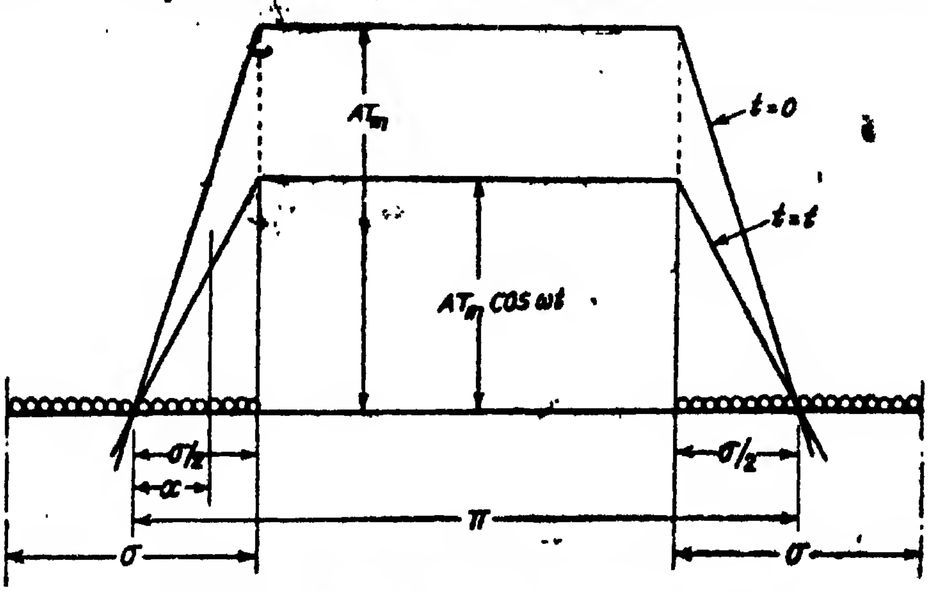


Fig. 6·78. Mmf distribution of a single phase winding.

Let q=slots per pole per phase and Z_s=conductors per slot,

and I_s=rms value of current in each conductor.

Let AT_m be the amplitude of phase mmf at $t=0$.

AT_m=maximum conductor current × turns per pole per phase.

$=\sqrt{2}\,I_s$ (slots per pole per phase) (conductors per slot/2) $=\sqrt{2}\,I_s\,q\,\dfrac{Z_s}{2}=qZ_s\dfrac{I_s}{\sqrt{2}}$

The variation of current, in a conductor, with time is, $i_s=\sqrt{2}\,I_s\cos\omega t$.

∴ The variation of AT_m, with time is $AT_m\cos\omega t$.

Value of mmf due to one phase at a distance x from the centre of a pole phase group at any time t is

(a) $$AT_{(x,\,t)}=\frac{x}{\sigma/2}\,AT_m\cos\omega t \text{ for } 0<x<\frac{\sigma}{2}$$

(b) $$AT_{(x,\,t)}=AT_m\cos\omega t \text{ for } \frac{\sigma}{2}<x<\pi-\frac{\sigma}{2}$$

$\rightarrow \pi - x$

(c) $$AT_{(x,\,t)}=\frac{\pi x}{\sigma/2}\,AT_m\cos\omega t \text{ for } \pi-\frac{\sigma}{2}<x<\pi.$$

Now $$AT_{(x,\,t)}=\sum_{n=0}^{\infty}a_n\sin nx \qquad \text{where} \quad a_n=\frac{4}{\pi}\int_0^{\pi/2}AT_{(x,\,t)}\sin nx\,dx$$

$$=\frac{4}{\pi}\,AT_m\cos\omega t\left[\int_0^{\sigma/2}\frac{2x}{\sigma}\sin nx\,dx+\int_{\sigma/2}^{\pi/x}\sin nx\,dx\right]=\frac{8AT_m\cos\omega t}{\sigma\pi n^2}\sin\frac{n\sigma}{2}$$

or $$AT_{(x,\,t)}=\frac{8AT_m\cos\omega t}{\sigma\pi}\sum_{n=0}^{\infty}\frac{1}{n^2}\sin\frac{n\sigma}{2}\sin nx=\frac{4\,AT_m\cos\omega t}{\pi}\sum_{n=0}^{\infty}\frac{1}{n}\times\frac{\sin n\sigma/2}{n\sigma/2}\sin nx$$

But $\dfrac{\sin n\,\sigma/2}{n\,\sigma/2}$ is equal to distribution factor K_{dn} for nth harmonic.

$$\therefore\quad AT_{(x,\,t)}=\frac{4AT_m\cos\omega t}{\pi}\sum_{n=0}^{\infty}K_{dn}\frac{\sin nx}{n} \qquad \ldots(6\cdot78)$$

Considering all the three phases, there is a displacement of $2\pi/3$ between the phases in space as well as in time. Taking a phase sequence RYB and writing expressions for the mmfs of all the three phases.

$$AT_{(x,\,t)R}=\frac{4AT_m\cos\omega t}{\pi}\sum_{n=0}^{\infty}K_{dn}\frac{\sin nx}{n}$$

$$AT_{(x,\,t)Y}=\frac{4AT_m\cos(\omega t-2\pi/3)}{\pi}\sum_{n=0}^{\infty}K_{dn}\frac{\sin n(x-2\pi/3)}{n}$$

$$AT_{(x,\,t)B}=\frac{4AT_m\cos(\omega t-4\pi/3)}{\pi}\sum_{n=0}^{\infty}K_{dn}\frac{\sin n(x-4\pi/3)}{n}$$

Let the total mmf be $AT_{(x,\,t)T}$ at any distance x and any time t.

The total mmf of all the phases is the summation of the individual mmfs.

$$\therefore \quad AT_{(x,\,t)T}=\frac{4}{\pi}\,AT_m\Bigg[\cos\omega t\sum_{n=0}^{\infty}K_{dn}\,\frac{\sin nx}{n}$$

$$+\cos\left(\omega t-\frac{2\pi}{3}\right)\sum_{n=0}^{\infty}K_{dn}\,\frac{\sin n\,(x-2\pi/3)}{n}$$

$$+\cos\left(\omega t-\frac{4\pi}{3}\right)\sum_{n=0}^{\infty}K_{dn}\,\frac{\sin n\,(x-4\pi/3)}{\pi}\Bigg]$$

Writing expression for the nth harmonic mmf

$$AT_{(x,\,t)n}=\frac{4}{\pi}\,AT_m\Bigg[\cos\omega t\,\frac{\sin nx}{n}\,.\,K_{dn}+\cos\left(\omega t-\frac{2\pi}{3}\right)\frac{\sin n\,(x-2\pi/3)}{n}\,K_{dn}$$

$$+\cos\left(\omega t-\frac{4\pi}{3}\right)\frac{\sin n(x-4\pi/3)}{n}\,K_{dn}\Bigg]$$

$$=\frac{4}{\pi}\,\frac{AT_m\,K_{dn}}{n}\Bigg[\cos\omega t\sin nx+\cos\left(\omega t-\frac{2\pi}{3}\right)\sin n\left(x-\frac{2\pi}{3}\right)$$

$$+\cos\left(\omega t-\frac{4\pi}{3}\right)\sin n\left(x-\frac{4x}{3}\right)\Bigg]$$

$$=\frac{4}{2\pi}\,\frac{AT_m\,K_{dn}}{n}\Bigg[\sin(nx+\omega t)+\sin(nx-\omega t)$$

$$+\sin\left\{nx+\omega t-\frac{2\pi}{3}\left(n+1\right)\right\}+\sin\left\{nx-\omega t-\frac{2\pi}{3}\left(n-1\right)\right\}$$

$$+\sin\left\{nx+\omega t-\frac{4\pi}{3}\left(n+1\right)\right\}+\sin\left\{nx-\omega t-\frac{4\pi}{3}\left(n-1\right)\right\}\Bigg] \quad \ldots(6\cdot79)$$

Let us now find out the amplitudes of individual harmonics.

Fundamental : Putting $n=1$

$$AT_{(x,\,t)1}=3\,.\,\frac{4}{2\pi}\,AT_m\,K_{d1}\sin(x-\omega t)=\frac{6}{\pi}\,AT_m\,K_{d1}\sin(x-\omega t) \quad \ldots(6\cdot80)$$

3rd harmonic : Putting $n=3$ $\quad AT_{(x,\,t)3}=0.$

5th harmonic : Puting $n=5$

$$AT_{(x,\,t)5}=3\,.\,\frac{4}{2\pi}\,\frac{AT_m}{7}\,K_{d5}\sin(5x+\omega t)=\frac{6}{\pi}\,AT_m\frac{K_{d5}}{5}\sin(5x+\omega t) \quad \ldots(6\cdot81)$$

7th harmonic : Putting $n=7$

$$AT_{(x,\,t)7}=3\,.\,\frac{4}{2\pi}\,\frac{AT_m}{7}\,K_{d7}\sin(7x-\omega t)=\frac{6}{\pi}\,AT_m\frac{K_{d7}}{7}\sin(7x-\omega t) \quad \ldots(6\cdot82)$$

$\therefore$ **The resultant mmf can now be expressed as**

$$AT_{(x,\,t)T}=AT_{(x,\,t)1}+AT_{(x,\,t)3}+AT_{(x,\,t)5}+AT_{(x,\,t)7}\ldots\ldots+AT_{(x,\,t)n}\ldots+$$

$$=\frac{6}{\pi}\,AT_m\left[K_{d1}\sin(x-\omega t)+\frac{K_{d5}}{5}\sin(5x+\omega t)+\frac{K_{d7}}{7}\sin(7x-\omega t)+\ldots\right] \quad \ldots(6\cdot83)$$

The values of distribution factors for a phase spread $\sigma=\pi/3$ are

$$K_{d_1}=\frac{\sin\sigma/2}{\sigma/2}=\frac{3}{\pi},\quad K_{d_5}=\frac{\sin 5\sigma/2}{5\sigma/2}=\frac{3}{5\pi},\quad K_{d_7}=\frac{\sin 7\sigma/2}{7\sigma/2}=\frac{-3}{7\pi}.$$

Substituting these values in Eqn. 6·83, resultant mmf :

$$AT_{(x,\,t)T}=\frac{6}{\pi}AT_m\left[\frac{3}{\pi}\sin(x-\omega t)+\frac{3}{5\pi}\cdot\frac{1}{5}\sin(5x+\omega t)-\frac{3}{7\pi}\cdot\frac{1}{7}\sin(7x-\omega t)+\ldots\ldots\right]$$

$$=\frac{18}{\pi^2}AT_m\left[\sin(x-\omega t)+\frac{1}{5^2}\sin(5x+\omega t)-\frac{1}{7^2}\sin(7x-\omega t)+\ldots\right]\qquad\ldots(6\cdot84)$$

Equation 6·84 represents a rapidly converging series. The principal term of this series is $\sin(x-\omega t)$ which is a travelling wave *i.e.* it represents a revolving field.

At time $t=0$ the distribution is sinusoidal as shown by curve $y=\sin x$ (Fig. 6·79) but at a later instant $t=t$ we have $y=\sin(x-\omega t)$ which is sine curve displaced from the former by the angle ωt. This displacement increases uniformly with time t, hence the wave moves forward with a constant speed travelling two pole pitches in a cycle *i.e.* revolving at synchronous speed.

This term is the fundamental component of armature reaction in a synchronous machine where it is seen to be advancing step in step with the rotor field. In an induction motor mmf waves of the stator and rotor currents combine to produce the revolving field which is responsible for the action of the motor.

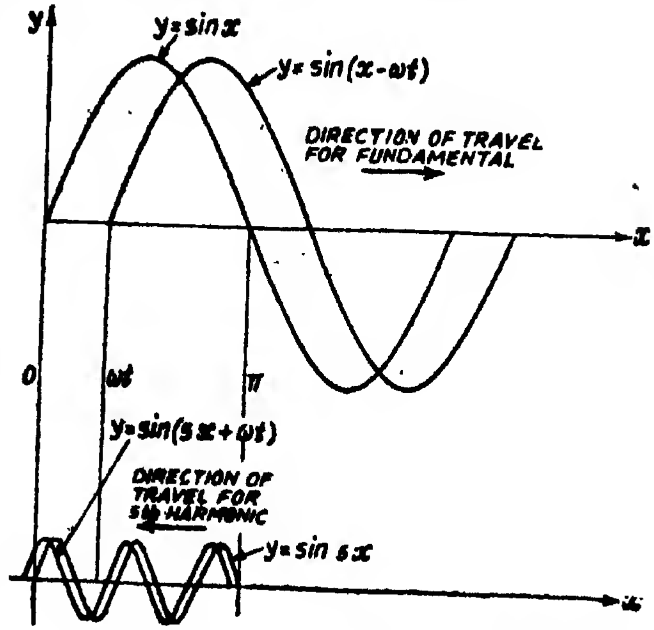


Fig. 6·79. Directions of travel of fundamental and 5th harmonic fields.

The 5th harmonic term is $\sin(5x+\omega t)$. Plotting this wave at $t=0$ (Fig. 6·79), we recognize it as a sinusoidally distributed field with five times as many poles as the fundamental field. As t increases this field is seen to be moving backwards at one fifth the synchronous speed.

The seventh harmonic is represented by the term $(7x-\omega t)$. It has seven times as many poles as the fundamental and moves forward with one seventh synchronous speed.

Thus Eqn. 6·84, demonstrates the presence of fundamental field and harmonic fields super-imposed, each moving with its own synchronous speed relative to the conductors. Summarizing above :

(*i*) The order of harmonics is 5, 7, 11, 13 or in general $6N\pm1$ where N is any positive integer. All multiples of 3 *i.e.* triplens are absent.

(*ii*) The harmonic fields move at a speed proportional to the reciprocal of their order with or against the fundamental. Those of order $6N+1$ move in the same direction as fundamental and those of order $6N-1$ move against it.

(*iii*) The amplitude of harmonic fields is inversely proportional to the square of their order. Thus amplitude of nth harmonics field is $1/n^2$ of the amplitude of fundamental field.

From Eqn. 6·80, amplitude of fundamental of armature mmf is

$$AT_a = AT_{m1} = \frac{6}{\pi} AT_m K_{d1}$$

$$= \frac{6}{\pi} qZ_s \frac{I_s}{\sqrt{2}} K_{d1} = \frac{2{\cdot}7\, I_{ph}\, T_{ph}}{p} K_{d1} \quad \text{...(6·85)}$$

as turns per phase $T_{ph}=qpZ_s/2$ and phase current $I_{ph}=I_s$.

When a short pitch double layer winding is used, the expression for armature mmf must include the pitch factor K_{p1}.

$$AT_a = AT_{m1} = \frac{2{\cdot}7\, I_{ph}\, T_{ph}}{p} K_{d1} K_{p1} = 2{\cdot}7 \frac{I_{ph}\, T_{ph}}{p} K_{w1} \quad \text{...(6·86)}$$

6·39. Field circuit equivalent of armature mmf in 3 phase machines. It is necessary to estimate the field mmf required to overcome the armature mmf in order to design the field windings of a.c. machines.

The field windings in the case of synchronous machines is of the "single phase" type and caries d.c. excitation. This makes the problem complicated and it is only possible to obtain the field circuit equivalent of "direct axis armature mmf."

In cylindrical rotor machines the field winding is distributed in slots and gives a trapezoidal mmf as shown. It has a constant amplitude as the winding carries d.c. For this case it suffices to equate the fundamental of field and armature mmfs.

For silent pole machines, the method of equating mmfs is not valid because the field coils are concentrated and the length of air gap is not uniform over the pole pitch. The armature mmf must be multiplied by a reduction factor in order to find the equivalent field mmf. This reduction factor for the direct axis mmf is

$$\rho_d = \frac{\alpha + \sin\alpha}{4\sin\alpha/2} \quad \text{...(6·87)}$$

where α = angle of span of pole arc.

The value ρ_d is normally equal to 0·85.

Field equivalent of direct axis armature mmf

$$AT_{ad} = \rho_d AT_a = \frac{2{\cdot}7\, I_d T_{ph} K_{w1}\, \rho_d}{p}$$

$$= 2{\cdot}7 \frac{I_{ph}\, T_{ph}\, K_{w1}\, \rho_d}{p} \sin\psi \quad \text{...(6·88)}$$

where I_d now is the armature current component causing direct axis armature mmf.

$$I_d = I_{ph} \sin\psi \quad \text{...(6·89)}$$

and $\psi=\phi+\delta$ for lagging power factors for generator where $\cos\phi$ is the power factor angle and δ is the load angle.

Example 6·38 *Plot to scale the mmf distribution over armature of a machine having a uniformly distributed winding with a phase spread of 60°, (a) when the current in phase R is maximum; (b) $\pi/6$ later than (a). Assume sinusoidal current variation and full pitch coils.*

Solution. The expression (Eqn 6·84) for mmf distribution of a uniformly distributed, 3 phase winding with 60° phase spread and full pitch coils and carrying sinusoidally varying currents is

$$AT_{(x,t)T} = \frac{18}{\pi^2} AT_m \left[\sin(x-\omega t) \frac{1}{5^2} \sin(5x+\omega t) - \frac{1}{7^2} \sin(7x-\omega t) + \dots \right]$$

(a) This expression is based upon the expression foru rrcent in phase R as :
$i = I_m \cos \omega t$.

Therefore current in phase R is maximum when $\omega t = 0$.

When $\omega t = 0$.

$$AT_{(x,\,t)T} = \frac{18}{\pi^2}\,AT_m\left[\sin x + \frac{1}{5^2}\sin 5x - \frac{1}{7^2}\sin 7x + \ldots\ldots\right]$$

At $x = 0$. $AT_{(x,\,t)T} = 0$.

At $x = \pi/6$, $AT_{(x,\,t)T} = \frac{18}{\pi^2}\,AT_m\left[\sin \pi/6 + \frac{1}{5^2}\sin 5\pi/6 - \frac{1}{7^2}\sin 7\pi/6\right]$
$= AT_m$.

At $x = \pi/2$, $AT_{(x,\,t)T} = \frac{18}{\pi^2}\,AT_m\left[\sin \pi/2 + \frac{1}{5^2}\sin 5\pi/2 - \frac{1}{7^2}\sin 7\pi/6\right]$
$= 2\,AT_m$.

Similarly $AT_{(x,\,t)T}$ at

$x = 5/6\pi$ is AT_m ; $x = \pi$ is 0 ; $x = 7\pi/6$ is $-AT_m$; $x = 3\pi/2$ is $-2\,AT_m$; $x = 11/6\pi$ is $-AT_m$ and $x = 2\pi$ is 0.

Thus the armature mmf distribution at $\omega t = 0$ is a pointed curve as shown in Fig. 6·80 (a).

When $\omega t = \pi/6$.

$$AT'_{(x,\,t)T} = \frac{18}{\pi^2}\,AT_m\left[\sin(x - \pi/6) + \frac{1}{5^2}\sin(5x + \pi/6) - \frac{1}{7^2}\sin(7x - \pi/6) + \ldots\right]$$

Substituting various values of x in the above expression, we have

At $x = 0$, $AT'_{(x,\,t)T} = -AT_m$

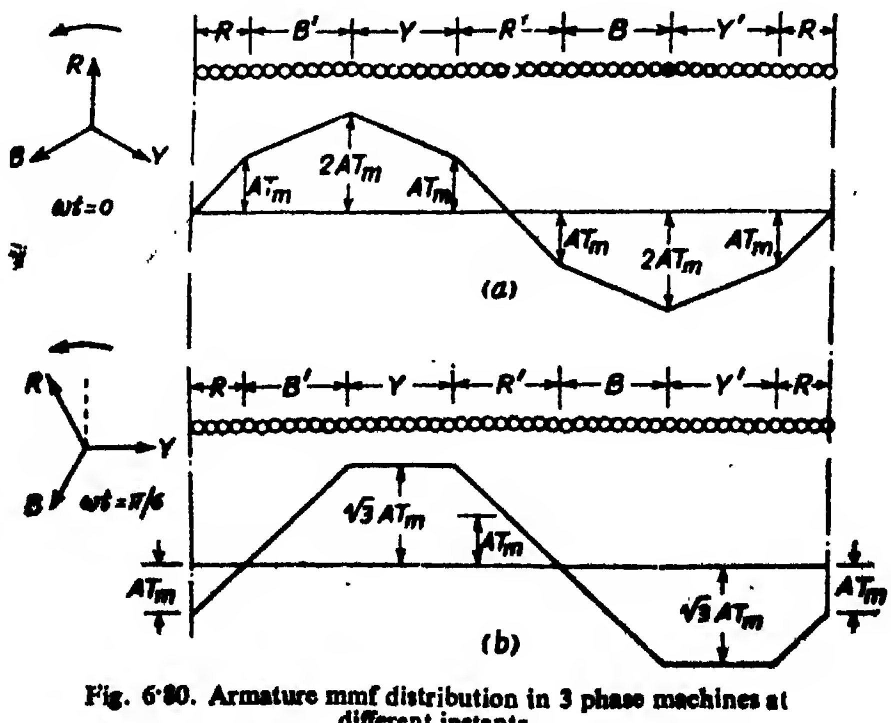


Fig. 6·80. Armature mmf distribution in 3 phase machines at different instants.

At $x=\pi/6$ $AT_{(x,\,t)T}=0$

The other values of $AT_{(x,\,t)T}$ are :

$\sqrt{3}\,AT_m$ at $x=\pi/2$; $\sqrt{3}\,AT_m$ at $x=5\pi/6$; AT_m at $x=\pi$; 0 at $x=7\pi/6$; $-\sqrt{3}\,AT_m$ at $x=3\pi/2$; $-\sqrt{3}\,AT_m$ at $x=11\pi/6$; $-AT_m$ at $x=2\pi$.

Thus the armature mmf disitribution is a flat topped curve at $\omega t=\pi/6$. This is shown in Fig. 6·80 (*b*).

Example 6·39. *A 1500 rpm 50-Hz, star connected salient pole, 3 phase alternator has a single-layer stator winding with 300 turns per phase and a spread of 60°. If there are 800 exciting turns on each pole, calculate the change in field current necessary to maintain the same terminal voltage when the output of the machine is varied from zero to 50 A at zero power factor assuming stator winding to be uniformly distributed. Take the field mmf to be a rectangle extending over polar arc, and the ratio pole : pole pitch = 2/3.*

Solution. Synchronous speed of machine $n_s=\dfrac{1500}{60}=25$ r.p.s.

Number of poles $=2f/n_s=2\times 50/25=4$.

The winding is uniformly distributed and therefore distribution factor for the fundamental $K_{d_1}=0{\cdot}955$.

The winding is single layer type and therefore full pitch coils are used or pitch factor for fundamental, $K_{p_1}=1$ and winding factor fundamental $K_{w1}=K_{d_1}\,K_{p_1}=0{\cdot}955$.

The machine operates at 'zero' power factor lagging and therefore the armature mmf acts only on the direct axis and it has no quadrature axis component.

The power factor is zero and therefore $\phi=90°$. The machine delivers no power and hence load angle $\delta=0$. Thus $\psi=\phi+\delta=90°$ and $\sin\psi=1$].

Armature current producing direct axis armature mmf $I_d=I_{ph}\sin\psi=50$ A.

Angle of span of pole arc $\alpha=2\pi/3$.

Reduction factor $\rho_d=\dfrac{\alpha+\sin\alpha}{4\sin\alpha/2}=\dfrac{2\pi/3+\sin 2\pi/3}{4\sin\pi/3}=0{\cdot}853$

Field mmf equivalent to direct armature mmf

$$AT_{ad}=2{\cdot}7\,\frac{I_{ph}\,T_{ph}}{p}K_{w_1}\,\rho_d\,\sin\psi=\frac{2{\cdot}7\times 50\times 300}{4}\times 0{\cdot}955\times 0{\cdot}853=8240\text{ A.}$$

$\therefore$ Field current required to overcome direct axis armature mmf at load

$$I_f=\frac{AT_{ad}}{T_f}=\frac{8240}{800}=10{\cdot}3\text{ A.}$$

Since the armature mmf is zero at no load, the field mmf or the field current required to overcome it is also zero. Therefore the change in excitation is 10·3 A.

Example 6·40. *Find the frequency of harmonic emfs induced in the stator and rotor windings by the 5th and 7th harmonic fields of armature of a 3 phase synchronous machine operating at normal speed and carrying balanced currents in its armature winding.*

Solution. The fifth harmonic of the armature revolves at 1/5th the synchronous speed of the fundamental with respect to the stator and its poles are 5 times those of the fundamental. Therefore frequency of harmonic emf induced in the stator winding by the 5th harmonic field $=\dfrac{5p\times(n_s/5)}{2}=\dfrac{pn_s}{2}=f_1$

where f_1 = fundamental frequency

n_{s1} = synchronous speed corresponding to fundamental.

Frequency of harmonic emf induced in the stator winding by the 7th harmonic field

$$=\frac{7p\times n_{s1}/7}{2}=\frac{pn_{s1}}{2}=f_1.$$

The 5th harmonic field revolves in the opposite direction to that of the rotor. Therefore speed of 5th harmonic field with respect to rotor winding

$$=n_{s1}+\frac{n_{s1}}{5}=\frac{6n_{s1}}{5}$$

∴ Frequency harmonic emf induced in the winding by the 5th harmonic field

$$=\frac{5p\times 6n_{s1}/5}{2}=\frac{6pn_{s1}}{2}=6f_1.$$

The 7th harmonic field revolves in the same direction to that of the rotor. Therefore relative speed of rotor and 7th harmonic field $=n_{s1}-\frac{n_{s1}}{7}=\frac{6}{7}n_{s1}$.

Frequency of harmonic emf induced in the rotor winding by the 7th harmonic

$$=\frac{7p}{2}\times\frac{6}{7}\ n_{s1}=6\frac{pn_{s1}}{2}=6f_1.$$

EDDY CURRENT LOSSES IN CONDUCTORS

6.39. **Skin Effect.** Let us consider an isolated conductor as shown in Fig. 6.81 (*a*). Let the conductor cross section be divided into four laminae of equal area and hence of equal resistance as shown in Fig. 6.81 (*b*). The current flowing in these laminae produces its own flux. It is clear that lamina 1 nearest to the centre links with maximum flux and the outer most lamina 4 links with minimum flux. Therefore, inductance and hence the reactance of all the laminae is not equal. The reactance of the inner most lamina 1 is highest and that of outermost lamina 4 is the lowest. The reactance of laminae goes on decreasing as we proceed outwards from centre.

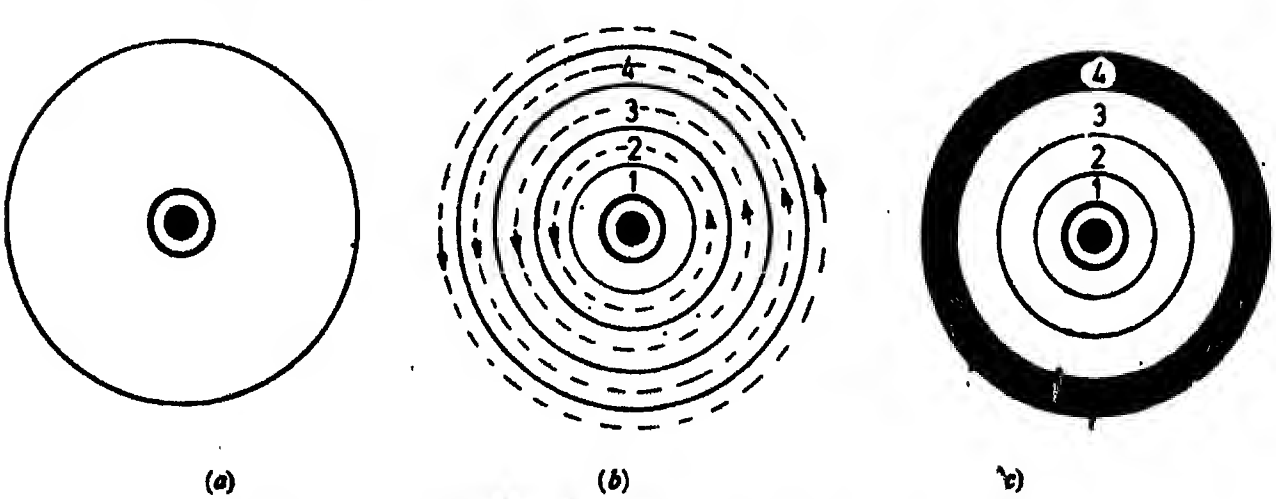


Fig. 6.81. Skin Effect.

If R is the resistance of one lamina and L its inductance, the impedance is $(R^2+\omega^2L^2)^{1/2}$

The current carried by each lamina is therefore,

$I=E/(R^2+\omega^2L^2)^{1/2}$ where E is the applied voltage.

At low frequencies the term ωL is small as compared with R and therefore $I=E/R$ (very nearly). Therefore the current distribution is uniform considering that the laminae have equal resistance.

At high frequencies, ωL is considerably larger than R and therefore current $I=E/\omega L$ (very nearly). Under these conditions, then, the difference in inductance between the central and outer laminae becomes very important. The inner laminae carry only a small current, on account of their high inductance and, the current in the conductor is almost entirely carried by the outermost lamina at the surface *i.e.* by the outer skin of the conductor as shown in Fig. 6·81 (*c*). Hence, this phenomenon is called **Skin Effect**. Therefore, effective cross-section of the conductor at high frequencies is the area of the outer skin. The reduction in the effective area results in increase of resistance of the conductor. Hence the a.c. resistance of the windings is higher than the d.c. resistance, the difference depending upon the cross-section of the conductor, and upon the resistivity of material.

The skin effect is obviously more predominant in conductors of large cross-section. The increased value of resistance due to skin effect causes additional I^2R loss. The skin effect may be interpreted another way. The current carrying conductor gives rise to eddy currents due to its own field. The eddy currents are produced on account of the difference in inductance of central portion (whose inductance is high) and the outer portion (whose inductance is low). The reactance of central portion is greater and therefore the current flows more readily in the outer portions of the conductor. This results in non-uniform current distribution over the cross section of the conductor, thereby producing non-uniform current density. Any departure from uniform current density increases the I^2R loss over its d.c. value since I^2R loss is proportional to square of current density. The greater self induced emf in the central portions of the conductor due to their high* inductance causes circulating currents (eddy currents) which, superimposed on the main current, cause additional I^2R loss known as eddy current loss.

The conductors in rotating machines and transformers however, are not isolated. Fig. 6·82 shows a conductor placed in a slot of a rotating machine.

The conductor has been divided into 5 laminae. The laminae produce leakage flux as shown. It is clear from the diagram that lamina 5 at the bottom of the slot links with maximum flux and hence has the highest inductance. The flux linking with and hence the inductance of the laminae goes on decreasing as we go upwards. Therefore, the emf induced in the laminae is highest in 5 and lowest in 1.

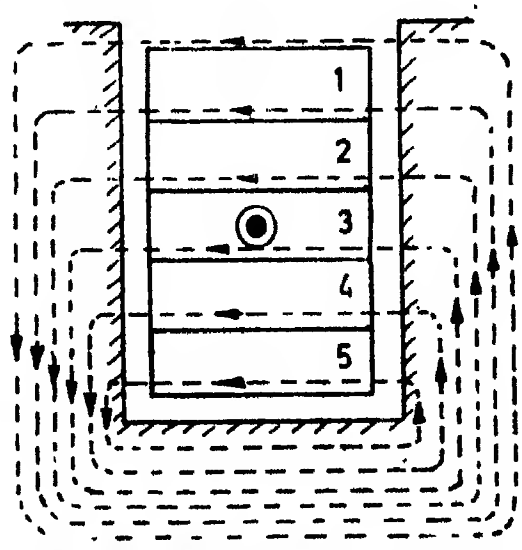


Fig. 6·82. Leakage field of a conductor placed in a slot.

This unequal induced emf in the different laminae of the same conductor causes eddy currents to flow. These eddy currents circulating over the cross-section of the conductor cause additional I^2R losses.

The eddy current losses due to leakage flux are not only produced in the conductors but also in adjacent machine parts. For example the core clamping plates which are made of ferro-magnetic material put in rotating machines to bind the core intensify the leakage flux. This increased magnetic leakage flux induces an emf in them thereby causing eddy currents to flow and cause additional I^2R loss. The tanks of transformers made of ferro-magnetic materials also increase, the leakage flux and therefore produce additional I^2R loss. The practice of using aluminium tanks has been started only with a view that aluminium being a non-ferro-magnetic material does not intensify the leakage field and therefore reduces the additional I^2R losses.

*Emf $e=-L\frac{di}{dt}$.

6·40. Eddy current loss. The mutual or useful flux in rotating machines and transformers induces emf in the windings. This induced emf is responsible for the transfer or conversion of energy. The leakage or non useful flux also induces emf. The induced emf creates a disturbance in the current distribution over the windings thereby causing additional I^2R loss in the windings. The additional I^2R loss in conductors caused by emfs produced by leakage flux is called **Eddy current loss.**

6·40·1. Eddy current loss in a single conductor placed in a slot. Fig. 6·82. (*a*) shows a single solid conductor having a height h and a width b in a slot of width W_s. A current flowing in the conductor sets up an mmf. Assuming a uniform current distribution over the whole cross-section of the conductor, the value of mmf at a height x from the bottom

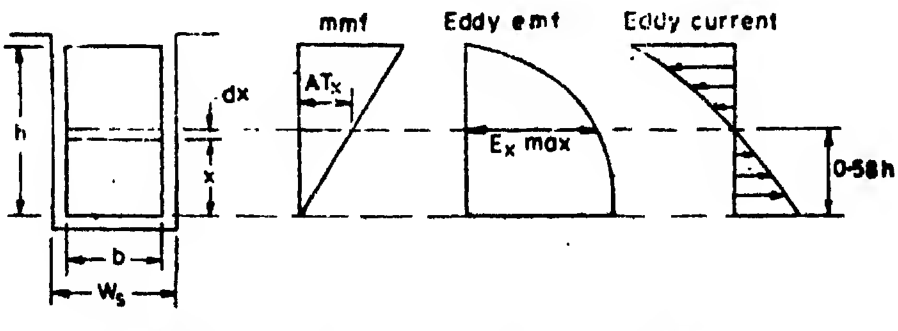


Fig. 6·82. Eddy current in single conductor in a slot.

of conductor, produced by current I flowing through the conductor, $AT_x = I\,x/h$.

As we go up from the bottom of the conductor x varies from 0 to h and the mmf varies accordingly. Fig 6·82 (*b*) shows the mmf at various heights. This mmf produces a flux passing across the slot. It is assumed that iron is infinitely permeable and the lines of force pass horizontally straight across the slot.

Flux at a distance x in a strip of width dx

$$d\Phi_x = \frac{\text{mmf}}{\text{reluctance}} = \frac{Ix/h}{W_s/\mu_0 L_s dx} = \mu_0 I \frac{L_s}{hW_s}\, x\, dx$$

Total flux from height 0 to height x is

$$\Phi_x = \int_0^x \mu_0 I \frac{L_s}{hW_s}\, x\, dx = \mu_0 I \frac{L_s}{hW_s} \cdot \frac{x^2}{2}.$$

Total flux across the slot from height 0 to height h is

$$\Phi_s = \int_0^h \mu_0 I \frac{L_s}{hW_s}\, x\, dx = \mu_0 I \frac{L_s h}{2W_s}.$$

An element of conductor at a height x is linked by a flux which is between height x and height h, or the element dx at a height x from bottom is linked by flux

$$\Phi_s - \Phi_x = \mu_0 I \frac{L_s}{2W_s}\left(h - \frac{x^2}{h}\right)$$

Considering the instant when the current is maximum, I is replaced by its maximum value I_m.

$$(\Phi_s - \Phi_x)_{max} = \mu_0 I_m \frac{L_s}{2W_s}\left(h - \frac{x^2}{h}\right)$$

$\therefore$ Peak value of emf of self-induction in this element

$$E_{x(max)}=2\pi f\,(\Phi_z-\Phi_x)_{max}=\pi\mu_0 f\,I_m\,\frac{L_s}{W_s}\left(h-\frac{x^2}{h}\right) \qquad ...(6\cdot90)$$

This emf is in quadrature with the current as it is on account of self induction.

From Eqn. 6·90 it is clear that the induced emf is maximum at the bottom and is zero at the top. The average value of induced emf over the conductor is obtained by averaging $E_x(max)$ over the height h.

Thus

$$E_{max(average)}=\frac{1}{h}\int_0^h E_{x(max)}\,dx=\frac{1}{h}\int_0^h \pi\mu_0 f\,I_m\,\frac{L_s}{W_s}\left(h-\frac{x^2}{h}\right)dx$$

$$=\pi\mu_0 f\,I_m\,\frac{L_s}{W_s}\cdot\frac{2h}{3} \qquad ...(6\cdot91)$$

Comparing Eqns. 6·90 and 6·91 the height at which the average emf occurs can be found by equating

$$h-\frac{x^2}{h}=\frac{2h}{3} \qquad \text{or} \qquad x=\frac{h}{\sqrt{3}}=0\cdot58\,h.$$

This height is a little above the middle of the conductor. Below this height the emf increases and above this height the emf decreases. Thus circulating currents flow owing to difference in the emfs. At $x=0\cdot58\,h$ there will not be any circulating current, but above it a circulating current will flow returning in the part of conductor below it.

The difference of emf at any height x and average emf causes the flow of current. This difference in emf is

$$E_{max(difference)}=\pi\mu_0 f\,I_m\,\frac{L_s}{W_s}\left(h-\frac{x^2}{h}-\frac{2h}{3}\right).$$

The resistance R_x of the element dx at a height x is $R=\rho L/b\;dx$

Hence the maximum value of eddy current

$$i_x=\frac{E_{max(difference)}}{R_x}=\frac{\pi\mu_0 f\,I_m\,\dfrac{L}{W_s}\left(h-\dfrac{x^2}{h}-\dfrac{2h}{3}\right)}{\rho\,L_s/b\;dx}$$

$$=\pi\mu_0 f\,I_m\,\frac{b}{\rho W_s}\left(\frac{h}{3}-\frac{x^2}{h}\right)dx.$$

The eddy current i_x summed over 0 to 0·58 h and 0·58 h to h gives two equal and opposite values of current.

The main armature current in the strip is $I_m\,dx/h$.

Therefore, for any strip

$$\frac{\text{Eddy current } I^2R \text{ loss}}{\text{Main current } I^2R \text{ loss}}=\left\{\frac{\dfrac{\pi\mu_0 f\,I_m\,b}{\rho W_s}\cdot\left(\dfrac{h}{3}-\dfrac{x^2}{h}\right)dx}{I_m\,dx/h}\right\}^2$$

$$=\frac{\pi^2\mu_0^2 f^2 b^2}{\rho^2 W_s^2}\left(\frac{h^2}{3}-x^2\right)^2$$

This expression integrated and averaged over height h gives the ratio of total eddy current I^2R loss to the main current I^2R loss.

$$\therefore \frac{\text{Eddy current } I^2R \text{ loss}}{\text{Main current } I^2R \text{ loss}} = \frac{1}{h} \cdot \frac{\pi^2\mu_0^2 f^2 b^2}{\rho^2 W_s^2} \int_0^h \left(\frac{h^2}{3} - x^2\right) dx$$

$$= \frac{4}{45} \cdot \frac{\pi^2\mu_0^2 f^2 b^2}{\rho^2 W_s^2} \cdot h^4 = \frac{4}{45}(\alpha h)^4 \quad \ldots(6\cdot 92)$$

where

$$\alpha = \frac{\pi\mu_0 f d}{\rho W_s} \quad \ldots(6\cdot 93)$$

Eddy loss ratio is defined as

$$K_e = \frac{\text{total } I^2R \text{ loss}}{I^2R \text{ loss due to main current}}$$

$$= \frac{I^2R \text{ loss due to main current} + I^2R \text{ loss due to eddy current}}{I^2R \text{ loss due to main current}} = 1 + \frac{4}{45}(\alpha h)^4 \quad \ldots(6\cdot 94)$$

At 60°C, $\rho = 2 \times 10^{-8}$ Ω m for copper and $4\cdot 5 \times 10^{-8}$ Ω m for aluminium. If $f = 50$ Hz

$$\alpha = 100\sqrt{b/W_s} \text{ for copper}$$

$$= 150\sqrt{b/W_s} \text{ for aluminium.}$$

Taking an example of a slot 5 mm wide containing a solid copper conductor 4 mm wide

$$\alpha = 100(4/5)^{1/2} \text{ and } \alpha^4 = (100)^4 \times 16/25.$$

∴ Eddy loss ratio

$$K_e = 1 + \frac{4}{45}(\alpha h)^4 = 1 + \frac{4}{45} \cdot \frac{16}{25}(100)^4 h^4 = 1 + 0\cdot 0568\,(100)^4 h^4.$$

For $h = 1$ mm,	$K_e = 1\cdot 0561$
$h = 2$ mm,	$K_e = 1\cdot 91$
$h = 30$ mm,	$K_e = 5\cdot 6$.

The above calculations show that if the height is above 30 mm, the losses are nearly 6 times as great as those calculated from main current and the d.c. resistance of the winding. Such an excessive I^2R loss cannot be tolerated, and the conductor depth has to be reduced. In practice it is unusual to keep $\alpha h > 0\cdot 7$ for a single solid conductor.

6·40·2. Eddy current loss in subdivided (laminated) conductors placed in slots. The production of excessive eddy currents can be avoided by properly sectionalizing or subdividing the conductor in the slot. These subdivisions are insulated from each other and thus the eddy currents of one cannot travel to the other.

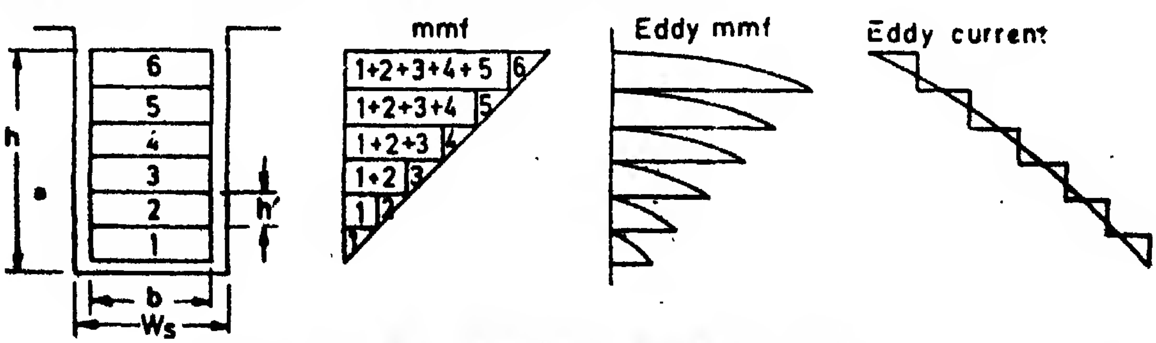


Fig. 6·83. Eddy current loss in subdivided conductors.

Let the conductor be divided into N layers, each of height h' (Fig. 6·83). Total height of all the layers, $h = Nh'$ if the thickness of insulation between layers is neglected.

Let I and I_m be rms and the maximum values respectively of armature current flowing through all the N layers.

Flux in a strip of height dx at a distance x from bottom

$$d\Phi_x = \frac{I\,x/h}{W_s\,\mu_0 L dx} = \mu_0 I \frac{L_s}{W_s}\,\frac{x}{h}\,dx$$

Total flux from height 0 to height x, $\Phi_x = \int_0^x \mu_0 I \frac{L_s}{W_s}\,\frac{x}{h}dx = \mu_0 I \frac{L_s}{W_s}\,\frac{x^2}{2h}$

Total flux across the slot from height 0 to height h

$$\Phi_s = \int_0^x \mu_0 I \frac{L_s}{W_s}\,\frac{x}{h}\,dx = \mu_0 I \frac{L_s}{W_s}\,\frac{h}{2}$$

A strip at a height x is linked by a flux in the height x to height h. This flux is equal to $\Phi_s - \Phi_x$ where

$$\Phi_s - \Phi_x = \mu_0 I \frac{L_s}{2W_s}\left(h - \frac{x^2}{h}\right)$$

Considering the instant when the current is maximum,

$$(\Phi_s - \Phi_x)_{max} = \mu_0 I_m \frac{L_s}{2W_s}\left(h - \frac{x^2}{h}\right)$$

The maximum induced emf in the strip $E_{x(max)} = 2\pi f\,(\Phi_s - \Phi_x)_{max}$

$$= \pi\mu_0 f I_m \frac{L_s}{W_s}\left(h - \frac{x^2}{h}\right) \qquad \text{...(6·95)}$$

Average emf in the first layer is obtained by averaging $E_{x(max)}$ from height 0 to height h'.

Average emf in 1st layer, $E_{max(average)1} = \frac{1}{h'}\int_0^{h'} \pi\mu_0 f I_m \frac{L_s}{W_s}\left(h - \frac{x^2}{h}\right)dx.$

In a similar manner the average emf in 2nd layer

$$E_{max(average)2} = \frac{1}{h'}\int_{h'}^{2h'} \pi\mu_0 f I_m \frac{L_s}{W_s}\left(h - \frac{x_2}{h}\right)dx$$

$$= \frac{1}{h/N}\int_{h/N}^{2h/N} \pi\mu_0 f I_m \frac{L_s}{W_s}\left(h - \frac{x^2}{h}\right)dx$$

and the average emf in pth layer

$$E_{max(average)p} = \frac{1}{h/N} \times \int_{h(p-1)/N}^{hp/N} \pi\mu_0 f I_m \frac{L_s}{W_s}\left(h - \frac{x^2}{h}\right)dx$$

$$= \frac{1}{h/N}\,\pi\mu_0 f I_m \frac{L_s}{W_s}\left[\frac{h^2 p}{N} - \frac{h^2 p^2}{3N^3} - \frac{h^2}{N}(p-1) + \frac{h^2(p-1)^2}{2N^3}\right]$$

$$= \pi\mu_0 f I_m \frac{L_s}{W_s}\left[h - \frac{h}{3N^2} - \frac{h}{N^2} \times p\,(p-1)\right] \qquad \text{...(6·96)}$$

Comparing Eqn. 6·95 and 6·96 the average value of emf in the pth layer occurs when

$$h-\frac{x^2}{h}=h-\frac{3}{3N^2}-\frac{h}{N^2}p(p-1).$$

The difference of the average emf and the emf at any height x causing the flow of current is given by :

$$E_{max\,(difference)}=\pi\mu_0\, f\, I_m\,\frac{L_s}{W_s}\left[h-\frac{x^2}{h}-h+\frac{h}{3N^2}+\frac{h}{N^2}p(p-1)\right]$$

$$=\pi\mu_0\, f\, I_m\,\frac{L_s}{W_s}\left\{\frac{h}{3N^2}[1+3p(p-1)]-\frac{x^2}{h}\right\}$$

The resistance of the strip dx at a height x, $R_x=\rho L_s/b\,dx$.

Hence, eddy current

$$i_e=\frac{\pi\mu_0\, f\, I_m\dfrac{L_s}{W_s}\left\{\dfrac{h}{3N^2}[1+3p(p-1)]-\dfrac{x^2}{h}\right\}}{\rho L_s/b\,dx}$$

$$=\pi\mu_0 f\frac{b}{\rho W_s}\, I_m\left\{\frac{h}{3N^2}[1+3p(p-1)]-\frac{x^2}{h}\right\}dx.$$

The ratio of I^2R loss due to eddy currents to that due to main current in the strip

$$=\frac{\left[\pi\mu_0 f\dfrac{b}{\rho W}\, I_m\left\{\dfrac{h}{3N^2}\left(1+3p\,\overline{p-1}\right)-\dfrac{x^2}{h}\right\}dx\right]^2}{(I_m\, dx/h)^2}$$

$$=\left(\pi\mu_0 f\, h\,\frac{b}{\rho W_s}\right)^2\left\{\frac{h}{3N^2}\left(1+3p(p-1)\right)-\frac{x^2}{h}\right\}^2$$

$$=\left(\pi\mu_0\,\frac{fb}{\rho W_s}\cdot\frac{h^2}{N^2}\right)^2\left[\frac{1+3p(p-1)}{3}-\frac{x^2N^2}{h^2}\right]^2$$

This expression when integrated over height of one layer gives the ratio of loss in a layer due to eddy currents to that due to main current.

$$\frac{\text{Eddy current } I^2R \text{ loss in } p\text{th layer}}{\text{Main current } I^2R \text{ loss in } p\text{th layer}}$$

$$=\left(\pi\mu_0 f\frac{b}{\rho W_s}\cdot\frac{h^2}{N^2}\right)^2\int_{(p-1)h/N}^{p\,h/N}\left[\frac{1+3p(p-1)}{3}-\frac{x^2N^2}{h^2}\right]^2 dx$$

$$=(\alpha h')^4\int_{(p-1)h'}^{ph'}\left[\frac{1+3p(p-1)}{3}-\frac{x^2}{h'^2}\right]^2 dx$$

$$=(\alpha h')^4\left[\frac{4}{45}+\frac{p(p-1)}{3}\right].$$

$$K_{ep}=\frac{\text{total } I^2R \text{ loss in } p\text{th layer}}{I^2R \text{ loss due to main current in } p\text{th layer}}$$

$$=1+(\alpha h')^4\left[\frac{4}{45}+\frac{p(p-1)}{3}\right] \qquad \ldots(6{\cdot}97)$$

If h' is less than 0·7, the effect of $\frac{4}{45}$ is negligible.

$$\therefore\quad K_{ep}=1+(\alpha h')^4\left[\frac{p(p-1)}{3}\right]. \qquad \ldots(6{\cdot}98)$$

The average loss ratio for N layers is

$$K_{e(av)}=1+(\alpha h')^4\frac{N^2}{9}\text{ (with sufficient accuracy)} \qquad ...(6\cdot 99)$$

6·40·3. Critical Depth. The I^2R loss due to eddy currents increases if the depth is increased while the I^2R loss due to main current is reduced with increase in depth. Hence for a given value of α there will be a critical depth for which the total losses are minimum.

Total I^2R loss

$$=[1+K(\alpha h')^4]\,(I^2R \text{ loss due to main current})$$

where $K=\frac{4}{45}$ for a single conductor,

$$=\frac{p(p-1)}{3} \text{ for } p\text{th layer,}$$

$$=\frac{N^2}{9} \text{ for } N \text{ layers.}$$

I^2R loss due to main current are proportional to $1/h'$.

$$\therefore \quad \text{Total loss} \propto \left[\frac{1}{h'}\,(1+K\,\alpha^4 h'^4)\right] \propto \left[\frac{1}{h'}+K\alpha^4 h'^3\right].$$

This is minimum when $\frac{d}{dh'}\left(\frac{1}{h'}+K\alpha^4 h'^3\right)=0$ or when $h'=\frac{1}{\alpha(3K)^{1/4}}$.

$$\therefore \quad \text{Critical depth} \quad h_c'=\frac{1}{\alpha(3K)^{1/4}} \qquad ...(6\cdot 100)$$

For critical depth $\quad h'=h_c'=\frac{1}{\alpha(3K)^{1/4}}$.

Eddy loss ratio for critical depth $K_{e(av)}=1+K\alpha^4.\frac{1}{\alpha^4\times 3K}=1\cdot 33.$

Therefore the eddy loss ratio with critical depth is always $1\cdot 33$.

6·41. Reduction of eddy currents in conductors in rotating machines. When the current carried by a conductor is large, it necessitates a large conductor cross-section. If a single solid conductor is used there would be considerable eddy current loss which is a source of inefficiency and heat. It then becomes essential to subdivide *i.e.* laminate the conductor into strips which should be sufficiently shallow in order to keep the loss factor as near unity as possible. The relationships derived above are valid only when the subdivisions are connected in parallel in such a manner as not to permit a path for eddy currents between subdivisions. A parallel connection between the subdivisions is necessary and therefore the subdivided conductors should be connected in parallel only when a complete balance between the subdivisions has been secured This may be obtained by twisting or transposing the conductors.

1. Transposed slot conductors. Let us consider a conductor subdivided into two parts. If the two subdivisions are placed in the slots as shown in Fig. 6·84 (*a*), they link with different values of flux. Therefore such an arrangement does not give any benefit because the two subdivisions have unequal emfs induced in them and since they are connected parallel eddy currents flow in the local circuit formed by them (subdivisions). Now consider the crossed conductor arrangement of Fig. 6·84 (*b*). Here the two parallel circuits are identical and therefore have equal emfs induced in them. This

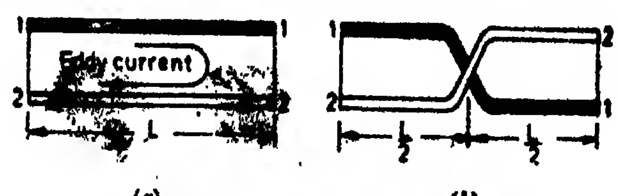


Fig. 6·84. Principle of transposition.

arrangement known as **transposition**, eliminates eddy circulating currents from one subdivision to another.

Slot conductors have their layers twisted and transposed in the slots so that every subdivision occupies all possible layer positions for the same length of slots. This gives symmetrical lengths for every subdivision and thus equalizes the eddy emfs in all the subdivisions. Therefore, the layers can be connected in parallel without producing eddy circulating currents. In long core turbo alternators the twisting may be carried out three to four times in a single slot. Fig. 6·85 shows the various methods of giving twists to conductors placed in slots.

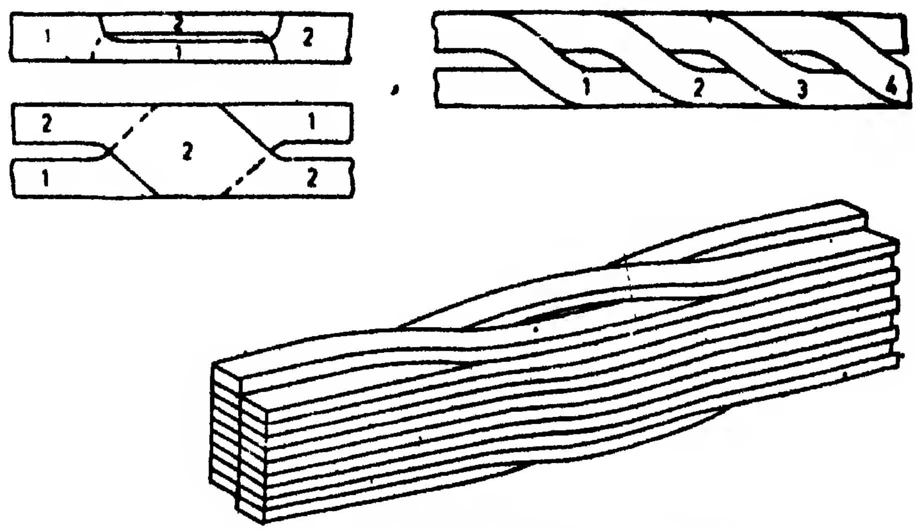


Fig. 6·85. Transposition of slot conductors.

2. **Transposed overhang.** The end connections should also be twisted in such a manner that the layers occupy all possible positions in the overhang. This equalizes the eddy emfs in the overhang. This transposition is automatically obtained in diamond type of coils, as the two coil sides of a coil occupy top and bottom positions. This may be sufficient for small and medium size machines. But for very large size machines, overhang transposition has also to be done to minimize the eddy losses.

6·42. Eddy current losses in transformer conductors and their reduction. Each coil of axial length L_c can be considered to be located in a slot of width H_w, the height of window. Comparing Fig. 6·86 for a transformer with that of a conductor in a slot as shown in Fig. 6·82 (*a*).

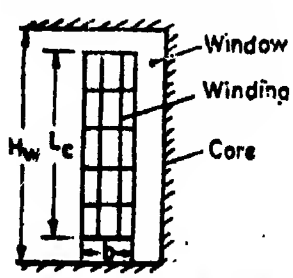


Fig. 6·86. Transformer winding.

$$\therefore \quad \alpha = 100\,(L_c/H_w)^{1/2} \text{ for copper}$$

$$= 150\,(L_c/H_w)^{1/2} \text{ for aluminium.}$$

Eddy loss ratio, for a single conductor

$$K_e = 1 + \frac{4}{45}(\alpha b)^4 \qquad \ldots(6\cdot101)$$

Eddy loss ratio for N layers

$$K_{e(av)} = 1 + (\alpha b)^4 \frac{N^2}{9} \qquad \ldots(6\cdot102)$$

From Eqn. 6·102, it is clear that the eddy current loss in conductors is proportional to b^4N^2 where b is the radial width of each conductor and N is the total number of radial conductors and therefore radial subdivisions of the conductor reduces eddy current loss. These conductor subdivisions must be continuously transposed in order to equalize their emfs and thus eliminate the possibility of circulating currents. The subdivisions are insulated with a thin film of synthetic enamel.

In transformers using aluminium foil coils, the low voltage winding is a spiral of full usable height of the window. Fig. 6·87 shows the current density distribution for the in-phase and quadrature components of current of low voltage winding. It is seen that the current density is much higher at the ends. However, the loss factor in foil windings rarely exceeds 1·1.

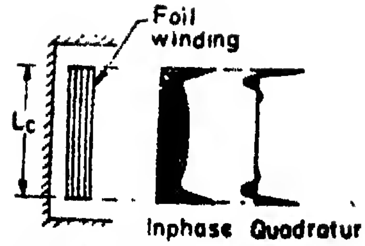


Fig. 6·87. Current density distribution of foil type transformer windings.

Example 6·41. *A slot, 20 mm wide, has five layers of solid copper conductors each 14 mm wide and 8 mm deep. Determine the eddy current loss ratio for each layer and the average loss ratio in five layers taken together. Find also the critical depth for minimum loss and the loss ratio for this depth. The frequency is 50 Hz.*

Solution For copper conductors at 50 Hz.

$$\alpha = 100\,(b/W_s)^{1/2}$$

$$\therefore \quad \alpha = 100\,(14/20)^{1/2} = 83{\cdot}6$$

$$\alpha h' = 83{\cdot}6 \times 0{\cdot}8 \times 10^{-2} = 0{\cdot}669,\ (\alpha h')^4 = 0{\cdot}2$$

Eddy loss factor for different layers from Eqn. 6·98 is :

1st layer $K_{e1} = 1$ 2nd layer $K_{e2} = 1 + 0{\cdot}2\ 2(2-1)/3 = 1{\cdot}13$.

3rd layer $K_{e3} = 1 + 0{\cdot}2\ 3(3-1)/3 = 1{\cdot}4$. 4th layer $K_{e4} = 1 + 0{\cdot}2\ 4(4-1)/3 = 1{\cdot}8$.

5th layer $K_{e5} = 1 + 0{\cdot}2\ 5(5-1)/3 = 2{\cdot}33$.

Average eddy current loss factor for all the five layers,

$$K_{e(av)} = 1 + (\alpha h)^4\, N^2/9 = 1 + 0{\cdot}2\ (5)^2/9 = 1{\cdot}55.$$

From Eqn. 6·100, critical depth $h_c' = \dfrac{1}{\alpha(3K)^{1/4}}$, where $K = \dfrac{N^2}{9} = \dfrac{25}{9}$.

$$h_c' = \frac{1}{83{\cdot}6(3 \times 25/9)^{1/4}}\,\text{m} \cong 7\,\text{mm}.$$

$$(\alpha h_c') = 83{\cdot}6 \times 0{\cdot}7 \times 10^{-2} = 0{\cdot}585 \text{ and } (\alpha h_c')^4 = 0{\cdot}117.$$

$\therefore$ Average loss ratio with 5 layers $K_{e(av)} = 1 + 0{\cdot}117 \times (5)^2/9 \cong 1{\cdot}33$.

Example 6·42. *Calculate the eddy current loss ratio in each layer and average loss ratio for the slot portion of copper conductors for the case shown in Fig. 6·88. All dimensions are in mm. Also calculate the overall average loss ratio if 40 per cent of a turn lies in the slot and eddy currents in the overhang are negligible. What is the value of effective a.c. resistance ? Frequency is 50 Hz.*

Solution.

Slot portion. For copper conductors at 50 Hz.

$$\alpha = 100(b/W_s)^{1/2} = 100(16/20)^{1/2} = 89{\cdot}4.$$

$$(\alpha h') = 89{\cdot}4 \times 8 \times 10^{-3} = 0{\cdot}715,\ (\alpha h')^4 = 0{\cdot}26.$$

From Eqn. 6·97, the eddy current loss ratio for pth layer

$$K_{e(p)} = 1 + (\alpha h')^4 \left[\frac{p(p-1)}{3} + \frac{4}{45}\right]$$

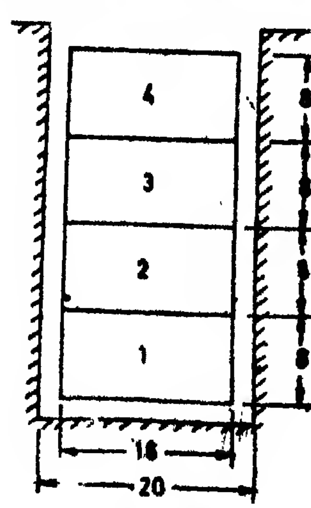


Fig. 6·88

Therefore,

$$K_{e1}=1+0.26\left(\frac{4}{45}\right) \qquad =1+0.023 \qquad =1.023$$

$$K_{e2}=1+0.26\left(\frac{4}{45}+\frac{2}{3}\right)=1+0.023+0.173 \qquad =1.196$$

$$K_{e3}=1+0.26\left(\frac{4}{45}+2\right)=1+2.023+0.52 \qquad =1.543$$

$$K_{e4}=1+0.26\left(\frac{4}{45}+4\right)=1+0.023+1.04 \qquad =2.063$$

$$\text{Total} \qquad =5.825$$

Average loss factor for four layers $K_{e(av)}=\dfrac{5.825}{4}=1.45$.

Overhang portion. The eddy current loss in the overhang is negligible and therefore the eddy current loss factor for the overhang is unity.

Total Conductor Length. Thus the overall eddy current loss factor for both slot and overhang portions of conductor

$$=\frac{1.45\times0.4+1\times0.6}{0.4+0.6}=1.18.$$

As the average overall loss factor is 1·18, the effective value of a.c. resistance 1·18 times d.c. resistance of the winding.

Example 6·4? *Find the eddy current loss factor for the conductor arrangement in the slots of an alternator shown in Fig. 6·89. All dimensions are in mm. Find the eddy current loss ratio for each layer and also the average loss factor.*

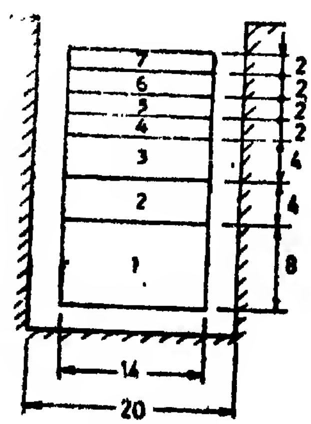


Fig. 6·89

Solution. For copper conductors at 50 Hz,

$$\alpha=100(h/W_e)^{1/2}=100(14/20)^{1/2}=83.6$$

Layer 1 :

$$\alpha h'=83.6\times8\times10^{-3}=0.669$$

or $\quad (\alpha h')^4=0.2$

$\therefore \quad K_{e1}=1+0.2\,(4/45)=1.0178$

Layers 2 and 3 ;

These are treated as layers $p=3$ and $p=4$ of a slot containing uniform layers of 4 mm height.

$$\alpha h'=83.6\times4\times10^{-3}=0.334 \quad \text{or} \quad (\alpha h')^4=0.0125.$$

From Eqn. 6·8?,

$$K_{e2}=1+(\alpha h')^4\frac{(p)(p-1)}{3}=1+0.0125\frac{(3)(3-1)}{3}=1.025$$

$$K_{e3}=1+0.0125\,\frac{4(4-1)}{3}=1.05$$

Layers 4, 5, 6 and 7 :

These layers are treated as layers $p=9$ to $p=12$ of a slot containing uniform layers of 2 mm height.

$$\alpha h'=83.7\times2\times10^{-3}=0.167 \qquad \text{or} \quad (\alpha h')^4=0.00078.$$

$$\text{Therefore } K_{e_4}=1+0{\cdot}00078\ \frac{9(9-1)}{3}=1{\cdot}0187$$

$$K_{e_5}=1+0{\cdot}00078\ \frac{10(10-1)}{3}=1{\cdot}0234$$

$$K_{e_6}=1+0{\cdot}00078\ \frac{11(11-1)}{3}=1{\cdot}0286$$

$$K_{e_7}=1+0{\cdot}00078\ \frac{12(12-1)}{3}=1{\cdot}0343.$$

The average loss ratio will depend on the areas over which the individual ratios obtain,

Area of layer 1=4×area of each of layers 4, 5, 6, 7.

Areas of layers 2 & 3=2×area of each of layers 4, 5, 6, 7.

$$\therefore\quad K_{eav}=\frac{4K_{e_1}+2K_{e_2}+2K_{e_3}+K_{e_4}+K_{e_5}+K_{e_6}+K_{e_7}}{12}$$

$$=\frac{4\times1{\cdot}017+2\times1{\cdot}025+2\times1{\cdot}05+(1{\cdot}0187+1{\cdot}0234+1{\cdot}0286+1{\cdot}0343)}{12}=1{\cdot}027.$$

UNSOLVED PROBLEMS

1. A 4 pole, 64 slot simplex lap wound d.c. armature has 64 slots and 1152 conductors. The number of commutator segments is 192. Determine the number of coil sides per slot, number of turns per coil and the winding pitche . Draw up the winding table. Find whether the winding is symmetrical or not.

[Ans. $u=6$; $T_c=3$; $y_b=97$; $y_f=95$; $y_c=+1$. Symmetrical]

2. A 4 pole armature has 103 commutator segments and 35 slots. Arrange a 2 circuit, wave connected winding for 00 V at 1000 rpm, with a flux of 7mWb.

[Ans. 103 active coils, 2 dummy coils ; 10 turns per coil]

3. Work out an arrangement for equalizer connections for a 8 pole d.c. machine having 240 coils. The number of equalizer rings is 10. [Ans. $Y_{eq}=60$ coils ; $Y_{ph}=6$ coils]

4. A 6 pole duplex lap wound armature has 36 single turn coils. Find the winding pitches and indicate the degree of re-entrancy of the winding. Draw up the winding table.

[Ans. $y_b=13$, $y_f=9$, $y_c=+2$; doubly re-entrant]

5. Find possible windings for a 4 pole, 2 circuit armature. The number of conductors in series must not be greater than 400 nor less than 360 ; the number of slots is to be between 40 and 45 and the number of commutator segments within 126±4. [Ans. (*a*) $C=123$, $y_c=61$ or 62, $u=6$, $S=41$

(*b*) $C=129$, $y_c=64$ or 65, $u=6$, $S=43$]

6. An 8 pole d.c. machine with a duplex wave winding requires about 720 conductors. Using single turn coils, find the nearest number of commutator segments and a suitable number of slots that can be used to give a symmetrical winding with (*a*) 2 coil sides per slot (*b*) 6 coil sides per slot.

[Ans. (*a*) $C=S=358$ or 362

(*b*) $C=354$, $S=118$ or $C=366$, $S=122$]

7. A 3 phase 16 pole synchronous machine is to be provided with a 3 phase single layer winding. Find the possible number of parallel circuits per phase. [Ans. 1, 2, 4, 8]

8. Work out the arrangement of a two layer winding for a 3 phase, 10 pole machine with 108 slots.

[Ans. Unit=5 poles and 18 slots ; 2 groups of 3 coils each and 3 groups of 4 coils each for every phase. Sequence of pole phase groups : 4, 4, 3, 4, 3, 4, 4, 3, 4, 3, 4, 4, 3, 4, 3]

9. A 12 pole generator has 6 armature circuits, 177 slots and 531 commutator sectors. Calculate (*a*) the commutator pitch (*b*) winding pitches. (*c*) State whether the winding is symmetrical. (*d*) Also find the tappings for a 3 phase voltage. Avoid use of split winding.

[Ans. (*a*) $y_c=88$ (*b*) $y_b=91$, $y_f=85$ (*c*) Yes (*d*) $Y_{eq}=177$, $Y_{ph}=59$]

10. A wave winding for the rotor of an 8 pole induction motor has 45 coils and 2 coil sides per slot. Work out the openings for connection to 3 slip rings. An internal star connection is required and the phase spread should be 60°.
[Ans. Joints to be opened : 1, 33, 31, 18, 16, 3
Phases : Coil sides : 1, 61, 31
Star points : Coil sides : 65, 35, 5
Connectors : Coil sides : 80—24, 56—20, 50—34]

11. A simplex wave wound armature has 4 poles, 135 coils and 6 coil sides per slot. Find (*a*) tappings off a closed winding for 3 phases (*b*) openings for a symmetrical 3 phase winding with a 60° phase spread.
[Ans. (*a*) Joints : phase *R*—1. Phase *Y*—46, Phase *B*—91.
(*b*) S_R—coil side 1=1 in slot 1 ; F_R—coil side 203=5 in slot 34
S_Y—coil side 91=1 in slot 16 ; F_Y—coil side=23=5 in slot 4
S_B—coil side 181=1 in slot 13 ; F_B—coil slide 113=5 in slot 19
Connect 6 in slot 34 to 4 in slot 23,
6 in slot 4 to 4 in slot 38
6 in slot 19 to 4 in slot 8] See Fig. 6.

12. The phase voltage of a 3 phase synchronous generator is expressed as ($2000 \sin \omega t + 100 \sin 3\omega t + 30 \sin 5\omega t$) volt. Deduce an expression for line voltage when the phases are connected in star and for the circulating current when they are connected in delta. Calculate also the r.m.s values of line voltage and circulating current. The leakage reactance per phase at fundamental frequency is 2 Ω. Neglect resistance of winding.
[Ans. ($3460 \sin \omega t + 52 \sin 5\omega t$) volt ; $16.66 \sin 3\omega t$ ampere ; 2440 5 volt ; 11.75 A]

13. The air gap flux distribution of an alternator contains 5th and 7th harmonics whose magnitudes are respectively 10 per cent and 5 percent of the fundamental. Find the corresponding percentages of harmonics in the terminal voltage. The machine has 105 stator slots, 14 poles and a 3 phase double layer winding with a coil span of 6 slots. [Ans. 0·65 percent of 7th harmonic]

14. The stator winding of a 3 phase, 4 pole induction motor has 36 slots and has a phase spread of 60°. Find the order of slot harmonics produced. Prove that the slot harmonics have the same distribution factor as the fundamental. [Ans. 17th, 19th, 35th, 37th]

15. A slot 50 mm wide is to have 5 equal layers of single solid conductors each 35 mm wide. What depth of conductor would give the minimum loss. [Ans. 8 mm]

7

Transformers

7·1. Introduction. A transformer is essentially a static electromagnetic device consisting of two or more windings which link with a common magnetic field. One of these windings, the **primary**, is connected to an alternating voltage source, an alternating flux is produced whose amplitude depends on the primary voltage and number of turns.

The primary induced voltage is $E_p = 4 \cdot 44 \, f \, \phi_m \, T_p$

where f=frequency, ϕ_m=mutual flux and T_p=number of turns in the primary winding

This flux linking with the **secondary** winding induces in it a voltage whose value depends on the amplitude of flux and the number of secondary winding turns. The induced voltage is the secondary winding is $E_s = 4 \cdot 44 \, f \, \Phi_m \, T_s$ where T_s=number of turns of secondary winding. Ratio of voltages is $E_s/E_p = T_s/T_p$. Therefore, any desired value of secondary voltage can be attained by using a suitable number of turns.

It must be understood that a transformer is not an energy conversion device but a device that transforms electrical energy from one or more primary a.c. circuits to one or more secondary a.c. circuits with changed values of voltage and current.

The main reason for extensive use of a.c. power systems is on account of transformers. This is because the transformers allow the power to be generated at the most economical generator voltage, power transfer at the most economical transmission voltage, and power utilization at the most suitable voltage required for different applications.

Presently, most of the electric power for industrial and utility purposes, is generated by large hydro-electric plants and steam power stations in the form of three phase a.c. at a frequency of 50 Hz. The voltages of the generators installed at the power plants is usually 6·6 kV or 11 kV. To transmit the power over long distances the voltage of the generators has to be increased depending upon quantum of power and the distance in order to reduce transmission losses and to effect economy. On the other hand, the voltage is decreased by the distribution substations to 3·3 kV, 6·6 kV and 11 kV in rural, urban and industrial areas. Ultimately, the voltage used in industrial and domestic premises has to be dropped to 433 V or 230 V. The raising and lowering of a.c. supply voltages is accompolished by **Power transformers**. Therefore, as regards this application, the transformers may be classified as

(i) **Step-up transformers**—transformers which raise the voltage, and

(ii) **Step-down transformers**—transformers which lower the voltage.

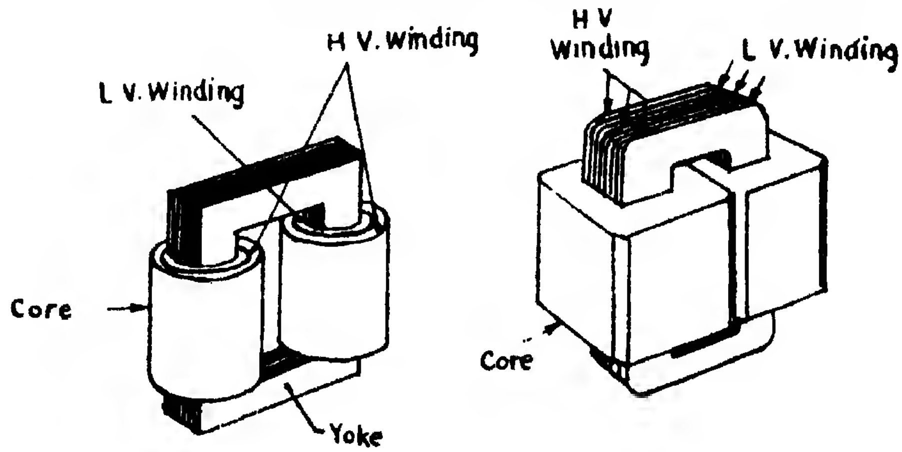
However, basically each transformer may be used as both a step-up and a step-down transformer because it is a reversible device.

The power tranformers have a remarkably high efficiency ranging from 95 to 99·5 percent, depending upon the power rating. The greater the power rating of the transformer, the higher the efficiency.

7·2. Core and Shell type Transformers. The transformer is basically a very simple device. It consists of windings wound on a laminated magnetic core and insulated from iron and from each other. The core is actually a magnetic circuit which serves as a path for the mutual flux. Therefore, the windings encircle the core and the core encircles the windings. There are two general types of constructions employed to achieve this in transformers. Consequently, depending upon the type of construction used, the transformers are classified into two categories as : (*i*) core type, and (*ii*) shell type.

Core type transformers. The magnetic core is built of laminations to form a rectangular frame and the windings are arranged concentrically with each other around the legs or limbs of the core as shown in Fig. 7·1(*a*).

The top and bottom members, called the yokes, connect the two limbs and have a cross-sectional area equal to or greater than that of the limbs.



(*a*) Core type (*b*) Shell type

Fig. 7·1. Single phase transformer connection.

A single phase transformer may be designed with primary winding wound on one limb and secondary winding on the other limb. This arrangement results in a large separation between the primary the secondary windings and hence a large leakage reactance. In actual practice, each limb carries one half of the primary winding and one half of the secondary winding so that the two windings can be closely coupled together to keep the leakage reactance low.

The low voltage (l.v.) winding is wound on the inside nearer to the core while the high voltage (h.v.) winding is wound over the l.v. winding away from core in order to reduce the amount of insulating materials required.

Shell type transformers. In shell type transformers the windings are put around the central limb and the flux path is completed through two side limbs as shown in Fig. 7·1 (*b*). The central limb carries total mutual flux while the side limbs forming a part of a parallel magnetic circuit carry half the total flux. Consequently, the cross-sectional area (and hence width) of the central limb is twice that of each of the side limbs.

Both high voltage (h.v.) and low voltage (l.v.) windings are divided into a number of coils. The h.v. and l.v. coils are shaped like pancakes and are arranged longitudinally along the core alternately. This gives rise to a **sandwich winding** with h.v. coils sandwiched between l.v. coils.

In the core type the impression is created that the windings surround the core, whereas with the shell type that the cores surround the windings.

Comparison of core and shell types of transformers

1. **Construction.** Core type transformers are much simpler in design and permit easier assembly and insulation of windings. Also, the core type of transformers are easier to dismantle for repair work.

2. The force produced between current carrying windings is proportional to the product of the currents carried by them. These currents tend to be very large under fault conditions. Consequentlyvery large electromagnetic forces are produced when the secondary winding is short circuited with the primary winding energized. Since, the windings carry currents in opposite direction, there exists a force of repulsion between them. Hence, the inner winding experiences a compressive force crushing it on to the core; the outer winding experiences a tensile force pulling it away as shown in Fig. 7·2.

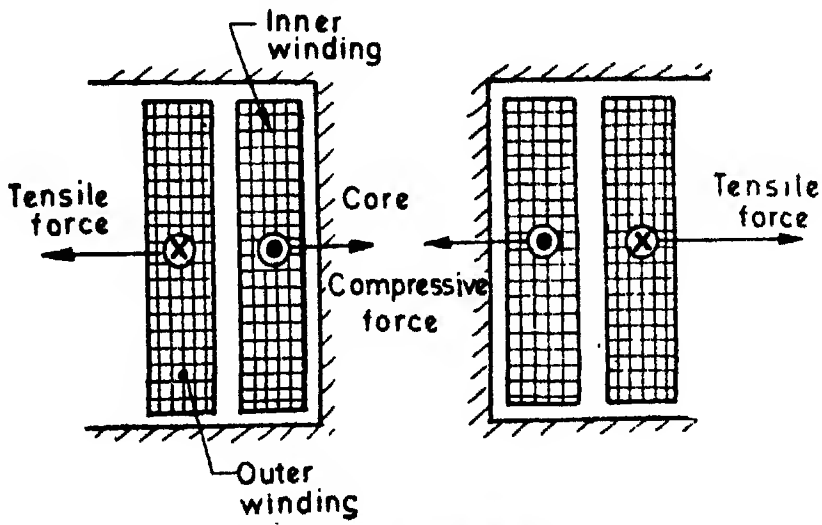


Fig. 7·2. Electromagnetic forces on transformer windings.

In modern power networks, the reliability of transformer operation is very important and therefore the design of the transformers should be such that the windings suffer no damage when short circuited. It is amply clear from Fig. 7·1 that windings in a shell type transformer have greater capability of withstanding forces produced under short circuit conditions as these windings are surrounded and thus braced (or supported) by the core over a large portion of length. On the other hand, the windings in core type construction have a poorer mechanical strength, because they (the windings) are not braced or supported. Therefore, the windings in core type transformers are more susceptible to damage under short circuit conditions, than the windings of a shell type transformer.

(*iii*) **Leakage Reactance.** Due to large space required between the high and low voltage windings, it is not easily possible to subdivide the windings to a great extent in the case of core type transformers, while, in the shell type, the windings can be easily subdivided by using sandwich coils. Thus it is possible to reduce the leakage reactance of shell type transformers to any desired value.

(*iv*) **Repairs.** The windings of a core type transformer are completely accessible except for a small portion in the window. This is of a great advantage in repair work because the coils can be easily inspected. Also, the core type transformer is easy to dismantle for repairs.

In the case of shell type transformers, the coils are surrounded by core for a large length and therefore there is great difficulty in inspection and repair of coils.

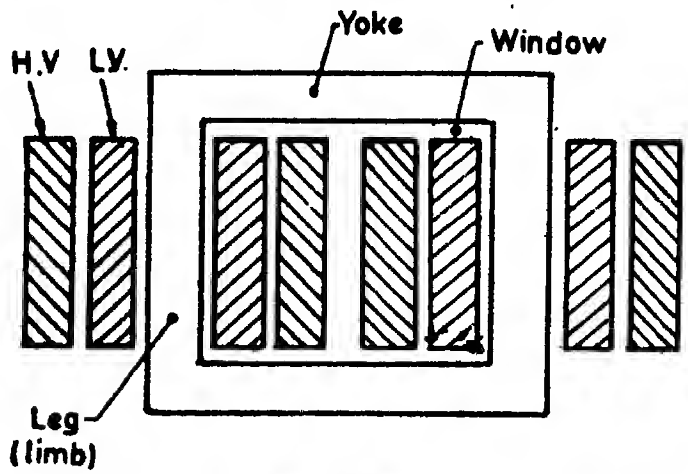
(*v*) **Cooling.** In the case of core type transformer, the windings surround the core. The windings are exposed and therefore the cooling is better in windings than in core.

In the case of shell type transformers, the core is exposed and therefore cooling is better in core than in windings.

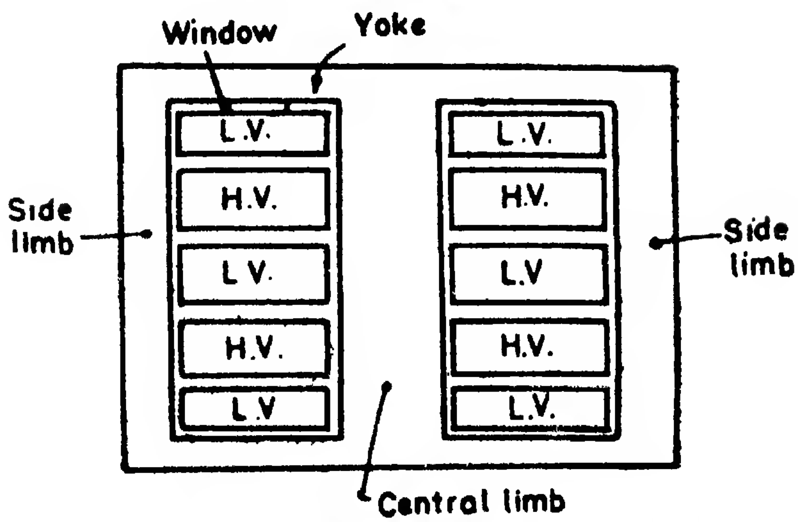
The most vulnerable part of a transformer is the insulation of windings. Therefore, core type of construction is universally followed because it affords better heat dissipation facilities from a part which is most prone to damage on account of heat developed.

7·3. Single and three phase transformers

Single phase transformers. The cross-sectional view of the windings and core of a 1-phase core type transformer is shown in Fig. 7·3(*a*). Single phase core type trans-



(*a*) Core type.



(*b*) Shell type.

Fig. 7·3. Single phase core and shell type transformers.

formers use two legged iron frame with one half of the primary winding and one half of the secondary winding wound on each leg. The low voltage (l.v.) and high voltage (h.v.) windings are concentric with each other with l.v. winding placed on the inner side nearer to the core.

Fig. 7·3(*b*) shows the cross-sectional view of a single phase shell type transformer. The low voltage (l.v.) and the high voltage (h.v.) coils are sandwiched between each other.

Three phase transformers. The generation, transmission and power utilization of a.c. electric energy almost invariably involves the use of three phase net works. The voltage transformation in a 3 phase network can be obtained by using :

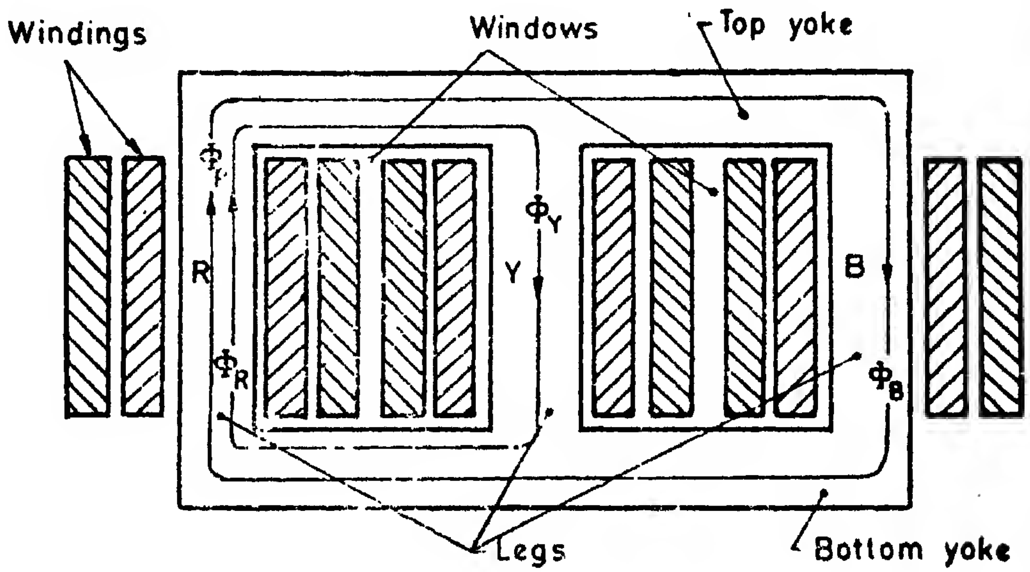
(*i*) three single phase transformers connected to form a 3-phase bank,

(*ii*) a 3-phase transformer.

A **transformer bank** used on 3 phase systems consists of three independent 1-phase transformers with their primary and secondary windings connected either in star or in delta.

The 3-phase transformation is also possible through the medium of one 3-phase transformer having a magnetic circuit common to all the three phases. A three phase magnetic circuit for transformation of three phase voltages may be obtained by combining three individual single phase core or shell type magnetic circuits into a common magnetic circuit with suitable changes in the configuration.

Fig. 7·4(*a*) shows a 3 phase core type of transformer. The core consists of three legs with the magnetic circuit completed through two yorks, one at the top and the other at the bottom. A primary and a secondary winding of one phase are wound on one leg. Flux flows up each leg in turn and down the other two legs in general, so that the magnetic circuits of different phases are in series and therefore independent. Fig. 7·4(*b*) shows the instant where the flux in the leg carrying the winding of phase *R* is positive (upwards)

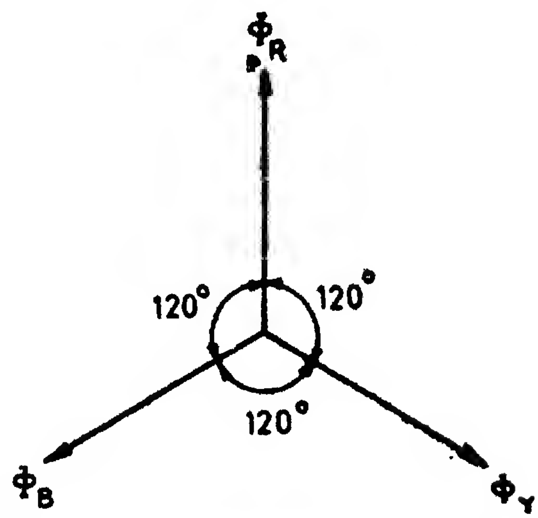


(*a*)

maximum while the flux in the other two legs carrying windings phases *Y* and *B* is half of the negative (downwards) maximum. It should be noted that the transformer has only two windows. Each of two windows contains two primary and two secondary windings.

Fig. 7·5 shows the less commonly used 3-phase shell type transformer whose construction appears like three 1-phase shell-type cores built on top of one another.

The windings of the middle core, phase *Y*, are reversed so that the parts of the core carry flux $\Phi_R/2+\Phi_Y/2$ or $\Phi_Y/2+\Phi_B/2$ instead of flux $\Phi_R/2-\Phi_Y/2$ or $\Phi_Y/2-\Phi_B/2$ in order to affect economy in core cross-section since $\Phi_R/2+\Phi_Y/2$ is smaller than $\Phi_R/2-\Phi_Y/2$ and also $\Phi_Y/2-\Phi_B/2$ $<\Phi_Y/2+\Phi_B/2$.



(*b*)

Fig. 7·4. Three phase core type transformer and its power diagram.

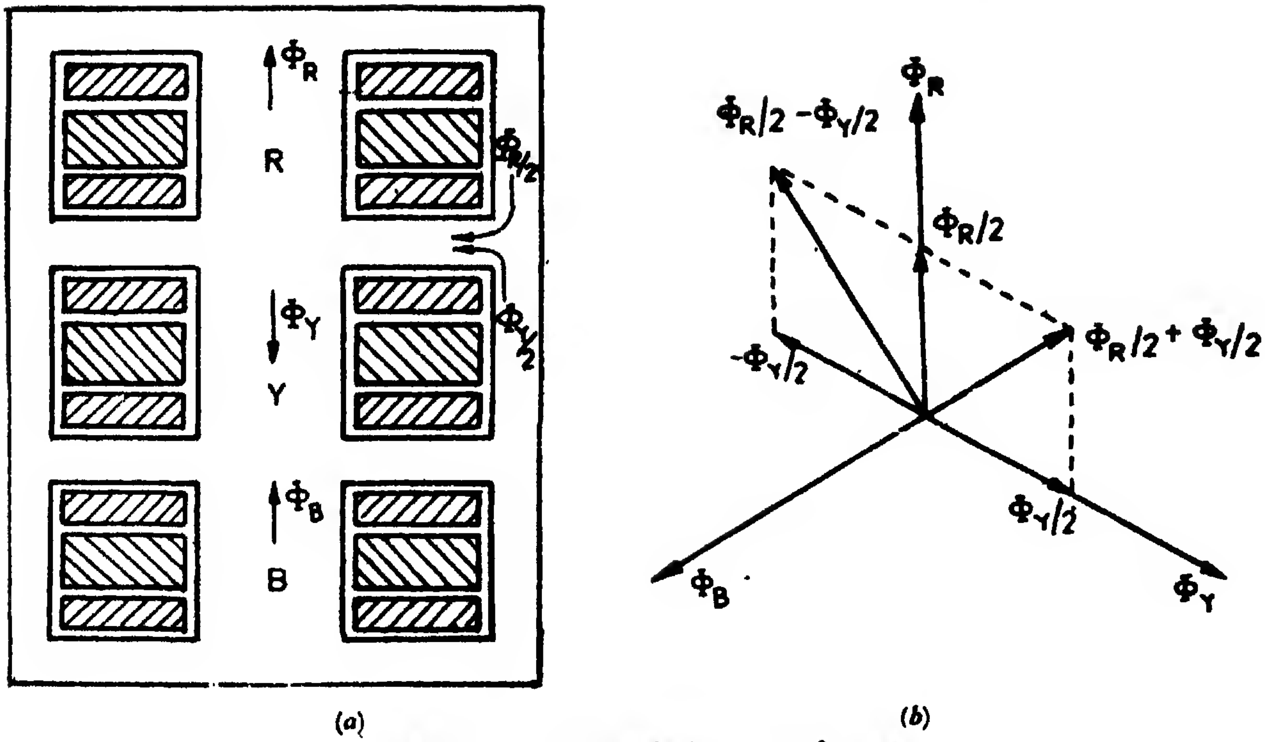


Fig. 7·5. Three phase shell type transformers.

The reversal in the flux Φ_Y in the central limb is brought about/by reversing the windings wound on the central limb so that the flux carried corresponds to the phasor sum of $\Phi_R/2$ and $\Phi_Y/2$ (or Φ_Y and $\Phi_B/2$) instead of their difference. This is illustrated by the phasor diagram of Fig. 75. It should be noted that $(\Phi_R/2+\Phi_Y/2)<(\Phi_R/2-\Phi_Y/2)$

In the case of 3 phase shell type of transformers the phase magnetic circuits are in parallel and therefore, independent, if saturation effects in the common paths are neglected.

The advantages of a 3-phase transformer over a bank of three 1-phase transformers are :

(*i*) A 3-phase transformer is lighter, occupies lesser space, cheaper and more efficient than a bank of 1-phase transformers. The reasons for lower cost of 3-phase transformers are : savings in cost of the iron core, of the tank and oil (since there is only one tank whose volume is much smaller) of the bushings (since the connections can be made internally and therefore the number of bushings is smaller) and of the auxiliary apparatus.

The higher efficiency of a 3-phase transformer is on account of the fact that it has a shorter magnetic path and consequently the volume of core and hence the core (iron) loss is smaller.

(*ii*) In case of 3-phase transformers, there is only one unit to install and operate. Hence, the installation and operational costs are smaller for 3-phase units.

The disadvantages of 3-phase transformers are :

The 3-phase transformers are built as a single unit whereas an equivalent bank of single phase transformers consists of three separate units. The weight and hence the cost per unit is thus higher in the case of 3 phase transformers. The three phase unit costs about 15% lesser than the bank.

(*i*) It is more difficult to transport a 3 phase transformer as the weight per unit is more,

(*ii*) In the event of a fault in any phase of a 3 phase transformer, the fault is transferred to the other two phases. Therefore, the whole unit needs replacement. Hence, in

3 phase transformer installations, a more expensive unit must be available for speedy replacement in the event of a breakdown. However, in the case of banks of single phase transformers, only a much smaller and less costlier single phase unit has to be kept in spare to replace the faulty unit. In the case of delta connected banks, the faulty unit is simply disconnected with continuity of supply maintained by operating the bank in open delta at 58% of the original power capacity till the faulty single phase unit is replaced. (The practice of operating two 1-phase transformers in open delta at 58% of rated load is quite common in U.S.A.)

The choice of transformer arrangement for 3-phase systems is mainly governed by the relative importance of the factors listed above. For example in mines three 1-phase units may be preferred to a 3-phase transformer on account of ease of transport.

7·4. Three Phase Transformer Connections. A 3-phase transformer or a bank of three 1-phase transformers may be connected in star or delta. When star connection is used, the phase voltage is $1/\sqrt{3}$ times the line voltage while in delta connection the phase voltage is equal to the line voltage.

Therefore, the number of turns per phase in a star connected winding are $1/\sqrt{3}$ times the number of turns per phase in a delta connected winding. On the other hand, the current in each phase (and hence in each conductor) of a star connected winding is $\sqrt{3}$ times the current in each phase (and hence in each conductor) of a delta connected winding.

Thus we conclude that a star connected winding is characterised by a small number of turns with conductors having a large area of cross-section while a delta connected winding has a large number of turns with a small area of cross-section. The larger number of turns of lower cross-sectional area used in delta connected transformers obviously require greater amount of insulation.

The advantages of star connected transformers are :

(*i*) the phase voltage is smaller, and hence the major insulation required between windings and earth is smaller. Another advantage of star connected winding with earthed neutral is that the maximum voltage to the core (which is at the earth potential) is limited to 58% of line voltage, whereas with a delta connected winding, in case of a line to earth fault the maximum voltage between windings and core increases to full line voltage.

(*ii*) the number of turns is smaller with the result the amount of insulation used is smaller.

The use of star connection at high voltages results in reduction in cost especially at higher voltages since at higher voltages the cost of insulation becomes significant. At very high voltages, the savings on account of star connection in the cost of insulation may amount to as much as 10%. The savings in the cost of insulation on account of connections may not be significant below a voltage of 11 kV.

(*iii*) In the case of star connection, both phase and line voltages are available for four wire supply. This is useful for distribution purposes at lower voltages where both 3-phase and 1-phase loads have to be supplied.

The delta connection behaves better under conditions of unbalance and in fact a delta connected primary is essential where the low voltage secondary is star connected four wire supply to mixed 3-phase and 1-phase loads.

7·5. Three Winding Transformers. The use of multi-circuit transformers in large power networks, the power generated at power stations is transmitted to consumers located at widely different distances. The most economic transmission voltage depends upon the distance of the load from the power station. Therefore, in situations where the

load centres are located at varying distances from the power generation centre, it becomes imperative (due to economic considerations) to transmit power at different voltages.

There are certain important consumers and systems which require to be fed from two or more independent distribution systems.

Also, it is often desirable to interconnect several systems having different voltages into a common grid network in order to economically distribute electric energy.

In all such applications **two-winding transformers** having two independent circuits *i.e.* primary and secondary, may be used. However, it is more convenient and economical to use **multi-circuit transformers**. These transformers have three or more independent circuits with different transformation ratios, so that the windings operate at three or more different voltage levels.

A **three winding transformer** consists of three sets of windings primary, secondary and tertiary. Three winding transformers may be either 3-phase units or 1-phase units connected in a three phase bank. These transformers have a large kVA rating. Since the three windings are operated at three different voltage levels, they may be called as ***high voltage*** (h.v.), ***medium voltage*** (m.v.) and ***low voltage*** (***l.v.***) ***windings***.

Two alternative arrangements of windings of a 3-phase transformer are shown in Figs. 7·6(*a*) and (*b*). The high voltage (h.v.) winding in both cases is the outermost winding away from the core because of insulation considerations. The l.v. and m.v. windings are kept adjacent to each other with either of two nearer to the core. The l.v. and m.v. windings are kept near to each other in order to reduce the leakage reactance between this pair of windings. Thus the two possible winding arrangements for windings starting from core outwards are :

(*i*) l.v., m.v. and h.v. and (*ii*) m.v., l.v. and h.v.

(*a*) (*b*)

Fig. 7·6. Winding arrangement of 3-winding transformers.

In two winding transformers, both the primary and the secondary windings have the same rated kVA rating. However, in a three winding transformer, the kVA ratings of three windings may be unequal. The rated kVA of the transformer is considered to be equal to the largest rated kVA of any of its windings.

There are many other reasons why three winding transformers are built with an additional winding, the **tertiary winding**, apart, of course, from the traditional primary and secondary windings of a two winding transformer. These reasons are :

(*i*) To supply small additional load at a different voltage.

(*ii*) To supply phase compensating devices such as capacitors required for power factor improvement which can be operated at different voltage.

(*iii*) In star/star or star/zigzag transformers, a delta connected teritary winding reduces the zero-phase sequence impedance and allows adequate earth fault current to flow

for the operation of protective device in order to limit the voltage imbalance which may be produced when the load is unbalanced.

(*iv*) To indicate voltage in a h.v. testing transformer.

The teritary winding is called an **auxiliary winding** when it is used for supplying an additional small load at a different voltage. On the other hand, it is called a **stabilizing winding**, when it is used for limiting the short circuit current as in (*iii*). Tertiary windings are normally connected in delta so that when line to earth or line to line faults occur on the primary or the secondary windings, the considerable unbalance in phase voltages may be comp nsated by the circulating currents flowing in the closed delta. The reactance of the windings should be large enough to limit the circulating currents in order that there is no overheating of the windings.

7·6. Distribution and Power Transformers. The transformers used in power systems may be divided into two categories depending upon the type of service. These are :

(*i*) Distribution transformers and (*ii*) Power transformers.

Distribution Transformers. Transformers upto a size of about 200 kVA, used to step down the distribution voltage to a standard service voltage or from transmission voltage to distribution voltage are known as *distribution transformers*. They are kept in operation all the 24 hours a day whether they are carrying any load or not. Energy is lost in iron losses throughout the day while the copper losses account for loss in energy when the transformer is loaded. Therefor , distribution transformers should have their iron losses small as compared with full load copper losses. In other words they should be designed to have maximum efficiency at a load much lower than full load (about 50 per cent). Owing to low iron loss, the distribution transformers have a good all day efficiency. Distribution transformers should have a good voltage regulation and therefore they should be designed for a small value of leakage reactance.

Power Transformers. They have a rating above 200 kVA and are used in generating stations and substations at each end of a power transmission line for stepping up or stepping down the voltage. They may be either single phase or three phase units. They are put in operation during load periods and are disconnected during light load periods. Therefore power transformers should be designed to have maximum efficiency at or near full load. Power transformers are designed to have considerably greater leakage reactance than is permissible in distribution transformers as in the case of power transformers inherent voltage regulation is less important than the current limiting effect of the higher leakage reactance.

7·7. Core. The transformer core is a closed magnetic circuit through the mutual flux *i.e.* the flux which links with both the windings passes. The core material and construction should be such that both the magnetizing current and the core losses are minimum. The cores of transformers are laminated in order to reduce the eddy current losses. The eddy current loss is proportional to the square of thickness of laminations. This apparently implies, that the thickness of the laminations should be extremely small in order to reduce the eddy current losses to a minimum. However, there is a practical limit beyond which the thickness of the laminations cannot be decreased further on account of mechanical considerations. This practical limit of thickness is 0·3 mm. The laminations are made 0·33—0·5 mm thick. The thickness should not be reduced below 0·3 mm because in that case, the laminations become mechanically weak and tend to buckle. These laminations are made of the so called transformer grade steel containing 3—5% silicon. The higher content of silicon increases the resistivity of the core, thereby reducing the eddy current core loss. High content silicon steel is a soft iron material having a narrow hysteresis loop and thus the hysteresis losses are also small. This material has a high permeability and hence the magnetizing current is also small. The steel used for transformer cores may be hot rolled or cold rolled. The hot rolled

steel which permitted a maximum flux density of 1·45 Wb/m² was in use for a considerable length of time. In recent years this type of steel has completely been superseded by 0·33 mm (or 0·35 mm) thick cold rolled steel allowing much higher flux densities upto 1·8 Wb/m² to be used. Although, cold rolled steel is 25—35% more expensive than the hot rolled steel, the increase in value of maximum flux density makes it possible to reduce the amount of core material.

However, the use of cold rolled steel involves a more complicated core construction and requires new methods for machining the laminations.

The hot rolled steel is sheared to size by power guillotines and then punched in multiple presses. With cold rolled steel, rolls of mass upto 2 tons are slit into widths by gang operated slitters. This working of cold rolled steel impairs its property and therefore it is annealed to relieve stresses. The annealing process involves heating sheets or complete small cores at 800°C in an inert atmosphere (to avoid oxidation and carbon contamination). The insulation on the surface of laminations is kaolin or varnish in the case of hot rolled steel but for cold rolled oriented steel, phosphate-base coating is used. This coating is done by the makers of the steel and can withstand the annealing process. Cores of small transformers need no further insulation if made of c.r.o.s. However, the transformers of capacity 10 MVA and above kaolin or varnish must be applied to the lamination (in addition to phosphate-base coating which already exists).

7·8. Core Cross-section. Small core type transformers have rectangular section limbs with rectangular coils as shown in Fig. 7·7. However, in large capacity transformers, the economic use of core material requires that the cross-section of the core should ideally be a circle since a circle has the minimum periphery for a particular area and hence, the windings which are put around the core have a minimum length of mean turn resulting

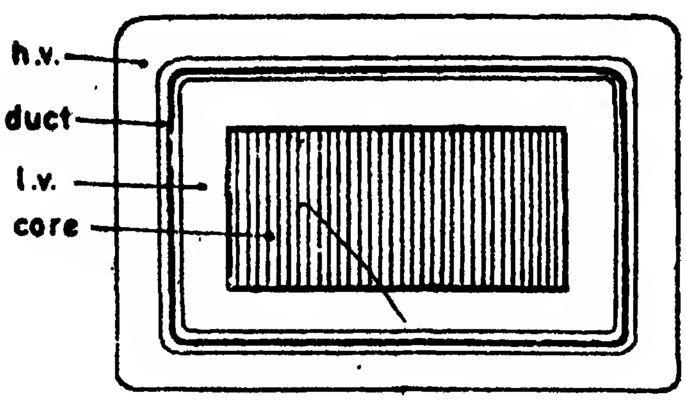


Fig. 7·7. Rectangular core.

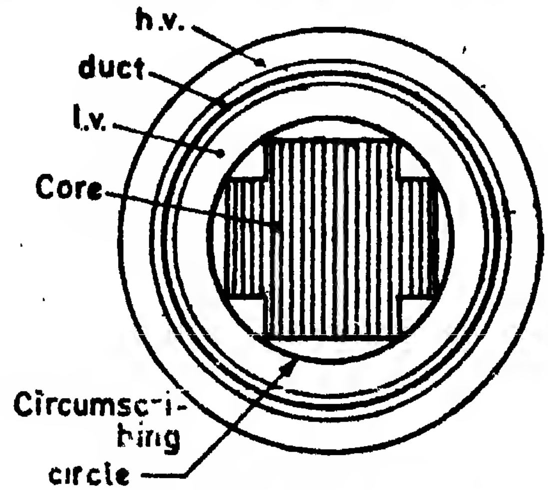
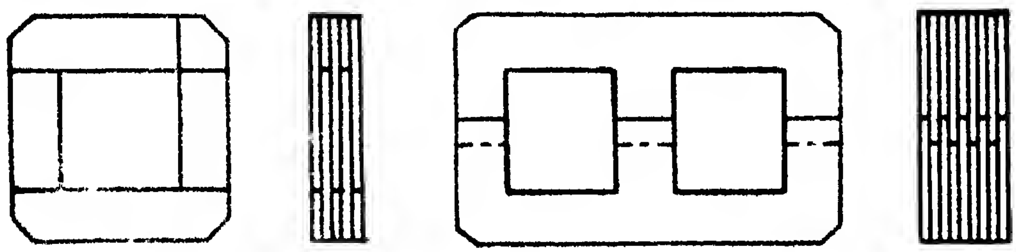


Fig. 7·8. Two-stepped (cruciform) core.

in reduced amount of conductor material thereby reducing the costs. A circular core, however, involves the use of an unmanageably large number of laminations of different sizes. The use of laminations of different sizes is possible but is highly time consuming and uneconomical on account of the obvious difficulties in core assembly and increased labour costs. A compromise is achieved by arranging the core section in steps in such a way that the net sectional area is maximum for the number of steps employed and the corners of the steps are so arranged that they lie on a circle known as a **circumscribing circle** of predetermined diameter. Fig. 7·8 shows a two-stepped core which is also known as **cruciform core.** A two stepped core requires two sizes of laminations. However as the number of steps increases, the number of different sizes of laminations also increases. With large number of steps, there is a reduction in the length of mean turn of windings and consequently there is a reduction in the cost of conductor material and the I^2R losses but

there is extra cost involved in shearing and assembling of different sizes of laminations. Therefore, while designing a core section, balance should be struck between the cost of conductors and core and also the labour charges. Cores for shell type transformers are usually of simple rectangular cross-section.

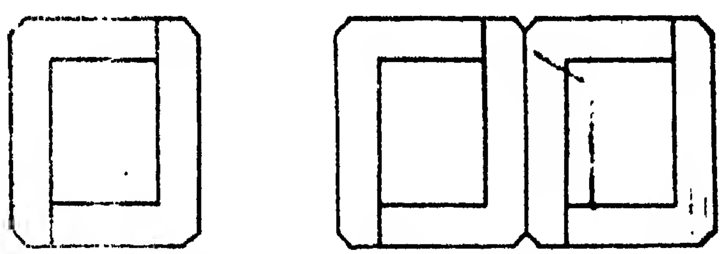
7·9. Core construction with hot rolled laminations. In small transformers, the complete magnetic circuit can be punched as a whole but this process would involve too much wastage of sheet if followed for large transformers. Moreover there would be great difficulty in putting the winding as each turn will have to be separately threaded through the window which is impossible for large transformers. Therefore, the coils are made separately and are then placed on assembled cores. The magnetic circuit



(*a*) I type laminations. (*b*) E type laminations.

Fig. 7·9. Core construction with different types of laminations.

is made of different types of laminations to give it a proper shape. Fig. 7·9 (*a*) shows a core built from I shaped laminations. Joints at the junctions of different strips introduce air gaps in the magnetic circuit resulting in increased magnetizing current as air has a much lower permeability than iron. Therefore the joints should be made as tight as

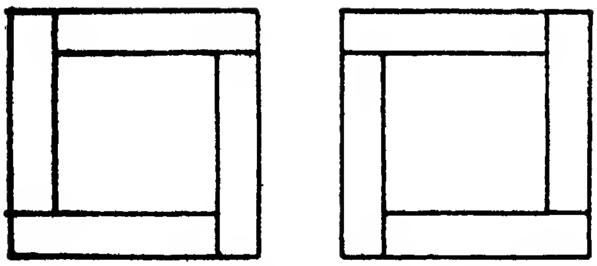


(*a*) Core type (*b*) Shell type

Fig. 7·10. Core built with L type laminations.

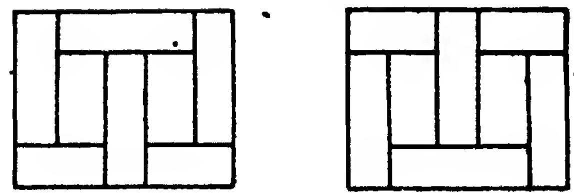
possible and the joints in adjacent laminations should be overlapped. This gives a smaller effective air gap for the flux to jump. In small transformers the path of flux through iron is small, and the air gap at the joints is of more relative importance than in large transformers. Therefore in order to decrease the magnetising current in small transformers, we have to make the laminations of special shape. In the case of shell type of transformers, the laminations can take the form of an E as shown in Fig. 7·9 (*b*). L type laminations can be used for building up cores of both core and shell of transformers as shown in Figs. 7·10 (*a*) and (*b*).

In the case of large core type transformers, I type laminations are used and the joints between limbs and yokes are interleaved. If the magnetic properties of the circuit are the only consideration, the best arrangement is obtained by interleaving one plate at a time. Fig. 7·11 shows the alternate layers of laminations of interleaved core of a single phase transformer while Fig. 7·12 shows the same for a three phase transformer. It is usually desirable for mechanical reasons to interleave the plates two, three or four at a time. As many as 20 widths of steel strip may be interleaved at a time for large power transformers as this acceler



(*a*) 1st position (odd numbered laminations) (*b*) 2nd position (even numbered laminations)

Fig. 7·11. Interleaving core of 1-phase core type transformers.



(*a*) 1st position (odd numbered laminations) (*b*) 2nd position (even numbered laminations)

Fig. 7·12. Interleaving core of 3-phase core type transformers.

rates the magnetic circuit assembly and minimizes the risk of imperfect interleaving on account of buckling of laminations. Fig. 7.13 shows the interleaving of core plates using a number of laminations at a time. In this case the interleaving patterns remain

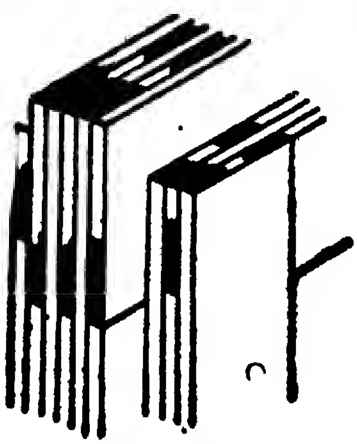


Fig. 7·13. Interleaving of core laminations.

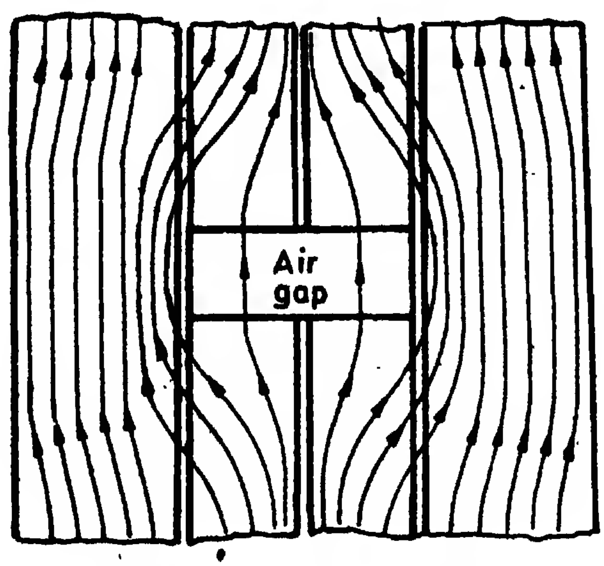


Fig. 7·14 Effect of air gap on flux patterns of core joints.

unchanged although the thickness of each layer grows accordingly two or three times. It should be pointed out here that the increased thickness of the layer increases the no load current and also the iron loss of the transformer because of increased reluctance offered to the magnetic lines of force bypassing the lamination joints as shown in Fig. 7·14. It is clear that larger the number of laminations in a layer, the greater is the number of magnetic lines of force which go through the air gaps between leg and yoke laminations. Since the flux has to pass through the air gaps in the joints of the laminations, the gaps left during the interleaving of the care result in large magnetizing current of the transformer.

Therefore, in order to reduce magnetizing current, the interleaving at the lamination joints should be done with utmost care. The gaps between laminations must not be greater than 1—2 mm.

7·10. Yoke cross-section. The cross-section of the yoke is made about 15 per cent greater than that of the core in transformers using hot rolled steel. This reduces the flux density in the yoke which in the reduces the magnetizing cur-ent and iron losses.

The core is made multi-stepped in order to reduce the length of mean turn of the windings. However, since there are no windings around the yoke it can be made of a larger cross-section in order to reduce no load current and also the iron losses. Also in order to simplify the core construction and to cut down labour costs, the yoke need not to be of multi-stepped cross-section. It can be made rectangular or two-stepped cross section in order to simplify the construction. This construction leads to **cross-fluxing** between core and yoke giving rise to additional iron losses. The phenomenon of cross-fluxing can be explained with the help of a analogy in hydraulic systems.

Let us consider three channels of equal area with equal water as shown in Fig. 7·15. Let them be joined together at a water head and then be trifurcated into three channels again. It is observed that water in a incoming channel goes into a corresponding outgoing channel straight across without cutting across and without causing turbulence in the common water head. Ideally there are no hydraulic losses since there is no turbulence of water.

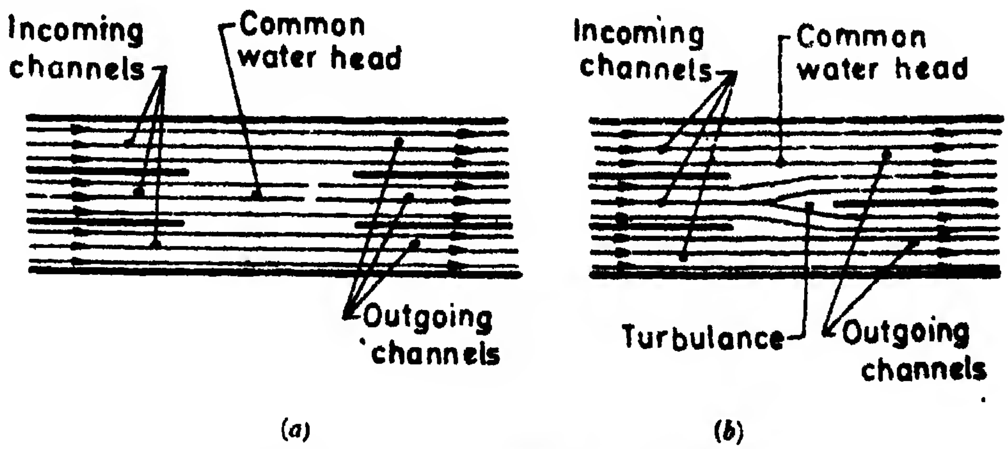


Fig. 7·15. Hydraulic analogy for cross-fluxing.

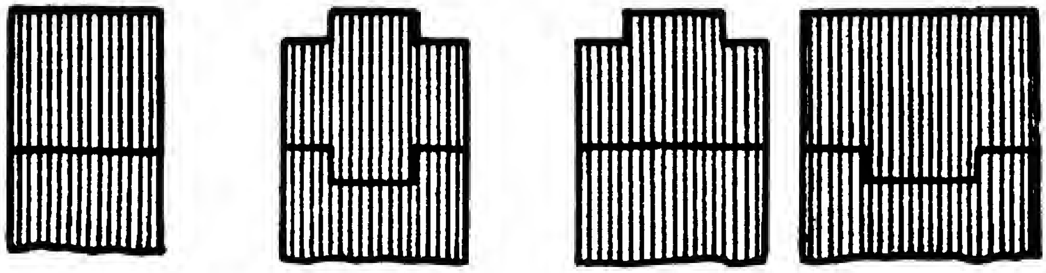
Let us consider another case wherein there are three incoming channels pouring water into a common water head with only two outgoing channels. In this case, since the water of three incoming channels has to go to two outgoing channels, water has to cut across the common water head and therefore increased turbulence is caused with resultant loss.

Similarly, when the core is three stepped and the yoke is two stepped, the flux will cut across the laminations at the joints of core and yoke resulting in *cross-fluxing* and consequent increase in iron loss.

The iron losses in a magnetic core, where the flux crosses from one part to another, are minimum if the flux density in all the packets comprising the magnetic circuit is the same. In a transformer, this is possible only if the core and the yoke are of the same shape so that the additional iron losses on account of cross-fluxing are avoided.

When the core and the yoke cross-sections are of different forms, cross fluxing occurs resulting in non-uniform flux density distribution which gives rise to additional iron losses and local overheating of the core. It is not always possible to use the same cross-section for both yoke and core owing to certain technological reasons. The yoke is made of much simpler cross-section which may be either rectangular or cruciform or with a step upwards or downwards as shown in Fig. 7·16. Yoke stepping, to some extent, equalises the flux density distribution in the core packets and reduces the additional iron loss as compared with the loss in rectangular section yokes.

Yokes with rectangular cross-section as shown in Fig. 7·16 (*a*) are used for small capacity transformers, while the yoke cross-sections shown in Figs. 7·16 (*b*), (*c*) and (*d*) are used for large capacity transformers.

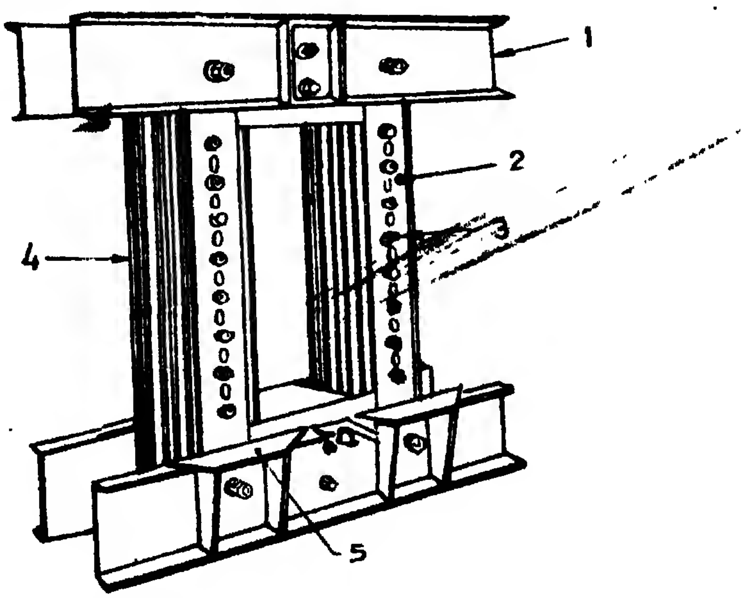


(*a*) Rectangular (*b*) Cruciform (*c*) With a step upwards (*d*) With a step downwards

Fig. 7·16. Yoke cross-sections.

The yoke cross-section with a step downwards as shown in Fig. 7·16 (*d*) is commonly used in large power transformers and has the advantage of providing additional space along the height for location of tappings in the winding.

7·11. Clamping of core. In small types of transformers, the laminations are held together by either string or by a strong cotton webbing. But this method does not give much strength and rigidity. Cores can also be clamped between iron frames. In bigger transformers, cores are kept in position by side plates bolted together at intervals along the limbs and the yoke. Holes are punched out in the laminations in order to accommodate the bolts. These bolts, which necessarily pass through the cores must be insulated both from the side plates and the laminations, while the side plates are insulated from the laminations. The isolation is necessary as otherwise the bolts would short circuit the laminations and would provide paths for the eddy currents. In order to provide more rigidity to the core and to prevent bulging of core between bolts, flitch plates are used. Fig. 7·17 shows the clamping arrangements for a core.



1. Channel section for clamping yoke.
2. Flitch plate.
3. Core clamping bolts.
4. Silicon steel laminations.
5. Bracket for supporting winding and end insulation.

Fig. 7·17. Clamping arrangement for transformer cores.

7·12. Core construction of modern core type power transformers. Cold rolled grain oriented steel laminations are used for cores of all modern power transformers. This is because it permits the use of flux densities between 1·6 to 1·8 Wb/m² as compared with 1·3 Wb/m² for hot rolled steel and consequently the weight of both core and windings is reduced.

Prior to introduction of grain oriented silicon steel, transformer core design was characterised by rectangular interleaved corners, larger yoke than core (limb) cross-sections, and clamping by bolts passing through the active core area. With the advent of cold grain oriented steel the three limbed construction for 3 phase transformers Fig. 7·4 use identical yoke and limb cross-sections.

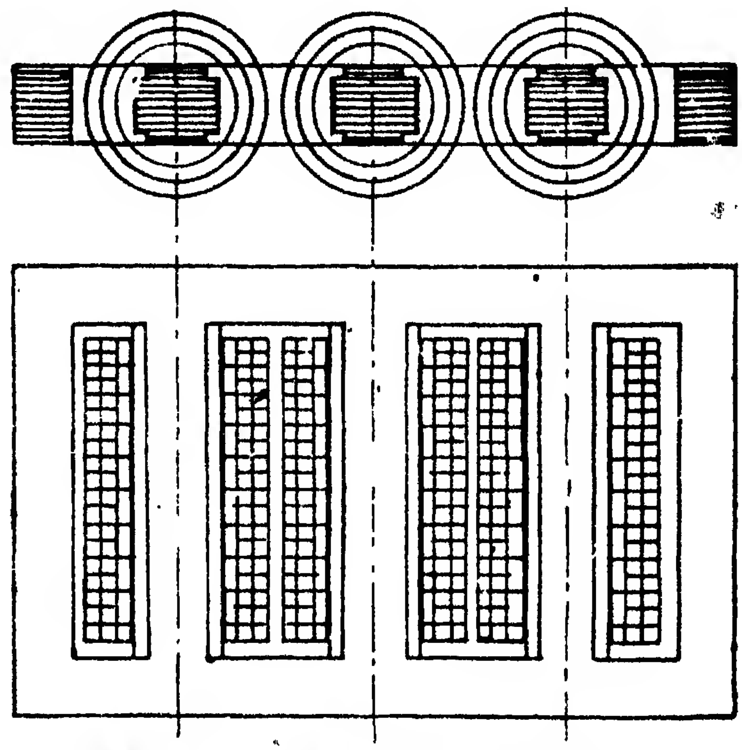


Fig. 7·18. Five limbed three phase transformer.

With very large 3 phase transformers, the yoke sections become ver heavy resulting in increased height of transformer. A limitation on the height of transformer may be imposed by the loading gauge of the railways route or the road along which the transformer is to be hauled. Thus it may become necessary to reduce the overall height of transformer. The height of yoke is reduced by using a 5 limbed construction in place of the conventional three limbed construction. In five limbed construction (Fig. 7·18) the top and the bottom yokes can be made of smaller of cross section (than the principal three limbs) by providing two extra limbs which provide extra return paths for the flux.

Top and the bottom yoke -sections are 58 percent and the vertical return limbs are 45 percent of principal limb cross-section.

The type of construction to be used whether say "core type" or "shell type" is usually an invariable function of the design policy, factory layout and construction. In India the core type of construction is used almost exculsively.

When very large transformers are manufactured, their overall dimensions must be taken into consideration from the point of view of transportation. The height of the tranformers has to be reduced on account of transportation difficulties like height of the bridges on the way to installation sites.

The reduction of overall height of transformers requires the height of yoke to be reduced. It is a common practice today to manufacture 1-phase generator transformers upto 800 MVA with cores of star form or cruciform plan ; three or four "two limb" cores are assembled with one central combined limb for the windings.

Fig. 7·19 shows a transformer core arrangement with a central leg around which both primary and secondory windings are wound, and the return path for the flux is through four yokes. This arrangement reduces the height of the yokes by four times. The reduction of height of yoke facilitates the use of either a circular or a square tank.

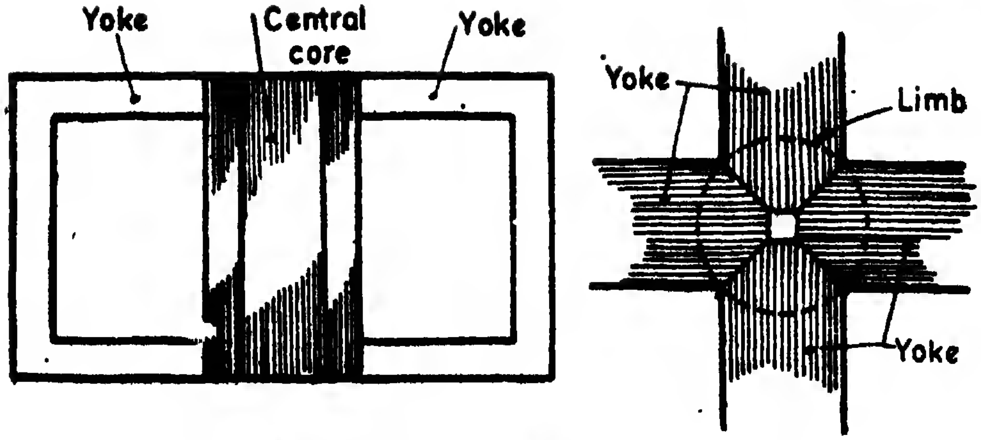
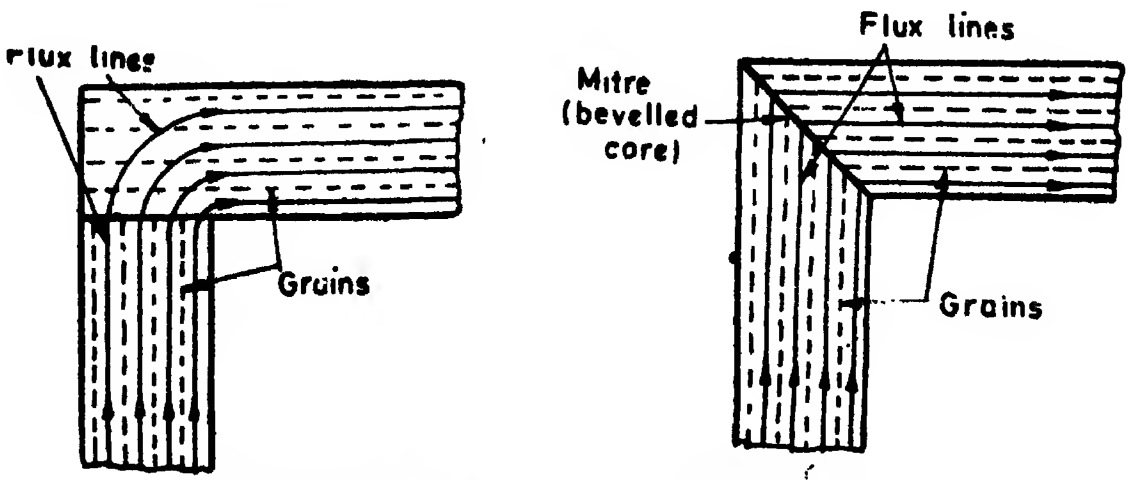


Fig. 7·19. Four yoke 1-phase core.

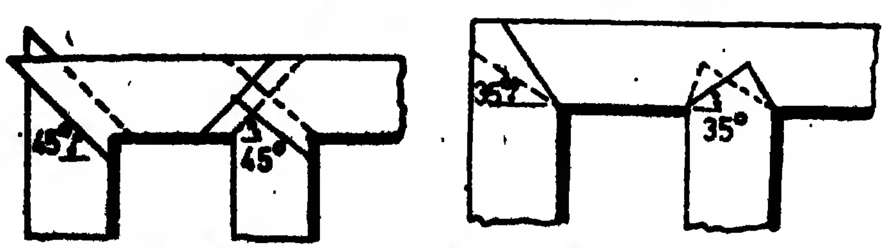
In cold rolled grain oriented steel the grains are along the direction of rolling. The cold rolled grain oriented steel has the maximum permeability in the direction of the grain orientation. Also the specific iron loss is minimum if the flux lines are along the direction of rolling. Therefore, if the cores and the yokes are stacked of interleaved right angled laminations the magnetic lines of force at the corners will turn at an angle to the direction of grain orientation as shown in Fig. 7·20 (*a*). This gives rise to increased iron loss and increased no load current.

The iron loss and the no load current can be decreased if the flux lines are made to flow along the direction of the grain orientation. This is possible if mitred (bevelled) joints are used for cores and yoke as shown in Fig. 7·20 (*b*).



(*a*) Conventional core. (*b*) Mitred core.

Fig. 7·20. Stacking of core using cold rolled oriented steel.



(*a*) 45° Mitre. (*b*) 35°/55° Mitre.

Fig. 7·21. Mitred joints for cores using cold rolled oriented steel (c.o.r.s.).

The commonly used forms of mitred joints are (*i*) 45° mitre and (*ii*) 35°/55° mitre. These joints are shown in Figs. 7·21 (*a*) and (*b*) respectively. The full lines show the joints in one layer of laminations and the dotted lines show the joints in the subsequent layer. The 35°/55° mitre improves the flow of the flux around the corner while the 45° mitre reduces waste of core material during the cutting process.

When the core (circumscribing) circle diameter exceeds about 0·8 m the commercially available laminations are too narrow to span the full diameter and therefore splitting of the core becomes essential. Therefore, bridging sections as shown in Fig. 7·22 are needed between the two halves of the core at the central limb. Incidently, an advantage of splitting the core in two halves is improvement in cooling of the core because the area of the laminations exposed to the coolant gets doubled.

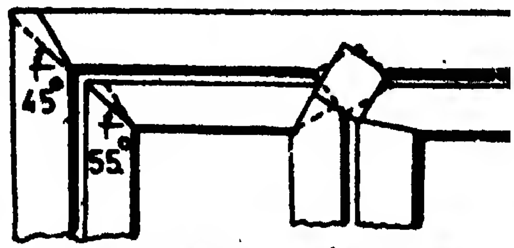


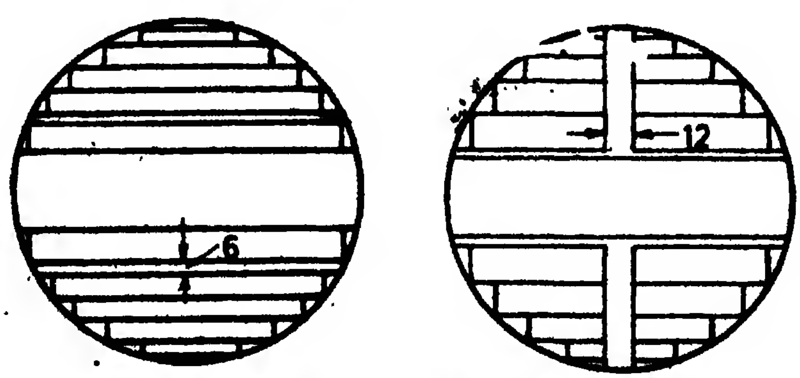
Fig. 7·22. Splitted core with bridge of 35°/55° mitre corners.

In order to make the best use of the grain oriented silicon steel, it is necessary to make the flux run parallel to the direction of rolling (*i.e.* the direction of grain orientation) for as much of the magnetic path as possible. This is made by designing yoke and limb sections of identical cross-section and shape in order to avoid cross fluxing and additional iron loss. The laminations are assembled with mitred (bevelled) joints.

The use of mitred joints assures that the flux flows along the direction of grain orientation thereby minimising both the core loss and the magnetising current. However, the use of mitred joints complicates the core construction. The core may be clamped with bolts. The bolts are insulated from the core by core of s.r.b p. or with larger cores with higher operating temperatures, these are often of epoxy resin bonded fibre glass.

However, in order to retain the advantages of cold rolled oriented steel, a boltless construction is used as the clamping bolts distort the flux path and increase the eddy current loss. The core is bound under pressure by synthetic-resin-bonded glass or Terelene which is then heated to set the resin. The outer packets of laminations are then bounded by a suitable adhesive such as Araldite in order to spread the pressure more evenly.

7·13. Cooling of cores. In transformers of medium and high capacity with diameter of circumscribing circle $D > 0·35$ m the cores have the relatively small surface/volume ratio so that the temperature gradient in the core is excessive. In such cases the cooling must be augmented by dividing the core into different stacks with longitudinal oil ducts (usually 6 mm wide) running parallel to the laminations as shown in Fig. 7·23 (*a*).



(*a*) longitudinal ducts. (*b*) longitudinal and transverse ducts.

Fig. 7·23. Core ducts.

In transformers of very high capacity ($D > 0{\cdot}8$ m) longitudinal ducts may not be sufficient and as heat flows more readily along the laminations, than between insulated laminations, it is necessary to increase the area of lamination edges by using transverse ducts which may be 10-12 mm wide as shown in Fig. 7·23 (*b*).

The magnetic circuit is therefore, divided into packets insulated from each other and to ensure good electrical continuity between packets, tinned copper strip bridging pieces are used.

7·14. Core Earthing. With the exception of individual laminations and core bolts, all internal metal parts of the transformer require earthing.

Due care must be taken in the design of the earthing system to avoid multiple paths which may initiate partial discharges because of the circulating currents inducing relatively high voltages across high impedance sections of an earth path.

7·15. Transformer Windings. The windings used in transformers are of different types and employ different arrangements for coils.

Shell type transformers use sandwich type of winding with coils shaped as pancakes. In this type of winding both low and high voltage windings are split up into a number of coils. Each high voltage coil lies (or is sandwiched) between two low voltage coils as shown in Fig. 7·3(*b*). The two low voltage coils at the ends have half the turns of a normal low voltage coil and therefore these coils are called *half coils*. The subdivision of low and high voltage windings into a number of coils gives a better coupling between the two windings and therefore results in lower leakage flux thereby reducing the leakage reactance. The leakage flux and leakage reactance of the windings depends upon the number of sections in which the windings are divided ; the larger the number of coils (and hence sections), the lower is the leakage reactance. Therefore, the advantage of sandwich coil is that with their use the leakage reactance of the transformer can be controlled to any desired value with a suitable division of windings.

Core type of transformers use concentric type of windings. Each limb is wound with a group of coils consisting of both primary and secondary windings which are concentric to each other as shown in Fig. 7·3(*a*). The low voltage winding is placed next to the core (which is at the earth potential) and the high voltage winding is placed on the outside. However, the low voltage and the high voltage windings can be alternately interlaced so as to reduce the leakage reactance.

The type and arrangement used for windings used for core type of transformers depends upon many factors. Some of these factors are :

(*i*) current rating, (*ii*) short circuit strength,

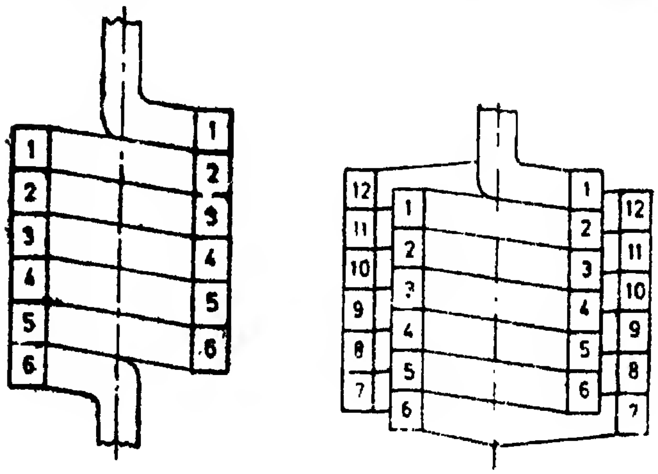
(*iii*) temperature rise, (*iv*) impedance,

(*v*) surge voltage and (*vi*) transport facilities.

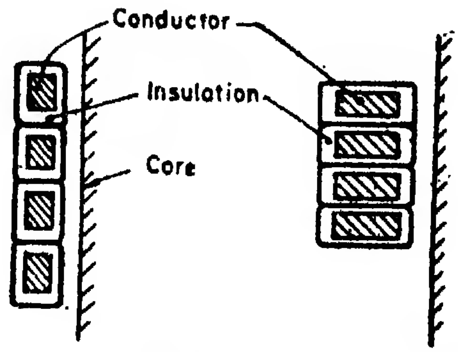
The windings used for core type of transformers are of the following types :

1. Cylindrical windings
2. Helical windings
3. Double helical windings
4. Multi-layer helical windings.
5. Cross-over windings
6. Disc and continuous disc windings
7. Aluminium foil windings.

1. Cylindrical windings. These windings are layered type and use either rectangular or round conductors. A cylindrical winding using rectangular conductors is



(a) (b)
Fig. 7·24. Cylindrical windings.



(a) on the flat side (b) on the rib side
Fig. 7·25. Positions of rectangular conductors.

shown in Fig. 7·24. The conductors are wound on the flat side with their longer sides parallel to the core axis as shown in Fig. 7·25 (a). However, sometimes they are wound on the rib side *i.e.* their longer sides are perpendicular to the core axis as shown in Fig. 7·25(b). The winding using rectangular conductors may be simultaneously wound from one or more parallel conductors placed flatwise or edgewise.

The layered winding may have to conductors wound in one, two or more layers and is, therefore, accordingly called the one, two or multilayer winding. The windings using rectangular conductors are usually two layered type because in this case it is easier to secure the lead-out ends. The two layers are separated by an oil duct. The windings designed for heavy currents are wound with a number of conductors connected in paralllel located side by side in one layer. The parallel conductors have the same length and are located in the magnetic field of leakage flux of almost the same flux density, and hence it is not necessary to make any transposition of the conductors. A wedge-shaped packing is used at each of the two extreme ends of winding in order to level it. The packing is made of either pressboard strips or rings cut from a bakelite cylinder.

Cylindrical windings employing rectangular conductors are used mainly as low voltage windings upto 6·6 kV for kVA ratings upto 600—750. However, their main use is for voltages upto 400V.

Cylindrical windings using circular conductors are multilayered as shown in Fig. 7·26. They are wound on a solid paper bakelite cylinder.

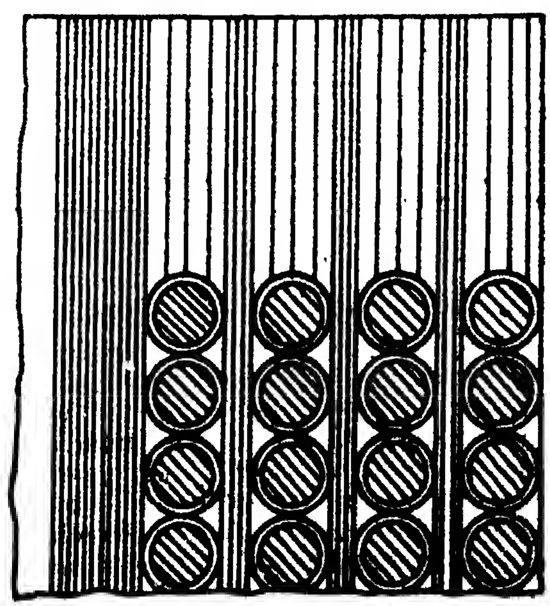


Fig. 7·26. Insulation between layers and on the butt-end of the cylindrical windings of circular conductors.

In order to improve the cooling conditions of the inner layer, the cylindrical windings using circular conductors are often wound on vertical strips forming an oil duct between winding and the insulating cylinder as shown in Fig. 7·27(*b*). Sometimes the winding is divided into two parts by an additional oil duct. This oil duct is usually located nearer to the inner winding as shown in Fig. 7·27(*c*).

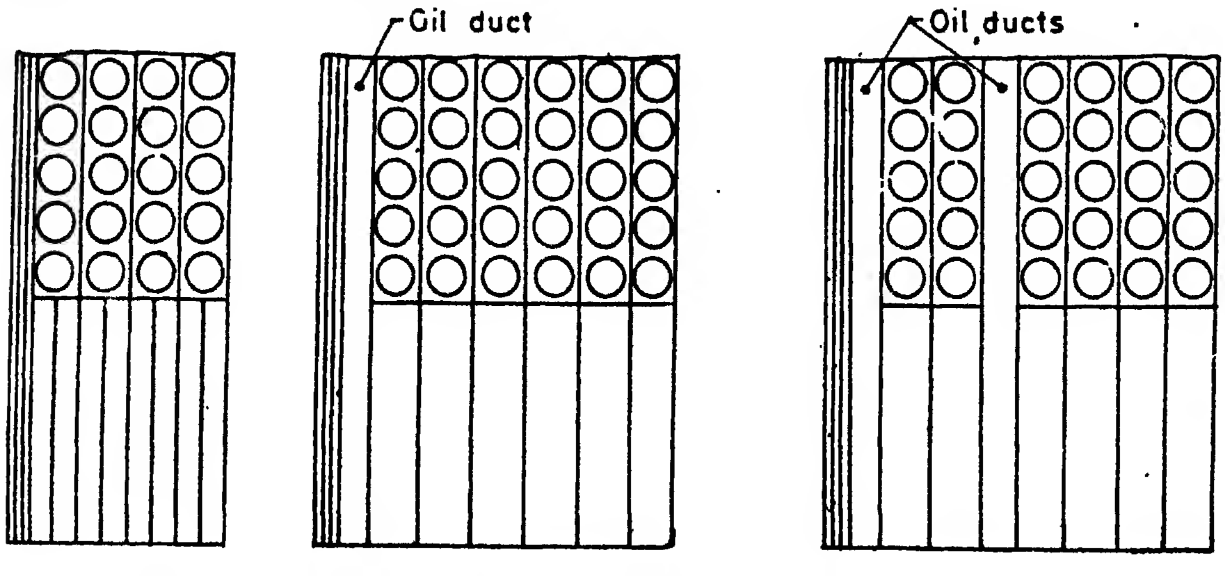


Fig. 7·27. Arrangements of the cylindrical windings with circular conductors

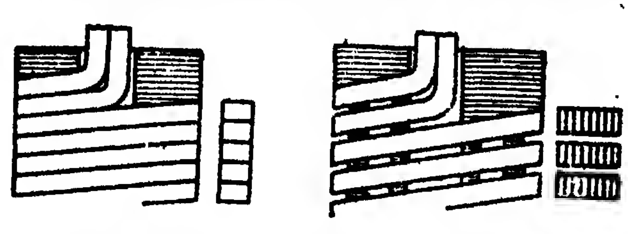
The cylindrical windings employing circular conductors are mainly used for high voltage windings with voltages 6·6, 11, and 33 kV for ratings upto 600—100 kVA

2. **Helical windings.** The helical windings are of two types

(*i*) single helical winding. (*ii*) double helical winding.

Single helical windings. The single helical or simply a helical winding has its turns wound in an axial direction along a screw line with an inclination corresponding to the height of a conductor and an oil duct between turns. There is only one turn in each winding layer as shown in Fig. 7;28(*a*). The winding consists of a single section conductor or a number of strands in parallel wound in the form of a continuous helix. The conductor is rectangular in cross-section and is paper covered. The oil is mounted on a thick pressboard or s.r.b.p. cylinder.

Helical windings are used for low voltage winding medium and high capacity transformers where the number of winding turns is small but the current is high. Therefore, low voltage windings of medium and high capacity transformers require the use of a conductor made of a large number of strips connected in parallel. The parallel connected strips are placed side by side in a radial direction so that each conductor occupies the total radial depth of the winding to form a *Disc-helical winding* as shown in Fig. 7·28 (*b*). The individual strips can be assembled in a radial pack, either as a single column or as two columns in parallel.



(*a*) Helix (*b*) Disc-helix

Fig. 7·28. Helical windings.

Helical windings are usually wound on the battens around the bakelite cylinder with insulating cylinders placed between the turns. The continuous helical winding exhibits high axial mechanical strength and therefore finds wide application in low voltage windings of large size power transformers.

A distinguishing feature of the helical winding is the use of transposed conductors by changing the relative position of individual conductors or groups of conductors. The transposition is essential for equalizing the resistance and leakage reactance of each of parallel conductors. In the absence of transposition these conductors will be of different length and, being situated in the leakage field having unequal flux densities, will have different resistance and leakage reactance. This would lead to non-uniform distribution of current in parallel conductors thereby overloading of portions of conductors and causing additional eddy current losses in conductors.

The transpositions in this winding are made in the following manner. The whole of the winding turns along axial length are subdivided into four equal sections. The trans-

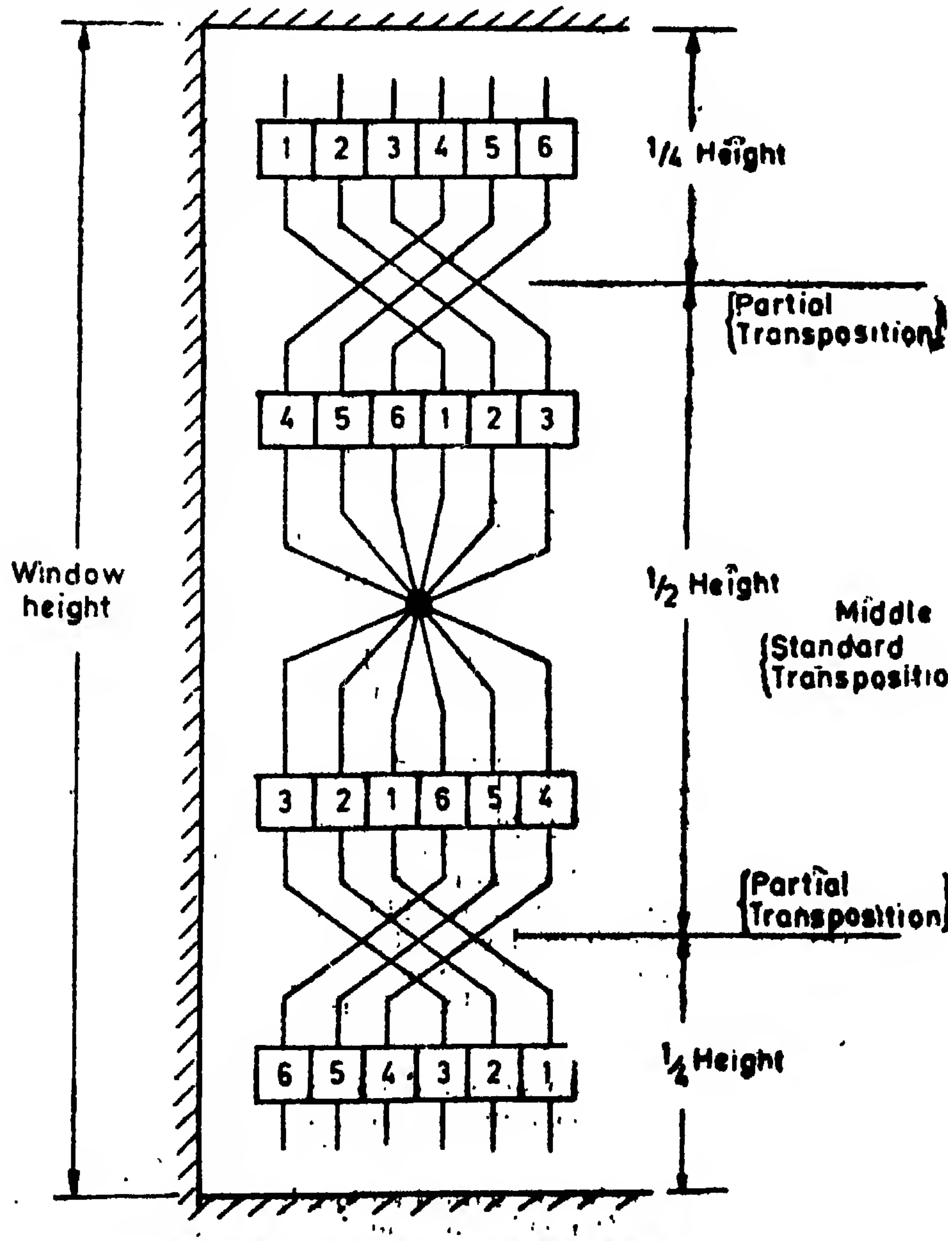


Fig. 7·29. Transpositions in Helical windings.

positions are made between these sections, at three different points of the winding. The three transpositions include one standard transposition and two partial transpositions as shown in Fig. 7·29. The standard transposition is done in the middle of winding with each conductor varying its position symmetrically relative to the middle point *i.e.* the conductor on the extreme right being transposed to the extreme left, the second conductor from right being transposed to second position from left and so on. The two partial transpositions are done at a distance of $\frac{1}{4}$ height from top and bottom ends of the windings. In partial transpositions two halves of parallel conductors are interchanged in positions with the right half the conductors being transposed to left half positions and *vice-versa*.

The disadvantage of the simplex helical winding is that ampere turns are as if thinned out at the points of transpositions, which leads to unequal distribution of mmf throughout the height of winding.

3. **Double-helical windings**. The double-helical winding is used in low voltage windings of high power ratings where the number of winding turns is small and a single

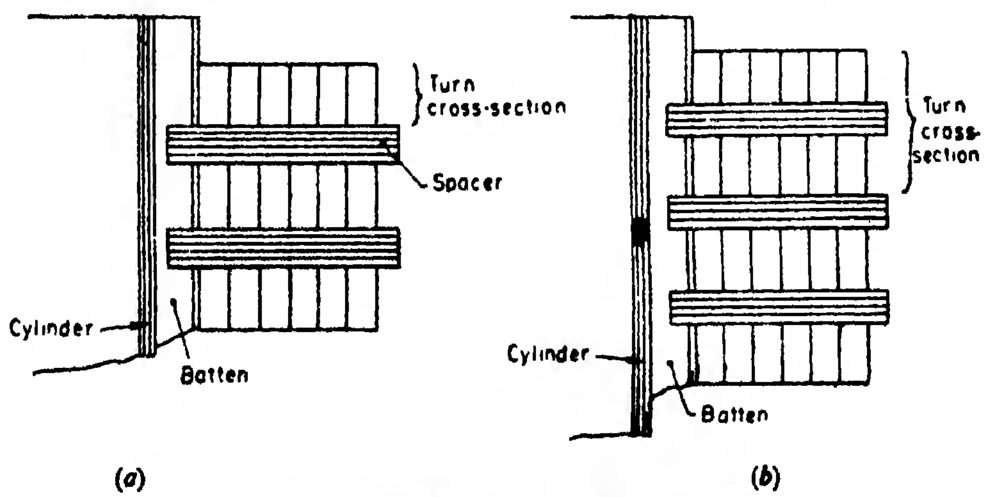


Fig. 7·30. Cross-section of helical windings.

helical winding with normal width of oil ducts does not fill up properly a window height, while the current and the number of parallel conductors required are very large.

Fig. 7·30 (*a*) shows a single helical winding. All the conductors forming one turn are situated side by side in one layer, the adjacent turns separated by spacers along the axial length. In contrast, the parallel conductors of a double helical winding are divided into two parallel circuits and are situated in two layers shifted in axial direction as shown in Fig. 7·30 (*b*). The advantage of double helical winding is the reduced eddy current loss in conductors. This is on account of the reduced number of parallel conductors situated in radial direction. This can be illustrated by a simple example. Suppose there are 8 parallel conductors which form one turn. If asingle helical winding is used, there will be 8 conductors placed radially, while there will be only 4 conductors in the radial direction if double-hexial wind-ing is employed. The magnetic field is non-uniform in the radial direction and, therefore, there is greater magnetic assemetry between the conductors when the single helix winding is employed, resulting in greater I^2R loss and increased leakage reactance.

In double-helix winding the transposition is obtained without using an axial space on interchauging the conductors, because the turn in its cross-section consists of two groups of conductors.

The transposition in this winding is termed uniformly distributed and is made in the following way. The entire winding (turns) is sub divided into equal sections. The number of these sections should be equal to the total number of parallel conductors. The conductors

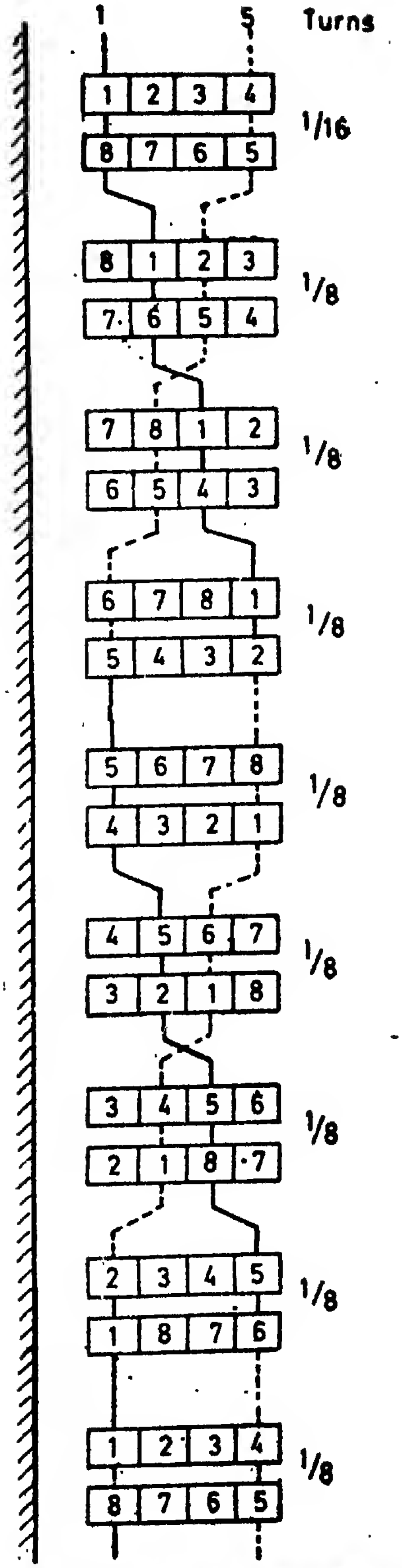


Fig. 7·31. Transpositions in double layer helical windings.

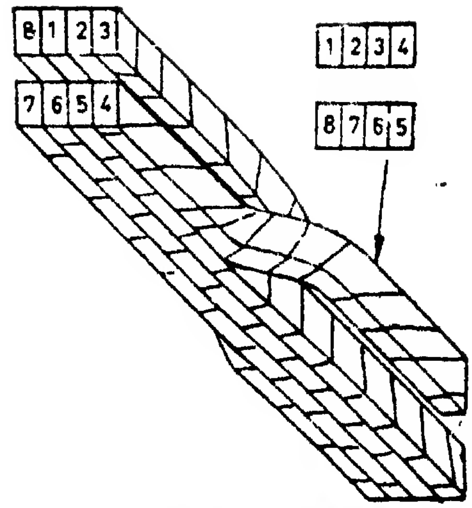


Fig. 7·32. Transposed conductors according to equally distributed transposition diagram.

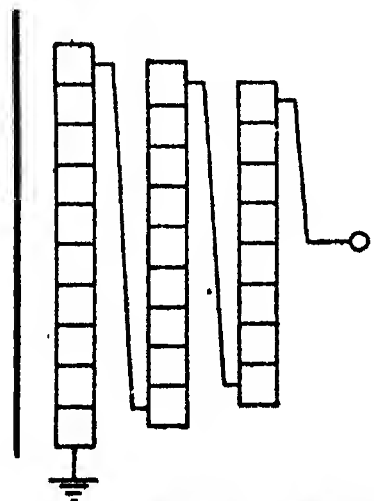


Fig. 7·33. Multi-layer helical winding.

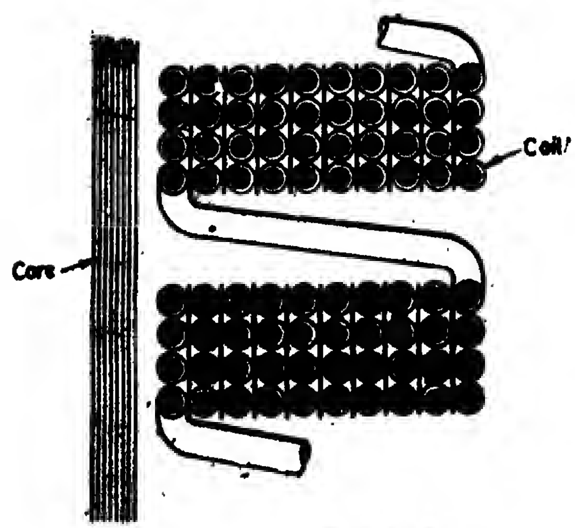


Fig. 7·34. Cross-over winding.

are then changed between these sections ; the top conductor of one group is placed on top of the second group while at the same time and in the same portion of the winding the bottom conductor of the second group is inserted underneath the first group.

The procedure for making a transposition in the double-helical winding is shown in Fig 7·31. The conductors are transposed in the gap between the spacers as shown in Fig. 7·32.

A double-helical winding is used for the same range of voltages as a single-helical winding. However, the current rating for the double helical winding is twice as that of single helical winding.

Helical windings are used in power transformers with outputs ranging from 150 kVA to 30 MVA at voltages from 400 V to 11 kV and sometimes upto 33 kV.

4. **Multilayer helical windings.** Multilayer helical windings are commonly used as high voltage windings for 110 kV and above.

This type of winding consists of several cylindrical layers concentrically wound and connected in series as shown in Fig. 7·33. The number of layers depends upon the voltage, higher the voltage, the larger the number of layers. All the layers are wound on paper cylinders and are separated from each other by vertical strips forming vertical oil ducts. One line terminal is connected to the outermost layer while the innermost layer adjoining the low voltage winding is grounded.

The outer layers are made shorter than the inner onesthereby distributing the capacitance uniformly. This winding is primarily used for improving the surge behaviour of transformers.

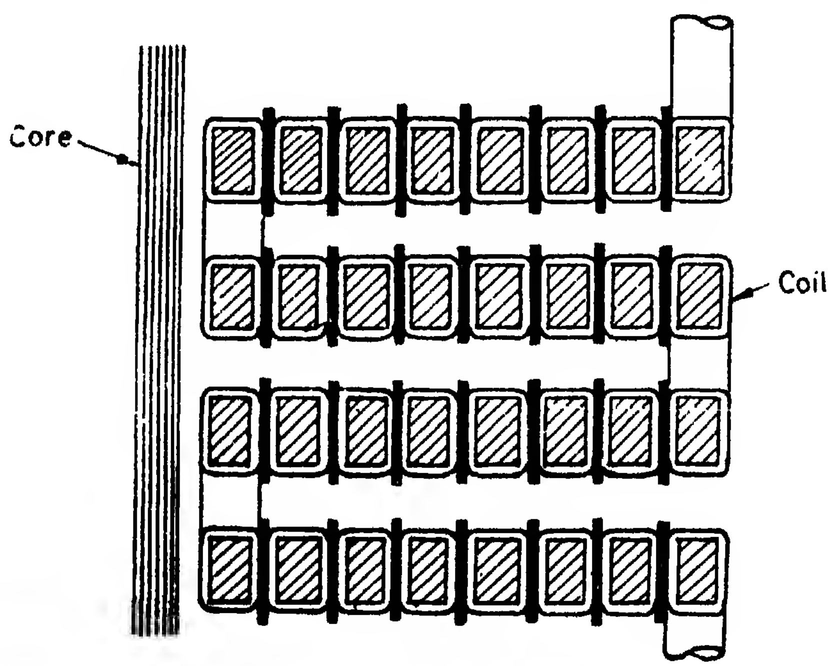


Fig. 7·35. Continuous disc winding.

The improvement in surge behaviour requires large capacitance, which means decrease in radial depth of winding. The decrease in radial dimensions means poor mechanical strength and also low leakage reactance. Therefore, multilayer helix windings are subject to large short circuit forces and since they are inherently weak (mechanically), their use is restricted.

5. **Cross-over windings** These windings are used for high voltage windings of small transformers. It has been mentioned earlier that cylindrical windings (especially cylindrical windings using circular conductors), are used for high voltage windings of low rating trans-

formers. However, the use of cylindrical multi-layer winding at high voltages, the voltages between adjacent layers become too high and it becomes difficult to select proper thickness for the interlayer insulation. Thus, it becomes imperative to reduce interlayer insulation and therefore the winding is axially separated into several multilayer coils. The winding is divided into a number of coils separated from each other by insulating washers or oil ducts formed by spacers.

In cross-over windings, the conductors are paper covered round wires or strips. The coils are wound over formers with side cheeks (U pieces) and each coil consists of a number of layers with a number of turns per layer. The complete winding consists of a number of coils connected in series. Two ends of each coil are brought out, one from inside and one from outside. The inside end of a coil is connected to the outside end of the adjacent coil. The actual axial length of each coil is about 50 mm while the spacing between the adjacent coils is about 6 mm to accommodate blocks of insulating material and to allow free circulation of oil. The width of coils is 25 to 50 mm. Fig. 7·34 shows cross-over coils.

Cross-over windings are used in the same range of ratings as the cylindrical windings. The cross-over winding has a higher strength than the cylindrical winding under normal operating conditions. However, this winding has a lower impulse strength than the cylindrical winding and also is more labour consuming.

6. **Disc and continuous disc windings.** Disc windings are primarily used in high capacity transformers. The winding consists of a number of flat coils or discs connected in series or parallel. The coils are formed with rectangular strips wound spirally from centre outwards in the radial direction as shown in Fig. 7·35. The conductor used is in such lengths as are sufficient for complete winding or section of winding between tappings. The conductor can be a single strip or a number of strips in parallel, wound on the flat side. This gives a robust construction for each of the discs. The discs are wound on a insulating cylinder spaced from it by strips along the length of cylinder. The discs are separated from each other with press-board sectors attached to vertical strips. The vertical and horizontal spacers provide radial and axial ducts for free circulation of oil which comes in contact with every turn.

The disc coils are usually assembled into double coils because this arrangement leads to a more convenient connection of the inner ends. The double disc windings are wound from an entire length of conductor. Each coil is wound starting from the middle, *i.e.*, from the connection point of coils, until a double coil is formed. This disposition of coils formed in pairs is achieved through pairs of coils with requisite turns for one disc being loosely wound so that the conductor finishes in a position to provide the start of the inside turn of the adjacent disc, which is then wound from inside outwards. The first disc is then rearranged in such a manner that the start is located as an outside turn.

In case, the winding consists of a number of discs connected in series, it can be wound continuously without breaking the conductor between the separate disc coil. Thus, following the formation of one disc, the procedure is repeated without cutting the conductor, thereby saving jointing and joint space. This is an advantage, especially for the winding which is placed on the inside.

A distinguishing feature of the continuous disc windings is the transposition of the coils. The purpose of these coils is clear from Fig. 7·36. These coils are initially wound in the ordinary manner, beginning from the cylinder and outward, and then these coils are transposed in the reverse order. The conductors are slackened somewhat in order to make the reversing, easier and the conductor running from the drum is again tensioned. This facilitates the continuous inter-connection of coils without any soldered joints.

The advantage of disc and continuous windings is their greater mechanical axial strength and cheapness.

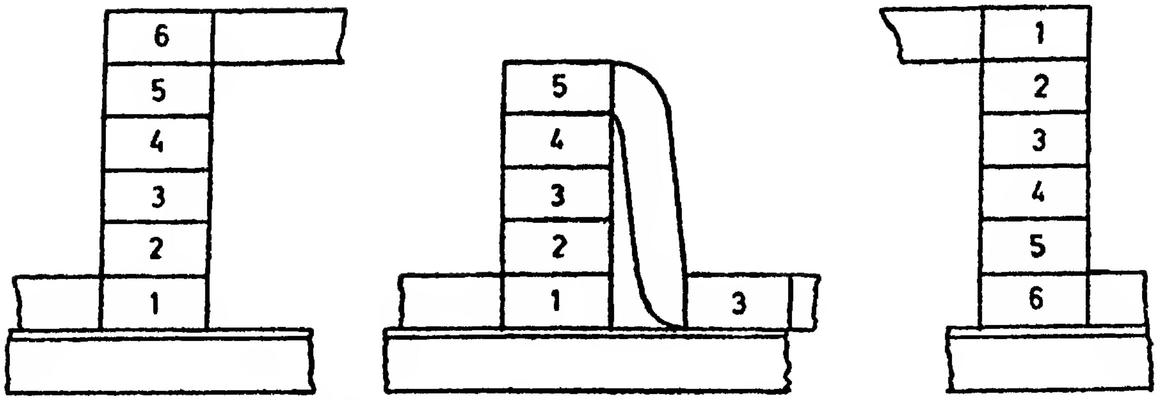


Fig. 7·36. Transposition of continuous disc winding.

7. Aluminium foil windings.

Aluminium, in place of copper can be used as a material for conductors in any of the above mentioned windings. Aluminium, when used as a single section has many disadvantages as compared with copper such as poor mechanical strength, poor workability and increased cross-secting.

However, it is uniquely employed in foil windings because it can be rolled to thinner and more flexible sheets than copper. The aluminium sheet is used in bobbin type coils of one or more turns per layer.

The types of windings used in core type of transformers are listed in Table 7·1.

Table 7·1. Windings used in core type transformers

Type of Service	*Rating* MVA	*H.V. Winding*		*L.V Winding*	
		Voltage kV	*Type*	*Voltage* kV	*Type*
Distribution	up to 1	11-33	Foil, Cross-over or Multilayer	0·43	Helix
System	1-30	33-66	Disc	11	Disc or Helix
Transmission	30 and upwards	132-500	Disc or Multilayer	11, 33, 66	Disc or Disc-Helix
Generator	30 and upwards	132-500	Disc or Multilayer	11-22	Disc-Helix

7·16. Continuously transposed conductor windings.

There has been a steady increase in the demand for electrical power. This situation calls for a commensurate increase in the power ratings and operating voltages of electrical power systems. The transformers used in power systems are now required to withstand higher electrical, mechanical and thermal ratings.

The development of continuously transposed conductor (CTC) has provided the transformer industry a winding material that can be used for ever rising system voltages and

ampacity. The windings constructed with CTC have been found beneficial for high voltage high power transformers. Continuously transposed conductor (CTC) used for transformer windings has a high thermo-mechanical strength, excellent insulation characteristics and low stray load loss. The stray losses in large transformers using conventional configuration for the conductors tend to be high. Therefore, a need arises to decrease these losses by suitably designing the windings of transformer.

Stray load loss has two components, (*i*) eddy current loss, and (*ii*) circulating current loss. Eddy current loss within each strand can be reduced by using thinner insulated conductors while the circulating current loss between strands can be reduced through frequent transposition of conductors. The advantage of using continuously transposed conductors is that the stray load loss is reduced since CTC uses a large number of conductor strips which are continuously transposed

The transposed conductor consists of odd number of copper strips connected in parallel. The number of strips ranges from 5 to 31. The cross-section of a CTC is shown in Fig. 7·37. Each strip is insulated with enamel. One layer of interleave paper is inserted in parallel between two parallel stacks of strips to avoid damage to insulation during transposition

The transposed conductor is butt-lapped with paper tape. The transposition of individual strips help in equalizing their lengths in the stack thereby reducing the circulating currents and the consequent loss. The transposition is done after 15 times the width of conductor or 50 mm whichever is greater.

The advantages of CTC over conventional paper covered conductors are :

(*i*) The use of CTC reduces the overall size of transformer on account of the compactness of strands and consequent reduction in the weight of core, windings, tank and oil.

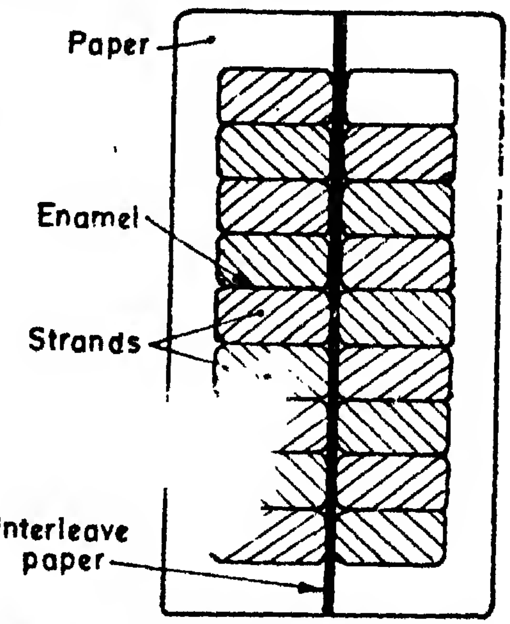


Fig. 7·37. Continuously transposed conductor (CTC).

(*ii*) The winding time is reduced due to the use of assembled transposed conductors in place of parallel strips.

(*iii*) The winding has an improved mechanical strength due to composite construction of the transposed conductor.

(*iv*) The individual strips use thin enamel insulation. Therefore, cooling of the conductors is improved on account of increased heat dissipation.

(*v*) The conductors have an increased mechanical strength.

7·17. Cooling of transformers

The transformer is a static device which converts energy at one voltage level to another voltage level. During this process of energy transfer, losses occur in the windings and core of the transformer. These losses appear as heat. The heat developed in the transformers is dissipated to the surroundings. The coolant useds in transformers are :

(*i*) air, and (*ii*) oil.

The transformer using air as the coolant are called **dry type transformers** while transformers which use oil as the coolant are called **oil immersed transformers**. In dry type transformers, the heat generated is conducted across the core and winding to be dissipated from the outer surfaces of windings to the surrounding air through convection.

In the case of oil immersed transformers, the heat produced inside the core and the windings is conducted across them to their surfaces. This heat is transferred by the oil to the walls of the tank through convection. Finally, the heat is transferred from the tank walls to the surrounding air by radiation and convection. It must be understood that cooling of transformers differs from that of rotating machines and presents greater problems since there are no moving parts in a transformer, that are responsible for inbuilt cooling of rotating machines.

7·18. Methods of cooling of transformers

There are a number of methods used for cooling of transformers. The choice of method depends upon the size, type of application and the type of conditions obtaining at the site where the transformer is installed.

The large number of methods used for dissipation of heat generated in transformers make it necessary to use a concise standard designation for them. The letter symbols used for designating these methods depend upon (*i*) medium of cooling used, and (*ii*) the type of circulation employed.

1. Medium. The cooling mediums (coolants) used for transformers along with the symbols used for designating them are : (*i*) Air—A, (*ii*) Gas—G, (*iii*) Synthetic oil—L, (*iv*) Mineral oil—O, (*v*) Solid insulation—S, and (*vi*) Water—W.

2. Circulation. The circulation of the cooling medium (coolant) may be through natural means or there may be a forced circulation of the coolant. Accordingly the symbols used are :

(*i*) Natural—N, and (*ii*) Forced—F

There are two ways of cooling a transformer.

(*i*) The coolant circulating inside the transformer comes in contact with the windings and cores and transfers all the heat entirely to the tank walls from where it is dissipated to the surrounding medium.

(*ii*) The coolant circulating inside the transformer comes in contact with windings and cores. The coolant partly transfers the heat generated to the transformer tank walls with the major portion of the heat generated inside the transformer being taken up by the coolant circulating inside the transformer, to be dissipated away later in an external heat exchanger.

The coolant circulating inside the transformer gets heated and is cooled in the heat exchanger. The heat exchanger may employ air or water in order to dissipate the heat of the coolant circulated inside the transformer.

The cooling methods are designated by symbols. Each of these letters is significant of some characteristic of the method of cooling. The cooling methods which do not employ an external heat exchanger are designated by two letters. The order in which letters are used to designate methods of cooling without external heat exchangers is :

(*i*) the medium in contact with the windings, and

(*ii*) the circulation of the coolant in contact with the windings

These methods, therefore, are designated by two letters. The order in which letters are used to designate methods of cooling using external heat exchangers is :

(*i*) the medium in contact with the windings,

(*ii*) the circulation of the coolant in contact with the windings,

(*iii*) the medium used in the external heat exchanger, and

(*iv*) the circulation of the coolant in the external heat exchanger.

The cooling methods used for ***dry type transformers*** are :

1. **Air Natural (AN)**. This method uses the ambient air as the cooling medium. The natural circulation of surrounding air is utilized to carry away the heat generated by natural convection. A sheet metal enclosure is used to protect the windings against mechanical damage. This method is used for small low voltage transformers. However, the development of insulating materials like glass and silicone resins class C materials which can withstand higher temperature (150°C) makes the method suitable for transformers of ratings up to 1.5 MVA. The high rating transformers are used in special applications like in mines where fire is a great hazard.

2. **Air Blast (AB)**. Cooling by natural circulation of air becomes inadequate to dissipate heat from large transformers and hence for circulation of air (air blast) is employed in order to keep the temperature rise within limits. The forced air circulation improves the heat dissipation.

In this method, the transformer is cooled by a continuous blast of cool air forced through the cores and the windings. The air blast is produced by external fans.

The improvement in heat dissipation caused by air blast allows higher specific loadings to be used in dry type transformers. The use of higher specific loading results in lower size for the transformers. The air supply must be filtered to prevent accumulation of dust particles in the ventilating ducts.

The cooling methods used for ***oil immersed transformers*** are :

1. **Oil Natural (ON)**. The cooling by air is not so effective and proves insufficient for transformers of medium sizes. Oil as a coolant has two distinct advantages :

(*i*) it is a better conductor of heat than air, and

(*ii*) it has a high co-efficient of volume expansion with temperature. Therefore, substantial circulation is easily obtained on account of the natural "thermal head" produced due to convection so long as the cooling ducts in the cores and windings are not unduly restricted.

Hence, almost all transformers (except for the transformers used for special applications like mines where there is a fire hazard) are oil immersed. The assembly of an oil immersed transformer is shown in Fig. 7.38.

The transformer is immersed in oil and the heat generated in cores and windings is passed on to oil by conduction. Oil in contact with the heated parts rises and its place is taken by cool oil from the bottom. The heated oil transfers its heat to the tank walls from where it (heat) is taken away to the ambient air. The heated oil thereby gets cooler and falls to the bottom. Therefore, a natural thermal head is created which transfers heat from the heated parts to the tank walls from where it is dissipated to the surrounding air.

The tank surface is the best dissipator of heat but in the case of large rating transformers the transformer tank will have to be excessively large, if used without any auxiliary means of heat dissipation. The reason for this is explained below :

Consider a transformer A with its linear dimensions k times the linear dimensions of another smaller but a similar unit B employing the same type of cooling technique. The rating of transformer A is k^4 times that of transformer B. The losses in a transformer are proportional to volume and hence losses in transformer A are k^3 times those in transformer B. The heat dissipating area of transformer A is k^2 times that of transformer B

The temperature rise $\theta = Qc/S$ where Q=losses, S=heat dissipating area, and c=cooling co-efficient. Therefore, the temperature rise of transformer A is $k^3/k^2 = k$ times that of B because the value of cooling co-efficient c remains the same if similar cooling techniques are used. Thus we conclude that temperature rise increases linearly with increase in dimensions.

The above can be explained with a simple example. Suppose, transformer B is designed for a rating of 10 kVA with temperature rise say, 40°C which is the maximum permissible. Let trans former A be designed with every linear dimension being twice of corresponding linear dimension of transformer B. Therefore, the rating of transformer A is $2^4=16$ times that of B, which is 160 kVA. The losses in transformer A are $2^3=8$ times that in B, and the surface area of A is $2^2=4$ times that of B. Consequently, the temperature rise of transformer A is $8/4=2$ times that of B, *i.e.*, the temperature rise of transformer A is therefore, 80°C. This temperature rise is twice than the maximum permissible. Hence, we conclude, that with increase in rating of transformers, the temperature rises and means must be adopted in order to keep the temperature rise within permissible limits.

The temperature rise is given by the relationship $\theta = Qc/S$. It it evident, that the temperature rise can be decreased and brought within limits by two means. These are :

(*i*) increasing S, the area of heat dissipation, and

(*ii*) decreasing the cooling co-efficient, c.

The temperature rise of a transformer is inversely proportional to S, its heat dissipating area. Thus if the size of the tank is increased, the dissipating area increases and hence the temperature rise decreases. Thus, in the example above if the size of the tank is increased such that its heat dissipating area is twice that which is otherwise demanded, the tem-

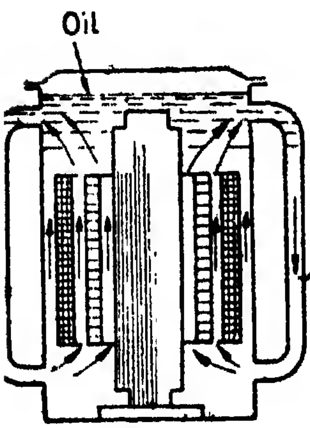


Fig. 7.38. Oil-circulation in a transformer using cooling tubes.

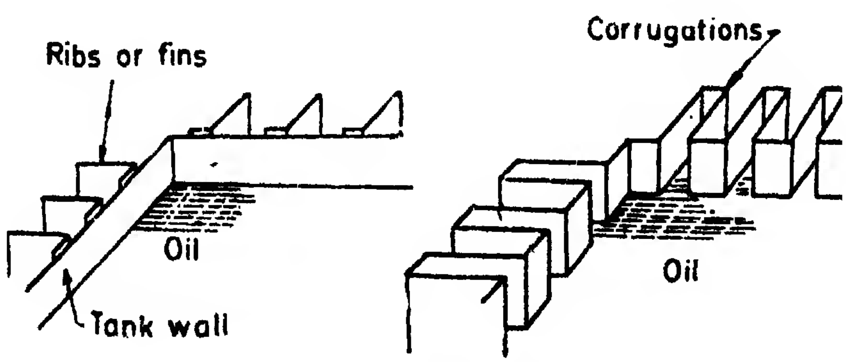


Fig. 7.39. Transformer tanks with fins and corrugations.

perature rise of the transformer is brought down to 40°C. Therefore, this calls for an excessively large tank. Increasing the size of tank out of proportion with the increase in transformer rating and consequent increase of dimensions, is obviously no solution for the

temperature rise. The plain walled tank cannot be used beyond a particular rating. This is because if the large rating transformers used plain walled tank, the size of the tank would become enormously large. A large sized tank needs large volume of oil and hence it will result higher cost and weight of transformer. Also, beyond a particular size it would become impossible to transport the transformer (due to its large size) from the place of manufacture to the site of installation due to the limitations imposed by the gauge of the rail line or the road.

The solution to the problem of decreasing the temperature rise of large transformers lies in decreasing *c*, cooling co-efficient. The value of cooling co-efficient can be decreased by augmenting the cooling by using auxiliary means. It should be understood that as the rating of the transformer increases, the value of cooling co-efficient has to be decreased. The reduction in the value of cooling co-efficient can be brought out by the use of sophisticated methods of cooling. Therefore, as the rating of transformers increases we have to use improved methods of cooling in order to keep the temperature rise within limits. The higher rating transformers require increasingly improved methods of cooling.

Oil immersed transformers of ratings upto 30 kVA use plain walled tanks. The heat dissipating capability of transformers of ratings higher than 30 kVA is increased by providing corrugations, fins, tubes and radiator tanks. Fig 7.39 shows fins and corrugations provided on four walls. Fig. 7.40 shows a transformer provided with cooling tubes. These tubes are welded to the tank walls at the top and bottom. The use of cooling tubes provides additional cooling surface but also improves the circulation of oil due to increase in thermal head.

For larger sizes of transformers, radiator tanks with fins or corrugations are employed. Fig. 7.41 shows a transformer provided with external radiators.

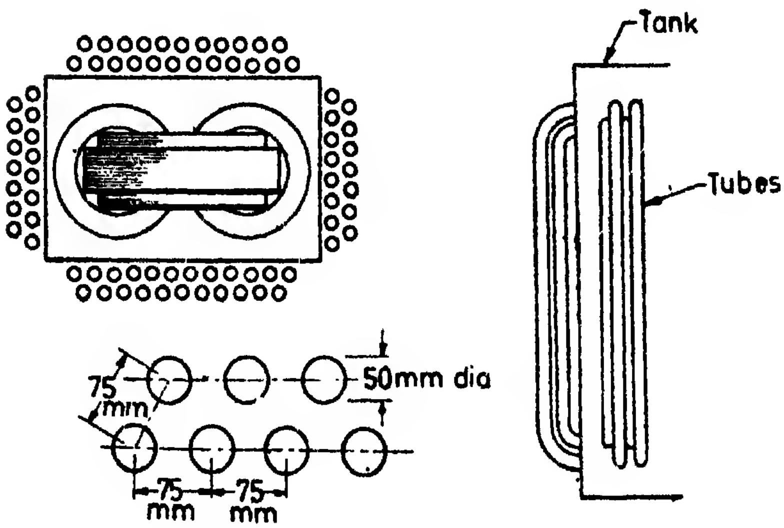


Fig. 7.40. Tank with tubes.

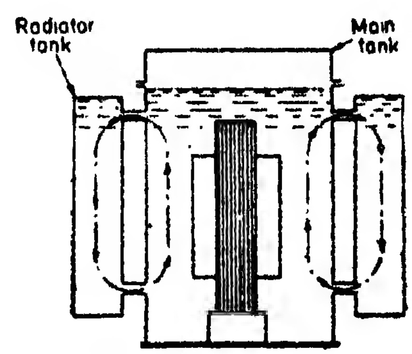


Fig. 7.41. Tank with external radiators.

It is clear, that in the methods described above the oil in the transformer is circulated on account of natural thermal head. The oil takes away the heat from inside the transformer to outside. The oil is cooled in tubes or external radiators by natural circulation of air. Therefore, the methods described above can be termed as **ONAN** (oil natural and air natural).

Transformers upto a capacity of about 5MVA or a loss of upto 50 kW use tanks with tubes. The tubes are usually round and are 50 mm in diameter and are arranged in one to three rows. Elliptical tubes are also used.

2. Oil Natural Air Forced (ONAF). In this method the oil circulating under natural head transfers heat to the tank walls. The transformer tank is made hollow and air is blown through the hollow space to cool the transformer. The heat removed from the inner tank walls can be increased to five or six times that dissipated by natural means and therefore very large transformers can be cooled by this method. However, the normal way of cooling the transformers by air blast is to use radiator banks of corrugated or elleptical tubes separated from the transformer tank and cooled by air blast produced by fans.

3. Oil Natural Water Forced (ONWF). In this method, copper cooling coils are mounted above the transformer core but below the surface of oil. Water is circulated through the cooling coils to cool the transformer.

This method proves to be cheap where a natural water head is already available.

The method has, however, the serious disadvantage that it employs a cooling system which carries water inside the oil tank. Since the water is at higher head than oil, therefore, in case of leakage water in the cooling tubes will enter the transformer tank contaminating oil and reducing its dielectric strength.

Since heat passes three times as rapidly from copper cooling tubes to water as from oil to copper tubes, the tubes are provided with fans to increase conduction of heat from oil to tubes. The water inlet and outlet pipes are lagged in order to prevent the moisture in the ambient air from condensing on the pipes and getting into the oil.

4. Forced Circulation of Oil (OF). In large transformers the natural circulation of oil is insufficient for cooling the transformer and forced circulation is employed. Oil is circulated by a motor driven pump from the top of a transformer tank to an external cooling plant (heat exchanger or refrigerator) where the oil is cooled. The cold oil enters the transformer at the bottom of the tank.

The method of cooling oil in the heat exchanger depends upon the condition obtained at the site. The methods of cooling transformers by forced circulation of oil ares classified accordingly as :

(i) Oil Forced Air Natural (OFAN). In this method oil is circulated through the transformer with the help of a pump and cooled in a heat exchanger by natural circulation of air. This method is not commonly used. However, this method proves very useful where the coolers have to be well removed from the transformer.

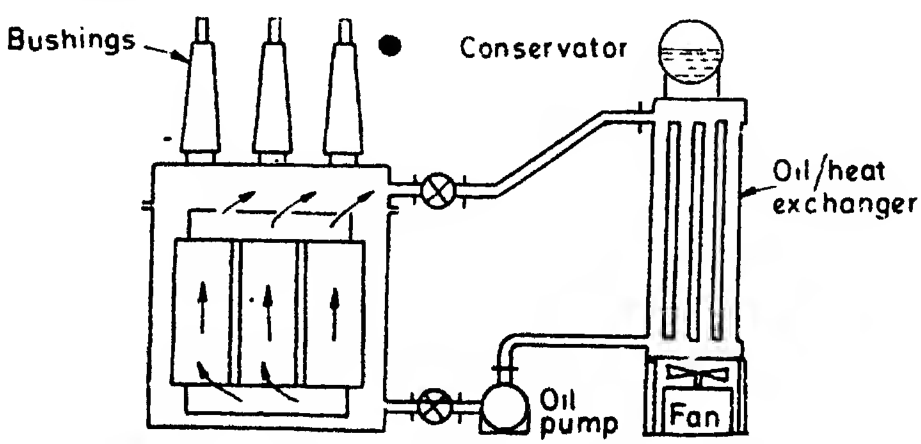


Fig. 7·42. Oil forced air forced cooling (OFAF).

(ii) Oil Forced Air Forced (OFAF). The method is depicted in Fig. 7·42. The oil is cooled in external heat exchangers using air blast produced by fans. It is interesting to note that the oil pump and fans may not be used all the time. At low loads, the losses are small and therefore natural circulation of oil with an ONAN condition may be sufficient to cool the transformer. At higher loads, the pump and the fans may be switched on by temperature-sensing elements. Therefore mixed cooling conditions are used, the transformer working with ONAN conditions upto 50% of rating and OFAF conditions at higher loads. This arrangement results in higher efficiency for the system.

(iii) Oil Forced Water Forced (OFWF). This method is shown in Fig. 3·34 on page 73. The heated oil is cooled in a water heat exchanger. In this method the pressure of oil is kept higher than that of water and therefore any leakage that occurs is from oil to water. Also there are no condensation problems. At sites, where the cooling water has a considerable head, it is usual to employ cascaded heat exchangers *i.e.* oil/water and water/water with the intermediate water circuit being at a low pressure. This cooling method is suitable for banks of transformers, but from the system reliability considerations not more than, say, three tanks should be connected in one cooling pump circuit. The advantages of OFWF method over ONWF are that the transformer is smaller and the transformer tank does not have to contain cooling coils carrying water.

The use of water as a coolant is common at generating stations, particularly hydro-electric stations, where large supply of water is available.

Transformers with a capacity of upto 10 MVA have a cooling radiator system with natural cooling.

The forced oil-and air circulation (OFAF) method is the usual one for transformers of capacities 30 MVA upwards.

As stated earlier, the forced oil and water (OFWF) is used for transformers designed for hydro-electric plants.

7·19. Transformer tank. Tank bodies for most of the transformers are made from rolled steel plates which are fabricated to form the container. Small tanks are welded from steel plates while larger ones are assembled from boiler plates. The tanks are provided with lifting lugs. Small transformers have cooling tubes let into the vertical sides, but large transformers require separate banks of cooling tubes. Such transformers have plain tanks with provision for pipe and valves to direct and control the oil flow.

While designing tanks for transformers, a large number of factors have to be considered. These factors include keeping the weight, stray load losses and cost a minimum, and it is obvious that these are requirements contradictory.

The tanks should be strong enough to withstand stresses produced by jacking and lifting. The size of the tank must be large enough to accommodate cores, windings, internal connections and also must give the requisite clearance between the windings and the walls.

Aluminium is increasingly being used for transformer tanks as a means of reducing weight. The use of aluminium in place of steel reduces the stray magnetic fields (since it is a non-magnetic material) and consequently the stray load loss. However, aluminium tanks are costlier. Also the use of aluminium necessitates special lifting arrangements in order prevent stressing of tank. However, usually aluminium tanks are made of cast aluminium parts mounted on a shallow mild steel tray. The mild steel tray is arranged to carry the main lifting and jacking members.

Where mild steel tanks are used for units with high leakage flux, electromagnetic screens or shunts are used to reduce eddy current losses.

7·20. Cooling ducts. In large transformers, the cooling surface of the cores must be augmented otherwise temperature rise will be excessive owing to small surface/volume ratio of the cores. The additional surface is provided by cooling ducts. The cooling may be (*i*) horizontal or (*ii*) vertical. The vertical cooling ducts are along the direction of laminations and hence can be easily provided. The horizontal ducts are across the laminations and therefore require special punching of core. The oil flowing through these ducts takes away the heat.

In the case of vertical ducts, the heat is conducted across the laminations and since the thermal resistivity across the laminations is high, there is a large temperature gradient between the hot spot in the core and the duct. Hence, provision of vertical ducts does not improve the cooling of core. However, when horizontal ducts are provided, the heat is conducted along the laminations and since the thermal resistivity along the laminations is only 1/20 of that across the core, the temperature gradients are extremely small and therefore cooling is significantly improved.

The oil must have a free access to all parts of winding. The multilayer helical windings have an advantage in this respect since they are made coils of relatively small radial depth and, therefore, the majority of the coil surface is exposed to oil in the vertical cooling ducts.

The large disc type of windings have a large radial depth and they have a disadvantage that the majority of the coil is exposed to oil in the horizontal ducts. Where the vertical ducts on each side of the winding are of equal width, the oil flows up these ducts under the influence of pumps or natural convection and will not tend to enter the horizontal ducts

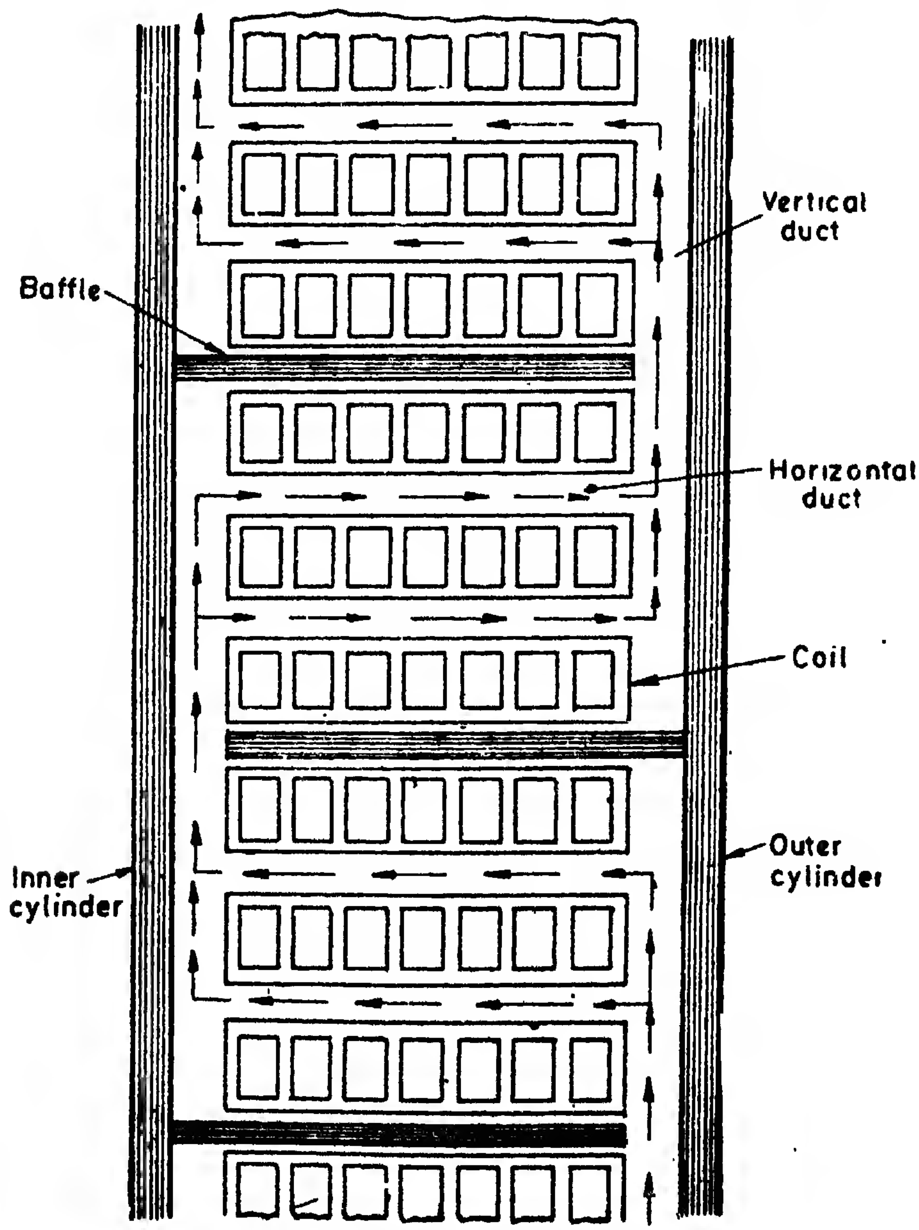


Fig. 7·43. Directed flow of oil in transformers.

particularly when the radial depth of the winding is large. Therefore, baffles are used to direct the rising stream of oil to flow in the horizontal ducts. This arrangement is shown in Fig. 7·43.

7·21. Transformer Oil. One of the most important factors which determine the life and satisfactory operation of a transformer is the oil in which it is immersed.

The transformer oil has two prime functions :

(*i*) To create an acceptable level of insulation in conjunction with insulated conductors and coils.

(*ii*) To provide a cooling medium capable of extracting quantities of heat without deterioration as an insulating medium.

Transformer oil is mineral oil (clean hydrocarbon oil) obtained by refining crude petroleum. Vegetable and animals oils are not used in transformers as they form fatty acids that attack the fibrous insulating materials used.

Some of the important characteristics necessary or desirable in transformer oil are described below :

1. Electric Strength. The transformer oil should have a high dielectric strength in order to minimize clearances between coils and from windings to tank. According to IS 335—1972 *"Specifications for new insulating oils for transformers and switchgear"*, the minimum electric strength of new oils should be 3 kV/mm (rms).

The resistivity of new transformer oils is more than $13 \times 10^{10}\ \Omega$m. However, if dust and small fibres are present in oil, they tend to align themselves along electric lines of force thereby forming paths of low resistivity which may cause electric failure.

2. Resistance to emulsion The oil should have a high resistance to emulsion in order to prevent holding water in suspension in it. This is because even a small trace of moisture severely reduces the electric strength of oil. The oil should not to be allowed to come in contact with moisture.

3. Viscosity. The viscosity of transformer oil should be small to permit rapid circulation of oil.

4. Purity. The oil must not contain any acid, alkali, and sulphur compounds as these cause corrosion of metal parts and insulation.

Sulphur compounds, if present, accelerate the production of sludge.

5. Flash Point. The flash point is the temperature at which oil vapour ignites spontaneously. The flash point of an oil characterises its tendency to evaporate. The lower the flash point the greater is the vaporization of oil. When an oil vapourises it loses in volume, its viscosity rises, and an explosive mixture may be formed. The flash point of transformer oil should be higher than 104°C.

6. Sludge formation. Sludging means the slow formation of solid hydrocarbons due to heating and oxidation. The sludge deposits itself on windings, tank walls and in cooling ducts. Sludge is a poor conductor of heat and therefore it produces temperature gradients across winding insulation causing overheating of conductors. The deposition of sludge in the oil duct does not allow the oil to circulate freely thereby impairing cooling. The resultant increased temperature produces more sludge. This process of sludge formation and consequent overheating will continue till the transformer becomes unserviceable. Therefore, contact of oil with air should be avoided in order to prevent sludge formation (since oxygen in air causes oxidation).

Transformer oil tends to deteriorate in service, but this tendency can be greatly reduced by paying attention to transformer operating conditions and to oil itself when this is shown to be necessary as the result of regular tests. The most important factors are :

(*i*) operating temperature, (*ii*) atmospheric conditions particularly inside stations, (*iii*) electric strength (*iv*) moisture, and other contamination, (*v*) sludge formation.

If the oil is found to contain moisture or suspended contaminants it should be filtered or if this treatment is considered to be inadequate, the oil should be replaced by fresh charge. Transformer oil is normally tested once every year and, if found below standard, may be treated by a centrifuge or filter unit.

7·22. Terminals and Leads. The connections to the windings are of insulated copper rods or bars. The shape and size of leads is important in high voltage transformers

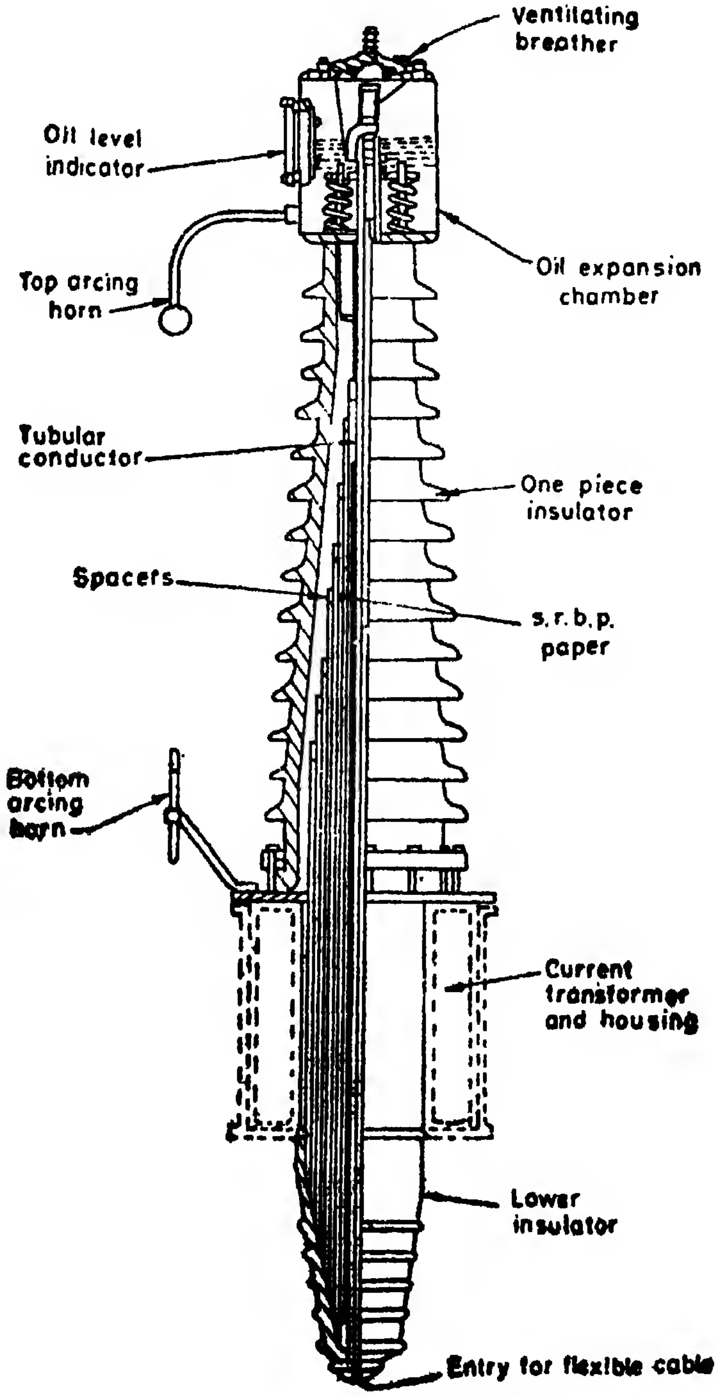


Fig. 7·44. High-voltage oil-filled bushing.

owing to dielectric stress and corona which are caused at bends and corners. Sharp edges and corners should be avoided

7·23 Bushings. Transformers are connected to high voltage lines, and, therefore, care must be taken to prevent flashover from the high voltage connection to the earthed bank. Connections for cables are made in cable boxes, but overhead connections must be brought through bushings specifically designed for various voltage classes.

The **bushing** consists of a current carrying part in the form of a conducting rod, bus or cable, a porcelain cylinder installed in a hole in the transformer cover and used for isolating the current carrying part.

The simplest bushing is a moulded, high quality glazed porcelain insulator with a conductor through its centre. This bushing is used for voltages upto 33 kV. The porcelain bushings used for indoor use have a smooth surface or slightly finned surface. The outside (upper part) of the bushing used for transformers working outdoors is made with petticoats to protect the lower fins against water in rainy weather.

The bushings used for transformers having voltages above 36 kV are either oil filled or capacitor type. The oil filled bushing (Fig. 7·44) consists of a hollow two part porcelain cylinder with a conductor, usually a cylinder, passing through its axis. The space between the conductor and the inner surface of the porcelain is filled with oil. The oil is contained separately from the oil in the transformer tank. The top of the bushing is connected to a small expansion chamber required to accommodate variations in the volume of the oil due to change in operating temperature. There is a provision for current transformer at the lower end of the bushing. The arrangement is such that the bushing can be removed without disturbing the current transformer. The capacitor bushing is made up of layers of synthetic resin bonded paper (s.r.b.p.) interleaved with thin layers of metal foil or paper impregnated with conducting material. The result is a series of capacitors with a capacitor formed by two layers of metal foil with s.r.b.p. cylinder inbetween. The variation in length of metal foils and the thickness of s.r.b.p. cylinders is so arranged that there is a uniform distribution of dielectric stress throughout the radial depth i.e. along the radius of the bushing.

7·24. Tappings and Tap changing. The voltage of power networks supplied by transformers can be controlled by changing the ratio of transformation of the transformers. The change in ratio of transformation can be affected by providing tappings on the transformer windings. The **tappings** are connections provided at different places in the windings and therefore, the number of turns included in the circuit at one tap is different from the number of turns at another tap. Hence, the turns ratio is different at different tappings and as different voltages are obtained at different tappings.

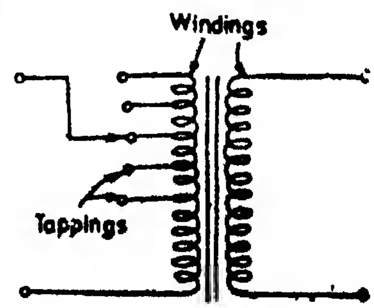


Fig. 7·45. Transformer tappings,

The tappings used in a transformer are shown in Fig. 7·45.

Consider a 3 phase, 11 kV/0.4 kV, distribution transformer. There is always a tapping on the h.v. winding which when connected to rated voltage (11 kV) gives rated voltage (400 volt in this case) on the l.v. side. This tapping is called the **principal tapping**. The principal tapping is that to which the rating of the winding is related.

The number of turns included at other taps may be either greater or lesser than the number of turns used at principal tapping. The tapping at which the turns included are more than that of the principal tapping is called positive tapping, while a tapping at which the number of turns included is less than the number of turns included at prinicipal tapping is called negative tapping.

The tappings are provided on the high voltage (h.v.) winding because of the following reasons :

(*i*) the number of the turns in h.v. winding is large and therefore a fine voltage regulation is obtained.

(*ii*) it may not be possible to provide correct voltage regulation by using tappings on the l.v. side because of the smaller number of turns. Suppose, it is desired to obtain $\pm 2\frac{1}{2}\%$ voltage regulation in a 3 phase, 6·6 kV/400V, delta/star connected transformer designed for 11.55V/turn. The voltage per phase on the l.v. side is $400/\sqrt{3}=231$V. The number of turns are, 231/11.55=20. It is possible to tap a whole number of turns. The minimum number of turns that is possible to tap is 1.Therefore, the minimum voltage regulation possible in this transformer by providing tappings on the l.v. side is $\pm 11{\cdot}55$V or $\pm 5\%$ and hence it is not possible to obtain a voltage regulation of $\pm 2\frac{1}{2}\%$ with taps on the secondary side.

(*iii*) The current on the l.v. side of high capacity transmission and generation transformers is very high. Therefore, provision of tappings on the l.v. windings for these transformers is impracticable on account of the difficulties encountered in interruption of high currents.

(*iv*) The l.v. winding is placed on the inside nearer to the cores while the h.v. winding is placed on the outside. Therefore, on account of the practical considerations, it is simpler to provide tappings on the high voltage winding.

(*v*) There is an additional advantage of providing tappings on high voltage winding of step down transformers. The voltage on low voltage side of these transformers increases on light loads. Therefore, in order to decrease this voltage, the tapping on the high voltage side is adjusted to such a position where the number of turns is large. The large number of turns decreases the flux and the flux density. This reduces the core loss which in turn increases the efficiency of the transformers at low loads.

The voltage control in electric supply networks is required on account of many reasons. These include :

(*i*) adjustment of voltage at consumers' premises within statutory limits.
(*ii*) control of active and reactive power, and
(*iii*) adjustments of short period (1—2%), daily (3—5%) and seasonal (5—10%) voltage variations in accordance with variations of load.

Location of tappings is partly a constructional question. The tappings can be provided at the phase ends, at the neutral point, or in the middle of the windings. The advantage of providing tappings at phase ends is that the number of bushing insulators is reduced. This is important, where the transformer cover space is limited. Some transformers have reinforced insulation at the phase ends, it is essential that in such cases either the tappings should not be provided or the re-inforcement should be carried beyond the lowest tap. When the tappings are made at the neutral point the insulation between various parts is small. This arrangement is economical especially in the case of high voltage transformers. When a large voltage variation is required, tappings should be near the centres of phase windings to reduce magnetic assymmetry. However, this arrangement cannot be used on l.v. windings placed next to the core.

The different methods of providing tappings on a transformer are shown in Fig. 7.46.

The tappings are potential source of axial magnetic assymmetry in a transformer. This is on account of the fact that the number of turns in the winding that is provided with tappings is altered while the number of turns in the other winding remain the same. Therefore, it is clear that cutting in or cutting out a part of the transformer winding would cause a mmf unbalance, thereby producing a magnetic assymmetry in the axial direction. This magnetic assymmetry produces large mechanical forces and consequent displacement of windings in the axial direction in case of faults.

The tappings are on one end of the windings as shown in Fig. 7·46 (*a*) in the case of small transformers while they are arranged at the centre of the windings of larger transformers as shown in (*b*) and (*c*). The axial mmf unbalance can be significantly reduced by thinning a

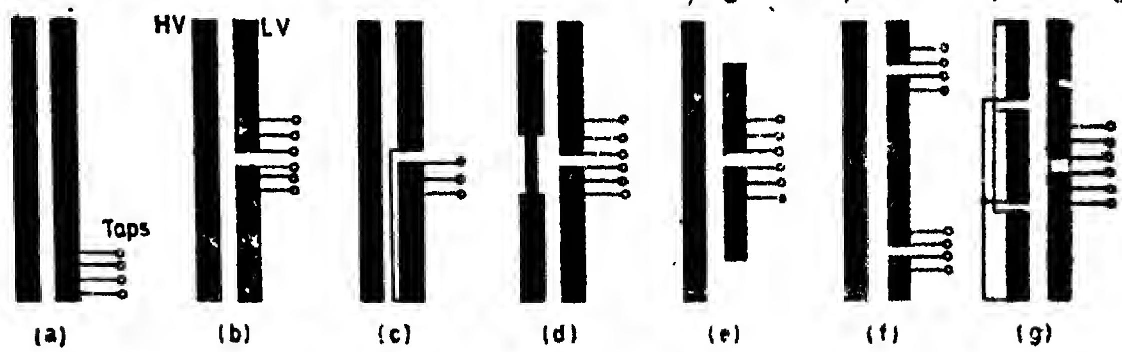


Fig. 7·46. Position of tappings.

part of the l.v. winding which is radically opposite to the part of the h.v. where the tappings are provided. This is shown in (*d*). The axial mmf unbalance is also reduced by adopting winding arrangements where parts of the winding are balanced more symmetrically as shown in (*d*) and (*e*).

The axial mmf balance may also be achevied by splitting the untapped winding into a number of parts and then connecting these parts in parallel This arrangement is shown in (*g*).

The tappings may be changed when the transformer is disconnected from the supply. This is called **off-circuit tap changing**. The off-circuit tap changing is used for occasional adjustments, as in distribution transformers which are provided with $\pm 5\%$ and $\pm 2\frac{1}{2}\%$ taps. The tappings may also be changed while the transformer is energized or on load. This is known as **on load tap changing**. Daily and short time voltage adjustments are done with the help of on-load tap-changing gear.

The off-circuit and on load tap changing arrangements are described below.

Off circuit tap changing. As the name suggests, the tappings are changed by disconnecting the transformer from the supply. This adjustment is carried out by tapping the respective windings as required and bringing the connections of tappings to some position near the top of the transformer. The change of tappings is done manually through hand holes provided in the cover. Another arrangement employs reconnection that can be made by carrying the tapping leads through the cover for changing either by hand or by manually operated switches. The commonly used switches are : (*i*) vertical tapping switches and (*ii*) faceplate switches.

One form of selector switch used for off-load tap changers is shown in Fig. 7.47. This arrangement is commonly used for providing $\pm 5\%$ tappings in steps of $\pm 2\frac{1}{2}\%$. Six brass or copper terminals are mounted on an insulating base and a contactor is mounted on an arm attached to the shaft. Taps are brought out of middle part of the winding and connected to the terminals of tap changer. The shaft is turned from one position to the next, the contactor connects adjacent pair of the stationary terminals.

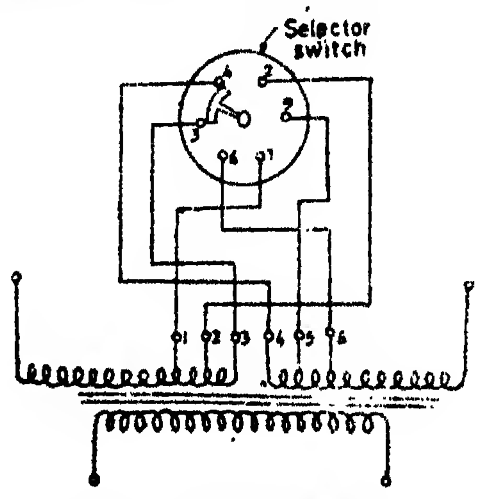


Fig. 7.47. Off-load tap changer.

With the contactor in the position shown, the selector switch connects taps 3 and 4 and therefore all the turns in both parts of the winding are in use. If the contactor is moved one point to the right, it makes a connection between taps 2 and 4, thus cutting out part of the winding between taps 2 and 3. The next step connects taps 2 and 5 and cuts part of the winding between taps 4 and 5. These parts of the windings are cut out in steps, until the

final position which connects taps 1 and 6 This position leaves minimum number of turns in the winding.

There are five positions of the switch. These positions correspond to five different voltages. Supposing it is desired to obtain ±5% regulation in steps of ±2½%. The switch which corresponds to normal voltage is on taps 2 and 5. Assuming that the winding which is tapped is the primary winding, the regulation at other switch positions is·

Taps	4, 2	[illegible]	5, 1	1, 6
Regulation	−2½%	−5%	+2½%	+5%

2. On-load tap changing. When a transformer is connected to a system it is necessary that arrangements be provided to vary the voltage on the secondary side in order to maintain normal voltage under load conditions. In the case of on-load tap changing, this variation is made when the load is on, and hence the tap changing gear must be capable of changing the turns ratio without interruption of supply. It is invariable practice to connect the tapings at the neutral end of high voltage windings of a generator transformer.

The tappings on the windings are brought out through a terminal board to separate oil filled compartment in which the on-load tap-changer switch is housed. The tap changer is in the form of a selector switch. The tap changer is operated by a motor operated driving mechanism by local or remote control and a handle is fitted for manual operation in case of an emergency.

The essential feature of an on-load tap-changing gear is the maintenance of circuit continuity throughout the tap changing operation. The circuit must not be broken otherwise there will be discontinuity of supply to load. Therefore, as the sector switch must not break current, a additional separate oil filled compartment is used to house the diverter switch which breaks the load current by an interrupted arc. This causes formation of carbon and therefore the oil in the diverter switch compartment must be prevented from mixing with the oil in the main tank. The oil in the selector switch tank may be connected directly to main transformer oil through the conservator.

As mentioned earlier, the tap changing takes place when the load is on and hence in order to maintain continuity of supply to the load, before one tapping is left opened, contact must of made to the next tapping. Therefore, the selectors witch is on-load tap changers is a *make before a break switch* and during the period of transition from one tap to another, momentary connection must be made between the adjacent taps. This results in short circuiting of turns between the adjacent tappings. Therefore, the short circuit current must be limited by including resistors or reactors. Reactors have been used for this purpose earlier, but in modern equipment it is usual to use two resistors for this purpose.

Fig. 7·48 shows a typical winding connection for a high speed resistor type on-load tap changer provided at the neutral end of each phase of windings of a star connected 3 phase transformer. One selector switch, S_1 is on tap 1 and the other S_2 on tap 2. The diverter switch, S_3, is shown connecting tap 1 to the neutral point of the transformer winding and the switching sequence for change over to tap 2 is as follows :

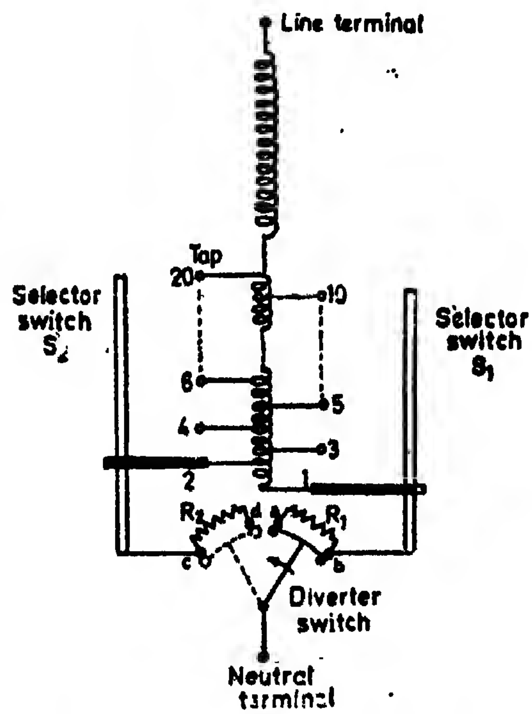


Fig. 7·48. On-load neutral terminal tap changer.

1. Contacts a and b are closed (resistance R_1 shorted) as shown. The load current flows from tap through contact b. This is the running position at tap 1.

2. An external mechanism moves the diverter switch S_3, contact b opens. The load current from tap 1 now flows through resistance R_1 and contact a.

3. As the moving contact of S_3 continues its travel further to the left, contact d closes and resistance R_1 is open circuited. Both the resistances R_1 and R_2 are connected across taps 1 and 2 and the load current flows through these resistances to their mid point (junction of a and d).

4. When S_3 moves still further to the left, contact a is opened. The load, current flows from tap 2 through resistance R_2 and contact d.

5. Finally, as the diverter switch S_3 reaches the extreme left position, contact c closes and resistance R_2 is short circuited. The load current from tap 2 flows through contact a. This is the running position for tap 2.

It is observed that tap change from tap 1 to tap 2 as described above does not involve any movement of selector switches S_1 and S_2.

However, a further tap change in the same direction, *i.e.* from tap 2 to tap 3, is required, the selector switch S_1 is moved to tap 3 before the diverter switch, S_3 moves. The diverter switch then follows the sequence described above but in the reverse order.

In order to limit the loss of energy, it is essential that the resistors be kept in circuit for as minimum of time as possible. The resistors are designed for short time rating for economical considerations and therefore, it is desirable to minimize their time of duty. Therefore, some form of energy storage must be incorporated in the driving mechanism to ensure that tap change, once initiated, shall be completed even in case of failure of auxiliary control supply. All modern on load tap changers use springs energy as storage elements. They reduce the time that a resistor is in the circuit to a few periods.

Such a tap changer is compact in size, and high speed breaking reduces the contact wear. The current breaking is eased by the fact that the short circuit resistor current has unity power factor.

7·24. Conservator and breather. The satisfactory operation of transformers depends so largely on the condition of oil and therefore devices and methods for keeping the oil clean and dry are of prime importance.

The oil level of a transformer changes with changes the temperature rise of oil which in turn depends upon the load on the transformer. The oil expands if the load increases and contracts when it decreases. Therefore, provision must be made to take up this expansion and contraction of oil.

Smaller transformers are not totally filled with oil and some space is left between oil level and the tank cover. This space is taken by air. The tank is connected to the atmosphere through a vent pipe. When the oil expands, air is expelled out while if it contracts air is drawn in from the atmosphere. This is called *breathing* of transformer. The air entering the transformer is passed through an apparatus called **breather** for the purposes of extracting moisture from it. A breather consists of a small container connected to the vent pipe and contains a dehydrating material like silica gel crystals impregnated with cobalt chloride. The material is blue when dry and a whitish pink when damp. The colour can be observed through a glass window provided in front of the container.

Breathers alone are not sufficient for protection of large and important power transformers because :

(*i*) these transformers are liable to overloads which may overheat the oil and consequently there is sludge formation if air is present.

(ii) Occasionally such transformers also suffer short circuits and temperature rise becomes very high. This causes vapourization of a part of the oil. The oil vapours form explosive mixture with air which ignites and can cause considerable damage.

For these reasons oil is prevented from having contact with air as well as moisture. *Conservators* are used for this purpose. The function of the conservator is to take up expansion and contraction of the oil without allowing it to come in contact with ambient air, from which it might absorb moisture.

The conservator is an airtight cylindrical drum mounted on or near the cover of the transformer and connected to it through a small pipe as shown in Fig. 7·49. The oil is set so that the transformer tank is entirly full with oil and the conservator is about half full. The interior of the conservator above the oil level is connected to the atmosphere through a breather having dehydrating material.

With the use of conservators, interchange of oil between conservator and main tank as a result of temperature changes is slow. Also dry is in contact with much smaller surface of relatively cool oil. Hence the sludge formation is considerably reduced and whatever sludge is formed remains in the conservator there being no sludge formation in the main tank. This is a great improvement over the ordinary tank with air space above the oil.

7·25. Explosion Vent. In order to guard against the possibility of a sudden high pressure caused by a breakdown or a short circuit in the transformer winding, a diaphragm relief device is used. This device consists of a large opening to the atmosphere covered by a thin non-metallic diaphragm. The diaphragm bursts if the pressure inside the tank becomes excessive. The relief device must be above the level of oil in the conservator in order to prevent an overflow of oil in case the device operates.

7·26. Temperature Indicators. The most obvious indicator of transformer temperature is the temperature of the hot oil. The oil temperature is measured by a dial type thermometer. The bulb of the thermometer is mounted in the oil and the dial is mounted outside the tank.

However, oil temperature is not a reliable measure of the winding temperature especially under sudden over-loads which cause the winding temperature to rise more rapidly than the oil temperature. Therefore, it is desirable to use an indicator which will show the actual temperature of hot spot in the windings.

Winding temperature indicator is a thermometer with a bulb. The thermometer is immersed in oil and the bulb is heated by heaters which carry a current proportional to the winding current. Therefore, the reading of the thermometer is an analogue indication of winding temperature.

7·27. Buchholz Relay. It is a gas and oil actuated protective device and is used practically in all oil immersed transformers with the exception of smaller distribution transformers. *Buchholz relay* is used for protection of transformer against faults developed in side the transformer. The device relies on the fact that an electrical fault in side the transformer tank is accompanied by generation of gas, and if the fault current is high enough by a surge of oil from the tank to the conservator.

The Buchholz relay is particularly useful in that it is capable of detecting fault conditions of very low magnitude such as interturn faults, incipient winding faults, and core faults due, for example, to core bolt insulation failure which gives rise to short circuit and subsequent arcing and gas.

The use of a Buchholz relay is possible only with transformers having conservators and the relay is placed between transformer tank and the conservator as shown in Fig. 7·49 (*a*).

A Buchholz relay consists of an oil filled chamber as shown in Fig 7 49 (b). It contains two floats, the top float F_1 and the bottom float F_2. Both the floats are hinged so as to be pressed by their buoyancy against two stops. When these floats sink, each of them short circuits two contacts thereby closing a circuit which gives a warning about the fault.

The operation of the Buchholz relay can be explained as under :

In case a fault occurs, gas bubbles are generated in the transformer tank on account of increased heating produced by faults currents. These bubbles rise and go towards the conservator through the pipe line. The gas bubbles are trapped in the upper part of the relay chamber and since the chamber is full of oil, the oil is displaced which lowers the top float F_1. The float sinks ultimately thereby causing the closure of contacts of the "*alarm circuit*".

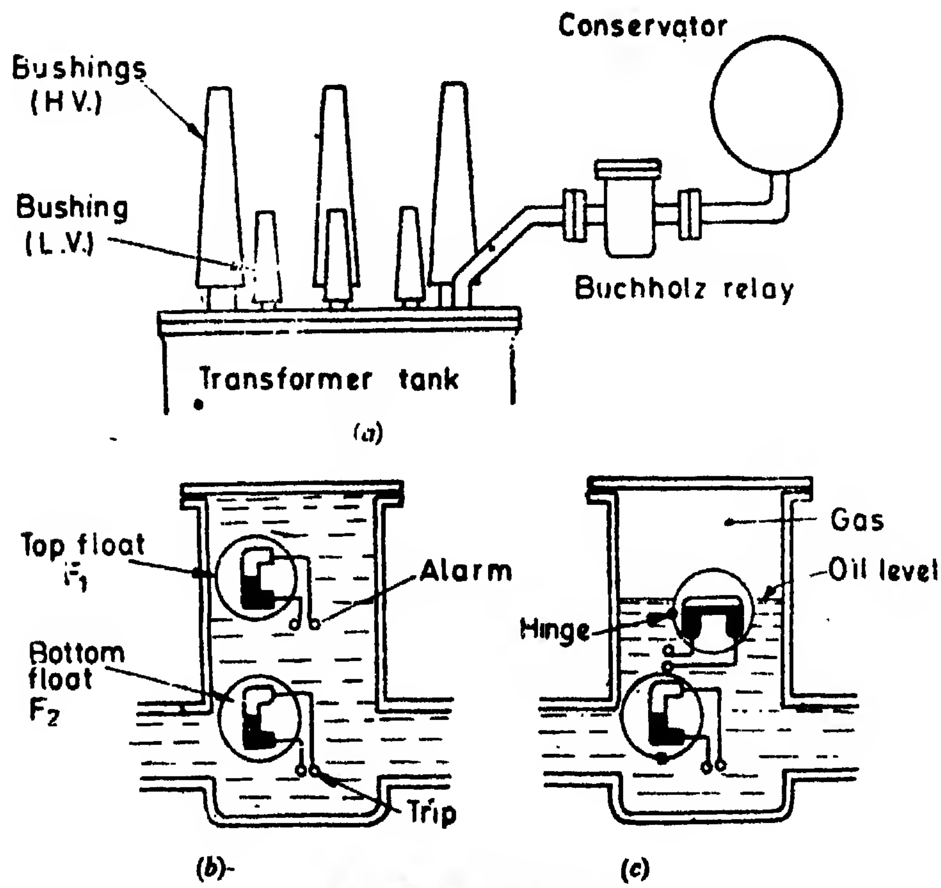


Fig. 7·49. Buchholz relay

A small window in the wall of the relay chamber shows the amount of gas trapped and its colour. A sample of the gas may be withdrawn and analysed. The amount of gas is indicative of the severity of the fault while the colour of the gas indicates the nature of the fault since the faults occurring in cores and windings produce gases of different colour.

In case the fault is not severe, the generation of gas is not sufficient so as to lower the level of oil to the extent to affect the bottom float F_2. Therefore, in case of mild faults inside the transformer, the bottom float F_2 remains unaffected.

However, in the case of severe faults, such as short circuits between phases or to earth, and faults in the tap changing gear, the gas production becomes violent and auses a surge which cause the bottom float F_2 to fall and close the contacts of a "*trip circuit*". This trip circuit energizes the relay of a circuit breaker which open circuits the transformer thereby clearing the fault.

7·28. Transformer Assembly. A complete assembly of a transformer with details of core, windings, tank, connections and major accessories is shown in Fig. 7·50.

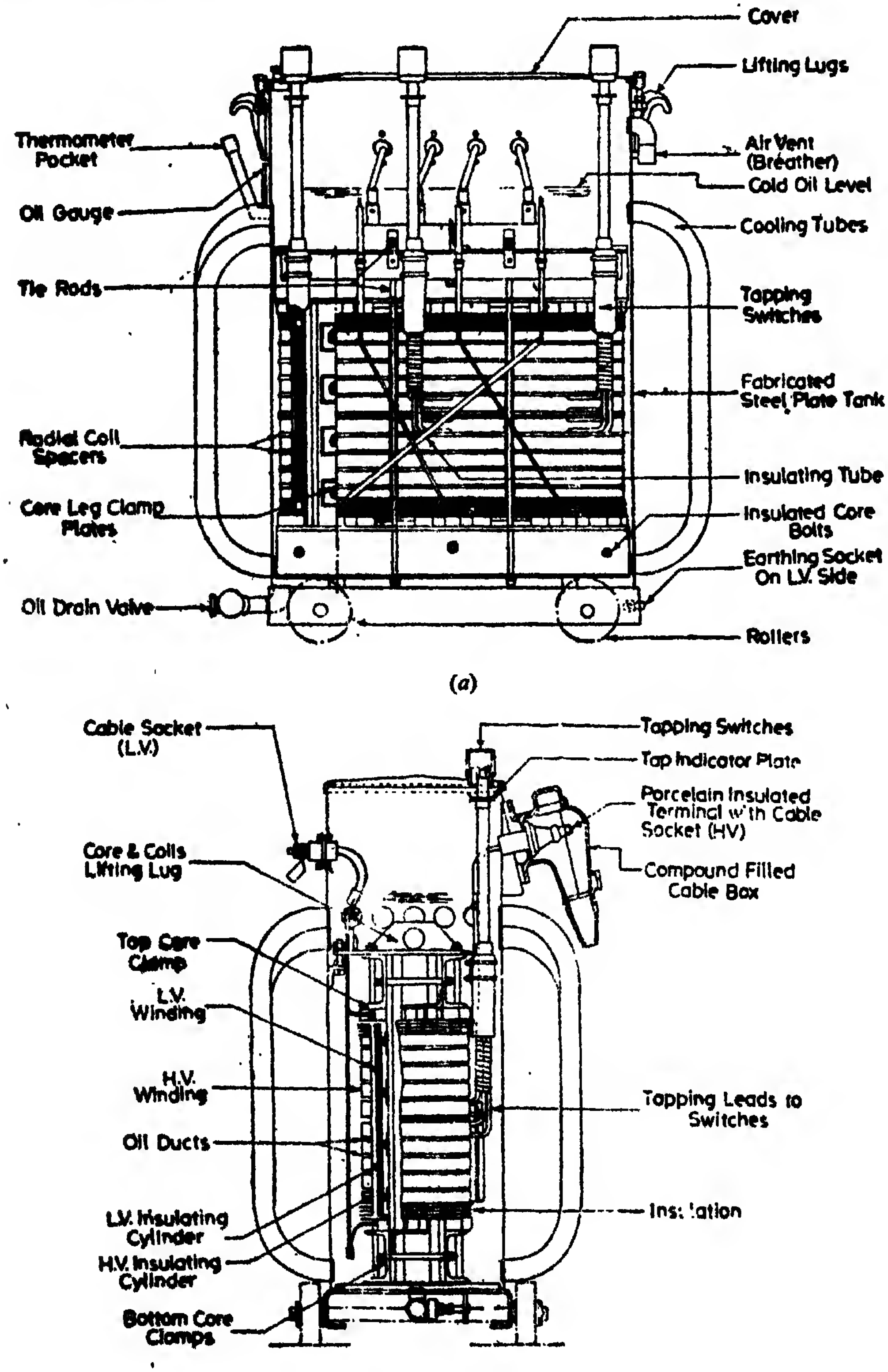


Fig. 7·50. Transformer Assembly

DESIGN

7·29. Output of Transformer. Let

Φ_m=main flux, Wb ; B_m=maximum flux density, Wb/m^2 ;
δ=current density A/m^2 ; A_{gi}=gross core area, m^2 ;
A_i=net core area, m^2=stacking factor×gross core area ;
A_c=area of copper in the window, m^2 ; A_w=window area, m^2 ;
D=distance between core centres, m ;
d=diameter of circumscribing circle, m ;
K_w=window space factor ; f=frequency, Hz, E_t=emf per turn, V ;
T_p, T_s=number o turns in primary and secondary windings respectively ;
I_p, I_s=current in primary and secondary windings respectively, A ;
V_p, V_s=terminal voltage of primary and secondary windings respectively, V ;
a_p, a_s=area of conductors of primary and secondary windings respectively, m^2 ;
l_i=mean length of flux path in iron, m ;
L_{mt}=length of mean turn of transformer windings, m ;
G_i=weight of active iron, kg ; G_c=weight of copper, kg ;
g_i=weight per m^3 of iron, kg ; g_c=weight per m^3 of copper, kg ;
p_i=loss in iron per kg, W ; p_c=loss in copper per kg, W.

(*i*) **Single Phase Transformers.** The voltage induced in a transformer winding with T turns and excited by a source having a frequency f Hz is given by :

Voltage per turn $E_t=\dfrac{E}{T}=4{\cdot}44\, f.\Phi_m.$ ...(7·1)

The window in a single phase transformer contains one primary and one secondary winding.

∴ Total copper area in window

A_c=copper area of primary winding+copper area of secondary winding,
=primary turns×area of primary conductor+secondary turns×area of secondary conductor
$=T_p\, a_p+T_s\, a_s.$

Taking the current density δ to be the same in both primary and secondary windings.

$a_p=I_p/\delta$ and $a_s=I_s/\delta$

∴ Total conductor area in window $A_c=T_p.\, I_p/\delta+T_s.\, I_s/\delta=(T_pI_p+T_sI_s)/\delta$

$$=\frac{2AT}{\delta} \quad ...(7{\cdot}2)$$

as $T_pI_p=T_sI_s=AT$ if we neglect magnetising mmf.

The window space factor K_w is defined as the ratio of copper area in window to total area of window,

or $$K_w=\frac{\text{conductor area in window}}{\text{total area of window}}=\frac{A_c}{A_w}$$

∴ Conductor area in window $A_c=K_wA_w$ ...(7·3)

From Eqn. 7·2 and 7·3, $2AT/\delta=K_wA_w$

or $$AT=\frac{K_wA_w\,\delta}{2}. \quad ...(7{\cdot}4)$$

Rating of a single phase transformer in kVA

$$Q=V_pI_p \cdot 10^{-3}=E_pI_p \cdot 10^{-3} \qquad \text{(as } V_p \text{ is approximately equal to } E_p\text{)}$$

$$=E_tT_pI_p \cdot 10^{-3}=E_t\,AT\,10^{-3}$$

$$=E_t\,\frac{K_wA_w\,\delta}{2}\cdot 10^{-3}=4{\cdot}44\,f\Phi_m\frac{K_wA_w\,\delta}{2}\cdot 10^{-3}$$

$$=2{\cdot}22\,f\,\Phi_m\,K_wA_w\,\delta \cdot 10^{-3} \qquad \text{...(7·5)}$$

But Φ_m = maximum flux density × net area of core $=B_mA_i$

$$\therefore \qquad Q=2{\cdot}22\,f\,B_m\,\delta\,K_wA_w\,A_i\times 10^{-3}\ \text{kVA} \qquad \text{...(7·6)}$$

(ii) **Three Phase Transformers.** In the case of three phase transformers, each window contains two primary and two secondary windings.

Proceeding as in the case of single phase transformers.

Total conductor area in each window $A_c=2(a_pT_p+a_sT_s)$

$$=2\,(I_pT_p/\delta+I_sT_s/\delta)=2(I_pT_p+I_sT_s)/\delta$$

$$=\frac{4\,AT}{\delta} \qquad \text{...(7·7)}$$

Total conductor area is also equal to K_wA_w. $\quad \therefore \quad 4AT/\delta=K_wA_w$

or
$$AT=\frac{K_wA_w\,\delta}{4} \qquad \text{...(7·8)}$$

Rating of a three phase transformer in kVA

$$Q=3\,V_pI_p\,10^{-3}=3\,E_pI_p\,10^{-3}=3\,E_tT_pI_p=3\,E_t\,AT$$

$$=3\times 4{\cdot}44\,f\,\Phi_m\times\frac{K_wA_w\,\delta}{4}\times 10^{-3}=3{\cdot}33\,f\,\Phi_m\,K_wA_w\,\delta\times 10^{-3} \qquad \text{...(7·9)}$$

$$=3{\cdot}33\,f\,B_m\,\delta K_wA_wA_i\times 10^{-3} \qquad \text{...(7·10)}$$

7·38. **Output Equation—Volt per turn.** Considering the output of one phase.

kVA rating of one phase

$$Q=I_pV_p \cdot 10^{-3}=I_p\times 4{\cdot}44\,f\,\Phi_m\,T_p \cdot 10^{-3}=4{\cdot}44\,f\,\Phi_m\,AT \cdot 10^{-3} \qquad \text{...(7·11)}$$

The ratio Φ_m/AT is a constant for a transformer of a given type, service and method of construction. Let $\Phi_m/AT=r$ where r is a constant.

From Eqn. 7·11

$$Q=4{\cdot}44\,\Phi_m\,f\,AT\,10^{-3}=4{\cdot}44\,\Phi_m f\frac{\Phi_m}{r}\cdot 10^{-3}$$

$$=4{\cdot}44\,\Phi_m^2\frac{f}{r}\cdot 10^{-3} \quad \text{or} \quad \Phi_m=\sqrt{\frac{r \cdot 10^3}{4{\cdot}44f}}\cdot\sqrt{Q}$$

Voltage per turn $E_t=4{\cdot}44\,f\,\Phi_m=4{\cdot}44\,f\left(\dfrac{r \cdot 10^3}{4{\cdot}44\,f}\cdot Q\right)^{1/2}$

$$=\sqrt{4{\cdot}44\,f\,r \cdot 10^3}\cdot\sqrt{Q}=K\sqrt{Q} \qquad \text{...(7·12)}$$

where
$$K=\sqrt{4{\cdot}44\,fr \cdot 10^3}=\left(4{\cdot}44\,f\frac{\Phi_m}{AT}\times 10^3\right)^{1/2} \qquad \text{...(7·13)}$$

As the ratio Φ_m/AT depends upon type of transformer and therefore K is also a constant whose value depends upon type, service conditions and method of construction.

Table 7·2 gives values of constant K for different types of transformers.

Table 7·2

Type	K
Sin le phase shell type	1 0 to 1·2
Single phase core type	0·75 to 0·85
Three phase shell type	1·3
Three phase core type (distribution)	0·45
Three phase core type (power)	0·6 to 0·7

7·31. Ratio of iron loss to copper loss. Copper loss per $m^3 = \rho\delta^2$ ...(7·14)

Taking resistivity of copper as $2·1\times10^{-8}$ Ωm at 75° and its density as $8·9\times10^3$ kg/m³.

Copper loss per kg at 75°C *i.e.* specific copper loss

$$p_c = \frac{2·1\times10^{-8}}{8·9\times10^3}\delta^2 = 2·36\times10^{-12}\,\delta^2 \text{ W/kg} \quad ...(7·15)$$

where δ = current density, A/m².

In addition to above we must take into consideration the stray load loss which may be 5 to 25 per cent of copper lo s. Total copper loss $W_c = p_c G_c$.

Total iron loss per kg *i.e.* specific iron loss p_i can be found from the loss curves. Total iron loss $W_i = p_i G_i$.

Ratio of iron loss to copper loss

$$\frac{P_i}{P_c} = \frac{p_i G_i}{p_c G_c} \quad ...(7·16)$$

When the densities in the iron and copper are fixed, the loss per kg for iron and copper can readily be determined. The ratio of weights for a given ratio of losses can easily be calculated from Eqn. 7·16. The ratio of weight of iron to weight of copper generally lies between 1·5 to 3·0 for distribution transformers. For small capacity, single phase core type transformers, the ratio of weights is often less than 1·5. For high voltage power transformers, it may be twice the values given above.

7·32. Relation between core area and weight of iron and copper. From Eqn. 7·6, kVA output of a single phase transformer

$$Q = 2·22\, f B_m \delta K_w A_w A_i \times 10^{-3}$$

Now weight of iron $G_i = A_i\, l_i\, g_i$ and weight of copper $G_c = 2\, a_p\, T_p\, L_{mt}\, g_c$ if the weights of the primary and the secondary windings are taken to be equal.

$$\text{Ratio} \qquad \frac{G_i}{G_c} = \frac{A_i\, l_i\, g_i}{2\, a_p T_p L_{mt}\, g_c} = \frac{A_i\, l_i\, g_i}{K_w A_w L_{mt}\, g_c} \quad \text{as} \quad K_w A_w = 2\, a_p T_p.$$

If the ratio of mean length of the magnetic circuit to the mean length of turn of the winding is assumed to be constant, which is approximately true for a given type of transformer, then

$$\frac{G_i}{G_c} = C_1 \frac{A_i}{K_w A_w} \quad \text{and} \therefore \quad K_w A_w = C_1 \frac{G_c A_i}{G_i} \quad \text{where } C_1 = \frac{l_i\, g_i}{L_{mt}\, g_c}.$$

Substituting the value of $K_w A_w$ in Eqn. 7·6,

$$Q=2{\cdot}22\, fB_m\delta\, C_1 \frac{G_c}{G_i} A_i^2 \times 10^{-3}$$

or area of core $A_i=\sqrt{\dfrac{Q}{fB_m}\cdot\dfrac{G_i}{G_c}}$ ...(7·17)

where $C=\left(\dfrac{10^3}{2{\cdot}22\, C_1}\right)^{1/2}=\left(\dfrac{1}{2{\cdot}22}\;\dfrac{L_{mt}}{l_i}\cdot\dfrac{g_c}{g_i}\times 10^3\right)^{1/2}$ ...(7·18)

Taking $g_i=7{\cdot}8\times10^3$ kg/m³ and $g_c=8{\cdot}9\times10^3$ kg/m³

$\therefore$ $C=22{\cdot}67\sqrt{L_{mt}/l_i}$ ...(7·19)

For three phase transformers

$$C=\frac{1}{\sqrt{3}}\times\left(\frac{1}{2{\cdot}22}\cdot\frac{L_{mt}}{l_i}\cdot\frac{g_c}{g_i}\times10^3\right)^{1/2}$$

$=\left(\dfrac{1}{6{\cdot}66}\;\dfrac{L_{mt}}{l_i}\;\dfrac{g_c}{g_i}\times10^3\right)^{1/2}$ ...(7·20)

$=13{\cdot}1\sqrt{L_{mt}/l_i}$ ...(7·21)

Typical values of ratio L_{mt}/l_i for different types of transformers are :

Single phase core type—0·3 to 0·55

Three phase core type—0·17 to 0·5

Single phase shell type—1·2 to 2.

7·33. Optimum designs. Transformers may be designed to make one of the following quantities as minimum.

(*i*) Total volume (*ii*) total weight, (*iii*) total cost, (*iv*) total losses. In general, these requirements are contradictory and it is normally possible to satisfy only one of them. All these quantities vary with ratio $r=\Phi_m/AT$. If we choose a high value of r, the flux becomes larger and consequently a large core cross section is needed which results in higher volume, weight, and cost of iron and also gives a higher iron loss. On the other hand owing to decrease in the value of AT the volume, weight and cost of copper required decreases and also the I^2R losses decrease Thus we conclude that the value of r is a controlling factor for the above mentioned quantities.

7·33·1. Design for minimum cost. Let us consider a single phase transformer. Its kVA output is :

$$Q=2{\cdot}22\, f\, B_m\delta K_w A_w A_i\times10^{-3}=2{\cdot}22\, fB_m\delta A_c A_i\times10^{-3}.$$

Assuming that the flux and current densities are constant, we see that for a transformer of given rating the product $A_c A_i$ is constant. Let this product $A_c A_i=M^2$ ...(*i*)

The optimum design problem is, therefore, that of determining the minimum value of total cost.

Now, $r=\Phi_m/AT$ and $\Phi_m=B_m A_i$ and $AT=\delta K_w A_w/2=\delta A_c/2$

$\therefore$ $r=\dfrac{2B_m A_i}{\delta A_c}$ or $\dfrac{A_i}{A_c}=\dfrac{\delta}{2B_m}r=\beta$ ...(*ii*)

where β is a function of r only as B_m and δ are constant.

Thus from (*i*) and (*ii*) we have

$A_i=M\sqrt{\beta}$ and $A_c=M/\sqrt{\beta}$

Let C_t=total cost of transformer active materials,

C_i=total cost of iron, and C_c=total cost of conductor

$\therefore$ $C_t=C_i+C_c=c_i G_i+c_c G_c$.

$$= c_i g_i l_i A_i + c_c g_c L_{mt} A_c.$$

where c_i and c_c are the specific costs of iron and copper respectively.

Now, $C_t = c_i g_i l_i M\sqrt{\beta} + c_c g_c L_{mt} M/\sqrt{\beta}$

Differentiating C_t with respect to β,

$$\frac{dC_t}{d\beta} = \frac{1}{2} c_i g_i l_i M(\beta)^{-1/2} - \frac{1}{2} c_c g_c L_{mt} M\beta^{-3/2}$$

For minimum cost $\frac{dC_t}{d\beta} = 0$

$\therefore \quad c_i g_i l_i = c_c g_c L_{mt} \beta^{-1}$ or $c_i g_i l_i = c_c g_c L_{mt} \frac{A_c}{A_i}$

or $\quad c_i g_i l_i A_i = c_c g_c L_{mt} A_c$ or $c_i G_i = c_c G_c$

or $\quad C_i = C_c$

Hence, for minimum total cost, the cost of iron must equal the cost of conductor.

Now $\quad G_i/G_c = c_c/c_i$ for minimum cost.

Knowing the value of specific costs of iron and conductor the ratio of weight of iron to conductor can be determined. This can be substituted in Eqn. 7·17 to determine the core area which gives minimum cost for the transformer.

Similar conditions apply to other quantities *e.g.*,

For minimum volume of transformer : Volume of iron = volume of conductor

$\therefore \quad G_i/g_i = G_c/g_c$ or $G_i/G_c = g_i/g_c$

For minimum weight of transformer

weight of iron = weight of conductor or $G_i = G_c$.

For minimum losses in transformer *i.e.*, for maximum efficiency,

iron loss $= I^2R$ loss in conductor or $P_i = x^2 P_c$

7·33·2. Design for minimum loss or maximum efficiency.

Total losses at full load $= P_i + P_c$

At any fraction x of full load, the total losses are $P_i + x^2 P_c$

If Q is the output at full load, the output at fraction x of full load is xQ.

$\therefore$ Efficiency at output xQ, $\eta_x = \dfrac{xQ}{xQ + P_i + x^2 P_c}$

This efficiency is maximum when $\dfrac{d\eta_x}{dx} = 0$

Differentiating η_x we have $\dfrac{d\eta_x}{dx} = \dfrac{(xQ + P_i + x^2 P_c)Q - xQ\,(Q + 2xP_c)}{(xQ + P_i + x^2 P_c)^2}$.

For maximum efficiency, $(xQ + P_i + x^2 P_c)\, Q - xQ\,(Q + 2xP_c) = 0$

or $\quad P_i = x^2 P_c$

So that the maximum efficiency is obtained when the variable losses are equal to the constant losses.

From Eqn. 7·16, we have :

$$\frac{P_i}{P_c} = \frac{p_i G_i}{p_c G_c}$$

$\therefore \quad x^2 = \dfrac{p_i G_i}{p_c G_c}$ or $\dfrac{G_i}{G_c} = x^2 \dfrac{p_c}{p_i}$ for maximum efficiency.

Now knowing the values of densities in iron and copper the specific losses p_i and p_c can be determined and the value of x i.e., the fraction of full load where the maximum efficiency occurs depends upon the service conditions of the transformer and is, therefore, known. Thus ratio G_i/G_c is known and its value is put in Eqn. 7·17 to get the core area for maximum efficiency.

7·34. Variation of output and losses in transformers with linear dimensions. Consider two transformers of same type with all their linear dimensions in the ratio x : 1 and having the same flux density, current density, frequency and window space factor. Let the transformer with dimensions x times be called A and the other transformer B.

Output. From Eqns. 7·6 and 7·10, output of single and three phase transformers is

$$\text{Output} \propto f B_m \delta \, K_w A_w A_i$$

Now f, B_m δ and K_w are constants, window area $A_w \propto x^2$ and net iron area $A_i \propto x^2$.

$$\therefore \quad \text{Output} \propto x^2 \times x^2 \propto x^4.$$

Hence output of transformer A is x^4 times that of B.

Losses. Total I^2R loss $= I^2R$ loss in primary $+ I^2R$ loss in secondary

$$= I_p^2 \frac{T_p \rho L_{mtp}}{a_p} + I_s^2 T_s \frac{\rho L_{mts}}{a_s}$$

(L_{mtp}, L_{mts} = length of mean turn of primary and secondary windings respectively)

Now $\quad I_p = \delta a_p$ and $I_s = \delta a_s$

$\therefore$ Total I^2R loss $= \delta^2 \rho \, (a_p T_p L_{mtp} + a_s T_s L_{mts}) = \delta^2 \rho \times$ volume of copper.

Now δ and ρ are constants and volume of copper $\propto x^3$.

Hence total I^2R loss $\propto x^3$

Thus I^2R loss of transformer A is x^3 times that of transformer B.

The specific iron loss i.e. loss per unit volume is constant if flux density and frequency are constant.

Total Iron loss = loss per unit volume × volume

$$\propto \text{volume} \propto x^3.$$

Both I^2R and iron losses vary as the third power of linear dimensions.

$\therefore$ Total losses $\propto x^3$.

This means that losses of transformer A are x^3 times that of transformer B.

7·35. Design of core. The core section for core type of transformers may be rectangular, square or stepped. Shell type transformers use cores with rectangular cross-section.

7·35·1. Rectangular core. For core type distribution transformers and small power transformers for moderate and low voltage, the rectangular shaped core section may be used. The ratio of depth to width of core varies between 1·4 to 2. Rectangular shaped coils are used for rectangular cores.

For a shell type transformer width of central limb is 2 to 3 times the depth of core

7·35·2. Square and Stepped cores. When circular coils are required for high voltage distribution and power transformers, square and stepped cores are used. Circular coils are preferred because of their superior mechanical characteristics. A transformer coil, under mechanical stresses produced by excessive leakage flux due to short circuits, tends to assume a circular form. On circular coils, these forces are radial and there is no tendency for the coil to change its shape ; on rectangular coils the forces are perpendicular to the conductors and tend to give the coil a circular form, thus deforming it.

With core type transformers of small sizes, simple rectangular core can be used with either circular or rectangular coils. As the size of the transformer increases, it becomes wasteful to use rectangular cores. For this purpose the cores are square shaped as shown in Fig. 7.51. The circle represents the inner surface of the tubular form carrying the windings. This circle is known as the **circumscribing circle.** Clearly a lot of useful space is wasted, the length of circumference of circumscribing circle being large in comparison with its cross-section. This means that the length of mean turn of winding is increased giving rise to higher I^2R losses and conductor costs.

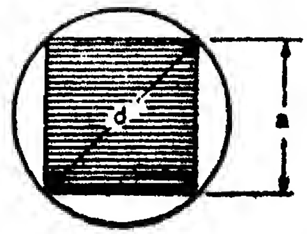


Fig. 7.51 Square core section

With larger transformers, cruciform cores, which utilize the space better, are used as shown in Fig. 7.51. As the space utilization is better with cruciform cores, the diameter of circumscribing circle is smaller than with square cores of the same area. Thus the length of mean turn of copper is reduced with consequent reduction in cost of copper. It should be kept in mind that two different sizes of laminations are used in cruciform cores.

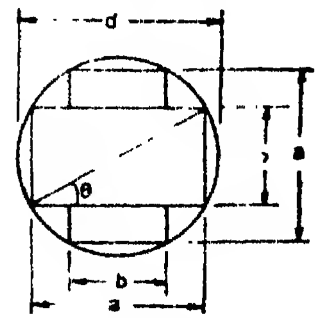


Fig 7.52. Cruciform core

With large transformers, further steps are introduced to utilize the core space which reduces the length of mean turn with consequent reduction in both cost of copper and copper loss.

It would seem that we can go on introducing steps with resultant reduction in cost of winding. However, with larger number of steps a large number of sizes of laminations have to be used. This results in higher labour charges for shearing and assembling different types of laminations. Thus the reduction in winding costs with a certain number of steps has to be balanced with the extra labour costs. The number of steps to be used for a particular transformer have to be decided by the above considerations.

1. **Square Cores.** Referring to Fig. 7.51

Gross area of core $A_{gi}=a^2=(0.71d)^2=0.5d^2$

where a=side of the square and d=diameter of circumscribing circle.

Net iron area A_i=stacking factor × gross iron area $=0.9\times0.5d^2=0.45d^2$

(taking stacking factor as 0.9).

$$\text{Ratio } \frac{\text{net core area}}{\text{area of circumscribing circle}}=\frac{0.45d^2}{(\pi/4\ d^2)}=0.58.$$

$$\text{Ratio } \frac{\text{gross core area}}{\text{area of circumscribing circle}}=\frac{0.5d^2}{\pi/4\ d^2}=0.64.$$

2. **Stepped Cores.** Fig. 7.52 shows a 2 stepped or a cruciform core. The dimensions of the two steps, to give maximum area for a given diameter are determined as given below.

Gross core area $A_{gi}=ab+b(a-b)=2ab-b^2$.

Now $a=d\cos\theta$ and $b=d\sin\theta$.

$\therefore$ $A_{gi}=2d^2\sin\theta\cos\theta-d^2\sin^2\theta=d^2(\sin 2\theta-\sin^2\theta)$

Differentiating the expression with respect to θ,

$$\frac{dA_{gi}}{d\theta}=d^2(2\cos 2\theta-2\sin\theta\cos\theta)=d^2(2\cos 2\theta-\sin 2\theta)$$

Equating $dA_{gi}/d\theta=0$, the value of θ which gives the maximum area is found out

or $d^2(2\cos 2\theta-\sin 2\theta)=0$ or $\tan 2\theta=2$ or $\theta=31°\ 45'$

Therefore $a=d\cos 31° 45'=0{\cdot}85d$; $b=d\sin 31° 45'=0{\cdot}53d$.

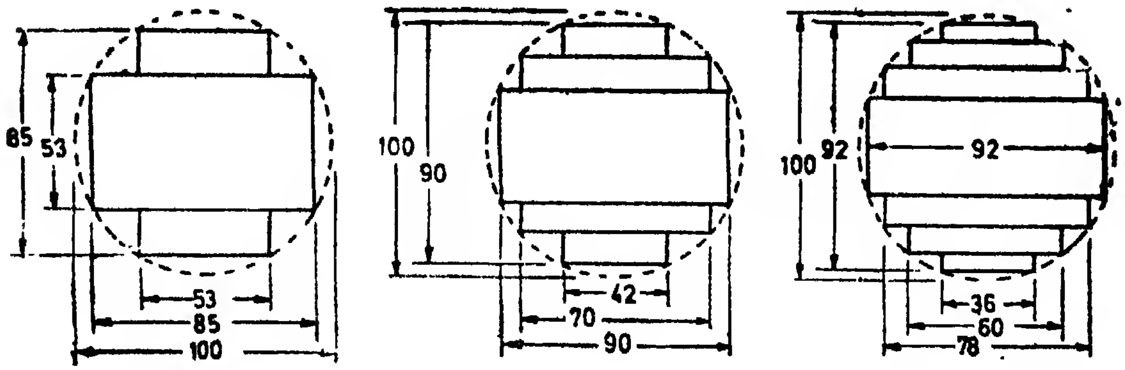
Gross core area $A_{gi}=2ab-b^2=0{\cdot}618d^2$.

Net core area $A_i=0{\cdot}9A_{gi}=0{\cdot}56d^2$.

a = 0·85 d
b = 0·53 d

$$\text{Ratio } \frac{\text{net core area}}{\text{area of circumscribing circle}}=\frac{0{\cdot}56d^2}{(\pi/4)d^2}=0{\cdot}71$$

$$\text{Ratio } \frac{\text{gross core area}}{\text{area of circumscribing circle}}=\frac{0{\cdot}618d^2}{(\pi/4)\,d^2}=0{\cdot}79.$$



(a) 2 Stepped (cruciform) (b) 3-stepped (c) 4-stepped

Fig. 7·53. Cross-section of stepped cores.

By increasing the number of steps, the area of circumscribing circle is more effectively utilised. The most economical dimensions of various steps for a multi-stepped core can be calculated. The results are tabulated in Table 7·3.

Table 7·3

Area percentage of circumscribing circle	*Square*	*Cruciform*	*Three stepped*	*Four stepped*
Gross core area A_{gi}	64	79	84	87
Net core area A_i	58	71	75	78
Net core area $A_i=k_c d^2$ k_c	0·45	0·56	0·6	0·62

The greatest theoretical cross-sectional area of a stepped core with same number of steps is obtained only at a definite ratios between the widths of laminations (for stacks) and for the diameter of circumscribing circle. These ratios are shown in Fig. 7·53. The dimensions are given as fraction of diameter of circumscribing circle. However, in actual practice the designer has to depart from the theoreticlly found sizes of laminations. The laminations are manufactured in standard size of width, normally 0·75 m or 1 m, This is done to avoid an excessively wide assortment of lamination sise and to minimise the wastage of steel during punching of laminations. Therefore, the core has to be composed of laminations of standard sizes only (and not excactly of the same sise as given by the ratios shown in Fig. 7.53).

7·36. Calculation of core area. The voltage per turn is calculated from Eqn. 7·12

$$E_t = K\sqrt{Q}.$$

A suitable value of K can be chosen from Table 7·2 and value of E_t determined.

Now, flux $\qquad \Phi_m = \dfrac{E_t}{4{\cdot}44 f}.$

Therefore, the value of flux in the core can be calculated. The area of the core is found out by assuming a suitable value of maximum flux density B_m

Net core area required $A_i = \dfrac{\Phi_m}{B_m}$

and gross core area $\qquad A_{gi} = \dfrac{A_i}{k_i}$

7·37. Choice of flux density. The value of flux density in the core determines the core area. Higher values of flux density give a smaller core area and therefore there is a saving in cost of iron. Also with the reduction in core area the length of mean turn of windings is also reduced. Thus there is a saving in conductor costs also. But with higher flux density, the iron losses become high resulting in considerable temperature gradient across the core. High flux density necessitates a large magnetizing current which contains objectionable harmonics.

The value of flux density to be chosen also depends upon the service conditions of the transformer. As a distribution transformer has to be designed for a high all day efficiency, and therefore the value of flux density should be low in order to keep down the iron losses.

The usual values of maximum flux density B_m for transformers using hot rolled silicon steel are :

Distribution transformer — 1·1 to 1·35 Wb/m².

Power transformer — 1·25 to 1·45 Wb/m².

Lower values should be used for small rating transformers.

For transformers using cold rolled grain oriented steel the following values may be used :

For transformers upto 132 kV — 1·55 Wb/m².

For 275 kV transformers — 1·6 Wb/m².

For 400 kV and generator transformers — 1·7 Wb/m².

7·38 Design of Windings. Number of turns in primary winding

$$T_p = \frac{\text{voltage of primary winding}}{\text{voltage per turn}} = \frac{V_p}{E_t}.$$

Number of turns in secondary winding $T_s = \dfrac{V_s}{E_t}$.

The number of turns of an l.v. winding is usually determined in a preliminary design by adjusting the voltage per turn to get the number of l.v. winding turns per phase as an integer.

$$T_{l.v.} = \frac{V_{l.v.}}{E_t} = \text{an integer.}$$

The number of h.v. winding turns per phase is therefore,

$$T_{h.v.} = \frac{V_{h.v.}}{V_{l.v.}} \cdot T_{l.v.}$$

If the tappings are located in the middle part of an h.v. winding, the number of winding turns must be even to ensure the symmetry of winding. For a winding with tappings it is necessary to have a proper turns ratio (or a voltage ratio) not only on the principal tapping but on the other taps as well. Therefore, turns should be selected judiously.

Current in primary winding $I_p = \frac{\text{kVA per phase} \times 10^3}{V_p}$

similarly $I_s = I_p \frac{V_p}{V_s}$

The area of conductors in primary and secondary windings is determined after choosing a suitable current density to be used in the windings. The permissible current density in the windings is limited by local hecting and efficiency. Temperature rise in the windings may become excessive if higher values of current density are chosen and this may cause injury to the insulation. The chooice of current density is important as the I^2R losses and hence the load at which maximum efficiency occurs depends on it. Therefore, current density in a winding should be chosen to guarantee the level of losses and cooling conditions required. The level of iron and I^2R losses required is different in distribution and power transformers. Thus the value of current density is different for different types of transformers (distribution and power).

For distribution and small power transformers, self oil cooled type upto 50 kVA :

$$\delta = 1{\cdot}1 \text{ to } 2{\cdot}3 \text{ A/mm}^2.$$

For large power transformers, self oil cooled type or air blast. $\delta = 2{\cdot}2$ to $3{\cdot}2$ A/mm^2.
For large power transformers with forced circulation of oil or with water cooling coils $\delta = 5{\cdot}4$ to $6{\cdot}2$ A/mm^2.

Area of each primary conductor $a_p = I_p/\delta_p$

and area of each secondary conductor $a_s = I_s/\delta_s$

The current densities in the two windings should be taken equal in order to have minimum copper loss. Let,

U_p, U_s = volume of conductors in primary and secondary windings respectively.

U_t = total volume of conductors $= U_p + U_s$.

Total volume of conductor is assumed constant.

I^2R loss in primary $= \rho\delta_p^2 U_p$.

I^2R loss in secondary $= \rho\delta_s^2 U_s = \rho\delta_s^2 (U_t - U_p)$.

Total I^2R loss $P_c = \rho[\delta_p^2 U_p + \delta_s^2 (U_t - U_p)]$.

Differentiating P_c w.r.t. U_p,

$$\frac{dP_c}{dU_p} = \rho[\delta_p^2 - \delta_s^2].$$

For minimum loss $\frac{dP_c}{dU_s} = \rho[\delta_p^2 - \delta_s^2] = 0$ or $\delta_p = \delta_s$.

Therefore, for minimum I^2R loss, the value of current density in each of the two windings should be equal.

In practice, however, the current density in the relatively better cooled outer winding is made 5 percent greater than the inner winding.

7·38·1 Selection of type of winding. It is first necessary to select proper types of windings to be used in the transformer. The design of the winding chosen must be such that the desired electrical characteristics and adequate mechanical strength is obtained. Sometimes, more than one type of winding may be suitable for the transformer. In this case, the winding which has simple constructional features should be used.

The high voltage windings are usually of the following types :

(*i*) Cylindrical winding with circular conductors,

(*ii*) Cross-over winding with either circular or small rectangular conductors.

(*iii*) Continuous disc type winding with rectangular conductors

The cylindrical and the cross-over windings are used for transformers of ratings upto 1000 kVA and 33 kV. The disc type winding is used for transformers of higher ratings ranging from 200 kVA to tens of MVA and voltages from 11 kV upwards.

The low voltage windings are usually of the following two types :

(*i*) cylindrical winding, (*ii*) helical winding (usually double helical).

Both these windings employ rectangular conductors Cylindrical windings are used for kVA ratings upto 800 and voltages upto 433 V. The helical winding can be used for ratings upto tens of MVA and voltages upto 15 kV and some times upto 33 kV.

It may be interesting to note that it may not be possible to use helical winding for l.v. of transformers having low kVA rating. This is explained as under :

The number of winding turns at given voltage increases when the kVA of a transformer decreases since a decreased value of voltage per turn is used for low kVA transformers ($E_t = K\sqrt{Q}$). Therefore, a helical winding, for example, may be easily used for a 6·6 kV, 25 MVA transformer, but it becomes difficult when the rating decreases to about 8 MVA, though the voltage in both cases is the same. In such cases, where the number of turns is a large a continuous disc type winding has to be employed.

The winding to be used in a transformer may be selected by referring to Table 7·4. This table shows the ranges of different windings. (The ratings refer to 3 phase transformers)

Table 7·4. Ranges of different windings types

Type of winding	*Rating* kVA	*Voltage* kV	*Maximum current/conductor* A	*Conductor cross-section* mm²	*No. of conductors (strips) in parallel*
1. Cylindrical (circular conductors)	5000-10,000	upto 33	upto 80	upto 30	1 to 2
2. Cylindrical (rectangular conductors)	5000-8000	upto 6 (usually 0·433)	10-600	5-200	1 to 4
3. Cross over	upto 1000	upto 33	upto 40	upto 15	1
4. Helical	From 160 to tens of thousands	upto 15 but sometimes upto 33	From 300 and above	75 to 100 and above	4 to 16 (sometimes more)
5. Continuous disc	From 200 to tens of thousands	3·3-220	12 and above	From 4 to 200 and above	1 to 4 (sometimes more)

7·38·2. Position of windings relative to core. The l.v. winding is placed on the inner side nearer to the core with h.v. winding on the outside. This arrangement is used because the potential difference between l.v winding and core (which is at earth potential) is small, there is less likelihood of a fault occurring between the two. Also, with l.v. winding placed nearer to the core, the insulation used between core and winding has a small thickness. However, in case h.v. winding is placed around the core, the insulation between the two has to be thick and this makes the length of mean turn large. This is clear from Fig. 7·54. The cost of insulation is also higher with h.v. on the under side as same amount of insulation has to be used between h.v. and l.v. as is used between h.v. and core while if l.v. is on the inner side, the major insulation is only between l.v. and core.

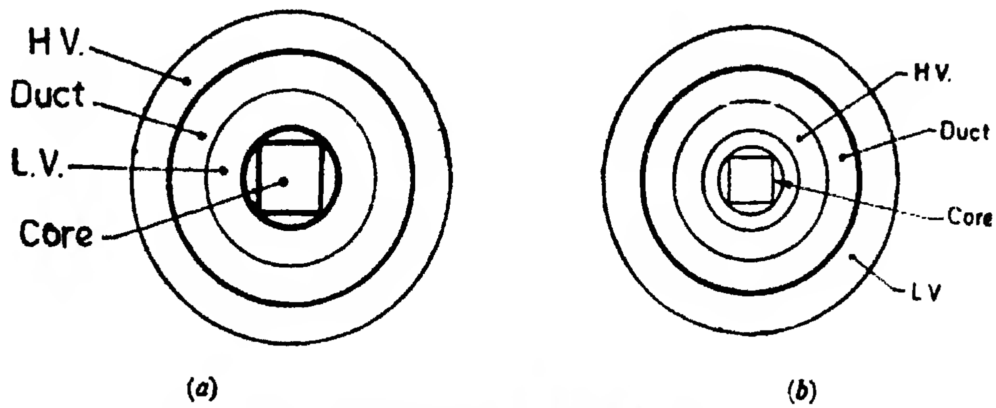


Fig. 7·54. Position of windings relative to core

The tappings are provided on the h.v. winding, therefore, it is very convenient to tap the winding as it is on the outside.

7·39. Design of insulation. During the processes of power transfer from one circuit to another electrical, mechanical and thermal phenomena take place in a transformer. The winding voltages produce an electrostatic field in the dielectric and therefore stress the insulation ; the currents in the windings set up magnetic fields which give rise to electromagnetic forces on the windings and to mechanical stressing of insulation ; finally the losses in the transformer produce temperature rise which produces thermal stressing of insulation.

Hence, the fundamental considerations in the design of insulation of transformers may be described as those of arranging core, windings and insulation to obtain satisfactory electrical, mechanical and thermal characteristics during the steady state as well as transient conditions. The three basic considerations in the design of insulation are :

1. **Electrical considerations.** The basic insulation structure is primarily determined from consideration of the magnitude and nature of voltages which appear between different parts of the transformer *i.e.* voltages between individual turns, between coil or layers, between windings and from windings to core and tank.

Tests like sustained frequency high voltage tests and impulse test are applied to check the strength of insulation between the various parts with a view to ensure that the transformer will have a reasonable life (average 20 years) and will be able to withstand damage under abnormal conditions imposed by lightning, switching surges and other transient phenomena. The electrical design should also take care of the eddy current losses in conductors and leakage reactance of windings.

. **Eddy current loss.** The windings should be so designed that the stray load loss is small. The stray load loss includes eddy current loss in conductors and connectors and also n tank walls and clamping structure. The conductors should be split into small strips to reduce eddy current losses in conductors. The radial width of strips should be small and they should be transposed

Leakage reactance. A given arrangement of core and windings determines the leakage reactance of the windings. The leakage reactance is adjusted by changing the winding configuration and brought within desired limits.

2. **Mechanical considerations.** The basic mechanical considerations in the design of insulation are of two types :

(*i*) The insulation must be capable of withstanding the mechanical stresses imposed on it during the manufacturing processes.

(*ii*) The insulation must be able to withstand the mechanical stresses which are developed in the winding due to electromagnetic phenomenon. The electromagnetic forces and mechanical stresses produced under normal conditions of operation are quite small and ordinarily are of minor importance. However, under fault conditions, particularly dead short circuit, the electromagnetic forces may be increased several hundred times. The insulation must be designed to withstand the stresses produced under abnormal conditions for a specified period of time.

The mechanical design of insulation should be such that hoop, bursting and compressive stresses are minimized. Also there should be axial balance between the windings and they should be adequately braced.

Thermal considerations. The thermal aspects of design of insulation are determined from the consideration of insulating materials used, selection of safe maximum operating temperatures and types of cooling method employed.

The transformer structure should be such that the losses developed in the core and windings produce temperature rises in the various parts which no where exceed the permissible limits both under normal and over load/fault conditions and which, in the interest of economy, approach those limits as nearly as possible.

The insulation used for conductors of oil immersed transformers is class A type. The conductors are usually paper covered. The dimensions of round and rectangular conductors are given in chapter 17. The increase in dimensions of rectangular conductors due to paper covering is 0·25 mm (minimum with double covering).

The low voltage windings of small and medium size transformers are insulated from the core by pressboard or a synthetic resin bonded paper (s. r. b. p.) cylinder. The cooling duct between the core and inside cylindrical surface of the core is formed by axial bars arranged around the cylinder. The bars may be placed around the outer surface of the l.v. winding and between layers of helical winding. The major insulation between the low voltage and high voltage windings is provided by another pressboard or s. r. b. p. cylinder and the bars are arranged around it. A practical formula for determining the thickness of insulation between a winding earth and between l. v. and h.v. windings is :

$$\text{Insulation thickness} = 5 + 0{\cdot}9 \text{ kV mm} \qquad \ldots(7{\cdot}22)$$

where kV is the voltage of the in kilo volt between windings and earth or between windings. This thickness includes the width of any oil duct provided in between. The width of an oil duct is about 6 mm in small capacity transformers and 7·5—12 mm in large capacity high voltage transformers. With disc and helical disc windings, the bars have wedge shaped section in order to enable the intercoil or interturn dovetailed spacers to be threaded to them. Insulation at the two ends of the windings consists of blocks keyed to the axial bars. These blocks are in line with the axial spacers and form a series of columns with help of which the winding can be clamped. The thickness of insulation at each end of the winding varies from 6 mm for windings below 500 V to about 150 mm for 66 kV transformers.

The insulation of a transformer is divided into four types :

(*i*) major insulation, (*ii*) minor insulation, (*iii*) insulation relative to tank and (*iv*) insulation between phases.

Major insulation. The insulation between windings and grounded core and the insulation between the windings of the same phase is called major insulation.

Major insulation. Insulation between different parts of one winding *i.e.* insulation between turns, coils and layers etc. is called minor insulation.

The insulation relative to the tank is called oil barrier insulation in oil immersed transformers. This insulation consists of oil ducts, barriers and coverings. Partitions of solid insulating materials placed inside an oil ducts are called *barriers*. For example pressboard s.r.b p. or cylinders placed between l.v. and h.v. windings and between windings and core are called barriers. Coverings, on the other hand closely cover a particular part of a transformer which is above the earth potential. The examples of coverings are turn and additional coil insulation. The barriers and coverings increase the electrical strength of an oil duct because they prevent the lining up of particles consisting of partly conducting impurities along the lines of force and thereby reduce the total clearance which would otherwise be necessary.

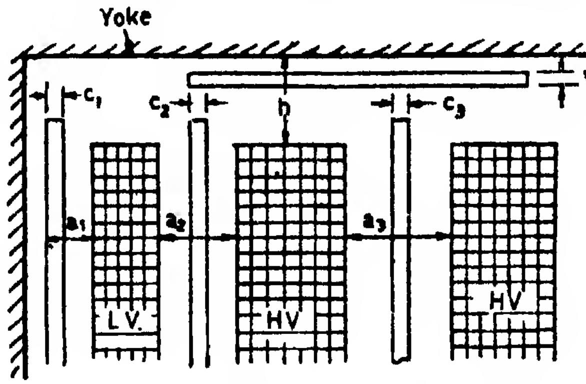


Fig. 7·55. Major insulation of transformers upto 33 kV.
C_1, C_2, C_3—thicknesses of insulating cylinders
w—thickness of horizontal solid insulation between winding ends and yoke

The major insulation for windings upto 33 kV is schematically shown in Fig. 7·55. The thickness of insulating cylinders, oil ducts, and the insulation between winding ends and yoke is given in Tables 7.5 and 7.6.

Table 7·5. Thickness of major insulation of H.V. windings upto 33 kV.
(All dimensions in mm)

Rating kVA	*Voltage* kV	*Between h.v. and l.v.*		*From winding end to yoke*		*Between phases*	
		a_2	C_2	h	w	a_3	C_3
25—100	3 3 and 6 6	8 5	2·5	20	—	10	2
125—630		12·0	2·5	20 to 30	—	10	2
800 and above		17·0	5 0	30—50	—	10	2
25—630	11	12 0	3 0	30	—	14	2
800 and above		17 0	5 0	30 to 50	—	14	2
25—630	15	15·0	3·5	40	—	17	2
800 and above		17·0	5·0	40—50	—	17	2
10—800	33	27 0	5·0	60	2	30	3
1000 and above		27·0	5 0	75	2	30	3

Table 7·6. Thickness of major insulation of L.V. windings upto 33 kV (All dimensions in mm)

Rating kVA	Voltage kV	From winding to core a_1	From winding to core C_1
25—630	upto 1	5	—
25—630	3·3 and 6·6	12	2·5
800 and above	upto 1, 3·3 and 6·6	15	5·0
25—630	11	18	3·0
800 and above	11	18	5·0
25—630	15	21	4·0
800 and above	15	23	5·0
Any kVA	33	27	5·0

In case of high voltage transformers, the insulation required between windings has a large thickness and it is customary to use a number of thin cylinders spaced by axial bars, the material being carried round the h.v. (outer) winding by flanged collars as shown in Fig. 7·56.

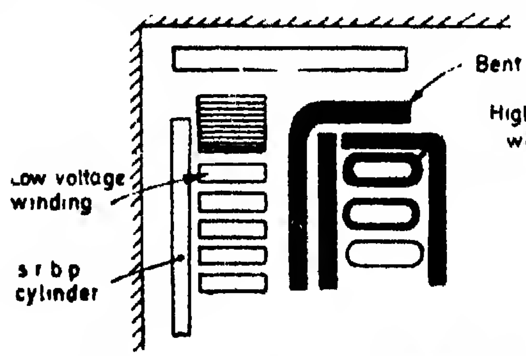


Fig. 7·56. Insulating cylinders and collars.

Some design features of different transformer windings are given below :

1. **Cylindrical windings.** The cylindrical windings with rectangular conductors are used for voltages upto 500 V and current rating between 10 to 600 A. The smallest cross-section is 5 mm² and the largest 200 mm² (4 strips in parallel).

The cylindrical windings using circular conductors are used for a current rating of upto 80 A. The maximum diameter used for the bare conductor is 4 mm. Single conductor is used for currents upto 40 A and above this range two conductors are used in parallel extending the range upto 80 A.

2. **Cross-over windings.** The cross-over winding is divided into a number of coils in order to reduce the voltage between adjacent layers. These coils are axially separated by a distance of 0·5 to 1 mm with the help of washers or axial ducts formed by spacers.

The voltage between adjacent coils in a cross-over winding should not be greater than 800-1000 V. Therefore, the number of coils used should be more than V/(800-1000) where V is the voltage of the winding. The number of coils used in 33 kV and 110 kV winding range from 60-80.

The number of layers used a cross-over winding are calculated on the basis that the maximum voltage between layers should not exceed 300 V. Fig. 7·57 shows four layers of a cross-over coil with 8 turns per layer. It is evident that the maximum voltage between two adjacent layers is equal to two times the voltage per turn times the turns per layer. (This occurs between conductors 1 and 16, between 9 and 24 and between 17 and 32 for Fig. 7·57).

Therefore, the winding should be so designed that

$$2N_{cl}E_t \not> 300 \text{ V}.$$

where N_{cl}=number of conductors per layer.

In order that the coils be cooled properly, the height and the radial width of the coil should be small.

The axial height of each coil is about 50 mm and the width of the coils is kept within 25—50 mm.

3. **Continuous disc type winding.** Transformers of high capacities often use single layer disc coil wound with rectangular conductors on their flat side. Each disc coil consists either of eight turns wound with a single conductor, or of two turns with four parallel strips, or one turn having eight parallel strips. Each coil rests on pressboard spacers forming horizontal ducts. The width of spacers is usually between 3·5 to 5 mm.

The width of radial oil duct is determined by the voltage between adjacent coils, specific thermal loading and the system of cooling employed.

The minimum width of oil duct is 5 mm for voltage less than 35 kV, 6 mm for 35 kV and 8 mm for 110 kV.

The area of conductor used in the winding varies from 4 mm² to 50 mm² and the limits for current are 12A to 600 A.

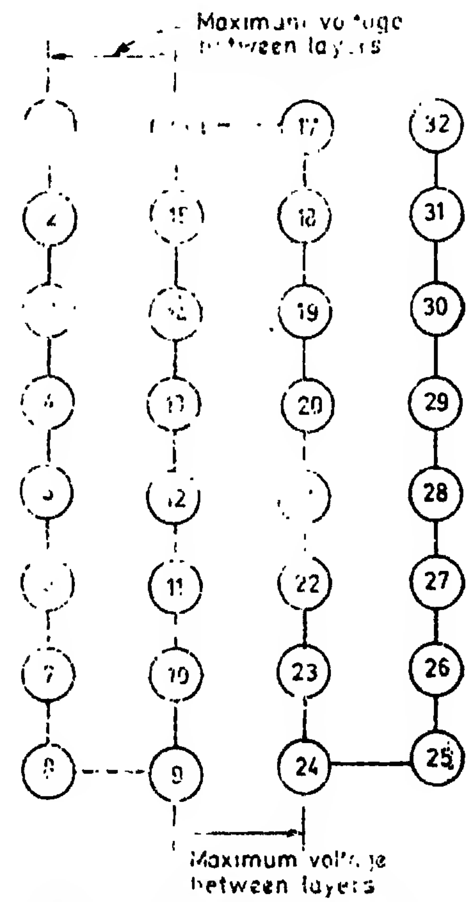


Fig. 7·57. Voltage between layers of cross-over winding.

In cases where a winding consists of several coils, for example, a cross-over or a continuous disc type, the voltage per coil should not be more than 800—1000 V. Therefore, the number of coils is $n_c \geqslant V/(800—1000)$.

When a continuous type of winding is used, the number of coils should be chosen in such a way as to obtain the necessary winding height utilizing the standard size of conductors. Windings designed for voltages between 33—110 kV normally use 60 to 80 coils. It should be noted that the number of coils of a continuous type winding with tapping at the middle should be a multiple of four.

4. **Helical winding.** A helical winding is used for l.v. windings of the power transformers with outputs ranging from 160 kVA to some ten thousand kVA at voltages from 230 V to 15 kV and sometimes upto and including 33 kV. In order to secure adequate mechanical strength the cross-sectional area of a strip not made less than 75—100 mm². The maximum number of strips used in parallel to make up a conductor is 16. Helical windings are used for current range 300 A to 2400 A, the maximum limit is based on the assumption that the maximum cross-sectional area of a strip normally does not exceed 50 mm².

A double helical winding is used for the same range of voltages as a single helical one, but the upper limit of currents for this winding is about two times higher.

In windings using rectangular conductors the radial width of a strip should not exceed 3·5 mm in order to limit the eddy current losses.

7·40. Surge Phenomenon. Transformers used in power systems are subject to surge overvoltages. These surge overvoltages, initiated on overhead transmission lines are caused by switching, faults or lightnig discharges. The surges may have a steep wave-front with a rate of rise 1000 times as great as the peak rate of normal operating frequency thereby imposing intense and rapidly changing electric stresses within the transformer winding. The surge phenomenon is particularly important in the case of high voltage transformers.

Not so long ago, design of high voltage windings of transformers exceeding 110 kV was governed wholly by power frequency voltages. The surge and impulse voltages were regarded as destructive and the designer was not expected to anticipate them. However, this attitude has undergone a radical change and the surge behaviour of the transformer has not only gained recognition as an essential part of designer's responsibility, but to assume such significance that it has become the foremost consideration in the design of high voltage windings.

The effect of a surge arriving at a transformer has an exceedingly complex effect on the transformer winding. The steeper the front and flatter the tail of the wave, the more severe is its effect, and therefore we will consider the effect of the application of a rectangular wave *i.e.* a step function voltage. Theoretically, the incidence of such a wave means that the line terminal is raised in potential at an infinite rate to the crest value and then held there. However, the transformer insulation may be considered as a network of capacitances with interturn insulation acting in series and each turn having a shunt capacitance to earth as shown in Fig. 7·58.

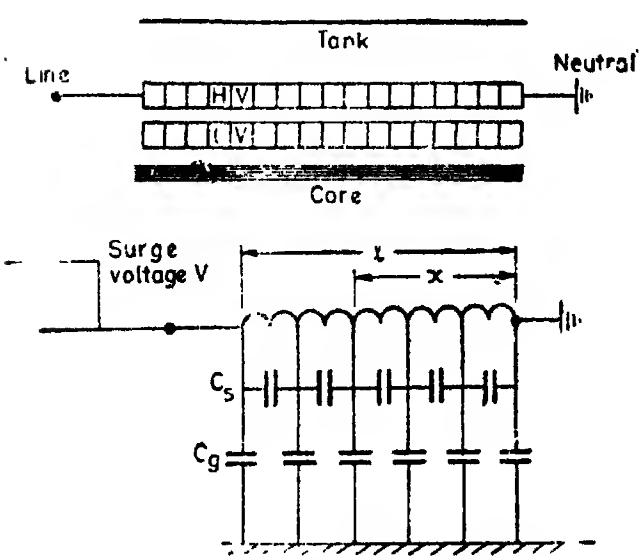


Fig. 7·58. Transformer model for analysis of surge phenomenon.

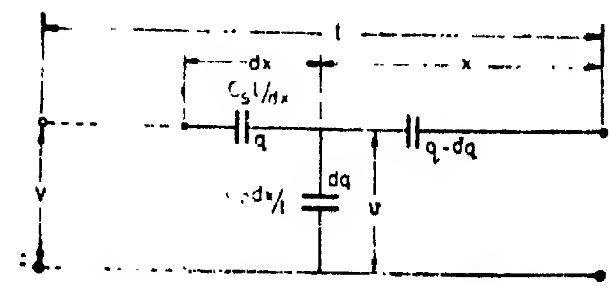


Fig. 7·59. Voltage and charge distribution in a winding element.

Let C_s=total series capacitance of the winding,

C_g=total shunt capacitance (*i.e.* capacitance of winding with respect to ground),

and l=total length (axial height) of winding.

Let us consider a winding element of length dx located at a distance x from the neutral end having a voltage v and a charge q as shown in Fig. 7·59.

The shunt capacitance of a winding element relative to ground is proportional to its length and the series capacitance is inversely proportional to the length. Therefore, the shunt capacitance of the winding element shown in Fig. 7·59 is $(C_g/l)dx$ and the series capacitance is $(C_s l)/dx$. If the charge of the adjacent winding element is $(q-dq)$, the charge of the shunt capacitance $(C_g/l)dx$ is, therefore, dq. The relationship between voltage and charge for any capacitance is $v=q/C$.

$\therefore$ The relationships for shunt and series capacitances of the element considered, are

$$v=\frac{dq}{(C_g/l)dx}=\frac{l}{C_g}\cdot\frac{dq}{dx} \text{ and } \frac{dv}{dx}=\frac{1}{C_s l}\,q.$$

$$\therefore \quad \frac{d^2v}{dx^2}=\frac{1}{C_s l}\quad\frac{dq}{dx}=\frac{1}{C_s l}\cdot\frac{C_g v}{l}=\frac{1}{l^2}\quad\frac{C_g}{C_s}\,v$$

or
$$\frac{d^2v}{dx^2}-\frac{1}{l^2}\quad\frac{C_g}{C_s}\,v=$$

Let $\alpha=\sqrt{C_g/C_s}$. Therefore, the above differential equation can be written as

$$\frac{d^2v}{dx^2}-\frac{\alpha^2}{l^2}\,v=0.$$

The solution of differential equation is

$$v=Ae^{\alpha x/l}+Be^{-\alpha x/l}.$$

The remote end (neutral end) of the winding is earthed

$\therefore \quad v=0 \quad$ at $\quad x=0$

$\therefore \quad 0=A+B \qquad$ or $\qquad A=-B.$

At the line end a step voltage of V is applied $\quad\therefore\quad$ At $x=l$, $v=V$

$\therefore \quad V=A(e^{\alpha}-e^{-\alpha}) \qquad$ or $\qquad A=V/(e^{\alpha}-e^{-\alpha})$

Hence, voltage at the element

$$v=\frac{V(e^{\alpha x/l}-e^{-\alpha x/l})}{(e^{\alpha}-e^{-\alpha})}=\frac{V\sinh(\alpha x/l)}{\sinh\alpha} \qquad \ldots(7·23)$$

Initial Distribution. Eqn. 7·23 shows that the voltage distribution along a winding depends upon the value of $\alpha=\sqrt{C_g/C_s}$. If the capacitance relative to ground is zero *i.e.* when $\alpha=0$, the expression $\dfrac{\sinh(x/l)}{\sinh\alpha}$ is of the form $\dfrac{0}{0}$. The equation can be solved by using *L' Hospitals'* rule, from which we have

$$v=V(x/l)\frac{\cosh\alpha x}{\cosh\alpha}\bigg|_{\alpha=0}=\frac{Vx}{l} \qquad \ldots(7·24)$$

From above $\dfrac{dv}{dx}=\dfrac{V}{l}$ constant.

Thus, it is evident from Eqn. 7·24 that if the shunt capacitance is zero, the voltage is uniformly distributed over the winding.

However, in modern power transformers α varies from 5 to 15 and hence the initial voltage distribution is considerably different from the linear distribution indicated by Eqn. 7·24.

The voltage gradient at the line-end of the winding is

$$\frac{dv}{dx}\bigg|_{x=l}=V\alpha(x/l)\frac{\cosh x/l}{\sinh\alpha}\bigg|_{x=l}=V\alpha\coth\alpha \qquad \ldots(7·25)$$

As $\alpha > 3$, $\coth \alpha \simeq 1$,

$$\therefore \quad \left.\frac{dv}{dx}\right|_{x=l} \simeq V\alpha \qquad \ldots(7{\cdot}25)$$

i.e. the voltage gradient at the line end is α times the value of gradient corresponding to the uniform voltage distribution (the voltage gradient corresponding to uniform distribution is V volt/metre).

Fig. 7·60 shows the initial distribution of a surge voltage over a uniform winding with neutral. The voltage distribution is uniform for $\alpha=0$ but in the presence of earth capacitances α may typically assume value of 10, and in that case, the voltage distribution becomes nonuniform with most of the voltage dropped across a fraction of the line end of the winding, in which the voltage between adjacent turns becomes excessive and may require additional insulation to withstand the high voltage stresses produced

Final Voltage Distribution. The tail of the step voltage is equivalent to a sustained d.c. voltage of magnitude V after the all transients have died down and the system has settled to steady state conditions. The voltage distribution therefore becomes

$$v = V(x/l)$$

This distribution is obtained with time $t = \infty$. It is clear from above, that the final voltage distribution is a straight line giving a uniform voltage gradient in the winding. This distribution is identical to the one obtained with $\alpha = 0$. (See Fig. 7·61).

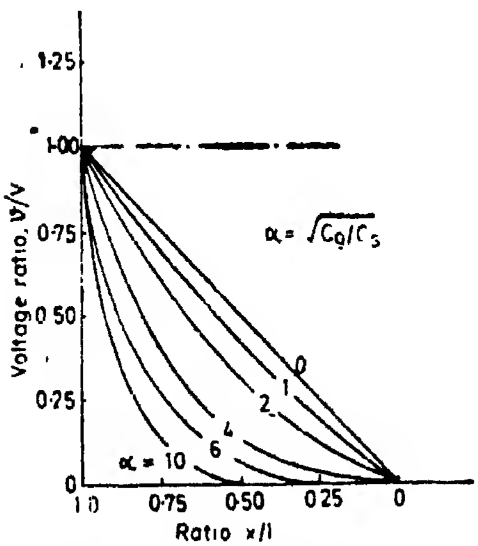


Fig. 7·60. Initial voltage distribution with earthed neutral.

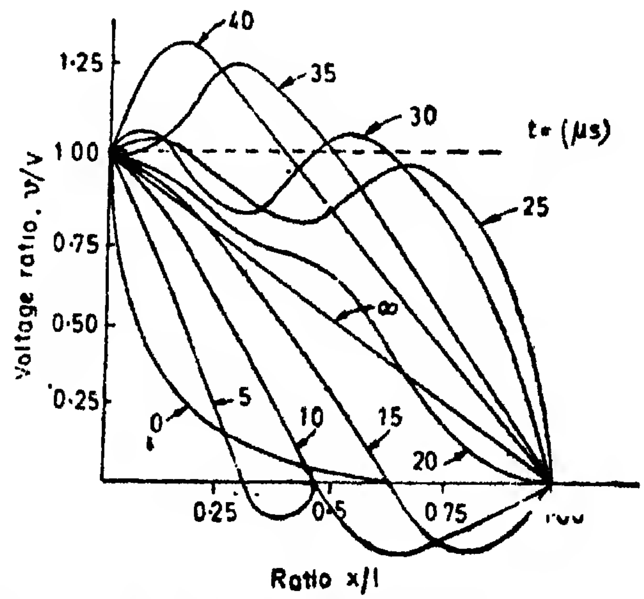


Fig. 7·61. Oscillations in winding subjected to voltage surge.

Intermediate Distribution. The transition from initial to final voltage distribution is always accompanied by oscillations on account of inductance and capacitance of transformer winding. There is a constant interchange of stored energy between capacitors and winding inductance resulting in complex oscillations at a variety of natural frequencies. As a result of damped transference of electrostatic and electromagnetic energy during complex oscillations, all parts of winding may be severely stressed (i.e have large voltage gradients) at different instants of time as shown in Fig. 7·61. Initially concentrations of voltage may appear at the line end of the winding; during transitional period concentrations may appear at the neutral end whilst voltages to earth considerably in excess of the incident surge may develop in the main body of winding. Therefore, reinforcement of end turns of the winding are of little assistance as far as the protection of windings against surge voltages is concerned.

7·40.1. Surge Protection. In a general case, under steady state conditions, equal voltages are induced between turns and consequently, ideally, equal amounts of insulation are required between turns. To utilize this uniformly disposed insulation to best advantage, the voltage appearing between turns throughout the winding under surge conditions should also be able to approach this ideal, in which case oscillation voltages are completely eliminated, the initial distribution, like the final must be uniform.

Before the effect of surge voltages was clearly understood, it was a universal practice to reinforce the insulation of a few turns (say 5%) on the line end of the transformer to withstand the impulsive voltage gradients. However, this results in decrease in the series capacitance resulting in increase of α and hence unequal voltage gradients. The use of reinforced turns at winding ends, instead of mitigating the trouble, intensifies it. External *surge absorbers* may be connected between the transmission line and the transformer terminals to reduce the steepness of wave front and to dissipate some of the energy of the surge wave.

However, the real solution to the problem of surge voltage distribution lies in designing the winding in such a way that the voltage distribution is more or less uniform and no part of the winding insulation is unduly stressed.

The initial distribution of voltage is determined wholly by the capacitance network. The voltage distribution depends upon α which is equal to $\sqrt{C_g/C_s}$. Consequently two circuit elements are available for controlling and improving the initial response. It has been stated earlier that when $\alpha=0$ (*i.e.* when the shunt capacitance $C_g=0$ or when series capacitance $C_s=\infty$), the initial distribution is uniform and is coincident with final distribution. Thus, it can be safely concluded that the distribution can be improved by decreasing shunt capacitance and/or by increasing the series capacitance. The methods adopted for securing uniform voltage distribution are :

1. **Shielded Windings.** The basic principle of protection of transformers rated at 110 kV and more involves the arrangement of an additional capacitance network connected to the line input terminals in order to neutralize the winding to earth capacitance.

An electrostatic shield is provided along the axial length of h.v. winding as shown in Fig. 7·62. The provision of the electrostatic shield introduces capacitances between the

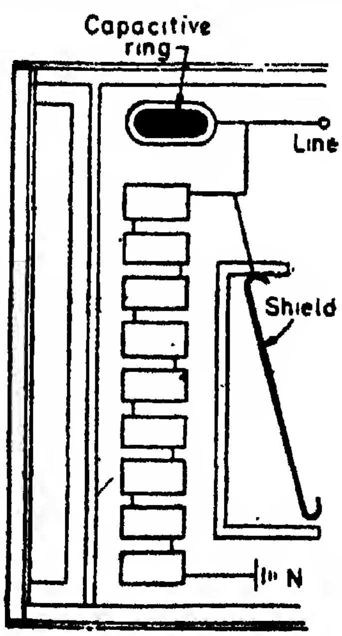
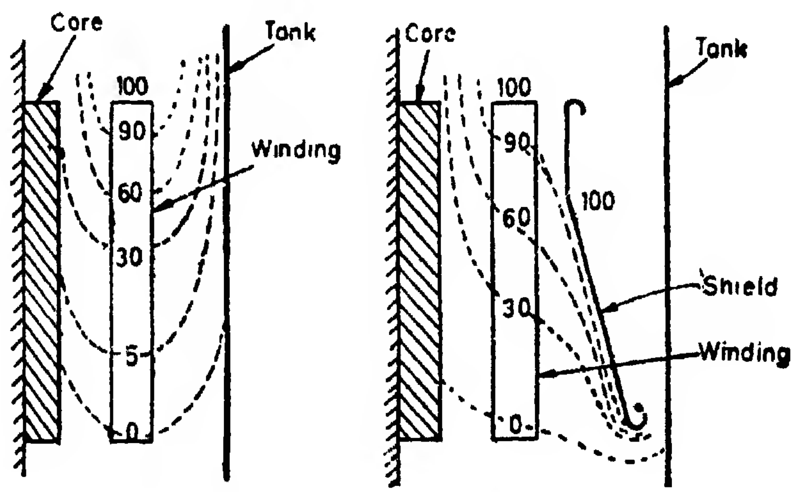


Fig. 7·62. Winding with static shield.



(*a*) Unshielded (*b*) Shielded

Fig. 7·63. Surge voltage distribution.

line terminal and the winding in a graded diminishing proportion as in a transmission line insulating string, thereby helping to neutralize the winding to ground capacitances

The use of the shield permits the shunt currents to flow directly from the line, thereby enabling the series currents to have the same value in all the sections. Hence a uniform voltage distribution is obtained. The surge voltage distribution with and without electrostatic shields is shown in Fig. 7·63, the figures indicating percentage equipotentials.

The surge protection of transformers by an electrostatic shield involves the following procedure :

(*i*) The shield is provided on the six terminal coils (for 110 to 220 kV) at increasing distances.

(*ii*) A capacitive ring is placed over the winding to equalize the voltages across the turns of the input coil. The ring (or collar) is of pressboard with copper or aluminium foil and is securely insulated. The capacitive ring is connected to the line terminal of the winding.

2. **Centre point disc winding**. In this method, the high voltage disc winding is divided in two halves which are connected in parallel. Each half starts from opposite end and finishes at the centre point. The centrepoint becomes the h v. terminal of the winding (Fig. 7·64). The h.v. winding is provided with radial and complete axial shields and the arrangement results in uniform initial and final distributions.

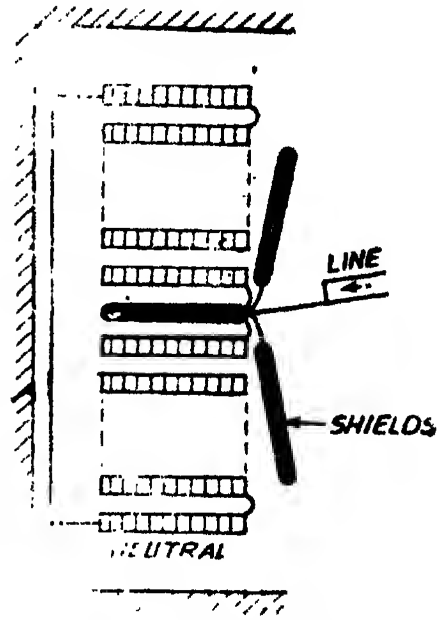


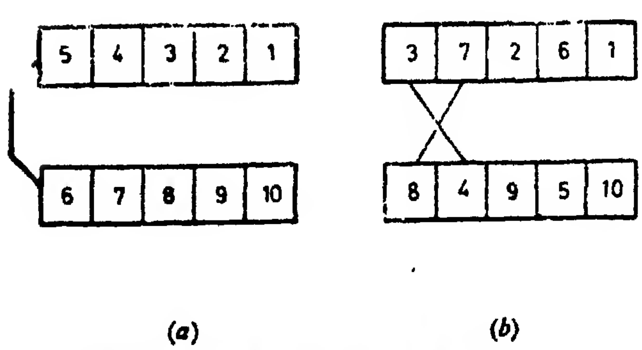
Fig. 7·64. Centre point disc winding.

3. **Interleaved windings** Two normal disc coils are shown in Fig. 7·65 (*a*). If the order of the turn interconnection is changed to have interleaved coils as in Fig. 7·65 (*b*), the interturn or series capacitance C_s increases thereby reducing α. Therefore, voltage distribution becomes more uniform.

Windings of interleaved double disc type can withstand impulse voltages better than the non-interleaved type. Partially interleaved windings are also used with interleaved coils only at the ends of h.v. windings.

4. **Layer type windings.** Recent developments in the design of high voltage transformers emphasise the difficulties of obtaining adequate impulse strength with continuous disc type winding. Modern trend is to use *layer type* of winding as shown in Fig. 7·66.

The high voltage winding is divided into concentric layers, separated by oil ducts,



(*a*) (*b*)

Fig. 7·65. Normal and Interleaved coils.

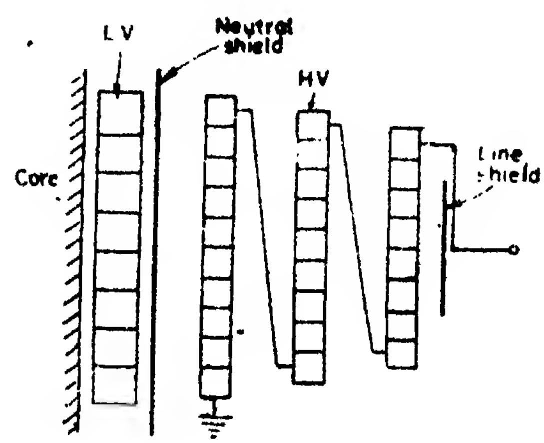


Fig. 7·66. Layer type winding.

each diminishing in length. Two shields are used, one at the line end and the other at the neutral end. On account of the provision of shields on the two ends, it is possible to neglect the capacitance between winding and earth (C_g) in comparison with capacitance between adjacent turns (C_s).

Therefore, α is small for these windings and hence the voltage distribution is almost uniform. Fig. 7·67 shows the voltage distribution in a layer type winding.

It is clear that in a layer type winding the voltage distribution initially and under transient conditions is almost uniform. The layer type of winding is thus lightning proof.

The same conditions exist in the windings of shell type power transformers because the capacitances between large flat coils considerably exceed the capacitances relative to earth and therefore, the initial and the final voltage distributions practically coincide.

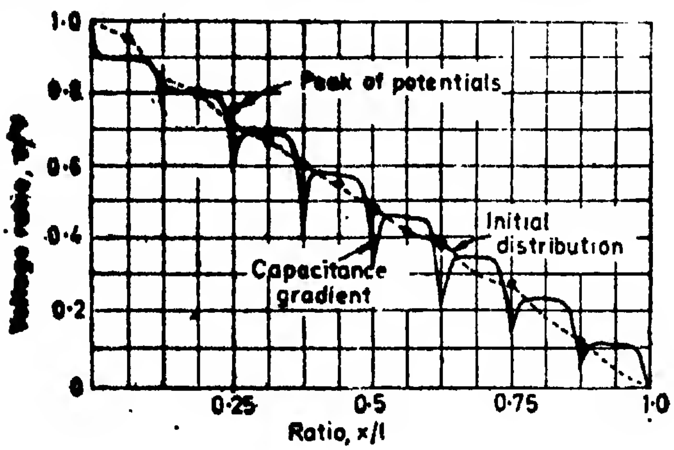
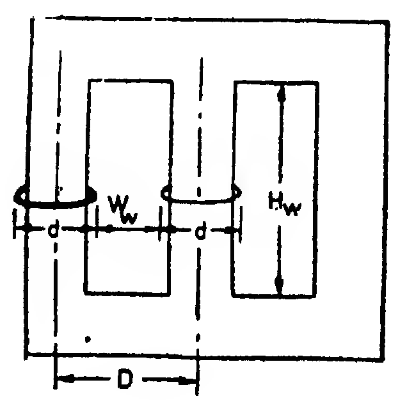


Fig. 7·67. Initial voltage distribution and peaks of potentials in layer type windings.



Fig, 7·68. Transformers frame.

7·41. Window Space Factor. The window space factor is defined as the ratio of copper area in the window of the total window area. It depends upon the relative amounts of insulation and copper provided, which in turn depends upon the voltage rating and output of transformers. The following empirical formulae may be used for estimating the value of window space factor :

$$K_w = 10/(30+\text{kV}) \quad \ldots(7{\cdot}29)$$

where kV is the voltage of h.v. winding in kilo-volt. The above formula is for transformers of ratings between 50 to 200 kVA.

Space factor is larger for large outputs and smaller for small outputs. For a transformer of about 1000 kVA rating $K_w = 12/(30+\text{kV})$ and for transformers of about 20 kVA rating $K_w = 8/(30+\text{kV})$. The values of space factor for intermediate ratings can be interpolated.

7·42. Window Dimensions. The leakage reactance is affected by the distance between adjacent limbs. If this distance is relatively small, the width of the winding is limited and this must be counter balanced by increasing the height of the winding. Thus the windings are long and thin. This arrangement leads to a low value of leakage reactance. If the height of the window is limited, the width of the window has to be increased in order to accommodate the coils. This results in short and wide coils giving a large value of leakage reactance.

The height and width of the window can be adjusted to give a suitable arrangement of windings and also to give a desired value of leakage reactance.

The area of the window depends upon total conductor area and the window space factor.

$$\text{Area of window } A_w = \frac{\text{total conductor area}}{\text{window space factor}}$$

$$= \frac{2\, a_p\, T_p}{K_w} \text{ for single phase transformers} \quad ...(7\cdot27)$$

$$= \frac{4\, a_p\, T_p}{K_w} \text{ for three phase transformers} \quad ...(7\cdot28)$$

Area of window A_w=height of window × width of window $=H_w \times W_w$

The ratio of height to width of window, H_w/W_w is between 2 to 4

Assuming a suitable value for ratio H_w/W_w, the height and width of window can be calculated.

7·43 Width of window for optimum output. Let D be the distance between adjacent limbs as shown in Fig 7·68. Now,

D=width of iron + width of bare conductors + width of insulation and clearance.

Let m be the space occupied by insulation and clearance etc. along the width

∴ Width occupied by copper plus iron $D'=D-m$.

Width of bare conductors in window

=width occupied by iron plus conductors − width occupied by iron $=D'-d$.

Let S be the output in VA of transformer per unit height of window.

∴ $S=E_t\, T_h\, I$ where T_h=turns per unit height.

Now $E_t=4{\cdot}44\, f\, \Phi_m=44{\cdot}4\, f\, B_m\, A_t=4{\cdot}44\, f\, B_m\, k_c d^2$.

∴ For a fixed value of frequency and flux density, E_t is proportional to d^2.

Now IT_h=mmf per unit height$=\delta\, a\, T_h$

where a=area of each conductor and δ=current density

Now aT_h=height of conductors × width of conductors

=width of conductors (as height is unity).

∴ IT_h is proportional to (δ × width of copper in window)

or IT_h is proportional to $(D'-d)$ for a constant value of current density.

∴ $S=Ad^2(D'-d)=Ad^2D'-Ad^3$ where A is a constant.

In order to determine the maximum output for a given value of D', S is differentiated with respect to d.

or $$\frac{dS}{dd}=2AD'd-3Ad^2. \text{ For maximum output, } \frac{dS}{dd}=0$$

or $2AD'd=3Ad^2$ ∴ $D=1{\cdot}5d$.

Now $D=D'+m$. The value of m can be taken as $0{\cdot}2d$ with normal designs.

or $D=1{\cdot}7\, d$

∴ The width of window which gives the maximum output is $W_w=D-d=0{\cdot}7d$.

7·44. Design of Yoke. The area of the yoke is taken as 15 to 25 percent larger than that of core for transformers using hot rolled silicon steel. This reduces the value of flux density obtaining in the yoke and therefore there is reduction in the iron losses and the manetising current. For transformers using cold rolled grain oriented steel in the area of yoke is taken equal to that of the core.

The section of the yoke can either be taken as rectangular or it may be stepped. The yoke sections are shown in Fig. 7·15. In the case of rectangular section yokes, the depth of the yoke is equal to the depth of the core. This depth of the core is equal to the width of the largest stamping when square or stepped cores are used. For example in the case of cruciform core (Fig. 7·52), the depth of yoke is equal to a.

For rectangular section yokes,

Area of yoke A_Y=depth of yoke×height of yoke$=D_Y \times H_Y$

where D_Y=width of largest core stamping$=a$

$A_Y=(1{\cdot}15$ to $1{\cdot}25)\ A_{gi}$ for transformers using hot rolled steel

$=A_{gi}$ for transformers using grain oriented steel.

7·45. Overall Dimensions. When dealing with overall dimensions in transformer problems, refer to the following details and diagrams.

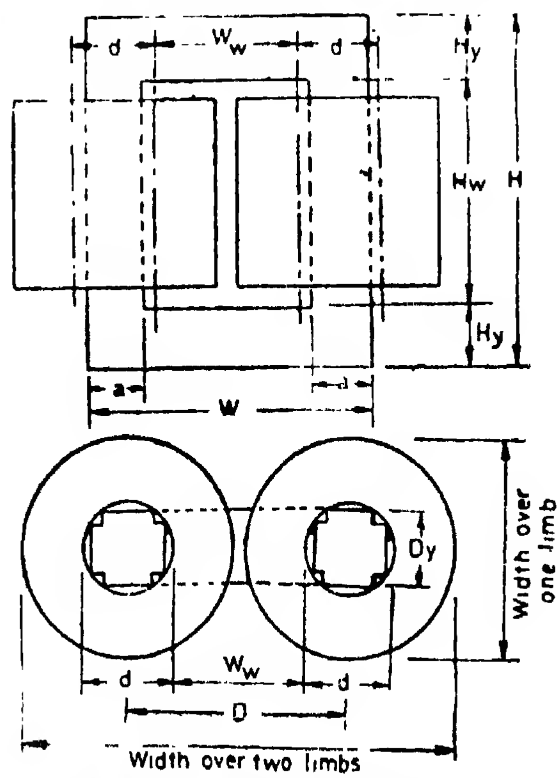


Fig. 7·69. Single phase core type transformer.

a=width of largest stamping,

d=diameter of circumscribing circle,

D=distance between centres of adjacent limbs,

W_w=width of window,

H_w=height of window,

=length of limb,

H_Y=height of yoke,

H=overall height of transformer over yokes or overall height of frame,

W=length of yoke=overall length of frame.

We have the following relations for single phase core type transformers (Fig. 7·69).

$D=d+W_w,\ D_Y=a,$

$H=H_w+2H_Y,\ W=D+a,$

Width over two limbs$=D+$outer diameter of h.v. winding,

Width over one limb=outer diameter of h.v. winding.

We have, for a 3 phase core type transformers (Fig. 7·70).

$$D=d+W_w\ ;\ D_Y=a\ ;\ H=H_w+2H_Y\ ;\ W=2D+a,$$

Width over 3 limbs$=2D+$outer diameter of h.v. winding.

Width over one limb=outer diameter of h.v. winding

For single phase shell type referring to Fig. 7·71

$$D_Y=b,\ H_Y=a,\ W=2W_w+4a,\ H=H_w+2a.$$

Example 7·1. ***Estimate the reduction in volume, expressed as a percentage of original volume of (i) iron core (ii) conductors, in a transformer when hot rolled silicon steel lamina-***

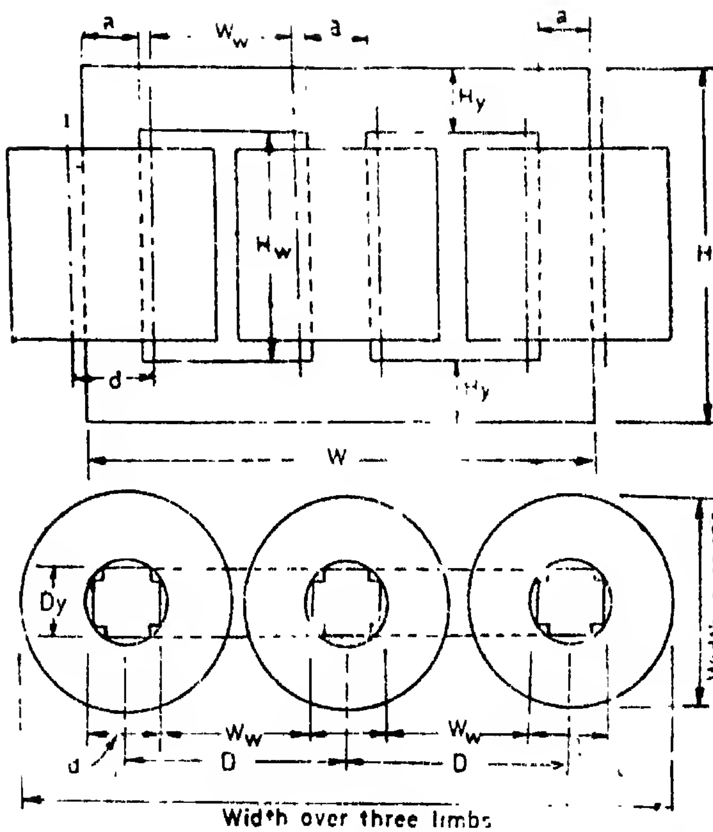


Fig. 7·70. Three phase core type transformer.

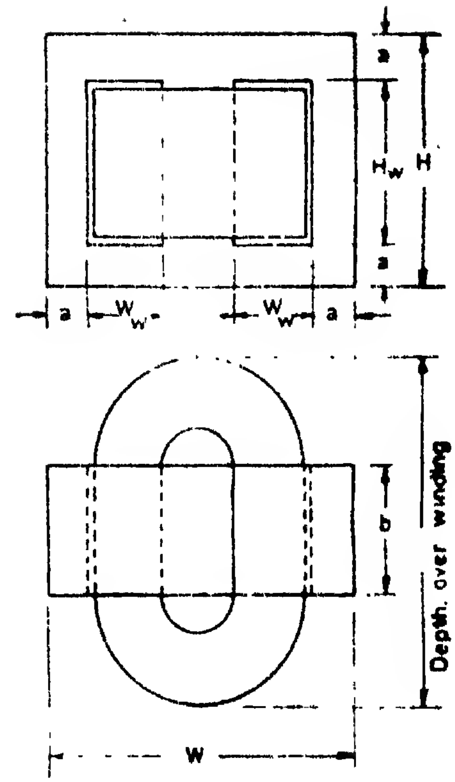


Fig. 7·71. Single phase shell type transformer.

tions worked at a flux density of 1·2 Wb/m² are replaced by cold rolled grain oriented silicon steel laminations worked at 1·8 Wb/m². Circular coils are used and the thickness of winding is negligible as compared with diameter of circumscribing circle. The length of flux path, area of conductor and the number of turns are the same for both cases.

Solution. Let A_{ih} and A_{ic} be the net iron areas with hot rolled and cold rolled laminations respectively.

The net iron area is inversely proportional to flux density.

$\therefore \quad A_{ic}=(1{\cdot}2/1{\cdot}8)\,A_{ih}=0{\cdot}667\,A_{ih}$

Hence, reduction in net iron area$=(1-0{\cdot}667)\times100=33{\cdot}3\%$

Let d_h and d_c be the diameters of circumscribing circle with hot rolled and cold rolled laminations respectively. Diameter of circumscribing circle is proportional to $\sqrt{A_i}$.

$\therefore \quad d_c=\sqrt{0{\cdot}667}\quad d_h=0{\cdot}817\,d_h.$

The length mean turn is πd. Therefore, length of mean turn of conductors wound on cold rolled steel is 0·817 times that of conductors wound on hot rolled steel. Since the number of turns and area of each conductor are the same in both cases, the reduction in volume of conductors with the use of cold rolled steel is $(1-0{\cdot}817)\times100=18{\cdot}3\%$.

Example 7·2. *Show that the output of a 3 phase core type transformer is :*

$$Q=5{\cdot}23\,f\,B_m\,Hd^2\,H_w\times10^{-3}\ \text{kVA}$$

where f=frequency, Hz ; B_m=maximum flux density, Wb/m² ; d=effective diameter of core, m ; H=magnetic potential gradient in limb, A/m ; H_w=height of limb (window), m.

Solution. kVA output of a three phase transformer $Q=3EI\times10^{-3}$

$$=3\times4{\cdot}44\,f\,\Phi_m\,TI\times10^{-3}=3\times4{\cdot}44\,f\,B_m\,A_i\,TI\times10^{-3}.$$

In a three phase core type transformer each limb has one primary and one secondary winding wound on it and therefore total mmf over one limb$=2TI$.

$\therefore$ Magnetic potential gradient $H=\dfrac{\text{mmf}}{\text{height of limb}}$

$$=\frac{2TI}{H_w} \quad \text{or} \quad TI=\frac{HH_w}{2} \qquad \text{Also } A_i=(\pi/4)\,d^2$$

Substituting the value of A_i and TI in the expression for Q, we have

$$Q=3\times4{\cdot}44\,f\,B_m\times\frac{\pi}{4}d^2\times H\,\frac{H_w}{2}\times10^{-3}=5{\cdot}23\,f\,B_m\,Hd^2\,H_w\times10^{-3}\text{ kVA}.$$

Example 7·3. *Calculate the kVA output of a single phase transformer from the following data :*

$$\frac{\textit{core height}}{\textit{distance between core centres}}=2{\cdot}8,\quad \frac{\textit{diameter of circumscribing circle}}{\textit{distance between core centres}}=0{\cdot}56,$$

$$\frac{\textit{net iron area}}{\textit{area of circumscribing circle}}=0{\cdot}7$$

current density$=2{\cdot}3$ *A/mm*^{2}, *window space factor*$=0{\cdot}27$,
frequency $=50$ *Hz*, *flux density of core*$=1{\cdot}2$ *Wb/m*^{2},
distance between core centres$=0{\cdot}4$ *m.*

Solution. Distance between core centres $D=0{\cdot}4$ m.

$\therefore$ Core height (window height) $H_w=2{\cdot}8\times0{\cdot}4=1{\cdot}12$ m.

Diameter of circumscribing circle $d=0{\cdot}56\times0{\cdot}4=0{\cdot}224$ m.

Width of window $W_w=D-d=0{\cdot}4-0{\cdot}224=0{\cdot}176$ m

$\therefore$ Area of window $A_w=H_w\times W_w=0{\cdot}112\times0{\cdot}176=0{\cdot}197$ m^2

Area of circumscribing circle is$=(\pi/4)\,d^2=0{\cdot}0394$ m^2

$\therefore$ Net iron area $A_i=0{\cdot}7\times0{\cdot}0394=0{\cdot}0276$ m^2

From Eqn. 7·6, for a single phase transformer

$$Q=2{\cdot}22\,f\,B_m\,K_w\,\delta\,A_w\,A_i\times10^{-3}\text{ kVA}$$

$$=2{\cdot}22\times50\times1{\cdot}2\times0{\cdot}27\times2{\cdot}3\times10^6\times0{\cdot}197\times0{\cdot}0276\times10^{-3}=450\text{ kVA}.$$

Example 7·4. *Determine the dimensions of core and yoke for a 200 kVA, 50 Hz single phase core type transformer. A cruciform core is used with distance between adjacent limbs equal to 1·6 times the width of core laminations. Assume voltage per turn 14 V, maximum flux density 1·1 Wb/m², window space factor 0·32, current density 3 A/mm², and stacking factor = 0·9. The net iron area is 0·56 d² in a cruciform core where d is the diameter of circumscribing circle. Also the width of largest stamping is 0·85 d.*

Solution. Voltage per turn $E_t=4{\cdot}44\,f\,\Phi_m=4{\cdot}44\,f\,B_m\,A_i$

$\therefore$ Net iron area $A_i=\dfrac{14}{4{\cdot}44\times50\times1{\cdot}1}=0{\cdot}0573$ m^2

$\therefore$ Diameter of circumscribing circle $d=\sqrt{A_i/0{\cdot}56}=\sqrt{0{\cdot}0573/0{\cdot}56}=0{\cdot}32$ m.

Width of largest stamping $a=0{\cdot}85\,d=0{\cdot}85\times0{\cdot}32=0{\cdot}272$ m.

Distance between core centres $D=1{\cdot}6\,a$ (given)$=1{\cdot}6\times0{\cdot}272=0{\cdot}435$ m.

Width of window $W_w=D-d=0{\cdot}435-0{\cdot}32=0{\cdot}115$ m.

From Eqn. 7·6, for a single phase transformer,

$Q = 2.22\, f\, B_m\, K_w\, \delta\, A_w A_i \times 10^{-3}$

$200 = 2.22 \times 50 \times 1.1 \times 0.32 \times 3 \times 10^6 \times A_w \times 0.0573 \times 10^{-3}$

$\therefore$ Window area $A_w = 0.0293$ m^2 $\therefore$ Height of window $H_w = 0.0298/0.115 = 0.26$ m.

Using the same stepped section for the yoke as for core

Depth of yoke $D_y = a = 0.272$ m and height of yoke $H_y = 0.272$ m.

Referring to Fig. 7.70,

Overall height of frame $H = H_w + 2H_y = 26 + 2 \times 0.272 = 0.804$ m.

Overall length of frame $W = D + a = 43.5 + 0.272 = 0.737$ m

Example 7.5. *Calculate approximate overall dimensions for a 200 kVA, 6600/440 V, 50 Hz, 3 phase core type transformer. The following data may be assumed : emf per turn = 10 V ; maximum flux density = 1.3 Wb/m^2 ; current density = 2.5 A/mm^2 , window space factor = 0.3 ; overall height = overall width ; stacking factor = 0.9. Use a 3 stepped core.*

For a three stepped core.

Width of largest stamping = 0.9 d, and

Net iron area = 0.6 d^2 where d is the diameter of circumscribing circle.

Solution. Net iron area $A_i = \dfrac{E_t}{4.44\, f B_m} = \dfrac{10}{4.44 \times 50 \times 1.3} = 0.0347$ m^2.

Diameter of circumscribing circle $d = \sqrt{0.0347/0.6} = 0.24$ m,

and width of largest stamping $a = 0.9 \times 0.24 = 0.216$ m.

Using a 3 stepped section for the yoke

Height of yoke $H_y = a = 0.216$ m, depth of yoke $D_y = a = 0.216$ m.

From Eqn. 7.10, for a 3 phase transformer,

$Q = 3.33\, f\, B_m\, K_w\, \delta\, A_w A_i \times 10^{-3}$

or $200 = 3.33 \times 50 \times 1.3 \times 0.3 \times 2.5 \times 10^6 \times A_w \times 0.0347 \times 10^{-3}$

$\therefore$ Window area $A_w = 0.0355$ m^2 or $H_w \times W_w = 0.0355$ m^2

The given condition is, overall height = overall width or $H = W$

Now, referring to Fig. 7.71

$H = H_w + 2H_y = H_w + 2 \times 0.216 = H_w + 0.432$

$W = 2D + a = 2(W_w + d) + a = 2W_w + 0.48 + 0.216 = 2W_w + 0.696$

As $H = W$, we have : $H_w + 0.432 = 2W_w + 0.696$

or $H_w = 2W_w + 0.264$

$\therefore$ $(2W_w + 0.264)W_w = 0.0355$ or $2W_w^2 + 0.264\, W_w - 0.0355 = 0$

or width of window $W_w = 0.083$ m and height of window $H_w = \dfrac{0.0355}{0.083} = 0.428$ m

Thus the dimensions of core are :

Distance between adjacent core centres $D = W_w + d = 0.323$ m.

Overall height $H = H_w + 2H_y = 0.86$ m.

Overall width $W = 2D + a = 0.862$ m.

Example 7.6. *The ratio of flux to full load mmf in a 400 kVA, 50 Hz, single phase core type power transformer is 2.4×10^{-6}. Calculate the net iron area and the window area of the transformer. Maximum flux density in the core is 1.3 Wb/m^2, current density 2.7 A/mm^2 and window space factor 0.26. Also calculate the full load mmf.*

Solution. From Eqn. 7.13,

$$K = \sqrt{4.44\, f\, (\Phi_m/AT)\, 10^3} = \sqrt{4.44 \times 50 \times 2.4 \times 10^{-6} \times 10^3} = 0.732$$

Voltage per turn $E_t = K\sqrt{Q} = 0.732\sqrt{400} = 14.64$ V.

$\therefore$ Flux $\quad \Phi_m = \dfrac{E_t}{4.44 f} = \dfrac{14.64}{4.44 \times 50} = 0.066$ Wb.

Net iron area $\quad A_i = \dfrac{\Phi_m}{B_m} = \dfrac{0.066}{1.3} = 0.0507$ m^2.

Window area of single phase transformer

$$A_w = \frac{Q}{2.22 f B_m K_w \delta A_i \times 10^{-3}}$$

$$= \frac{400}{2.22 \times 50 \times 1.3 \times 0.26 \times 2.7 \times 10^6 \times 0.0507 \times 10^{-3}} = 0.0777 \text{ m}^2.$$

Full load mmf $AT = \dfrac{\Phi_m}{2.4 \times 10^{-6}} = \dfrac{0.066}{2.4 \times 10^{-6}} = 27500$ A.

Example 7.7. *Determine the main dimensions of the core, the number of turns and the cross-section of the conductors for a 5 kVA, 11000/400 V, 50 Hz, single phase core type distribution transformer. The net conductor area in the window is 0.6 times the net cross-section of iron in the core. Assume a square cross-section for the core, a flux density 1 Wb/m², a current density 1.4 A/mm², and a window space factor 0.2. The height of window is 3 times its width.*

Solution. Given that :

Net conductor area $=0.6 \times$ net iron area $\quad$ or $\quad K_w A_w = 0.6 A_i$

$\therefore$ Window area $\quad A_w = \dfrac{0.6}{K_w} A_i = \dfrac{0.6}{0.2} A_i = 3A_i$.

From Eqn. 7.6, for a single phase transformer,

$Q = 2.22 f B_m K_w \delta A_w A_i \times 10^{-3}$

or $\quad 5 = 2.22 \times 50 \times 1.0 \times 0.2 \times 1.4 \times 10^6 \times 3 A_i \times A_i \times 10^{-3}$

or net iron area $A_i = 0.00732$ m^2. Gross iron area $A_{gi} = \dfrac{0.00732}{0.9} = 0.00814$ m^2.

$\therefore$ Width of core $a = \sqrt{0.00814} \simeq 0.09$ m. Gross iron area provided $= 0.0081$ m^2.
Net iron area provided $= 0.00729$ m^2, Window area $A_w = 3 \times 0.00729 = 0.02187$ m^2.
Height of window $H_w = 3W_w \quad \therefore \quad 3W_w^2 = 0.02187$, $\quad$ But $H_w \times W_w = A_w$,
or width of window $W_w \simeq 0.085$ m, and height of window $H_w = 3 \times 0.085 \simeq 0.255$ m.
The yoke has the same gross area as the core. Gross area of yoke $A_y = 0.081$ m^2.

Depth of yoke $D_y = a = 0.09$ m, $\quad \therefore$ Height of yoke $H_y = \dfrac{0.0081}{0.09} = 0.09$ m.

Flux $\Phi_m = B_m A_i = 1.0 \times 72.9 \times 10^{-4} = 7.29 \times 10^{-3}$ Wb.

Voltage per turn $E_t = 4.44 f \Phi_m = 4.44 \times 50 \times 7.29 \times 10^{-3} = 1.625$ V.

Primary turns $= \dfrac{\text{primary voltage}}{\text{secondary voltage}} \times$ secondary turns $= \dfrac{11000}{400} \times 246 = 6765$

(The turns of the low voltage should be calculated first and that of high voltage afterwards by using the voltage ratio).

Secondary turns $T_s = \dfrac{\text{secondary voltage}}{\text{voltage per turn}} = \dfrac{400}{1.625} = 246$.

Primary winding current $I_p = \dfrac{5000}{11000} = 0.455$ A.

Area of primary winding conductor $a_p = \frac{I_p}{\delta} = \frac{0.455}{1.4} = 0.384$ mm^2.

Using circular conductors, diameter of primary conductor $= \sqrt{0.324 \times 4/\pi} = 0.642$ mm

Secondary winding current $I_s = \frac{5000}{400} = 12.5$ A.

Area of secondary winding conductor $a_s = \frac{I_s}{\delta} = \frac{12.5}{1.4} = 8.93$ mm^2.

Using a square conductor 3×3 mm^2.

Area of secondary conductor $a_s = 9$ mm^2.

Overall dimensions of core—

Referring to Fig. 7·6.

Distance between core centres $D = a + W_w = 0.09 + 0.085 = 0.175$ m.

Length of frame $W = D + a = 0.175 + 0.09 = 0.265$ m.

Height of frame $H = H_w + 2H_y = 0.255 + 2 \times 0.09 = 0.435$ m.

Example 7·8. *Calculate the main dimensions and winding details of a 100 kVA 2000/400 volt, 50 Hz, single phase shell type, oil immersed, self cooled transformer. Assume : Voltage per turn, 10 V flux density in core, 1·1 Wb/m^2 ; current density, A/mm^2 window space factor, 0·33.*

The ratio of window height to window width and ratio of core depth to width of central limb = 2·5. The stacking factor is 0·9.

Solution. Net iron area $A_i = \frac{E_t}{4.44\, f\, B_m} = \frac{10}{4.44 \times 50 \times 1.1} = 0.441$ m^2.

Gross iron area $A_{gi} = \frac{0.041}{0.9} = 0.0555$ m^2.

Referring to Fig. 7·71, $\frac{b}{2a} = 2.5$ (given). We have, gross iron area $A_{gi} = 2a \times b$

$\therefore$ $2.5\,(2a)^2 = 0.04555$ or width of central limb, $2a = 0.135$ m.

Core depth $b = 2.5 \times 0.135 = 0.3475$ m.

The yoke carries half of the flux in the central limb. Assuming the same flux density in the core as in the limb, the area of yoke is equal to half the area of the central limb.

Gross area of yoke $A_y = \frac{0.04555}{2} = 227.75 \times 10^{-4}$ m^2.

Depth of yoke $D_Y = b = 0.3375$ m. $\therefore$ Height of yoke $H_y = \frac{22.275 \times 10^{-3}}{0.3375} = 0.0675$ m.

The side limbs carry half of the flux in the central limb. Therefore, the width of side limb is half of the width of central limb. Width of side limb $a = 0.0675$ m.

From Eqn. 7·6 for a single phase transformer

$Q = 2.22\, f\, B_m\, K_w\, \delta\, A_i\, A_w \times 10^{-3}$

$100 = 2.22 \times 50 \times 1.1 \times 0.33 \times 2 \times 10^6 \times 0.041 \times A_w \times 10^{-3}$

We have, $H_w \times W_w = 0.0303$ and $\frac{H_w}{W_w} = 3$.

$\therefore$ Window area $A_w = 0.0303$ m^2.

or $3W_w^2=303\times10^{-4}$. Thus width of window $W_w=0{\cdot}1$ m

Height of window $H_w=0{\cdot}3$ m

Referring to Fig 7·71

Overall height of frame $H=H_w+2H_Y=0{\cdot}\ \ +2\times0{\cdot}0675=0{\cdot}435$ m.

Overall length of frame $W=2W_w+4a=2\times0{\cdot}1+4\times0{\cdot}0675=0{\cdot}47$ m.

Overall depth of frame $=b=0{\cdot}3375$ m.

Windings

H.V. winding turns $T_p=\dfrac{2000}{10}=200$, L.V. winding turns $T_s=\dfrac{400}{10}=400$.

H.V. winding current $I_p=\dfrac{100\times1000}{2000}=50$ A.

H.V. winding conductor area $a_p=\dfrac{50}{2}=25$ mm².

L.V. winding current $I_s=\dfrac{100\times1000}{400}=250$ A.

L.V. winding area $a_s=\dfrac{250}{2}=1.5$ mm².

Example 7·9. *Calculate the core and window areas of a 400 kVA, 50 Hz, single phase, core type power transformer. The following data may be assumed : Ratio of weight of iron to weight of copper=4, Ratio of length of mean turn of copper to length of mean flux path=0 5 ; maximum flux density=1 5 Wb/m² ; current density=2·2 A/mm² ; density of copper=8·9×10³ kg/m³ ; density of iron=7·8×10³ kg/m³, copper space factor=0·12.*

Solution. From Eqn. 7·18,

$$C=\left(\frac{1}{2{\cdot}22}\cdot\frac{L_{mt}}{l_i}\cdot\frac{g_c}{g_i}\times10^3\right)^{1/2}=\left(\frac{1}{2{\cdot}22}\times0{\cdot}5\times\frac{8{\cdot}9\times10^3}{7{\cdot}65\times10^3}\times10^3\right)^{1/2}=16$$

From Eqn. 7·17, net iron area

$$A_i=C\times\left(\frac{Q}{fB_m\,\delta}\cdot\frac{G_i}{G_c}\right)^{1/2}$$

$$=16\times\left(\frac{400}{50\times15\times2{\cdot}2\times10^6}\times4\right)^{1/2}=0{\cdot}0478\ \text{m}^2$$

From Eqn. 7·6, window area

$$A_w=\frac{Q}{2{\cdot}22\,f\,B_m\,K_w\,\delta A_i\times10^{-3}}$$

$$=\frac{400}{2{\cdot}22\times50\times1{\cdot}5\times0{\cdot}12\times2{\cdot}2\times10^6\times0{\cdot}0498\times10^{-3}}=0{\cdot}183\ \text{m}^2.$$

Example 7·10. *Calculate the ratio of weight of iron to weight of copper, net iron area, voltage per turn and the constant K (where voltage per turn $E_t=K\sqrt{kVA}$) for a 500 kVA, 50 Hz single phase core type power transformer for (i) maximum efficiency to occur at 90 percent of full load (ii) minimum cost (iii) minimum weight (iv) minimum volume. Assume : maximum flux density 1 5 Wb/m² ; current density=2·75 A/mm² ; resistivity of copper at 75°C=2 1×10⁻⁸ Ωm ; density of iron=7·65×10³ kg/m³ ; density of copper=8 9×10³ kg/m³ ; ratio of specific cost of copper to specific cost of iron for built up cores=4 ; ratio of length of mean length of turn of windings to length of flux path=0·5 ; stray load loss=10 percent of full load copper loss ; iron loss per kg for 1·5 Wb/m²=1 23 W Assume an extra loss for joints=20 percent of total iron loss.*

Solution. Specific iron loss $p_i = 1{\cdot}2 \times 1{\cdot}23 = 1{\cdot}475$ W/kg.

Specific copper loss $= \dfrac{\delta^2 \rho}{8{\cdot}9 \times 10^3} = \dfrac{(2{\cdot}75 \times 10^6)^2 \times 2{\cdot}1 \times 10^{-8}}{8{\cdot}9 \times 10^3} = 17{\cdot}8$ W/kg.

Specific copper loss including stray load loss $p_c = 1{\cdot}1 \times 17{\cdot}8 = 19{\cdot}6$ W/kg.

Ratio of weight of iron to weight of copper to give maximum efficiency at 90% full load.

$$\frac{G_i}{G_c} = \frac{x^2 p_c}{p_i} = (0{\cdot}9)^2 \times \frac{19{\cdot}6}{1{\cdot}475} = 10{\cdot}57.$$

Now, $$C = \left(\frac{1}{2{\cdot}22} \cdot \frac{L_{mt}}{l_i} \cdot \frac{g_c}{g_i} \times 10^3\right)^{1/2}$$

$$= \left(\frac{1}{2{\cdot}22} \times 0{\cdot}5 \times \frac{8{\cdot}9 \times 10^3}{7{\cdot}65 \times 10^3} \times 10^3\right)^{1/2} = 16$$

Net iron area $$A_i = C\left(\frac{Q}{fB_m\delta} \cdot \frac{G_i}{G_c}\right)^{1/2} = 16\left(\frac{500}{50 \times 1{\cdot}5 \times 2{\cdot}75 \times 10^6} \times 10{\cdot}75\right)^{1/2}$$
$$= 0{\cdot}0819 \text{ m}^2.$$

Voltage per turn $E_t = 4{\cdot}44\, f\, B_m\, A_i = 4{\cdot}44 \times 50 \times 1{\cdot}5 \times 0{\cdot}0819 = 27{\cdot}27$ V.

$\therefore$ Constant $$K = \frac{E_t}{\sqrt{Q}} = \frac{27{\cdot}27}{\sqrt{500}} = 1{\cdot}22.$$

(*ii*) Ratio weight of iron to weight of copper to give minimum cost is $G_i/G_c = c_c/c_i = 4$

$\therefore$ Net iron area $$A_i = 16\left(\frac{500 \times 4}{50 \times 1{\cdot}5 \times 2{\cdot}75 \times 10^6}\right)^{1/2} = 0{\cdot}05 \text{ m}^2$$

$$E_t = 4{\cdot}44 \times 50 \times 1{\cdot}5 \times 0{\cdot}05 = 16{\cdot}65 \text{ V}$$

and $$K = \frac{16{\cdot}65}{\sqrt{500}} = 0{\cdot}745$$

(*iii*) For minimum weight, $\dfrac{G_i}{G_c} = 1$

$\therefore$ $A_i = 0{\cdot}025$ m², $E_t = 8{\cdot}34$ V, and $K = 0{\cdot}375$

(*iv*) For minimum volume

$$\frac{G_i}{G_c} = \frac{g_i}{g_c} = \frac{7{\cdot}65}{8{\cdot}9} = 0{\cdot}86$$

$\therefore$ $A_i = 0{\cdot}023$ m² $E_t = 7{\cdot}7$ V and $K = 0{\cdot}344$.

Example 7·11. *A h.v. disc winding has 9 coils each separated by an oil duct as shown in Fig 7·62 page 394. Each coil has 10 turns as shown Compare the series capacitance of the interleaved disc coil pair as shown in Fig. 7 65 (b) page 3·95 with that of normal pair as shown in Fig. 7·65 (a).*

Solution. (*i*) *Normal Coil*. Let E be the voltage per coil pair and C_t the turn to turn capacitance.

$\therefore$ Voltage between adjacent turns $E_t = E/10$ as there are 10 turns in each coil.

$\therefore$ Electric field energy between successive and adjacent turns

$$W_t = \frac{1}{2} C_t \left(\frac{E}{10}\right)^2$$

There are 8 complete pair of ducts (as there are 9 coils) and therefore the total electric field energy

$$W = 8 \times \tfrac{1}{2} C_t (E/10)^2 = 0{\cdot}04\, C_t E^2 \qquad \ldots(i)$$

Let C_{s1} be the total effective series capacitance between turns 1 and 10.

$\therefore$ Total electric field energy $W = \frac{1}{2} C_{s1} E^2$ ...(*ii*)

Equating (*i*) and (*ii*), we have : $C_{s1} = 0{\cdot}08\ C_t$.

(*ii*) ***Interleaved coil*** The geometry of the turns is unchanged and therefore C_t remains the same. However, the interleaving effect increases the voltage between adjacent (but no longer successive) turns to 5 E_t (*e.g* turn 1 to 6, 2 to 7 etc.) or to 4 E_t (*e.g.* 6 to 2 and 7 to 3 etc).

$\therefore$ Total electrostatic field energy

$$W_t = \frac{1}{2} C_t\,[4(5E/10)^2 + 4(4E/10)^2] = 0{\cdot}82\ C_t E^2 \qquad \text{...(iii)}$$

Let C_{s2} be the effective series capacitance in this case.

Therefore, total field energy $= \frac{1}{2} C_{s2} E^2$ ...(*iv*)

Equating (*iii*) and (*iv*), the effective series capacitance $C_{s2} = 1{\cdot}64\ C_t$

Hence, ratio of effective series capacitance for interleaved and normal coils is

$$\frac{C_{s2}}{C_{s1}} = \frac{1{\cdot}64\ C_t}{0{\cdot}08\ C_t} = 20{\cdot}5$$

Therefore, α for the interleaved coils is $1/\sqrt{20{\cdot}5} = 0{\cdot}22$ times that for normal coils. Hence the voltage distribution in interleaved coils with surge voltages is much more uniform than that in normal coils.

OPERATING CHARACTERISTICS

7·46. Resistance of winding.

Let L_{mtp}, L_{mts} = length of primary and secondary windings respectively, m ;

r_p, r_s = resistance of primary and secondary windings respectively, m.

$$\therefore \qquad r_p = \rho\,\frac{T_p\,L_{mtp}}{a_p} \quad \text{and} \quad r_s = \rho\,\frac{T_s\,L_{mts}}{a_s}$$

Total I^2R loss in windings $P_c = I_p^2 r_p + I_s^2 r_s$

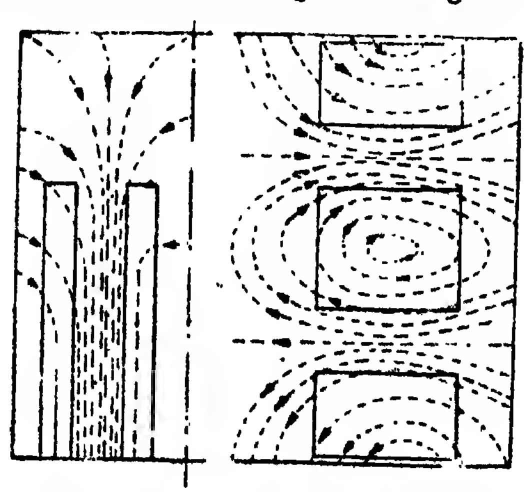
$\therefore$ Total resistance (per phase) of transformer referred to primary side

$$R_p = \frac{P_c}{I_p^2} = r_p + \left(\frac{I_s}{I_p}\right)^2 r_s = r_p + \left(\frac{T_p}{T_s}\right)^2 r_s \qquad \text{...(7·30)}$$

Per unit resistance $\quad \varepsilon_r = \dfrac{I_p R_p}{V_p}$ ...(7·31)

7·47. Leakage reactance of winding. The estimation of leakage reactance is primarily the estimation of the distribution of leakage flux and the resulting flux leakages of the primary and the secondary windings. The distribution of the leakage flux depends upon the geometrical configuration of the coils and the neighbouring iron masses, and also on the permeability of the latter.

The most common arrangements of the core and the windings are of those of core and shell types as shown in Fig. 7·72. Fig. 7·72 (*a*) shows the case of cylindrical core type of windings of equal axial length. The leakage field is mainly packed into the space between the windings (*i.e.* into the duct) and runs parallel with the core for nearly the full length of the coils. The distribution of leakage flux in the case of shell type of transformers having sandwich coils is shown in Fig. 7·72 (*b*). The field is mainly packed into the ducts between the windings, and runs parallel along the width of the coils in this case.



(*a*) Concentric winding (*b*) Sandwitch coils

Fig. 7·72. Leakage flux in transformer windings.

In both of the above two cases, the field is sufficiently symmetrical and geometrical and therefore considerable simplifications can be used to evolve usab'e approximate mathematical relationships.

7·47·1. Leakage reactance of core type transformers. The arrangement of windings of a core type transformer is shown in Fig. 7·73. The problem of calculation of leakage flux and consequent leakage reactance is greatly simplified by making the following *assumption* :

1. The primary and the secondary windings have an equal axial length.
2. The flux paths are parallel to the windings along the axial height.
3. The permeance of the leakage flux path external to the winding L_c is taken to be so large as to require the expenditure of a ngligible mmf i.e. whole of the winding mmf is expended on the length of flux path of length L_c ; this path being entirely through air. Therefore, in fact, the assumption is that the mmf requird for iron parts is negligible.
4. The primary winding mmf I_pI_p is equal to secondary winding mmf I_sT_s Therefore, the magnetizing mmf and hence magnetizing current is equal to zero.

   The total mmf $AT=I_pT_p=I_sT_s$.
5. Half of the leakage flux in the duct links with each winding.
6. The lengths of mean turn of the windings are equal
7. The reluctance of flux path through yokes is negligible. Therefore, the reluctance of yoke does not affect the flux distribution.
8. The windings are uniformly distributed and therefore the winding mmf varies linearly from zero from one end to AT at the other end.

Three leakage flux paths Φ_p, Φ_s and Φ_o are shown Φ_p, Φ_s are the leakage fluxes in the primary and the secondary windings respectively while Φ_o is the flux through the duct. Let :

L_o=mean circumference of duct,
L_c=axial height of windings,
b_p, b_s=radial width of primary and secondary windings respectively,
a=width of radial duct.

The flux leakages of the windings can be found as under.

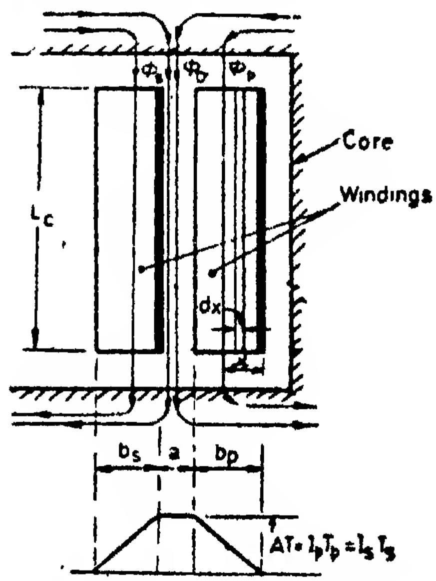


Fig. 7·73. Leakage flux and mmf distribution in core type transformers.

Conductor portion. Consider an infinitesimal strip of width dx at a distance x from the edge of primary winding along its width.

Mmf acting across the strip

$$=\frac{x}{b_p}.I_pT_p=(I_pT_p)\frac{x}{b_p}.$$

Permeance of strip$=\mu_0\frac{L_{mtp}}{L_c}dx$

∴ Flux in the strip

$$=(I_pT_p)\frac{x}{b_p}\times\mu_0\frac{L_{mtp}}{L_c}dx$$

$$=\mu_0\frac{L_{mtp}}{L_c}I_pT_p\frac{x}{b_p}dx.$$

This flux links with $(x/b_p)T_p$ turns.

∴ Flux linkages of the strip

$$=d\psi_1=\mu_0\frac{L_{mtp}}{L_c}I_pT_p\frac{x}{b_p}dx\times\frac{x}{b_p}T_p=\mu_0\frac{L_{mt}}{L_c}I_pT_p^2\left(\frac{x}{b_p}\right)^2dx$$

Hence, flux linkages of primary winding due to flux in the strip

$$\psi_1=\int d\psi_1=\mu_0 \frac{L_{mtp}}{L_c} I_p T_p^2 \int_0^{b_b}\left(\frac{x}{b_p}\right)^2 dx=\mu_0 \frac{L_{mtp}}{L_c} I_p T_p^2 \frac{b_p}{3}.$$

Duct portion. Mmf acting across duct$=I_pT_p$. Permeance of duct$=\mu_0 \frac{L_o}{L_c}a$

Flux in duct $\Phi_o=I_pT_p\times\mu_o \frac{L_o}{L_c}a=\mu_o I_pT_p\frac{L_o}{L_c}a.$

Half of the duct flux links with each of the two windings

or duct flux linking with primary winding$=\frac{1}{2}\mu_o I_pT_p \frac{L_o}{L_c}a.$

This flux links with the entire primary winding.

$\therefore$ Flux linkages of primary winding due to duct flux $\psi_o=\frac{1}{2}\mu_o I_pT_p\frac{L_o}{L_c}a\times T_p$

$$=\frac{1}{2}\mu_o I_pT_p^2 \frac{L_o}{L_c}a$$

Hence, total flux linkages of primary winding

$$\psi_p=\psi_1+\psi_o=\mu_o \frac{I_pT_p^2}{L_c}\left(L_{mtp}\frac{b_p}{3}+bL_o\frac{a}{2}\right)$$

The above expression is simplified by assuming $L_{mt}=L_{mtp}=L_o$

$$\therefore \quad \psi_p=\mu_o I_pT_p^2\frac{L_{mt}}{L_c}\left(\frac{b_p}{3}+\frac{a}{2}\right)$$

Leakge inductance of primary winding $=\frac{\psi_p}{I_p}=\mu_o \frac{L_{mt}}{L_c}\left(\frac{b_p}{3}+\frac{a}{2}\right)$

Leakage reactance of primary winding

$$x_p=2\pi f\mu_o T_p^2 \frac{L_{mt}}{L_c}\left(\frac{b_p}{3}+\frac{a}{2}\right) \quad ...(7{\cdot}73)$$

The leakage reactanee of the secondary winding can be similarly calculated by assuming $L_{mts}=L_o=L_{mt}$.

$\therefore$ Leakage reactance of secondary winding

$$x_s=2\pi f\mu_o T_s^2\frac{L_{mt}}{L_c}\left(\frac{b_s}{3}+\frac{a}{2}\right) \quad ...(7{\cdot}74)$$

Leakage reactance of secondary winding referred to primary side

$$x_s'=x_s\left(\frac{T_p}{T_s}\right)^2=2\pi f\mu_o T_p^2\frac{L_{mt}}{L_c}\left(\frac{b_s}{3}+\frac{a}{2}\right)$$

$\therefore$ Total reactance of transformer (per phase) referred to primary side

$$X_p=x_p+x_s'=2\pi f\mu_o T_p^2 \frac{L_{mt}}{L_c}\left(a+\frac{b_p+b_s}{3}\right) \quad ...(7\ 75)$$

Per unit reactance $\varepsilon_x=\frac{I_pX_p}{V_p}=2\pi f\mu_o\frac{I_pT_p^2}{V_p}\frac{L_{mt}}{L_c}\left(a+\frac{b_p+b_s}{3}\right)$

$$=2\pi f\mu_o \frac{AT}{E_t}\frac{L_{mt}}{L_c}\left(a+\frac{b_p+b_s}{3}\right) \quad ...(7{\cdot}76)$$

as $T_p/V_p = E_t$ and $I_pT_p = AT$ where AT is the total mmf per limb of either coil.

In some cases, one of the windings, usually the h.v. is split up into two parts as shown in Fig. 7·74.

The leakage reactance of transformer (per phase) referred to primary side in this case can be written as :

$$X_p = 2\pi f\mu_o T_p^2 \frac{L_{mt}}{L_c}\left(a + \frac{b_p' + b_p'' + b_s}{3} + \frac{a}{4}\right) \quad ...(7·77)$$

as the p.u. reactance

$$\varepsilon_x = 2\pi f\mu_o \frac{AT}{E_t}\frac{L_{mt}}{L_c}\left(a + \frac{b_p' + b_p'' + b_s}{3} + \frac{a'}{4}\right) \quad ...(7·78)$$

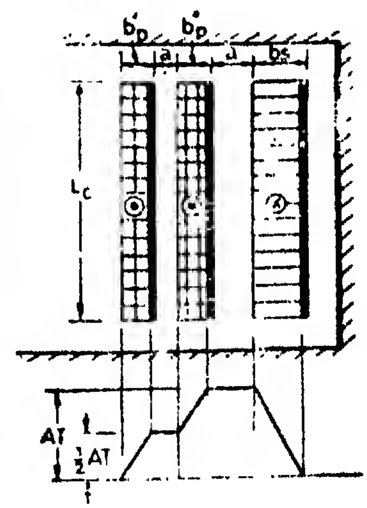


Fig. 7·74. Mmf distribution in transformers with h.v. winding split in two parts.

where a close regulation of voltage is essential as in distribution transformers used for purely domestic purposes, it is necessary to make the leakage reactance small so that the voltage remains within 5% of the rated value. The obvious way to achieve this is to sandwich the h.v. winding between two sections of l.v. winding as shown in Fig. 7·75. For the purposes of analysis the actual arrangement of the windings on each leg can be considered as consisting of two groups of coils connected in series, with each group consisting of half the total number of turns per phase for each winding.

From Eqn. 7·76, total leakage reactance of transformer (per phase) referred to primary side

$$X_{p1} = 2\pi f\mu_o \left(\frac{T_p}{2}\right)^2 \frac{L_{mt}}{L_c}\left(a + \frac{b_p + b_s}{6}\right)$$

and the total leakage reactance of transformer (per phase) referred to primary side

$$X_p = 2X_{p1} = \pi f\mu_o T_p^2 \frac{L_{mt}}{L_c}\left(a + \frac{b_p + b_s}{6}\right) \quad ...(7·79)$$

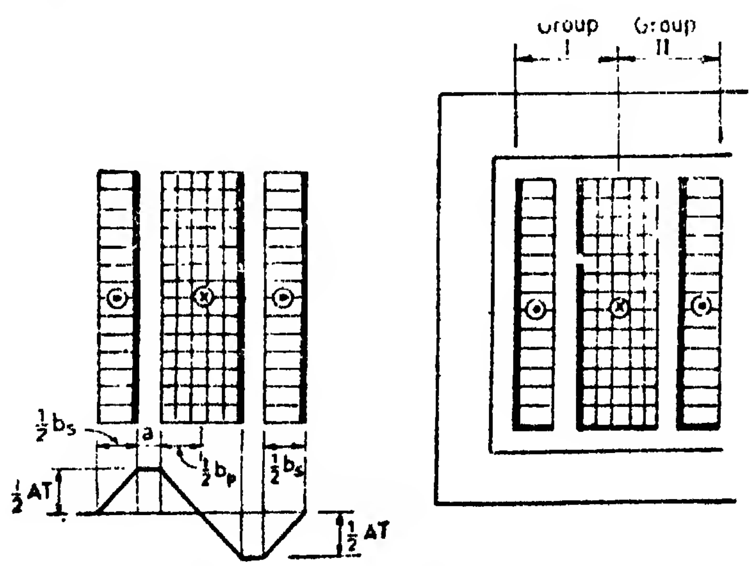


Fig. 7·75. Mmf distribution in core type transformers with h v. winding sandwiched between two sections of l.v. winding.

Comparing Eqn. 7·79 with Eqn. 7·75, it can be safely concluded that the leakage reactance is reduced to nearly one fourth by subdivision and interlacing.

7·47·2. Leakage reactance of sandwich coils. The idealized flux distribution in shell type transformers using sandwich coils is shown in Fig. 7·76.

Let the h.v. winding have n coils. Each of the n coils is sandwiched between two coils of l.v. winding. This requires the l.v. winding to have two half coils, one at each end of the winding and $n-1$ full coils spread in between. Each half coil of l.v. winding contains half the number of turns of that of a full l.v. coil.

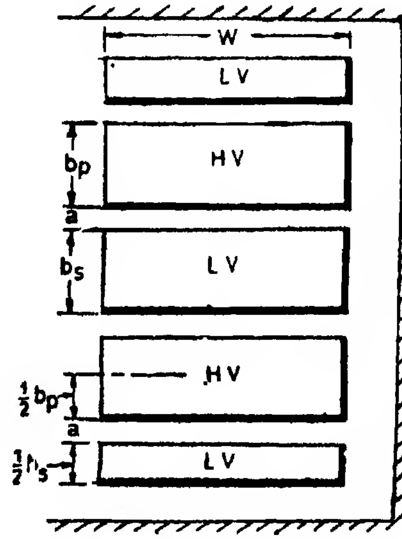


Fig. 7·76. Sandwich winding.

Thus, if there are n coils in the h.v. winding, there will be $n-1$ full coils and 2 half coils in the l.v. winding. The half coils have half the turns of a full coil. The winding can be considered as consisting of $2n$ units connected in series with each unit consisting of a half l.v. coil and a half h.v. coil. Each of these units can be treated on the same basis as that of cylindrical concentric winding. The width of the coil, W, can be taken analogous to axial length L_c, of the cylindrical windings.

Leakage reactance of each unit (per phase) referred to primary side with analogy to Eqn. 7·75

$$X_u=2\pi f\mu_0\left(\frac{T_p}{2n}\right)^2\frac{L_{mt}}{W}\left(a+\frac{b_p+b_s}{6}\right)$$

as $T_p/2n$=number of turns in each half coil of primary winding.

$\therefore$ Total reactance of transformer (per phase) referred on the primary side

$$X_p=2n\times 2\pi f\mu_0\left(\frac{T_p}{2n}\right)^2\frac{L_{mt}}{W}\left(a+\frac{b_p+b_s}{6}\right)$$

$$=\pi f\mu_0\frac{L_{mt}}{W}\;\frac{T_p^2}{n}\left(a+\frac{b_p+b_s}{6}\right) \quad \ldots(7\cdot80)$$

Per unit reactance $$\varepsilon_x=\frac{\pi f\mu_0}{n}\cdot\frac{AT}{E_t}\cdot\frac{L_{mt}}{W}\left(a+\frac{b_p+b_s}{6}\right) \quad \ldots(7\cdot81)$$

Example 7·12. *The current densities in the primary and secondary windings of a transformer are 2·2 and 2·1 A/mm² respectively. The ratio of transformation is 10 : 1 and the length of mean turn of the primary is 10 per cent greater than that of the secondary. Calculate the resistance of the secondary winding given that primary winding resistance is 8 Ω.*

Solution. Resistance of primary winding

$$r_p=\rho\frac{T_p L_{mtp}}{a_p}=\rho\frac{T_pL_{mtp}}{I_p/\delta_p}=\rho\frac{T_pL_{mtp}\delta_p}{I_p}$$

Similarly, resistance of secondary winding

$$r_s=\rho\frac{T_sL_{mts}\delta_s}{I_s}.$$

$$\therefore\quad \frac{r_s}{r_p}=\rho\left(\frac{T_s L_{mts}\delta_s}{I_s}\right)\Big/\rho\left(\frac{T_p L_{mtp}\delta_p}{I_p}\right)$$

$$=\frac{T_s}{T_p}\cdot\frac{L_{mts}}{L_{mtp}}\cdot\frac{\delta_s}{\delta_p}\cdot\frac{I_p}{I_s}$$

$$=\left(\frac{T_s}{T_p}\right)^2\left(\frac{\delta_s}{\delta_p}\right)\left(\frac{L_{mts}}{L_{mtp}}\right) \quad \text{as} \quad \frac{I_p}{I_s}=\frac{T_s}{T_p}$$

Hence $$r_s=\left(\frac{T_s}{T_p}\right)^2\left(\frac{\delta_s}{\delta_p}\right)\left(\frac{L_{mts}}{L_{mtp}}\right) r_p=\left(\frac{1}{10}\right)^2\times\left(\frac{2\cdot1}{2\cdot2}\right)\left(\frac{1}{1\cdot1}\right)\times 8$$

$$=0\cdot0694\ \Omega.$$

Example 7·13. *A 300 kVA, 6600/400 V, 50 Hz, delta/star 3 phase core type transformer has the following data :*

width of h.v. winding=25 mm ; width of l.v. winding=16 mm ; height of coils=0·5 m ;
length of mean turn=0·9 m ; h.v. winding turns=830 ;
width of duct between h.v. and l.v. windings=15 mm.

(a) Calculate the leakage reactance of the transformer referred to the h.v. side.

(b) If the l.v. coil is split into two parts with one part on each side of the h.v. coil, calculate the leakage reactance referred to the h.v. side. Assume that there is a duct 15 mm wide between h.v. winding and each part of l.v. winding.

Solution. Leakage reactance referred to the primary side

$$X_p=2\pi f \mu_0 T_p^2 \frac{L_{mt}}{L_c}\left(a+\frac{b_p+b_s}{3}\right)$$

$$=2\pi\times50\times4\pi\times10^{-7}\times(830)^2\times\frac{0\cdot9}{0\cdot5}\left(0\cdot015+\frac{0\cdot025+0\cdot016}{3}\right)=14\ \Omega$$

(*b*) The l.v. winding divided into two parts, one on each side of h.v. winding and therefore Eqn. 7·79 is used.

$$X_p=\pi f \mu_0 T_p^2 \frac{L_{mt}}{L_c}\left(a+\frac{b_p+b_s}{3}\right)$$

$$=\pi\times50\times4\pi\times10^{-7}\times(830)^2\times\frac{0\cdot9}{0\cdot5}\left(0\cdot015+\frac{0\cdot025+0\cdot016}{6}\right)=5\cdot36\ \Omega.$$

Example 7·14. *A, 100 kVA. 2000/400 V, 50 Hz, single phase shell type transformer has sandwich coils. There are two full h.v. coils, one full l.v. coil and, two half l.v. coils. Calculate the value of leakage reactance referred to h.v side. Also calculate p. u. leakage reactance. The data given is :*

depth of h.v. coil=40 mm, depth of l.v. coil=36 mm,
depth of duct between h.v. and l.v.=16 mm,
width of winding=0·12 m, length of mean turn=1·5 m.
The number of turns in h.v. winding are 200.

Solution. From Eqn. 7·80, leakage reactance referred to h.v. side,

$$X_p=\pi f \mu_0 \frac{L_{mt}}{W}\frac{T_p^2}{n}\left(a+\frac{b_p+b_s}{6}\right)$$

$$=\pi\times50\times4\pi\times10^{-7}\times\frac{1\cdot5}{0\cdot12}\times\frac{(200)^2}{2}\left(0\cdot016+\frac{0\cdot04+0\cdot036}{6}\right)=1\cdot41\ \Omega.$$

H.V. winding current at full load $=\dfrac{100\times1000}{2000}=50$ A.

$\therefore$ Per unit leakage reactance $=\dfrac{50\times1\cdot41}{2000}=0\cdot0353.$

7·48. Regulation. Fig. 7·77 (*a*) shows an approximate equivalent circuit of transformer with parameters referred to primary side.

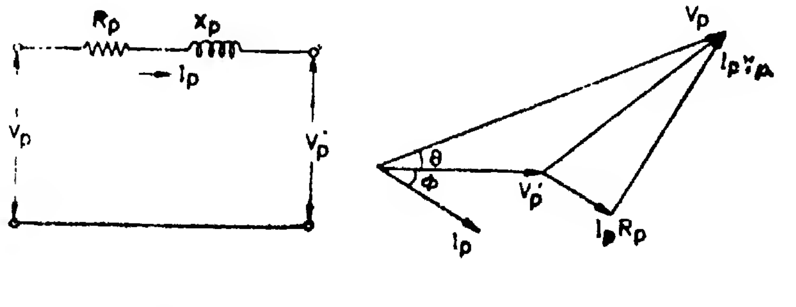


Fig. 7·77. Simplified equivalent circuit and phasor diagram with lagging load.

On no load the secondary terminal voltage $V_p'=V_p$. The drop in secondary terminal voltage from no load to full load can be calculated by drawing the phasor diagram. Fig. 7·77 (*b*) shows the phasor diagram at a lagging power factor $\cos\phi$. From the phasor diagram :

$$V_p=\sqrt{(V_p'+I_pR_p\cos\phi+I_pX_p\sin\phi)^2+(I_pX_p\cos\phi-I_pR_p\sin\phi)^2}$$

Assuming that the angle θ between V_p and V_p' is very small, we have :

$$V_p=V_p'+I_pR_p\cos\phi+I_pX_p\sin\phi \quad \text{or} \quad V_p-V_p'=I_pR_p\cos\phi+I_pX_p\sin\phi.$$

The p.u. regulation, for full load rated output Q and full load current I_p, is :

$$\varepsilon=\frac{V_p-V_p'}{V_p}=\frac{I_pR_p\cos\phi+I_pX_p\sin\phi}{V_p}=\varepsilon_r\cos\phi+\varepsilon_x\sin\phi \qquad \ldots(7\cdot81)$$

If the regulation is large and the phase shift between V_p and V_p' is not justified. For this case the following relationship should be used.

$$\varepsilon=\varepsilon_r\cos\phi+\varepsilon_x\sin\phi+\tfrac{1}{2}(\varepsilon_x\cos\phi-\varepsilon_r\sin\phi)^2. \qquad \ldots(7\cdot82)$$

Example 7·15. *Estimate the per unit regulation, at full load and 0·8 power factor lagging, for a 300 kVA, 50 Hz, 6600/400 V, 3 phase, delta/star, core type transformer. The data given is :*

H.V. winding—

outside diameter = 0·36 m, inside diameter = 0·29 m, area of conductor = 5·4 mm².

L.V. winding—

outside diameter = 0·26 m, inside diameter = 0·22 m, area of conductor = 170 mm².

Length of coils = 0·5 m, voltage per turn = 8V, resistivity = 0·21 Ω/m/mm².

Solution. L.V. voltage per phase $V_s=\dfrac{400}{\sqrt{3}}=231$ V.

L.V. turns per phase $T_s=\dfrac{V_s}{E_t}=\dfrac{231}{8}\simeq 29$. H.V. voltage per phase $V_p=6600$ V.

H.V. turns per phase $T_p=\dfrac{6600}{231}\times 29=826$

Mean diameter of l.v. winding $=\dfrac{0\cdot26+0\cdot22}{2}=0\cdot24$ m.

Length of mean turn of l.v. winding $=\pi\times0\cdot24=0\cdot752$ m.

Resistance of l.v. winding $r_s=\dfrac{0\cdot021\times29\times0\cdot752}{170}=0\cdot00269\ \Omega.$

Mean diameter of h.v. winding$=(0{\cdot}36+0{\cdot}29)/2=0{\cdot}325$ m.
Length of mean turn of h.v. winding$=\pi\times0{\cdot}325=1{\cdot}02$ m.

Resistance of h.v. winding$=\dfrac{0{\cdot}021\times826\times1{\cdot}02}{5{\cdot}4}=3{\cdot}28\ \Omega$.

Resistance of transformer referred to primary $R_p=3{\cdot}28+0{\cdot}00269(826/29)^2=5{\cdot}47\ \Omega$.

H.V. winding current per phase $I_p=\dfrac{300\times1000}{3\times6600}=15{\cdot}1$ A

$\therefore$ P.U. resistance $\varepsilon_r=\dfrac{15{\cdot}1\times5{\cdot}47}{6600}=0{\cdot}0126$.

Mean diameter$=(0{\cdot}36+0{\cdot}22)/2=0{\cdot}29$ m.
Length of mean turn $L_{mt}=\pi\times0{\cdot}29=0{\cdot}91$ m.
Width of l.v. winding $b_s=(0{\cdot}26-0{\cdot}22)/2=0{\cdot}02$ m.

Width of h.v. winding $b_p=(0{\cdot}36-0{\cdot}29)/2=0{\cdot}035$ m, Width of duct $a=(0{\cdot}29-0{\cdot}26)/2=0{\cdot}015$ m.

Leakage reactance of transformer referred to primary side

$$X_p=2\pi\times50\times4\pi\times10^{-7}\times(826)^2\times\frac{0{\cdot}91}{0{\cdot}5}\left(0{\cdot}015+\frac{0{\cdot}035+0{\cdot}02}{3}\right)=17{\cdot}3\ \Omega.$$

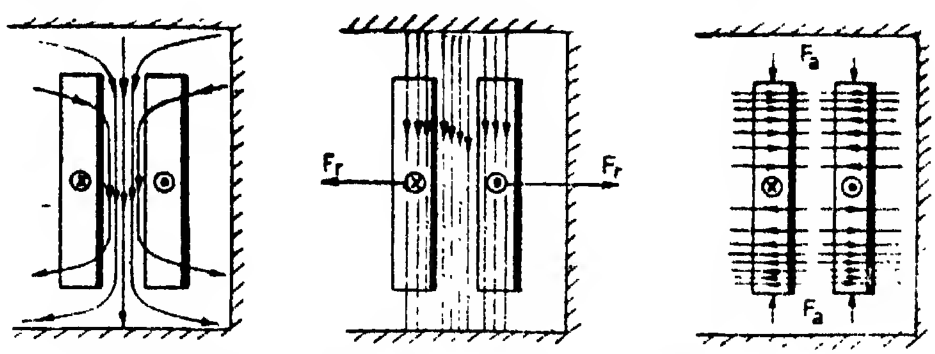
and p.u. leakage reactance $\varepsilon_x=\dfrac{15{\cdot}1\times17{\cdot}3}{6600}=0{\cdot}0395$

From Eqn. 7·81, per unit regulation

$$\varepsilon=\varepsilon_r\cos\phi+\varepsilon_x\sin\phi=0{\cdot}0126\times0{\cdot}8+0{\cdot}0395\times0{\cdot}6=0{\cdot}0338.$$

7·49. Mechanical forces. It is well known that parallel conductors repel or attract each other according to whether the currents they carry are in the opposite or in the same direction. Mechanical forces exist in transformers (even though they are stationary) on account of the interaction of the current in the windings with the leakage field surrounding them (windings). It should be noted that the two windings carry currents in the opposite direction.

The leakage field in a core type transformer is shown in Fig. 7·78(*a*). The leakage



(*a*) Leakage field (*b*) Axial leakage field (*c*) Radial leakage field
Fig. 7·78. Leakage fields and menachical forces.

field can be resolved into its axial and radial components as shown in Fig. 7·78 (*b*) and (*c*). The mechanical forces produced on account of interaction of two components of leakage flux with current are consequently in two different perpendicular directions. The forces in two different directions are :

(*i*) radial forces and (*ii*) axial forces.

1. **Radial Forces**. Fig. 7·78 (*b*) shows the interaction of axial component of leakage flux with the current carrying windings. Applying Fleming's left hand rule it is found that the force acts in the radial direction. The nature of radial force is such that it tries to burst the outer winding (subjecting it to tensile force) and to crush the inner winding (subjecting it to compressive force). This is obvious, because the primary and the secondary windings of a transformer carry currents in the opposite direction and therefore produce a force of repulsion tending to pull the outer winding and to compress the inner winding.

2. **Axial Forces** Fig 7·78 (*c*) shows the interaction of radial component of leakage flux with the current carrying windings. Applying Fleming's left hand rule, it is found that forces produced due to this interaction act in the axial direction. The forces acting on the transformer windings in the axial direction are compressive in nature. However, if there is an axial magnetic assymmetry between the two windings, a tensile axial force is produced.

The magnitude of force acting on a conductor is proportional to the product of current in it and the intensity of magnetic field due to currents in the neighbouring conductors, and since the latter is itself proportional to the current, the forces are proportional to square of the current. The mechanical forces with normal load currents are low compared to the strength of windings and hence, are not noticeable. However, when large transformers are subjected to short circuit conditions at full voltage, the short circuit currents may reach 10 to 25 times full load value with forces increasing to 100 to 625 times under normal conditions. Under fault conditions (such as short circuits or other severe faults) the mechanical forces may become so great as to damage the transformer permanently unless the windings are solidly *braced* *i.e.* mehanically reinforced.

7·50. Calculation of mechanical forces. The mechanical forces produced in a transformer may be classified as :

(*i*) radial forces, (*ii*) axial forces, and (*iii*) unbalanced axial forces due to magnetic assymmetry.

7·50·1. Radial forces. Consider a core type transformer with cylindrical windings as shown in Fig. 7·79. The winding has T turns and carries an instantaneous current i. The instantaneous mmf acting across the duct is iT. The flux density in the duct is

$$B_a=\mu H=\mu_0\, iT/L_c.$$

The flux density in the windings varies from 0 to B_a. Therefore, the average value of flux density in the duct is $B_a/2$ or $B_{av}=\mu_0\, iT/2L_c$.

$\therefore$ Radial force acting on a strip situated at a mean radius R within an angle $d\theta$ of a coil is

$$dF_r=\text{turns}\times\text{flux density}\times\text{current}\times\text{length of strip}$$

$$=T\times B_{av}\times i\times R d\theta=T\times\frac{\mu_0\, iT}{2L_c}\times i\times Rd\theta=\frac{\mu_0}{2}(iT)^2\frac{R}{L_c}\,d\theta$$

Hence, total instantaneous radial fore acting on the coil

$$F_r=\int_0^{2\pi} dF_r=\frac{\mu_0}{2}\,(iT)^2\frac{R}{L_c}\int_0^{2\pi} d\theta=\frac{\mu_0}{2}\,(iT)^2.\frac{2\pi R}{L_c}$$

But $2\pi R=$ mean periphery of coil $=L_{mt}$

$$\therefore \qquad F_r=\frac{\mu_0}{2}\,(iT)^2\,\frac{L_{mt}}{L_c} \qquad \ldots(7{\cdot}83)$$

Now, short circuit fault current is I/ε_x where I is the rms value of full load current and ε_x is the per unit leakage reactance. (The resistance of the winding is neglected). On worst fault conditions the instantaneous value of current may become equal to twice the maximum value of current under short circuit conditions.

∴ under worst fault conditions

$$i=2\sqrt{2}I/\varepsilon_x$$

Hence, maximum radial force under worst fault conditions is

$$F_{r(max)f}=\frac{\mu_0}{2}\left(2\sqrt{2}\frac{I}{\varepsilon_x}\right)^2\frac{L_{mt}}{L_c}$$

$$=4\mu_0\left(\frac{I}{\varepsilon_x}\right)^2\frac{L_{mt}}{L_c}$$

$$=2\mu_0\left(\frac{\sqrt{2}I}{\varepsilon_x}\right)^2\cdot\frac{L_{mt}}{L_c} \quad \ldots(7\cdot84)$$

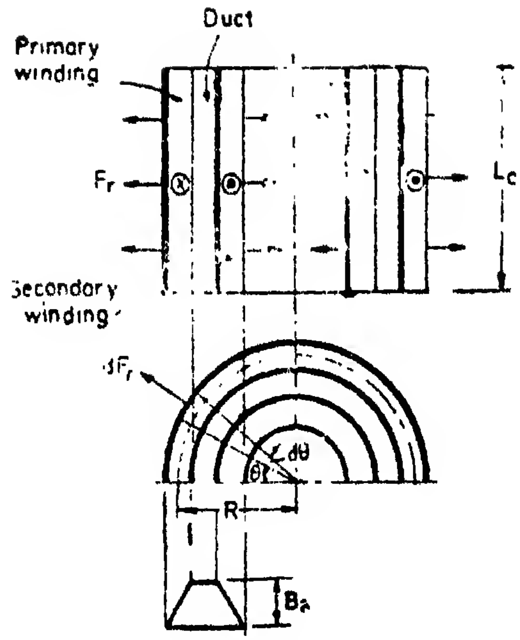


Fig. 7·79. Radial forces on windings

The ratio of maximum radial force under worst possible fault conditions to maximum radial force under normal conditions is :

$$\frac{F_{r(max)f}}{F_{r(max)n}}=\frac{2\mu_0(\sqrt{2}I/\varepsilon_x)^2L_{mt}/L_c}{\mu_0(\sqrt{2}I)^2L_{mt}/L_c}=\frac{2}{\varepsilon_x^2}.$$

It is clear from above, that the radial force is inversely proportional to square of the p.u. reactance. Let consider a transformer with $\varepsilon_x=0\cdot05$. The maximum radial force acting in this transformer is $2/(0\cdot05)^2=800$ times that maximum radial force under normal operation.

The average radial force under normal operating conditions is

$$F_{r(av)}=\frac{\mu_0}{2}\ \frac{L_{mt}}{L_c}\cdot\frac{1}{2\pi}\int_0^{2\pi} i^2T^2\,d(\omega t)$$

$$=\frac{\mu_0}{2}\ \frac{L_{mt}}{L_c}\ \frac{1}{2\pi}\int_0^{2\pi}\sqrt{2}\,I\sin^2\omega t\,d(\omega t)$$

$$=\frac{\mu_0}{2}\,(IT)^2\,\frac{L_{mt}}{L_c} \quad \ldots(7\cdot85)$$

The windings of a transformer must be designed to withstand radial forces under worst fault conditions. It is clear that circular coils should be preferred as they are the strongest to withstand radial forces. This is because the tendency of the radial forces is to make the coils circular and as they are already circular there is no deformation. However, if rectangular or square coils are used, they would be deformed by the radial forces. In shell type transformers, the coils are rectangular but these coils are mechanically braced for a considerable part of their length and hence are mechanically strong.

7·50·2. Axial Forces. The winding currents interact with the radial component of leakage flux to produce axial compressive forces tending to squeeze the windings together at

the middle. With symmetrical windings these forces are not noticeable even at short circuits.

In shell type transformers having sandwich coils symmetrically arranged as in Fig 7.80, there is a force of repulsion between the coils of each of pairs 1-2, 3-4, 5-6, and 7-8. The force on coil 2 is opposite in direction to that acting on 3, likewise for coils 4—5 and 6—7. Thus there is force acting on coils 1 and 8 only. The magnitude of this force is computed as follows :

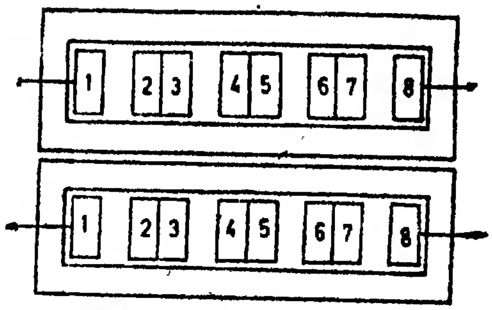


Fig. 7·80. Axial forces on windings.

Average value of flux density in the winding,

$$B_{av}=\frac{1}{2}\ \mu_0\ \frac{iT_n}{W}$$

T_n=number of turns in the end half coil.

Instantaneous axial force due to interaction of magnetic field and the current i in T_n turns of end coil is :

$$F_a={}_{av}\ L_{mt}\ T_n\ i=\frac{\mu_0}{2}\ iT_n\ .\ \frac{L_{mt}}{W}\ T_n\ i=\frac{\mu_0}{2}\ .\ (iT_n)\ \frac{L_{mt}}{W}.$$

But $\qquad T_n=\dfrac{T}{2n}$ as $2n$ is the total number of half coils.

$$\therefore \qquad F_a=\frac{\mu_0}{2}\left(i\ \frac{T}{2n}\right)^2\left(\frac{L_{mt}}{W}\right)=\frac{\mu_0}{8n^2}\ (iT)^2\left(\frac{L_{mt}}{W}\right) \qquad ...(7{\cdot}86)$$

Average value of this force is : $F_{a(av)}=\dfrac{\mu_0}{8n^2}\ .\ (IT)^2\ \dfrac{L_{mt}}{W}.$ $\qquad ...(7{\cdot}87)$

7·50.3. Forces due to assymmetry. The analysis carried out till now has been for symmetrical arrangements of windings. The axial length of windings is assumed equal with windings placed symmetrically with respect to each other. The mmf distribution in a symmetrical arrangement is uniform and therefore the magnetic centres of the windings are coincident. Fig 7·81 shows a winding arrangement where the two windings have an axial assymetry as a result of which the magnetic centres are not coincident. The force, F, between the two windings can be resolved into its radial and axial components. The radial component of force is $F \cos \alpha$ and the axial component is $F \sin \alpha$. The latter may attain large values under fault conditions causing movement of winding in the axial direction which would increase the axial force still further. Therefore, in order to prevent axial movement of the windings, adequate *bracing* of the winding is necessary.

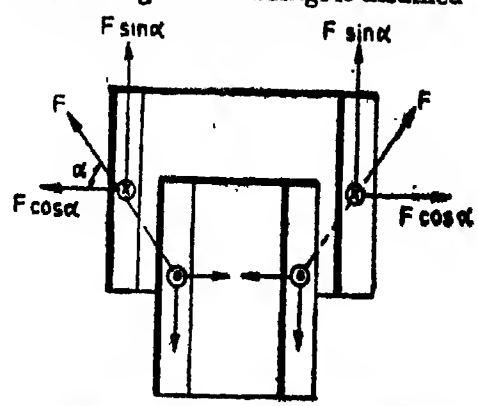


Fig. 7·81. Forces due to magnetic assymmetry.

The causes of axial assymmetry between windings of a transformer are provision of tappings for voltage regulation, manufacturing process of certain types of windings and non-uniform turn insulation. These are explained as under :

1. One of the windings of the transformer, usually the h.v. winding, is provided with tappings. These tappings include and exclude a part of the winding during service. This causes mmf unbalance between the windings in the axial direction.

2. When calculating the axial length of helical windings, one turn is added to the number of turns due to helical wind of the turns. However, at a greater width h_t of the turn, an empty space, free of a conductor material, is formed at the ends of helical (or cylindrical) windings as shown by hatched area. This causes shortening of winding height by approximately $h/2$.

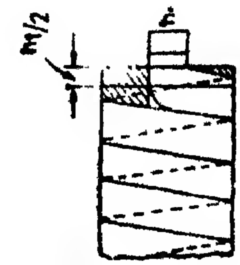


Fig. 7·82. Helical winding.

3. The end turns of the high voltage are reinforced to withstand the effect of surge voltages. Therefore, the end coils of the windings have lesser turns and hence the mmf distribution of the h.v. windings is not uniform along axial direction.

The case of axial assymmetry caused by provision of tappings in the windings is shown in Fig. 7·83. Due to cutting out of a section of one of the windings to provide voltage regulation, the axial length of the winding provided with tappings becomes smaller as compared with the axial length of the other winding. The axial forces produced are on account of radial leakage field resulting from axial assymmetry of windings. Fig 7·83 (*a*) represents worst conditions where the tappings are provided at one end of winding. The coils get further displaced due to assymmetry resulting in further intensification of forces. If the tappings are provided at both the ends, we have an arrangement shown in Fig. 7·83 (*b*). There is thus ideally a lesser tendency to aggrevate the effect. Fig. 7·83 (*c*) shows the effect of providing tappings at the centre of a winding.

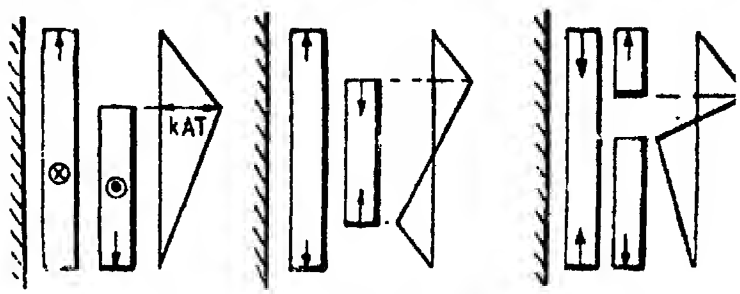


Fig. 7·83. Axial forces of assymmetry.

The force acting between coils of unequal lengths can be calculated by resolving the leakage flux into its axial and radial components. Fig. 7·84 (*a*) shows the case of two windings having unequal axial length, with each winding having an mmf AT. The arrangement is resolved as in (*b*) into two superposed mmf distributions (*i*) and (*ii*) in accordance with the length wise unbalance k. The first case (*i*) is dealt with as if it comprised balanced windings of equal axial lengths, and latter (*ii*) in a corresponding way but with different orientation and appropriate physical dimensions. The windings are assumed to be enclosed between two iron surfaces separated by a distance $2l$ as shown in Fig 7 84 (*c*).

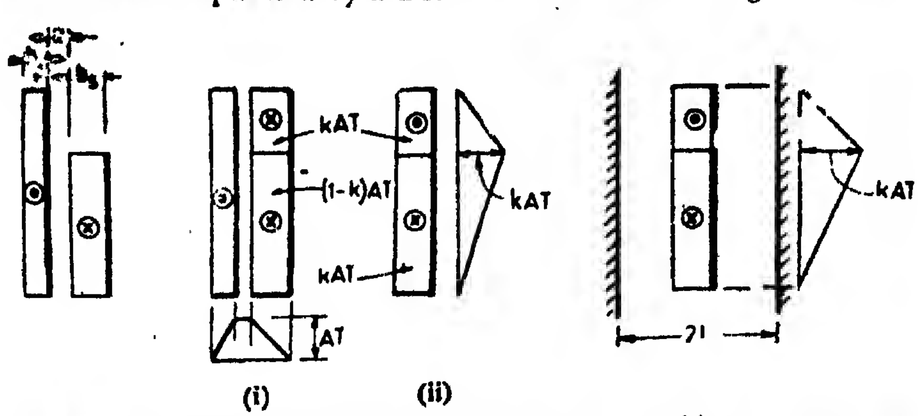


Fig. 7·84. Resolution of mmf of axially assymmetrical windings into two superposed mmfs.

where $l=a+b_p+b_s$.

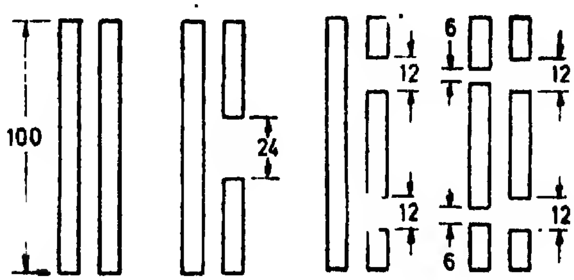
Average value of radial flux density $B_{av(r)}=\frac{1}{2}\mu_0 \frac{k(iT)}{2l}$

∴ Total instantaneous axial force acting on the coil

$$F_a=\frac{\mu_0}{2}\ \frac{k(iT)}{2l}\ L_{mt}\ (iT)=\frac{1}{2}\ \mu_0\ k(iT)^2\frac{L_{mt}}{2(a+b_p+b_s)} \quad ...(7\cdot88)$$

Average axial force $F_{a(av)}=\frac{1}{2}\ \mu_0\ k(IT)^2\frac{L_{mt}}{2(a+b_p+b_s)}$ ...(7·89)

Fig. 7·85 shows the different cases of assymmetry in windings along axial length on account of provision of tappings. Both radial and axial forces are produced. The values of p.u reactance and the values of radial and axial forces produced compared with those produced with symmetrical windings are given.



p.u. reactance	0·1	0·14	0·11
Relative p.u. stress : radial	100	50	83
axial	0	18	9

Fig. 7·85. Axial assymmetry in cylindrical coils.

7·51. Bracing of windings. Inward axial forces are taken by windings themselves. Inward radial forces are passed on to the formers, packing pieces and cores. Outward axial forces must be withstood by end insulation. A well constructed transformer should have suitable choice of conductor dimensions and interturn insulation, and the coils well supported and braced with compressive stresses kept in view. The end supports are not easily arranged to give good insulation and at the same time good mechanical strength. Therefore, the need for second quality (good mechanical strength) should be avoided as far as possible by having good magnetic symmetry and the unpreventive asymmetry caused by provision of tappings should be suitably distributed.

Example 7·16 *A 1000 kVA 6600/400 V, 50 Hz, 3 phase delta/star, core type, oil immersed transformer has 500 turns on the h.v. winding. The height of winding is 0 6 m and the length of mean turn 1·3 m. Calculate the instantaneous radial force on the h v. winding if a short circuit occurs at the terminals of the l.v. winding with h.v energised. The leakage impedance is 5 percent. Take the doubling effect multiplier as 1 8. Also calculate the force at full load.*

Solution. Full load current per phase on h.v. side $=\frac{100\times1000}{3\times6600}=50\cdot5$ A.

∴ Instantaneous peak value of short circuit current

$$=1\cdot8\times\sqrt{2}\times(1/0\cdot05)\times50\cdot5=2580\text{ A}.$$

From Eqn. 7·52, the total instantaneous radial force on the h.v. coil :

$$F_r=\frac{\mu_0}{2}(iT)^2\frac{L_{mt}}{L_c}=\frac{4\pi\times10^{-7}}{2}\ (2580\times500)^2\times\frac{1\cdot3}{0\cdot6}=2\cdot33\times10^6\text{ N}.$$

Force at full load, $F_r=\frac{4\pi\times10^{-7}}{2}(50\cdot5\times500)^2\times\frac{1\cdot3}{0\cdot6}=866$ N.

This shows that the forces under short circuit conditions are considerably large as compared with forces at full load.

Example 7 17. *A 575 kVA, 7500/435 V, 50 Hz, single phase, core type transformer has the following data :*

Width of h.v. winding=27 mm, width of l.v. winding=23 mm, width of duct=15 mm, height of coils =0·35 m, length of mean turn =1·25 m, number of turns in h.v. winding =190, per unit impedance =0·036 ; doubling effect multiplier=1 8.

(a) Find the instantaneous radial force on h.v. winding under short circuit conditions if the height of h.v. and l.v. winding is equal. (b) Find the instantaneous axial force on h.v. winding under short circuit conditions if the h.v. winding is 5 percent shorter than the l.v. winding at one end.

Solution. Rms value of full load current=575×1000/7500= 6·7 A.

Peak current under short circuit conditions,

$$i=1\cdot8\times\sqrt{2}\times1/0\cdot036\times76\cdot7=5420 \text{ A}.$$

(a) From Eqn. 7·52, radial force $F_r=\frac{\mu_0}{2}(iT)^2\frac{L_{mt}}{L_c}$

$$=\frac{4\pi\times10^{-7}}{2}\times(5420\times190)^2\times\frac{1\cdot25}{0\cdot35}=2\cdot38\times10^6 \text{ N}.$$

(b) From Eqn. 7·58, axial force $F_a=\frac{\mu_0}{2}\,k\,(iT)^2\,\frac{L_{mt}}{2\,(a+b_p+b_s)}$

$$=\frac{4\pi\times10^{-7}}{2}\times0\cdot05\times(5420\times190)^2\times\frac{1\cdot25}{(0\cdot015+0\cdot027+0\cdot023)}=3\cdot2\times10^6 \text{ N}.$$

This shows that there is a very large axial force, even though one of the windings is only 5 percent shorter than the other at one end.

7 52. No load current. The no load current I_0 consists of two components : (*i*) magnetizing current I_m (*ii*) loss component I_l and its value is given by $I_0=(I_m^2+I_l^2)^{1/2}$

Thus the estimation of no load current I_0 requires the calculation of its two components I_m and I_l.

7·52·1. No load current of single phase transformers. Let

l_c, l_y=length of flux path through core and yoke respectively ($l_c=H_w$, $l_y=w$),

at_c, at_y=mmf/metre for flux densities in core and yoke respectively.

∴ Total magnetizing mmf $AT_0=2\,at_c\,l_c+2\,at_y\,l_y$+mmf required for joints ...(7·90)

The values of at_c and at_y are taken from the $B-H$ curves for transformer steel (given in chapter 4). The joints in a magnetic circuit may be taken as short air gaps in parallel with iron paths.

The calculation of total mmf is based upon the maximum value of flux density.

∴ Rms value of magnetising current $I_m=AT_0/(\sqrt{2}\,T_p)$ ...(7·91)

But the magnetising is not sinusoidal and therefore the peak factor K_{pk} should be used in place of $\sqrt{2}$.

$$I_m=AT_0/(K_{pk}\,T_p) \qquad ...(7\cdot92)$$

Let the iron losses be P_i, the loss component

$$I_l=P_i/V_p \qquad ...(7\cdot93)$$

The iron losses are calculated by finding the weight of cores and yokes. The loss per kg i taken from relevant curves given in chapter 4.

7·52 2. No load current of three phase transformers.

Total magnetising mmf required for the transformer

$$=3\,at_c\,l_c+2\,at_y\,l_y+\text{mmf required for joints.}$$

Total magnetising mmf required per phase

$$AT_0=\frac{3at_cl_c+2at_yl_y+\text{mmf required for joints}}{3} \qquad \ldots(7\cdot94)$$

$\therefore$ Magnetising current per phase $I_m=AT_0/(\sqrt{2}\,T_p)$ or $I_m=AT_0/(K_{pk}\,T_p)$ ...(7·95)

The values of at_c and at_y are taken from relevant graphs given in chapter 4.

Let P_i be the total iron loss for the three phases. $\therefore$ $I_l=P_i/3V_p$

It is not usual to calculate the no load current in transformers as in a normally designed transformer, the no load current would be below 5 percent and a considerable variation can be made in its value without effecting the transformer performance.

7 51·3. Magnetizing volt-ampere. We have $E_p=4{\cdot}44\,f\,T_p\,B_m\,A_i$.

Magnetizing volt ampere $(VA)_m=E_pI_m=(4{\cdot}44\,f\,T_p\,B_mA_i)\left(\dfrac{AT_0}{\sqrt{2}\,T_p}\right)$

$$=\frac{4{\cdot}44\,f\,B_mA_i(AT_0)}{\sqrt{2}}.$$

Now, AT_0=magnetizing mmf per metre × length of path in iron$=at_ml_i$.

$$\therefore \qquad (VA)_m=\frac{4{\cdot}44\,f\,B_m\,A_i\,at_m\,l_i}{\sqrt{2}}.$$

Weight of iron$=A_i\,l_i\times7{\cdot}8\times10^3$, if the density of iron is assumed as $7{\cdot}8\times10^3$ kg/m³

$\therefore$ Magnetizing volt-ampere/kg

$$=\frac{4{\cdot}44\,f\,B_m\,at_m}{\sqrt{2}\times7{\cdot}8\times10^3}\simeq0{\cdot}4\,fB_mat_m\times10^{-3}.$$

Now a curve can be plotted between B_m and magnetizing volt ampere/kg from B—H curve of the material (value of at_m for different values of B_m will be known from this B—H curve) Manufactures supply the magnetizing VA/kg—B_m characteristics and from it the magnetizing current can be known.

Magnetizing current

$$I_m=\frac{\text{magnetizing }VA\text{/kg}\times\text{weight of core}}{\text{number of phases}\times\text{voltage/phase}} \qquad \ldots(7\cdot96)$$

Example 7·18. *A single phase, 400 V, 50 Hz, transformer is built from stampings having a relative permeability of 1000. The length of the flux path is 2·5 m, the area of cross-section of the core is $2{\cdot}5\times10^{-3}$ m² and the primary winding has 800 turns. Estimate the maximum flux and no load current of the transformer. The iron loss at the working flux density is 2·6 W/kg. Iron weighs $7{\cdot}8\times10^3$ kg/m³. Stacking factor is 0 9.*

Solution. Net iron area $A_i=0{\cdot}9\times2{\cdot}5\times10^{-3}=2{\cdot}25\times10^{-3}$ m²

We have, $E_p=4{\cdot}44f\Phi_m\,T_p=4{\cdot}44\,f\,B_m\,A_i\,T_p$.

$$\therefore \qquad B_m=\frac{400}{4{\cdot}44\times50\times2{\cdot}25\times10^{-3}\times800}\doteq1{\cdot}0\ \text{Wb/m}^2.$$

$\therefore$ Flux in the core$=B_m\,A_i=1\times2{\cdot}25\times10^{-3}=2{\cdot}25\times10^{-3}$ Wb.

Mnagetizing mmf AT_0=reluctance×flux $= \frac{l_i}{\mu A_i} \times \Phi_m = \frac{l_i}{\mu_r \mu_0} B_m$

$$= \frac{2.5 \times 1}{1000 \times 4\pi \times 10^{-7}} = 1980 \ A.$$

Magnetising current $I_m = \frac{AT_0}{\sqrt{2}\, T_p} = \frac{1980}{\sqrt{2} \times 800} = 1.75$ A.

Volume of core$=2.25\times 10^{-3}\times 2.5=5.625\times 10^{-3}$.

∴ Weight of core$=5.625\times 10^{-3}\times 7.8\times 10^{3}=43.8$ kg and iron loss$=2.6\times 43.8$
$=144$ W.

Loss component of no load current $I_l = \frac{P_i}{V\phi} = \frac{114}{400} = 0.2\ 5$ A.

∴ No load current $I_0 = \sqrt{1.75^2 + 0.285^2} = 1.77$ A.

Example 7·19 *Calculate the active and reactive components of no load current of a 100 V, 50 Hz, single phase transformer having the following particulars :*

Core of transformer steel ; Stacking factor=0.9 ; density=7.8×10³ kg/m³ ; length of mean flux path 2.2 m ; gross iron section 10×10⁻³ m² ; primary turns 200 ; joints equivalent to 0.2 mm air gap. Use the following data :

B_m Wb/m²	0·9	1·0	1·2	1·3	1·4
Mmf A/m	130	2.0	420	660	1300
Iron loss W/kg	0·8	1·3	1·9	2·4	2·9

Solution. Gross iron area$=10\times 10^{-3}$ m². Net iron area$=9\times 10^{-3}$ m².

Flux in core $\Phi_m = \frac{400}{4.44 \times 50 \times 200} = 9.02 \times 10^{-3}$ Wb.

Flux density $B_m = \frac{\Phi_m}{A_i} = 9.02 \times 10^{-3} / 9 \times 10^{-3} = 1$ Wb/m².

Corresponding to $B_m=1.0$ Wb/m², mmf/metre$=210$ A, loss per kg$=1.3$ W.

∴ Mmf for iron path$=210\times 2.2=462$ A.

Mmf for joints$=800{,}000\ Bl_g=800{,}000\times 1 \times \frac{0.2}{1000}=160$ A.

∴ Total magnetising mmf, $AT_0=462+160=622$ A.

∴ Reactive (magnetising) current, $I_m = \frac{622}{\sqrt{2} \times 200} = 2.2$ A.

Volume of core $=2.2\times 0.009=0.0198$ m³.

Weight of core $=7.8\times 10^3\times 0.0198=155$ kg. Total iron loss$=155\times 1.3=201.5$ W.

Loss (active) component of no load current $I_l = \frac{201.5}{400} \simeq 0.5$ A.

∴ No load current$=\sqrt{I_m^2+I_l^2}=\sqrt{2.2^2+0.5^2}=2.26$ A.

Example 7·20. *A 6600 V, 60 Hz single phase transformer has a core of sheet steel. The net iron cross sectional area is 22.6×10⁻³ m², the mean length is 2.23 m, and there are*

four lap joints. Each lap joint takes 1/4 times as much reactive mmf as is required per metre of core. If $B_m=1{\cdot}1$ Wb/m², determine (a) the number of turns on the 6600 V winding and (b) the no load current. Assume an amplitude factor of 1·52 and that for given flux density, mmf per metre = 232 A/m ; specific loss = 1·76 W/kg. Specific gravity of plates = 7·5.

Solution (a) Number of turns

$$T=\frac{E}{4{\cdot}44\,f\,B_m\,A_i}=\frac{6600}{4{\cdot}44\times60\times1{\cdot}1\times22{\cdot}6\times10^{-3}}=1100.$$

(b) Mmf required for iron parts $=232\times2{\cdot}23=517$ A.

Mmf required for joints $=4\times\frac{1}{4}\times232=232$ A.

Total magnetizing mmf $AT_0=517+232=749$ A.

Magnetizing current $I_m=\dfrac{AT_0}{K_{pk}T_p}=\dfrac{749}{1{\cdot}52\times\sqrt{2}\times1100}=0{\cdot}318$ A.

(as peak factor K_{pk} = amplitude factor $\times\sqrt{2}$).

Weight of core $=2{\cdot}23\times22{\cdot}6\times10^{-3}\times7{\cdot}5\times10^3=378$ kg

Total iron loss $P_i=1{\cdot}76\times378=665$ W. Loss component $I_l=\dfrac{665}{6600}=0{\cdot}1$ A.

No load current $I_0=\sqrt{(0{\cdot}318)^2+(0{\cdot}1)^2}=0{\cdot}333$ A.

Example 7·21. *A 15000 kVA, 33/6·6 kV, 3 phase star/delta, core type transformer has the following data :*

Net iron area of each limb = $1{\cdot}5\times10^{-3}$ m², net area of yoke = $1{\cdot}8\times10^{-3}$ m², mean length of flux path in each limb = 2·3, mean length of flux path in each yoke = 1·6, number of turns in h.v. winding = 450.

Calculate the no load current. Use the data given in example 7·19 for mmf per metre and loss per kg.

Solution. H.V. winding voltage per phase $=33300/\sqrt{3}=19100$ V.

Flux $\Phi_m=\dfrac{19100}{4{\cdot}44\times50\times450}=0{\cdot}191$ Wb

Flux density in the limbs, B_m (limb) $=\dfrac{0{\cdot}191}{0{\cdot}15}=1{\cdot}27$ Wb/m².

Corresponding to this flux density, $at_l=560$ A/m ; $p_i=2{\cdot}25$ W/kg.

Flux density in the yokes, B_m (yoke) $=\dfrac{0{\cdot}191}{0{\cdot}18}=1{\cdot}06$ Wb/m².

For this flux density, $at_y=260$ A/m ; $p_i=1{\cdot}4$ W/kg.

Total mmf for three limbs $=3\times560\times2{\cdot}3=3860$ A.

Total mmf for two yokes $=2\times260\times1{\cdot}6=832$ A.

Total mmf for limbs and yokes $=3860+832=4692$ A.

Magnetizing mmf per phase $AT_0=4692/3=1564$ A.

∴ Magnetizing current per phase $I_m=1564/(\sqrt{2}\times450)=2{\cdot}46$ A.

Volume of 3 limbs $=3\times2{\cdot}3\times0{\cdot}15=1{\cdot}035$ m³.

Weight of 3 limbs $=1{\cdot}035\times7{\cdot}8\times10^3=8{\cdot}08\times10^3$ kg.

Volume of two yokes $=2\times1{\cdot}6\times0{\cdot}18=0{\cdot}576$ m³.

Weight of two yokes $=0{\cdot}576\times7{\cdot}8\times10^3=4{\cdot}49\times10^3$ kg.

Loss in limbs $=8{\cdot}08\times10^3\times2{\cdot}25=18{\cdot}2\times10^3$ W.

Loss in yokes $=4.49\times10^3\times1.4=6.3\times10^3$ W.

Total iron loss $=18.2\times10^3+6.3\times10^3=24.5\times10^3$ W.

Loss per phase $=24.5\times10^3/3=8.16\times10^3$ W.

$\therefore$ Loss component per phase $I_l=8.16\times10^3/19100=0.427$ A.

No load current $I_0=\sqrt{2.46^2+0.427^2}=2.5$ A.

7·53. Change of parameters with change of frequency.

1. **Effect on Losses.** The specific iron loss is given by $p_i=K_hfB_m^2+K_ef^2B_m^2$ W/kg.

From this relationship it is evident that change of either f or B_m or both, will, in general, result in change of iron losses. As the transformer is designed on the basis of a definite heat dissipation, the I^2R loss should be re-adjusted if the total losses are to remain the same *i.e.* if the core loss under changed conditions is greater than that in the original design the I^2R loss will have to be made smaller, and *vice versa*. With the new I^2R loss, and effective resistance of winding, the new current rating is easily obtained.

Let us examine the effect of change of frequency on iron losses if the voltage remains the same. We have voltage $E=4.44\,f\,B_mA_iT$ and eddy current loss $P_e=K_ef^2B_m^2$.

Therefore as long as E remains constant product (fB_m) remains constant and therefore, eddy current losses remain constant even though the frequency is changed.

We have hysteresis loss $P_h=K_hfB_m^2$. Now if applied voltage is kept constant product fB_m remains constant. Let $fB_m=K$.

$\therefore$ Hysteresis loss$=K_h(fB_m)B_m=K_hKB_m=K_hK.\dfrac{K}{f}=\dfrac{K_hK^2}{f}$.

From above it is clear that the hysteresis losses will decrease with increase in frequency if the voltage is kept constant.

Therefore, the total iron losses decrease if the frequency is increased and the applied voltage is kept constant. Hence with increased frequency, we can afford to have more I^2R loss and thus for the same loss (*i.e.* the same temperature rise) the rating of the transformer can be increased. This fact is illustrated by example 7 23.

2. **Effect on voltage.** If the flux density is also changed, the new voltage rating is determined from the emf equation. For the new voltage total iron loss and the magnetising current and the no load current can be calculated.

3. **Effect on leakage reactance and resistance of windings.** The change in frequency will not have much effect on leakage inductance and therefore leakage reactance will increase linearly with frequency.

Due to skin effect, the effective resistance increases with increase in frequency. This effect is not normally important with small changes in frequency.

Example 7·22. *An 11 kV, 25 Hz transformer has I^2R, hysteresis and eddy current losses 1·6, 0·6 and 0·4 per cent of the output. What will be the percentage losses if the transformer is connected to 22 kV, 50 Hz supply assuming the full load current to remain the same ?*

Solution. Subscript 1 refers to case with 11 kV and 25 Hz.

Subscript 2 refers to case with 22 kV and 50 Hz.

(*a*) ***I^2R loss.*** As the full load current is same in both the cases, the copper loss remains the same.

$\therefore$ I^2R losses$=P_{c2}=P_{c1}=1.6$ per cent of output with 11kV.

The transformer is assumed to be single phase.

$$E_1=4{\cdot}44\,f_1\Phi_{m1}\,T \text{ and } E_2=4{\cdot}44\,f_2\Phi_{m2}T.$$

$$\therefore \quad \frac{E_2}{E_1}=\frac{\Phi_{m2}f_2}{\Phi_{m1}f_1} \quad \text{or} \quad \frac{22}{11}=\frac{\Phi_{m2}}{\Phi_{m1}}\times\frac{50}{25}$$

or $$\Phi_{m2}=\Phi_{m1} \quad \therefore \quad B_{m2}=B_{m1}.$$

(*b*) *Hysteresis loss.* $P_h=K_hB_m^2f$ or $\dfrac{P_{h2}}{P_{h1}}=\dfrac{K_hB_{m2}^2f_2}{K_hB_{m1}^2f_1}=\dfrac{f_2}{f_1}$

or $$P_{h2}=0{\cdot}6\times\frac{50}{25}=1{\cdot}2\% \text{ of output with 11kV.}$$

(*c*) *Eddy current loss.* $P_e=K_eB_m^2f^2$ or $\dfrac{P_{e2}}{P_{e1}}=\dfrac{K_eB_{m2}^2f_2^2}{K_eB_{m1}^2f_1^2}=\dfrac{f_2^2}{f_1^2}$

or $$P_{e2}=0{\cdot}4\times\frac{50^2}{25^2}=1{\cdot}6\% \text{ of output with 11 kV.}$$

The output with 22 kV is double that with 11 kV as the current is the same in both the cases. Therefore,

I^2R, hysteresis and eddy current losses are respectively 0·8%, 0·6% and 0·8% of output with 22 kV.

Example 7 23. *A 40 Hz transformer is to be used on a 50 Hz system Assuming the Steinmetz's co-efficient as 1·6 and losses at lower frequency 1·2%, 0 7% and 0 5% for I^2R, hysteresis and eddy currents respectively. Find (a) losses on 50 Hz for the same supply voltage and current (b) output at 50 Hz for the same total losses as on 40 Hz.*

Solution. (*a*) The voltage and current at both 40 Hz and 50 Hz are the same and therefore the output in both the cases is the same. Subscript 1 refers to 40 Hz and subscript 2 refers to 50 Hz.

The current at both 40 Hz and 50 Hz is the same and therefore the I^2R loss is same in both the cases.

$$\therefore \quad P_{c2}=P_{c1}=1{\cdot}2\%.$$

We have, $\dfrac{E_2}{E_1}=\dfrac{4{\cdot}44\,f_2\Phi_{m2}T}{4{\cdot}44\,f_1\Phi_{m1}T}$ and as $E_2=E_1$

$$\therefore \quad \Phi_{m2}=\Phi_{m1}\times\frac{f_1}{f_2}=\frac{40}{50}\Phi_{m1}=0{\cdot}8\,\Phi_{m1} \quad \text{or } B_{m2}=0{\cdot}8\,B_{m1}.$$

Now, hysteresis loss $P_h=K_hB_m^{1{\cdot}6}f$ or $\dfrac{P_{h2}}{P_{h1}}=\dfrac{K_hB_{m2}^{1{\cdot}6}f_2}{K_hB_{m1}^{1{\cdot}6}f_1}=\dfrac{(0{\cdot}8\,B_{m1})^{1{\cdot}6}f_2}{B_{m1}^{1{\cdot}6}f_1}$

or $$P_{h2}=P_{h1}\times0{\cdot}8^{1{\cdot}6}\times\frac{50}{40}=0{\cdot}875\,P_{h1}=0{\cdot}875\times0{\cdot}7=0{\cdot}61\%.$$

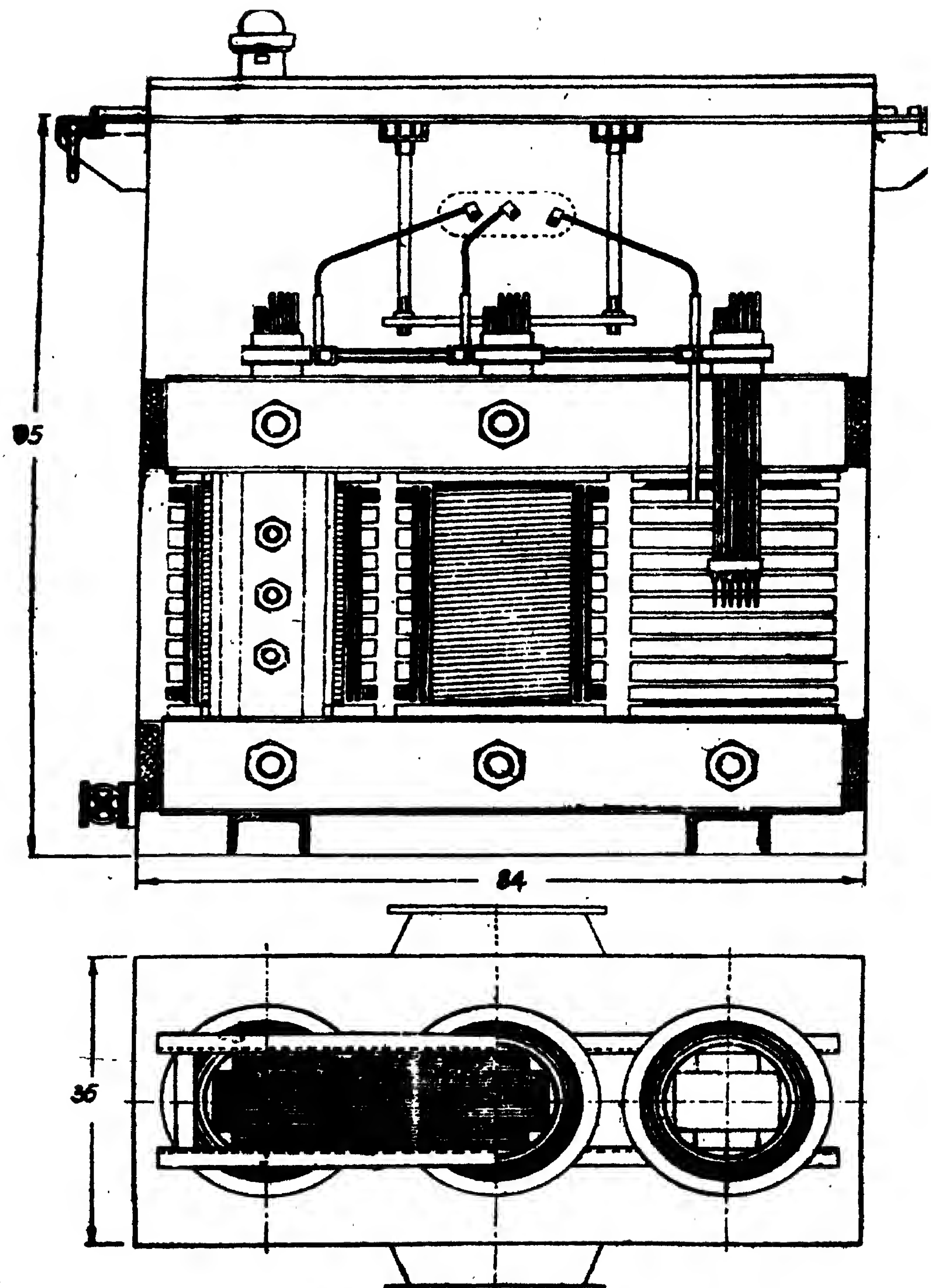
Eddy current loss $P_e=K_eB_{m2}^2f_2^2$ or $\dfrac{P_{e2}}{P_{e1}}=\dfrac{K_eB_{m2}^2f_2^2}{K_eB_{m1}^2f_1^2}=(0{\cdot}8)^2\left(\dfrac{50}{40}\right)^2=1$

$$\therefore \quad P_{e2}=P_{e1}=0{\cdot}5\%.$$

(*b*) If the rating of the transformer is changed, eddy current and hysteresis losses remain the same while there is a change in I^2R loss.

Total loss in 1st case $=1{\cdot}2+0{\cdot}7+0{\cdot}5=2{\cdot}4\%$

Total loss in 2nd case $=1{\cdot}2+0{\cdot}61+0{\cdot}5=2{\cdot}31\%$.



25 KVA, 3 PHASE, 1100/433 VOLT, 50 Hz, CORE TYPE
DISTRIBUTION TRANSFORMER

FIG. 7.60

∴ In order that the loss should be equal in both the cases, the I^2R loss has to be increas- by $2{\cdot}4-2{\cdot}31=0{\cdot}09\%$ in the 2nd case.

∴ I^2R loss in 2nd case $=1{\cdot}2+0{\cdot}09=1{\cdot}29\%$.

I^2R loss varies as the square of the output $\dfrac{P_{c_2}}{P_{c_1}}=\left(\dfrac{\text{output in 2nd case}}{\text{output in 1st case}}\right)^2$

output in second case $=$ output in 1st case $\times\sqrt{P_{c_2}/P_{c_1}}=$ output in 1st case $\times\sqrt{1{\cdot}29/1{\cdot}2}$

$=1{\cdot}038\times$ output in 1st case.

Thus an increase in frequency results in increase in rating.

7·54. Temperature rise of transformers. The problem of temperature rise and cooling of transformers is essentially the same as that of rotating machinery. Similar to the latter, the losses developed in the transformer cores and windings during conversion are converted into thermal energy and cause heating of corresponding transformer parts. From a source the heat is directed, due to thermal gradients, to the places where it may be transferred to a cooling medium *i.e.* to air, or water, depending upon the method of transformer cooling. Heat dissipation occurs in the same way as in electrical machines, *i.e.* by way of radiation and convection.

The path of heat flow is :

(*i*) From the internal most heated spots of a given part (of core or winding) to their outer surfaces in contact with the oil.

(*ii*) From the outer surface of a given transformer part to the oil that cools it.

(*iii*) From the oil to the walls of a cooler, for instance, of the tank.

(*iv*) From the walls of the cooler to the cooling medium—air or water.

In section (*i*) the heat is transferred by conduction In sections (*ii*) and (*iii*), the heat is transferred by convection of the oil

In section (*iv*), the heat is dissipated by both convection and radiation.

7·54·1. Transformer oil as a cooling medium. Tests have shown that an average working temperature of the oil $\theta_o=50$ to 60°C and oil viscosity corresponding to this temperature, the specific heat dissipation due to convection of oil is

$$\lambda_{conv}=40{\cdot}3(\theta/H)^{1/4} \quad \text{W/m}^2\text{—°C}$$

where $\theta=$ temperature difference of the surface relative to oil, °C

$H=$ height of dissipating surface, m.

If we assume an average $\theta=20$°C and $H=0{\cdot}5$ to 1 m, $\lambda_{conv}=80$ to 100 W/m²—°C.

The corresponding figure for convection due to air is 8W/m²—°C. Thus the convection due to oil is 10 times and above that with air. This constitutes a major valuable property of oil as a cooling medium.

7·54·2. Temperature rise in plain walled tanks. The walls of tank dissipate heat by both radiation and convection. It has been found experimentally that a plain tank surface dissipates 6 and 6·5 W/m²—°C by radiation and convection respectively (for a temperature rise of nearly 40°C above an ambient temperature of 20°C). Thus the total loss dissipation is 12·5 W/m²—°C.

$$\therefore \quad \text{Temperature rise } \theta=\frac{\text{total loss}}{\text{specific heat dissipation}\times\text{surface}}=\frac{P_i+P_c}{12{\cdot}5\,S_t} \qquad \ldots(7{\cdot}97)$$

where $S_t=$ heat dissipating surface of tank. The surface, to be considered in applying the above formula, is the total area of the vertical sides plus one half area of the cover, unless the oil is in contact with the cover in which case whole area of the lid should be taken. The area of bottom of the tank should be neglected as it has very little cooling effect.

For transformers of low output, the plain walled tank ge are enough to accommodate the transformer and oil have sufficient surface to keep t emperature rise within limits. But for transformers of large output, the plain walled ta re not sufficient to dissipate losses. This is because volume and hence losses increase be of linear dimensions while the dissipating surface increases as the square of linear nsions. Thus an increase in rating results in an increase in loss to be dissipated per t area giving a higher temperature rise.

Modern oil immersed power transformers with natural oil cooling and a plain tank may be produced for outputs not exceeding 20—30 kVA. Transformers rated for larger outputs must be provided with means to improve the conditions of heat dissipation. This may be done by providing corrugations, tubing or radiators where feasible.

7·55. Design of tank with tubes. If the temperature rise as calculated with plain tank exceeds the specified limits, it can be brought down by provision of tubes. The provision of tubes increases the dissipating area but the increase in dissipation of heat is not proportional to area because tube screen some of the tank surface preventing radiation from there. So there is no change in surface as far as dissipation of heat due to radiation is concerned. But the increase in dissipation of heat is more than what is justified by the increase in surface area. The circulation of oil is improved due to more effective heads of pressure produced by columns of oil in tubes. An addition of about 35 per cent should be made to tube area in order to take into account this improvement in dissipation of loss by convection.

Let the dissipating surface of the tank be S_t

It will dissipate $(6+6.5)\,S_t = 12{\cdot}5\,S_t$ W/°C. Let the area of tubes $= xS_t$.

Loss dissipated by tubes by convection $= 1{\cdot}35 \times 6.5\,xS_t = 8{\cdot}8\,x\,S_t$ W/°C

$\therefore$ Total loss dissipated by tank wall and tubes $= 12{\cdot}5\,S_t + 8.8\,xS_t$

$= S_t\,(12{\cdot}5 + 8.8\,x)$ W/°C

Total area of tank walls and tubes $= S_t + xS_t = S_t\,(1+x)$

$$\therefore \quad \text{Loss dissipated} = \frac{(12.5 + 8{\cdot}8\,x)}{x+1}\ \text{W/m}^2\text{—°C} \qquad \ldots(7.98)$$

$$\text{Temperature rise with tubes } \theta = \frac{P_i + P_c}{S_t\,(12{\cdot}5 + 8{\cdot}8\,x)} \quad \text{or} \quad x = \frac{1}{8.8}\left(\frac{P_i+P_c}{S_t\,\theta} - 12.5\right) \qquad \ldots(7{\cdot}99)$$

$$\text{Total area of tubes} = \frac{1}{8.8}\left(\frac{P_i+P_c}{\theta} - 12.5\,S_t\right)$$

Let l_t and d_t be the length and diameter of each tube respectively.

Area of each tube $= \pi d_t l_t$.

$$\text{Hence, number of tubes } n_t = \frac{1}{8{\cdot}8\,\pi d_t l_t}\left(\frac{P_i+P_c}{\theta} - 12.5\,S_t\right) \qquad \ldots(7{\cdot}100)$$

The area of the tubes can be found out by using the above expression. The diameter of tubes, normally used, is 50 mm and they are spaced at 75 mm. Elliptical tubes with pressed radiators are increasingly being used as they give a greater dissipating surface for smaller volume of oil.

The inner dimensions of the transformer tank are fixed by the active dimensions of the transformer and clearances between windings and grounded parts of transformer.

Width of tank $W_t = 2D + D_e + 2b$ (for three phase)

$= D + D_e + 2b$ (for single phase)

where D = distance between adjacent limbs, D_e = external diameter of h.v. winding and

b=clearance between h.v. winding and tank.

Length of tank $L_t=D_e+2l$

where l=clearance on each side between the winding and tank along the width.

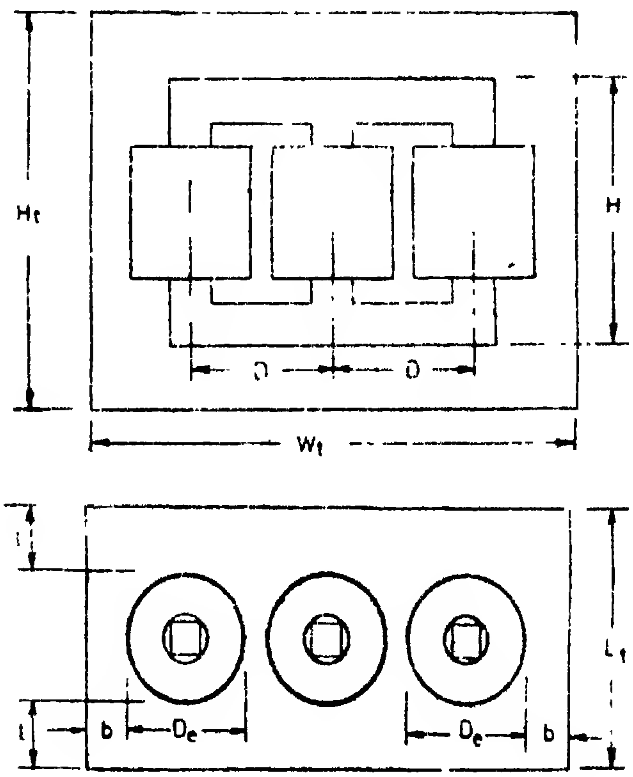


Fig. 7·86. Tank dimensions.

Height of transformer tank $H_t=H+h$

where H=height of transformer frame and h=clearance (height) between the assembled transformer and the tank. This includes the clearance at the base, oil height above the assembled transformer space for terminals and tap changing gear. Typical values of clearances b, l and h are given below :

Voltage kV	*Rating* kVA	*Clearance* mm		
		b	l	h
11 kV or less	less than 1000	40	50	450
	1000—5000	70	90	420
about 11 kV and upto 33 kV	less than 1000	75	100	550
	1000—5000	85	125	550

7·56. Air blast cooling. At present time the radiators are forced cooled by small fans mounted on each radiator. Compared with natural cooling, air blast cooling of a tank increases the heat dissipation by 50 to 60 per cent or

λ=loss dissipated W/m²—°C by both convection and radiation with air blast.

$=(1·5 \text{ to } 1·6)\times 12·5=20$ W/m²—°C.

Transformers upto a capacity of 10 MVA have a cooling radiator system with natural ooling. For 10 MVA upwards air blast cooling of radiator is used.

7·57. Forced oil circulation. The velocity with which natural oil circulation takes lace is very low, of the order of a few mm/s. Investigations of the problem have hown that with an increase in velocity of oil circulation by m times, the transformer output, or the same winding temperature rise, increases $(m)^{1/4}$ times. If for example $m=3$, the ransformer output rises by about 30 per cent. An excessive increase of oil circulating rate unsuitable because this involves large energy losses in the pumping unit. To cool the il, it is circulated through a special oil cooler.

In an oil cooler with natural air cooling the flow rate of the circulating oil is of the rder of 12 litre per minute per kW of losses.

When the cooler is air blast cooled, the transformer output increases roughly to the ame extent as when the tank is air blast cooled.

In transformers with water cooling of the circulating oil, the pipe coils and tube oolers are employed working on the counterflow principle. Cooler surface per 1 kW of osses ranges from 0·18 to 0·25 m². The flow rate of the circulating oil per kW of losses, quals approximately 6 to 8 litre per minute. The water flow rate is about 1.5 litre per minute. The temperature difference between the incoming and outgoing water is usually aken to be nearly 10°C.

Example 7·24. *A 250 kVA, 6600/400 V, 3 phase core type transformer has a total oss of 4800 W at full load. The transformer tank is 1·25 m in height and 1 m × 0·5 m in plan. Design a suitable scheme for tubes if the average temperature rise is to be limited to 35°C. The diameter of tubes is 50 mm and are spaced 75 mm from each other. The average height of tubes is 1·05 m.*

Specific heat dissipation due to radiation and convection is respectively 6 and 6·5 W/m²—°C. Assume that convection is improved by 35 per cent due to provision of tubes.

Solution. Area of plane tank $S_t=2(1+0·5)\times 1·25=3·75$ m²

Let the tube area be xS_t.

∴ Total dissipating surface$=(1+x)\,S_t=3·75\,(1+x)$

$$\text{Specific loss dissipation} = \frac{4800}{3·75(1+x)\times 35}=\frac{36·5}{1+x}\ \text{W/m}^2\text{—°C}$$

From Eqn. 7·98 loss dissipated$=\dfrac{12·5+8·8x}{1+x}$ W/m²—°C

$$\frac{12·5+8·8x}{1+x}=\frac{36·5}{1\times x} \quad \text{or} \quad x=2·73$$

∴ Area of tubes$=2·73\times 3·75=10·23$ m².

Wall area of each tube$=\pi d_t\, l_t=\pi\times 0·05\times 1·05=0·165$ m².

∴ Total number of tubes to be provided$=10·23/0·165=62$.

The tubes are spaced 75 mm apart. Therefore in 1 m along the width of tank, we can accommodate 12 tubes leaving 90 mm on each side. In 0·5 m along the depth of tank we can accommodate 5 tubes with 100 mm space on each side. The total tubes provided in the first row along width and depth are $2\times 12+2\times 5=34$. The balance $62-34=28$ tubes can be provided in second row at the back.

11 tubes can be provided in a staggered fashion in each of the two long sides and 4 in each of the two short sides.

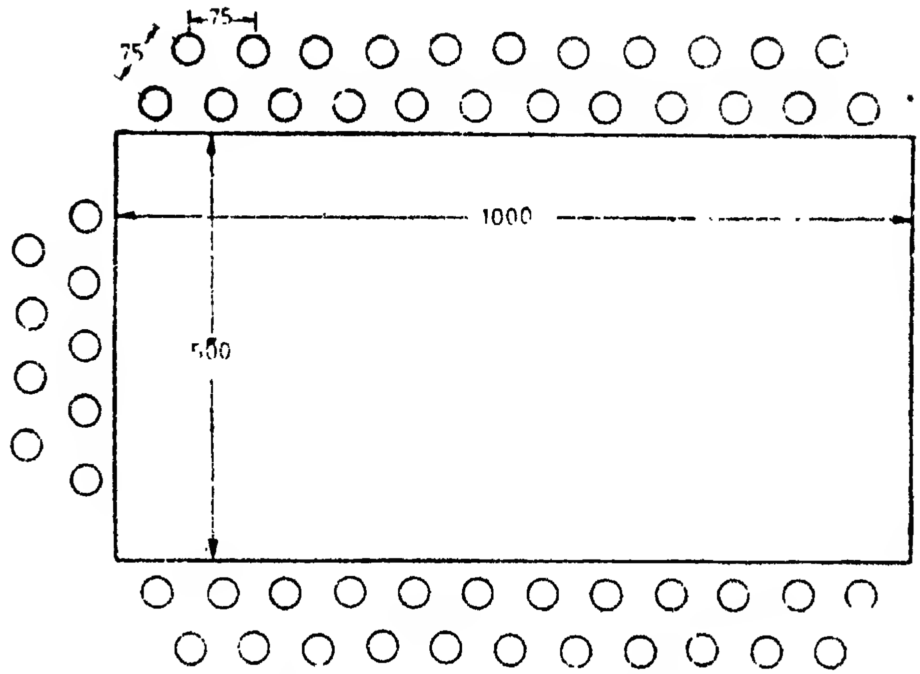


Fig. 7·87. Arrangement of cooling tubes

$\therefore$ Total tubes provided $=2\times12+2\times11+2\times5+2\times4=64$

Fig. 7·87 shows the arrangement for tubes.

Example 7·25 *A 1000 kVA, 6600/440 V, 50 Hz, 3 phase, delta/star, core type, oil immersed natural cooled (ON) transformer. The design data of the transformer is :*

distance between centres of adjacent limbs = 0·47 m, outer diameter of high voltage winding = 0·44 m, height of frame = 1·24 m.

Core loss = 3·7 kW and I^2R loss = 10·5 kW.

Design a suitable tank for the transformer. The average temperature rise of oil should not exceed 35°C.

The specific heat dissipation from the tank walls is 6 W/m²–°C and 6·5 W/m²–°C due to radiation and convection respectively. Assume that the convection is improved by 35% due to convection.

Solution. The clearance between windings and tank is assumed as 70 mm and 90 mm on each side along width and length respectively.

$\therefore$ Width of tank $W_t=2D+D_e+2b=2\times0·47+0·44+2\times0·07=1·52$ m.

Length of tank $L_t=D_e+2l=0·44+2\times0·09=0·62$ m.

Allowing 50 mm for the base and 300 mm for oil above the frame,

Height of oil level from the bottom of tank $=1·24+0·05+0·3\simeq1·6$ m.

A further height of 300 mm is necessary for the leads etc.

$\therefore$ Height of tank $H_t=1·6+0·3=1·9$ m.

Dissipating surface of plain tank $S_t=2(1·52+0·62)\times1·9=8·132$ m².

Let the area of tubes be $x\,S_t$.

$\therefore$ Total dissipating area $=(1+x)\,S_t=8·32\,(1+x)$ m².

Specific loss dissipation $\lambda=\frac{6+6.5+(1+0.35)xS_t}{1+x}=\frac{12.5+8.8x}{1+x}$ W/m²$-$°C.

Total loss$=3.7+10.5=14.2$ kW. The temperature rise is to be kept below 35°C.

Specific loss dissipation$=\frac{14.2\times10^3}{8.132(1+x)\times35}=\frac{12.5+8.8x}{1+x}$

Hence, $x=4.25$. $\therefore$ Area of tubes$=x\,S_t=4.25\times8.132=34.56$ m².

The average length of tubes is approximately 1.4 m and the diameter of each tube is 50 mm.

$\therefore$ Dissipating area of each tube$=\pi\,d_t\,l_t=\pi\times0.05\times1.4=0.22$ m².

Hence, number of tubes required $N_t=\frac{34.56}{0.22}=153$

The tubes are spaced 75 mm apart.

The arrangement of tubes is :

Along length : 4 rows on each side with 18, 17, 16 and 15 tubes

Along width : 4 rows on each side with 7, 6, 5 and 4 tubes

Total number of tubes used$=2(18+17+16+15)+2(7+6+5+4)=156$.

7·58. Thermal rating. The rating of a transformer is based exclusively on thermal basis, the limitation being imposed by the maximum working temperature of the winding which will afford a reasonable life to the insulation. The critical component of most transformer insulation is cellulose, which deteriorates physically or mechanically at a rate determined by moisture content, electric stress and oil purity. The most important factor is temperature. Insulating materials immersed in oil maintained continuously at a temperature 75°C may have a life of 50 years, but low limit of temperature would be uneconomical from the rating view point. If the insulation is kept in oil maintained at a temperature of 110°C continuously, the life may be less than 5 years. Thus this would require transformer replacement after a very short time. Therefore, the transformer should not be operated at a high temperature (for the sake of getting higher output) otherwise its life will be very short.

The reference ambient temperatures assumed are :

(*a*) maximum ambient air temperature—45°C

(*b*) maximum daily average air temperature—35°C

(*c*) maximum yearly average air temperature—30°C.

Operation of a transformer at its rated kVA provides normal life expectancy if the temperature of the cooling air does not, at any time, exceed the reference ambient temperature. This results in copper in the winding attaining a temperature of 90° to 100°C (*i.e.* 55°C above maximum reference ambient temperature of 35°C daily average and 45°C maximum) and an assumed hot spot temperature in the windings of about 105°C. A rough and ready rule concerning the deterioration of insulating materials at high temperatures which has received wide acceptance is that the rate of deterrioation of class A insulation in oil roughly doubles for every 8°C rise in operating temperature. From this it is apparent that operation of a transformer for any given period in a cooling air temperature in excess of reference ambient temperatures such as to produce a rise in the temperature of insulation of 8°C above normal is equivalent to its operation under standard ambient conditions for roughly double the period. This would, of course, mean a corresponding reduction in the life expectancy of transformer.

On the other hand, it is possible to operate a transformer in cooling air temperatures exceeding reference ambient temperatures whilst still maintaining its normal life expectancy by suitably reducing its load from its rated kVA. As a very rough guide, it may be stated

that for a transformer covered by ISI. specification 2026—1962, a reduction in load from its rated kVA of approximately 2 per cent is necessary for this purpose for each 1°C by which the actual cooling temperature exceeds the reference ambient temperature. This rule does not hold good if the cooling air temperature exceeds the reference ambient temperature by more than 10°C.

If the cooling air temperature is less than the reference ambient temperature it is possible to operate the transformer at a load above its rated kVA by about 1 per cent per degree centigrade by which cooling air temperature is less than the reference air ambient temperature. This holds good if the cooling air temperature does not differ from the reference ambient temperature by more than 10°C.

Whenever the transformer has a temperature rise less than its specified limit, it may be subjected to overloads on thermal considerations. The time for which such an overload can be sustained depends obviously on the initial temperature and on the time constant. The greater the weight of the transformer, the greater will be its thermal capacity and time constant. Thus a transformer with large weight has a smaller temperature rise for short time overloads. It is possible to construct a transformer of small weight to confirm with the standard rating in respect of temperature rise of windings and oil by providing ample tank cooling surface and adequate ducts in the windings. But the performance of such a low weight transformer will be unsatisfactory on short time overloads owing to its small time constant.

However, it is incorrect to compare two transformers in this respect solely on the basis of weights without a knowledge of their cooling. The ability of a transformer to withstand overloads depends upon the efficiency of the cooling system as well as on weight.

The overload that can be imposed upon a transformer depends upon the ratio full load copper loss P_c to iron loss P_i.

Let us compare two transformers, one with $P_c/P_i=1$ and other with $P_c/P_i=2$.

At 200 per cent overload, the first transformer has a loss ratio of.

$$\frac{\text{total loss at overload}}{\text{total loss at full load}}=\frac{(2)^2\times 1+1}{1+1}=2\cdot 5$$

and the second transformer has a loss ratio of

$$\frac{\text{total loss at overload}}{\text{total loss at full load}}=\frac{(2)^2\times 2+1}{2+1}=3.$$

Therefore overload losses in two cases are 2·5 : 3 ; so that a transformer with a greater P_c/P_i ratio is less capable of sustaining overloads.

7·59. Momentary overloads. Transformers in service must be capable of withstanding short-circuits at normal line voltage without injury. The duration of short circuits according to IS ; 2026—1962 is :

Percent impedance	4 or less	5	6	7 and above
Duration of short circuit s	2	3	4	5

The calculated winding temperature must not exceed 25 °C starting from an initial value of 90°C for water cooling and 105°C for air cooling. It is assumed that the heat generated is stored in the copper and there is no dissipation as the interval is very small.

Temperature rise $\theta = at\left[\frac{2T_1+at}{2T_1}+\frac{620\,K_e}{2T_1+at}\right]$°C ...(7·101)

where t=time, s ; $T_1=\theta_1+235°$;

θ_1=initial temperature, °C

K_e=eddy current loss ratio at 75°C

$a=0.0025\times$loss in W/kg at $\theta_1=1.9\,\delta^2T_1\times10^{-5}$

and δ=current density, A/mm².

7·60. Heating time constant of transformers. The heating time constant is $T_h=Gh/S\lambda$ (see Chapter 3).

This means that the heating time constant T_h is directly proportional to weight G and specific heat h while it is inversely proportional to specific heat dissipation λ. The value of heating time constant lies between one to five hours. Natural cooling and high voltage insulation, which try to give a reduced value of specific heat dissipation λ, produce longer time constants. A transformer with large weight has a longer time constant and is thus capable of withstanding momentary overloads better than a light weight transformer (as in a transformer with a longer time constant, it takes longer time for the temperature to rise).

A transformer is not a homogeneous body and therefore the aboue relationship is not strictly applicable to it. As a transformer consists essentially of a core, a winding and oil, all having different weights, specific heats etc. we have to deal with the heating time constants of core T_c, the winding T_w and the oil T_o.

Example 7·26 shows how greatly the heating time constants differ for the different parts of a transformer.

Example 7·26. *A 3 phase 5000 kVA, 33 kV, 50 Hz, oil immersed transformer has the following data :*

Weight : core=5200 kg, copper=1200 kg, oil=5500 kg

Core losses=18 kW, copper losses=57 kW.

The specific heats of core steel, copper and oil are respectively 480, 390 and 1670 J/kg-°C.

Assuming that the temperature rise of core above oil θ_c=20°C, of the winding above oil θ_w=20°C, and the temperature rise of oil θ_o=40°C. Find the heating time constants for core, winding and oil.

Comment upon the results.

Solution. Heating time constant $T_h=Gh/S\lambda$

but $Q=S\lambda\theta$ or $S\lambda=Q/\theta$ $\therefore$ $T_h=Gh\theta/Q$

Q stands for loss in the above relationship.

Heating time constant for :

Core : $T_c=\frac{5200\times480\times20}{18000}$=2710 s=46·3 minutes.

Winding : $T_w=\frac{1200\times390\times20}{57000}$=164 s=2·74 minutes.

Oil : $T_o=\frac{5500\times1670\times40}{18000+57000}$=4900 s=81·7 minutes.

Thus the heating time constant for oil is 30 times that of windings. This makes it legitimate to assume that during rapid changes in load, we may without any significant error, neglect the oil temperature variation as compared with that in the windings. Therefore, the oil temperature is no indicative of the winding temperature during rapid changes

of in load. Hence, in addition to a temperature indicator immersed in oil another temperature indicator must be used to monitor the temperature of the windings.

DESIGN PROBLEMS

Problem I *Design a 25 kVA, 11000/433 V, 50 Hz, 3 phase, delta/star, core type, oil immersed natural cooled distribution transformer. The transformer is provided with tappings ± 2½ ±5% on the h v winding. Maximum temperature rise not to exceed 45°C with mean temperature rise of oil 35°C.*

Solution.

Core Design. The value of K is taken from Table 7·2

$K=0.45$ for three phase core type distribution transformers

Voltage per turn $E_t=K\sqrt{Q}=0.45\ \sqrt{25}=2.25$ V.

$\therefore$ Flux in the core $\Phi_m=\dfrac{E_t}{4.44\times f}=\dfrac{2.25}{4.44\times 50}=0.010135$ Wb.

Hot rolled silicon steel grade 92 is used. The value of flux density B_m is assumed as 1·0 Wb/m².

Net iron area $A_i=\dfrac{0.010135}{1.0}=0.01035\ \text{m}^2=10.135\times 10^3\ \text{mm}^2$

Using a cruciform core, $A_i=0.56\ d^2$

Diameter of circumscribing circle $d=\sqrt{10.135\times 10^3/0.56}=134.5$ mm

Reference Figs. 7·52 and 7·53(*a*), widths of laminations :

$a=0.85\ d=0.85\times 135.8=114.8$ mm, $b=0.53\ d=0.53\times 135.8=71.6$ mm.

The laminations are punched from 750 mm wide plates and the nearest standard dimensions are $a=114$ mm and $b=73$ mm

Window Dimensions. The window space factor for a small rating transformer is $K_w=8/(30+\text{kV})$.

$\therefore$ $K_w=8/(30+71)=0.195$. The value assumed is $K_w=0.18$

The current density in the windings is taken as 2·3 A/mm².

Output of transformer.

$Q=3.33\ f\ B_m\ K_w\ \delta\ A_w\ A_i\times 10^{-3}$

$25=3.33\times 50\times 1\times 0.18\times 2.3\times 10^6\times A_w\times 0.010135\times 10^{-3}$

or window area $A_w=0.0358\ \text{m}^2=35.8\times 10^3\ \text{mm}^2$.

Taking the ratio height to width of window as 2·5,

$$H_w\times W_w=35.8\times 10^3 \text{ or } 2.5\ W_w^2=35.8\times 10^3$$

$\therefore$ Width of window $W_w=120$ mm and height of window$=300$ mm.

Area of window provided $A_w=300\times 120=36\times 10^3\ \text{mm}^2=0.036\ \text{m}^2$.

Distance between adjacent core centres $D=W_w+d=120+135=255$ mm.

Yoke Design. The area of yoke is taken as 1·2 times that of limb

$\therefore$ Flux density in yoke$=1/1.2=0.833$ Wb/m².

Net area of yoke$=1.2\times 10.135\times 10^3=12.16\times 10^3$ mm².

Gross area of yoke$=12.16\times 10^3/0.9=13.5\times 10^3$ mm².

Taking the section of the yoke as rectangular.

Depth of yoke $D_y=a=114$ mm $\qquad\therefore$ Height of yoke $H_y=13.5\times 10^3/114\cong 114$ mm

Overall Dimensions of Frame. Reference Fig. 7·70·

Height of frame $H=H_w+2H_y=300+2\times 114=528$ mm.

Width of frame $W=2D+a=2\times255+114=624$ mm.

Depth of frame $D_y=a=114$ mm.

L.V. Winding :

Secondary line voltage $=433$ V ; Connection=star.

Secondary phase voltage $V_s=433/\sqrt{3}=250$ V.

Number of turns per phase $T_s=V_s/E_t=250/2{\cdot}25=111$.

Secondary phase current $I_s=\dfrac{25\times1000}{3\times250}=33{\cdot}3$ A.

A current density of $2{\cdot}3$ A/mm^2 is used.

Area of secondary conductor $a_s=33{\cdot}3/2{\cdot}3=14{\cdot}48$ mm^2.

From Table 17·1 (IS : 1897—1962), using a bare conductor of $7{\cdot}7\times2{\cdot}2$ mm.

Area of bare conductor $a_s=14{\cdot}9$ mm^2.

Current density in secondary winding $\delta_s=33{\cdot}4/14{\cdot}9=2{\cdot}23$ A/m^2.

The conductors are paper covered. The increase in dimensions on account of paper covering is 0·5 mm.

∴ Dimensions of insulated conductor $=7{\cdot}5\times2{\cdot}7$ mm^2.

Using three layers for the winding.

Helical winding is used. Therefore space has to be provided for $(37+1)=38$ turns along the axial depth.

Axial depth of l.v. winding

$L_{cs}=38\times$axial depth of conductor$=38\times7{\cdot}5=285$ mm.

The height of window is 300 mm. This leaves a clearance of $(300-285)/2=7{\cdot}5$ on each side of the winding. (The minimum clearance should be 6 mm for windings having voltages below 500 V.)

Using 0·5 mm pressboard cylinders between layers.

Radial depth of l.v. winding

$b_s=$number of layers $\times$radial depth of conductor+insulation between layers

$=3\times2{\cdot}7+2\times0{\cdot}5=9{\cdot}1$ mm.

Fig. 7·88 shows a cross-section through l.v coil.

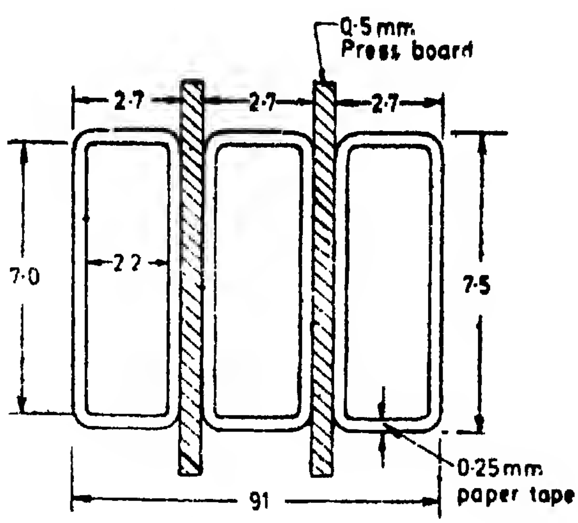


Fig. 7·88. L.V. winding (All dimensions in mm).

Diameter of circumscribing circle $d=135$ mm.

Using pressboard wraps 1·5 mm thick as insulation between l v. winding and core.

Inside diameter of l.v. winding$=135+2\times1·5=138$ mm.

Outside diameter of l v. winding$=138+2\times9·1=156·2$ mm.

H.V. Winding

Primary line voltage$=11000$ V. Connection$=$Delta

Primary phase voltage $V_p=11000$ V.

∴ Number of turns per phase $T_p=11000\times111/250=4884$.

As $\pm$ 5% tappings are to be provided, therefore, the number of turns is increased to $T_p=1·05\times4884=5128$.

The voltage per coil is about 1500 V. ∴ Using 8 coils.

Voltage per coil$=11000/8=1375$ V. Turns per coil$=5158/8=641$.

Using 7 normal coils of 672 turns and one reinforced coil of 424 turns.

Total h.v. turns provided $T_p=7\times672+424=5128$.

Taking 24 layers per coil. Turns per layer$=674/24=28$.

Maximum voltage between layers$=2\times28\times2·25=126$ V, which is below the allowable limit.

H.V. winding phase current $I_p=\dfrac{25\times1000}{3\times11000}=0·757$ A.

As the current is below 20 A, cross-over coils are used for h v. winding

Taking a current density of 2·4 A/mm^2

Area of h v conductor $a_p=0·757/2·4=0.316$ mm^2.

Diameter of bare conductor$=(4/\pi\times0·316)^{1/2}=0·635$ mm.

Using paper covered conductors. From Table 17·4 (IS : 3454–1966) the nearest standard conductor size has .

bare diameter$=0·63$ mm, insulated diameter$=0·805$ mm with fine covering.

Modified area of conductor $a_p=\pi/4\times(0·63)^2=0·312$ mm^2.

Actual value of current density used $\delta_p=0·757/0·312=2·43$ A/mm^2.

Axial depth of one coil$=28\times8·05=22·6$ mm.

The spacers used between adjacent coils are 5 mm in height.

Axial length of h v. winding :

$L_{cp}=$number of coils$\times$axial depth of each coil$+$depth of spacers

$=8\times22·6+8\times5=221$ mm.

The height of window is 300 mm and therefore, the space left between winding and window is 300—221=79 mm. This is occupied by insulation and axial bracing of the coil. The clearance left on each side is 39·5 mm, which is sufficient for 11 kV transformers.

The insulation used between layers is 0·3 mm thick paper.

∴ Radial depth of h.v. coil $b_p=24\times0·805+23\times0·3=26·22$ mm.

From Eqn. 7·22 the thickness of insulation between h.v. and l.v. winding is

$=5+0·9$ kV$=5+0·9\times11\cong15$ mm. This includes the width of oil duct also.

The insulation between h.v. and l v. winding is a 5 mm thick bakelized paper cylinder. The h.v. winding is wound on a former 5 mm thick and the duct is 5 mm wide, making the total insulation between h.v. and l.v. windings 15 mm.

∴ Inside diameter of h.v. winding
=outside diameter l.v. winding $+2\times$ thickness of insulation
$=156.2+2\times15=186.2$ mm.

Outside diameter of h.v. winding $=186.2+2\times26.22=238.64$ mm $\simeq239$ mm.

Clearance between windings of two adjacent limbs $=255-239=16$ mm.

Resistance

Mean diameter of primary winding $=\dfrac{186.2+239}{2}\simeq212$ mm.

Length of mean turn of primary winding $L_{mtp}=\pi\times212\times10^{-3}=0.666$ m.

Resistance of primary winding at 75°C $r_p=\dfrac{T_p\,\rho\,L_{mtp}}{a_p}$

$$=\frac{4884\times0.021\times0.666}{0.312}=219.2\ \Omega.$$

Mean diameter of secondary winding $=\dfrac{138+156.2}{2}\simeq149$ mm.

Length of mean turn of secondary winding $L_{mts}=\pi\times149\times10^{-3}=0.468$ m.

Resistance of secondary winding at 75°C, $r_s=\dfrac{111\times0.021\times0.468}{14.9}=0.0732\ \Omega$

∴ Total resistance referred to primary side $R_p=219.2+\left(\dfrac{4884}{111}\right)^2\times0.0732=364\ \Omega.$

P.U. resistance of transformer $\varepsilon_r=\dfrac{I_p\,R_p}{V_p}=\dfrac{0.757\times364}{11000}=0.025$

Leakage Reactance

Mean diameter of windings $=(138+239)/2=188.5$ mm.

Length of mean turn $L_{mt}=\pi\times188.5\times10^{-3}=0.592$ m.

Height of winding $L_c=(L_{cp}+L_{cs})/2=(221+285)/2=253$ mm.

Leakage reactance of transformer referred to primary side

$$X_p=2\pi f\,\mu_0\,T_p^2\frac{L_{mt}}{L_c}\left(a+\frac{b_p+b_s}{3}\right)$$

$$=2\pi\times50\times4\pi\times10^{-7}\times(4884)^2\times\left(\frac{0.592}{0.235}\right)\times\left(15+\frac{26.22+9.1}{3}\right)\times10^{-3}=590\ \Omega.$$

P.U. leakage reactance $\varepsilon_x=0.757\times590/11000=0.0406$.

P.U. impedance $\varepsilon_z=\sqrt{(0.025)^2+(0.0406)^2}=0.0477$.

Regulation. P.U. regulation $\varepsilon=\varepsilon_r\cos\phi+\varepsilon_x\sin\phi$.

∴ Per unit regulation at unity power factor $\varepsilon=\varepsilon_r=0.025$, at zero p.f lagging $\varepsilon=\varepsilon_x=0.0406$, at 0.8 p.f. lagging $\varepsilon=0.025\times0.8+0.0406\times0.6=0.0444$.

Losses

I^2R Loss

I^2R loss at 75°C, $=3I_p^2\,R_p=3\times0.757^2\times364=626$ W.

Taking stray load loss 15% of above.

Total I^2R loss including stray load loss $P_c=1.15\times626=720$ W.

Core Loss

Taking the density of laminations as 7.6×10^3 kg/m³,

weight of 3 limbs $=3\times0.3\times0.010135\times7.6\times10^3=69.3$ kg

The flux density in the limbs is 1 Wb/m^2 and corresponding to this density, specific core loss is 1·2 W/kg (See Fig. 4·29 page 148 for 92 grade.)

$\therefore$ Core loss in limbs $= 69{\cdot}3 \times 1{\cdot}2 = 83{\cdot}2$ W

Weight of two yokes $= 2 \times 0{\cdot}624 \times 0{\cdot}01216 \times 7{\cdot}6 \times 10^3 = 115{\cdot}3$ kg

Corresponding to a flux density of 0·833 Wb/m^2 in the yoke, the specific core loss $= 0{\cdot}85$ W.

$\therefore$ Core loss in yoke $= 115{\cdot}3 \times 0{\cdot}85 = 98$ W.

Total core losses $P_i = 83{\cdot}2 + 98{\cdot}0 \simeq 181$ W.

Efficiency. Total losses at full load $= 181 + 720 = 901$ W.

Efficiency at full load and unity p.f $= \dfrac{25000}{25000+901} \times 100 = 96{\cdot}5$ per cent

For maximum efficiency $x^2 P_c = P_i$ $\therefore$ $x = \sqrt{P_i/P_c} = \sqrt{181/720} = 0{\cdot}0501$

Thus maximum efficiency occurs at 50·1 percent of full load. This is a good figure for distribution transformers.

No load current. Corresponding to flux densities of 1 Wb/m^2 and 0·833 Wb/m^2 in core and yoke respectively $at_c = 120$ A/m and $at_y = 80$ A/m (See Fig. 4·3 page 122)

$\therefore$ Total magnetizing mmf $= 3 \times 120 \times 0{\cdot}3 + 2 \times 80 \times 0{\cdot}624 = 207$ A.

Magnetizing mmf per phase $AT_0 = 207/3 = 69$ A

Magnetizing current $I_m = AT_0/(\sqrt{2}\, T_p) = 62/(\sqrt{2} \times 4884) = 5{\cdot}5 \times 10^{-3}$ A

Loss component of no load current $I_l = 181/3 \times 11000 = 5{\cdot}5 \times 10^{-3}$ A

No load current $I_0 = \sqrt{(10 \times 10^{-3})^2 + (5{\cdot}5 \times 10^{-3})^2} = 11{\cdot}4 \times 10^{-3}$ A.

No load current as a percentage of full load current $= \dfrac{11{\cdot}4 \times 10^{-3}}{0{\cdot}757} \times 100 = 1{\cdot}5\%$

Allowing for joints etc. the no load current will be about 2·5% of full load current.

Tank Height over yoke $H = 528$ mm Allowing 50 mm at the base and about 150 mm for oil. Height of oil level $= 528 + 50 + 150 = 728$ mm. Allowing another 200 mm height for leads etc., height of tank $H_t = 728 + 200 = 928$ mm. The height of tank is taken as 0·95 m or $H_t = 0{\cdot}95$ m.

Assuming a clearance of 40 mm along the width on each side

Width of tank $W_t = 2D + D_e + 2l = 2 \times 255 + 239 + 2 \times 40 = 829$ mm.

The width of tank W_t is taken as 0·84 m.

The clearance along the length of the transformer is greater than that along the width. This is because additional space is needed along the length to accommodate tappings etc. The clearance used is approximately 50 mm on each side.

$\therefore$ Length of tank $L_t = D_e + 2b = 239 + 2 \times 50 = 239$ mm.

The length of tank L_t is taken as 0·35 m.

Total loss dissipating surface of tank $S_t = 2(0{\cdot}84 + 0{\cdot}35) \times 0{\cdot}95 = 2{\cdot}26$ m^2.

Total specific loss dissipation due to radiation and convection is 12·5 W/m^2—°C

$\therefore$ Temperature rise $= 901/(2{\cdot}26 \times 12{\cdot}5) = 31{\cdot}9$°C

This is below 35°C and therefore plain tank is sufficient for cooling and no tubes are required.

Fig. 7·89 shows core and winding details of the transformer.

DESIGN SHEET

kVA 25 Phase 3 Frequency—50 Hz Delta/Star

Line voltage [h.v. 11000 V; l.v. 433 V.] Phase voltage [h.v. 11000 V.; l.v. 250 V.]

Line current [h.v. 1·31 A.; l.v. 36 A.] Phase current [h.v. 0·757 A.; l.v. 36 A.]

Type—core Type of cooling—ON

	Core		
1.	Material		0·35 mm thick 92 Grade
2.	Output constant	K	0·25
3.	Voltage per turn	E_t	2·25 V
4.	Circumscribing circle diameter	d	135 mm
5.	Number of steps	...	2
6	Dimensions	...	
		a	114 mm
		b	73 mm
7.	Net iron area	A_i	$10{\cdot}135 \times 10^3$ mm^2
8.	Flux density	B_m	1·0 Wb/m^2
9.	Flux	ϕ_m	10·13 5 m Wb
10.	Weight		69·3 kg
11.	Specific iron loss		1·2 W/kg
12.	Iron loss		83·2 W
	Yoke		
1.	Depth of yoke	D_y	114 mm
2.	Height of yoke	H_y	114 mm
3.	Net yoke area		$12{\cdot}16 \times 10^3$ mm^2
4.	Flux density		0·833 Wb/m^2
5.	Flux		10·135 m Wb
6.	Weight		115·3 kg
7.	Specific iron loss		0·8 W/kg
8.	Iron loss		98 W
	Windows		
1.	Number		2
2.	Window space factor	K_w	0·18
3.	Height of window	H_w	300 mm
4.	Width of window	W_w	120 mm
5	Window area	A_w	36×10^3 mm^2
	Frame		
1.	Distance between adjacent limbs	D	255 mm
2.	Height of frame	H	536 mm
3.	Width of frame	W	624 mm
4.	Depth of frame	D_y	114 mm

Windings	L.V.	H.V.
1. Type of winding	Helical	Cross-over
2. Connections	Star	Delta
3. Conductor		
Dimensions—bare	7·0 × 2·2 mm²	Diameter = 0·63 mm
insulated	7·5 × 2·7 mm²	Diameter = 0·814 mm
Area	14·9 mm²	0·312 mm²
Number in parallel	None	None
4. Current density	2·23 A/mm²	2·43 A/mm²
5. Turns per phase	111	4884 (5128 at—5% tap)
6. Coils		
total number	3	3 × 8
per core leg	1	8
7. Turns		
per coil	111	7 of 672 turns, 1 of 424 turns
per layer	34	28
8. Number of layers	3	24
9. Height of winding	285 mm	221 mm
10. Depth of winding	9·1 mm	26 mm
11. Insulation		
Between layers	0·5 mm press board	0·3 mm paper
Between coils		5·0 mm spacers
12. Coil Diameters		
Inside	138 mm	186·2 mm
Outside	156·2 mm	239 mm
13. Length of mean turn	0·468 m	0·666 m
14. Resistance at 75°C	0·0732 Ω	219·2 Ω

Insulation

1. Between l.v. winding and core = press board wraps 1·5 mm
2. Between l.v. winding and h.v. winding = bakelized paper 5·0 mm
3. Width of duct between l.v. and h.v = 5 mm

Tank		
1. Dimensions		
height	H_t	0·95 m
length	L_t	0·35 m
width	W_t	0·84 m
2. Oil Level		0·728 m
3. Tubes		Nil
4. Temperature rise		31·9°C
Impedance		
1. P.U. Resistance		0·025
2. P.U. Reactance		0·0406
3. P.U. Impedance		0·444
Losses		
1. Total core loss		181 W
2. Total copper loss		720 W
3. Total losses at full load		901 W
4. Efficiency at full load and u.p.f.		96·5%

Problem II. *Design a 500 kVA, 50 Hz, 6600/400 V, single phase core type, oil immersed, natural cooled power transformer. The mean temperature rise of oil not to exceed 35°C.*

Solution. Core Design

For this transformer we use 53 Grade (G.K.W.) cold rolled oriented steel laminations. The following assumptions are made :

$B_m=1.5$ Wb/m^2 ; $\delta=2.75$ A/mm^2 ; $g_c=8.9\times10^3$ kg/m^3 ; $g_i=7.65\times10^3$ kg/m^3 ; ratio L_{mt}/l_t 0.5. The values of constant K for various conditions have been worked out in Example 7.10, page 404.

The value of K is respectively 1.22, 0.745, 0.375 and 0.344 for maximum efficiency at 90% full load, minimum cost, minimum weight and minimum volume.

It is clear from above that if the transformer is designed for maximum efficiency to occur at 90% full load, the cost would be exhorbitant. Therefore, keeping the cost of the transformer the main consideration and keeping in view to some extent, the efficiency, volume and weight of transformer the value of K is assumed as 0.8.

Voltage per turn $E_t=K\sqrt{Q}=0.8\sqrt{500}=17.9$ V.

Flux $\Phi_m=17.9/(4.44\times50)=0.0806$ Wb

Net iron area $A_i=0.0806/1.5=0.0537$ m^2$=53.7\times10^3$ mm^2.

A 3-stepped core is used. From Table 7.3, for a 3-stepped core

$A_i=0.6\,d^2$ or diameter of circumscribing circle $d=\sqrt{53.7\times10^{-3}/0.6}=300$ mm.

The dimensions of the laminations are :

$a=0.9\times300=270$ mm, $b=0.7\times300=210$ mm, $c=0.42\times300=126$ mm.

(See Fig. 7.53 page 382).

The standard sizes of laminations used are 270 mm, 215 mm and 135 mm.

Window Dimensions. As the transformer is rated at 500 kVA, we take window space factor

$$K_w=\frac{10}{30+\text{kV}}+\frac{10}{30+6.6}=0.27.$$

From Eqn. 7.6, for a single transformer, $Q=2.22\,f\,B_m\,K_w\,\delta\,A_w\,A_i\times10^{-3}$

or $500=2.22\times50\times1.5\times0.27\times2.75\times10^6\times A_w\times0.0537\times10^{-3}$

$\therefore$ Window area $A_w=75.3\times10^3$ mm^2

Taking the ratio height to width of window$=2.5$ $\therefore$ $2.5\,W_w^2=75.3\times10^3$.

$\therefore$ Width of window $W_w=175$ mm and height of window $H_w=437.5$ mm

Distance between adjacent limbs $D=d+W_w=300+175=475$ mm.

Yoke Design

The area of yoke is taken to be the same as that of core. Assuming rectangular cross-section for the yoke. Height of yoke $H_y=a=270$ mm and depth of yoke $D_y=a=270$ mm.

Frame

Height of frame$=H_w+2H_y=437.5+2\times270=977.5$ mm.

Width of frame$=D+a=475+270=745$ mm. Depth of frame $a=270$ mm.

Windings.

L.V. Winding

L.V. winding turns $T_s=400/17.9=22.3$ say 22.

Modified value of core flux density $B_m=1.5\times22.3/22=1.52$ Wb/m^2.

L. V. winding current $I_s = \dfrac{500 \times 1000}{400} = 1250$ A

Area of l.v. conductor $a_s = \dfrac{1250}{2.75} = 455$ mm^2.

Helical winding is used. The l.v. conductor consists of 20 strips connected in parallel. Each strip is 6×4 mm in cross-section with an area of 23.4 mm^2 (see Table 17.1).

Total area provided $a_s = 20 \times 23.4 = 468$ mm^2.

Actual current density used $\delta_s = 1250/468 = 2.67$ A/mm^2

The arrangement of twenty conductors is shown in Fig 7.90

The insulated conductors have dimensions 6.5×4.5 mm.

Axial depth of one turn $= 5 \times 6.5 + 1$ mm slack $= 33.5$ mm.

Radial width of one turn $= 5 \times 4.5 + 1$ mm slack $= 19$ mm.

The winding is applied on both the limbs with 11 turns per limb. Therefore space has to be provided for 11+1 *i.e.* 12 turns.

Axial height of l.v. coil $L_{cs} = 12 \times 33.5 = 402$ mm.

The height of window is 437.5 mm and the balance height of 35.5 mm is occupied by end insulation and bracings etc.

Radial depth or width of l v. coil $b_s = 19$ mm.

The insulation between core and l.v. winding is bakelite former 5 mm thick.

Inside diameter of former $d = 300$ mm.

Outside diameter of former $= 310$ mm.

Inside diameter of l.v. winding $= 310$ mm.

Outside diameter of l.v. winding $= 310 + 2 \times 19$
$= 348$ mm

Mean diameter of l.v. winding
$= (310 + 348)/2 = 329$ mm.

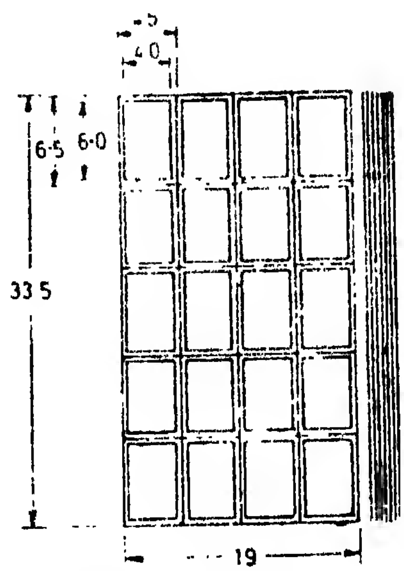


Fig. 7.90. L V. conductor.

Length of mean turn of l v winding $L_{mts} = \pi \times 0.329 = 1.03$ m.

Resistance of l.v winding $r_s = \dfrac{22 \times 0.021 \times 1.03}{468} = 0.001\ \Omega$.

I^2R losses in l v winding $= (1250)^2 \times 0.001 = 1562$ W.

H.V. Winding

H.V. winding turns $T_p = (6600/400) \times 22 = 363$.

Using 182 turns on one limb and 181 on the other

H.V. winding current $I_p = \dfrac{500 \times 1000}{6600} = 75.75$ A.

Area of h.v. conductor $a_p = 75.75/2.75 = 27.5$ mm^2.

The available height of window is not sufficient for a single coil.

Therefore a double helical winding is used with 91 turns in helix. (On one limb there are 91 turns per helix but on the other there are 91 in one and 90 in the other helix).

The cross-section of bare conductor is $3 \cdot 5 \times 8$ mm. It is wound on edge with 0·5 mm thick press board between turns.

Area of h.v. conductor $a_p = 27 \cdot 4$ mm². (see Table 17 1).

Radial width of each coil=8 mm. Axial height of each turn=3·5+0·5=4 mm.

Axial depth of 91 turns=92×4	368 mm
Insulation and bracing etc.	60 mm
Slack	9·5 mm
Window Height	437·5 mm

Axial height of h.v. coil, $L_{cp} = 368 + 9 \cdot 5 = 377 \cdot 5$ mm.

Insulation between h.v. and l.v. windings

$$= 5 + 0 \cdot 9 \text{ kV} = 5 + 0 \cdot 9 \times 6 \cdot 6 = 11 \text{ mm.}$$

Using a duct 6 mm wide and a former 5 mm thick for first coil (helix) of h.v. winding.

Inside diameter of first h.v. former $= 348 + 2 \times 6 = 360$ mm.

Inside diameter of 1st h.v. coil = outside diameter of first h v. former

$$= 360 + 2 \times 5 = 370 \text{ mm.}$$

Outside diameter of first h.v. coil $= 370 + 2 \times 8 = 386$ mm.

Allow a duct 8 mm wide including the former for second h.v. coil.

Inside diameter of second h.v. coil $= 386 + 2 \times 8 = 402$ mm.

Outside diameter of second h.v. coil $= 402 + 2 \times 8 = 418$ mm.

Distance between h.v. windings on adjacent limbs $= 475 - 418 = 57$ mm.

Mean diameter of h.v. winding $= (360 + 418)/2 = 389$ mm.

Length of mean turn of h.v. winding $L_{mtp} = \pi \times 389$ mm $= 1 \cdot 22$ m.

Resistance of h.v. winding at 75°C, $r_p = \dfrac{363 \times 0 \cdot 021 \times 1 \cdot 22}{27 \cdot 4} = 0 \cdot 34\ \Omega$

I^2R loss in h.v. winding $= (75 \cdot 75)^2 \times 0 \cdot 34 = 1951$ W.

Resistance

Total I^2R loss $= 1562 + 1951 = 3513$ W.

This is increased by about 10 percent to account for stray load loss.

∴ Total I^2R loss including additional loss $P_c = 1 \cdot 1 \times 3513 = 3865$ W.

∴ Resistance of transformer referred to primary (h.v.) side

$$R_p = \frac{3865}{(75 \cdot 75)^2} = 0 \cdot 674\ \Omega.$$

P.U. resistance of transformer $\varepsilon_r = \dfrac{75 \cdot 75 \times 0 \cdot 674}{6600} = 0\ 0077$

Leakage Reactance

Length of mean turn $L_{mt} = \pi \dfrac{(310 + 418)}{2} \times 10^{-3} = 1 \cdot 14$ m.

Mean height of coil $L_c = \frac{0.775+0.402}{2} = 0.39$ m

The windings are divided equally over two limbs and therefore the leakage reactance of the transformer is equal to twice the leakage reactance of windings arranged on one limb. Further the h.v. coil is divided into two parts and therefore Eqn. 7.78 is used.

Leakage reactance of winding on each limb

$$X_{p1} = 2\pi f \mu_0 T_p^2 \frac{L_{mt}}{L_c}\left(a + \frac{b_p' + b_p'' + b_s}{3} + \frac{a'}{4} \right)$$

$$= 2\pi \times 50 \times 4\pi \times 10^{-7} \times 182^2 \times \frac{1.14}{0.39}\left(\frac{0.011+0.008+0.008+0.019}{3} + \frac{0.008}{4} \right) = 0.982\ \Omega$$

∴ Total leakage reactance of transformer referred to primary

$X_p = 2 \times 0.982 = 1.964\ \Omega$.

P.U. leakage reactance $\varepsilon_x = \frac{75.75 \times 1.964}{6600} = 0.025$.

P.U. Impedance $\varepsilon_z = \sqrt{0.0082^2 + 0.025^2} = 0.0236$.

Losses

I^2R loss. Total copper loss $P_c = 3865$ W.

Core loss. As mentioned earlier 0.33 mm thick 56 grade (G.K.W.) laminations are used

Length of mean flux path $l_i = 2(437.5+475+270) \times 10^{-3} = 2.37$ m

Weight of iron $= 2.37 \times 0.0537 \times 7.65 \times 10^3 = 974$ kg.

The specific loss for $B_m = 1.52$ Wb/m², from Fig. 4.26, is 1.25 W/kg.

Core loss $= 1.25 \times 974 = 1217.4$ W.

Making an allowance of 20% for joints, total core loss $P_i = 1.2 \times 1217.5 = 1460$ W.

Total loss at full load $= 3865 + 1460 = 5325$ W.

Efficiency

Efficiency at full load and unity p.f.

$$= \frac{500 \times 10^3}{500 \times 10^3 + 5325} \times 100 = 98.95\%$$

Regulation

Regulation at full load and 0.8 p.f. lagging $\varepsilon = \varepsilon_r \cos\phi + \varepsilon_x \sin\phi$.

$$= 0.0077 \times 0.8 + 0.0225 + 0.6 = 0.0197 \text{ p.u. or } 19.7\%.$$

Tank

Height of frame $H = 977.5$ mm. Allowing 50 mm for the base and 250 mm for oil

∴ Oil level $= 977.5 + 50 + 250 \cong 1300$ mm. Allowing 300 mm further for leads etc.

Height of tank $H_t = 1300 + 300$
$= 1600$ mm $= 1{\cdot}6$ m.

Allowing a clearance of 80 mm at each along the length.

Length of tank

$L_t = 475 + 418 + 2 \times 80$ mm $= 1{\cdot}05$ m

A clearance of about 200 mm is allowed along the width to accommodate taps etc.

∴ Width of tank

$W_t = 418 + 200$ mm $\simeq 0{\cdot}62$ m

Loss dissipating surface of tank

$S_t = 2(1{\cdot}05 + 0{\cdot}62) \times 1{\cdot}6 = 5{\cdot}34$ m²

The mean temperature rise of oil is to be limited to 35°C.

Let the area of tubes be xS_t.

∴ Specific loss dissipation

$$= \frac{5325}{5{\cdot}34(1+x) \times 35} = \frac{28{\cdot}5}{1+x} \text{ W/m}^2\text{–°C}$$

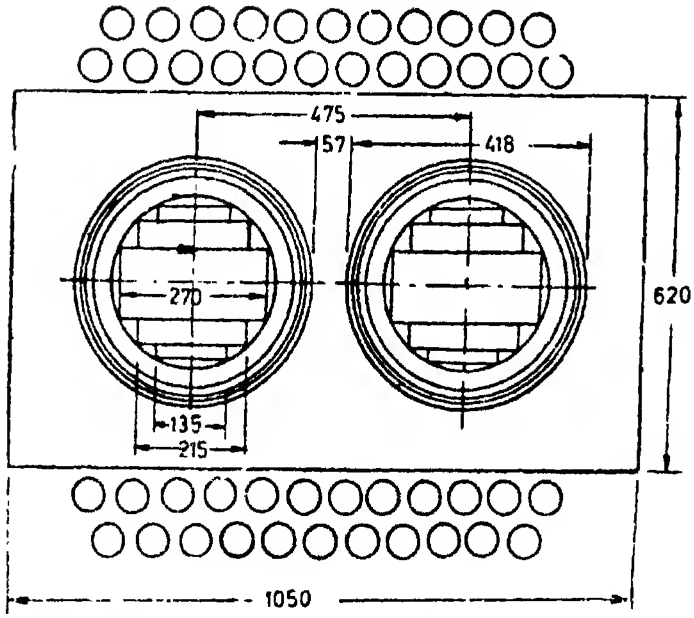


Fig. 7·91. Core and winding details of 500 kVA, Single Phase 6600/400 volt, 50 Hz core type power transformer (dimensions in mm)

or $$\frac{28{\cdot}5}{1+x} = \frac{12{\cdot}5 + 8{\cdot}8x}{1+x} \text{ (see Eqn. 7.8)} \quad \therefore \quad x = 1{\cdot}82$$

∴ Area of tubes needed $= 1{\cdot}82 \times 5{\cdot}34 = 9{\cdot}72$ m²

Using 50 mm diameter tubes spaced 75 mm apart. The average length of tubes is assumed as 1·35 m.

Dissipating area of each tube $= \pi \times 0{\cdot}05 \times 1{\cdot}35 = 0{\cdot}212$ m²

∴ Number of tubes to be provided $= 9{\cdot}72/0{\cdot}212 = 46$.

Arrangement of tubes :

Along length—2 rows—12 and 11 tubes.

The details of core windings and tank are given in Fig. 7·91.

Problem III. *Design a 150 kVA, 6600/400 V, 50 Hz, single phase shell type, oil immersed, self cooled power transformer. Average temperature rise of oil not to exceed 35°C.*

Solution. Core Design. From Table 7·1, the value of output coefficient K is taken as 1.

Voltage per turn $E_t = K\sqrt{Q} = 1 \times \sqrt{150} = 12{\cdot}25$ V.

$$\text{Flux } \Phi_m = \frac{E_t}{4{\cdot}44 f} = \frac{12{\cdot}25}{4{\cdot}44 \times 50} = 0{\cdot}0552 \text{ Wb.}$$

Taking the value of maximum flux density $B_m = 1{\cdot}25$ Wb/m².

Net iron area $A_i = 0{\cdot}0552/1{\cdot}25 = 0{\cdot}0442$ m² $= 44{\cdot}2 \times 10^3$ mm²

Taking the iron stacking factor $= 0{\cdot}9$. Gross iron area $A_{gi} = 44{\cdot}2/0{\cdot}9 = 49 \times 10^3$ mm².

The ratio of $b/2a$ is taken as 2·5 (See Fig. 7·71).

∴ $2{\cdot}5(2a)^2 = 49 \times 10^3$ or width of central limb $2a = 140$ mm.

Depth of frame $b = 2{\cdot}5 \times 140 = 350$ mm

The side limbs carry half of the flux and therefore their width is equal to half of the width of central limb or width of side limbs $a=70$ mm

Window Dimensions.

Window space factor $K_w = \frac{10}{30+kV} = \frac{10}{30+6.6} = 0.27$.

For the sake of calculation of window area, a current density of 2·3 A/mm² is assumed.

Now, $Q = 2.22\, f\, B_m\, K_w\, \delta\, A_i\, A_w \times 10^{-3}$

or $150 = 2.22 \times 50 \times 1.25 \times 0.27 \times 2.3 \times 10^6 \times 0.0442 \times A_w \times 10^{-3}$.

$\therefore$ Window area $A_w = 0.0394$ m² $= 39.4 \times 10^3$ mm².

The height of the window is taken as twice the width of window, or $2W_w^2 = 39.4 \times 10^3$

$\therefore$ Width of window $=140$ mm, and height of window $H_w = 280$ mm.

Yoke Design. The flux in the yoke is half of the flux in the central limb. The flux density in the yoke is taken same as that for limbs. $\therefore$ Height of yoke $H_y = a = 70$ mm.

Frame. Reference Fig. 7·92.

Height of frame $H = H_w + 2H_y = 280 + 2 \times 70 = 420$ mm.

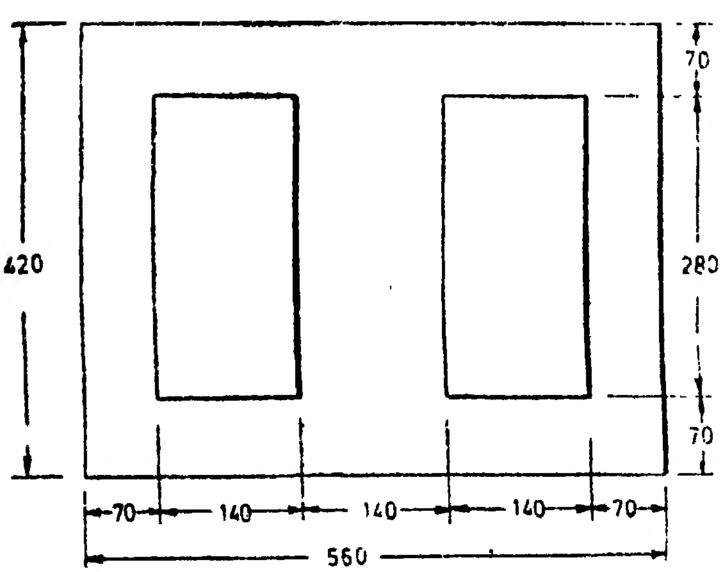


Fig. 7·92. Frame of shell type transformer (All dimensions in mm)

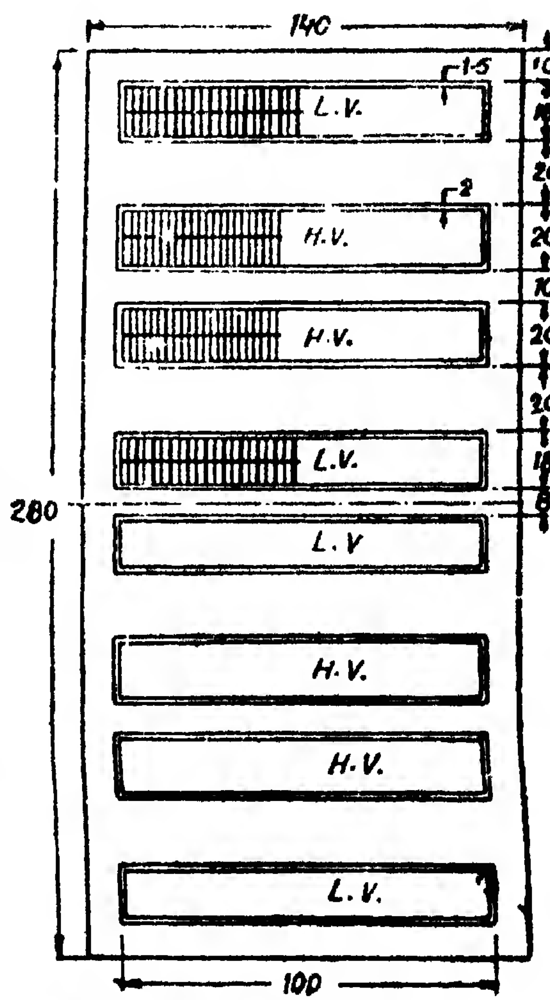


Fig. 7·93. Arrangement of coils. (All dimensions in mm)

Width of frame $W=4a+2W_{w}=4\times70+2\times140=560$ mm.

Depth of frame $D_y=b=50$ mm.

Windings.

Number of turns in l.v winding $T_s=440/12\cdot25=36$.

This number is divisible by four to allow the subdivision of l.v. winding into 4 sections of 9 turns each.

Number of turns in the h.v winding $T_p=36\times6600/440=540$.

This is divided into two coils of 150 turns each and two of 129 turns each with reinforced end turns.

The arrangement of the coils is shown in Fig. 7·93.

The main insulation, height wise in the window is given below :

Insulation between end l.v. coils and core $=2\times10=20$ mm.

Spacers between h.v. and l.v. coils $=4\times20=80$ mm.

Spacers between l.v coils $=1\times8=8$ mm. Spacers between h.v. coils $=2\times10=20$ mm.

Total height of insulation $=20+80+8+20=128$ mm.

$\therefore$ Height available for conductors $=280-128=152$ mm.

Total number of h v. and l.v. coils $=8$. $\therefore$ Height of each coil $=152/8=19$ mm.

H.V. Winding.

H.V. winding current $I_p=\dfrac{150\times1000}{6600}=22{\cdot}7$ A.

Area of h.v. winding conductor $a_p=22{\cdot}7/2{\cdot}3=9{\cdot}9$ mm^2.

The h.v. conductor comprises of two paper covered strips $8\times0{\cdot}6$ mm bare in parallel.

Area of h.v. conductor used $a_p=2\times8\times0{\cdot}6=9{\cdot}6$ mm^2.

Actual value of current density used in h.v. winding $=22{\cdot}7/9{\cdot}6=2{\cdot}37$ A/mm^2.

Space taken by h.v. conductor in a coil height wise $=2\times8=16$ mm.

The balance height $(19-16)=3$ mm is taken up by paper.

Space taken by h v. bare conductors width wise $=150\times0{\cdot}6=90$ mm.

The space taken up by insulation between h.v. and core width wise $=2\times20=40$ mm.

Space for winding along width of window $=140-40=100$ mm.

or width of h.v. coil $=100$ mm.

The balance space $=100-90=10$ mm is taken by insulation between h.v. conductors.

L. V. Winding

L. V. winding current $I_s=\dfrac{150\times1000}{440}=341$ A.

Area of l.v. winding conductor $a_s=\dfrac{341}{2{\cdot}3}=148$ mm^2.

Using 4 strips of $7{\cdot}5\times5$ mm each in parallel.

Area of l.v. winding conductor used $a_s=4\times7{\cdot}5\times5=150$ mm^2.

Actual value of current density used in l.v. winding $\delta_s=341/150=2{\cdot}28$ A/mm^2.

Space occupied by bare conductors in a coil height wise $=2\times7{\cdot}5=15$ mm.

The balance space $(19-15)=4$ mm is taken by insulation etc.

Space occupied by bare conductors, in a section, width wise

$=9\times2\times5=90$ mm.

Using 1 mm thick press board width wise between each conductor.

∴ Width of l.v. coil $=94+9\times 1 \simeq 100$ mm.

The balance space $140-100=40$ mm along the width is taken up by insulation between core and l.v. winding with 20 mm on each side.

Resistance. The approximate length of mean turn $=L_{mtp}=L_{mts}=1\cdot 58$ m.

Resistance of h.v. winding $r_p=\dfrac{0\cdot 021\times 540\times 1\cdot 58}{9\cdot 6}=1\cdot 87\ \Omega$.

Resistance of l.v. winding $r_s=\dfrac{0\cdot 021\times 36\times 1\cdot 58}{150}=0\cdot 008\ \Omega$.

Resistance of l.v. winding referred to h.v. side $r_s'=0\cdot 008\times(540/36)^2=1\cdot 8\ \Omega$.

Total resistance of transformer referred to primary side $R_p=1\cdot 87+1\cdot 8=3\cdot 67\ \Omega$

$=3\cdot 85\ \Omega$ (taking loss in connections etc.)

P.U. resistance $\varepsilon_r=\dfrac{22\cdot 7\times 3\cdot 85}{6600}=0\cdot 0133$.

Leakage Reactance. For sandwich coils, leakage reactance of transformer referred to primary side :

$$X_p=\pi f\mu_0\frac{L_{mt}}{W}\frac{T_p^2}{n}\left(a+\frac{b_p+b_s}{6}\right) \qquad \text{(See Eqn. 7·80)}$$

$$=\pi\times 50\times 4\pi\times 10^{-7}\times\frac{1\cdot 58}{0\cdot 1}\times\frac{540^2}{2}\left(0\cdot 0235+\frac{0\cdot 040+0\cdot 042}{6}\right)=17\cdot 3\ \Omega.$$

P.U. reactance $\varepsilon_x=\dfrac{22\cdot 7\times 17\cdot 3}{6600}=0\cdot 0595$. P.U. impedance $\varepsilon_z=\sqrt{0\cdot 012^2+0\cdot 0595^2}=0\cdot 061$

Regulation. The regulation of transformer at full load and 0·8 p.f. lagging.

$\varepsilon=\varepsilon_r\cos\phi+\varepsilon_x\sin\phi=0\cdot 0125\times 0\cdot 8+0\cdot 0595\times 0\cdot 6=0\cdot 0457=4\cdot 57\%$

Losses

I²R Loss

I^2R loss in h.v. winding $=22\cdot 7^2\times 1\cdot 87=964$ W.

I^2R loss in l.v. winding $=341^2\times 0\cdot 008=228$ W.

Total I^2R loss $=964+928=1892$ W.

The total I^2R loss including loss in connections etc. may be taken as $P_c=2000$ W.

Core Loss. Referring to Fig. 7·98.

Volume of iron $=14\times 35\times 28+2\times 7\times 35\times 56+2\times 7\times 35\times 28=0\cdot 05488$ m³

Weight of iron $=0\cdot 05488\times 7\cdot 6\times 10^3=417$ kg.

The specific iron loss for a flux density of 1·25 Wb/m² is 1·8 W.

Total core loss $=417\times 1\cdot 8=750$ W.

The total core loss including additional losses may be taken as 840 W.

Efficiency

Total loss at full load $=2000+840=2840$ W

Efficiency at full load unity power factor $=\dfrac{150\times 10^3}{150\times 10^3+2840}\times 100=98\cdot 3\%$

P.U. load for maximum efficiency $=\sqrt{840/2000}=0\cdot 65$

Load for maximum efficiency $=0\cdot 65\times 150=97\cdot 5$ kVA.

DESIGN OF SMALL SINGLE PHASE TRANSFORMERS

7·61 Introduction. The design of small low voltage transformers of ratings from 10 to 1000 VA is being given here in order to help the students in their project work.

7·62. Core Design The starting point in the design of small transformers is the choice of turns per volt. The values of turns per volt are given in Table 7·4.

Table 7·7. Turns per volt T_e.

VA	*Turns per volt*	*VA*	*Turns per volt*
10	23·3	200	3·5
15	17·5	250	2·8
20	14·0	300	2·8
25	11·7	400	2·3
50	7·0	500	2·0
75	5·6	750	1·7
100	4·6	1000	1·6
150	4·0		

$\therefore$ Now $E=4{\cdot}44\, f\, \Phi_m\, T$ $\therefore$ Turns per volt $T_e=T/E=1/4{\cdot}44\, f\, \Phi_m$.

Flux in the core $\Phi_m=1/4{\cdot}44\, f\, T_e$.

The frequency of the transformer is specified and the value of turns per volt T_e is taken from Table 7·7. Therefore flux Φ_m in the core is known.

Net area of core $A_i=\Phi_m/B_m$ The value of maximum flux density B_m is assumed to be 1 Wb/m^2.

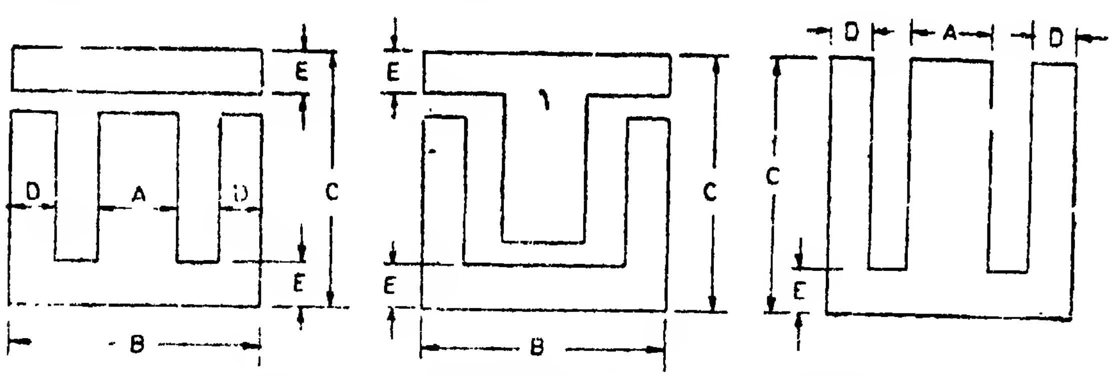
Gross area of core $A_{gi}=A_i/0{\cdot}9$. (Assuming a stacking factor of 0·9)

A shell type of construction is normally used for small transformers.

The core is made up of any of the following combination of stampings.

(*i*) E and I (Fig. 7·94(*a*)), (*ii*) T and U (Fig. 7·94(*b*))

(*iii*) E used in pairs (Fig. 7·94(*c*)).



(*a*) E-I type. (*b*) T-U type. (*c*) E type

Fig. 7·94. Laminations for small transformers.

Tables 7.8, 7·9, and 7·10 give information about standard stampings manufactured by Pre... ion Pressing Division of M/s Guest Keen Williams for small transformers and chokes.

Table 7·8. E—I Stampings [(*Fig. 7·94(a)*)]

No.	Dimensions					Remarks
	A	*B*	*C*	*D*	*E*	
17	1/2"	1·1/2"	1·1/4"	1/4"	1/4"	
12 A	5/8"	1·7/8"	1·9/16"	5/16"	5/16"	
21	5/8"	2"	2·1/8"	5/16"	3/8"	
10	5/8"	2·3/8"	2·1/8"	3/8"	3/8"	
10 A	5/8"	2·3/8"	2·1/8"	3/8"	3/8"	
1	31/32"	2·17/32"	2·1/4"	5/16"	5/16"	
74	11/16"	2·1/16"	1·23/32"	11/32"	11/32"	
23	3/4"	2·1/4"	1·7/8"	3/8"	3/8"	
11	3/4"	3"	2·1/4"	3/8"	3/8"	
11 A	3/4"	3"	2·5/8"	3/8"	3/8"	
2	3/4"	3"	3"	3/8"	3/8"	
30	20 mm	60 mm	50 mm	10 mm	10 mm	4 holes 5/32" dia.
31	7/8"	2·5/8"	2·3/16"	7/16"	7/16"	
45	7/8"	2·5/8"	2·3/16"	7/16"	7/16"	4 holes 5/32" dia.
15	1"	3"	2·1/2"	1/2"	1/2"	4 holes 7/32" dia.
44	1"	3"	2·1/2"	1/2"	1/2"	4 holes 7/32" dia.
14	1"	3·5/16"	2·5/8"	17/32"	1/2"	4 holes 7/32" dia.
4	1"	3·13/16"	3·13/16"	17/32"	1/2"	4 holes 7/32" dia.
33	28 mm	84 mm	70 mm	14 mm	14 mm	4 holes 11/64" dia.
3	1·1/4"	3·3/4"	3·1/8"	5/8"	5/8"	4 holes 7/32" dia.
13	1·1/4"	4"	3·1/2"	1/2"	1/2"	4 holes 7/32" dia.
4 A	1·5/16"	4·1/8"	3·7/16"	21/32"	21/32"	4 holes 3/16" dia.
16	1·1/2"	4·1/2"	3·3/4"	3/4"	3/4"	4 holes 7/32" dia.
5	1·1/2"	4·3/4"	3·3/4"	3/4"	3/4"	4 holes 17/64" dia.
6	1·1/2"	5"	4·1/2"	3/4"	3/4"	4 holes 9/32" dia.
7	2"	6"	4·15/16"	1"	1"	4 holes 17/64" dia.
43	2"	6"	5"	1"	1"	4 holes 17/64" dia.
8	2"	7·1/4"	6·3/4"	1"	1"	4 holes 3/8" dia.

Table 7.9. T—U Stampings [(*Fig.* 7·94(*b*)].

No.	Dimensions *A*	*B*	*C*	*D*	*E*	*Remarks*
34	5/8"	3 1/8"	2 1/2"	1/2"	1/2"	4 holes 5/32" dia.
9	7/8"	3 1/4"	2 1/2"	7/16"	7/16"	4 holes 7/32" dia.
9 A	7/8"	3 1/4"	2 1/2"	7/16"	7/16"	4 holes 5/32" dia.
4 AX	15/16"	3 9/16"	3 3/16"	7/16"	7/16"	4 holes 5/32" dia.
75	1"	4"	3 3/8"	1/2"	1/2"	4 U holes 1/4" dia.
35 A	1 1/2"	6 1/4"	5 1/4"	3/4"	3/4"	4 holes 1/4" dia.

Table 7.10. E—E Stampings [(*Fig.* 7·94(*c*)].

No	Dimensions *A*	*B*	*C*	*D*	*E*	*Remarks*
32	1/4"		1/2"	1/8"	1/8"	used in pair
	1/4"	1"	1/4"	1/8"	1/8"	
12 B	5/8"	1·7/8"	2 3/16"	5/16"	5/16"	
12 C	5/8"	1·7/8"	2"	5/16"	5/16"	
20	5/8"	2·1/8"	1·5/8"	3/8"	3/8"	
41	0·79"	30·9"	1·313"	0·475"	0·475"	7—3/32" dia holes
	0·79"	3·09"	0·585"	0·485"	0·475"	4—3/32" dia holes
						1—3/16" dia hole

A square section is normally used for the central limb, *i.e.* the depth of the core is made equal to the width of the central limb

or width of central limb $A=\sqrt{A_{gi}}$.

A standard stamping giving a width A nearly equal to the value calculated above may be used.

7·63. Winding Design. Current in the primary winding $I_p=VA/\eta$.

The efficiency of small transformers varies from 80 to 96 per cent.

Area of primary winding conductors $a_p=I_p/\delta_p$ mm², where δ_p is the current density in the primary winding conductors in A/mm². A value of 2·3 A/mm² may be used.

Enamelled round conductors are used for the windings of small transformers.

A standard size of conductor is selected by referring to relevant tables given in chapter 17.

Turns in primary winding $T_p = V_p T_e$. Current in secondary winding $I_s = VA/V_s$.

Area of secondary winding conductor $a_s = I_s/\delta_s$ mm^2.

When calculating the number of secondary winding turns, an allowance of 5 percent exrta turns is made to compensate for the voltage drop in the windings

Secondary winding turns $T_s = 1{\cdot}05\ V_s T_e$.

7 4. **Window Area.** Space is required in the window for :

(*i*) primary winding (*ii*) secondary winding (*iii*) insulation and the former (bobbin) on which the windings are supported.

Space required for primary winding $= \dfrac{T_p a_p}{\text{space factor}}$

Space required for secondary winding $= \dfrac{T_s a_s}{\text{space factor}}$

Space factor $S = 0{\cdot}8\ (d/d_1)^2$

where d = diameter of bare conductor and d_1 = diameter of insulated conductor.

The space required for insulation and former is estimated as 20% of that required for the windings.

$\therefore$ Window area required

$A_w = 1{\cdot}2$ (window area required for primary and secondary windings).

It should be checked that the stamping used, gives a higher value of window area than the value calculated above.

Design Problem IV. *Design a single phase transformer to be connected to a 230 V, 50 Hz supply. The transformer is to deliver 3A at 50 V.*

Solution.

Core. Volt ampere rating of transformer $= 50 \times 3 = 150$ VA.

From Table 7·7, turns per volt $T_e = 4{\cdot}0$.

$$\Phi_m = \frac{1}{4{\cdot}44\, f\, T_e} = \frac{1}{4{\cdot}44 \times 50 \times 4} \text{ Wb} = 1{\cdot}125 \text{ m Wb.}$$

Taking a flux density of 1·0 Wb/m^2,

Net iron area of core. $A_i = \dfrac{\Phi_m}{B_m} = \dfrac{1{\cdot}125 \times 10^{-3}}{1}$ m$^2 = 1{\cdot}125 \times 10^3$ mm^2.

Gross core area $A_{gi} = 1{\cdot}125 \times 10^{-3}/0{\cdot}9 = 1{\cdot}255 \times 10^3$ mm^2.

Taking a square section for the central limb.

Width of central limb $A = 12{\cdot}5 \times 10^2 = 35{\cdot}4$ mm^2.

Primary winding. The efficiency of this transformer is assumed as 92%.

Primary winding current $I_p = \dfrac{VA}{\eta V_p} = \dfrac{150}{0{\cdot}92 \times 230} = 0{\cdot}71$ A

Taking a current density of 2·3 A/mm^2.

Area of primary winding conductor $a_p = \dfrac{0{\cdot}71}{2{\cdot}3} = 0{\cdot}309$ mm^2.

Diameter of bare conductor=0·626 mm.

Using enamelled conductors. From Table 17·7, the nearest standard conductor has bare diameter=0·63 mm The diameter of insulated conductor is 0·707 mm.

Space factor for primary winding=0·8(0·63/0·707)²=0·635.

Area of primary conductor used $a_p=\frac{\pi}{4}(0\cdot63)^2=0\cdot312$ mm².

Number of primary winding turns $T_p=V_pT_e=230\times4=920$.

Window space required by primary winding$=\frac{T_pa_p}{S_f}=\frac{920\times0\cdot312}{0\cdot635}=452$ mm².

Secondary winding. Secondary winding current $I_s=3$ A.

Area of secondary winding conductor $a_s=\frac{3}{2\cdot3}=1\cdot3$ mm².

Diameter of bare conductor=1·285 mm.

Using enamelled conductor for secondary winding.

The nearest standard conductor has bare diameter=1·32 mm.

diameter of insulated conductor=1·42 mm.

Space factor for secondary winding=0·8 (1·32/1·42)²=0·69.

Area of secondary winding conductor $a_s=\frac{\pi}{4}(1\cdot32)^2=1\cdot37$ mm².

Number of secondary winding turns $T_s=1\cdot05\ V_sT_e=1\cdot05\times50\times3=210$.

Window space required by secondary winding$=\frac{T_sa_s}{S_f}=\frac{210\times1\cdot37}{0\cdot69}=417$ mm².

Stamping Size. Total window space required

$A_w=1\cdot2$ (space required for primary and secondary winding)

$=1\cdot2(452+417=1045$ mm²

We have now to select a stamping which gives :

width of central limb $A=35\cdot4$ mm$=1\cdot39''$

and area of window $A_w=1045$ mm²$=1\cdot62$ sq. inch.

Using a combination of E and I stampings. From Table 7·8, seclecting No. 16

$A=1\frac{1}{2}''$, $B=4\frac{1}{2}''$, $C=3\cdot3/4''$, $D=3/4''$, $E=3/4''$.

From Fig. 7·94(a), width of window $W_w=\frac{B-A-2D}{2}=3/4''$, and

height of window $H_w=C-2E=2\frac{1}{4}''$.

∴ Area of window provided $A_w=W_w\times H_w=3/4''\times2\frac{1}{4}=1\cdot69$ sq. inch.

This is more than the required window space.

Width of central limb $A=1\frac{1}{2}''=38\cdot1$ mm.

UNSOLVED PROBLEMS

1. Calculate the core and window areas required for a 1000 kVA, 6600/400 V, 50 Hz, single phase core type transformer. Assume a maximum flux density of 1·25 Wb/m² and a current density of 2·5 A/mm². Voltage per turn=30 V, and window space factor=0·32. [Ans. 108 × 10³ mm² ; 83·4 × 10³ mm²]

2. A 3 phase, 50 Hz, oil cooled core type transformer has the following dimensions : distance between core centres=0·2 m ; height of window=0·24 m ; diameter of circumscribing circle=0·14 m. The flux density in the core is 1·25 Wb/m² and the current density in the conductors is 2·5 A/mm². Estimate the kVA rating. Assume a window space factor of 0·2 and a core area factor=0·56. The core is 2 stepped. [Ans. 16·5 kVA]

3. Determine the dimensions for core and yoke for a 5 kVA, 50 Hz, single phase core type transformer. A rectangular core is used with long side twice as long as short side. The window height is 3 times the width. Voltage per turn is 1·8 V ; space factor 0·2 ; current density 1·8 A/mm² ; flux density 1 Wb/m².
[Ans. A_i=8100 mm² ; core 135×67·5 mm² ; A_w . 15·5×10³ mm² , window 72·5×217·5 mm²]

4. Estimate the main dimensions including winding conductor areas of a 3 phase delta/star core type transformer rated at 300 kVA, 6600/440 V, 50 Hz. A suitable core with three steps having a circumscribing circle of 0·25 m diameter and a leg spacing of 0·4 m is available. The emf per turn is 8·5 V. Assume a current density of 2·5 A/mm², a window space factor of 0·28, and a stacking factor of 0·9.
[Ans. T_p=776, a_p=6 mm² ; T_s=30, a_s=157·5 mm² ; window 0·15×0·45 m² ; core=37·5×10³ mm²]

5. Determine the dimensions of the core, the number of turns and the cross-sectional area of conductors in the primary and secondary windings of a 100 kVA, 2200/480 V single phase core type transformer to operate at a frequency of 50 Hz, assuming the following data ; approximate voltage per turn, 7·5 V ; maximum flux density, 1·2 Wb/m² ; ratio of effective crosssectional area of core to square of diameter of circumscribing circle, 0·6 ; ratio of height to width of window, 2 ; window space factor, 0·28 ; current density, 2·5 A/mm². [Ans. 28·2×10³ mm² ; 294, 64 18·2 mm²; 83·2 mm²]

6. Calculate the dimensions of the core the number of turns and the cross sections of the conductors for a 100 kVA, 2300/400 V, 50 Hz single phase shell type transformer, assuming : ratio of magnetic and electric loadings (*i.e.* flux and secondary mmf at full load), 480×10⁻⁸ ; maximum flux density 1·1 Wb/m² ; current density, 2·2×10⁶ A/m² ; window space factor, 0·3 ; ratio of depth of stacked core to width of central limb, 2·6 ; ratio of height to width of window, 2·5 ; stacking factor 0·9. [Ans. 135×340 mm² ; 230, 40 ; 19·8 mm², 114 mm²]

7. Two single phase transformers with *AN* cooling having linear dimensions in the ratio x : 1, are designed to work at the same current density, flux density and frequency. Calculate relative ratings, losses and total weights per kilovolt ampere of the two tranformers.

Comment briefly on the temperature rise of the first transformer for increasing values of x, assuming that the second transformer operates with permissible temperature rise on full load. [Ans x^4 : 1; x^3 ; 1 x^{-1} ; 1]

8. A 750 kVA, 6600 V, 50 Hz, three phase delta/star core type transformer has the following data ; width of l.v. winding, 30 mm ; width of h.v. winding, 25 mm; width of dust between h.v. and l.v. windings, 15 mm ; height of windings, 0·4 m ; length of mean turn 1·5 m, h.v. winding turns, 217. Estimate the leakage reactance of the transformer referred to h v. side. Estimate the per unit regulation of the transformer at full load and 0·8 power factor lagging if the resistance per phase referred to the h.v. side is 0·8 Ω.
[Ans. 2·32 Ω, 0·0347]

9. Derive an expression for the leakage inductance per limb of the windings of a single phase transformer referred to the primary side in terms of the relevant quantities, and calculate this inductance in a given case where the windings have a mean length of turn 1·5 m and each winding has a radial depth of 30 mm. The radial clearance between windings on the same limb is 35 mm and the axial height of each winding is 0·6 m. The primary winding consists of 240 turns per limb. [Ans. 9·?8 mH]

10. In a transformer having a 10 : 1 turns ratio, the copper loss in the primary winding is 15 per cent less than that in the secondary winding. The resistance per turn of the primary is 0·00394 Ω. and there are 1485 more turns in the primary than in the secondary. Calculate the resistance of secondary winding.
[Ans. 0·0675 Ω]

11. A 100 kVA, 2000/400 V 50 Hz, single phase shell type transformer has two full h.v. coils, one full l.v. coil and two half l.v. coils. Calculate the value of average axial force acting on the end coils under short circuit at full voltage if its leakage reactance is 0·035 p u. Length of mean turn 1·5 m, width of coils, 0·12 m ; h.v. winding turns, 200 ; p.u. resistance of transformer, 0·01. [Ans. 35·2×10³ N]

12. A 220/110 V, 1 kVA, 50 Hz single phase transformer has a core with a uniform cross sectional area of 2500 mm², an effective magnetic core length of 0·4 m and a core weight of 8 kg. If the core is worked at a maximum flux density, B, of 1·2 Wb/m², the corresponding magnetizing force, H, is 200 A/m and the specific core loss is 1·0 W/kg, determine (*a*) the transformer no load current when the h v. is fed at 220 V, and (*b*) the corresponding magnetizing reactance and the equivalent shunt resistance to represent the core loss. [Ans. 0·175 A ; 1287 Ω ; 6600 Ω]

13. A 300 kVA, 6600/440 V, three phase delta/star core type transformer has a maximum flux density of 1·35 Wb/m² and the total weight of core is 650 kg. The magnetizing VA/kg and the iron loss/kg corresponding to 1·35 Wb/m² are 30 and 2·5 W respectively. Calculate the no load current if the mmf required for joints is 2·5 percent of that for iron. [Ans. 0·695 A per phase]

14. The tank of a 1250 kVA natural oil cooled transformer has the dimensions length, width and height as 1·55 m×0·65 m×1·85 m respectively. The full load loss is 13·1 kW. Find the number of tubes for this transformer assuming : W/m²—°C due to radiation=6 ; W/m²—°C due to convection =6·5 ; improvement in convection due to provision of tubes=40 per cent ; temperature rise=40°C ; length of each tube=1 m ; diameter of tubes=50 mm. Neglect the top and bottom surfaces of the tank as regards cooling.
[Ans. 190 tubes]

15. With 100 A taken from the terminals of a certain transformer connected to a 25 Hz primary supply circuit, the resulting total reactive drop is 5 per cent. Calculate the percentage reactive drop if the same transformer is rewound with 15 per cent more turns in both windings and connected to a 60 Hz circuit with a load drawing 110 A from secondary winding. Assume the terminal voltage and the dimensions of all leakage paths to remain unaltered. [Ans. 17·5 per cent]

8

General Concepts and Constraints of Design of Rotating Machines

8·1. Relation between rating and dimensions of rotating machines. The purpose of this section is to try to relate the rating of rotating machines to their main dimensions. A few general equations are developed which are applicable to all types of rotating machines *i.e.* d.c., induction and synchronous machines. However, it may be emphasised here that design is a complex process and many factors which affect design of different types of machines cannot be incorporated into a set of few general equations. Some design concepts and constraints are being introduced here not to demonstrate the complete process of design of all types of machines, but to show in a general manner, how the size and shape of a machine are related to its rating. The detailed design and influencing factors of different types of machines are given later in this text.

8·1·1. Symbols. The various symbols used in this section are as follows :

D=armature diameter or stator bore, m ; L=stator core length, m ;

n=speed r.p.s. ; n_s=synchronous speed r.p.s. ;

p=number of poles ; a=number of parallel paths ;

τ=pole pitch, m ; Z=total number of armature or stator conductors ;

T_{ph}=turns per phase ; I_z=current in each conductor, A ;

K_w=winding factor ;

I_a=armature current, A ; I_{ph}=current per phase, A ; E=back emf, V ;

E_{ph}=induced emf per phase, V ; P=rating of machine, kW ;

P_a=power developed by armature, kW ; Q=kVA rating of machine.

8·1·2. Main Dimensions. The armature diameter (or stator bore) D and armature (or stator) core length L are known as the main dimensions of a rotating machine. Refer to Fig. 8·1.

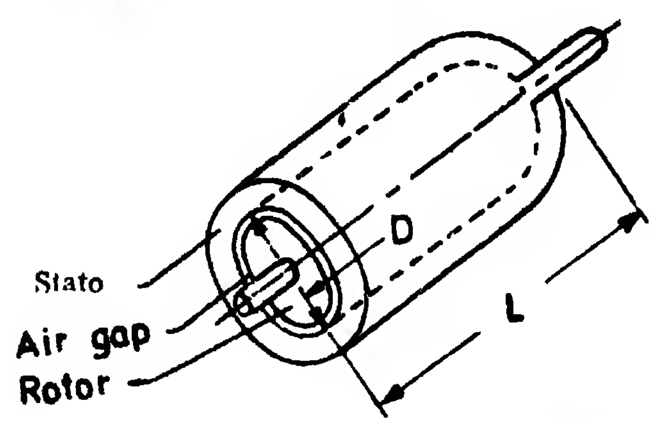


Fig. 8·1. Main dimensions of rotating machines.

8·1·3. Total Loadings

Total Magnetic Loading. The total flux around the armature (or stator) periphery at the air gap is called the total magnetic loading.

$$\text{Total magnetic loading}=p\Phi \quad \ldots(8·1)$$

Total Electric Loading. The total number of ampere conductors around the armature (or stator) periphery is called the total electric loading.

$$\text{Total electric loading}=I_z Z \quad \ldots(8·2)$$

8·1·4. Specific Loadings. Two types of loadings are specified which are the starting point in the design of rotating electrical machines.

1. *Specific Magnetic Loading.* The average flux density over the air gap of a machine is known as **specific magnetic loading.**

Specific magnetic loading

$$B_{av}=\frac{\text{total flux around the air gap}}{\text{area of flux path at the air gap}}=\frac{p\Phi}{\pi DL} \quad \ldots(8\cdot3)$$

$$=\frac{\Phi}{\tau L} \quad \ldots(8\cdot4)$$

2. *Specific Electric Loading.* The number of armature (or stator) ampere conductors per metre of armature (or stator) periphery at the air gap is known as **specific electric loading.**

Specific electric loading

$$ac=\frac{\text{total armature ampere conductors}}{\text{armature periphery at air gap}}=\frac{I_z Z}{\pi D} \quad \ldots(8\cdot5)$$

8·1·5. Output Equation. The output of a machine can be expressed in terms of its main dimensions, specific magnetic and electric loadings and speed ; the equation describing this relationship is known as **Output Equation.** The output equations of the more important machines are given below :

1. **D.C. Machines.** Power developed by armature in kW

$$P_a=\text{generated emf}\times\text{armature current}\times10^{-3}=EI_a\times10^{-3}$$

But $$E=\Phi Zn\frac{p}{a}$$

$$P_a=\Phi Zn\frac{p}{a}\cdot I_a\times10^{-3}=(p\Phi)\left(\frac{I_a}{a}Z\right)n\times10^{-3}=(p\Phi)(I_zZ)n\times10^{-3} \quad \ldots(8\cdot6)$$

(as $I_z=I_a/a$)

Hence P_a=(total magnetic loading)(total electric loading)(speed in r.p.s.)$\times10^{-3}$

Therefore, basically the output of a d c. machine is determined by the total loadings. This is true of a.c. machines as well. A suitable design is not, however, determined by total loadings ; intensity of loadings is a major influencing factor. This is because active materials are iron and copper and the extent to which they are utilized is determined by specific loadings. We have,

Specific magnetic loading $B_{av}=\dfrac{p\Phi}{\pi DL}$ or $p\Phi=\pi DL\,B_{av}$.

Specific electric loading $ac=\dfrac{I_zZ}{\pi D}$ or $I_zZ=\pi D\,ac$.

Substituting the values of $p\Phi$ and I_zZ in Eqn. 8·6, we get

$$P_a=(\pi DL\,B_{av})(\pi D\,ac)n\times10^{-3}$$

$$=(\pi^2B_{av}\,ac\times10^{-3})D^2Ln \quad \ldots(8\cdot7)$$

$$=C_0D^2Ln \quad \ldots(8\cdot8)$$

where $$C_0=\pi^2B_{av}\,ac\times10^{-3} \quad \ldots(8\cdot9)$$

Eqn. 8·8 is the output equation of a d.c. machine. Quantity C_0 is termed as the ***output co-efficient.***

We should not confuse the power P_a developed by the armature with rated output P of the machine. Also distinction must be made between generator and motor action. Consider a d.c. shunt machine acting as a generator. The prime mover must supply friction,

windage and iron losses which do not exist in the absence of rotation. The armature supplies its own copper loss and also the field copper loss.

$\therefore$ Power developed by armature for a generator

$$P_a = \text{output power} + \text{armature } I^2R \text{ loss} + \text{field } I^2R \text{ loss.}$$
$$= P + \text{armature } I^2R \text{ loss} + \text{field } I^2R \text{ loss}$$
$$= \text{input power} - \text{friction, windage and iron loss}$$
$$= P/\eta - (\text{friction, windage and iron loss}) \qquad ...(8.10)$$

With the same machine acting as a motor, the I^2R losses are taken from supply and the friction, windage and iron losses are supplied by the armature. Therefore for a motor :

$$P_a = P + (\text{friction, windage and iron loss}) \qquad ...(8.11)$$

The difference between armature power and the rated output is not large especially in the case of large machines. Therefore in the case of large machines, for initial calculations, we can neglect friction, windage and iron losses.

$$\therefore \quad P_a = P/\eta \text{ for generators} \qquad ...(8.12)$$
$$= P \text{ for motors} \qquad ...(8.13)$$

But in small machines there is a considerable difference between the armature power and rated output as the friction, windage and iron losses are relatively large and thus cannot be neglected. Let us compute the armature power of a small motor having an output P and efficiency η.

$$\text{Output} = P \text{ and input} = P/\eta.$$

$$\therefore \quad \text{Total losses} = \frac{P}{\eta} - P = P\left(\frac{1-\eta}{\eta}\right).$$

The friction, windage and iron losses of a small motor may be taken as $\frac{1}{3}$rd of the total losses.

$$\therefore \quad \text{Friction, windage and iron losses} = \frac{1}{3}\,P\left(\frac{1-\eta}{\eta}\right)$$

Hence power developed by armature of a motor.

$$P_a = P + (\text{friction, windage and iron losses})$$
$$= P + \frac{1}{3}\,P\left(\frac{1-\eta}{\eta}\right) = \left(\frac{1+2\eta}{3\eta}\right)P \qquad ...(8.14)$$

Similarly for a small generator $P_a = \left(\dfrac{2+\eta}{3\eta}\right)P. \qquad ...(8.15)$

2. A.C. Machines. Consider an 'm' phase machine having one circuit (parallel path) per phase. kVA rating of machine

$$Q = \text{number of phases} \times \text{output voltage per phase} \times \text{current per phase} \times 10^{-3}$$
$$= m\,E_{ph} I_{ph} \times 10^{-3}.$$

Terminal voltage of each phase may be taken equal to the induced emf per phase.

We have,

Induced emf per phase $E_{ph} = 4.44\, f\, \Phi\, T_{ph}\, K_w$. $\therefore$ $Q = m \times 4.44\, f\, \Phi\, T_{ph} K_w \times I_{ph} \times 10^{-3}$

But $\quad f = pn_s/2$

Therefore we can write,

$$Q = m \times 4.44\,(pn_s/2)\,\Phi\, T_{ph}\, K_w\, I_{ph} \times 10^{-3}$$
$$= 1.11\, K_w\,(p\Phi)(2m\, I_{ph}\, T_{ph})\, n_s \times 10^{-3}.$$

Now current in each conductor $I_z = I_{ph}$ (as there is only one circuit per phase).

Total number of armature conductors

$$Z = \text{number of phases} \times (2 \times \text{turns per phase}) = 2m\, T_{ph}.$$

$\therefore$ Total electric loading $= I_z Z = 2m\, I_{ph}\, T_{ph}$.

Hence, $$Q = 1.11\, K_w\, (p\Phi)(I_z Z) n_s \times 10^{-3} \quad \ldots(8.16)$$

$= 1.11\, K_w$ (total magnetic loading)(total electric loading) (synchronous speed $\times 10^{-3}$

But $p\Phi = \pi DL\, B_{av}$ and $I_z Z = \pi D\, ac$

Substituting these values in Eqn. 8.16, we have

$$Q = 1.11\, K_w\, (\pi DL\, B_{av})(\pi D\, ac) n_s \times 10^{-3}$$

$$= (1.11\, \pi^2\, B_{av}\, ac\, K_w \times 10^{-3}) D^2 L n_s$$

$$= (11\, B_{av}\, ac\, K_w \times 10^{-3})\, D^2 L n_s \quad \ldots(8.17)$$

$$= C_0\, D^2 L\, n_s \quad \ldots(8.18)$$

where $$C_0 = 11\, B_{av}\, ac\, K_w \times 10^{-3} \quad \ldots(8.19)$$

Eqn. 8.18 is known as the output equation of an a.c. machine. Quantity C_0 is called the output co-efficient.

8.1.6. Factors affecting size of rotating machines. Examining output equations (Eqns. 8.8 and 8.18) of d.c. and a c. machines, we observe that product D^2L will decrease with increase of speed and/or increase of output co-efficient. The volume of active parts of a rotating machine is $(\pi/4)D^2L$ and evidently therefore the volume of active parts and hence the size and the cost of the machine decreases with increase in speed and/or increase in the value of output co-efficient.

Let us elaborate on these points further :

1. **Speed**. It is clear from Eqns. 8.8 and 8.18, that the volume of active parts varies inversely as the speed. Thus for the same output a machine designed with greater speed will have smaller size and hence lesser cost as compared to a machine designed with smaller speed. Therefore, whenever a choice has to be made (when the speed is not specified and is left for the designer to decide), the highest practical speed rating should be selected. However, in special circumstances, the maximum speed may be limited by mechanical stresses in the armature materials.

2. **Output co-efficient**. From Eqns. 8.8 and 8.18 we gather that the volume of active parts is inversely proportional to the value of output co-efficient C_0. Thus an increase in the value of C_0 results in reduction in size and cost of machine and so looking from the economics point of view the value of output co-efficient C_0 should be as high as possible. Now the output co-efficient is,

$$C_0 = \pi^2\, B_{av}\, ac \times 10^{-3} \text{ for a d.c. machine}$$

$$= 11\, B_{av}\, ac\, K_w \times 10^{-3} \text{ for an a.c. machine}$$

or in general $$C_0 = K\, B_{av} \times ac, \text{ where } K \text{ is a constant.}$$

Since the output coefficient is proportional to product of specific magnetic and specific electric loadings we conclude that the size and hence also the cost of machine decreases if increased values of specific magnetic and specific electric loadings are used. Thus from commercial standpoint, it is desirable to push the values of specific loadings as high as possible to reduce the dimensions of the machine. How much high they should be pushed is decided by the designer by analysing the effect of increased loadings on performance characteristics of machine as the cost of machine is not the only important aspect of machine design. If high values of loadings are used some performance characteristics like temperature rise, efficiency, power

factor (in case of induction motors) and commutation conditions (in case of d.c. machines) are adversely affected and this point cannot be lost sight of. In fact such values of specific loadings should be selected which give a design that complies with specifications relating to performance required and at the same time gives a machine having maximum reliability and efficiency together with minimum cost.

8·1·7. Choice of specific magnetic loading. The choice of specific loading is influenced by certain factors. Some of these factors are general in nature *i.e.* apply to all types of machines and some are specific and apply to individual machines. The aim here is to discuss some general factors that influence the choice of specific loadings for all types of machines. The factors which affect the choice of loadings for a particular type of machine *i.e.* whether d c or induction type etc. are discussed in details along with its design later in this book.

Basically, the specific magnetic loading is determined by :

(*i*) maximum flux density in iron parts of machine,

(*ii*) magnetizing current, and (*iii*) core losses.

(*i*) **Maximum flux density in iron.** The maximum flux density in any iron part of magnetic circuit of the machine must be definitely below a certain limiting value depending on the material used. The flux density in iron parts is directly proportional to the average flux density in the air gap *i.e* specific magnetic loading as is shown by Eqn. 8·20. In a well designed machine the maximum flux density occurs in the teeth of the machine and therefore let us relate the flux density in the teeth with flux density in the air gap.

Relation between flux density in teeth and average flux density in air gap. Let us consider a non-salient pole machine having S armature slots.

$$\text{Flux over one slot pitch}=\frac{p\Phi}{S}=p\,.\,B_{av}\,\frac{\pi DL}{p}\,.\,\frac{1}{S}=B_{av}\frac{\pi D}{S}\,L=B_{av}\,y_s\,L$$

where y_s=slot pitch=$\pi D/S$.

If we neglect saturation the entire flux over a slot pitch is carried by the tooth (Fig. 8·2).

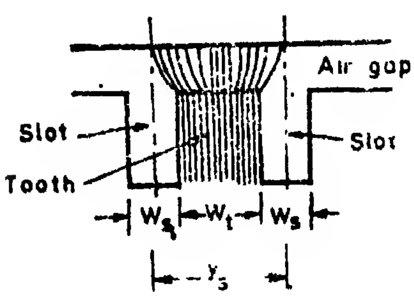


Fig. 8·2. Flux over a slot pitch.

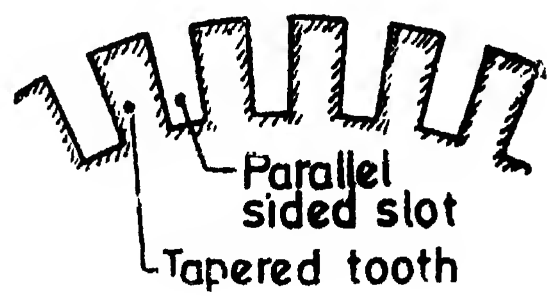


Fig. 8·3. Armature with tapered teeth.

Area of flux path in each tooth=width of tooth × core length=$W_t L$.

$$\therefore \quad \text{Flux density in teeth } B_t=\frac{\text{flux in each tooth}}{\text{area of each tooth}}=B_{av}\,.\,\frac{y_s L}{W_t L}=B_{av}\,.\,\frac{y_s}{W_t} \qquad \ldots(8\cdot20)$$

In a salient pole machine, the flux is concentrated over the pole arc and therefore the teeth which are under the pole arc carry whole of the flux and hardly any flux is carried by the teeth lying outside the pole arc.

Hence flux density in the teeth of salient pole machines is $B_t=\dfrac{B_{av}}{\psi}\,.\,\dfrac{y_s}{W_t}$ **...(8·21)**

$$= B_g \cdot \frac{y_s}{W_t} \qquad \ldots(8{\cdot}22)$$

where B_g = maximum flux density in the air gap, and

ψ = ratio of pole arc to pole pitch

From Eqns 8·20 and 8·21 it is clear that the flux density in the teeth (and for that matter flux density in any part of the magnetic circuit) is directly proportional to specific magnetic loading. Let us consider the case of a salient pole machine. We have,

$$B_t/B_{av} = y_s/\psi W_t.$$

Thus for a machine of given dimensions ratio B_t/B_{av} is constant and therefore if B_t is not to exceed a maximum specified limit (2·0 to 2·2 Wb/m² in the case of d.c. machines), specific magnetic loading has to be kept below a certain limiting value. Taking a specific example of a d.c. machine with tooth width equal to slot width and ψ=0·66, we have

$$W_t = y_s/2 \text{ as } y_s = W_t + W_s \text{ and } W_t = W_s$$

and therefore $B_t = (y_s/\psi W_t)\, B_{av} = (2/\psi)\, B_{av} = (2/0{\cdot}66)\, B_{av} = 3\, B_{av}$

and if B_t is to be limited to 2·1 Wb/m², the value of B_{av} should not exceed 2·1/3 = 0·7 Wb/m².

Machines using rectangular parallel sided slots have tapered teeth and therefore the tooth width is not the same over the entire height of tooth (see Fig. 8·3). This gives different values of flux density in teeth at different heights. The maximum value of flux density in teeth occurs where the tooth width is smallest *i.e.* at the root of the teeth in the case of d.c. machines and at a section near the air gap for synchronous machines.

In big machines which have large diameters, the taper of teeth is not significant and therefore the width of teeth is almost the same over their entire height. However in small machines which have smaller diameters, the taper of teeth is very pronounced and consequently the ratio B_t/B_{av} is very large at the section where the teeth have the smallest width and hence for a given maximum value of B_t it follows that B_{av} must be reduced. In general, therefore, small machines have lower specific magnetic loadings.

(*ii*) **Magnetising current.** The magnetising current of a machine is directly proportional to the mmf required to force the flux through the air gap and iron parts of the machine. The mmf required for air gap is directly proportional to the gap flux density *i.e.* the specific magnetic loading. As far as the iron parts are concerned, we have seen earlier that the value of flux density in them depends upon the value of specific magnetic loading. If a small value of specific magnetic loading is chosen, the flux density in the iron parts is low and therefore these parts are worked on the linear or knee portion of the *B—H* curve (Fig. 8·4). This requires a small or even negligible value of mmf for iron parts, as *H*, the mmf per metre length is very small for flux densities on the linear and knee portions of the curve. However, if a large value of specific magnetic loading is assumed, the flux density in iron parts (especially teeth) may be such as to work these parts in the saturation region of the *B—H* curve. It is clear from Fig. 8·4 that if iron parts are worked in the saturation region, the mmf per metre length and consequently the mmf required for iron parts is excessively large. Thus a large value of specific magnetic loading results in increased values of magnetising mmf and hence of magnetising current.

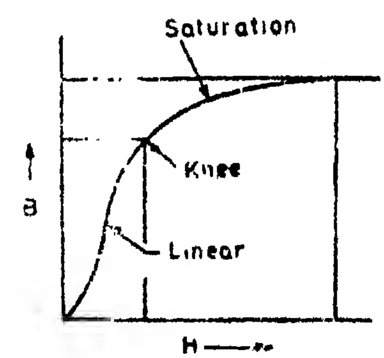


Fig. 8·4. *B-H* curve.

The value of magnetising current is not usually a serious design consideration in d.c. machines as there is ample space on salient poles to accommodate the required number of field turns. However, in induction motors, the consideration of magnetising current is very important as an increased value of magnetising current means a low operating power factor. Therefore specific magnetic loading in the case of induction motors is lower than that in d.c. machines. For synchronous machines the magnetising current is not so critical and the value of specific magnetic loading intermediate between d.c. and induction machines may be used.

(ii) **Core loss.** The area of cross-section of iron parts of the magnetic circuit of a machine is

$$=\frac{\text{flux carried by the part}}{\text{flux density in the part}}\propto\frac{1}{B}$$

If this part is subjected to alternating magnetization, there will be core loss in it. The specific core loss i.e. loss per unit volume or weight is approximately proportional to square of the flux density or iron loss per unit volume $\propto B^2$.

Now total core loss = loss per unit volume $\times$ volume

$$=\text{loss per unit volume}\times\text{area}\times\text{length}\ \propto B^2\times(1/B)\propto B$$

(as the length of flux path is constant though not strictly).

Thus we find that the core loss in any part of the magnetic circuit is directly proportional to flux density for which it is going to be designed. (**It should be noted that it is not true for an existing machine for which the iron loss is proportional** to B^2). Since the flux density in any part of the magnetic circuit is proportional to the specific magnetic loading, we conclude that the core loss in a machine varies directly as the specific magnetic loading. Thus a large value of specific magnetic loading indicates an increased core loss and consequently a decreased efficiency and an increased temperature rise.

With a given specific magnetic loading, the core loss increases as the frequency of reversals is increased. This is because the hysteresis loss is directly proportional to the frequency and eddy current loss is proportional to the square of the frequency. It follows that for high speed d.c. machine or high frequency a.c. machines, specific magnetic loadings must be reduced in order to get lower iron loss so that reasonable values of efficiency may be maintained. For example B_{av} in 50 Hz induction motors is about 0·45 Wb/m², while in 400 Hz induction servomotors it is about 0·25 Wb/m² and even this value is made possible only by using more expensive lamination materials having lower specific core loss.

The specific magnetic loading may be increased slightly with increase in size of machine for the following reason :

For a given frequency and flux density, the specific iron loss of the material is constant. Thus for two machines having linear dimensions in the ratio of $x:1$, core loss would be in the ratio $x^3:1$ (see Eqn. 8·30). But the outputs of the two machines are approximately in the ratio $x^4:1$ (see Eqn. 8·27). Thus for the same specific magnetic loading, the percentage core loss is proportional to $\frac{x^3}{x^4}$ or $\frac{1}{x}$. Therefore percentage core loss decreases with increase in size. It follows, that for the same percentage core loss in the two machines, specific magnetic loading may be increased slightly for the larger machine.

8·18. Choice of specific electric loading

The following factors influence the choice of specific electric loading :

1. Premissible temperature rise. An armature of a ratating mochine is shown in Fig 8·5. For this machine, let

Z=total number of armature conductors,

S=number of armature slots.

a_z=area of each conductor,

ρ=resistivity of conductor material, and

δ=current density.

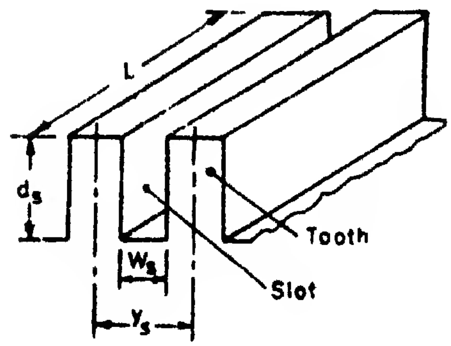


Fig. 8·5. Slots and teeth of armature of a rotating machine.

Therefore, if we consider a slot pitch, ampere conductors per metre for this portion are

$$ac=\frac{I_z Z}{\pi D}=\frac{I_z Z/S}{\pi D/S}=\frac{I_z Z_s}{y_s}$$

where $Z_s=Z/S$=number of conductors per slot.

Resistance of slot portion of each conductor$=\rho L/a_z$

I^2R loss in slot portion of each conductor$=I_z^2\,\frac{\rho L}{Z}$

$\therefore$ I^2R loss in each slot$=Z_s\,\frac{I_z^2\,\rho L}{a_z}$.

Heat produced in a slot is dissipated over the surface over one slot pitch. Considering only the cylindrical surface,

heat dissipating surface $S=y_s L$.

Loss dissipated per unit area of armature surface $q=\frac{\text{loss}}{\text{surface}}=\frac{Z_s I_z^2 \rho L}{a_z y_s L}=\frac{I_z Z_s}{y_s}\cdot\frac{I_z}{a_z}\cdot\rho$

Now $ac=I_z Z_s/y_s$ and $\delta=I_z/a_z$

$\therefore$ $=ac\,\delta\,\rho$ ...(8·23)

Therefore, other things being equal, heat dissipated per unit area of armature surface is proportional to specific electric loading.

It should be noted that we have not taken into consideration the heat dissipating surface of the overhang. This is quite reasonable since Eqn. 8·23 is normally not used quantitatively.

Temperature rise $\theta=Qc/S$ (See Eqn. 3·52) where Q is the loss to be dissipated, S the dissipation surface and c the cooling co-efficient.

Loss dissipated per unit area $q=Q/S$.

Hence temperature rise $\theta=qc=ac\;\rho\;\delta\;c$.

$\therefore$ Specific electric loading $ac=\theta/\rho\;\delta\;c$. ...(8·24)

From Eqn. 8·23, it can be concluded that the limiting value of specific electric loading ac is fixed by maximum allowable temperature rise θ and the cooling co-efficient c. The effects of different factors is discussed in details below.

(i) Temperature rise It is apparant from Eqn. 8·24 that a high value of specific electric loading can be used in a machine where a high maximum temperature rise is allowed. **The maximum allowable temperature rise of a machine is determined by the type of insulating.**

materials used in it. For example organic materials like cotton, paper and many varnishes may be worked upto a maximum temperature (not temperature rise) of 105°C while inorganic materials like mica, asbestos, and glass fibre bounded with silicone can withstand a temperature of 180°C without deterioration. Hence when better quality insulating materials, which can withstand high temperature rises, are used in machines, increased values of specific electric loading can be used resulting in reduction in the size of the machine.

(*ii*) **Cooling coefficient**. Examining equation 8·24, we find that if the cooling co-efficient of the machine is small, a high value of specific electric loading may be used in the machine.

The value of cooling co-efficient, c, depends upon the ventilation conditions in the machine. A machine with better ventilation has a lower value of cooling co-efficient and therefore, a high value of specific electric loading may be used in it. So for the same reason in a high speed machine a high value of ac may be used as due to high speed the ventilation conditions in the machine are improved owing to natural fanning action of the rotor.

. **Voltage**. Area of each slot=height of slot×width of slot$=d_s W_s$.

Total area of all slots$=Sd_s W_s=\dfrac{\pi D}{y_s}\cdot d_s W_s=\pi D\, d_s \left(\dfrac{W_s}{y_s}\right)$

$\therefore$ Total area of conductors (bare) in slots$=\pi D\, d_s\, (W_s/y_s)\, S_f$ ...(*i*)

where S_f=space factors for slots

Also total area of conductors (bare)$=Za_z=ZI_z\delta=\pi D\, ac/\delta$ ...(*ii*)

as $a_z=I_z/\delta$ and $I_z Z=\pi D\, ac$.

Equating (*i*) and (*ii*), we have : $\pi D\, \dfrac{ac}{\delta}=\pi D d_s \left(\dfrac{W_s}{y_s}\right) S_f$

$$ac=d_s\left(\frac{W_s}{y_s}\right)\delta\, S_f \qquad ...(8\cdot 25)$$

It follows from Eqn. 8·25, that for a fixed ratio of slot width to slot pitch and fixed values of depth of slot and current density, the specific electric loading is directly dependent upon space factor S_f *i.e.* the ratio of bare conductor area to total slot area. In high voltage machines, greater insulation thickness is required and therefore the space factor for these machines is lower. Hence an increase in voltage will, in general, necessitate a reduction in specific electric loading '*ac*'.

Incidently the value of space factor also depends upon shape of the conductors (round or rectangular). The space factor for round conductors is lower than that of rectangular conductors and consequently lower values of '*ac*' are used for machines employing round conductors.

3. **Size of machine.** It follows from Eqn. 8·25 that provided it can be assumed that depth of slot, ratio of width of slot to slot pitch, current density and slot space factor are the same for all machines, the specific electric loading is constant. In practice, of course, these assumptions are not quite valid and slightly varying values must be used throughout a range of sizes. For example, the assumption of constant slot depth is not true. The larger the machine the greater the slot depth and greater the specific electric loading. Actually if the current density and the slot space factor are assumed constant, then specific electric loading is proportional to the diameter as slot depth usually depends upon the diameter.

4. **Current density**. From Eqn. 8·23 it is clear that a higher value of specific electric loading can be used in a machine which employs lower current density in its conductors.

Typical values of current density are in the range of 2—5 A/mm^2. The temperature rise is usually 40°C for normal applications and cooling co-efficient is between 0·02 to 0·035 °C W—m^2.

If we know the insulating material and the temperature it can withstand, the ambient temperature, and the conductor resistivity ρ at operating temperature, a compatible set values for current density δ and specific electric loading 'ac' can be derived from Eqn. 8·23.

Example 8·1. *A 350 kW, 500 V, 450 rpm, 6 pole d.c. generator is built with an armature diameter of 0·87 m and a core length of 0·32 m. The lap wound armature has 660 conductors. Calculate the specific electric and magnetic loadings.*

Solution. Armature current $I_a = \frac{350 \times 1000}{500} = 700$ A.

Current in each conductor $I_z = \frac{I_a}{a} = \frac{700}{6} = 116·6$ A. ($a = p$ for a lap winding).

Specific electric loading $ac = \frac{I_z Z}{\pi D} = \frac{116·6 \times 660}{\pi \times 0·87} = 28200$ ampere conductors per metre.

We have $E = \Phi Z n \frac{p}{a}$ $\therefore$ Flux per pole $\Phi = \frac{E \times a}{Znp} = \frac{500 \times 6}{660 \times (450/60) \times 6}$

$= 0·101$ Wb.

Specific magnetic loading $B_{av} = \frac{p\Phi}{\pi D L} = \frac{6 \times 0·101}{\pi \times 0·87 \times 0·32} = 0·693$ Wb/m^2.

Example 8·2. (*a*) *The output co-efficient of 1250 kVA, 300 rpm. synchronous generator is 200 kVA/m^3—r.p.s (a). Find the values of main dimensions (D, L) of the machine if the ratio of length to diameter is 0·2. Also calculate the value of main dimensions if :*

(*b*) *specific loading are decreased by 10 percent each with speed remaining the same ;*

(*c*) *speed is decreased to 150 rpm with specific loading remaining the same as in part (a);*

Assume the same ratio of length to diameter. Comment upon the results.

Solution. (*a*) Speed $= 300/60 = 5$ r.p.s.

From Eqn. 8·18, $D^2L = \frac{Q}{C_0 n_s} = \frac{1250}{200 \times 5} = 1·25$ m^3.

But $L/D = 0·2$ (given) $\therefore$ $0·2\ D^3 = 1·25$

(*b*) From Eqn. 8·19, output co-efficient $C_0 = 11\ B_{av}\ ac\ K_w \times 10^{-3}$.

This means that the output co-efficient C_0 is directly proportional to the product of specific electric and specific magnetic loadings. The specific loadings are each decreased by 10 per cent and therefore new value of output co-efficient is : $C_0 = 0·9 \times 0·9 \times 200 = 162$.

Hence $D^2L = \frac{1250}{161 \times 5} = 155$ m^3. $\therefore$ $0·2\ D^3 = 1·55$

or $D = 1·98$ m and $L = 0·4$ m.

The results indicate that the size of the machine increases with decrease in specific loadings.

(*c*) Speed $n_s = 150/60 = 2·5$ and output co-efficient $C_0 = 200$.

$\therefore$ $D^2L = \frac{1250}{200 \times 2·5} = 2·5$ m^3 or $0·2\ D^3 = 2·5$

or $D = 2·3$ m and $L = 0·46$ m.

The results show that the size of machine increases with decrease in operating speed.

Example 8 3. *Prove that in a d.c. machine the volume of active parts is proportional to torque of the machine.*

Solution Torque $T = \dfrac{\text{power developed by armature}}{\text{angular velocity}} = \dfrac{EI_a}{2\pi n}$

$$= \frac{\Phi Z n\, p/a}{2\pi n} \cdot I_a = \frac{1}{2\pi}\Phi \frac{p}{a} I_a Z.$$

But $\Phi = B_{av}\, \pi DL/p$, $\quad I_z = I_a/a$ and $\quad I_z Z = \pi D\, ac$

$$\therefore \qquad T = \frac{1}{2\pi}\left(B_{av}\frac{\pi DL}{p}\right) p(\pi D\, ac) = \frac{\pi}{2} B_{av}\, ac\, D^2L$$

$$\text{or} \qquad D^2L = \frac{2T}{\pi\, B_{av}\, ac}$$

Volume of active parts of machine $= \dfrac{\pi}{4} D^2L = \dfrac{T}{2\, B_{av}\, ac}$

Since B_{av} and ac are constant, the volume of active parts is proportional to the torque.

Example 8 4 *Prove that for a 'm' phase synchronous machine, the effective rotor volume is given by :*

$$volume = \frac{Q\times 10^3}{\sqrt{2}\ \pi^2 B_{av}\, ac\, n}$$

(b) A rough estimate of the dimensions and windings of 100 MVA 11 kV, 3000 rp star connected 3 phase turbo-alternator is required The maximum value of flux density in th air gap of a machine is to be limited to 1 0 Wb/m² The specific electric loading is 80,00 ampere conductors per metre

(i) Determine the approximate volume of the cylindrical part of the rotor

(ii) The peripheral speed of rotor is to be limited to 200 m/s Estimate the required diameter and length.

(iii) Calculate the number of turns per phase.

Solution kVA rating $Q = m\, E_{ph}\, I_{ph}\times 10^{-3}$

Voltage per phase $E_{ph} = 4{\cdot}44\, T_{ph}\, \Phi f\, K_w = \sqrt{2}\,\pi\, T_{ph}\, \Phi f\, K_w$

$$\therefore \qquad Q = m\times\sqrt{2}\,\pi\, T_{ph}\, \Phi f\, K_w \times I_{ph}\times 10^{-3}$$

Putting $\qquad \Phi = B_{av}\cdot\dfrac{\pi DL}{p},\ I_{ph}\, T_{ph} = \dfrac{\pi D\, ac}{2m}$ and $f = \dfrac{pn_s}{2}$, we have :

$$Q = m\times\sqrt{2}\,\pi\left(B_{av}\frac{\pi DL}{p}\right)\left(\frac{pn_s}{2}\right)\left(\frac{\pi D\, ac}{2m}\right)K_w\times 10^{-3}$$

$$= \left(\frac{\pi^2}{2\sqrt{2}} B_{av}\, ac\, n_s\, K_w\times 10^{-3}\right)D^2L \quad \therefore\ D^2L = \frac{2\sqrt{2}\, Q\times 10^3}{\pi^3 B_{av}\, ac\, n_s\, K_w}$$

Volume of rotor $= \dfrac{\pi}{4} D^2L = \dfrac{\pi}{4}\times\dfrac{2\sqrt{2}\, Q\times 10^3}{\pi^3 B_{av}\, ac\, n_s\, K_w} = \dfrac{Q\times 10^5}{\sqrt{2}\,\pi^2 B_{av}\, ac\, n_s}$

as value of winding factor kW may be assumed as 1.

(b) Assuming a sinusoidal flux distribution : $B_{av} = (2/\pi)\, B_m = (2/\pi)\times 1 = 0{\cdot}637\ \text{Wb/m}^2$

Synchronous speed $n_s = 3000/60 = 50$ rps.

(i) Volume of rotor $= \dfrac{Q\times 10^3}{\sqrt{2}\,\pi^2 B_{av}\, ac\, n_s} = \dfrac{100000\times 10^3}{\sqrt{2}\,\pi^2\times 0{\cdot}637\times 80000\times 50} = 2{\cdot}75\ \text{m}^3$.

(ii) Peripheral speed $= \pi D\, n_s = 200$ m/s (given)

Maximum permissible rotor diameter $= \dfrac{200}{\pi\times 50} = 1{\cdot}275$ m.

Taking rotor diameter $D=1{\cdot}25$ m. $\therefore$ Core length $=\frac{4}{\pi\times(1{\cdot}25)^2}\times 0{\cdot}75=2{\cdot}24$ m.

(*iii*) Number of poles $=\frac{2f}{n_s}=\frac{2\times 50}{50}=2$.

Flux per pole $\Phi=B_{av}\frac{\pi DL}{p}=0{\cdot}637\times\frac{\pi\times 1{\cdot}25\times 2{\cdot}24}{2}=2{\cdot}8$ Wb.

Voltage per phase $E_{ph}=11000/\sqrt{3}=6350$ V.

Now $E_{ph}=4{\cdot}44\ T_{ph}\ \Phi f K_w$. or $6350=4{\cdot}44\times T_{ph}\times 2{\cdot}8\times 50\times 1$

$\therefore$ Turns/phase $\simeq 10$

Example 8·5. *A 125 W, 230 V, 5000 r.p.m., universal motor has a full load efficiency of 50 per cent. Calculate, the power developed by armature of the motor if the sum of iron, friction, and windage losses is approximately 1/3 of total losses.*

Solution. Efficiency $\eta=\frac{\text{output}}{\text{output}+\text{losses}}$ or $0{\cdot}5=\frac{125}{125+\text{losses}}$

or total losses $=125$ W and constant losses $=125/3\simeq 42$ W

$\therefore$ Power developed by armature $P_a=125+42=167$ W.

(The same result is obtained by using Eqn. 8·14).

Example 8·6. *In a d.c. machine the maximum flux density in the armature teeth is to be limited to 2·0 Wb/m². The ratio of minimum width of tooth to slot pitch is 0·4. Calculate the limiting value of specific magnetic loading if the ratio pole arc to pole pitch = 0·7.*

Solution. From Eqn. 8·20, for a salient pole machine limiting value of specific magnetic loading is,

$$B_{av}=\psi\times\frac{W_t}{y_s}\times B_t=0{\cdot}7\times 0{\cdot}4\times 2=0{\cdot}56\ \text{W/m}^2.$$

Example 8·7. *The core loss in the teeth of an induction motor when designed with a specific magnetic loading of 0·46 Wb/m² is 920 W. If the machine is redesigned with a specific magnetic loading of 0·5 Wb/m², calculate the approximate value of the core loss in teeth. Assume that specific core losses vary as square of flux density and the length flux path and also the ratio of width of tooth to slot pitch remain constant.*

Solution. When dealing with effect of specific magnetic loading on core losses we have seen that core losses in any part of machine vary approximately linearly with flux density. The flux density in any part of the magnetic circuit is proportional to specific magnetic loading, B_{av}.

$\therefore$ Core loss with $B_{av}=0{\cdot}5$ Wb/m² is $920\times 0{\cdot}5/0{\cdot}46=1000$ W

Example 8·8. *The IEC recommendations allow a maximum temperature rise of 50°C (as measured with thermometer) for windings of small machines employing class A insulation. Calculate the maximum permissible value of specific electric loading which can be used for a machine designed with class A insulation and using copper conductors with a current density of 3·5 A/mm². Assume a maximum ambient temperature of 40°C and a cooling co-efficient of 0·02. The resistivity of copper is $1{\cdot}734\times 10^{-8}\ \Omega$ m at 20°C. The resistance temperature co-efficient of copper is 0·00393/°C at 20°C.*

Supposing the above machine is redesigned with class E insulating materials for which maximum allowable temperature rise is 65°C above an ambient temperature of 40°C, calculate the value of maximum allowable value of specific electric loading. The values of current density and cooling co-efficient remain the same.

Solution.

Class A Insulation. Operating temperature of copper conductors

$=40^\circ+50^\circ=90^\circ C,$

Resistivity at operating temperature $\rho=1.734\times10^{-8}[1+0.00393(90-20)]$

$=2.2\times10^{-8}\ \Omega$ m.

From Eqn. 8·23, maximum allowable specific electric loading

$$ac=\frac{\theta}{\rho\,\delta\,c}=\frac{50}{2.2\times10^{-8}\times3.5\times10^{6}\times0.03}$$

=21600 ampere conductors per metre.

Class E Insulation. Operating temperature of conductors $=40^\circ+65^\circ=105^\circ C.$

Resistivity at operating temperature

$$\rho=1.734\times10^{-8}[1+0.00393(105-20)]=2.32\times10^{-8}\ \Omega\ \text{m}$$

Maximum permissible specific electric loading

$$ac=\frac{65}{2.32\times10^{-8}\times3.5\times10^{6}\times0.03}$$

=26700 ampere conductors per metre.

Example 8·9. *For a certain d.c. generator the core loss is 1000 W and the armature resistance is 0·025 Ω. The core and windings form a cylinder 0·25 m long and 0·25 m in diameter. Specific loss dissipation is 230 W/m²—°C. Calculate the specific electric loading which would result in windings and core having a temperature rise of 40°C. The machine is wave wound with 270 armature conductors.*

Assume that the heat is dissipated from the cylindrical surface only.

Solution. Cooling co-efficient $c=1/\lambda=1/230=0.00435.$

Dissipating surface $S=\pi DL=\pi\times0.25\times0.25=0.1965$ m².

Maximum allowable power dissipation from armature surface

$$Q=\frac{S\theta}{c}=\frac{0.1965\times40}{0.00435}=1800\ \text{W}.$$

Maximum allowable I^2R loss$=1800-1000=800$ W.

$\therefore\ (I_a)^2\times0.025=800$ or armature current $I_a=180$ A.

Current in each conductor $I_z=I_a/a=180/2=90$ as $a=2$ for wave windings.

Specific electric loading $ac=\dfrac{I_zZ}{\pi D}=\dfrac{90\times270}{\pi\times0.25}$

=31000 ampere conductors per metre.

8·2. Variation of output and losses with linear dimensions. Consider two machines of the same type with all their linear dimensions in the ratio $x:1$ and having the same speed, flux density and current density.

Let the machine with linear dimensions x times be called A and other machine B.

Output. From Eqns. 8·8 and 8·17, we have for any type of machine :

Output $\propto B_{av}\,ac\,D^2L\,n.$

Since flux density B_{av} and speed n are constant therefore, output $\propto ac\,D^2L.$

Now $ac = \frac{I_z Z}{\pi D} = \frac{(\delta a_z)Z}{\pi D}$ $\therefore$ $ac \propto \frac{x^2}{x} \propto x$

since area of each conductor $a_z \propto x^2$, diameter $D \propto x$ and current density δ is constant and π and Z have just numerical values.

We have $D^2 \propto x^2$ and $L \propto x$

$\therefore$ Output $\propto x \times x^2 \times x \propto x^4$.

Therefore, the output of machine A is x^4 times the output of machine B.

Losses

I^2R loss = number of conductors × copper loss in each conductor

$$= Z \times (I_z)^2 \rho \frac{l}{a_z} = Z(\delta a_z)^2 \rho \frac{l}{a_z} = \delta^2 \rho \times (Z a_z l)$$

$= \delta^2 \rho \times$ volume of active portion of conductors.

Now current density δ and resistivity ρ are constant.

$\therefore$ I^2R loss $\propto$ volume of conductors $\propto x^3$.

Thus the copper loss of machine A is x^3 time that of machine B.

The specific iron loss (i.e. loss per unit volume) remains constant if the flux density is constant.

Total iron loss = loss per unit volume × volume of iron $\propto$ volume of iron $\propto x^3$.

Both I^2R and iron losses vary as the third power of linear dimensions.

$\therefore$ Total losses $\propto x^3$.

This means that total losses of machine A are x^3 times those of B.

Efficiency. Efficiency $\eta = \frac{\text{output}}{\text{output} + \text{losses}} \propto \frac{x^4}{x^4 + Kx^3} \propto \frac{1}{1 + K/x}$.

From above it is clear that η increases with increase in x. It follows, therefore, that the larger machines are intrinsically more efficient. This explains in part why fractional horsepower motors have efficiency of the order of 60 percent or less and large turbo-alternators have efficiencies of the order of 98 per cent.

Cooling. The heat dissipating surface is proportional to the square of linear dimensions or $S \propto x^2$.

We have,

temperature rise, $\theta = \frac{Qc}{S} \propto \frac{x^3 c}{x^2}$ as losses $\propto x^3$

$\propto x\,c$.

If we assume cooling co-efficient c to remain constant, $\theta \propto x$.

Therefore it is evident that the temperature rise of machine A is x times that of machine B. The variation of temperature rise directly with linear dimensions (although it is strictly not correct as with increase in dimensions, the rotor diameter increases and so does the peripheral speed and therefore the ventilation conditions in the machine are improved which lower the value of c) This is certainly a serious situation as normally the machines are operated over very narrow range of temperature rises say about 60°C for class A insulation, about 70°C for class E insulation and, therefore, we must resort to some outside methods to bring down the value of cooling co-efficient c for big machines in order that their temperature rise is within maximum allowable limits. Although to a limited extent the increased natural fanning action of rotors of big machines due to their high peripheral speeds, helps in improving the ventilation conditions in the machine but forced cooling eventually becomes essential. Thus as the size of machine increases,

better ventilation and cooling conditions have to be provided in the machine in order to keep its temperature rise within limits. ***This explains in part why fractional horsepower motors can be entirely self-cooled while large turbo-alternators require more elaborate and sophisticated cooling schemes.***

It may be pointed out here that unless we incorporate better cooling methods in big machines due to which the heat generated in the machine is taken away it will be impossible to work these machines to deliver higher outputs as the high temperature rise brought about by higher losses will damage the insulation. Thus simply increasing the dimensions of the machine will not help us in taking higher output from it, but we have to provide better cooling facilities to protect the insulation from deterioration caused by higher losses that follow higher outputs.

Current Density. In the analysis done till now we have assumed a constant value for current density. But it is not possible to maintain constant value of current density for larger sizes of machines even though we employ forced cooling and if current density δ is not maintained constant the specific electric loading ac is not therefore proportional to x. The current density is greatest in small machines, diminishing considerably with increase in size. However, in large turbo-alternators which employ direct cooling of conductors the value of current density can be appreciably increased.

Table 8·1 gives normal values of specific magnetic loading, B_{av}, specific, electric loading, ac, and current density, δ, usually used in practice. These values are for a temperature rise 40°C. For a temperature rise of 50°C, the values given in Table 8·1 may be increased by 7·5% for B_{av}, 7·5% for ac, and 15% for δ.

The values are also based upon peripheral speeds between 15 to 40 m/s.

8·3. Separation of D and L. The value of product D^2L can be obtained by using Eqns. 8·8 and 8·17 but additional data is required before this product can be further split up into its components D and L. The factors which influence the relative values of D and L are different for different types of machines.

8·3·1. Separation of D and L for d.c. machines.

(i) **Machine proportions.** One factor in determining the value of core length is the ratio of core length to pole pitch as it determines the proportions of the pole.

The pole section which necessitates the smallest weight of copper for the field winding is circular, since its periphery is the smallest for a given area. Therefore, the mean length of turn is smallest with circular poles than with any other form of pole section. However, circular poles necessitate the use of solid pole section. Modern practice is to use laminated poles as their use results in reduction of production costs and therefore rectangular section poles are used. The dimensions of the machine are decided by the square pole criterion. This states that, for given flux and cross area of pole, the length of mean turn of field winding is minimum when the periphery forms a square.

This means the length L must be approximately equal to the pole arc or $L=b=\psi\tau$. The value of ψ is usually between 0·64 to 0·72 and therefore the ratio L/τ is 0·64 to 0·72. However, in practice L is slightly greater than pole arc b and therefore L/τ is usually between 0·7 to 0·9.

***(ii)* Peripheral speed.** The peripheral speed of armature ($V_a=\pi Dn$) is sometimes a limiting factor to the value of diameter.

The peripheral speed should not exceed about 30 m/s as this speed does not call for any special rotor construction. However, if this speed is exceeded, the banding wires on the overhang have to be made especially strong.

***(iii)* Moment of Inertia.** For machines used in control systems, a small moment of inertia is desirable. As moment of inertia is approximately proportional to D^4L, the dia-

Table 8·1. Specific Loadings.

		3 Phase, 50 Hz Machines									D.C. Machines				
		Slip Ring Induction			Squirrel Cage Induction			Synchronous							
D	L_i/D maximum	B_{av}	ac	δ	B_{av}	ac	δ	B_{av}	ac	δ	D	L_i/D maximum	poles	B_{av}	ac
0·10	0·60	0·30	6000	3·8	0·30	11000	4·0	—	—	—	0·10	0·78	2	0·30	6000
0·15	0·75	0·35	10000	3·6	0·35	15000	3.7	—	—	—	0·15			0·36	10000
0.20	0·70	0·40	13000	3·4	0.40	18000	3·6	0·40	13000	3·4	0·20	0·7	4	0·43	13000
0·30	0·65	0·43	17500	3·3	0·43	22500	3·5	0·43	17500	3·3	0·30			0·50	17500
0·40	0·62	0·45	21500	3·2	0.45	26000	3·5	0·46	21500	3,2	0·40			0·54	21500
0·50	0·60	0·46	25000	3·2	0.46	29000	3.5	0·48	25000	3·2	0·50	0·32	6	0·59	25000
0·75	0·50	0·47	30000	3·2	0·47	33000	3·5	0·52	35000	3·2	0·75			0·64	31000
1·00	0·42	0·48	32500	3·2	0·48	35000	3.5	0·54	36000	3·2	1·00	0·24	8	0·66	35000
1·50	0·33	0·50	34000	3·2				0·55	37000	3·2	1·50			0·70	37000
2·00	0·30	0·51	35000	3·2				0·56	38000	3·2	2·00	0·19	10	0·72	38000
2·50	0·30	0·52	36000	3·2				0·57	39000	3·2	2·50	0·16	12	0·74	39000
3·00	0·30	0·53	37000	3·2				0·58	40000	3·2	3·00	0·12	16	0·75	40000

meter should be made as small as feasible for machines meant control system applications. Conversely, a high inertia machine may be required for impact-load applications and such machines are designed for large diameter.

(*iv*) **Voltage between adjacent segments.** The maximum core length is fixed by the maximum voltage that can be allowed between adjacent segments.

The maximum voltage between adjacent segments

$$E_{cm}=2B_{gm}LV_aT_c$$

where B_{gm}=maximum air gap flux density under load conditions,

and T_c=turns per coil.

Taking typical limiting values $E_{cm}=30$ V, $B_{gm}=1{\cdot}2$ Wb/m², $V_a=30$ m/s and $T_c=1$ (single turn coils are used for large machines), we have :

$$30=2\times1{\cdot}2\times L\times30\times1 \text{ or } L\simeq0{\cdot}4 \text{ m}.$$

This is only an indication of value obtained, but it must be clear that large d.c. machines should have large diameters rather than large core lengths.

8·3·2. Separation of D and L for induction motors. The operating characteristics of an induction motor are mainly influenced by the ratio L/τ *i.e.* by the ratio L/D for a fixed number poles. The factors influencing this choice are :

For minimum cost $L/\tau=1{\cdot}5$ to 2, for good power factor $L/\tau=1{\cdot}0$ to $1{\cdot}25$

For good efficiency $L/\tau=1{\cdot}5$, and for good overall design $L/\tau=1$.

Actually largest values of L/τ ratio apply to high voltage machines, where due to the relatively longer core length the flux per pole is increased and thereby number of conductors is reduced. This greatly reduces the cost of insulation.

8·3·3. Separation of D and L for synchronous machines.

(*i*) **Peripheral speed.** For large high speed machines D is fixed by the limiting peripheral speed. If this peripheral speed is exceeded, the rotor may become distorted owing to increased centrifugal stresses. It, therefore, becomes logical to express the output equation in terms of peripheral speed.

The output equation for a synchronous machine is :

$$Q=(11\ B_{av}\ ac\ K_w\times10^{-3})\ L\ D^2\ n_s \text{ (Eqn. 8·16)}$$

and peripheral speed of rotor $V_a=\pi Dn_s$

∴ We have.

$$Q=(11\ B_{av}\ ac\ K_w\times10^{-3})\left(\frac{V_a}{\pi n_s}\right)^2 L$$

$$=(1{\cdot}11\ B_{av}\ ac\ K_w\times10^{-3})\ \frac{V_a^2L}{n_s} \qquad \ldots(8{\cdot}32)$$

An increase in machine rating will necessitate an increase in D (and L) until the maximum permissible peripheral speed is reached. Once this happens, the value of D cannot be increased further and the only way to get increased output is to increase the length L. This presents many difficult mechanical engineering problems connected with critical speed of rotor and also with cooling arrangements for the machine.

The peripheral speed V_a does not necessarily correspond to the synchronous speed of the machine as overload speed must be allowed for, especially in the case of hydro-electric generators. The peripheral speed must be calculated for run away speed which may be as much 100 percent in excess of normal speed.

The peripheral speed may reach 175 m/s for forged steel rotors of turbo-alternators, and a value of about 60 m/s may be the limit for slow speed salient pole alternators.

(ii) Number of poles. The diameter of the machine depends upon the number of poles. If the number of poles is large the pitch is small and if a small diameter is used for this machine, there may not be sufficient room for field coils. Therefore, a large diameter is advisable for machines having large number of poles (*i.e.* for machines working at low speeds). Actually the following empirical relationship may be used

$$\tau/L=0\cdot5+6/p.$$

(iii) Short circuit ratio. A major factor influencing the design of synchronous machines is their short circuit ratio (SCR). This is discussed in details in the chapter on synchronous machines. In order to obtain ∝ high value for short circuit ratio, long core length should be used.

However, a short core length is advantageous since it is easy to cool, has a higher critical speed and reduces the leakage between pole bodies.

8 4. Standard frames. Apart from a few special machines, the manufacture of all modern motors for industrial applications is concentrated into a series of standard frames to cover a wide range of power ratings. The general practice is to limit the number of rated outputs Table 8 2 lists the recommended ratings as per IEC 72

Table 8·2 Recommended ratings (kW)

0·06	0·37	2·2	15	45	132	220	335	450	600	800
0·09	0·55	3·7	18·5	55	150	250	355	475	630	850
0·12	0·75	5·5	22	75	160	280	375	500	670	900
0·18	1·1	7·5	30	90	185	300	400	530	710	950
0·25	1·5	11	37	110	200	315	425	560	750	1000

These ratings can be provided with a limited number of standard frames. A **frame** is the mechanical structure required to house a stator of given outside diameter D_o along with its bearings, end covers and terminal box, and maximum core length L as shown in Fig. 8·6. A variation in rating can be obtained by using alternative core lengths less than L such as 0·7 L or 0·5 L. Suppose a 45 kW 750 rpm motor is designed to be housed in a certain frame, the same frame can be used for 37 kW, 750 rpm motor if the length of core is made 0 7 L (as 45×0·7 ≃ 37).

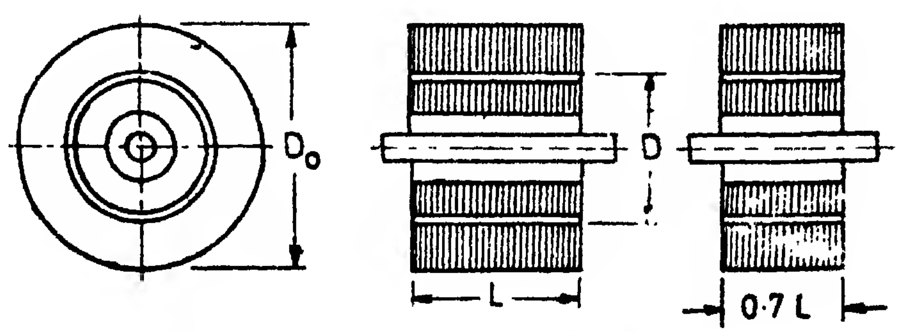


Fig. 8·6. Stator dimensions relevant to frame.

The standardization of frame sizes leads to economy. All modern motors of small and medium sizes are built with standard frame sizes as specified in IEC 72 which lists a coherent range of main structural dimensions with centre heights between 56 to 1000 mm. (See Fig. 8·7)

A **frame size** is designated by a number which is its centre height (H) expressed in mm. Thus frame designated 180 has a centre height (H) of 180 mm. The frame sizes with heights (H) between 56 to 1000 mm as recommended by IEC are listed in Table 8·3. The outputs from the standard frames are periodically assessed to take into account the latest technological advancements.

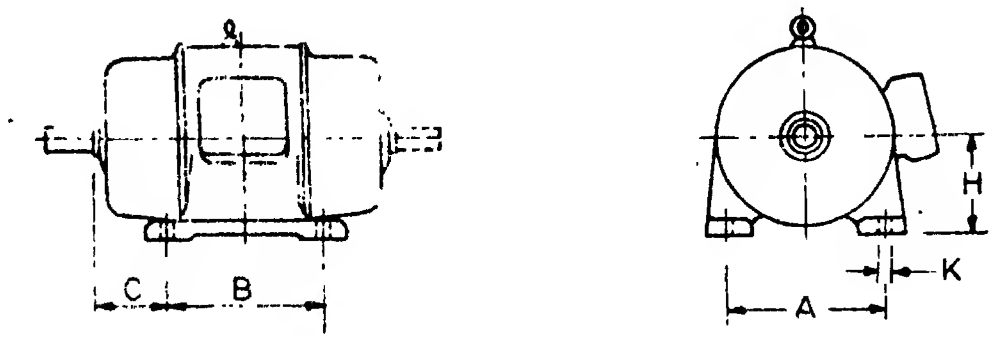


Fig. 8·7. Structural dimensions of standard frames

Table 8·3 Frame sizes for foot mounted motors.
(H=distance centre from shaft to mounting surface).

56	63	71	80	90	100	112	132	160	180	200	225	250
[illegible]	315	355	400	450	500	630	710	800	900	1000		

The frame sizes used for foot mounted 3 phase a.c. induction motors are given in Table 8·4. The structural dimensions are indicated in Fig 8·7.

The frame size in Table 8·4 consists of two parts, the first part giving the figures relate to the actual shaft centre and the second part giving letters indicating the frame lengths, the letters being :

S for short core length
M for medium core length
L for long core length.

Table 8·4 shows that the frame numbers (shaft centre H) are associated with three different dimensions values for B, giving small, medium and long core lengths. **However,** in actual practice, only two values are adopted for B for each frame size.

Example 8·12. *In two d.c. motors running at the same speed and having the same number of poles, the physical dimensions are in the ratio of 3 : 2. Compare the outputs, armature iron losses, and copper losses in the two machines. State any assumptions made.*

Solution. We have already analyzed that for two machines having same speed and having linear dimensions in the ratio of x : 1,

outputs are in the ratio x^4 : 1 and losses are in the ratio x^3 : 1.

These results are based on the assumption that the specific magnetic loading and the current density are the same for the two machines.

In the given problem, the machines have their linear dimensions in the ratio 3 : 2 or 1·5 : 1 and therefore

outputs are in the ratio $(1·5)^4$: 1 or 5.06 : 1,
iron losses are in the ratio $(1·5)^3$: 1 or 3·37 : 1,
and copper losses are in the ratio $(1·5)^3$: 1 or 3·37 : 1.

Table 8.4. Dimensions for foot-mounted a.c. induction machines with shaft heights 112—315 mm.

Frame (number)	Millimetres				
	H	A	B	C	K max.
112 S	112	190	114	70	12
112 M	112	190	140	70	12
(112 L)	112	190	159	70	12
132 S	132	216	140	89	12
132 M	132	216	178	89	12
(132 L)	132	216	203	89	12
160 S	160	254	178	108	14
160 M	160	254	210	108	14
160 L	160	254	254	108	14
180 S	180	279	203	121	14
180 M	180	279	241	121	14
180 L	180	279	279	121	14
200 S	200	318	228	133	18
200 M	200	318	267	133	18
200 L	200	318	305	133	18
225 S	225	356	286	149	18
225 M	225	356	311	149	18
(225 L)	225	356	356	149	18
250 S	250	406	311	168	22
250 M	250	406	349	168	22
(250 L)	250	406	406	168	22
280 S	280	457	368	190	22
280 M	280	457	419	190	22
(280 L)	280	457	457	190	22
315 S	315	508	406	216	27
315 M	315	508	457	216	27
(315 L)	315	508	508	216	27

Example 8.13. *Compare the outputs of two synchronous machines having linear dimensions in the ratio $x : 1$ and running at different speeds but subjected to equal centrifugal stress. Assume that the specific magnetic loading and current density are the same in the two machines.*

Solution. kVA output $Q = C_0 D^2 L n_s = (11\ K_w B_{av}\ ac \times 10^{-3}) D^2 L n_s$

Now $\qquad B_{av}$ = constant and $ac \propto x$ if current density is assumed constant

It is given in the problem that the centrifugal stresses remain the same and therefore the peripheral speed V_a of the two machines is the same.

Peripheral speed $V_a = \pi D n_s$ or speed $n_s \propto 1/D \propto 1/x$

$$\therefore \quad \text{Output } Q \propto x \times x^2 \times x \times \frac{1}{x} \propto x^3$$

as $ac \propto x$, $D^2 \propto x^2$, $L \propto x$ and $n_s \propto 1/x$

Hence the ratio of outputs of two machines is : $x^3 : 1$.

Example 8.14. *It has been established from test results that the ratio of losses to output of small rotating machines vary approximately as $D^{-1} L^{-1/2} n^{-1/2}$ where D, L, n are respectively the diameter, length and speed of the machine. Prove that if the temperature rise remains constant the output of a small machine varies as $D^3 Ln$. Assume cooling co-efficient $c \propto V_a^{-1/2}$ where V_a is the peripheral speed of the rotor. The effective heat dissipating surface of the machine is proportional to the geometric mean of the end surfaces and the cylindrical surface of the rotor.*

Solution. Temperature rise $\theta = \dfrac{\text{losses} \times \text{cooling co-efficient}}{\text{dissipating surface}}$

or losses $= \dfrac{\theta \times \text{dissipating surface}}{\text{cooling co-efficient}}$.

End surface $= 2 \times (\pi/4)\, D^2 \propto D^2$ and

cylindrical surface $= \pi DL \propto DL$

$\therefore$ Dissipating surface $\propto \sqrt{D^2 \times DL} \propto D^{3/2} L^{1/2}$.

Cooling co-efficient $c \propto (V_a)^{-1/2} \propto (\pi D n_s)^{-1/2} \propto D^{-1/2} n^{-1/2}$.

Hence, $\qquad$ losses $\propto \dfrac{D^{3/2} L^{1/2}}{D^{-1/2} n^{-1/2}} \propto D^2 L^{1/2} n^{1/2}$. $\qquad$ (as θ is constant.)

Now, $\qquad \dfrac{\text{losses}}{\text{output}} \propto D^{-1} L^{-1/2} n^{-1/2}$ (given)

$\therefore$ Output $\propto$ losses $\times D L^{1/2} n^{1/2} \propto (D^2 L^{1/2} n^{1/2}) \times D L^{1/2} n^{1/2} \propto D^3 Ln$

Example 8.15. *The losses in a 11 kW, 3 phase, 4000 V, 50 Hz, 1000 rpm induction motor are :*

I^2R losses = 950 W, iron losses = 500 W, and friction and windage losses = 110 W. Find the output losses and efficiency of a similar motor designed with each linear dimension 1.5 times the linear dimensions of the given motor.

Compare the values of efficiency of the two motors.

Solution. Total losses of the given motor = 950 + 500 + 110 = 1560 W.

Efficiency of the given motor $= \dfrac{11000}{11000 + 1560} = 0.876$

Ratio of friction and windage loss to output$=110/11000=0\cdot01$ or 1%

Output varies as fourth power of linear dimensions

$\therefore$ Output of a motor whose linear dimensions of 1·5 times those of the given motor$=(1\cdot5)^4\times11=55\cdot7$ kW.

I^2R and iron losses vary as the cube of the linear dimensions.

$\therefore$ I^2R and iron losses$=(950+500)\times(1\cdot5)^3=4893$ W.

Friction and windage losses$=1\%$ of output$=\frac{1}{100}\times55\cdot7\times10^3=557$ W

Total losses$=4893+557=5450$ W.

Efficiency $\eta=\frac{55700}{55700+5450}=91\cdot5\%$

Example 8 16. *A 1·1 kW, 3 phase, 50 Hz 1500 srpm delta connected induction motor has : stator bore $D=0\cdot15$ m and core length $L=0\cdot06$ m.*

Estimate the main dimensions of a 3·7 kW, 3 phase, 50 Hz, 1000 srpm delta connected motor having the same loadings as the previous one. The efficiency and power factor are same. Assume the same L/τ ratio.

Solution Let subscripts 1 and 2 refer to the Ist and 2nd machine respectively.

Synchronous speed $n_{s_1}=\frac{1500}{60}=25$ rps. Number of poles $p_1=\frac{2\times50}{25}=4$

$$n_{s_2}=\frac{1000}{60}=16\cdot67 \text{ rps}, \quad p_2=\frac{2\times50}{16\cdot67}=6$$

Pole pitch, $\tau_1=\frac{\pi\times0\cdot15}{4}=0\cdot118$. Ratio $L/\tau=0\cdot06/0\cdot118=0\cdot51$

Output $Q=C_0D^2L\,n_s$ or $\frac{\text{kW}}{\eta\cos\phi}=C_0\,D^2L\,n_s$ or $D^2L=\frac{\text{kW}}{\eta\cos\phi\,n_s}$

$$\therefore \quad \frac{D_2^2L_2}{D_1^2L_1}=\frac{\text{kW}_2}{\text{kW}_1}\cdot\frac{\eta_1\cos\phi_1}{\eta_2\cos\phi_2}\cdot\frac{C_{0_1}}{C_{0_2}}\cdot\frac{n_{s_1}}{n_{s_2}}$$

Now the power factor, efficiency and output co-efficients are the same in both cases,

$$\therefore \quad D_2^2L_2=\frac{\text{kW}_2}{\text{kW}_1}\cdot\frac{n_{s_1}}{n_{s_2}}(D_1^2L_1)=\frac{3\cdot7}{1\cdot1}\cdot\frac{25}{16\cdot67}\times(0\cdot15)^2\times(0\cdot06)=6\cdot8\times10^{-3}\text{ m}^3$$

Number of poles is 6 and ratio $L/\tau=0\cdot51$

Thus $\frac{L_2}{\pi D_2/6}=0\cdot51$ or $L_2=0\cdot267D_2$ $\therefore$ $0\cdot267\,D_2^3=6\cdot8\times10^{-3}$

or $D_2=0\cdot29$ m and $L_2\simeq0\cdot08$ m.

Example 8·17. *A control motor is required to operate from a 24 V d.c. supply and to provide a torque of 0·5 Nm at 200 rps. The armature length is to be twice of armature diameter in order to achieve a low polar moment of inertia.*

(a) Estimate the main dimensions of the armature. Assume average gap density$=0\cdot4$ Wb/m² and ampere conductors per metre$=8000$ since the machine has a small rating.

(b) Assuming that the average density of rotor material is about 8000 kg/m³, estimate the polar moment of inertia of the armature. It may be assumed that the polar moment of inertia of commutator, shaft and overhang is approximately equal to that of cylinder of magnetic material.

Solution. (*a*) Power developed by armature

$$P_a = T\omega = 0.5 \times 2\pi \times 20 \times 10^{-3} = 0.628 \text{ kW}.$$

Output co-efficient $C_0 = \pi^2 B_{av}\, ac \times 10^{-3} = \pi^2 \times 0.4 \times 800 \times 10^{-3} = 31.3.$

Now $$D^2L = \frac{P_a}{C_0 n} = \frac{0.628}{31.3 \times 204} = 0.984 \times 10^{-6} \text{ m}^3.$$

We have $L = 2D$ (given) $\therefore\ 2D^3 = 0.984 \times 10^{-6}$

or $D = 36.5$ mm and $L = 73$ mm

(*b*) Polar moment of inertia

$$J = \text{mass density} \times \frac{\pi}{16} D^4 L = 8000 \times \frac{\pi}{16} (0.0375)^4 \times 0.073.$$

$$= 23.3 \times 10^{-6} \text{ kg-m}^2$$

Example 8·18. *A 500 kW, 375 rpm d.c. generator is designed with $B_{av} = 0.6$ Wb/m² and ac = 35000 ampere conductors per metre and ratio pole arc to pitch = 0·66. The armature is lap connected and single turn coils are used. Find suitable values for diameter and length of armature if the maximum value of voltage between adjacent segments is not to exceed 30 V at full load and the peripheral speed is not to exceed 30 m/s. Assume the maximum value of gap density at full load to be 1·3 times the maximum value of flux density at no load. Efficiency at full load = 0·91.*

Solution. Neglecting rotational losses, power developed by armature at full load

$$P_a = \frac{P}{\eta} = \frac{500}{0.91} = 550 \text{ kW}.$$

Speed $n = 375/60 = 6.25$ rps.

Output co-efficient $C_0 = \pi^2 B_{av}\, ac \times 10^{-3} = \pi^2 \times 0.6 \times 35000 \times 10^{-3} = 207.5.$

Now $$D^2L = \frac{P_a}{C_0 n} = \frac{550}{207.5 \times 6.25} = 0.423 \text{ m}^3.$$

Taking peripheral speed $V_a = 20$ m/s

Diameter $$D = \frac{V_a}{\pi n} = \frac{20}{\pi \times 6.25} \approx 1 \text{ m and } L = 0.42 \text{ m}.$$

Maximum value of flux density at no load

$$B_g = \frac{B_a}{\psi} = \frac{0.6}{0.66} = 0.9 \text{ Wb/m}^2.$$

Maximum flux density at load $B_{gm} = 1.3 \times 0.9 = 1.17$ Wb/m².

Maximum voltage between adjacent segments at full load

$$E_{cm} = 2\, B_{gm}\, LV_a\, T_c = 2 \times 1.17 \times 0.42 \times 25 \times 1$$

$$= 24.6 \text{ V. Within limits.}$$

Example 8·19. *Select dimensions from the following range for a 25 h.p., 400 V, 3 phase, 6-pole, 50-Hz induction motor. The mean gap density is not to exceed 0·45 Wb/m² and specific electric loading is not to exceed 25000 ampere conductor per metre. Calculate also the turns per phase for the stator winding. The product of efficiency and power factor may be taken as 0·72 and the motor must be suitable for star delta starting.*

Stator bore	*m*	*0·25*	*0·30*	*0·36*
Core lengths	*m*	*0·10*	*0·12*	*0·19*
		0·14	*0·16*	*0·18*

Assume winding factor = 0·955.

Solution. kVA input to induction motor

$$Q=\frac{\text{h.p.}\times 0.746}{\eta\cos\phi}=\frac{25\times 0.746}{0.72}=25.9.$$

Output coefficient $C_0=11\ K_w\ B_{av}\ ac\times 10^{-3}$

$$=11\times 0.955\times 0.45\times 25000\times 10^{-3}=118.5.$$

Synchronous speed $n_s=\frac{2f}{p}=\frac{2\times 50}{6}=16.66$ rps

$\therefore$ $$D^2L=\frac{Q}{C_0n_s}=\frac{25.9}{118.5\times 16.65}=13.1\times 10^{-3}\ \text{m}^3.$$

The D^2L product for different frame sizes is tabulated below :

D	0·25	0·25	0·3..	0·30	0·360	0·36
L	0·10	0·14	0·12	0·16	0·19	0·20
D^2L	6.25×10^{-3}	8.75×10^{-3}	10.1×10^{-3}	14.4×10^{-3}	24.6×10^{-3}	23.3×10^{-3}

From the above table we observe that the suitable frame size which gives the nearest value of D^2L is :

$$D=0.30\ \text{m}\qquad\text{and}\qquad L=0.16\ \text{m}$$

Flux per pole $\Phi=\frac{\pi DL}{p}\ B_{av}=\frac{\pi\times 0.3\times 0.16}{6}\times 0.45=0.0113$ Wb.

Since the machine is to be started by a star delta starter, it must be designed for delta connection and therefore voltage per phase $E_{ph}=400$ V.

$\therefore$ Turns per phase $$T_{ph}=\frac{E_{ph}}{4.44\ f\ \Phi K_w}$$

$$=\frac{400}{4.44\times 50\times 0.0113\times 0.955}\simeq 168$$

Example 8·20. *Determine the stator bore and length of armature for 750 kVA, 50 Hz, 2200 V, 3 phase star connected 500 rpm alternator, given that the length of armature is equal to the pole pitch. The relation between the output co-efficient and the diameter is :*

D (*metre*)	0·8	1·0	1·2	1·4	1·6	1·8
C_0	154	174	188	198	205	210

Solution. Synchronous speed $n_s=\frac{500}{60}=8.33$ rps.

Number of poles $p=\frac{2f}{n_s}=\frac{2\times 50}{8.33}=12.$

It is given that core length=pole pitch. $L=\pi D/p=\pi D/12=0.262D.$

Now $$Q=C_0D^2Ln_s=C_0(0.262\ D^3)\times 8.33$$

or $$D^3C_0=\frac{750}{0.262\times 8.33}=344$$

The product D^3C_0 for various diameters is tabulated below :

D	0·8	1·0	1·2	1·4	1·6	1·8
D^3C_0	78·7	174	325	545	840	1225

A graph is plotted between D and product $D^3\ C_0$. It is observed from this graph that a diameter of 1·22 m gives $D^3C_0=344$, the required value.

$\therefore$ $D=1.22$ m and $L=0.262\times 1.22=0.32$ m.

Example 8·21. *The design details of two d.c. machines are tabulated below. Compare their relative outputs.*

Machine	*Diameter* D *m*	*Core length* L *m*	*Speed* n *rps.*	*Slots* S	*Area of each slot* A_s mm^2	*Slot space factor* S_f	B_{av} Wb/m^2	*Current density* δ A/mm^2
A	*0·75*	*0·31*	*10*	*72*	*48×11*	*0·5*	*0·6*	*4·5*
B	*0·55*	*0·25*	*7·5*	*61*	*44×10*	*0·43*	*0·56*	*5·2*

Solution. Specific electric loading $ac=I_z\ Z/\pi D$.

Current in each conductor I_z=current density×area of each conductor$=\delta\ a_z$.

Total armature conductors Z=conductors per slot×slots

$$=\frac{\text{area of copper in each slot}}{\text{area of each conductor}}\times\text{slots}=\frac{S_fA_s}{a_z}\cdot S$$

$$I_zZ=(\delta\ a_z)\times\left(\frac{S_fA_s}{a_z}\right)S=\delta S_fA_s\ S$$

and $$ac=\frac{I_zZ}{\pi D}=\frac{\delta\ S_f\ A_s\ S}{\pi D}$$

We have, $\text{output}=(\pi^2\ B_{av}\ ac\times 10^{-3})\ D^2Ln=\pi B_{av}\ \delta\ S_f\ A_s\ SDLn\times 10^{-3}$

$\therefore$ Ratio of outputs

$$\frac{(\text{output})_A}{(\text{output})_B}=\frac{(B_{av})_A}{(B_{av})_B}\times\frac{(\delta)_A}{(\delta)_B}\times\frac{(S_f)_A}{(S_1)_B}\times\frac{(A_s)_A}{(A_s)_B}\times\frac{(S)_A}{(S)_B}\times\frac{(D)_A}{(D)_B}\times\frac{(L)_A}{(L)_B}\times\frac{(n)_A}{(n)_B}$$

$$=\left(\frac{0.6}{0.56}\right)\times\left(\frac{4.5}{5.2}\right)\times\left(\frac{0.5}{0.43}\right)\times\left(\frac{48\times 11}{44\times 10}\right)\times\left(\frac{72}{61}\right)\times\left(\frac{0.75}{0.55}\right)$$

$$\times\left(\frac{0.31}{0.25}\right)\times\left(\frac{10}{7.5}\right)=3.44$$

UNSOLVED PROBLEMS

1. Calculate specific electric and magnetic loadings of a 100 h.p., 3000 V, 3 phase 50 Hz 8 pole star connected flame proof induction motor having stator core length = 0·5 m, stator bore = 0·66 m and turns per phase = 286. Assume full load efficiency = 0·938 and power factor = 0·86.

[Ans. $B_{av} = 0{\cdot}22$ Wb/m²; $ac = 14750$]

2. (*a*) Prove that the effective volume of the rotor of a *m* phase 2 pole synchronous machine having a sinusoidally distributed winding is given by :

$$\text{Volume} = \frac{2\sqrt{2}Q \times 10^3}{\pi\omega B_m ac}\ \text{m}^3.$$

where Q is the kVA rating, ω is the angular frequency, B_m is the maximum flux in air gap and ac is the specific electric loading.

(*b*) Show that the volume for a p pole machine is $p/2$ times the value calculated in (*a*).

(*c*) If the stator has a large number (m) of phase windings, each of which consists of one concentrated coil, show that the volume is reduced to $\pi/4$ of the value given in part (*a*).

(*d*) Using the expression derived above estimate the main dimensions of a 1, MVA, 2300 V, 3 phase, 60 Hz, 6 pole star connected synchronous machine. A maximum gap density of 0·9 Wb/m² and a linear current density of 40 000 ampere conductors per metre may be assumed. The axial length of machine may be taken equal to one pole pitch.

Calculate also the number of turns per phase if the winding is sinusoidally distributed.

[Ans. $L = 0{\cdot}89$ m, $D = 1{\cdot}7$ m, $T_{ph} = 14$]

3. A 300 watt d.c motor has a full load efficiency of 60 per cent. Calculate the power developed by the armature at full load if the sum of iron, friction and windage losses is 1/3 of total losses.

[Ans. 366 W]

4. The cooling co-efficient for armature of a machine using a particular type of construction is given by

$$c = \frac{0{\cdot}09}{1 + 0{\cdot}1\,V_a}$$

where V_a is the rotor peripheral speed in metre per second. Calculate the maximum allowable value of current density that can be used for the machine if the specific loading is 20,000 ampere conductors per metre, maximum allowable temperature rise is 40°C and the armature peripheral velocity is 20 metre per second. The resistivity of copper may be taken as $2{\cdot}2 \times 10^{-8}\ \Omega$m.

Suppose the machine is redesigned with a peripheral speed of 40 metre per second, calculate the new value of current density that can be used for the machine [Ans. 3·03 A/mm²; 5·05 A/mm²]

5. Prove from first principles that for a rotating machine output in volt ampere $= C_0 D^2 Ln$. Show fully how and why the output co-efficient C_0 changes with size and type of machine, and show that in all designs it approaches a fixed maximum value.

6. Find the minimum permissible conductor area for a 1000 kW, 500 V, 8 pole, d.c. generator if the permissible copper loss is not to exceed 3000 watt per m² of armature surface. The armature diameter is 2·1 m, and there are 760 lap connected armature conductors. Assume value of resistivity for copper conductors to be 2×10^{-8} ohm metre. [Ans. 48 mm²]

7. Two a.c. generators having similar proportions are designed to work with the same flux densities and current densities and at the same speeds. The linear dimensions of one are x times those of the other. Calculate their relative outputs, the relative iron and copper losses, and the relative heating in loss dissipated per unit area of cooling surface. Are the above conditions those used in practice ?

[Ans, $x^4 : 1$; $x^3 : 1$; $x : 1$]

8. Two a.c generators having similar proportions are designed to work at same speeds and same flux densities. The linear dimensions of one are x times those of the other. Assuming the current density to vary as $x^{-1/4}$, calculate, for the machines, their relative outputs, their relative iron and copper losses.

[Ans. $x^{3.5} : 1$; $x^3 : 1$; $x^{2.75} : 1$]

9. A 50 kW, 800 rpm d.c. generator has full load efficiency of 88 per cent. If now another similar d.c. generator having two times the linear dimensions of 50 kW generator is built to work at 800 rpm find the output, losses and efficiency of the new generator. Assume that flux densities and current densities are the same for the two machines.

[Ans. 800 kW ; 54·4 kW ; 0·937]

10. It has been established from test results that the ratio of total losses to outpt of large machines vary approximately as $D^{-1/8}\,n^{-1/8}$ where D, n are respectively the diameter, and speed of the machine. Prove that if the temperature rise remains constant the output of a large machine varies as $D^{9/8}\,L^{1/8}\,n$. Assume cooling co-efficient $c \propto V_a^{-1/8}$ where V_a is the peripheral speed of rotor. The effective heat dissipating surface of the machine is proportional to the geometric mean of the end surfaces and the cylindrical surface of the rotor

11. A 40 h.p., 1000 rpm, d.c. motor has a specific electric loading of 30,000 ampere conductors per metre and a specific magnetic loading of 0·44 Wb/m². Estimate the horse power of an 800 rpm, d.c. motor which has a specific magnetic loading of 0·5 Wb/m², a current density 10 per cent greater than that of the 40 h.p. machine, and linear dimensions which are all, including those of slots, 20 per cent greater. Assume the two motors to have the same efficiency.

[Ans. 83 h.p.]

12. Determine the maximum allowable core length for a d.c. machine which has the following data.

Maximum gap flux density at no load=0·54 Wb/m², maximum gap flux density at full load=1·3 times no load value. Turns per coil=1; Peripheral speed=30 m/s. The maximum allowable voltage between adjacent segment at full load is 30 V. [Ans. 0·356 m]

13. Determine the stator core dimensions for a 20 MVA, 3000 rpm 3 phase turbo-alternator, with the following data: average gap flux density=0·5 Wb/m²; ampere conductors per metre=56000; permissible peripheral speed of rotor=150 m/s; length of air gap=30 mm. Assume winding factor=0·955.

[Ans. D=1·015 m, L=1·32 m]

14. A 20-hp., 440 volt, 4 pole, 50 Hz, 3 phase induction motor is built with a stator bore of 0·25 m and core length of 0·16 m. The specific electric loading is 23000 ampere conductors per metre. Find the specific magnetic loading of the machine. Assume full load efficiency of 84 percent and a power factor of 0·82.

Using the data of the above machine determine the main dimensions for a 15 h.p., 460 volt, 6 pole, 50 Hz motor.

[Ans. B_{av}=0·36 Wb/m²; D=0·8 m, L=0·125 m]

15. The design data of two d.c. machines is tabulated below. The power developed by armature of machine A is 650 kW. Find the power developed by armature of machine B. Assume the current densities and slot space factors to be the same for the two machines.

Machine	*Diameter* D *m*	*Length* L *m*	*Speed* n *rps*	*Slots* S	*Slot area* A_s *mm²*	B_{av} *Wb/m²*
A	1·05	0·37	6·25	111	42×14	0·64
B	0·55	0·21	7·5	61	38×12	0·55

[Ans. 120 kW]

16. A 600 r.p.m., 50 Hz, 10,000 volt, 3 phase synchronous generator has the following design data.

Specific magnetic loading=0·48 Wb/m²; current density=2·7 A/mm²; slot space factor=0·35, number of slots=144; slot size=120×20 mm; stator bore=1·92 m; stator core length=0·4 m

[Ans. 4000 kVA]

9

D.C. Machines

9·1. **Introduction and applications**. D.C. machines can work as generators, motors and brakes. In the *generator mode*, the machine is driven by a prime mover with the mechanical power converted into electrical power while in the *motor mode*, the machine drives a mechanical load with the electrical power supplied converted into mechanical power. In the *brake mode*, the machine (which functions as a motor before the application of braking action) works as a generator and the electrical power developed is either pumped back to the supply as in regenerative braking or is dissipated in the machine system as in dynamic braking. Hence, in the brake mode the machine decelerates on account of the power supplied or dissipated by it and, therefore, produces a mechanical braking action

There are almost no modern uses of d.c. machines as generators although in the earlier stages of electrical power generation and distribution, d.c. generators were the principal means of supplying electrical power to industrial and domestic consumers. Presently, all the land based electric power supply networks are a.c systems of generation, transmission and distribution. The almost universal use of a.c. systems is on account of their lower generation and transmission costs, higher efficiency (large bulk of a.c. power can be transmitted and distributed over wide areas and long distances at much higher voltages that are impossible in d.c. systems), greater reliability on account of interconnection and control.

The decline of d.c. generators as source of power supply in the electric utilization especially in the field of electric drives is due to the reasons discussed below :

1. The d.c. motor is not used as extensively in industrial electric drives as it was used earlier. With the advent of variable voltage variable frequency static inverters, it has become feasible and economically viable to use a.c. electric motors over a wide speed range which was earlier the exclusive preview of d.c. motors. In fact, it is an accepted practice in industry to employ a.c. motors wherever their use is inherently suitable. Thus the decline of d.c. generator as a source is obvious.

However, it should not be conveniently assumed that the d c motors are on their way out. On the other hand, there are many fields of application where the d.c. machines offer many distinct economic and technological advantages on account of their greater versiality viz. d.c. machines can be designed for wide ranges of voltage/current or speed/torque characteristics for both steady state as well as transient state conditions. But the d.c. supply needed to feed these motors is not obtained from rotating d.c. generators as earlier but is obtained from the output of static power rectifiers whose input is the existing a.c. power supply networks.

No doubt, applications like aerocrafts, ships and road mounted vehicles which are isolated from land based a.c. networks employ d.c. sources including d.c. generators and secondary batteries for power supply but the modern trend is to use a.c. generators with the d.c. supply being obtained by rectification with the help of static power rectifiers.

D.C. generators are still being used to produce power in small back up and standby generating plants driven by windmills and mountain streams (mini hydro-electric plants) to provide uninterrupted power supply. A.C. generators cannot be used with

these fluctuating and intermittent loads because they would produce a variable frequency output (on account of wide variations in the speed of prime mover) which are totally unacceptable.

As apart from d.c. generators, the d.c. motors are finding increasing applications and that too in diverse fields. The principal applications of d.c. motors are as industrial drive motors, especially where large magnitude, and precisely controlled torque is required. Such motors are used in rolling mills, in overhead cranes and for traction purposes like in fork lift trucks, electric vehicles, and electric trains. They are also used in portable machine tools supplied from batteries, in automotive vehicles as starter motors, blower motors, and in many control applications as actuators and as speed and position sensing devices. Some of the important applications of d.c. motors are elaborated below :

(*i*) *Traction*. The d.c. series motor is admirably suited to traction applications like inter-city and rapid transit trains on account of its high starting torque, matching torque speed characteristics, where the torque decreases as the speed increases and the ability of a number of series motors operating in parallel on one locomotive to draw current at any given speed varying from that of any other motor within very close limits.

(*ii*) *Drives for process industry*. The control of speed of d.c. motors can be exercised to close limits with a high degree of precision. This aspect is particularly useful in process drives and mine winding machinery.

(*iii*) *Battery driven vehicles*. The primary source of supply to road driven vehicles like fork trucks and delivery vans and even to passenger tram cars is secondary battery. The battery drives chopper fed d.c. series motors. The use of battery driven vehicles is on the increase

(*iv*) *Machine tools*. D.C. machines find applications in drives for machine tools where a wide range of speed control with precision is desired.

(*v*) *Appliances*. Battery driven miniature d.c. motors are used in shaving razors, tape recorders and cameras.

(*vi*) *Automatic control*. The advent of automatic control has again brought the use of d.c. machines into limelight. Small motors designed with high energy permanent magnets, encapsuled epoxy resin winding and electronic commutation are cheaper and highly reliable as compared with their a.c. counterparts. These machines together with power transistors are used in feedback control systems.

D.C servomotors are used in purely d.c. control systems.

D.C stepper motors are employed in applications where digital control is desired.

9·2. Classification. The d.c. commutator motor is a very versatile device and is built in wide range of sizes, from small control devices in the mW power rating up to very large motors of MW rating used in rolling mill applications. The d c. motors may be classified according to applications as under :

(*i*) **Industrial motors**. Large motors used in rolling mills, cranes, machine tools and industrial drives of ratings upto a few kW are classified as industrial motors. Modern industrial motors are usually supplied by rectifiers working on a.c. mains.

(*ii*) **Small motors**. Motors used in domestic gadgets, hand tools and the motors used for starting automobiles are classified as small motors.

(*iii*) **Traction motors**. These motors are used in locomotives, multiple unit trains and battery driven road vehicles.

(*iv*) **Miniature motors**. These motors have a power rating of a few watts and are used for intermittent duty in applications which do not require precision control.

(*v*) **Control motors**. These motors are used for servo applications in both open loop and closed loop control schemes.

(*vi*) **Special motors**. This classification of motors includes motors which have special applications like use as, actuators, eddy current brakes or special design features like linear motion, printed circuited machines and machines with superconducting field windings.

The d.c. machines may be either homopolar or heteropolar in construction. There may be an excitation system provided by permanent magnets or electromagnets. The homopolar machines are used only for special applications while permanent magnet motors are primarily used in control applications. These machines are not used in industrial electric drives. The present text is devoted to design of d.c. machines used for industrial applications and, therefore, d.c. machines referred herein are heteropolar machines with excitation provided by electromagnets.

9·3. Constructional details. The d.c. commutator machines used for industrial applications have essentially three major parts :

(*i*) field system, (*ii*) armature and (*iii*) commutator.

The **field system** is located on the stationary part of the machine called *stator*. The field system is designated for producing magnetic flux and, therefore, provides the necessary excitation for operation of machine. The stator of a d.c. machine comprises of :

(*i*) *Main poles*. These poles are designed to produce the main magnetic flux.

(*ii*) *Interpoles*. These poles are placed in between the main poles and are designed to improve commutation conditions to ensure sparkless operation of machine. Interpoles are not used in very small machines as these machines are not prone to commutation problems.

(*iii*) *Frame*. This provides support for the machine. In many machines the frame is also a part of the magnetic circuit.

The **armature** is the rotating part (rotor) of a d.c. machine where the process of electromechanical energy conversion takes place. The armature is a cylindrical body which rotates between the magnetic poles. The armature and the field system are separated from each other by an air gap. The armature consists of (*i*) armature core with slots and (*ii*) armature winding accommodated in slots.

The commutator is mounted on the rotor of a d.c. machine and it performs with the help of brushes a mechanical rectification of power : from a.c. to d.c. in case of generators and d.c. to a.c. in case of motors.

9·4. Stator. The stator of a d.c. machine consists of a frame or yoke, and poles which support the field windings. The frame or yoke in addition to being a part of the magnetic circuit serves as a mechanical support for the entire assembly. Earlier, cast iron was used for the construction of yoke but it has been replaced by cast steel. This is because cast iron has a saturation density of 0·8 Wb/m² while saturation occurs in cast steel at a density of approximately 1·5 Wb/m² and therefore, cast steel can be worked at a density which is twice that in the case of cast iron. Thus, the cross-section of the cast steel frame or yoke is half that of cast iron and hence cast steel is used in case it is desired to reduce the weight of machine. Another disadvantage of cast iron is that its mechanical and magnetic properties are uncertain on account of presence of blow holes in the castings.

Fabricated steel yokes are commonly used as they are economical (due to absence of pattern costs as in the case of cast iron and cast steel) and have consistent magnetic and mechanical properties. For very small sized machines it may still be advantageous to use cast iron frames but for medium and large sizes rolled steel is used

However, all modern motors which are fed from static power converters with fluctuating power supplies and harmonic currents, laminated silicon steel yokes are used.

9·4·1. Poles. The poles were formerly cast integral with the yoke. This practice is still being followed for small machines. But in present day machines it is universal practice to use completely laminated poles. However, in some machines laminated pole shoes attached to solid steel poles are used. The laminations of a completely laminated pole have rivet holes for the assembly of poles. The laminations are assembled on steel rods and the whole assembly is then pressed between thick steel end plates The rods are riveted and the rivet heads are spot welded Fig. 9·1 shows a pole lamination. The pole laminations are 0·4—0·5 mm thick. The advantage of a completely laminated pole construction is the economy which is achieved when it is desired to use the same frame size to obtain a range of ratings. This is elaborated below :

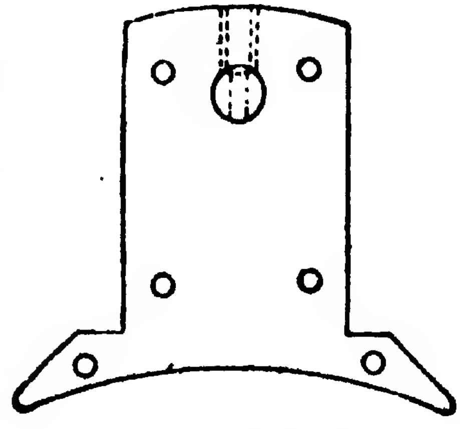


Fig. 9·1. Pole lamination.

The output of a d.c. machine is proportional to product D^2L where D is the diameter of armature and L is the length of core. Therefore, it is possible to design a range of d c. machines having different ratings using the same frame size (same diameter of laminations) but with different core lengths. A laminated construction lends itself admirably to commercial designs, where it becomes necessary to manufacture a range of machines with a single frame with different core lengths. This construction makes it very convenient to take different outputs from the same frame with a process of simply stackings together different lengths of cores built with laminations unlike in the case of cast field construction where different patterns are required for different core lengths.

Different methods are used for attaching poles to the yoke. In case of smaller sizes, the back of the pole is drilled and tapped to receive the pole bolts (Fig. 9·2). In larger sizes, a circular or a rectangular pole bar is fitted to the pole. This pole bar is drilled and tapped and the pole bolts passing through laminations screw into the tapped bar (Fig. 9·2).

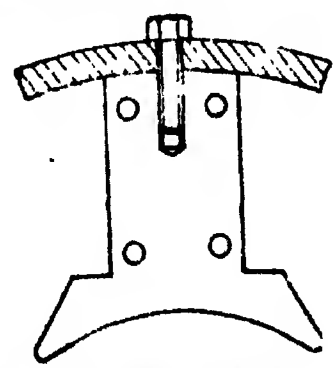


Fig. 9·2. Fixing pole to yoke.

In the case of machines with solid poles, the poles are made from solid steel forgings. These are then bolted to the yoke. With such poles, separate pole shoes are necessary. These pole shoes are built up of sheet steel laminations lightly insulated from each other in order to reduce the pulsation losses in the pole faces. The laminations are securely held together by steel rivets, and the whole shoe is fixed to the pole body by means of counter sunk screws In case of machines having compensating windings, the pole face is slotted to accommodate the windings.

9·4·2. Interpoles. The interpoles are made from laminated steel or from low carbon steel. Laminated interpoles are used in machines with severe commutation problems. In small machines it is usual to use solid low carbon steel poles. Interpoles may be parallel sided or tapered. In large machines, tapered interpoles are used in order to ensure that there is no saturation at the root of the pole at heavy overloads.

9·4·3. Main field winding. The shunt field coils of d.c. machine are wound with round copper wire for small and medium, size machines. But for large size machines rectangular conductors are used and these conductors are wound on a rectangular former.

Fig. 9·3 shows a shunt field coil. The series coils may be placed on the top of the shunt coils or they can be arranged as a separate winding.

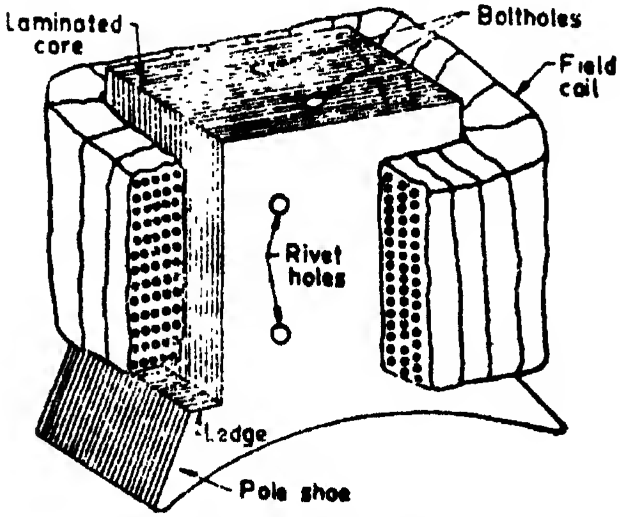


Fig. 9·3. Pole and field coil of a d.c. machine.

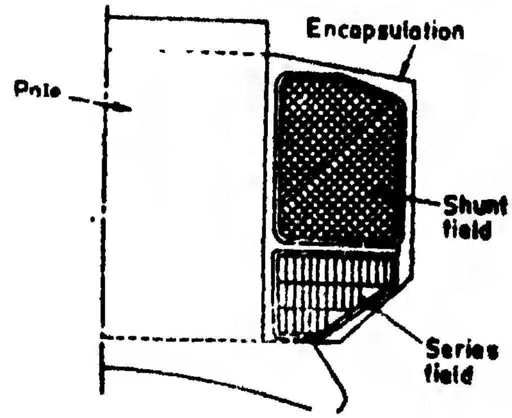


Fig. 9·4. Encapsuled field coils.

In modern electrical motors, the shunt and the series field coils are insulated from earth by silicon varnish bonded mica splittings wound half lap on a glass backing. In another arrangement enamelled conductors are used. These conductors are encapsuled in an epoxy resin moulding and fixed to the pole in the same operation This arrangement is shown in Fig. 9·4.

The series field coil is placed below the shunt field because the former has a higher mechanical strength on account of larger cross section of conductors.

9·4·4. Interpole winding. The interpole winding may consist of round or square wire in the case of small machines.

In large machines, the coils are edge wound from flat copper strip in one or two layers.

Modern machines use epoxide resin through bonded coils with reconstituted mica for main insulation. The assembly of this type of coil is shown in Fig. 9·5.

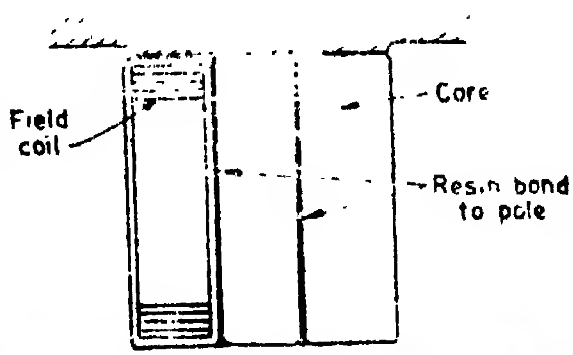


Fig. 9·5. Interpole field coil and pole assemblies (through-bonded class F insulation)

9.5. Armature. The armature of d.c. machines is built up of thin laminations of low loss silicon steel. The laminations are usually 0·4 to 0·5 mm thick.

In small machines, the armature laminations are fitted directly on to the shaft and are clamped tightly between end flanges which also act as supports for the armature winding. One end flange rests against a shoulder on the shaft, the laminations are fitted and other end flange is pressed on the shaft and is retained by a key (Fig. 9·6).

Except in small sizes, the core is divided into number of packets by radial ventilation spacers. These spacers are usually I sections welded to thick steel laminations and arranged to pass centrally down each tooth.

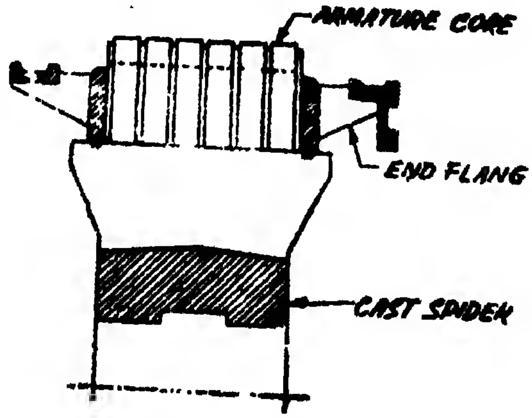


Fig. 9·6. Armature core clamping.

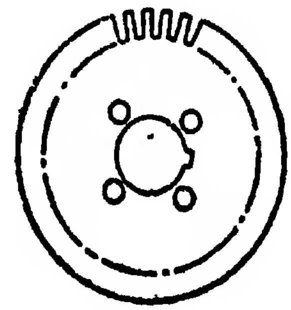


Fig. 9·7. Armature lamination.

With small machines, the laminations are punched in one piece (Fig. 9·7). These laminations are built up directly on the shaft. With such an arrangement, it is necessary to provide axial ventilation holes so that air can pass into ventilating ducts.

Medium size machines having more than four poles have their armature laminations built up on a spider. The spider may be cast or fabricated. Laminations upto a diameter of about 1 m are punched in one piece and are directly keyed on to the spider. Fig. 9·6 shows such a construction with a cast spider.

In large machines it is not possible to punch the laminations in one piece, and it becomes economical to punch the laminations in segments. Fig. 9·8 shows a segmental lamination. In such cases the spider arms have dovetailed grooves, the laminations have also similar grooves and keys are used to attach the segments to the spider. In other cases, the segments have dovetails which fit into spider dovetail grooves (Fig. 9·9).

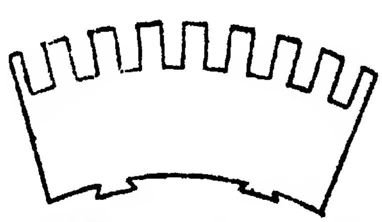


Fig. 9·8. Segmental lamination.

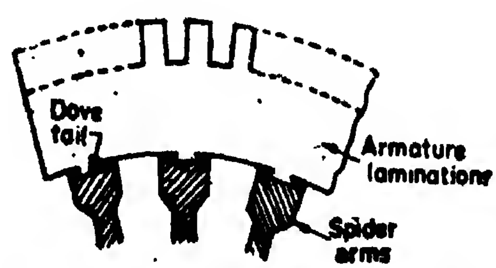


Fig. 9·9. Fixing armature core to spider.

9·5·1. Armature winding. In d.c. machines two layer winding with diamond shaped coils is used. The coils which are preformed and insulated have one coil side at the bottom of a slot and the other at the top of a slot approximately one pole pitch away as shown in Fig. 9·10.

In large machines there is only one turn per coil and the conductors are made of rectangular strip wound on edge as shown in Fig. 9·11. However, in smaller machines, multiturn coils having conductors of round cross-section are used.

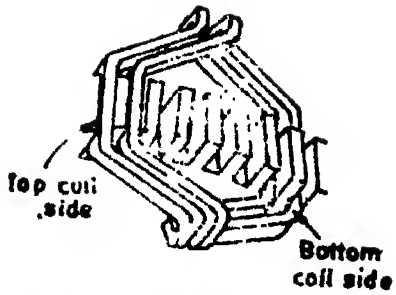


Fig. 9·10. Position of armature coils in slots.

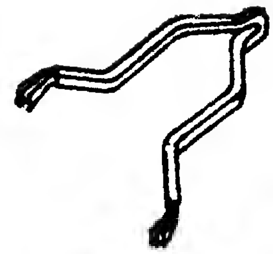


Fig. 9·11. Preformed single turn coil (six coil sides per slot).

In small machines, the coils are held in position by bands of steel wire wound under tension along the core. But in large machines, it is useful to employ wedge of fibre or wood to hold the coils in place in the slots. Wire bands are employed for holding the overhang as shown in Fig. 9.12.

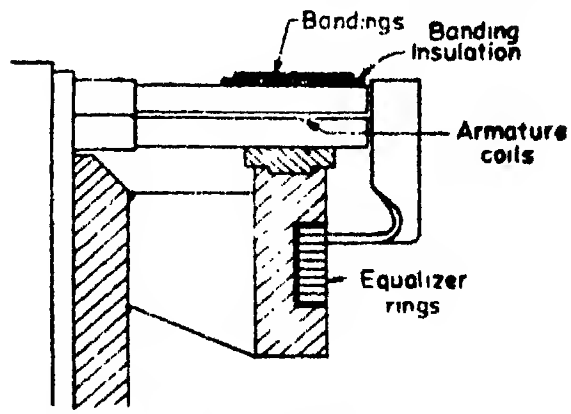


Fig. 9·12. Ring type equalizers.

However, with the increasing use of continuous fibre glass banding in present day machine armatures, the necessity for banding insulation is now decreasing, but where steel wire is used, the banding over the armature overhang is insulated from the windings by flexible micanite and asbestos layers enclosed in a woven glass cloth hood bound down with impregnated woven glass tape.

The equalizer connections are located under the overhang on the side of the commutator. Fig. 9·12 shows a typical arrangement for equalizers. The equalizers can be accommodated on the other end of the armature also.

9·6. Commutator. The function of the commutator is to rectify the alternating current induced in the ... [illegible] ...ductors. It is cylindrically shaped and is placed at one end

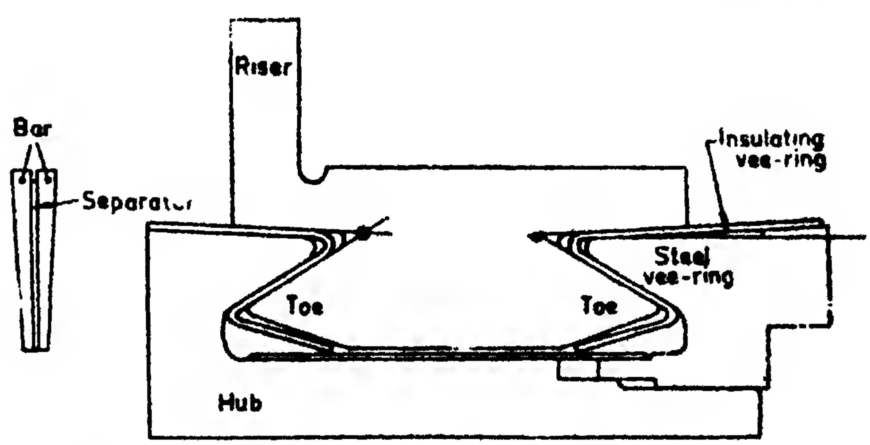


Fig. 9·13. Commutator segment.

of the armature. The essentials of the construction are a number of copper bars or segments separated from one another by a suitable insulating material and connected to the armature conductors. The connection of armature conductors to the commutator is made with the

help of **risers.** The risers connecting the segments to the armature coils are made of copper strips for large machines. Before assembling the segments, these strips are reveted and soldered into the saw cuts made into the segments. The outer end of the riser is shaped so as to form a clip into which the armature conductors are soldered.

For small commutators a lug is provided, to form a part of the segment, and the armature conductors are soldered directly into the lug. Separate risers are therefore not necessary in small machines. The segments are normally wedge shaped (Fig. 9.13) so that the construction forms a circular assembly. The assembly of copper segments together with the insulation between them is insulated from and held together by steel clamping rings. Fig. 9.14 shows the assembly of a typical commutator for a d.c. machine.

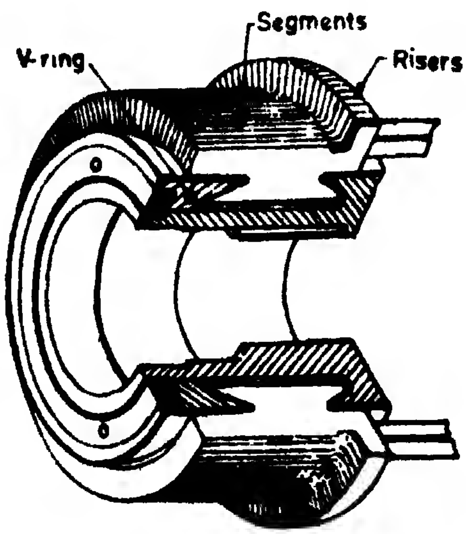


Fig. 9.14. Commutator assembly.

Satisfactory performance of d.c. machines is dependent under good mechanical stability of the commutator under all conditions of speed and temperature within the operating range. A mechanically unstable commutator manifests itself in a poor commutation performance and results in unsatisfactory brush life as well as necessitating frequent machining of commutator to restore the cylindrical surface.

The material used for commutator segments for earlier machines was hard drawn copper. The modern d.c. machines used commutator segments made from silvered copper. The segments are of extruded copper containing about 0.05% silver. The advantage of using silvered copper over hard drawn copper is that it can withstand the flood soldering of the armature coil ends to the risers at 300°C.

The separators used for insulating the commutator segments from each other should possess consistent mechanical properties because good commutation is dependent on accurate bar-to-bar spacing which in turn is dependent upon consistency of thickness of spacers. In earlier machines, thin sheets of mica or micanite, usually 0.8 mm thick were used to separators. Modern machines using class *B* insulation employ separators made from shellac bonded micanite. Machines using class *H* insulation use melamine resin bonded asbestos paper laminate separators because of better thermal stability.

The segments with separators are clamped between two steel V-rings insulated from the segments by high quality micanite consisting of mica splittings bonded with shellac and with no paper covering. In recent years, use of micanite V rings has given way to the use of epoxy-glass V rings.

9.7. Brush gear. The brush gear is an assembly which is used for commutation. It consists of a set of brush holders (usually equal to the number of poles) which are fastened together and bolted to a yoke of strong insulating material. The brush holders are fitted with brushes of suitable grade and hardness.

9.7.1. Brush holders. A brush holder is essentially a metal box, insulated from the frame and rigidly mounted close to the commutator surface. Fig. 9.15 shows a box type brush-holder. An arm is clamped to the brush spindle. At the outer end of the arm, a brush

box, open at top and bottom, is attached. The brush must be able to slide in the box and to be pressed against the commutator surface by a stainless steel *or* phosphor bronze spring. The pressure can be adjusted by a lever arrangement provided with the spring The brush is connected to a flexible conductor called *pig tail*. The flexible conductor may be attached to the brush by a screw or may be soldered.

The brush boxes are usually made of bronze casting or sheet brass. In low voltage d c machines where the commutation conditions are easy, galvanized steel boxes may be used.

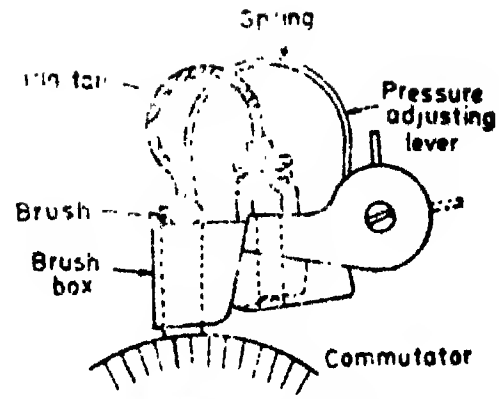


Fig. 9·15. Box type brush assembly.

Some manufacturers use individual brush holders while others use multiple **holders** *i e.* a number of single boxes built up into one long assembly.

9·7·2. Brush rockers. Brush holders are fixed to brush rockers with bolts. The brush rocker is arranged concentrically round the commutator. Cast iron is usually used for brush rockers.

9·7·3. Brushes The brushes used for d.c. machines are divided into four classes.

(*i*) Natural Graphite (*ii*) Hard Carbon

(*iii*) Electrographitic (*iv*) Metal Graphite.

1. Natural Graphite. When used with non-conducting resin binder, natural graphite gives a high voltage drop which helps the commutation process. Therefore, brushes made of natural graphite are used in small machines which do not employ interpoles. Natural graphite brushes have a good lubricating effect at high speeds and therefore a low noise level. However, they are fragile and cause excessive wear of commutator.

2. Hard Carbon. The brushes made from hard carbon are hard but are cheap. The rate of wear caused by them is the same both for the segments and the insulating separators placed between them. Hence, these brushes find extensive application in small machines of fractional kilowatt ratings and also in larger slow speed machines with comparatively easy commutation conditions.

3. Electro-graphitic. Electrographitic brushes are widely used in industrial machines. They have a low rate of wear and therefore extensively used in traction applications.

4. Metal Graphite. The advantages of metal graphite brushes are :

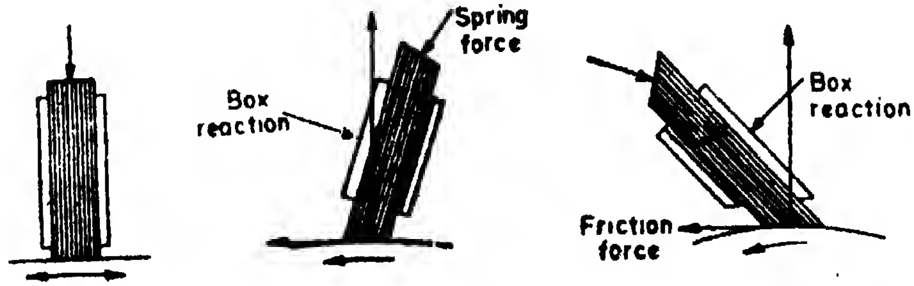
(*i*) They have a low contact voltage drop and (*ii*) they can be used for high values of current density. Therefore metal graphite brushes are suitable for low voltage high current machines such as battery vehicle motors and automobile starters.

9·7·4. Methods of applying brushes to commutators. There are three common methods of applying brushes to the commutator :

(*i*) radial (*ii*) trailing (*iii*) reaction.

Radial. In most of the d.c. machines, the brushes are set so that their centre line is radial to the commutator [Fig. 9·16 (*a*)]. This means that operation of the machine in both the directions is similar. In radial brush arrangement, the frictional force operating in the direction of rotation tends to cause a tilting action which might result in side wear of the brush. Therefore to prevent this, the brushes should be close fitting in a very deep brush box. Thus it is difficult to obtain stability with radial brushes especially in high speed machines.

In non reversing machines the brushes are inclined and for this purpose the brush box is given either a *trailing* or a *reaction* rake.



(*a*) Radial (*b*) Trailing (*c*) Reaction

Fig. 9·16. Positioning of brushes.

Trailing. The brushes are inclined in a trailing direction as shown in Fig. 9·16 (*b*). The angle inclination to vertical is 10-15°. The advantage is that the spring, friction and radial forces in the trailing box combine to maintain contact of the brush with the left hand face of the box, giving rise to good stability. But this results in increased frictional resistance to the sliding of the brush because of commutator surface irregularities.

Reaction. The brushes are inclined in the leading direction as shown in Fig. 9·16(*c*). The brushes slide more easily in the reaction box. The right hand side of the brush is in contact, with the box, however there is an apposing frictional force. The angle of inclination to vertical is kept 30°—40° in order to maintain the side pressure. When narrow brushes are employed the reaction type is preferred.

As stated earlier, radial boxes ase used in reversible machines as the stability conditions are not affected. However, there is a tendency of the brush to lie diagonally and this tendency is countered by using brushes of greater height placed in close fitting boxes. Brush holders inclined at 15—20° to vertical with trailing in one direction and reaction in the other have been found to be useful.

9·7·5. Staggering of brushes. In order to prevent formation of ridges on the commutator surface (also called grooving of commutator), due to all brushes following the same track, the brushes are staggered *i.e.* set in slightly different axial positions as shown in Fig. 9·17. Each set of positive brushes is staggered in the axial direction with relation to the previous set of positive brushes, and the negative brushes are also similarly arranged. Each track or part on the commutator surface is swept past by the same number of positive brushes and negative brushes resulting in uniform wear of the commutator with consequent absence of ridges.

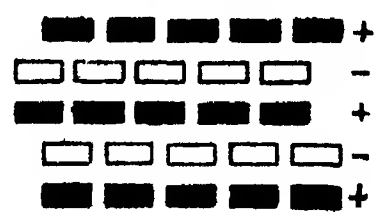


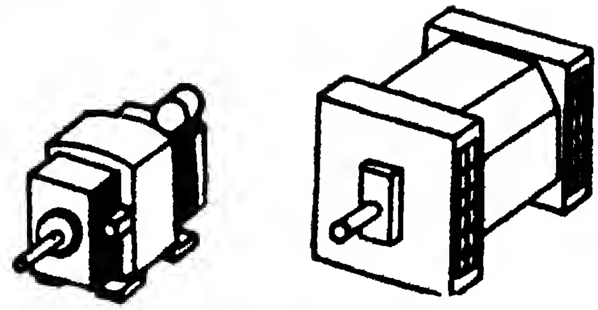
Fig. 9·17. Axial staggering.

9·8. Frames. Fig. 9·18 shows two typical frame sizes. The frame of a low power machine is shown in (*a*) while that for a medium power machine is shown in (*b*). The frame contains the entire field system, provides bearings for the armature in end

covers and accommodates feet or base plate, lifting lugs and cable entry. The frames shown are for screen protected machines which provide inlet and outlet for the air for ventilation.

Small rating machines employ roller bearings at both the ends. For larger machines roller bearings are used at the driving end and ball bearings are used at the non-driving end. It should be mentioned here that the commutator is placed at the non-driving end.

A tachometer generator is usually provided at the non-driving of the motor for the purposes of indication and control of speed.



(a) Low power rating (b) Medium power rating

Fig. 9·18. Frames for d.c. machines

9·9. Constructional features of motors fed from static converters. The d.c. machines being manufactured are mainly d.c. motors and are meant for industrial applications. As explained earlier, these motors are used in control systems for industrial processes. Many industrial processes require the motor to be operated over a wide speed range. The speed of a d.c. motor can be varied by :

(i) armature control which requires varying the armature voltage ;

(ii) field control which requires variation of voltage across the field circuit to control the field current which in turn controls the flux.

The speed of the motors for industrial applications can thus be controlled by either changing the voltage across the armature or the voltage across the field circuit or both.

The motors used in industrial applications are either shunt or compound type. In shunt motors the armature and field circuits are separately connected. The speed of the motor is preferably controlled through armature control (if it is within normal limits) and if wider range of speed control is required, it can be augmented (if the speed is desired to be increased beyond normal) through field control. The majority of the present day d.c. motors are fed from single phase or three phase bridge rectifiers which use thyristors (SCRs-silicon controlled rectifiers). The field and the armature are fed from separate rectifier circuits and therefore the motors are designed for different voltages for field and armature circuits. In case only armature control is used, controlled rectifier bridge circuits are used for the armature while a uncontrolled rectifier circuit (using diodes in place of SCRs) may be used for the field circuit. However, if the speed control is exercised through both the armature and the field circuit, then controlled rectifiers circuits using SCRs are employed for both the circuits.

The output voltage from the rectifier bridge circuits using SCRs is not pure d.c. but contains ripples. Consequently, the armature current also has a substantial ripple content. Also many a times the motor draws a current which is discontinuous in nature. The following problems arise in d.c. motors which are fed from thyristor bridge circuits :

1. Increased I^2R losses. The I^2R losses in a winding are proportional to the square of the form factor of the current. The form factor of pure d.c. is unity while the form factor of the pulsating armature current is greater than unity. Hence, the I^2R losses in the armature windings, fed from rectifiers using SCRs are much higher than when fed from d.c. generators whose output voltage is almost free from ripples.

2. Increased core losses. The substantial ripple content in the armature current distorts the flux field form resulting in increased eddy current core losses in armature, interpoles and yoke.

The increase in losses in the armature winding and in the magnetic circuit cause increased heating. Therefore, motors designed to be fed from d.c. generators have to be de-rated when fed from static power rectifiers using SCRs.

3. Poor commutation. The fluctuating armature current impairs the commutation conditions in the motor because of the following reasons :

(*i*) The value of the current to be commutated is the peak value of the armature current and hence there is an increase in the reactance voltage which in turn causes delayed commutation.

(*ii*) The flux produced by the interpoles is used for neutralizing the flux produced by the armature current and in addition produce a rotational voltage in the coil undergoing commutation which neutralizes the reactance voltage. The complete neutralization of both armature reaction and reactance voltage is essential for good commutation in the machine. Now, the interpole flux, armature reaction and reactance voltage are proportional to armature current and for effective neutralization the variations in interpole flux must be in synchronism with variations in armature reaction and reactance voltage. But interpolar flux produces eddy currents which cause it to be displaced with respect to armature current and therefore the interpole flux is no longer in synchronism with armature reaction and reactance voltage. Hence, the reactance voltage cannot be wholly neutralized. This results in increased sparking.

4. Change in motor parameters. The fluctuating armature current causes a wide variation in the value of armature resistance and inductance thereby rendering the performance of the motor unpredictible. The value of armature resistance changes because of (*i*) *heating*—the increased heating in the motor increases the resistance of the armature winding. The heating and consequently value of armature resistance depend upon the armature current waveform and (*ii*) *skin effect*—the increase in armature resistance caused by skin effect is unpredictible.

The increase in the value of armature resistance depends upon magnitude and order of harmonics in the armature current. The waveform of armature current is quite complex and is dependent upon many factors and therefore, it is almost impossible to predict the value of armature resistance under the actual operating conditions.

The value of armature inductance is dependent upon the degree of saturation in the motor which in turn depends upon the waveform of armature current and therefore, it is not possible to predict its value.

Some of the effects of problems associated with motors fed from rectifiers using SCRs can be minimized by using the following special design techniques for these motors :

(*i*) The effect of heating produced by increased I^2R and core losses can be taken care of by providing better ventilation.

(*ii*) An inductor is included in series with the motor circuit. The series inductor helps to smoothen the armature current thereby improving its waveform by bringing it nearer to pure d.c.

(*iii*) The core losses can be reduced by laminating the poles, yoke and the interpoles in order to reduce eddy current losses. The additional advantage of having a motor with a completely laminated structure is that it results in reduction of the time constant of the windings. The time constant $\tau = L/R$. Now, by using a laminated magnetic circuit, the value of effective resistance of windings, R, decreases on account of decrease in eddy current losses and therefore, time constant, τ, decreases. The dynamic response of motors used for present day applications is very important. The lower is the time constant, the faster is the response. Therefore, motors used for control purposes and also involving rapid changes in load should be designed for a small time constant. This certainly requires a completely laminated armature and field structure. The improvement in time constant by utilizing a completely laminated magnetic structure can be visualised from that a motor with a solid magnetic structure has a time constant of 0·5 s or more while that of motor with a laminated structure varies between 0·005 s to 0·05 s depending upon the thickness of laminations used, the lower the thickness of laminations, the lower the time constant.

(*iv*) The improvement in the commutation conditions of a motor designed for static power rectifiers can be brought about by using a motor with an inherent lower reactance voltage. The reactance voltage can be written as :

$E_r = LI_z/\tau_c$ where L is the armature inductance, I_z is the armature current per parallel path and τ_c is the time of commutation.

The reactance voltage is directly proportional to armature inductance and also to armature current and inversely proportional to time of commutation. The armature current per parallel path is dependent upon the loading conditions of motor over which the designer has no control. The designer certainly has a control over the other two factors : armature inductance and time of commutation.

The armature inductance can be decreased by decreasing the number of armature conductors. This requires a very high value of flux density and this flux density cannot be increased beyond a certain limit. Therefore, a certain minimum number of turns has to be used. The inductance and hence the reactance voltage of the coil undergoing commutation can be reduced if the number of turns in the coil is small. (This is because the inductance of a coil is proportional to the square of number of turns in it). Thus given an armature with a specified number of turns, the obvious solution to reduce the reactance voltage is to decrease the number of turns per coil. Therefore, the number of coils used in a motor fed rectifiers employing SCRs has to be more than that in a motor fed from a source supplying pure d.c. However, since the number of coils is equal to the number of commutator segments, a motor designed to be fed from static power rectifiers has a commutator of much greater diameter than that of a motor fed from a d.c. generator. However, the larger diameter of the commutator results in greater inertia of the moving parts increasing the time constant of the machine. Therefore, the resulting machine has a poor transient response.

DESIGN

9·10. Output Equation. The output equation of a d.c. machine is derived in Art. 8·1.5. page 455.

Power developed by armature $P_a = C_0 D^2 L n$ kW ...(9·1)

where output co-efficient $C_0 = \pi^2 B_{av}\, ac \times 10^{-3}$ ...(9·2)

Sometimes, the value of maximum gap density B_g is specified instead of the value of average gap density B_{av}. The expression for C_0 has to be modified in that case.

Maximum gap density $B_g = B_{av}/K_f \cong B_{av}/\psi$.

where K_f = field form factor and ψ = ratio of pole arc to pole pitch.

Rewriting expression for output co-efficient

$$C_0 = \pi^2 \psi\, B_g\, ac \times 10^{-3} \qquad ...(9·3)$$

Power developed by the armature, P_a, is different from the rated power output P, of the machine. The relationships between the two are

$$P_a = P/\eta \text{ for generator and } P_a = P \text{ for motors}$$

where η = efficiency of the machines. (See Eqns. 8·12 and 8·13)

9·10·1. Choice of average gap density. Some of the factors affecting the choice of average flux in the air gap in rotating electrical machines have already been discussed in Art 8·1·7 page 458. Some factors which are specifically relevant to d.c. machines are discussed below :

(*i*) Flux density in teeth. If a high value of flux density is assumed for the air gap, the flux density in armature teeth also becomes high. The value of the air gap density should be so chosen that the flux density at the root of the teeth (where the tooth section is minimum) does not exceed a value of 2·2 Wb/m² as otherwise the mmf required for the teeth would become excessively large which would mean that the field mmf

will have to be made large which in turn results in higher field copper loss and higher cost of copper The iron losses in the teeth are determined by the flux density and consequently they would increase if a higher flux density is used in the teeth.

(*ii*) **Frequency.** When the machine rotates, the armature magnetic circuit alternately comes under the influence of north and south poles. The frequency of reversals is $f=pn/2$. If the frequency of reversals is high, iron losses in armature core and teeth would be high. Therefore we should not use a high value of flux density in the air gap of machines which have a high frequency.

(*iii*) **Size of machine.** It is possible to use increased values of flux density as the size of the machine increases. As the diameter D of the machine increases, the width of the tooth also increases, permitting a increased value of gap flux density without causing saturation in the machine. The values of gap flux density, for different armature diameters are as plotted shown in Fig. 9·19.

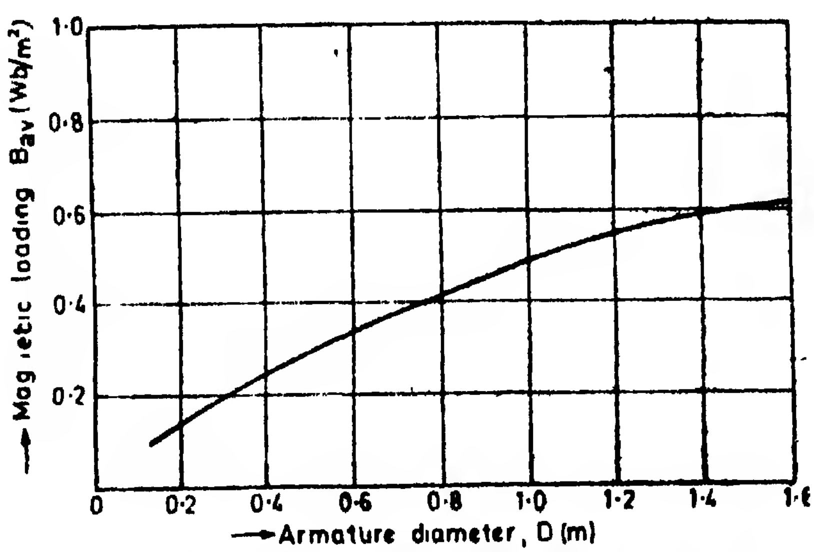


Fig. 9·19. Typical values for specific magnetic loading.

The value of B_g varies between 0·55 to 1·15 Wb/m² and the corresponding values of B_{av} are 0·4 to 0·8 Wb/m²..

Table 9·1 should be consulted for values of B_g.

Table 9·1

Approximate Values of B_g

Output kW	B_g Wb/m²	Output kW	B_g Wb/m²
5	0·58	500	0·92
10	0·65	1000	0·96
50	0·78	2000	0·98
100	0·82	5000	1·05
200	0·87	10000	1·15

9·10·2. Choice of ampere conductors per metre. The choice of specific electric loading has been discussed in Art. 8.18 page 460.

(*i*) **Temperature rise.** A higher value of '*ac*' results in a high temperature rise of windings. A high value '*ac*' can be used for machines using insulating materials which can withstand high temperature rises. For example in machines using class *F* insulation which can withstand a temperature of 155°C, the value of *ac* can be approximately 40% higher than that used in machines designed for class *A* insulation which can withstand a temperature of only 105°C. The type of enclosure used and cooling techniques employed determine the temperature rise in a machine and, therefore the choice of value for *ac* A totally machine has poor ventilation as compared with that of a semi-enclosed machine. Therefore, it is possible to use a higher of *ac* in a semi-closed machine as compared to that in a totally enclosed machine.

(*ii*) **Speed of machine** If the speed of machine is high, the ventilation of the machine is better and therefore, greater losses can be dissipated. Thus a higher value of *ac* (which would mean higher I^2R losses) can be used for machine having high speed.

(*iii*) **Voltage.** In high voltage machines, larger space is required for insulation and therefore there is less space for conductors. This means that in high voltage machines, the space left for conductors is less and therefore we should use a small value of ampere conductors per metre.

(*iv*) **Size of machine.** In large size machines it is easier to find space for accommodating conductors. In fact, for a given geometry of armature core, the slot area which can be provided to accommodate armature windings is proportional to the square of armature diameter *i.e.* to D^2. Therefore, for a given current density in armature conductors, the current is proportional to D^2 and the specific electric loading, *ac*, to *D*. Therefore the greater the diameter of machine the greater is the value of *ac* which can be employed in it.

However, the relationship between *ac* and *D* cannot kept linear because the temperature rise increases with increase of linear dimensions requiring a more elaborate cooling system to keep down the temperature rise. Therefore, it is expected that the value of *ac* increases with increase in armature diameter though the relationship is not linear. Fig. 9·20 shows the variation of *ac* with armature diameter *D*. The rate of increase of *ac* with diameter *D* becomes smaller as diameter increases on account of increasing temperature rise due to decreasing heat dissipation capability.

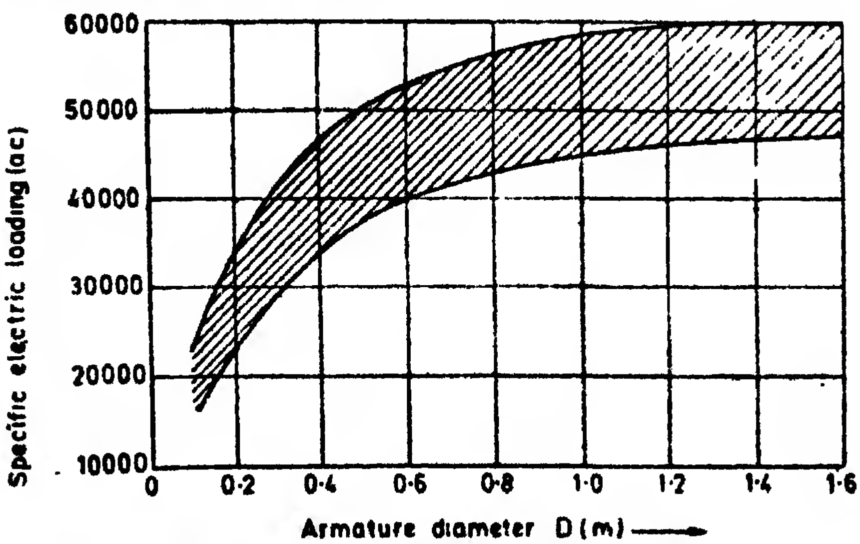


Fig. 9·20. Typical values of specific electric loading.

There is another contributing factor for the decreased rate of increase in *ac* with increase in diameter. We started with the assumption that the conductors area is aproportional to D^2. This assumes that both the width and the depth of slot increase linearly with diameter D. This is true of width of slot which can be made to vary linearly with diameter D. However, the slot depth cannot be made to vary linearly with D because an excessively deep slot, greatly increases the leakage reactance giving rise to high reactance voltage which has a deleterious effect on commutation conditions.

Therefore, specific electric loading is not proportional to D^2 because the slot depth cannot be increased beyond a certain value, the maximum value being limited by the slot leakage and the consequent reactance voltage the fact the slot depth is of the order of 30 mm both for a machine with $D=2{\cdot}5$ m and for a standard armature with $D=0{\cdot}25$ m. However, specific electric loading can be increased with increase in linear dimensions. This is because with machines designed with increased dimensions it is possible to use wider slots (giving a lower slot permeance) on account of reduced taper without increasing the flux density in teeth.

(*v*) **Armature reaction**. If we use a high value of *ac*, the armature mmf becomes high or in other words the armature becomes magnetically stronger. This means that under load conditions there will be greater distortion of field form resulting in a large reduction in the value of flux. In order to prevent this, the field will have to be made stronger. Therefore the mmf required for the field will have to be increased in order to prevent decrease in flux. This means that the cost of conductors used in the machine would go up.

(*vi*) **Commutation**. The armature ampere conductors per metre $ac=I_z Z/\pi D$. Now a machine designed with a high value of *ac* will have either a large number of armature conductors, Z, or a small armature diameter D.

The machine designed with a large number of coils will naturally have a large number of turns in each coil (the number of armature coils cannot be increased beyond certain value, the limitation being imposed by the thickness of commutator segments). Now, the inductance of the coils is proportional to the square of the number of turns. Therefore, the use of high value of *ac* results in coils having high inductance.

Let us look at the second alternative to use an armature with a small diameter. When the diameter is small, it is not possible to use wide slots because otherwise the space left for teeth will become smaller giving rise to high flux density in them. The only way to accommodate the conductors is to use deeper slots. But the use of deeper slots increases the inductance of armature.

Thus, it is clear that the use of increased value of specific electric loading (resulting from the use of either large number of armature conductors or small diameter) increases the inductance of armature coils. The reactance voltage in coils undergoing commutation is directly proportional to the inductance. Thus, the reactance voltage is high in case a high value of *ac* is used. The reactance voltage delays the commutation and therefore a high value of *ac* worsens the commutation conditions in the machine. Hence from the point of view of commutation a small value of *ac* is desirable. The value of *ac* usually lies between 15,000 to 50,000 ampere conductors per metre. Typical values of *ac* used for machines having class A insulation are given in Table 9·2.

Table 9·2. Approximate values of 'ac

(ampere conductors per metre)

Output kW	*ac*	*Output* kW	*ac*
5	15000	500	35000
10	17500	1000	40000
50	25000	2000	43000
100	27500	5000	49500
200	31000	10000	51000

The values given in Table 9·2 can be increased by approximately 40% in case class *F* insulation is used.

9·11 Interdependence of specific magnetic and electric loadings The output of a d.c. machine is proportional to the product of their specific magnetic and electrical loadings *i.e.* $P_a \propto (B_{av} \times ac)$ The same power output can be obtained by using a high value of B_{av} and a low value of *ac* or vice versa. For a particular output, the values of specific magnetic and electric loadings are interdependent *i.e.* if the value of one is chosen higher, the value of other has to be assumed lower.

The specific magnetic loading is limited by the saturation in the magnetic circuit. The air gap density, B_{av}, can be increased only if sufficient area is available in the magnetic circuit. The most critical part of the magnetic circuit is the armature teeth as far as the saturation is concerned. The primary factor is the ratio of tooth width to slot pitch The gap density cannot be allowed to increase above a value where the flux density at any section of the tooth increase beyond 2·2 Wb/m^2. Supposing it is desired to increase the gap density. This can only be done by increasing the tooth width (to keep the density in the teeth below 2·2 Wb/m^2). The widening of teeth allows B_{av} to be increased. However, the slot width becomes smaller on account of the increased tooth width as the slot pitch remains the same Thus, the space left for conductors becomes smaller. Therefore, this requires that a smaller value of *ac* should be used for the machine. It is evident, that the specific loadings B_{av} and *ac* are interrelated and the increase in one calls for decrease in other.

The product $B_{av} \times ac$ is a major factor and the increase of one at the expense of other may not lead to increase in rating. In fact the values of specific loadings should be so chosen that an optimized design is obtained.

9·12. Selection of number of poles. The value of output co-efficient can be obtained after suitable choice of values for B_{av} and *ac* and then, using Eqn. 9·2. The value of product D^2L is obtained from Eqn. 9·1.

It now remains to select appropriate values of *D* and *L* which correspond to the calculated value of D^2L. It is the aim of the designer to select such dimensions as will result in the minimum cost and yet at the same time meet the desired specifications.

In order to obtain suitable proportions for the machine it is necessary to consider both the magnetic as well as the electric circuits. So far as the magnetic circuit is concerned, it is necessary to choose a suitable number of poles and also to suitably proportion them. A proper design of the electric circuit requires suitable dimensions which result in satisfactory arrangements for winding and commutator.

The number of poles used in a d.c. machine has an important bearing upon both the magnetic and the electric circuits.

In the case of alternating current machines, the number of poles is fixed by the supply frequency and the speed of the machine. But in the case of d.c machines, any number of poles can be used. However, there is always a very small range of number of poles that gives a design, which is sound from the commercial point of view.

Coming to choice of number of poles consider that the length and the diameter of the machine, the specific magnetic and electric loadings are fixed and the number of poles can be varied. This means that,

Φ_T = total flux around the air gap $= p\Phi = B_{av} \times \pi DL$

and AC = total ampere conductors over the armature periphery

$= I_z Z = I_a/a \,.\, Z = ac\ \pi D$

are both constant.

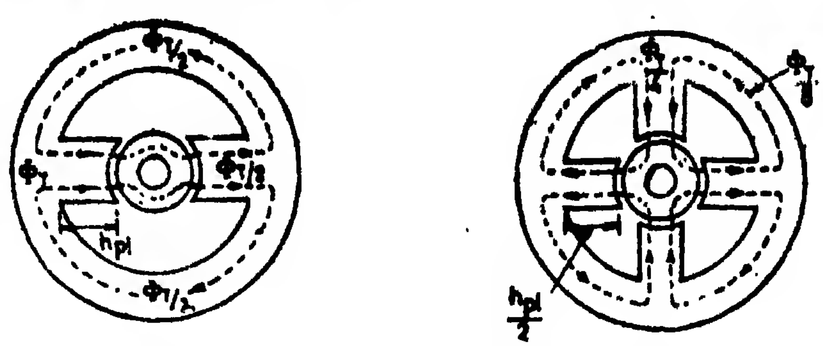
1 Frequency. The frequency of flux reversals is given by $f = pn/2$. Therefore, if we choose a large number of poles, the frequencis high. The frequency of alterations of magnetic flux in d.c. machines should not be very high as it would give rise to excessive iron losses in armature teeth and core. Generally, the value of frequency, f, lies between 25—50 Hz, but may be more in small machines typically high speed series motor designed with a low gay air density.

In certain cases, frequency might be a deciding factor in the choice of number of poles. For example in the case of high speed d.c. turbo-generators, the number of poles used is 2 as otherwise the frequency will become high giving rise to excessive iron losses.

2. Weight of iron parts. The number of poles effects the weight of the various parts of the magnetic circuit as explained below :

(*a*) *Yoke area.* The total flux around the air gap remains constant considering the assumptions we have made. This flux is Φ_T. Consider the case of a 2 pole machine as shown in Fig 9·21 (*a*). The flux per pole is $\Phi_T/2$. At the yoke, this flux divides itself in two parts and therefore the yoke has to carry a flux $\Phi_T/4$.

Now if the number of poles of this machine is raised to 4, the flux per pole becomes $\Phi_T/4$ and the flux in the yoke is $\Phi_T/8$ as shown in Fig. 9·21 (*b*). Thus if the number of poles is doubled, the flux carried by yoke is halved.



(*a*) 2 Pole Machine (*b*) 4 Pole Machine

Fig. 9·21. Decrease in machine dimensions with increase in number of poles.

Generalizing the above, the flux carried by yoke is inversely proportional to number of poles. Therefore by using greater number of poles, the area of cross-section of yoke is proportionately decreased.

(*b*) *Armature core area.* The flux per pole divides itself in two paths in the armature core as shown in Fig. 9·21. Therefore in the case of a 2 pole machine, the flux in the armature core is $\Phi_T/4$ while in the case of a 4 pole machine, it is $\Phi_T/8$.

The yoke carries a steady flux and therefore there are no iron losses in it. Thus we can safely use a large number of poles so as to reduce the weight of iron in the yoke. But this is not true in the case of armature core. We can no doubt decrease the weight of iron in the core by using a large number of poles but the increase in number of poles would result in higher iron loss in armature owing to increased frequency of flux reversals. Let us examine the effect of using increased number of poles on the armature core area keeping in view the increased iron losses caused by increased frequency.

We compare here, two machines one with 2 poles and the other with 4 poles.

Let B_c = flux density in armature core ; f = frequency of flux reversals ;

n = speed in rps, and A_2 and A_4 equal area of core for 2 pole and 4 pole machines respectively.

Considering the ***eddy current loss*** :

2 pole machine :

$$\text{Eddy current loss in armature core} \propto B_c^2 f^2 \propto B_c^2 \left(\frac{pn}{2}\right)^2$$

$$\propto \left(\frac{\Phi_T}{4A_2}\right)^2 \times \left(\frac{2\times n}{2}\right)^2 \propto \frac{\Phi_T^2 n^2}{16A_2^2}. \qquad \ldots(i)$$

4 pole machine :

$$\text{Eddy current core loss in armature core} \propto B_c^2 f^2 \propto B_c^2 \left(\frac{pn}{2}\right)^2$$

$$\propto \left(\frac{\Phi_T}{8A_4}\right)^2 \times \left(\frac{4\times n}{2}\right)^2 \propto \frac{\Phi_T^2 n^2}{16 A_4^2} \qquad \ldots(ii)$$

It is evident from relations (*i*) and (*ii*), that for the same eddy current core loss A_2 should be equal to A_4. Thus in order to obtain a constant value of eddy current loss in the core irrespective of the number of poles, the armature core section has to be kept the same. In case the cross-sectional area of core is decreased for higher number of poles, the eddy current loss in core would increase.

Considering the ***hysteresis loss*** :

2 pole machine :

$$\text{Hysteresis loss in core} \propto B_c^2 f \propto B_c^2 \left(\frac{pn}{2}\right) \propto \left(\frac{\Phi_T}{4A_2}\right)^2 \times n \propto \left(\frac{\Phi_T}{A_2}\right)^2 \times \frac{n}{16} \qquad \ldots(iii)$$

4 pole machine :

$$\text{Hysteresis loss in core} \propto B_c^2 f \propto B_c^2 \left(\frac{pn}{2}\right)$$

$$\propto \left(\frac{\Phi_T}{8A_4}\right)^2 \times \left(\frac{4n}{2}\right) \propto \left(\frac{\Phi_T}{A_4}\right)^2 \times \frac{n}{32} \qquad \ldots(iv)$$

Comparing the expressions for hysteresis loss for the 2 pole and 4 pole machine it is clear that if we keep the core area same for both the cases (*i.e.* $A_2 = A_4$) the hysteresis loss in the case of 4 pole machine would be 50 per cent of that in the case of 2 pole machine. Thus the hysteresis loss decreases with increase in number of poles. In other words, if we have to keep a constant value of hysteresis loss, we can decrease the area of cross-section of the machine with larger number of poles. Therefore, by increasing the number of poles the weight of iron in the armature core can be decreased.

(*c*) ***Overall diameter***. In all logically designed machines, the total field mmf bears a constant ratio to total armature mmf. Let this ratio be K.

$\therefore$ Total field mmf $= K \times$ total armature mmf $= K\, AC/2$.

But the total armature mmf ($AC/2$) is constant irrespective of the number of poles and therefore the total field mmf is constant.

Now field mmf per pole $AT_f = KAC/2p$. Since we have a field coil on each pole. Therefore the mmf developed by each field coil varies inversely as the number of poles. Hence for the same depth of winding, the winding height and thus the actual height of pole decreases as the number of poles is increased.

Comparing the case of two machines, one with 2 poles and the other with 4 poles, the field mmf per pole in the 2 pole machine is double that in the 4 pole machine. Therefore the height of field winding and hence the height of pole in the case of 2 pole machine is double of that in the case of 4 pole machine. This is shown in Fig. 9·21 wherein the heights are indicated as h_{pl} and $h_{pl}/2$.

It is clear that the height of pole is smaller in machines with large number of poles as indicated by Fig 9·21. Since the overall diameter of a d.c. machine is dependent upon the height of pole, lesser the pole height lesser the overall diameter of machine. Therefore the overall diameter of the machine decreases as the number of poles is increased. This means that the total weight of iron required by the machine decreases with an increase in number of poles.

3. Weight of copper

(*a*) ***Armature copper.*** The portion of conductors responsible for emf production in generators or torque production in motors is called the *active* copper or active portion of conductors. Therefore, the portion of conductors embedded in slots is called the active portion while the portion of conductors in the overhang which just provides a connection between the active portions of conductors and thus does not take part in the energy conversion process) is called ***inactive copper*** or inactive portion of conductors.

It should be made clear here that lesser the ratio of inactive conductor material (which does not take part in the electromechanical energy conversion process) to active conductor material (which is responsible for the electromechanical energy conversion process, the cheaper is the machine.

Let us consider the inactive copper. Diameter of the machine remains constant and therefore the pole pitch varies inversely as the number of poles. It means that the length of conductors in the *overhang* decreases with increase in number of poles and to a first approximation we can take that the length and hence weight of inactive copper varies inversely as the number of poles.

Fig. 9·22 shows the active and inactive portions of conductors of a 2 pole and a 4 pole machine.

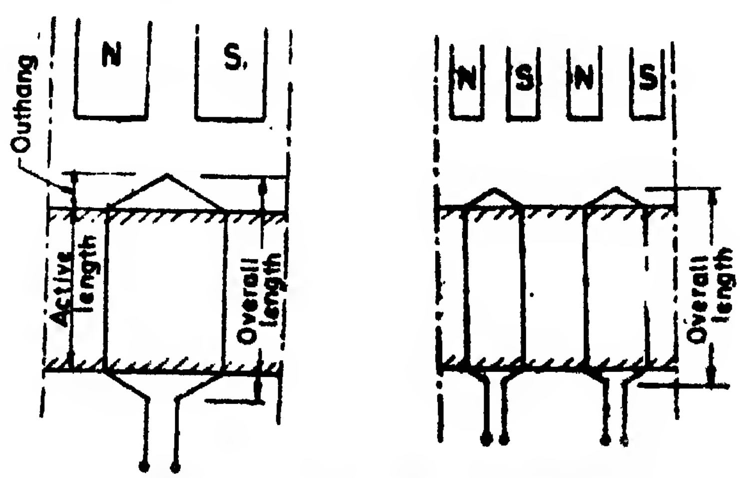


Fig. 9·22. Decrease in length of overhang with increase in number of poles.

Comparing the two machines, we find that the amount of copper used in the overhang of the 2 pole machine is greater than that in the 4 pole machine. It is also clear from Fig. 9 22 that the *outhang i.e.* the distance through which the overhang projects outside the core, is more in the case of 2 pole machine giving a larger overall length for the machine.

Hence, increase in number of poles reduces the weight of copper in the armature and at the same time reduces the outhang of the winding so that the overall length of machine is reduced.

(*b*) *Field copper*. We have already noted that the field mmf per pole varies inversely as the number of poles--Consider the case of two machines, one with 2 poles and the other with 4 poles. The area of cross-section of each pole in the 2 pole machine is double that in the 4 pole machine. Also the field mmf per pole in the case of 2 pole machine is double that in the case of 4 pole machine. It is therefore clear that the mean length of field turn is greater in the case of 2 pole machine as shown in Fig 9·23. This means that the weight of field copper is more in the 2 pole machine.

Therefore, the total weight of field copper decreases with increase in number of poles. The mean length of field turn decreases and therefore the field winding resistance decreases with increase in number of poles. This means that the field copper losses become smaller and therefore for the same loss we can decrease the area of field conductors so that saving in field copper due to increase in number of poles is quite appreciable.

Thus the weight of copper of both armature and field windings decreases with increase in number of poles.

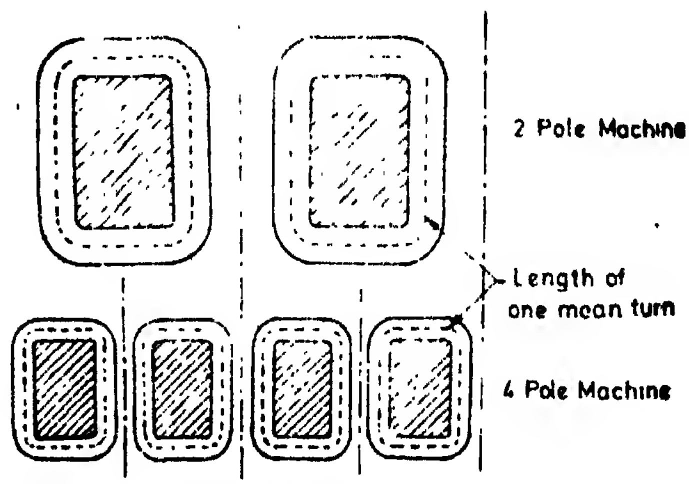


Fig. 9·23. Decrease in length of mean turn of field winding with increase in number of poles

4 **Length of Commutator**. The number of brush arms employed is equal to the number of poles in the case of machines with lap wound armatures. In machines with wave winding the number of brush arms required is 2 but usually the brush arms provided are equal to the number of poles. Consider two lap wound machines, one with 2 poles and the other with 4 poles.

2 pole machine

There are two brush arms in the machine placed as shown in Fig. 9·... current per parallel path is $I_z = I_a/2$ and since there are two parallel paths meeting at brush arm the current in each brush arm

$$I_b = 2I_z = I_a.$$

4 pole machine :

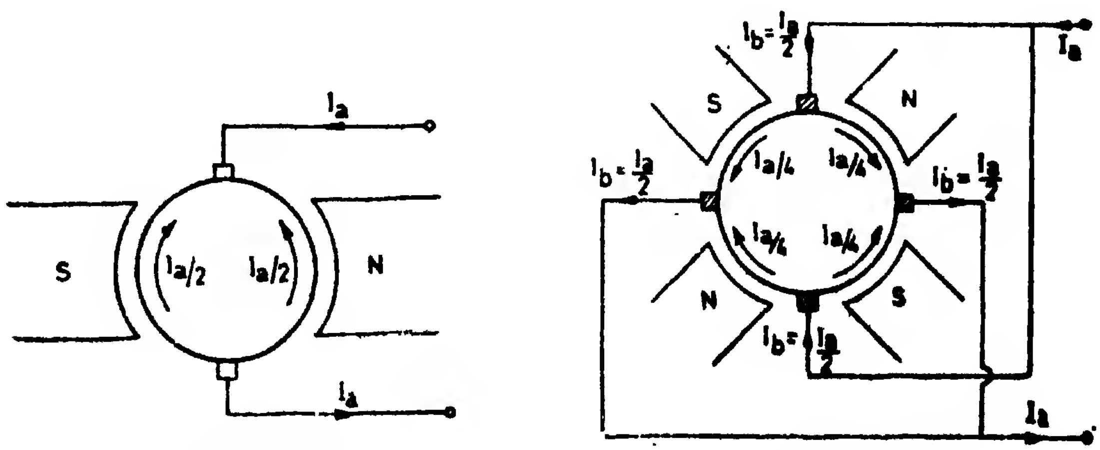
There are four parallel paths and the current in each parallel path $I_z = I_a/4$.

Therefore current in each brush arm $I_b = 2I_z = I_a/2$.

Therefore, the current brush arm for 4 poles is half of that for 2 poles.

Generalizing the current is collected by each brush arm varies inversely as the number of poles. This means that the area of brushes in each arm decreases if the number of poles increases. In most cases it is not necessary to reduce appreciably the thickness of the brushes when the number of poles is increased. Therefore the length of brushes

required in each brush arm is reduced with increase in number of poles. This results in reduction in the length of the commutator and incidentally results in reduction of overall length of machine.



(a) 2 pole machine (b) 4 pole machine

Fig. 9·24. Current in each brush arm

5. Labour charges

(a) *Armature coils.* Considering the emf equation of a d.c. machine $E=\Phi Zn\ p/a$.

Since $p\Phi$ is assumed to be constant $E \propto Zn/a$.

In a lap winding the number of parallel paths is equal to the number of poles *i.e.* $a=p$.

$\therefore$ $E \propto Zn/p$ for a lap winding or $Z \propto Ep/n$.

Thus the number of conductors Z increases in direct proportion to the number of poles. In multipolar lap wound machines, single turn coils are invariably used. Therefore, the number of armature coils increases with increase in number of armature conductors. Hence the number of armature coils increases with increase in number of poles

It should, however, be noted that in a wave winding the number of conductors and hence number of coils remain the same irrespective of the number of poles.

Since the number of commutator segments is equal to the number of armature coils the former increase with increase in number of poles.

(b) *Field coils.* The number of field coils is equal to the number of poles. Therefore, there are more field coils to be assembled if the number of poles is higher.

From above, it is clear that the labour charges will increase as the number of poles increases because there are more armature coils to wind, insulate and connect to commutator, and there are commutator segments to the make insulate and assemble, and more poles to assemble and more field coils to wind.

6. Flash over. This is a very important factor when deciding about the number of poles to be used. The use of a large number of poles results in increased danger of flashover between adjacent brush arms as explained below.

The number of brush arms is equal to the number of poles. For the same diameter of the commutator, the distance between the adjacent brush arms decreases as the number of poles is increased. This increases the possibility of flash over between the adjacent brush arms. Alternatively, to avoid possibility of a flash over the diameter of the commutator will have to be increased as the number of poles increases.

7. **Distortion of field form.** Armature mmf per pole

$$AT_a = \frac{ac}{2} \times \text{pole pitch} = \frac{ac}{2} \times \frac{\pi D}{p} = \frac{AC}{2p}.$$

Therefore, the armature mmf per pole varies inversely with number of poles. Hence with a smaller number of poles, the armature mmf per pole increases resulting in distortion of field form and reduction in flux under load conditions. The distortion in flux causes poor commutation conditions and sparking while the reduction in flux causes lower generated emf. Hence, with smaller number of poles, it may become necessary to use compensating winding to obviate the effects of increased armature reaction. The provision of compensating winding complicates the construction and increases the cost of machine.

Summarizing, the advantages of having large number of poles are :

There is a *reduction* in :

(*i*) *weight of armature core and yoke*

(*ii*) *cost of armature and field conductors*

(*iii*) *overall length and diameter of machine*

(*iv*) *length of commutator, and*

(*v*) *distortion of field form under load conditions.*

The disadvantages of larger number of poles are :

There is an *increase* in :

(*i*) *frequency of flux reversals*

(*ii*) *labour charges*

(*iii*) *possibility of flash over between brush arms.*

We have seen that the cost of materials *i.e.* iron and copper decreases and the labour charges increases with increase in number of poles. In general, the number of poles should be so chosen, that good operating characteristics are obtained with minimum weight of active materials and minimum cost of construction. The number of poles depends upon the relative charges for materials and labour, where labour is cheap and materials are expensive, it is advisable to use larger number of poles.

9·12·1. Guiding factors for choice of number of poles. The following may be taken as guiding factors for the choice of number of poles :

(*i*) The frequency of flux reversals in the armature core generally lies between 25 to 50 Hz. Lower values of frequency are used for large machines

(*ii*) The value of current per parallel path is limited to about 200 A. Thus the current per brush arm should not be more than 400 A.

(*iii*) The armature mmf should not be excessively large. Table 9·3 gives the normal values of armature mmf per pole These values should not be exceeded under average conditions.

Table 9·3. Armature mmf per pole.

Output kW	*Armature mmf per pole* A
upto 100	5,000 or less
100 to 500	5,000 to 7,500
500 to 1500	7,500 to 10,000
over 1500	Upto 12,500

The armature mmf per pole $AT_a = \frac{ac}{2} \cdot \tau = \frac{ac}{2} \cdot \frac{\pi D}{p}$.

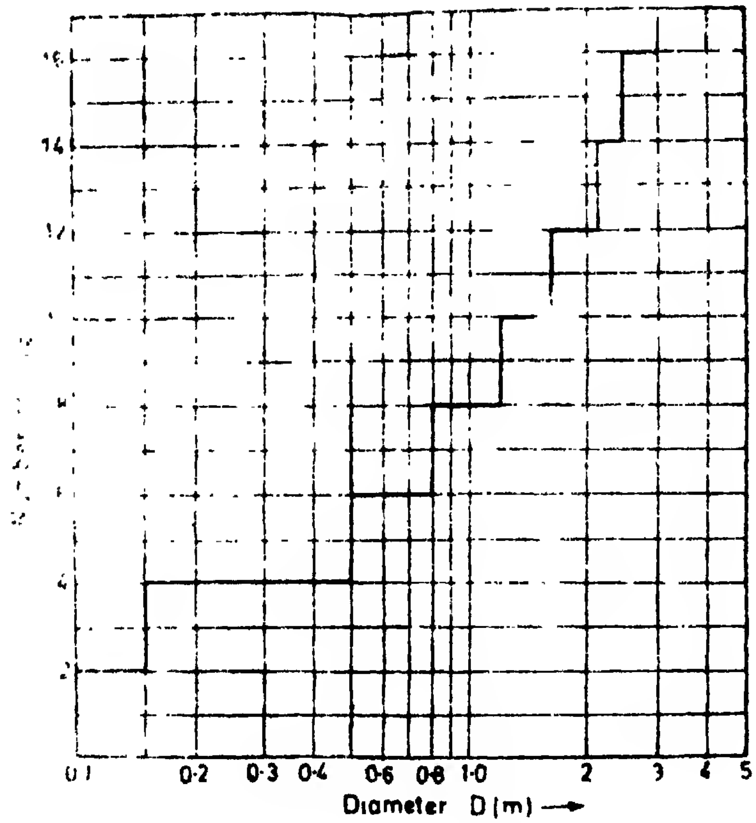


Fig. 9·25. Number of poles in a d.c. machine.

It is seen that the armature mmf is directly proportional to diameter of armature and inversely proportional to number of poles. Therefore, in order to keep the armature mmf within limits (about 10,000 A), the number of poles should increase as the diameter of machine increases. Fig. 9·25 shows the number of poles as a function of armature diameter.

9·13 Core length Following factors should be considered when selecting a suitable value of core length.

(*i*) **Cost** The manufacturing costs of a machine with large core length are less. This is because the proportion of inactive copper to active copper is smaller, greater the length of core. Thus from the point of view of cost, it is desirable to have a large core length.

(*ii*) **Ventilation** The ventilation of machines with large core lengths is difficult because the central portion of the core tends to attain a high temperature rise. The length of core cannot be increased beyond a certain value on account of ventilation problems. If long armatures are necessary, as in high speed machines, special means for ventilation of core must be provided.

9·13·1. Limiting value of core length. It has been derived in Art. 9·24 page 521 that the emf induced in a conductor e_z should not exceed $7{\cdot}5/T_c N_c$ in order that the maximum voltage between adjacent segments at load is to be limited to 30 V.

The voltage in a conductor at no load $e_s = B_{av} LV_a$.

$\therefore$ For a limiting case : $B_{av}\, L\, V_a = \dfrac{7.5}{T_c N_c}$

or limiting (maximum) value of core length $L = \dfrac{7.5}{B_{av}\, V_a\, T_c\, N_c}$ ...(9.4)

where B_{av} = average gap density, Wb/m^2 ;

V_a = peripheral speed of armature, m/s ;

T_c = turns per coil ;

N_c = number of coils between adjacent segments, 1 for a simplex lap winding and $p/2$ for a simplex wave winding.

Taking the case of a simplex lap winding with single turn coils, we have :

$N_c = 1$ and $T_c = 1$.

$\therefore$ Maximum permissible conductor emf at no load $e_s = 7.5$ V.

Limiting (maximum) value of core length, $L = \dfrac{7.5}{B_{av}\, V_a}$ ...(9.5)

For normal designs $B_{av} = 0.7$ Wb/m^2 and $V_a = 30$ m/s.

$\therefore$ Maximum permissible core length $L = \dfrac{7.5}{0.7 \times 30} = 0.36$ m.

In actual practice the values of average gap density and peripheral speed may be lower than the above mentioned values and therefore values of core lengths of about 0.4 to 0.5 m are possible with obviously the proper ventilation conditions required to keep the temperature rise of central parts of the machine within specified limits.

9.14. Armature diameter. Following factors should be considered when selecting a suitable value for armature diameter :

(*i*) **Peripheral speed.** The peripheral speed is sometimes a limiting factor for the diameter. The peripheral speed lies between 15 to 50 m/s, the lower values correspond to low speed machines.

The value of peripheral speed should not, normally, exceed 30 m/s. This speed does not call for any special rotor construction but if this peripheral speed is exceeded, the bands of binding wire for the overhang will have to be made extra strong In other ... it may be necessary to provide special method to prevent the overhang from flying out, due to excessive centrifugal force.

(*ii*) **Pole pitch.** The pole pitch obtained after selecting a suitable diameter, may be used as a check for the number of poles. Table 9.4 gives the usual values of pole pitch.

Table 9.4. Pole pitch

Poles	*Pole Pitch* mm
2	Upto 240
4	Between 240 and 350—400
6	Betweeen 350 to 400—450
Above 6	450—500

9.14.1. Limiting value for armature diameter.

Output $P = EI_a \times 10^{-3}$ kW.

We have : E = emf per conductor × conductors per parallel path $= e_s Z/a$.

or $$P = \left(e_s \frac{Z}{a}\right) I_a \times 10^{-3} = e_s \frac{I_a}{a} Z \times 10^{-3} = e_s \pi D\, ac \times 10^{-3}$$

$$\therefore \quad D = \frac{P \times 10^{-3}}{\pi\, ac\, e_s}. \quad \text{...(9·6)}$$

If we assume limiting values for $ac = 40,000$ and $e_s = 7{\cdot}5$ V.

Minimum permissible armature diameter

$$D = \frac{P \times 10^{3}}{\pi \times 40,000 \times 7{\cdot}5} \cong 0{\cdot}001\ P.$$

Hence a 1000 kW machine will have a diameter not less than $0{\cdot}001 \times 1000 = 1$ m or we can say that maximum permissible output from machine of 1 m diameter is 1000 kW.

9·15. Pole proportions. The cross-section for the poles should be circular in order that the length of mean turn of the field winding is minimum as a circle gives minimum periphery for a given area. Thus a circular pole section would result in reduction in cost of copper and also in reduced field copper loss. A circular pole requires a solid casting for the pole. As stated earlier, the majority of present day d.c. machines are motors which are used in industrial drives fed from static power converters using SCRs. The solid field structure cannot be used in these machines because of increased core losses, slow response (due to high core losses), and commutation problems. A laminated field structure is a must on account of operating conditions. Also, a laminated field structure lends itself admirably when it is desired to manufacture a range of machines from a given frame. A laminated field structure requires the use of either a square or a rectangular pole section. A square pole section gives minimum cost of field winding.

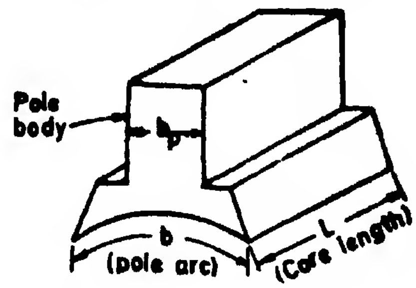


Fig. 9·26. Pole proportions.

In a square section the width of pole body is equal to the length of the machine or referring to Fig. 9·26, $b_p = L$. As the width of pole body is about 0·45 to 0·55 of pole pitch, a square section for the pole would result in a machine of short core length. Some manufacturers use a core length which is even twice the width of pole body.

Thus we have, $L = b_p$ to $2\,b_p = 0{\cdot}45\tau$ to $1{\cdot}1\tau$ or $L/\tau = 0{\cdot}45$ to $1{\cdot}1$.

Usually the ratio L/τ lies between 0·7 to 0·9.

Some designers prefer a square pole face. This means that the value of pole arc b, is equal to the length of the machine or $L = b = \psi\tau$

$\therefore \quad L = 0{\cdot}64\tau$ to $0{\cdot}72\tau$ as $\psi = 0{\cdot}64$ to $0{\cdot}72$.

When designing a range of d.c. machines, the armature diameters should be so chosen that as many ratings as possible can be obtained with the same diameter. Thus by using the same frame diameter and varying the length, different ratings would be obtained. This cuts down the manufacturing costs appreciably.

9·15. Number of ventilating ducts. Radial ventilating ducts, for cooling the armature, are used if the length of core exceeds about 0·12 m. A radial duct is provided for approximately every 70 mm or 0·07 m of core length. The width of ducts is usually 10 mm.

9·16. Length of air gap. The length of air gap can be fixed by considering the following :

(*i*) **Armature reaction**. In order to prevent excessive distortion of field form by the armature reaction, the field mmf must be made large in comparison with the armature mmf. A machine designed with a long air gap requires large field mmf. Thus the distorting effect of armature reaction can be reduced if the length of air gap is made large.

However, the increase in field mmf results in increase in size and cost of machine.

(*ii*) **Circulating currents**. The air gap, in the case of multipolar lap wound machines, should be long, because if the gap is small, a slight irregularity in air gap would result in large circulating currents. (See Art. 6·12 page 243).

(*iii*) **Pole face losses**. If the length of air gap is made large, the variations in air gap flux density due to slotting is small. Therefore, the pulsation loss in the pole faces decreases if the length of air gap is increased.

(*v*) **Noise**. The operation of machine with large air gap lengths is comparatively quiet.

(*vi*) **Cooling**. Machines designed with a large value of air gap length have better ventilation.

(*vii*) **Mechanical considerations**. Before the advent of commutating poles, machines were designed to have a large value of air gap length in order to prevent distortion of field form. But the use of commutating poles has brought down the values of air gap length to the extent considered reasonable minimum from mechanical point of view. With smaller length of air gap there is a greater possibility of appreciable unbalanced magnetic pull developing and causing the rotor to foul with the stator. The length of air gap should be large to prevent any such possibility. (See pages 179 to 189 for unbalanced magnetic pull).

9·16·1. Estimation of air gap length. Mmf required for air gap of salient pole machines is : $AT_g = 800{,}000\, B_g K_g l_g$ ...(*i*)

and armature mmf per pole $AT_a = \dfrac{ac\tau}{2}$

The value of gap mmf is normally between 0·5 to 0·7 of armature mmf. The usual value is 0·55.

$$\therefore \quad AT_g = (0·5 \text{ to } 0·7)\, AT_a = (0·5 \text{ to } 0·7)\, \frac{ac\tau}{2} \qquad ...(ii)$$

Equating (*i*) and (*ii*), we have, length of air gap, $l_g = \dfrac{(0·5 \text{ to } 0·7)\, ac\,\tau}{1600{,}000\, K_g\, B_g}$.

The gap contraction factor K_g may be assumed as 1·15.

Usually the value of air gap length lies between 0·01 to 0·015 of pole pitch.

9·17. Pole face profile. The field form (air gap flux distribution curve) must have such a shape that the commutation conditions are improved. In order to have good commutation, the flux density in the air gap must decrease gradually from maximum value under the centre of the pole to zero on the interpolar axis. A field form that drops off abruptly from maximum value to zero leads to commutation difficulties and magnetic noise.

In order to achieve a good field form, the length of air gap should not be uniform under the entire pole face (or pole arc) but the face must be shaped to give a gradually increasing distance towards both pole tips. The air gap length at the pole tips is generally 1·5 to 2 times the gap length at the centre of the pole. The usual shape of pole face is given in Fig. 9·27.

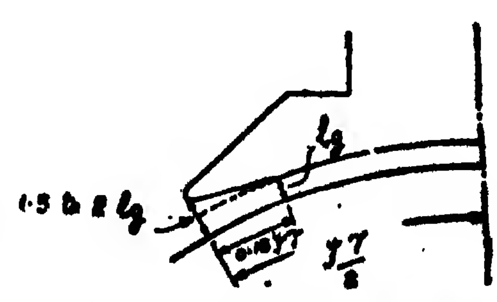


Fig. 9·27. Profile and chamfering of pole face.

Example 9·1 *A 5 kW, 250 V, 4 pole, 1500 r.p.m. shunt generator is designed to have a square pole face. The loadings are :*

average flux density in the gap=0·42 Wb/m² and ampere conductors per metre=15,000.

Find the main dimensions of the machine. Assume full load efficiency=0·87 and ratio of pole arc to pole pitch=0·66.

Solution

Armature power $P_a=\frac{5}{0\cdot87}=5\cdot75$ kW ; Speed $n=\frac{1500}{60}=25$ r.p.s

Output co-efficient $C_0=\pi^2 B_{av}\, ac\, 10^{-3}=\pi^2\times0\cdot42\times15000\times10^{-3}=62\cdot1$

$$\therefore \qquad D^2L=\frac{P_a}{C_0\, n}=\frac{5\cdot75}{62\cdot1\times25}=3\cdot7\times10^{-3}\ \text{m}^3.$$

For a square pole face, $\frac{\text{core length}}{\text{pole arc}}=1$ or $\frac{L}{\psi\tau}=1$

or $\qquad L=\psi\frac{\pi D}{p}=0\cdot66\times\frac{\pi D}{4}=0\cdot518\,D \quad\therefore\quad 0\cdot518\,D^3=3\cdot7\times10^{-3}$

or $\qquad D=0\cdot193$ m and $L=0\cdot1$ m.

Example 9·2. *A design is required for a 50 kW, 4 pole, 600 r.p.m. d.c. shunt generator, the full load terminal voltage being 220 V. If the maximum gap density is 0·83 Wb/m² and the armature ampere conductors per metre are 30,000, calculate suitable dimensions of armature core to give a square pole face.*

Assume that the full load armature voltage drop is 3 per cent of the rated terminal voltage, and that the field current is 1 percent of rated full load current. Ratio of pole arc to pole pitch is 0·67.

Solution. Output coefficient $C_0=\pi^2\psi\, B_g\, ac\times10^{-3}$

$$=\pi^2\times0\cdot67\times0\cdot83\times30000\times10^{-3}=167.$$

Speed $\qquad n=600/60=10$ r.p.s.

Back e.m.f at full load $E=220+0\cdot03\times220=226\cdot6$ V.

Full load current $=\frac{50\times1000}{220}=227$ A.

$\therefore$ Field current $=0\cdot01\times227=2\cdot27$ A.

Armature current $\quad I_a=227+2\cdot27=2\cdot27=229\cdot27$ A.

Power developed by armature $P_a=EI_a\times10^{-3}=229\cdot27\times10^{-3}=5\cdot18$ kW

$$\therefore \qquad D^2L=\frac{P_a}{C_0 n}=\frac{51\cdot8}{167\times10}=0\cdot0311\ \text{m}^3.$$

For a square pole face $\frac{L}{\psi\tau}=1$ or $\frac{L}{0\cdot67\times\pi\times D/4}=1$ or $L=0\cdot526\,D.$

Hence, $\qquad D^3=\frac{0\cdot0311}{0\cdot526}=0\cdot0591\ \text{m}^3$

$\therefore \qquad D=0\cdot389\ \text{m}\simeq0\cdot39$ m and $L=0\cdot21$ m.

Example 9·3. *Determine the main dimensions, number of poles and the length of air gap of a 600 kW, 500 V, 900 r.p.m. generator. Assume average gap density as 0·6 Wb/m² and ampere conductors per metre as 35000. The ratio of pole arc to pole pitch is 0·75 and the efficiency is 91 per cent.*

The following are the design constraints : peripheral speed ≯ 40 m/s, frequency of flux reversals ≯ 50 Hz, current per brush arm ≯ 400 A and armature mmf per pole ≯ 7500 A.

The mmf required for air gap is 50 per cent of armature mmf and gap contraction factor is 1·15.

Solution. Output coefficient $C_0 = \pi^2 B_{av}\, ac\, 10^{-3} = \pi^2 \times 0.6 \times 35000 \times 10^{-3} = 207.$

Power developed by armature $P_a = \dfrac{600}{0.91} = 660$ kW.

Speed $\quad n = \dfrac{900}{60} = 15$ r.p.s

$$\therefore \quad D^2L = \frac{P_a}{C_0\, n} = \frac{660}{207 \times 15} = 0.2126 \text{ m}^3.$$

Number of Poles :

The choice of number of poles for this size of machine lies between 4, 6 or 8. Comparing the results :

(*i*) ***Frequency.*** Frequency of flux reversals, $f = \dfrac{pn}{2}$ Hz.

The frequency is 30 Hz for 4 poles, 45 Hz for 6 poles and 60 Hz for 8 poles.

Therefore we cannot use 8 poles since the frequency exceeds the maximum allowable limit of 50 Hz

(*ii*) ***Current per brush arm.*** Neglecting the field current, armature current

$$I_a = \frac{600 \times 1000}{500} = 1200 \text{ A}.$$

Current per brush arm $I_b = 2\, I_a/p$.

With $p=4$, $I_b = \dfrac{2 \times 1200}{4} = 600$ A. With $p=6$, $I_b = \dfrac{2 \times 1200}{6} = 400$ A.

The current per brush arm should not exceed about 400 A. This means that 4 poles cannot be used for this machine. Therefore the number of poles is taken as 6.

Main Dimensions :

Now $\quad L/\psi\tau = 1 \quad \therefore \quad L = 0.75 \times \pi D/6 = 0.393\, D.$

$\therefore \quad 0.393\, D^3 = 0.2126 \quad$ or $\quad D = 0.815$ m.

Taking diameter $D = 0.8$ m ; length $L = 0.33$ m.

Checks :

(*i*) ***Peripheral speed :*** $V_a = \pi Dn = \pi \times 0.8 \times 15 = 37.7$ *m/s*. This is within specified limits.

(*ii*) ***Armature mmf :*** Pole pitch $\tau = \dfrac{\pi \times 0.8}{6} = 0.42$ m.

Armature mmf per pole $AT_a = \dfrac{ac\,\tau}{2} = \dfrac{35000 \times 0.42}{2} = 7350$ A.

This is within the specified limit.

Length of Air Gap :

Maximum flux density in the gap $B_g = \dfrac{0.6}{0.75} = 0.8$ Wb/m².

Mmf required for air gap $AT_g = 800{,}000 \times 0{\cdot}8 \times 1{\cdot}15\ l_g = 736{,}000\ l_g$ ...(*i*)

Also, air gap mmf per pole $AT_g = 0{\cdot}5\ AT_a = 0{\cdot}5 \times 7350 = 3675$ A. ...(*ii*)

Equating (*i*) and (*ii*), length of air gap $l_g = \dfrac{3675}{736000}$ m $\simeq 5$ mm.

Example 9·4. *Calculate the diameter and length of armature for a 7·5 kW, 4 pole, 1000 r.p.m. 220 V shunt motor. Given : full load efficiency = 0·83 ; maximum gap flux density = 0·9 Wb/m² ; specific electric loading = 30,000 ampere conductors per metre ; field form factor = 0·7. Assume that the maximum efficiency occurs at full load and the field current is 2·5% of rated current. The pole face is square.*

Solution. Power input $= \dfrac{P}{\eta} = \dfrac{7500}{0{\cdot}83} = 9040$ W.

Total losses at full load $= 9040 - 7500 = 1540$ W.

Since the maximum efficiency occurs at full load, the constant losses and armature I^2R loss are equal at full load.

$\therefore$ Constant losses $= \dfrac{1540}{2} = 770$ W.

Motor current at full load $= \dfrac{7500}{0{\cdot}83 \times 220} = 41{\cdot}1$ A.

Field current $= 0{\cdot}025 \times 41{\cdot}1 = 1{\cdot}03$ A. Field I^2R loss $= 220 \times 1{\cdot}03 = 227$W

$\therefore$ Friction and windage plus iron loss $= 770 - 227 = 543$ W.

Hence, power developed by armature $P_a = 7{\cdot}5 + 0{\cdot}543 = 8{\cdot}1$ kW.

Average gap density $B_{av} = K_f\ B_g = 0{\cdot}7 \times 0{\cdot}9 = 0{\cdot}63$ Wb/m².

Output coefficient $C_0 = \pi^2 \times 0{\cdot}63 \times 30{,}000 \times 10^{-3} = 186{\cdot}5$.

Speed $n = \dfrac{1000}{60} = 16{\cdot}67$ r.p.s.

$$\therefore \quad D^2L = \frac{P_a}{C_0\, n} = \frac{8{\cdot}1}{186{\cdot}3 \times 16{\cdot}67} = 2{\cdot}6 \times 10^{-3}\ \text{m}^3$$

For a square pole face $L/\psi\tau = 1$

or $$L = 0{\cdot}7 \times \frac{\pi D}{4} = 0{\cdot}55\ D. \quad (\text{taking } \psi = K_f)$$

$\therefore$ $0{\cdot}55\ D^3 = 2{\cdot}61 \times 10^{-3}$ or $D = 0{\cdot}17$ m and $L = 0{\cdot}09$ m.

Example 9·5. *A 150 kW, 230 V, 500 r.p.m., d.c. shunt motor has a square field coil. Find its number of poles and the main dimensions and air gap length. Assume the average gap density over the pole arc as 0·85 Wb/m² and the ampere conductors per metre as 29000. The ratio of width of pole body to pole pitch is 0·55 and the ratio of pole arc to pole pitch is 0·7. The efficiency is 91 per cent. Assume that the mmf required for air gap is 55 per cent of armature mmf and the gap contraction factor is 1·15.*

Solution. Using Eqn. 8·14, power developed by armature

$$P_a = \left(\frac{1+2\eta}{3\eta}\right) P = \left(\frac{1+2\times 0{\cdot}91}{3\times 0{\cdot}91}\right) 150 = 155 \text{ kW}.$$

Speed $n = \dfrac{500}{6} = 8{\cdot}33$ r.p.s.

It has been given that the average flux density over pole arc is 0·85 Wb/m². If we examine the field form of a d.c. machine given in Fig. 4·12 page 127, we find that the average value of flux density over the pole arc is approximately equal to B_g. (We have to

differentiate between B_g and B_{av}. The value of gap flux density under pole arc is approximately constant and is equal to B_g. Beyond the pole arc the value of gap density falls and is zero at interpolar axis. B_{av} is the average flux density over the entire pole pitch while B_g is the average gap density over the pole arc).

Output coefficient $C_0=\pi^2\psi B_g\, ac\; 10^{-3}=\pi^2\times 0\cdot7\times 0\cdot85\times 29000\times 10^{-3}=170$.

Speed $\quad n=\dfrac{500}{6}=8\cdot33$ r.p.s. $\therefore\quad D^2L=\dfrac{P_a}{C_0\, n}=\dfrac{155}{170\times 8\cdot33}=0\cdot1095\ \text{m}^3$.

Number of Poles. We can use either 4 or 6 poles for a medium rating machine (150 kW) designed for medium speed (500 r.p.m.) Comparing the two designs.

(i *Frequency* :

The frequency with 4 poles is $f=\dfrac{4\times 8\cdot33}{2}=16\cdot67$ Hz.

With 6 poles $f=\dfrac{6\times 8\cdot33}{2}=25$ Hz.

(ii) *Current per brush arm* : Line current $=\dfrac{150}{0\cdot91\times 302}=715$ A

The armature current can be approximately taken equal to line current.

With $p=4$, current per brush arm $=2\times 715/4=357\cdot5$ A and

with $p=6$, current per brush arm $=2\times 715/6=238\cdot33$ A.

The values of frequency and current per brush arm are within limits for both the designs. The value of frequency is reasonable with $p=6$ while for $p=4$, the frequency is on the lower side. (Low frequency is no disadvantage but this implies that a higher number of poles can be used without iron loss getting excessive. Higher number of poles should be used wherever possible because their use results in reduction in the cost of materials). Thus we choose $p=6$ as this would result in reduction in the cost of copper and iron.

Main Dimensions. The machine is to be designed with a square field coil. This means that the length of machine is equal to the width of pole body. It is given that the width of pole body is 0·55 of pole pitch.

$\therefore$ Length of machine $L=$ width of pole body $=0\cdot55\ \tau=0\cdot55\times\pi D/6=0\cdot287\ D$.

$\therefore\quad 0\cdot287\ D^3=0\cdot1095$ or $D=0\cdot725$ m and $L=0\cdot21$ m.

Air Gap Length. Pole pitch $\tau=\pi\times 0\cdot725/6=0\cdot38$ m.

Armature mmf per pole $AT_a=\dfrac{ac\ \tau}{2}=\dfrac{29000\times 0\cdot38}{2}=5510$A

Mmf required for air gap $AT_g=0\cdot55\ AT_a=0\cdot55\times 5510=3030$ A. ...(i)

Also gap mmf $AT_g=800{,}000\times 0\cdot85\times 1\cdot15\ l_g=782{,}000\ l_g$ ...(ii)

Equating (i) and (ii), length of air gap $l_g=\dfrac{3030}{782{,}000}=3\cdot87\times 10^{-3}$ m.

The length of air gap is taken as 4 mm.

Example 9·6. ***Find the diameter and length of armature core for a 75 kW, 400 V, 14 rps, 4 pole d.c. motor having a full load efficiency of 91 per cent. The product of specific magnetic and electric loadings is related to diameter as follows :***

D metre	0·1	0·2	0·3	0·4	0·5	0·6
$B_{av}\times ac$	5200	9300	12700	15500	18000	20000

B_{av} is expressed in Wb/m^2 and ac in A/m.
Take ratio of pole arc to pole pitch=0·7. Select the design which gives a square pole face.

Solution Power developed by armature

$$P_a=P\left(\frac{1+2\eta}{3\eta}\right)=75\left(\frac{1+2\times0{\cdot}91}{3\times0{\cdot}91}\right)=77{\cdot}5\text{ kW}.$$

Now $\quad P_a=C_0D^2Ln=\pi^2 B_{av}\,ac\,D^2Ln\times10^{-3}=\pi^2(B_{av}\,ac)\,D^2L\times14\times10^{-3}$
$=138\,(B_{av}\,ac)\,D^2L\times10^{-3}$

$$\therefore \quad L=\frac{P_a\times10^3}{138(B_{av}\,ac)D^2}=\frac{77{\cdot}5\times10^3}{138(B_{av}\,ac)D^2}=\frac{562}{(B_{av}\,ac)D^2}$$

From the above relation we can find the lengths for various diameters and also the ratio of length to pole pitch. The results are tabulated below :

D m	0·1	0·2	0·3	0·4	0·5	0·6
Product $(B_{av}\,ac)D^2$	52	372	1143	2480	4500	7200
$L=\frac{562}{(B_{av}\,ac)D^2}$ m	10·8	1·51	0·49	0·227	0·125	0·078

From above we find that cores with diameters 0·1 m, 0·2 m and 0·3 m are too long while core with diameter 0·6 m is too short. Machines with diameters 0·4 m and 0·5 m have normal designs.

The details of designs with two diameters are given below :

Diameter D	Length τ	Pole pitch τ	pole arc $b=\psi\tau$ $\psi=0{\cdot}7\tau$	Ratio L/b
0·4	0·227	0·314	0·22	1·03
0·5	0·125	0·471	0·33	0·38

The design with 0.4 m diameter gives a square pole face and therefore, it is adopted for the machine.

Hence $\qquad D=0{\cdot}4$ m, and $L=0{\cdot}227$ m

Example 9·7. *The maximum permissible value of emf induced in a conductor at no load is 7 volt in a simplex lap wound machine using single turn coils. Find the maximum permissible core length for the machine if the maximum gap density is 1 Wb/m^2; form factor is 0·67 and the armature peripheral speed is 40 m/s*

Solution. Average gap density $B_{av}=K_fB_g=0{\cdot}67\times1=0{\cdot}67$ Wb/m^2

Referring to Art. 9·13·1 (page 504), maximum permissible core length

$$L=\frac{7}{B_{av}V_a}=\frac{7}{0{\cdot}67\times40}=0{\cdot}26\text{ m}.$$

(An expression similar to Eqn. 9·5 has been used here except that maximum allowable voltage in the conductor is 7V in place of 7·5 V as used in Eqn. 9·5.

Example 9·8. *Find the maximum permissible output from a machine frame having a diameter of 2 m. The maximum permissible specific electric loading is 50,000 and the emf generated in a conductor at no load is to be limited to 7·5 V.*

Solution. From Eqn. 9·6, maximum permissible output

$$P=\pi D\, ac\, e_s\times 10^{-3}=\pi\times 2\times 50{,}000\times 7{\cdot}5\times 10^{-3}$$
$$=2350 \text{ kW}.$$

ARMATURE REACTION

9·18. Flux distribution at load. The preliminary details about armature reaction in d.c. machines have been discussed in Art. 6·37 page 308. It has been shown that for all d.c. machines, the armature mmf approximates to a symmetrical triangular wave with an amplitude, $AT_a=(I_a/a)(Z/2p)$. The axis of the armature mmf is the interpole axis so that the armature mmf and field mmf are displaced 90° in space. Therefore the armature reaction has a cross magnetization effect.

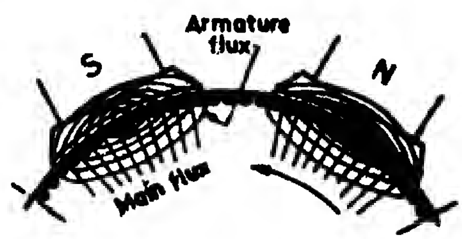


Fig. 9. 28. Path of armature flux.

Fig. 9·28 shows the distribution of armature current in a d.c motor. (It must be borne in mind that the direction of current in a d.c. motor is opposite to that of the generated emf). The path of the flux produced by the armature current is through the air gap at pole tips, armature teeth and core, and the pole shoe. The same diagram is shown in developed form in Fig 9·29 (*a*). The armature mmf is zero at the pole centres and maximum at the pole centres at the interpole axis. If the gap dimensions are constant over the whole pole arc the flux distribution curve due to the sole action of armature currents is as shown in Fig. 9·29 (*c*). The flux density in the interpolar region is small because of large reluctance offered by the long air paths in this region.

Fig. 9·29 (*b*) shows the flux distribution at no load due to main field.

If the magnetic saturation is entirely absent, the value of flux density at any point in the air gap may be taken to be proportional to the total mmf acting at that point. Therefore, the resultant flux distribution curve due to the joint action of field mmf and armature mmf at load may be drawn by adding the corresponding ordinates at every point. Fig. 9·29 (*d*) shows the resultant field distribution curve. From this we gather the following information :

(*i*) The resultant field distribution curve is much distorted and is no longer symmetrical about the pole axis.

(*ii*) In the case of a motor, the flux density under the leading pole tip is increased whereas it is decreased under the trailing pole tip. (The opposite is true in the case of a generator).

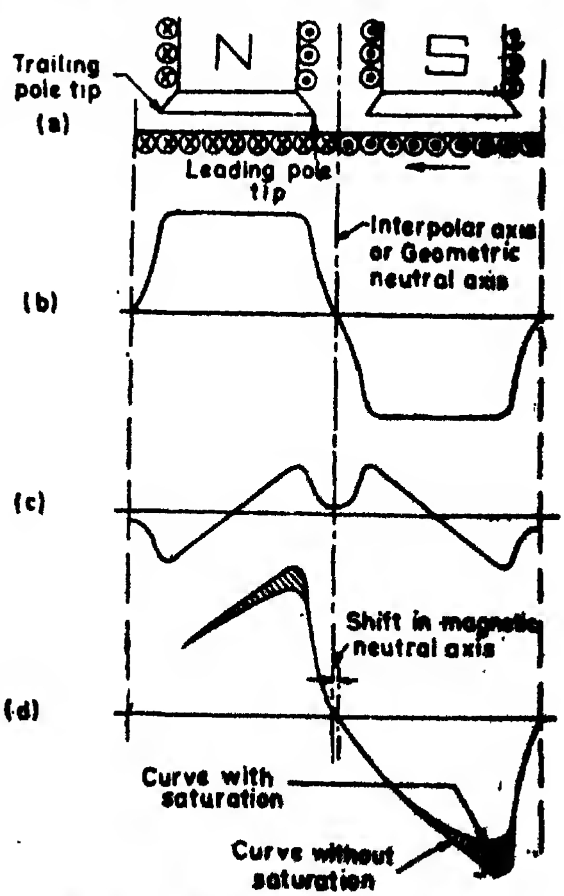


Fig. 9·29. Effect of armature reaction.

(*iii*) The flux density at the interpolar axis (or the geometric neutral axis) is no longer zero but has an appreciable finite value which depends upon the armature mmf.

(*iv*) The magnetic neutral axis (point of zero flux density) which was at the geometric neutral axis at no load is now shifted backwards (against the direction of rotation for a motor) The amount of shift is dependent upon the armature mmf.

(*v*) For this case where we have considered no saturation, the value of the useful flux per pole remains the same or the flux per pole is unaltered from its value at no load; it is merely the distribution of flux that is altered.

The direct addition of field curves of the main field and the armature mmfs as in Fig. 9·29 (*d*), can be considered as correct as far as the interpolar region is concerned because there is no saturation in this region owing to large length of air paths. This addition would be incorrect for the region under the pole arc where the armature teeth are considerably saturated. Let us consider the mmfs at different places. They are :

at trailing pole tip $AT_{min}=AT_{f0}-\psi AT_a$; at leading pole tip $AT_{max}=AT_{f0}+\psi AT_a$;
at pole centre $AT_0=AT_{f0}$ where AT_{f0}=field mmf at no load.

Fig. 9·30 shows the magnetization curve. B_0 is the value of flux density in air gap at pole centre under no load-conditions corresponding to mmf AT_{f0}. This is also the value of flux density at no load at the two pole tips. From Fig. 9·30, the value of flux density at trailing pole tip is B_{min} and at the leading pole tip is B_{max}. Therefore, it is clear, that with saturation, the increase in the value of flux density at the leading pole tip is not as much as the decrease in the value of flux density at the trailing pole tips. Thus there is reduction in the value of flux when the machine is loaded This reduction in the value of flux depends upon the degree of saturation, the value of flux per pole will be seriously reduced when the machine is heavily loaded.

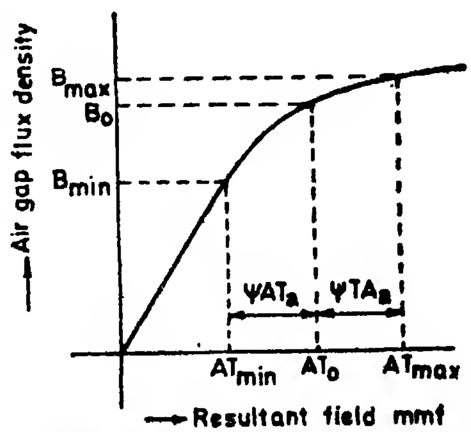


Fig. 9·30. Gap flux density under pole tips and centre under load conditions.

The resultant field form considering saturation is also shown in Fig. 9·29 (*d*). The reduction in flux is indicated by the shaded area.

9·19. Effect of armature reaction

(*i*) **Reduction in emf.** It has been shown above that in a machine working under saturation conditions the value of flux per pole decreases, owing to the effect of armature reaction. With the decrease in the value of flux per pole, the emf generated in the machine decreases as the machine is loaded.

(*ii*) **Increase in iron losses.** The value of the iron losses in the teeth and pole shoes is determined by the maximum value of flux density at which they work. The value of maximum density at load increases considerably above the no load value (it may be about 1·3 times the no load value) owing to the field distorting effect of the armature reaction. Therefore the iron losses particularly in teeth, are greater on load than on no load. The iron losses at load are approximately 1·5 times the iron losses at no load.

(*iii*) **Sparking and ring fire.** The armature reaction increases the maximum value of gap flux density and therefore the maximum voltage between adjacent segments at load is higher than at no load. If the voltage between adjacent segments goes beyond 30 V, there is every possibility of a spark between adjacent segments. This spark over may take the form of a ring fire around the commutator.

(*iv*) **Delayed commutation.** In order to have good commutation conditions, the coils which undergo commutation should not have any generated emf in them. With the brushes placed at the geometric neutral axis, the coils undergo commutation there. In order that the coils undergoing commutation, do not have a generated emf, the flux density at the neutral axis should be zero. It is observed from Fig. 9·29 (*d*), that due to armature reaction, the flux density in the interpolar axis is not zero but has a small finite value. This flux density generates an emf, in the coils undergoing commutation, of a polarity which tries to maintain the current in the original direction. As commutation means the reversal of current from one direction to a direction opposite to the original, the armature reaction has a tendency to maintain the current in its original direction and thus delay commutation.

9·20. Brush shift and its effect. It has been stated before that the armature reaction not only distorts the main field but also shifts the magnetic neutral axis (axis of zero flux density) against direction of rotation for motors. Therefore, the coils short-circuited by the brushes are no longer in the magnetic neutral zone. In order to avoid delayed commutation heavy short circuits and sparking at the bruhes as explained before, the brushes should be shifted backwards opposite to the direction of rotation (for motors) in order to bring

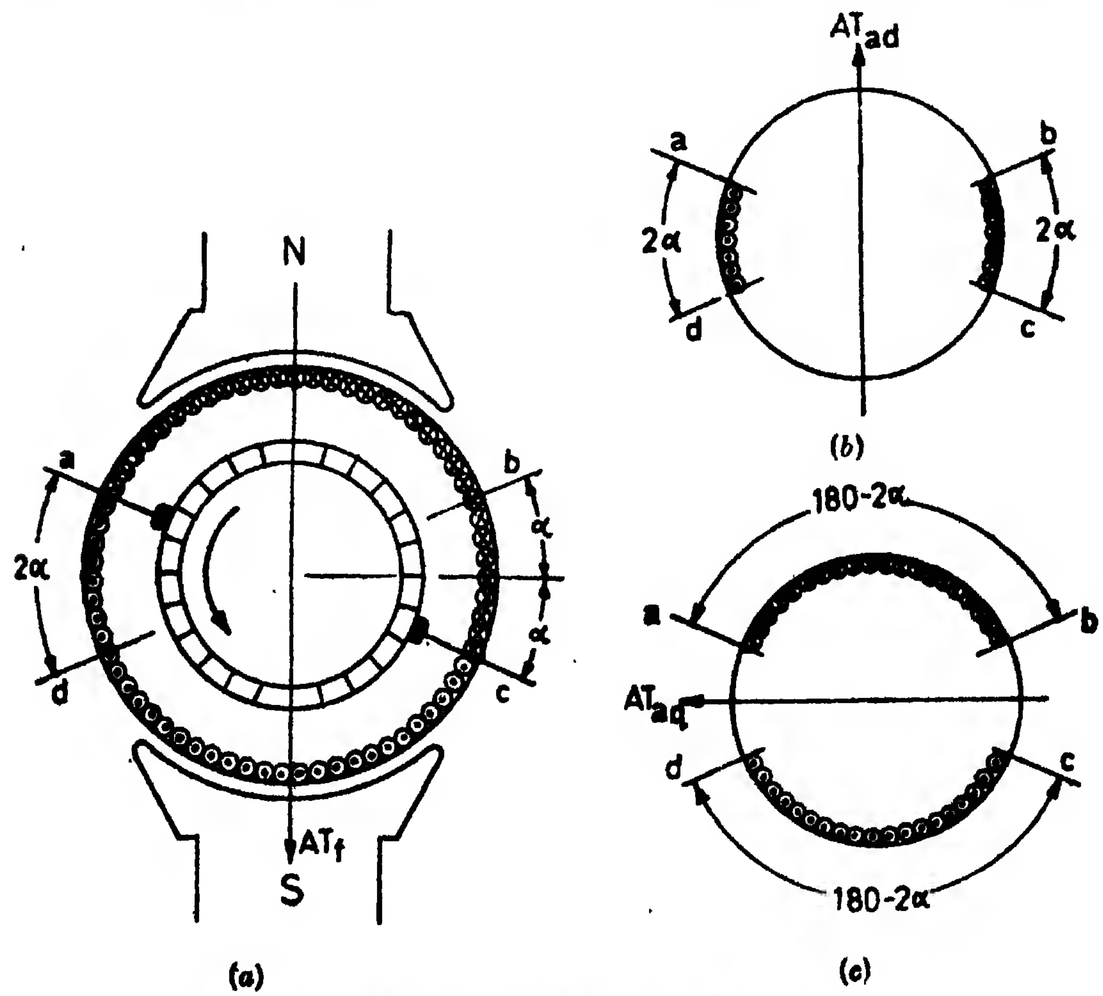


Fig. 9·31. Distribution of armature current with brush shift.

them in the magnetic neutral zone. Fig. 9·31 (*a*) shows the distribution of armature current in a bipolar machine when the brushes are given a shift of angle α. The effect of this brush shift is to resolve the armature winding into two component windings as shown in Fig. 9·31 (*b*) and (*c*). As shown in Fig. 9·31 (*b*) part *ad—bc* of the winding produces an mmf AT_{ad} which acts directly against the field mmf AT_f or in other words AT_{ad} is the *demagnetising*

component of armature mmf. Similarly Fig. 9·31 (*c*) shows part *ab*—*cd* of the armature winding produces an mmf AT_{aq} whose axis is at 90° with respect to the main field and therefore AT_{aq} is called the ***cross-magnetizing*** component of armature reaction.

Demagnetizing mmf per pole

$$AT_{ad}=AT_a\times\frac{2\alpha}{180} \qquad \text{...(9·7)}$$

$$=\left(\frac{I_a}{a}\right)\left(\frac{Z}{2p}\right)\left(\frac{2\alpha}{180}\right)=\frac{I_aZ}{ap}\left(\frac{\alpha}{180}\right) \qquad \text{...(9·8)}$$

The cross-magnetizing mmf per pole is equal to the difference between the total armature mmf per pole and the demagnetizing mmf per pole.

∴ Cross magnetizing mmf per pole $AT_{aq}=AT_a-AT_{ad}$ ...(9·9)

$$=AT_a\left(1-\frac{2\alpha}{180}\right)=\frac{I_aZ}{2ap}\left(1-\frac{2\alpha}{180}\right) \qquad \text{...(9·10)}$$

9·21. Reduction of effects of armature reaction. The following are the different methods adopted to reduce the effects of armature reaction.

(*i*) **Increase in length of air gap at pole tips.** In the case of motors having an air gap of constant value over the whole of the pole arc, it is evident that with armature teeth worked normally at a high degree of saturation, extreme saturation will be attained in the teeth under the leading pole tip This would result in a serious reduction in the value of total flux per pole when the machine is loaded.

The distorting effect of armature reaction can be reduced if the reluctance of the path of the cross-magnetizing field is increased. As stated earlier, the armature teeth and air gap at the pole tips offer reluctance to path of armature cross flux ; therefore if the length of air gap at the pole tips is increased, the effect of armature reaction is reduced. The length of air gap at the pole tips is made 1·5 to 2 times the length of air gap at the pole centre as shown in Fig. 9·27. Increasing the length of air gap at the pole tips is known as ***chamfering*** of pole faces.

(*ii*) **Increasing reluctance of pole tips.** The distorting effect of armature reaction can be reduced by increasing the reluctance of pole tips. This is done by adopting a special construction for shoes. In this type of construction leading and trailing pole tips portions of laminations are alternately omitted as shown in Fig. 9·32.

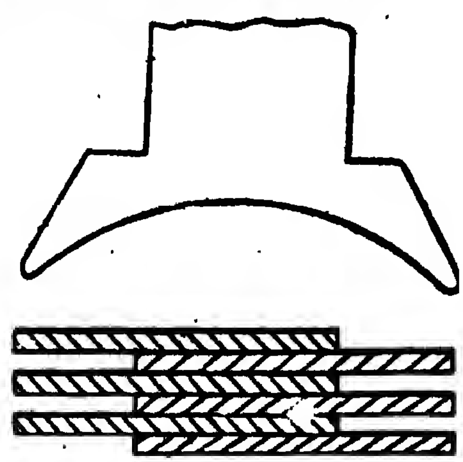


Fig. 9·32. Omission of alternate tips of pole laminations to reduce effect of armature reaction.

(*iii*) **Compensating winding.** Compensating windings are used to neutralize the effect of armature reaction. These windings are of concentric type and are housed in axial slots in the pole faces as shown in Fig. 9·33. The conductors of this winding carry currents in opposite direction to that of the adjacent armature conductors in order to nullify the effect of armature mmf. To be effective at all load, the compensating winding is connected in series with armature winding.

In order to obtain perfect compensation of the armature mmf under the pole shoe, it is necessary that the compensating winding ampere conductors are equal to the total armature ampere conductors under the pole shoe.

$\therefore$ Compensating winding mmf per pole

$$AT_c = \frac{\text{pole arc}}{\text{pole pitch}} \times \text{armature mmf per pole} = \psi\, AT_a \qquad ...(9\cdot11)$$

Owing to large costs involved, compensating winding is used in very special cases where there are violent changes in the load due to which there is a great possibility of flash over, for example in motors of reversing rolling mills.

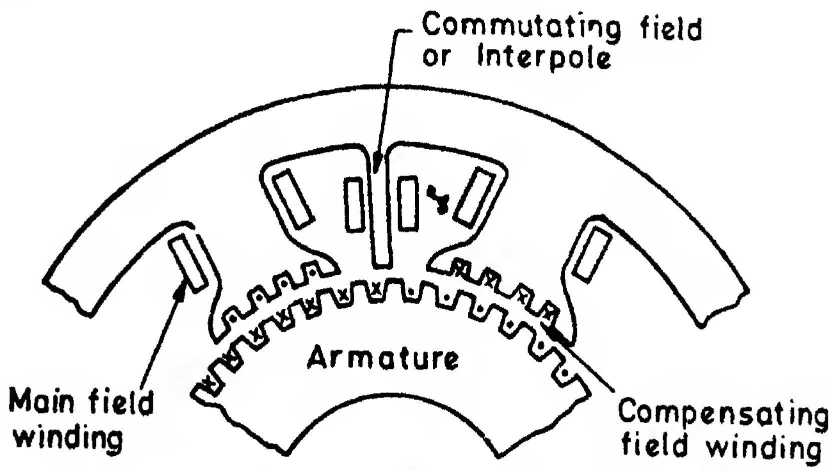


Fig. 9·33. Section of a d.c. machine showing compensating winding.

(iv) Interpoles. It has been shown in Art. 9·18 that the armature reaction shifts the magnetic neutral axis backwards against the direction of rotation in the case of motors and therefore there is a finite value of flux density at brush axis which produces a generated emf in the coils under-going commutation. The nature of polarity of this field is such that it generates an emf which tries to maintain the current in the original direction thus causing delayed commutation.

In order that the commutation takes place at magnetic neutral axis (at zero flux density), the armature reaction at the brush axis must be neutralized. This means that another mmf should be applied at the brush axis which is equal and opposite to that of armature mmf. This mmf is applied by *interpoles or commutating poles.* These poles are placed at the geometric neutral axis mid-way between the main poles as shown in Fig. 9·34.

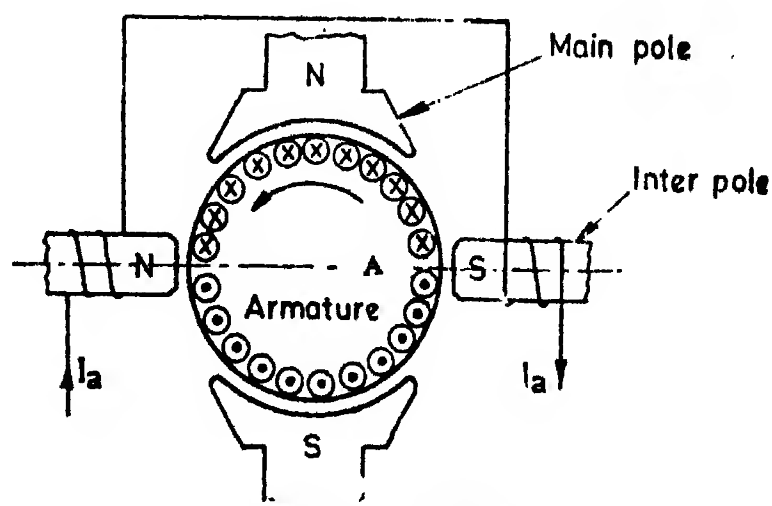


Fig. 9·34. Interpoles.

In the case of motors, the armature reaction establishes a field at the geometric neutral axis which has the same nature as the pole ahead it and therefore, the interpole should produce a field which has a nature corresponding to the main pole behind of it in order to neutralize the armature reaction. This point is elaborated as under :

Consider Fig. 9·34. The coils undergo commutation at A. The armature reaction shifts the magnetic neutral axis behind A and establishes a field corresponding to north pole

(the main pole ahead of it). In order to produce zero field at A, the interpole mmf should be equal and opposite to the armature mmf. Therefore the interpole should have a south polarity (a polarity corresponding to main pole behind of it).

In addition to neutralizing the armature mmf at the brush axis, the interpoles should produce a commutating field as explained later in in this chapter. The production of this field requires additional mmf, and therefore, the mmf required for the interpoles must be greater than the armature mmf per pole AT_a.

The interpoles must serve their purpose correctly at all loads and therefore the interpole winding is connected in series with the armature.

Example 9·9. *A 4 pole generator supplies a current of 140 A. It has 480 armature conductors (a) wave connected (b) lap connected. The brushes are given an actual lead of 10°. Calculate the cross and demagnetizing mmf per pole in each case. The field winding is shunt connected and takes a current of 10 A, find the number of extra shunt field turns to neutralize the demagnetization.*

Solution.

(*a*) *Wave connected* :

Armature current $I_a=140+10=150$ A.

With wave winding number of parallel paths $a=2$.

$\therefore$ Armature mmf per pole $AT_a=\dfrac{I_a}{a}\,\dfrac{Z}{2p}=\dfrac{150}{2}\times\dfrac{480}{2\times4}=4500$ A.

Angle of lead $\alpha=\dfrac{p}{2}\times$mech. degrees$=\dfrac{4}{2}\times10=20°$ electrical.

Demagnetizing mmf per pole $AT_{ad}=AT_a.\dfrac{2\alpha}{180}=4500\times\dfrac{2\times20}{180}=1000$A.

Cross magnetizing mmf per pole $AT_{aq}=AT_a-AT_{ad}=4500-1000=3500$ A.

Mmf for neutralizing demagnetizing mmf$=1000$ A.

$\therefore$ Extra turns required on the shunt field$=1000/10=100$.

(*b*) *Lap connected* :

With lap winding, number of parallel paths $a=p=4$.

Armature mmf per pole $AT_a=\dfrac{I_a}{a}\times\dfrac{Z}{2p}=\dfrac{150}{4}\cdot\dfrac{480}{2\times4}=2250$ A.

Demagnetizing mmf per pole $AT_{ad}=2250\times\dfrac{2\times20}{180}=500$ A.

Cross-magnetizing mmf per pole $AT_q=2250-500=1750$ A.

Extra turns required on the shunt field for neutralizing demagnetizing mmf

$=500/10=50$.

Example 9·10 *A 500 kW, 375 r.p.m. 8 pole, d.c. generator has a flux per pole of 0·0885 Wb. Determine the armature demagnetizing and cross magnetising mmf per pole if the brushes are given a lead of 5 per cent of pole pitch. Assume power developed by armature to be equal to rating of machine.*

Solution. Speed $n=375/60=6·25$ r.p.s.

Power output $P=EI_a\times10^{-3}=\Phi Zn\dfrac{p}{a}I_a\times10^{-3}$

$$=(\Phi np\times 10^{-3})\times 2p\times\left(\frac{I_a}{a}\ \frac{Z}{2p}\right)=(2\Phi\ np^2\times 10^{-3})\ AT_a.$$

$\therefore$ Armature mmf per pole $AT_a=\dfrac{P}{2\Phi\ np^2\times 10^{-3}}$

$$=\frac{500}{2\times 0\ 0885\times 6\cdot 25\times (8)^2\times 10^{-3}}=7060\ \text{A}.$$

Brush shift $\alpha=5/100\times 180=9°$ electrical.

Demagnetizing mmf per pole $AT_{ad}=AT_a\times\dfrac{2\alpha}{180}=7060\times\dfrac{2\times 9}{180}=706$ A.

Cross magnetizing mmf per pole $AT_{ac}=7060-706=6354$ A.

Example 9 11. *The armature of a 12 pole, 500 kW, 550 V generator has a simplex lap winding consisting of 2,484 conductors. There are 621 commutator segments. The ratio of pole arc to pole pitch is 0 7.*

(*a*) *Calculate the demagnetizing mmf per pole at rated full load current if the brushes are shifted through three segments from the geometric neutral axis. What is the cross magnetising mmf per pole ?*

(*b*) *Calculate the number of conductors that must be provided in each pole face if a compensating winding is used for the machine.*

Solution. Neglecting the field current, the rated armature current

$$I_a=\frac{500\times 1000}{550}=909\ \text{A}.$$

For a simplex lap winding $a=p$ $\therefore$ Number of parallel paths $a=12$.

Armature mmf/pole $ATa=\dfrac{I_a}{a}\cdot\dfrac{Z}{2p}=\dfrac{909}{12}\times\dfrac{2484}{2\times 12}=7840$ A.

(*a*) Angle of brush shift $=\dfrac{3}{621}\times 360=1\cdot 74°$ mechanical.

$\therefore$ $\alpha=\dfrac{12}{2}\times 1\cdot 74=10\cdot 44°$ electrical.

Demagnetizing mmf/pole $AT_{ad}=AT_a.\dfrac{2\alpha}{180}=7840\times\dfrac{2\times 10\cdot 44}{180}=910$ A.

$\therefore$ Cross mgnetizing mmf/pole $AT_{ac}=AT_a-AT_{ad}=7840-910=6930$ A.

(*b*) Compensating winding mmf/pole $AT_c=\psi AT_a=0\cdot 7\times 7840=5488$ A.

Turns/pole for compensating winding $=\dfrac{5488}{909}=6$.

$\therefore$ Number of pole face conductors for compensating winding $=2\times 6=12$.

ARMATURE DESIGN

9·22. Choice of armature winding. D.C. armature windings have been discussed in details in chapter 6.

Simplex windings are invariably used in preference to multiplex windings owing to the limitations imposed by equalizer connections upon the later. The choice usually lies between simplex lap and wave windings. The following points are considered for comparison :

1. In the case of simplex lap winding the number of parallel paths is equal to the number of poles. Therefore, the current in each parallel path is $1/p$th part of the rated armature current. Each of the $a=p$ parallel paths in a simplex lap winding must develop an emf almost equal to the rated voltage.

In the case of simplex wave winding, the number of parallel paths is equal to 2. Therefore the conductors have to carry half of the rated current and each of the two parallel paths must develop a voltage almost equal to the rated voltage.

This means that the number of conductors required in a simplex lap winding is $p/2$ times that in a simplex wave winding but the cross-sectional area of each conductor in a lap winding is $2/p$ times that of each conductor in wave winding and therefore the total volume of copper used in both the windings is the same.

2. It has been pointed out above that the number of conductors in a simplex lap winding is $p/2$ times that in a simplex wave winding. The larger number of conductors, with smaller area of cross-section, in the case of lap winding implies that the necessary insulation will occupy a larger percentage of the slot space than is the case with smaller number of conductors with large cross-sectional area when wave winding is used. Therefore wave windings are preferred for small and medium capacity machines and for high voltage and slow speed machines.

3. The wave winding has smaller number of conductors and therefore there would be smaller number of coils. This reduces the cost of manufacture and repairs.

4. Equalizer connections have to be used for a simplex lap winding while they are not at all required for a simplex wave winding. Provision of equalizer rings adds to the cost of the machine and complicates its construction. Therefore, wave winding is usually used for small machines.

5. The wave winding, for its satisfactory operation is independent of precise spacing of brush gear, the main and the commutating poles, and of irregularities in air gaps.

On account of the important advantages explained above it is still the practice to use wave windings so long as the output and duty cycle of the machine make it possible.

The factors discussed above do not mean that a wave winding is always superior to a lap winding. There is a limit beyond which a lap winding is superior to a wave winding. It has been found that commutation difficulties are experienced when the current per parallel path exceeds about 200 A. This means that wave windings can be used for machines which have their rated current upto 400 A and beyond this limit of 400 A rating a lap winding should be used.

The lap winding, owing to the necessity for equalizer connections, is more expensive and complicated than a wave winding, but it has many distinct advantages as under.

1. The winding can be made short pitched. This results in a reduction in overhang length at both the front and the back. This reduction is particularly important in machines like traction motors where every means of reducing dimensions and weight are significant.

2. Lap windings generally use even coils per slot. Therefore, it is possible to use more than one arrangement of the conductors in the slot in order to have the best compromise between reduction of eddy current loss and convenience in coil formation and winding.

3. In a lap winding each coil closes on itself at the commutator end, therefore the coils are easier to form and to wind in the core than is the case with the wave winding.

9·23. Number of armature conductors. Generated emf in the armature :

$$\left.\begin{aligned} E &= V + I_a r_m \text{ for a generator} \\ &= V - I_a r_m \text{ for a motor} \end{aligned}\right\} \quad \ldots(9.12)$$

where V = terminal voltage and r_m = internal resistance.

$I_a r_m$ is called the internal voltage drop. This can be found from Fig. 9·35, where its value in percentage of rated voltage is given as a function of $P \times$ r.p.m.

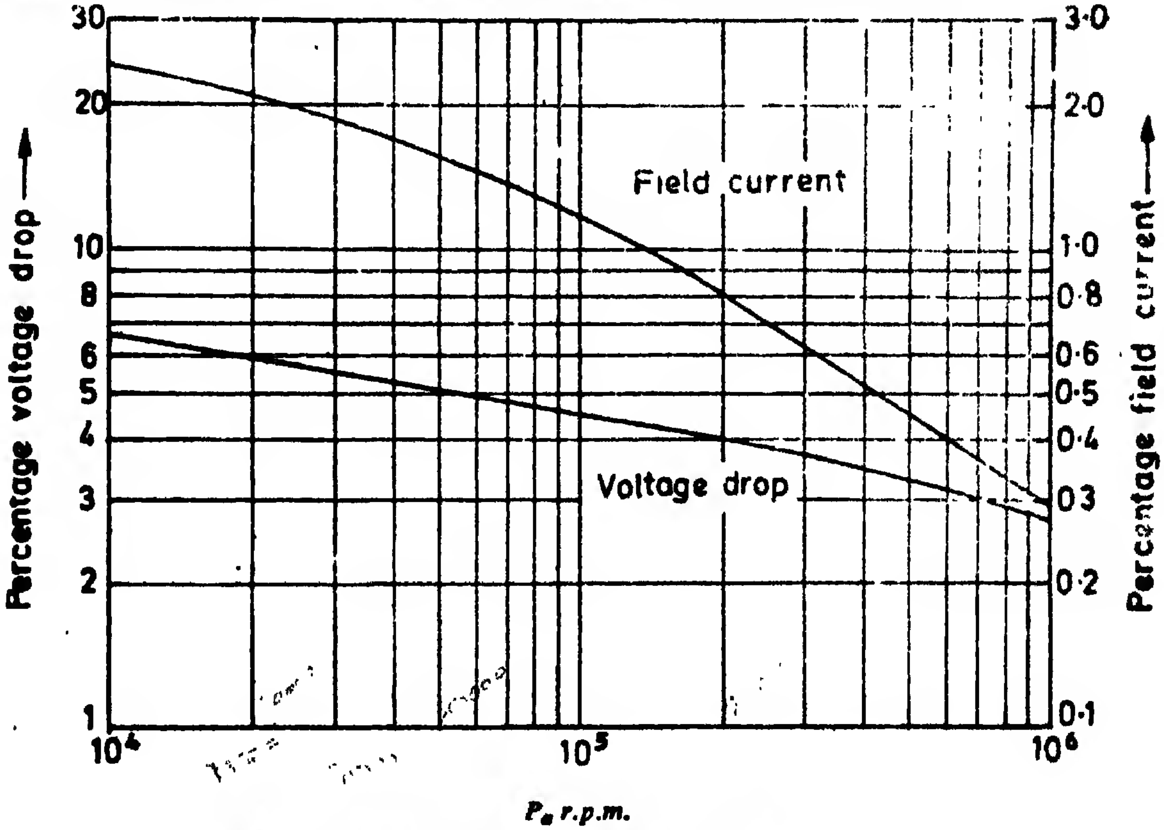


Fig. 9·35. Values of internal voltage drop and field air.

The number of armature conductors is given by the expression $Z = Ea/\Phi np$.

9·24. Number of armature coils. Whenever possible, single turn coils are used for lap windings, as a simplex wave winding is perferred to a lap winding with multi-turn coils. For small currents, wave winding with multi-turn coils is used. The number of turns per coil and the number of coils are so chosen that the voltage between adjacent commutator segments is limited to a value where there is no possibility of a flashover. Large machines flash over when the maximum voltage between adjacent segments exceeds about 28 V. For medium capacity machines there is no flashover with 35V maximum between adjacent segments. For very small machines this limit may rise to 60V owing to their high internal resistance. Normally, the maximum voltage between adjacent segments at load should not exceed 30V.

Average voltage between adjacent segments at no load

E_s = number of conductors between adjacent segments × voltage per conductor at no load

= 2 × turns between adjacent segments × average voltage per conductor at no load

= 2 × turns per coil × number of coils between adjacent segments × average voltage per conductor at no load.

$$= 2\, T_c N_c e_c \qquad \ldots(9·13)$$

where

T_c = turns per coil ; e_c = average voltage per conductor at no load ;

N_c = coils connected between adjacent segments

= $p/2$ or 1 for simplex wave or lap winding respectively.

Average votage per conductor at no load, $e_c = B_{av} L V_a$ ...(9·14)

$\therefore$ Average voltage between adjacent segments at no load,

$$E_s = 2T_c N_c B_{av} L V_a \qquad \ldots(9\text{·}15)$$

Maximum voltage between adjacent segments at no load $= 2T_c N_c B_g L V_a$.

Let B_{gm} be the maximum value of flux density at load.

$\therefore$ Maximum voltage between adjacent segments at load

$$E_{cm} = 2\, T_c N_c\, B_{gm}\, L V_a \qquad \ldots(9\text{·}16)$$

$$= 2\, T_c\, B_{gm}\, L V_a \text{ for a lap winding} \qquad \ldots(9\text{·}17)$$

$$= p T_c\, B_{gm}\, L V_a \text{ for a wave winding} \qquad \ldots(9\text{·}18)$$

Now B_{gm} is usually 1·3 times B_g.

$$\therefore \quad E_{cm} = 2\, T_c N_c \times 1\text{·}3\, B_g\, L V_a = 2\, T_c N_c \times 1\text{·}3\, \frac{B_{av}}{K_f}\, L V_a.$$

Taking $K_f = 0\text{·}66$, we have :

$$E_{cm} \simeq 4\, T_c N_c\, B_{av}\, L V_a \simeq 4\, T_c N_c e_s \qquad \ldots(9\text{·}19)$$

$$\therefore \quad e_s = \frac{E_{cm}}{4T_c\, N_c}.$$

We have said that the maximum value of voltage between segments at load, E_{cm}, should not exceed 30 V.

$\therefore$ Average voltage per conductor at no load $e_s \ngtr 7\text{·}5/T_c N_c$.

The number of turns per coil $T_c = Z/2C$ where C is the number of armature coils.

From Eqn. 9·19,

$$E_{cm} = 4 \times \frac{Z}{2C} \times N_c\, e_s = 2Z \frac{N_c}{C}\, e_s. \qquad \ldots(9\text{·}20)$$

Thus the minimum number of coils required in order that the maximum voltage between segments should not go beyond 30 V at load is, (from Eqn. 9·20)

$$C = \frac{2ZN_c e_s}{E_{cm}} = \frac{2ZN_c e_s}{30} = \frac{ZN_c}{15} \cdot e_s$$

Now $\quad e_s = B_{av}\, L V_a$

$$\therefore \quad C = \frac{ZN_c}{15}\, B_{av}\, L V_a = \frac{ZN_c}{15}\, B_{av}\, L n \pi D n$$

$$= (\Phi Z n p) \frac{N_c}{15} = \frac{E a N_c}{15}$$

as $p\Phi = B_{av}\, \pi D L$ and $E_a = \Phi Z n p$.

For a *wave winding* $a = 2$ and $N_c = p/2$.

Minimum number of coils required $C = \dfrac{E \times 2 \times p/2}{15} = \dfrac{Ep}{15}$.

For a *lap winding*, $a = p$ and $N_c = 1$.

$\therefore$ Minimum number of coils required $C = \dfrac{E \times p \times 1}{15} = \dfrac{Ep}{15}$.

Therefore, the minimum number of coils or commutator segments required is $Ep/15$. Now Ep/C is the voltage between each commutator segment at no load. Consequently, the average voltage between adjacent commutator segments at no load should not go beyond 15 V in order to limit the maximum voltage at load between adjacent segments to about 30 V.

The number of coils should not be decided without a reference to the mechanical design of the commutator. Normally it is not possible to work with commutator segments less than 3 to 4 mm thick at the outer circumference. The thickness of the mica between adjacent commutator segments about 0·8 mm and therefore the minimum pitch of commutator segments is about $3+0{\cdot}8\simeq4$ mm. Thus the number of commutator segments should be such that the commutator pitch is at least 4 mm or pitch of segment $\beta_c=\pi D_c/C\geqslant4$ mm, where D_c is the diameter of the commutator. The diameter of the commutator varies from about 0·62 of the armature diameter for 350/700 V machines to 0·68 for 200/250 V machines and 0·74 for 100/125 V machines. A preliminary value of the commutator diameter may be calculated for checking the number of coils.

9·25. Number of armature slots. The following factors should be considered when selecting the number of armature slots.

1. **Mechanical difficulties.** If we use a larger number of slots, the slot pitch becomes smaller and consequently the width of tooth gets smaller. This may lead to difficulties in construction for the reason that it will be difficult to support the teeth at the ventilating ducts without obstructing the ventilation.

2. **Cooling of armature conductors.** If we choose larger number of slots, the number of conductors per slot is less and therefore only a few conductors are bunched together. Thus the cooling of armature conductors is better if a larger number of slots are taken.

3. **Flux pulsations.** The flux pulsations *i.e.* changes in air gap flux due to slotting give rise to eddy current losses in the pole faces and produce magnetic noise. With larger number of slots, the flux pulsations are reduced and therefore there is a reduction in pole face losses and in the noise level of the machine.

Consider Fig. 9·36 (*a*) where the pole shoe covers 5 teeth and the flux passes from pole to armature through 5 teeth. In Fig. 9·36 (*b*), the armature has moved through ½ slot pitch to the right, and the flux from the pole passes into the armature through 6 teeth. Therefore, the reluctance of the air gap is greater for position shown in Fig. 9·36 (*a*) and consequently the flux per pole is smaller than that for position of armature shown in Fig. 9·36 (*b*). Therefore, there would be flux pulsations in the air gap flux as the armature rotates.

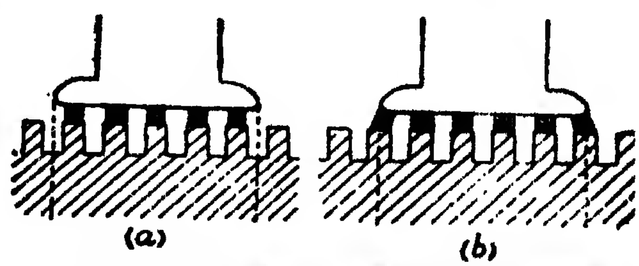


Fig. 9·36. Flux pulsations with integral number of slots per pole arc.

Let us now consider a case where pole shoe cover 5-1/2 slots as shown in Fig. 9·37 (*a*). In Fig. 9·37 (*b*), the armature has rotated through ½ slot pitch to the right. It is clear from Fig. 9·37 (*a*) and Fig. 9·37 (*b*) that the total reluctance of the flux path and the flux per pole remain approximately the same for all positions of the armature. Therefore the number of slots under the pole shoe should be an integer plus 1/2. From Fig. 9·37 (*a*) and Fig. 9·37 (*b*), it may be seen that the reluctance and the flux under the pole tips is not the same for all positions of armature and that when the armature rotates, the flux under the pole *oscillates* between the pole tips. The oscillations of the flux would be small if the pole shoes are well bevelled, and when the length of air gap is large and the slot openings narrow.

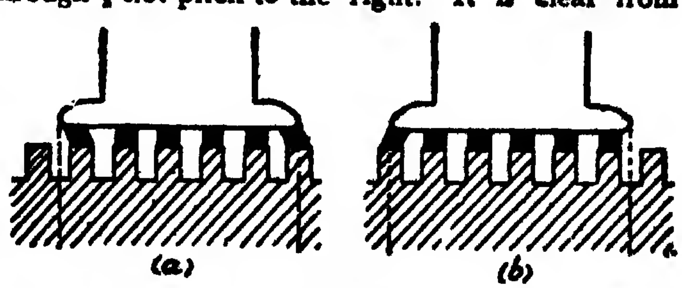


Fig. 9·37. Reduction of flux pulsations using integer +1/2 slots per pole arc.

In order to avoid flux pulsations and oscillations, the air gap reluctance per pair of poles should remain practically constant. This is attained when the number of slots per pole is an integer plus 1/2. This means that the number of slots per pole pair should be an odd integer.

It is clear from the above discussion that the pulsations and oscillations of air gap flux are reduced to minimum when

(*i*) the number of slots under the pole shoe is equal to an integer plus 1/2

(*ii*) the number of slots per pole is equal to an integer plus 1/2.

In practice, however, it is normally neither possible nor desirable to fulfil both the conditions indicated above. Generally the slots under the pole shoe are an integer while the slots per pole are equal to integer plus 1/2.

4. **Commutation.** Pulsations and oscillations of the flux under the interpole must be avoided, as they cause sparking. A large air gap under the interpole and a large number of slots helps to reduce the effect of slotting upon the flux under the interpole.

In fact, the number of slots in the region between the tips of two adjacent poles should be at least 3, or

$$(1-\psi)\times \text{slots per pole} \geq 3$$

Taking $\psi=0.67$, number of slots per pole, $\frac{S}{p}>\frac{3}{1-0.67}>9.12$.

Therefore, from the point of view of commutation, the number of slots per pole should at least be equal to 9.

With a large number of slots, the number of conductors per slot decreases. This is a desirable feature from the point of view of commutation.

5. **Cost.** A smaller number of slots are desirable considering the cost as the charges for punching the slots increase with their number. Further with smaller number of slots, there are fewer slots to insulate and therefore the cost of insulation also goes down.

9·25·1. Guiding factors for choice of number of armature slots

1. **Slot pitch.** The value of slot pitch lies between 20 to 40 mm as extreme limits. The usual limit is between 25 to 35 mm except in the case of very small machines, where it may be 20 mm and even less.

2. **Slot loading.** The slot loading *i.e.* number of ampere conductors per slot should not exceed about 15000 A or $I_z . Z_s < 1500$ A. Where Z_s=number of conductors per slot.

3. **Flux pulsations.** As stated earlier the number of slots per pole pair should be an odd integer in order to minimize pulsation losses.

4. **Commutation.** It has been mentioned earlier that the slots per pole should be at least 9 in order to prevent sparking. The number of slots per pole usually lies between 9 to 16. In very small machines the number may go down to 8, as the internal resistance is high in their case.

5. **Suitability for winding.** When selecting the number of slots, we must confirm that the number selected suits the armature winding as regards the total number of coils and the coil sides per slot.

The number of slots per pole should not be less than that given in Table 9·5 :

Table 9·5. Number of armature slots.

Rating	*Slots per pole*
upto 5 kW	8
5 kW to 50 kW	10
50 kW and over	12 and over

D.C. windings are always 2 layer type. Therefore the number of slots should be so chosen that the number of conductors per slot is an even integer. Also the number of conductors per slot should be such that they are divisible by the number of coil sides per slot. If this is not done we would get turns per coil as a mixed number and not an integer.

In the case of lap windings, equalizer connections are used. Therefore, the winding must be symmetrically arranged with respect to poles, that is, the number of slots must be a multiple of the number of pair of poles.

For a wave winding $y_c=\dfrac{C\pm 1}{p/2}$ must be an integer in order to avoid dummy coil (See Eqn. 6·11 page 239)

or $\qquad y_c=\dfrac{\frac{1}{2}uS\pm 1}{p/2}$ should be an integer.

where S=number of slots and u=number of coil sides per slot.

Therefore the number of slots S and the number of coil sides per layer $u/2$, should not be multiple of pair of a poles for wave winding in order that y_c is an integer, as otherwise the winding would be unsymmetrical *i.e.*, it would involve the use of a dummy coil.

It should be noted that whereas in the case of simplex lap windings the number of slots should be a multiple of pole pairs, with wave wound machines, the number of slots should *not* be a multiple of pole pairs.

9·26. Cross-section of armature conductors.

Armature current $\qquad I_a=\dfrac{P_a}{E}\,.\,10^3$

The armature current may be calculated from

$$\left.\begin{aligned} I_a&=I_l+I_f \text{ for a generator}\\ &=I_l-I_f \text{ for a motor}\end{aligned}\right\} \qquad \ldots(9\cdot22)$$

where $\qquad I_l$=line current and I_f=shunt field current.

The value of I_f is taken from Fig. 9·35 page 521. Here I_f is expressed as a percentage of full load current. Multiply the values of I_f given in Fig. 9·35 by 0·5 for constant speed motors.

Area of each armature conductor $a_z=I_z/\delta_a=I_a/a\delta_a$ mm^2 $\qquad \ldots(9\cdot23)$

where δ_a=current density in armature conductors, A/mm^2.

The current density in armature conductors should be taken as high as efficiency and temperature rise conditions permit. This is because a large value of current density reduces the size of conductors and therefore there is saving in the cost of copper. Also the slot area required becomes small and therefore shallow slots can be employed and use of shallow slots is beneficial for commutation conditions.

The following values of current density may be used :

(*i*) Large strap wound armatures with very good normal ventilation—4·5 A/mm^2.

(*ii*) Small wire wound armatures with very good normal ventilation—5A/mm^2.

(*iii*) High speed fan ventilated machines—6 to 7 A/mm^2.

For conductors having small area of cross-section, round wires are used. The required cross-sectional area may be obtained by using two wires connected in parallel. The standard size of conductor is taken from Tables in Chapter 17.

For cross-sectional area above 10 mm^2, square or rectangular conductors, are used. See Table 17·1 for sizes of bare copper strip.

The maximum depth of conductor that can be used in d.c. armatures is about 19 mm where the frequency is 25 Hz. In cases where the frequency is relatively high, the conductor depth should not exceed 15 mm. If these dimensions are exceeded there would be appreciable copper loss in the conductors because of eddy currents. In case the depth exceeds the above limits as with large high speed machines, it is necessary to laminate the conductors.

9·27. Insulation of armature winding. Earlier d.c. machines were designed to withstand class A temperatures. These days the d.c. machines are designed for classes E, B and F in order to meet the demand for higher outputs from a given frame size. It is interesting to note that armature winding of medium and large size machines are frequently provided with class F insulation even if the rating of the machine is based upon class B insulation because it then becomes possible to overload the machine.

The windings of all modern low voltage machines of small ratings are made on high speed winders and are transferred mechanically to pre-insulated armature slots. The conductors are coated with polyvinyl acetal (p.v a) enamel. Sometimes the wires are double coated with the outer surface being abrasion resistant. In order to reduce the friction which makes high speed winding difficult the wire is coated with micro-crystalline wax.

The conductors for larger sizes of machines are rectangular in cross-section. The conductor insulation is fibre glass applied as braiding and impregnated with silicone or polyester varnish to reduce abrasion resistance during forming. The interlayer insulation is made of polynamide. Built up mica wrap backed by fine weaves of glass cloth is used as the slot insulation.

The armature coils are retained in the slots by wedges made of an insulating material having a high mechanical strength. The wedges are made of bakelized former. For large machines especially those used for arduous duty wedges made of phenolic-resin-bonded asbestos felt are used. These wedges have a high mechanical strength which is retained over the life of the winding.

Fig. 9·38 shows the cross-section of an armature slot.

The insulation arrangement for windings used for large machines having rectangular conductors and single turn coils is explained here.

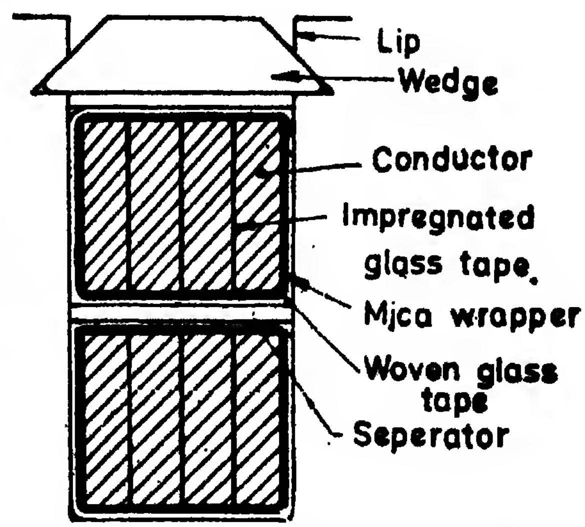


Fig. 9·38. Cross-section of an armature slot.

The individual conductors can be insulated by one of the various methods described below :

1. The insulating coverings on copper are applied by the manufacturer of conductors and the insulated conductors are then supplied to the machine manufacturer. The most important of these coverings are :

(i) Glass fibre applied as braiding and impregnated with silicone, polyester or synthetic varnish

(ii) Polyester or polyvinyl acetal enamel combined with a single or double glass fibre lapping, the latter being impregnated with silicone, polyester or synthetic varnish.

2. Bare uninsulated copper conductors are supplied by the conductor manufacturer. The machine manufacturer applies a half lap layer of a tape on the conductors after *forming* and *finishing* the bare copper. The commonly used tapes are :

(*i*) A silicone-resin impregnated 0·075 mm thick woven glass tape.

(*ii*) A single layer of mica splittings bonded with silicone resin on 0·03 mm thick backing of woven glass or between two layers of 0·013 mm thick polyester film.

(*iii*) A 0·05 mm thick layer of mica paper bonded with silicone resin to 0·05 mm thick backi[illegible] of woven glass.

The greatest advantage of forming and finishing the bare copper before applying the conductor insulation is that a complete uniformity of insulation over the entire length of conductor is ensured, which is otherwise not possible with a pre-insulated conductor. The insulation of a pre-insulated conductor is invariably is damaged during the *forming* operation thereby causing additional expenditure.

Conductors using mica tapes are used for high voltage machines (3000 V) while conductors using impregnated glass tapes are used for low voltage machines (upto 750V).

The insulated conductors are assembled and bound together by a spiral layer by suitable woven synthetic fibre tape. The bound conductors are pressed and cured.

After the first pressing operation of the assembled conductors, the main earth insulation is applied to the straight portions of the coil. The main insulation consists of several turns of a wrapper which consists of a number of layers of mica splittings bonded to a backing of woven glass cloth. The thickness of a layer of mica splittings is 0·1 mm. Shellac is used as bonding resin for class H insulation.

If mica paper is used as conductor tape, the same along with backing of woven glass cloth may be used as main insulation.

The wrapped coil is then dipped in a suitable thermal setting synthetic, epoxide or silicone resin and transferred to a heated press. The insulation is consolidated and cured to predetermined dimensions.

In order to protect the insulation particularly in the slot portion, an overlapped layer 0·075—0·125 mm thick woven glass tape lightly treated with silicone is applied throughout the length of coil.

The coils are transferred to the armature after treatment and pressing.

Glass banding is used for securing the armature end windings. This is in the form of a narrow tape comprising of unidirectional glass-fibre strands impregnated with acrylic-polyester resin bond. Glass banding has the following advantages over steel banding :

1. The processing of glass banding is cheaper as no extra insulation is required (steel wire used for banding has to be insulated from the armature overhang with flexible micanite and asbestos layers enclosed in a woven glass hood bound with impregnated woven glass tape). Also glass banding is self-setting.

2. The absence of steel wire results in reduction of overhang leakage flux, giving a slight improvement in commutation.

3. When glass bands are used there is considerably less damage in case there is a banding failure.

The thickness of conductor insulation may [illegible] mm. The insulation thicknesses widthwise and depthwise are given in Table 9·6.

9·28. Slot dimensions. The following points should be considered when fixing up the dimensions of the slot.

Table 9·6. Insulation thickness.

Insulation	*Width* mm	*Depth* mm
Main insulation	2·0	3·2
Insulation between layers (separator)	—	1·0
Wedge and lip	—	3·5
Slack	0·5	1·0
Total	2·5	8·5

1. **Excessive flux density.** The dimensions of the armature slots should be so chosen that they will accommodate the armature conductors with the necessary insulation without producing excessive flux density in the armature teeth. The slots being parallel sided, the teeth are tapered, the value of slot depth should be so chosen that the flux density at 1/3 height from root of tooth does not exceed 2·1 Wb/m².

2 **Flux pulsations.** Wide open slots produce greater pulsations of the flux in the air gap than slots with narrow openings. Since flux pulsations cause extra iron loss and are detrimental to the commutation the slots should not be too wide.

3. **Eddy current loss in conductors.** With conductors having large depths the eddy current loss in conductors increases. Therefore, the depth of conductors should be limited to the values mentioned earlier *i.e.* 19 mm for 25 Hz and 15 mm for higher frequencies. The depth of slot in normal large size machines should be limited to such a value that is just sufficient to accommodate a 2 layer winding in which the conductor depth is about 19 mm. The depth of conductor usually varies between 12·5 to 19 mm.

4. **Reactance voltage.** With deep slots, the specific permeance goes up and consequently the reactance voltage is high. This reactance voltage retards commutation. A high value of reactance voltage is particularly objectionable in non-interpolar machines. Therefore in non-interpolar machines, the ratio of slot depth to slot width is not allowed to exceed 4. For machines with interpoles deeper slots may be used because the interpoles produce a commutating field to compensate for reactances voltage.

5. **Mechanical difficulties.** With deep slots, the thickness of teeth at the root may become too small, in which case they have to be supported at the ducts and it may not be possible to support them without impairing the ventilation.

It is possible to use increasing values for slot depth as the diameter increases because the increased tooth width which does not allow the flux density to go beyond permissible limit and gives adequate mechanical strength.

Table 9·7 shows typical values of slot depth.

Table 9·7. Slot Depth.

Diameter of armature m	*Slot Depth* mm
0·15	22·5
0·20	27·5
0·25	32·5
0·30	37·5
0·40	42·5
0·50	45·0

9·29. Armature voltage drop. The length of mean turn of armature winding can be estimated from the following relationship :

Length of mean turn of armature winding $L_{mt}=2L+2\cdot3\tau+5\ d_s$ ...(9·24)

Projection of coil outside the core $L_o=0\cdot3\tau+1\cdot25\ d_s$ ...(9·25)

where L, τ and d_s are expressed in metre.

$$\text{Resistance of each conductor}=\frac{1}{2}\cdot\frac{\rho L_{mt}}{a_s}.$$

$$\text{Resistance of each parallel path}=\frac{Z}{a}\cdot\frac{\rho\ L_{mt}}{2a_s}.$$

The 'a' paths are connected in parallel.

$$\therefore\quad \text{Resistance of armature } r_a=\frac{1}{a}\left(\frac{Z}{a}\cdot\frac{\rho\ L_{mt}}{2a_s}\right)=\frac{Z}{2}\cdot\frac{\rho\ L_{mt}}{a^2\times a_s} \qquad \text{...(9·26)}$$

The value of resistivity of copper under normal cold conditions and for a temperature rise of 50°C is respectively 0·017 and 0·021 Ω/m and mm².

Armature voltage drop$=I_a r_a$.

9·30. Depth of armature core. Flux in the armature $\Phi_c=\frac{1}{2}$ flux per pole$=\frac{1}{2}\ \Phi$.

The flux density B_c in the armature core can be taken between 1·0 to 1·5 Wb/m². The core is usually worked at a density of 1·25 Wb/m²

$$\text{Area of armature core } A_c=\frac{\Phi_c}{B_c}=\frac{1}{2}\ \frac{\Phi}{B_c}=L_i\times d_c \qquad \text{...(9·27)}$$

$$\therefore\quad \text{Depth of armature } d_c=\frac{1}{2}\ \frac{\Phi}{L_iB_c}. \qquad \text{...(9·28)}$$

Example 9·12 *Find the armature voltage drop of a 300 kW, 500 volt, 6 pole lap connected d.c. generator having 150 slots with 8 conductors per slot. Area of each conductor is 25 mm² and length of mean turn is 2·5 m. The resistivity is 0·021 Ω/m and mm².*

Solution. Parallel paths $a=p=6$. Armature conductors $Z=150\times8=1200$.

$$\text{Resistance of armature } r_a=\frac{Z}{2}\cdot\frac{\rho\ L_{mt}}{a^2\times a_s}=\frac{1200}{2}\times\frac{0\cdot021\times2\cdot5}{(6)^2\times25}=0\cdot035\ \Omega.$$

$$\text{Armature current } I_a=\frac{300\times1000}{500}=600\ \text{A}.$$

$\therefore$ Armature voltage drop$=600\times0\cdot035=21$ V.

Example 9·13. *A 250 kW, 500 volt, 600 r.p.m. d.c. generator is built with an armature diameter of 0·75 m and a core length of 0·3 m. The lap connected armature has 720 conductors. Using the data obtained from this machine, determine the armature diameter, core length, number of armature slots, armature conductors and commutator segments for a 350 kW, 440 volt, 720 r.p.m., 6 pole d.c. generator.*

Assume a square pole face with ratio of pole arc to pole pitch equal to 0·66. The full load efficiency is 0·91 and the internal voltage drop is 4 per cent of rated voltage.

The diameter of commutator is 0·7 of armature diameter. The pitch of commutator segments should not be less than 4 mm.

The voltage between adjacent segments should not exceed 15 V at no load.

Solution.

250 kW Generator

Power developed by armature $P_a = \frac{250}{0.91} = 275$ kW

Speed $n = \frac{600}{60} = 10$ r.p.s.

Output co-efficient $C_o = \frac{P_a}{D^2 Ln} = \frac{275}{(0.75)^2 \times 0.3 \times 10} = 163.$

Generated voltage $E = V + I_a r_a = 500 + 0.04 \times 500 = 520$ V.

Now $E = \Phi Zn \frac{p}{a} = \left(B_{av} \cdot \frac{\pi DL}{p}\right) Zn \frac{p}{a} = \pi B_{av} DLZ \frac{n}{a}$.

$\therefore$ Average flux density $B_{av} = \frac{E\,a}{\pi DLZn}$

$$= \frac{520 \times 6}{\pi \times 0.75 \times 0.3 \times 720 \times 10} = 0.61 \text{ Wb/m}^2.$$

350 kW Generator

Power developed by armature $P_a = \frac{350}{0.91} = 385$ kW.

Speed $n = \frac{720}{60} = 12$ r.p.s.

$$D^2L = \frac{P_a}{C_o n} = \frac{385}{163 \times 12} = 0.198 \text{ m}^3$$

(Using the same value of C_o as obtained for 250 kW generator)

For a square pole face $L = \psi\tau$. $\therefore$ $L = 0.66 \times \pi D/6 = 0.345\, D$.

Thus $0.345\, D^3 = 0.198$

$\therefore$ $D = 0.835$ m and $L = 0.29$ m.

Flux per pole $= B_{av} \times \frac{\pi DL}{p} = 0.61 \times \frac{\pi \times 0.835 \times 0.29}{6} = 77.3 \times 10^{-3}$ Wb.

$\therefore$ Generated voltage $E = 440 + 0.04 \times 440 = 457.6$ V.

Number of armature conductors $Z = \frac{Ea}{\Phi np} = \frac{457.6 \times 6}{77.3 \times 10^{-3} \times 12 \times 6} = 493.$

Number of Slots :

1. The slot pitch varies from 25 to 35 mm. Therefore number of slots vary from

$$\frac{\pi \times 0.835}{0.035} = 73 \text{ to } \frac{\pi \times 0.835}{0.025} = 102.$$

2. The slots per pole should lie between 9 to 16 for proper commutation

Therefore the number of slots lie between $6 \times 9 = 54$ and $6 \times 16 = 96$.

Therefore the range of number of slots is between 73 and 96.

3. In a lap winding the number of slots should be a multiple of pole pairs. It is desirable that the number of slots/pole = integer + $\frac{1}{2}$. This requires that the number of slots be an odd integer which is multiple of $p/2 = 3$.

Thus we can select the number of slots from the following (multiples of 6 *i.e.* number of poles) : 75, 81, 87, 93.

4. In order to reduce flux pulsations, the number of slots per pole arc should be an integer$+\frac{1}{2}$.

With $S=87$, we obtain slots per pole arc nearly an integer$+\frac{1}{2}$

as $\psi S/p=0{\cdot}66\times 87/6=9{\cdot}57$.

Winding. In order that voltage between adjacent segments should not exceed 15 V at no load, minimum number of coils required

$$=\frac{Ep}{15}=\frac{440\times 6}{15}=176.$$

Conductors per slot $Z_s=493/87=5{\cdot}67$.

The conductors per slot have to be an even integer. Taking conductors per slot $Z_s=6$.

$\therefore$ Number of conductors used $Z=6\times 87=522$.

Now we have to find out the number of coil sides per slot.

The number of coils is $\frac{1}{2}uS$. Therefore, the number of coils is 87, 174 and 252 with 2, 4, 6 coil sides per slot respectively.

The minimum number of coils is 176 and therefore we cannot use $u=2$. With $u=4$ the number of conductors per slot are not divisible by number of coil sides per slot and therefore cannot be used. With $u=6$ the number of conductors per slot is divisible by number of coil sides per slot.

Using $u=6$ and $C=252$. Number of commutator segments$=192$.

The diameter of commutator $D_c=0{\cdot}7\times 0{\cdot}835=0{\cdot}585$ m.

Pitch of commutator segments $\beta_c=\frac{\pi\times 585}{252}=7{\cdot}3$ mm.

Example 9·14. *A 100 kW, 500 V, 6 pole 450 r.p.m., d.c. shunt motor has the following data :*

armature diameter$=0{\cdot}54$ m ; armature core length$=0{\cdot}245$ m ;

average flux density in the gap$=0{\cdot}55$ Wb/m^2 ; number of ducts$=2$;

width of each duct$=10$ mm ; stacking factor$=0{\cdot}92$.

Find the number of armature slots and work out the details of a suitable armature winding. Also work out the dimensions of the slot. The flux density in the teeth at 1/3 height from root should not exceed 2·1 Wb/m^2. Assume a full load efficiency of 0·89, an armature voltage drop of 5 per cent of rated voltage and field current 1 per cent of line current and a current density of 4·7 A/mm^2.

Check for the following :

(i) The slot loading should not exceed 1500 A, (ii) the pitch of commutator segments should not be less than 4 mm (The diameter of commutator is 0·65 of armature diameter) and (iii) the voltage between adjacent segments should not exceed 15 V at no load.

The ratio of pole arc to pole pitch is 0·66.

Solution.

Type of Winding

Back emf $E=V-I_a r_m=500-0{\cdot}05\times 500=475$ V.

Line current $\quad I_l = \dfrac{100\times10^3}{500\times0{\cdot}89} = 225$ A.

Field current $\quad I_f = 0{\cdot}01\times225 \cong 2$ A.

$\therefore$ Armature current $I_a = 225-2 = 223$ A.

It has been discussed earlier that a wave winding is preferred unless the current per parallel path exceeds 200 A with wave winding.

With wave winding, $a=2$. $\therefore$ Current per parallel path $=223/2=111{\cdot}5$ A.

It is below 200 A and therefore a wave winding is used for the armature.

Number of Conductors

Pole pitch $\quad \tau = \dfrac{\pi D}{p} = \dfrac{\pi\times0{\cdot}54}{6} = 0{\cdot}283$ m.

Flux per pole $\Phi = B_{av}\,\tau\,L = 0{\cdot}55\times0{\cdot}283\times0{\cdot}245 = 38{\cdot}2\times10^{-3}$ Wb.

Speed $\quad n = \dfrac{450}{60} = 7{\cdot}5$ r.p.s.

$\therefore$ Number of armature conductors

$$Z = \frac{475\times2}{38{\cdot}2\times10^{-3}\times7{\cdot}5\times6} = 553.$$

Number of Slots. 1. The slot pitch varies from 25 mm to 35 mm.

Therefore the number of slots lie between

$$\frac{\pi\times0{\cdot}54}{0{\cdot}035} = 48 \text{ and } \frac{\pi\times0{\cdot}54}{0{\cdot}025} = 68.$$

The number of slots per pole should be atleast 9.

$\therefore$ Minimum number of slots $=6\times9=54$.

Thus the range in which the number of slots lie is between 54 and 68.

3. In a wave winding the number of slots should not be a multiple of pole pairs. Therefore in this case the number of slots should not be a multiple of 3.

The numbers thus eliminated are : 54, 57, 60, 63 and 66.

The numbers from which the choice is to be made are :

55, 56, 58, 61, 62, 64, 65, 67, 68.

4. We have seen that in order to reduce flux pulsations the number of slots per pole should be an integer $+\frac{1}{2}$. If we do that the number of slots would become a multiple of pair of poles. This is not desirable in a wave winding as in such a case a dummy coil has to be used.

In order to reduce flux pulsations we should take the slots per pole arc as an integer $+\frac{1}{2}$.

Number of slots/pole arc $=\psi\, S/p = 0{\cdot}66/6 = 0{\cdot}11\, S$.

With $S=58$ and 67 we obtain the number of slots per pole arc as nearly an integer plus $\frac{1}{2}$.

Therefore in order to reduce flux pulsations we must either choose $S=58$ or 67 from the range of slots.

Winding. In order that voltage between adjacent segments should not exceed 15 V at no load,

Minimum number of coils required $C=\frac{E\times p}{15}=\frac{500\times 6}{15}=200$.

The number of slots and coils should be so chosen that there is no appreciable change in the number of conductors provided and the number of conductors calculated. We must bear in mind that the number of coil sides per layer in a wave winding should not be a multiple of pair of poles *i.e.* 3 in this case.

Let us work out the details with $S=58$ and $S=67$.

With S=58. Conductors per slot $Z_s=\frac{553}{58}=9{\cdot}56$.

But the number of conductors per slot should be an even integer. We can take $Z_s=10$.

Number of conductors used $Z=10\times 58=580$. Therefore there is only a small change (about 5%) in the value of conductors calculated and used.

We should now find out the coil sides per slot.

With $u=2$, $C=\frac{1}{2}\times 2\times 58=58$. $u=4$, $C=\frac{1}{2}\times 4\times 58=116$.

$u=6$, $C=\frac{1}{2}\times 6\times 58=174$. $u=8$, $C=\frac{1}{2}\times 8\times 58=232$.

$u=10$, $C=\frac{1}{2}\times 10\times 58=290$.

The minimum number of coils is 200 and therefore we cannot use $u=2$, $u=4$ and $u=6$ as their use results in number of coils smaller than the required. We also cannot use $u=6$ because in a wave winding, the number of coil sides per layer should not be a multiple of pair of poles, *i.e.* 3 in this case. Thus we are left with $u=8$ and $u=10$.

With $u=8$, we have $C=232$.

Turns per coil $T_c=\frac{Z}{2C}=\frac{580}{2\times 232}$, a mixed number (not an integer) and therefore we cannot use $u=8$.

(This should become apparent from the fact that the number of conductors per slot should be divisible by the number of coil sides per slot).

With $u=10$, we have $C=290$.

Turns per coil $T_c=\frac{580}{2\times 290}=1$. Therefore we get single turn coils.

We cannot use values of u higher than 10 as in that case the number of conductors per slot is not divisible by coil sides per slot and this would result in fractional turn coils which are not possible in practice.

Thus we have,

number of slots $S=58$, number of coils $C=290$, number of conductors $Z=580$, coil sides per slot $u=10$, conductors per slot $Z_s=10$, turns per coil $T_c=1$.

With S=67 :

Conductors per slot $Z_s=\frac{553}{67}=8{\cdot}26$.

Conductors per slot has to be an even integer and therefore we take $Z_s=8$.

Number of conductors used $Z=8\times 67=536$.

Let us find out the coil sides per slot.

With $u=2$; $C=\frac{1}{2}\times 2\times 67=67$; $u=4$, $C=\frac{1}{2}\times 4\times 67=134$;

$u=6$; $C=\frac{1}{2}\times 6\times 67=201$; $u=8$, $C=\frac{1}{2}\times 8\times 67=268$;

$u=10$, $C=\frac{1}{2}\times 10\times 67=$ [illegible]

The minimum number of coils is 200 and therefore we cannot use $u=2$ and $u=4$. Also $u=6$ is already ruled out.

We should use $u=8$, as with this, the number of coils is more than the minimum required and also the number of conductors per slot is divisible by coil sides per slot. With $u=10$ and higher, the number of conductors per slot is not divisible by number of coil sides per slot and therefore they are scored out.

With $u=8$, we have, $C=268$.

Turns per coil $T_c=\dfrac{536}{2\times 261}=1$. Thus we get single turn coils.

Hence we have,

number of slots $S=67$, number of coils $C=268$, number of conductors $Z=536$, conductors per slot $Z_s=8$, coil sides per slot $u=8$, turns per coil $T_c=1$.

Before finally deciding about the number of slots, we must apply the following checks :

1. **Slot loading.** Current per conductor $I_z=125$.

With $Z_s=10$, slot loading$=I_z Z_s=111{\cdot}5\times 10=1115$ A.

With $Z_s=8$, slot loading$=111{\cdot}5\times 8=892$ A.

The slot loading in both the cases is below the maximum allowable limit of 1500 ampere conductors.

2. **Commutator segment pitch.**

Diameter of commutator $D_c=0{\cdot}65\times 0{\cdot}54=0{\cdot}351$ m.

With $C=290$, pitch of segments $\beta_c=\dfrac{\pi\times 351}{290}=3{\cdot}8$ mm.

With $C=268$, $\beta_c=\dfrac{\pi\times 351}{268}=4{\cdot}1$ mm.

The minimum pitch of segments is 4 mm. Therefore we cannot use the design which gives $C=290$. (The design with $S=58$ and $C=290$ can also be adopted by using a commutator of little larger diameter).

Thus the design with $S=67$ is used.

Winding Layout.

Back pitch $y_b=\dfrac{2C}{p}\pm K=\dfrac{536}{6}\pm K=89$.

We take $y_b=89$ as with this $\dfrac{y_b-1}{u}$ is an integer and therefore we do not get split coils.

Commutator pitch $y_c=\dfrac{C\pm 1}{p/2}=\dfrac{268\pm 1}{3}=89$.

Resultant pitch $Y=2\,y_c=178$. Front pitch $y_f=Y-y_b=89$.

Slot Design. Current in each conductor $I_z=111{\cdot}5$ A.

Area of each conductor $a_z=\dfrac{111{\cdot}5}{4{\cdot}7}=23{\cdot}8$ mm^2.

Rectangular conductors are used for the machine.

The armature diameter is 0·54 m and therefore a slot depth of about 46 mm is used (See Table 9·7). The conductors are arranged as shown in Fig. 9·39. Let us work out the preliminary details of the slot in order to see that the arrangement is feasible.

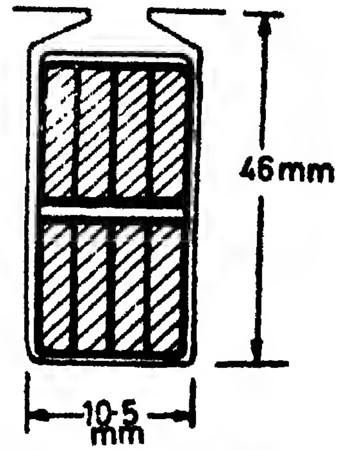


Fig. 9·39. Armature slot of Example 9·14.

Out of the slot depth of 46 mm the insulation including lip and wedge but excluding the conductor insulation etc. would take up a depth of 8·5 mm. The conductor insulation is 0·7 mm thick and therefore space taken by conductor insulation depthwise is $2\times2\times0·35=1·4$ mm.

Thus the space left for bare conductors depthwise

$$46-8·5-1·4=36·1$$

∴ Maximum depth of each conductor (bare) = 18 mm.

Using a conductor with depth = 17 mm.

Width of each conductor $=\frac{23·5}{18}=1·3$ mm.

Space taken by base conductors width wise $=1·3\times4=5·2$ mm.

Space taken by insulated conductors widthwise $=5·2+2\times4\times0·35=8$ mm.

Space taken by other insulation widthwise $=2·5$ mm. (See Table 9·6).

∴ Width of slot $=8+2·5=10·5$ mm

Net iron length $L_i=0·92(0·24\,5-2\times0·01)=0·207$ m.

Armature diameter at $\frac{1}{3}$ height from narrow end $=0·54-\frac{4}{3}\times0·046=0·479$ m.

Tooth width at $\frac{1}{3}$ height from narrow end

$$W_{t1/3}=\frac{\pi\times0·479}{67}-10·5\times10^{-3}=11·95\times10^{-3}\text{ m.}$$

Thus density in teeth at $\frac{1}{3}$ height from narrow end

$$B_{t1/3}=\frac{p\Phi}{\psi S W_t L_i}=\frac{6\times38·2\times10^{-3}}{0·67\times67\times11·95\times10^{-3}\times0·207}=2·06\text{ Wb/m}^2.$$

This is below the maximum allowable limit of 2·1 Wb/m².

The exact details of conductors and slot are :

Bare conductor cross-section $=1·3\times18$ mm².

Insulated conductor cross-section $=2\times18·7$ mm²

The dimensions of slot are :

Slot Width		
Insulated conductor	4 x 2	= 8·0 mm
Main insulation	2 x 1	= 2·0 mm
Slack		= 0·5 mm
Total slot width		W_s = 10·5 mm

Slot Depth		
Insulated conductor	2×18.7	$=37.4$ mm
Main insulation	3×1	$=3.0$ mm
Separator		$=1.0$ mm
Wedge and Lip		$=3.5$ mm
Slack		$=1.1$ mm
Total slot depth		$=46$ mm

Thus the slot size is 46×10.5 mm. Refer to Fig. 9·39 for insulation details.

Example 9·15. *A 500 kW, 460 V, 8 pole, 375 r.p.m. compound generator has an armature diameter of 1·1 m and a core length of 0·33 m. Design a symmetrical armature winding, giving the details of equalizers. The ampere conductors per metre are 34000. The internal voltage drop is 4 per cent of terminal voltage and the field current is 1 per cent of output current.*

The ratio of pole arc to pole pitch is 0·7. The voltage between adjacent segments at no load should not exceed 15 V and the slot loading should not exceed 1500 A. The diameter of commutator is 0·65 of armature diameter and the minimum allowable pitch of segments is 4 mm.

Make other suitable assumptions.

Solution.

Type of winding. Generated emf $E=V+I_a r_a=1.04\times460=478$ V.

Load current $=\dfrac{500\times1000}{460}=1087$ A.

Field current $=0.01\times1087\approx11$ A.

$\therefore$ Armature current $I_a=1087+11=1098$ A.

With wave winding : Current per parallel path $I_z=\dfrac{1098}{2}=549$ A.

This exceeds the limit of 200 A per parallel path. Therefore a simplex wave winding cannot be used for this machine.

With lap winding : Current per parallel path $I_z=\dfrac{1098}{8}=136$ A.

This is within limits and therefore a simplex lap winding is used for this machine.

Number of conductors

Armature ampere conductors per metre $ac=\dfrac{I_a}{a}\cdot\dfrac{Z}{\pi D}$

$\therefore$ Number of armature conductors $Z=a\cdot\dfrac{\pi D\,ac}{I_a}=8\times\dfrac{\pi\times1.1\times34000}{1098}=854.$

Number of Slots

1. The slot pitch varies from 25 mm to 35 mm. Therefore the number of slots vary from $\frac{\pi\times1\cdot1}{0\cdot035}=98$ to $\frac{\pi\times1\cdot1}{0\cdot025}=138$.

2. The number of slots per pole vary from 9 to 16.

$\therefore$ The minimum number of slots is $8\times9=72$ and the maximum number is $8\times16=128$.

Thus the slots should be taken from the range 98 to 128.

3. In order to have a symmetrical lap winding, the number of slots should be a multiple of pair of poles, *i.e.* 4 in this case.

Thus we can select the slots from the following

100, 104, 108, 112, 116, 120, 124, 128.

4. In order to reduce flux pulsations the slots per pole should be an integer $+\frac{1}{2}$. Thus the slots should be selected from 100, 108, 116, 124.

Further reducing the flux pulsations, we should make the slots per pole arc as an integer $+\frac{1}{2}$.

$S=108$ gives slots per pole arc approximately equal to an integer $+\frac{1}{2}$.

Winding.

In order that the voltage between adjacent segments be limited to 15 V at no load,

minimum number of coils $C=\frac{Ep}{15}=\frac{460\times8}{15}=245$.

With $S=108$:

Conductors per slot $=\frac{854}{108}=7\cdot92$.

But the number of conductors per slot has to be an even integer. Taking conductors per slot $Z_s=8$, there will be a difference of nearly 1 per cent in the calculated value and the value used for conductors.

$\therefore$ Number of conductors used $=8\times108=864$.

Now we have to find out the number of coil sides per slot.

With $u=2$, $C=108$; $u=4$, $C=216$;
$u=6$, $C=324$; $u=8$, $C=432$.

Thus we cannot use $u=2$ or $u=4$ as they give the number of coils lower than the required minimum.

We cannot use $u=6$ as in that case the number of conductors per slot is not divisible by the number of coil sides per slot.

Taking $u=8$ we have, $C=432$.

$\therefore$ Turns per coil $T_c=\frac{864}{2\times432}=1$.

Before we finally decide the winding details we must apply the following checks :

1. Slot Loading :

Current in each conductor $I_z=136$ A.

$\therefore$ Slot loading$=I_z \cdot Z_s=136\times8=1088$ ampere conductors.

This is within the limit of 1500 ampere conductors.

2. Commutator segment pitch

Diameter of commutator $D_c=0.65\times1.1=0.715$ m.

$\therefore$ Pitch of commutator segments $=\dfrac{\pi\times715}{432}=5.2$ mm.

(higher than the minimum allowable value of 4 mm)

Thus we have,

number of slots	$S=108$,	*number of coils*	$C=432$,
number of segments	$C=432$,		
number of coil sides per slot	$u=8$,		
number of conductors per slot	$Z_s=8$,		
number of turns per coil	$T_c=1$.		

Winding layout. Back pitch $y_b=\dfrac{2C}{p}\pm K=\dfrac{2\times432}{8}+K=105$.

We take $y_b=105$ as with this $\dfrac{y_b-1}{u}$ is an integer and therefore we do not get split coils.

Taking a progressive winding : front pitch $y_f=y_b-2=105-2=103$.

Commutator pitch $y_c=+1$ and total pitch $Y=+2$.

Equalizer Connections.

The number of rings should be between 10 to 20. In order to obtain symmetry the number of coils should be divisible by number of rings.

Taking number of rings $m=12$.

Number of tappings $=m\dfrac{p}{2}=12\times\dfrac{8}{2}=48$.

Equipotential pitch $Y_{eq}=\dfrac{2C}{p}=\dfrac{2\times432}{8}=108$ coils.

Phase pitch $Y_{ph}=\dfrac{2C}{m}=\dfrac{2\times432}{12\times8}=9$ coils.

The layout of the equalizer connections is

Ring No, $Y_{ph}\longrightarrow$	*I*	*II*	*III*	*IV*	*V*	*VI*	*VII*	*VIII*	*IX*	*X*	*XI*	*XII*
Coil No.	1	10	19	28	37	46	55	64	73	82	91	100
	109	118	127	136	145	154	163	172	181	190	199	208
$Y_{eq}\downarrow$	217	226	235	244	253	262	271	280	289	298	307	316
	325	334	343	352	361	370	379	388	397	406	415	424

DESIGN OF FIELD SYSTEM

9.31. Pole Design. The design of poles involves, the determination of area of cross-section of the poles, their height and the design of field windings.

9.31.1. Area of poles. Flux in pole body

Φ_p = leakage co-efficient × useful flux per pole $= C_l \times \Phi$.

Thus the determination of flux in the field poles involves the knowledge of leakage co-efficient. The leakage flux can be determined by the application of Eqn. 4.111 (page 176). This relationship involves the knowledge of total flux and the useful flux which can be determined if the dimensions of the main poles are known (See Eqns. 4.107 and 4.108). But these dimensions are not known at this stage. Thus, in order to proceed, we have to assume a suitable value of leakage co-efficient. Table 9.7 gives the approximate values of leakage coefficient.

Table 9.7. Leakage co-efficient.

Output kW	*Leakage co-efficient* C_l
50	1.12 to 1.25
100	1.11 to 1.22
200	1.10 to 1.20
500	1.09 to 1.18
1000	1.08 to 1.16

The flux density in the pole shank (pole body) can be assumed to lie between 1.2 to 1.7 Wb/m² for laminated pole.

$$\therefore \quad \text{Area of pole shank} \quad A_p = \frac{\Phi_p}{B_p} = \frac{C_l \Phi}{B_p} \qquad \ldots(9.29)$$

The length of pole is taken less than that of armature in order to permit end play and avoid magnetic centring. The difference in length is about 10 to 15 mm.

$$\text{Length of pole } L_p = L - (0.001 \text{ to } 0.015) \qquad \ldots(9.30)$$

$$\therefore \quad \text{Width of pole shank (body)} \quad b_p = \frac{A_p}{L_{pi}} = \frac{\Phi_l}{B_p L_{pi}} \qquad \ldots(9.31)$$

where L_{pi} = net iron length of pole core $= 0.9\, L_p$.

However, for machines using laminated core for poles, the axial length of poles is equal to that of armature core and therefore $L_{pi} = L_i$.

9.31.2. Height of pole. The height of pole is decided by the mmf to be provided on the pole at full load. So the first thing would be to find out the field mmf required at full load. In order to do so we must know the open circuit characteristics (magnetization curve) of the machine and in order to draw the magnetization curve for the machine, we must know the mmf required for the pole and the yoke, which can only be known if the height of the pole is known. Thus the field mmf required at full load and the height of pole are inter-related and can be known only when one of them is known. But at present neither of the two is known.

In order to proceed with the design we assume the value of field mmf at full load.

Full load field mmf. In order to prevent the effects of armature reaction becoming too excessive, the field system is so designed that the mmf developed by the field coils is sufficiently powerful in comparison with the mmf developed by the armature at full load.

In the case of all logically designed machines, there is thus a certain minimum value for the ratio of field mmf at full load, AT_{fl}, to armature mmf at full load AT_a, that is regarded as a safe limit.

Depending upon the duty which the machine has to perform, we should decide upon the value of this ratio. The normal designs are based upon a ratio,

$$\frac{AT_{fl}}{AT_a}=\frac{\text{field mmf at full load}}{\text{armature mmf at full load}}=1{\cdot}1 \text{ to } 1{\cdot}25. \qquad \ldots(9{\cdot}32)$$

9·32 Tentative design of field winding. Taking the case of a cylindrical coil, the end surfaces may be neglected while considering cooling. The inner and outer surfaces may be considered effective for cooling. Let,

AT_{fl}=mmf developed by field winding at full load

h_f=height of winding, m ; d_f=depth of winding, m ;

S=cooling surface of field coil, m^2 ; L_{mt}=length of mean turn of field winding, m ;

R_f=resistance of each field coil, Ω ; T_f=number of turns in each field coil ;

a_f=area of each conductor of field winding, m^2 ;

I_f=current in the field winding, A ;

δ_f=current density in the field winding, A/mm^2 ;

Q_f=copper loss in each field coil, W ;

q_f=permissible loss per unit winding surface for normal temperature rise, W/m^2 ;

S_f=copper space factor ; ρ=resistivity, Ω m.

Cooling surface of field winding, neglecting top and bottom surfaces $S=2L_{mt}\,h_f$.

∴ Permissible copper loss in each field coil$=Sq_f=2L_{mt}\,h_f\,q_f$ ...(9·33)

Area of cross-section of field coil$=h_f\,d_f$ ...(9·34)

Area of copper in each field coil$=S_f\,h_f\,d_f$ ...(9·35)

But area of copper in each field coil is also equal to $T_f\,a_f$

or $$T_f\,a_f=S_f\,h_f\,d_f \qquad \ldots(9{\cdot}36)$$

Copper loss in each field coil $$Q_f=I_f^2\,R_f=I_f^2\,\frac{T_f\,\rho\,L_{mt}}{a_f}=\frac{(a_f\,\delta_f)^2\,T_f\,\rho\,L_{mt}}{a_f}$$

$$=\delta_f^2\,\rho\,L_{mt}\,T_f\,a_f \qquad \ldots(9{\cdot}37)$$

But $L_{mt}\,T_f\,a_f$=volume of copper in each field coil

∴ Copper loss $Q_f=\delta_f^2\,\rho\times$volume of copper.

This incidentally proves that *for a particular volume of copper, the copper loss is proportional to square of current density.*

From Eqns. 9·36 and 9·37, $Q_f=\delta_f^2\,\rho\,L_{mt}\,S_f\,h_f\,d_f$ ...(9·38)

In order that the temperature rise in the field coil does not exceed the specified limits, the copper loss should be equal to the permissible loss as given by Eqn. 9·33.

Equating Eqns. 9·33 and 9·38, we have : $2\,L_{mt}\,h_f\,q_f=\delta_f^2\,\rho\,L_{mt}\,S_f\,h_f$

or $$\delta_f=\sqrt{\frac{2\,q_f}{\rho\,S_f\,d_f}} \qquad \ldots(9{\cdot}39)$$

Mmf per metre height of field winding$=\dfrac{AT_{fl}}{h_f}=\dfrac{I_f\,T_f}{h_f}$

$$=\frac{\delta_f\,a_f\,T_f}{h_f}=\frac{\delta_f\,S_f\,d_f\,h_f}{h_f}$$ (Substituting the value of $T_f\,a_f$ from Eqn. 9·36)

$$=\delta_f S_f d_f = \sqrt{\frac{2 q_f}{\rho S_f d_f}} \cdot S_f d_f \qquad \text{(from Eqn. 9·39)}$$

$$=\sqrt{\frac{2 q_f S_f d_f}{\rho}} \qquad \ldots(9·40)$$

$$=10^4 \times \sqrt{q_f S_f d_f} \qquad \ldots(9·41)$$

with $\rho = 2\times10^{-8}$ Ω-m and q_f expressed in W/m² and d_f expressed in m.

Therefore, for a particular temperature rise, the mmf per metre of height of field winding is proportional to under root of product of three factors :

(i) q_f—permissible loss per m² of cooling surface excluding the top and the bottom surfaces. Its value may be taken as 700 W/m².

(ii) S_f—copper space factor. Its value may be as low as 0·4 for small wires increasing to 0·65 for large round wires and to 0·75 for large rectangular conductors.

(iii) d_f—depth of winding. It should not exceed 40 to 50 mm otherwise the temperature difference between the hot spot and outside would be very large.

Table 9·8 gives the usual values of winding depth.

Table 9·8. Depth of shunt field winding.

Armature diameter m	*Winding depth* mm
0·20	30
0·35	35
0·50	40
0·65	45
1·00	50
1·00 and above	55

The height of field winding h_f can be calculated after assuming suitable values of d_f, S_f and q_f.

Height of field $h_f = \dfrac{AT_{fl}\times10^{-4}}{\sqrt{q_f S_f d_f}}$. ...(9·42)

Total height of pole

$h_{pl} = h_f + h_s$ + height taken by insulation and height wasted due to curvature of yoke.

The height of pole shoe, h_s, is assumed to be 0·1 to 0·2 of height of pole.

The height of insulation and the space wasted because of curvature may be taken as about 0·1τ to 0·15τ depending upon the size of the machine.

9·33. Yoke. The dimensions of yoke are determined by the value of flux Φ_y carried by the yoke. The leakage coefficient for yoke is a little higher than that for the poles. From Fig. 9·40 it is clear that the yoke carries half of the total flux.

Flux in the yoke Φ_y = leakage co-efficient × ½ useful flux per pole = ½ $C_l\,\phi$.

The flux density in cast steel yokes is normally equal to 1·2 Wb/m² while in laminated yokes it is about 1·5 Wb/m².

Area of yoke $A_y=\Phi_y/B_y$ and depth of yoke $dy=\Phi y/B_yL_{yi}$ ...(9·43)

where B_y=flux density in yoke and L_{yi}=net axial length of yoke

In the case of cast steel yokes, the net axial length of yoke is equal to the actual axial length of yoke.

For machines with laminated yokes the net axial length of yoke is equal to net iron length of armature.

9·33. **Magnetic circuit**. The fundamental relationships for magnetic circuit calculations have aleady been discussed in Chapter 4. The magnetic circuit for a 6 pole d.c. machine is illustrated in Fig. 9·40, which shows that the magnetic circuit per pair of poles comprises the yoke, the pole the air gap, the armature teeth and the armature core below the teeth.

Mmf required for air gap AT_g=800,000 B_g K_g l_g. (See Eqn. 4·37 page 128)

The method of calculating mmf required for tapered teeth is explained in Art. 4·4·3. page 131.

Mmf for teeth $AT_t=at_t\times d_s$

where at_t=mmf per metre corresponding to flux density at ⅓ height from narrow end and d_s=depth of slot.

Mmf required for armatute core $AT_c=at_c\times l_c$ where at_c=mmf per metre corresponding to flux density in core, and

$$l_c=\text{length of flux path in core}=\frac{\pi(D-2d_s-d_c)}{2}. \qquad ...(9·44)$$

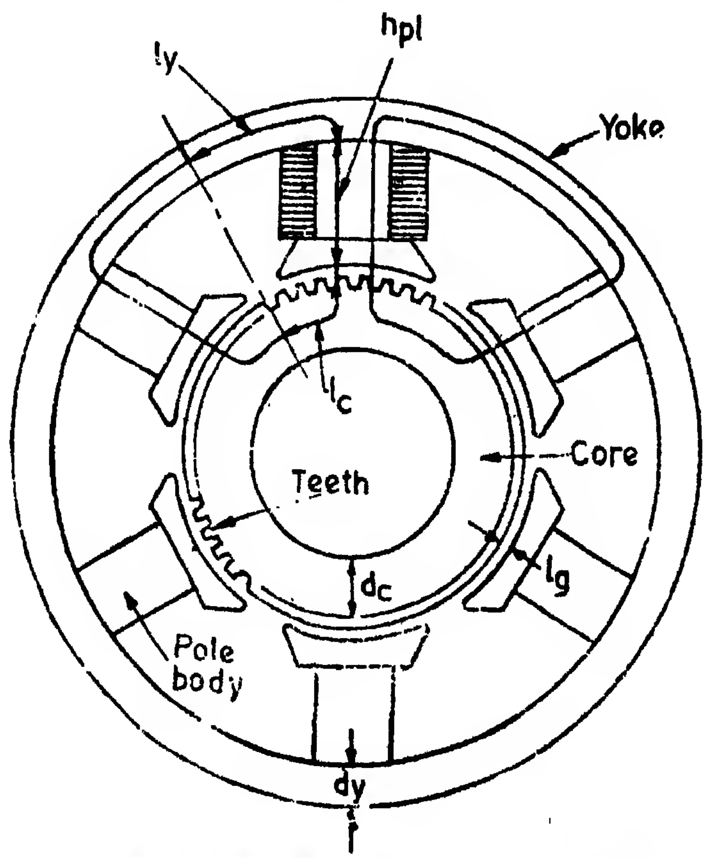


Fig. 9·40. Magnetic circuit of d.c. machine.

Mmf required for pole body $AT_p=at_p\times h_{pl}$

where at_p=mmf per metre corresponding to flux density in pole body.

Mmf required for yoke $AT_y = at_y \times l_y$

where at_y = mmf per metre corresponding to flux density in yoke,

and l_y = length of flux path in yoke $= \dfrac{\pi(D+2\,l_g+2\,h_{ss}+d_y)}{2p}$ ...(9·45)

The values of at_t, at_c, at_p and at_y are taken from B-H curves of the materials used.

Total mmf per pole at no load and normal voltage

$$AT_{fo} = AT_g + AT_t + AT_c + AT_p + AT_y.$$

Example 9·16. *A multi-pole d.c. machine has the following dimensions :*

Cross-section of pole body = 0·08 m²; height of pole = 0·25 m; cross section of yoke = 0·05 m²; mean flux path in yoke = 0·9 m (pole to pole); cross section of armature core = 0·04 m²;

length of flux path in core = 0·45 m (pole to pole); area of pole face = 0·12 m²; length of air gap = 5 mm.

There are 12 slots per pole and the width of each tooth is 15 mm (40 mm long). The length of machine is 0·33 m and the ratio of pole arc to pole pitch is 0·67.

Find the mmf per pole to give a flux of 0·1 Wb. The relative permeability for teeth is 15 and for rest of magnetic circuit is 1200. Assume a stacking factor of 0·9. Neglect leakage.

Solution. Mmf required for air gap $AT_g = \Phi_g \dfrac{l_g}{\mu_0 A_g} = \dfrac{0{\cdot}1 \times 0{\cdot}005}{4\pi \times 10^{-7} \times 0{\cdot}12} = 3320$ A.

Mmf required for pole $AT_p = \Phi_p \dfrac{h_{pl}}{\mu_r \mu_0 A_p} = \dfrac{0{\cdot}1 \times 0{\cdot}25}{1200 \times 4\pi \times 10^{-7} \times 0{\cdot}08} = 200$ A.

Flux in yoke $\Phi_y = 0{\cdot}1/2 = 0{\cdot}05$ Wb. Length of flux path in yoke $l_y = 0{\cdot}9/2 = 0{\cdot}45$ m

Mmf required for yoke $AT_y = 0{\cdot}05 \times \dfrac{0{\cdot}45}{1200 \times 4\pi \times 10^{-7} \times 0{\cdot}05} = 290$ A.

Flux in core $\Phi_c = 0{\cdot}1/2 = 0{\cdot}05$ Wb. Length of flux path in core $l_c = 0{\cdot}45/2 = 0{\cdot}225$ m.

Mmf required for core $AT_c = 0{\cdot}05 \times \dfrac{0{\cdot}225}{1200 \times 4\pi \times 10^{-7} \times 0{\cdot}04} = 180$ A.

Teeth per pole arc $= \psi \dfrac{S}{p} = 0{\cdot}67 \times 12 = 8.$

Flux in each tooth $= \dfrac{0{\cdot}1}{8} = 0{\cdot}0125$ Wb.

Mmf required for teeth $AT_t = 0{\cdot}0125 \times \dfrac{40 \times 10^{-3}}{15 \times 4\pi \times 10^{-7} \times 0{\cdot}9 \times 0{\cdot}33 \times 0{\cdot}15} = 1200$ A.

$\therefore$ Total mmf required per pole = 3320 + 200 + 290 + 180 + 1200 = 5190 A.

9·34. Magnetization curve (O.C.C.). The emf is directly proportional to flux if the speed of machine is kept constant. In order to construct the magnetization curve, it is necessary to calculate the mmf per pole required to establish different values of useful flux. It is normally sufficient to carry out the calculations for different values of flux ranging from 70 per cent to about 120 per cent of the value of the flux at rated voltage. Below 70 percent of the useful flux at rated value, the reluctance of iron parts is insignificant in comparison with that of air gap and therefore the curve merges into a straight line. The values of mmf per pole are normally calculated for voltage between 70 to 120 percent of normal rated voltage in tabular form and the magnetization curve is plotted from the results obtained.

9.35. Design of shunt field winding. Shunt field windings for small machin employ coils of a few hundred turns of small section varnish covered wire. The insulate conductors are encapsuled in an epoxy resin moulding and fixed to the pole in a singl operation. In large machines, the field current is large and therefore rectangular conducto may be used.

With fan ventilated machines, and high speed machines, sectionalized coils may b used. With these coils, instead of having conductors in one solid mass, they are woun in sections. These sections are separated by distance pieces so that ventilation spaces ar formed between them. The sectionalized coils, therefore, result in improved ventilatio conditions.

In shunt machines, the entire winding space along the height of pole is taken up b the winding while in compound machines, 80% of the winding space is taken up by shur field and the rest 20% by series field.

Design Procedure.

1. Assume a suitable depth d_f, for the winding from Table 9.8.

2. Calculate the length of mean turn. The length of mean turn is

$$L_{mt}=2(L_p+b_p)+2d_f \qquad \ldots(9.46$$

3. In order to allow for voltage regulation, in generators assume that 15 to 20 pe cent of rated voltage is absorbed by the field rheostat. In the case field controlled motor this allowance depends upon range of speed control.

∴ Voltage across the shunt field winding=(0.8 to 0.85) V and voltage across eac shunt field coil is :

$$E_f=\frac{(0.8 \text{ to } 0.85)\,V}{p} \qquad \ldots(9.47$$

as there are as many shunt field coils as the number of poles and they are all connected in series.

4. Resistance of each field coil $R_f=\dfrac{T_f\,\rho\,L_{mt}}{a_f}=\dfrac{E_f}{I_f}$

or area of shunt field conductor $a_f=\dfrac{I_fT_f\,\rho\,L_{mt}}{E_f}=\dfrac{AT_f\,\rho\,L_{mt}}{E_f}$ ...(9.48

Thus the area of shunt field conductor is known from Eqn. 9.48.

5. Choose a suitable cross-section for the conductor. For small cross-section standar round wire sizes should be used as given in Chapter 17. For larger cross-sections square or rectangular conductors should be used.

6. Calculate the winding height.

The space for winding along the radial height is,

$h_f=h_{pl}$−height of shoe−insulation and clearance

$$=h_{pl}-(0.1 \text{ to } 0.2)\,h_{pl}-(0.1 \text{ to } 0.15)\,\tau \qquad \ldots(9.49$$

7. Number of turns provided $T_f=\dfrac{h_f\times d_f}{\delta_f}$ ...(9.50

where S_f=space factor for the winding. The space factor for round insulated conductor is 0.8 $(d/d_1)^2$ where d and d_1 are diameters of bare and insulated conductors respectively. (See pages 200 & 201).

In order to account for the additional space required for winding insulation, the space factor for the entire winding is assumed as $S_f = 0.75\ (d/d_1)^2$. The increase in diameter on account of varnish coating on the conductors is assumed as 0.1 mm.

8. Find the resistance of each coil with T_f turns and area of cross-section of conductors provided.

Resistance of each field coil $R_f = \dfrac{T_f\, \rho\, L_{mt}}{a_f}$.

9. Find the value of the shunt field current with the above value of resistance.

Field current $I_f = E_f / R_f$.

10. Check for current density in the shunt field winding. Current density $\delta_f = I_f / a_f$.

The value of current density in shunt field should be between 1.2 to 2.5 A/mm². In case of windings with air ducts it may go upto 3.5 A/mm².

11. Find the mmf provided $AT_f = I_f T_f$.

Check whether the mmf provided on the shunt field is nearly equal to the value required. If it is less than the required value, the depth of winding should be increased but if it is in excess the depth should be decreased. The calculations should be repeated till the value of mmf provided is nearly equal to the value required.

12. Calculate the I^2R loss in each field coil.

I^2R loss in each field coil $Q_f = I_f^2 R_f$.

13. Calculate the cooling surface of the coil.
Cooling surface of each coil $S = 2L_{mt}\,(a_f + d_f)$. See Fig. 3.59 page 112.

14. Assume a suitable value of cooling co-efficient from Table 3.6. (Page 111)

Cooling co-efficient $c = \dfrac{0.14 \text{ to } 0.16}{1 + 0.1\, V_a}$.

15. Calculate the temperature rise of the coil.

Temperature rise $\theta_m = \dfrac{Q_f\, c}{S}$. (See Eqn. 3.52 page 76)

The value of temperature rise should be within the limits specified. If the temperature rise exceeds the specified limits, assume a greater depth for winding and repeat the calculations.

9.36. Design of series field. The series field winding is wound with rectangular conductors. The conductors may be flat wound or wound on edge. The insulation between the turns of series field depends mainly upon whether the coil is flat-wound or wound on edge. In flat wound copper coils, the inter turn insulation consists of a tape slightly wider than the copper strip. The tape is composed of dry asbestos paper with thickness of asbestos varying between 0.2 to 0.4 mm in thickness. In flat-wound coils, which usually consist of two or more sections, the insulation between sections is a washer of about 0.1 to 0.5 mm thickness, consisting of micanite or of a composite material made up from asbestos paper and woven glass cloth polyester varnish bonded and pressed to size.

In order to provide the main insulation, the series field conductors along with those of shunt field are epoxide moulded or cast resin insulated (encapsulated).

The mmf to be provided on series field at full load usually lies between 15 to 25 per cent of armature mmf and for normal machines it may be taken as 20 per cent.

Mmf of series field per pole $AT_s = (0.15 - 0.25)\, AT_a$.

$\therefore$ Number of series field turns $T_s = AT / I_s$

Area of series field conductors $a_s = I_s / \delta_s$.

The current density used for the series field has a somewhat higher value than that for the shunt field owing to better cooling conditions.

SOLVED EXAMPLES

Design of field coils

The method for solving problems on electromagnet coils is given in Art. 5·4 page 198. A few examples have also been given. The student is advised to first go through the examples given there before solving problems on field coils of d.c. machines. The equations derived earlier for coils of electromagnets are valid for d.c. machine field coils. In fact there is no fundamental difference between them. Refer to Fig. 3.59 (page 112) for reactangular coils and to Fig. 5 6 (page 198) for circular coils.

In the following examples, area a_f has been expressed in mm² and the rest of the dimensions are in m. The value of ρ used is in Ω/m and mm². While using different units proper conversions should be made.

Example 9·17. *The following particulars refer to the shunt field coil for a 440 V, 6 pole, d.c. generator :*

Mmf per pole = 7000 A ; depth of winding = 50 mm ; length of inner turn = 1·1 m ; length of outer turn = 1·4 m ; loss radiated from outer surface excluding ends = 1400 W/m² ; space factor = 0·62 ; resistivity = 0·02 Ω/m and mm².

Calculate (a) the diameter of wire (b) length of coil (c) number of turns and (d) exciting current.

Assume a voltage drop of 20 per cent of terminal voltage across the field regulator.

Solution.

Voltage across the shunt field winding $=0{\cdot}8\times440=352$ V.

Voltage across each field coil $E_f=352/6=58{\cdot}7$ V.

Length of mean turn $\quad L_{mt}=\dfrac{L_o+L_i}{2}=\dfrac{1{\cdot}4+1{\cdot}1}{2}=1{\cdot}25$ m.

Area of field conductor $\quad a_f=\dfrac{AT_f\,\rho\,L_{mt}}{E_f}=\dfrac{700\times0{\cdot}02\times1{\cdot}25}{58{\cdot}7}=2{\cdot}98$ mm².

Diameter of bare conductor $d=1{\cdot}95$ mm.

Number of turns $T_f=\dfrac{S_f\,h_f\,d_f}{a_f}\times10^6=\dfrac{0{\cdot}62\,h_f\times0{\cdot}05}{2{\cdot}98}\times10^6=1{\cdot}04\times10^4\,h_f$ ...(i)

Area of outer surface excluding ends $=L_o\,h_f=1{\cdot}4\,h_f$.

∴ Permissible loss $Q_f=1400\times1{\cdot}4\,h_f=1960\,h_f$.

Field current $I_f=\dfrac{Q_f}{E_f}=\dfrac{1960\,h_f}{58{\cdot}7}=33{\cdot}4\,h_f$.

Now $\quad AT_f=I_f\,T_f=33{\cdot}4\,h_f\,T_f=7000$ (given) $\quad$ or $\quad T_f=210/h_f$ ...(ii)

Equating (i) and (ii) we have,

$$T_f^2=1{\cdot}04\times10^4\,h_f\times\frac{210}{h_f}=2{\cdot}184\times10^6.$$

∴ Number of turns in each field coil $T_f=1475$.

Resistance of each field coil $R_f=T_f\,\dfrac{\rho\,L_{mt}}{a_f}=\dfrac{1475\times0{\cdot}02\times1{\cdot}25}{2{\cdot}98}=12{\cdot}4$ Ω

Field current $\quad I_f=E_f/R_f=58{\cdot}7/12{\cdot}4=4{\cdot}73$ A.

Hence, height of field coil $\quad h_f=\dfrac{T_f}{1{\cdot}04\times10^4}=\dfrac{1475}{1{\cdot}04\times10^4}=0{\cdot}142$ m.

Example 9·18. *A shunt field coil has to develop an mmf of 9000 A. The voltage drop in the coil is 40 V, and the resistivity of round wire used is 0·021 Ω/m and mm². The depth of winding is 35 mm approximately and the length of mean turn is 1·4 m. Design a coil so that the power dissipated is 700 W/m² of the total coil surface (i.e. outer, inner, top and bottom). Take the diameter of the insulated wire 0·2 mm greater than that of bare wire.*

Solution.

Area of conductor $a_f=\dfrac{AT_f\,\rho\,L_{mt}}{E_f}=\dfrac{9000\times0{\cdot}021\times1{\cdot}4}{40}=6{\cdot}6\text{ mm}^2.$

Diameter of bare conductors $d=2{\cdot}9$ mm.

Diameter of insulated conductor $d_1=2{\cdot}9+0{\cdot}2=3{\cdot}1$ mm.

Space factor $S_f=0{\cdot}75(d/d_1)^2=0{\cdot}75(2{\cdot}9/3{\cdot}1)^2=0{\cdot}66.$

Total winding area $A_w=d_f\times h_f=0{\cdot}035\,h_f.$

$\therefore$ Total area of conductors$=0{\cdot}66\times0{\cdot}035\,h_f=0{\cdot}0231\,h_f.$

Winding area is also equal to $T_f\,a_f\times10^{-6}$ (as a_f is expressed in mm²).

$$\therefore\quad T_f\,a_f\times10^{-6}=0{\cdot}0231\,h_f \quad\text{or}\quad T_f=\frac{0{\cdot}0281\times10^6}{6{\cdot}6}\,h_f=3{\cdot}5\times10^3\,h_f. \qquad ...(i)$$

Total dissipating area of a coil considering all surfaces

$$=2\,L_{mt}\,(h_f+d_f)=2\times1{\cdot}4\,(h_f+0{\cdot}035)=2{\cdot}8\,h_f+0{\cdot}098$$

$$\therefore\quad \text{Permissible loss } Q_f=700(2{\cdot}8\,h_f+0{\cdot}098)=1960\,h_f+68{\cdot}6. \qquad ...(ii)$$

$$\text{Permissible loss}\quad Q_f=I_f^2\,R_f=\frac{E_f^2}{R_f}=\frac{E_f^2\,.\,a_f}{T_f\,\rho\,L_{mt}}$$

$$=\frac{(40)^2\times6{\cdot}6}{T_f\times0{\cdot}021\times1{\cdot}4}=\frac{0{\cdot}36\times10^6}{T_f} \qquad ...(iii)$$

Equating (*ii*) and (*iii*), we have,

$$1960\,h_f+68{\cdot}6=\frac{0{\cdot}36\times10^6}{T_f}\quad\text{or}\quad T_f=\frac{0{\cdot}36\times10^6}{1960\,h_f+68{\cdot}6} \qquad ...(iv)$$

Equating (*i*) and (*iv*)

$$3{\cdot}5\times10^3\,h_f=\frac{0{\cdot}36\times10^6}{1960\,h_f+68{\cdot}6}\quad\text{or}\quad h_f=0{\cdot}15\text{ m}.$$

$$\therefore\quad \text{Number of turns } T_f=\frac{0{\cdot}36\times10^6}{1960\times0{\cdot}15+68{\cdot}6}=933.$$

$$\text{Permissible loss}\quad Q_f=\frac{0{\cdot}36\times10^6}{933}=386\text{ W}.$$

$$\therefore\quad \text{Field current}\quad I_f=\frac{Q_f}{E_f}=\frac{386}{40}=9{\cdot}65\text{ A}.$$

Example 9·19. *A 8 pole, 500 V d.c. shunt generator with all the field coils connected in series requires an mmf of 5000 A/pole. The poles are of rectangular dimensions, 120×200 mm² and the available winding cross section is 120×25 mm². Determine :*

(a) the cross-section area of wire, (b) the number of turns,

(c) the dissipation in W/m² based upon the area of the outside surface and the two end surfaces of the coil.

A conductor of round cross-section is used. The resistivity is 0·02 Ω/m and mm² and the insulation on the wires increases the diameter by 0·2 mm. Allow a voltage drop of 50 V in the field regulator.

Solution.

Voltage across the shunt field winding $=500-50=450$ V.

Voltage across each field coil $E_f=450/8=56{\cdot}25$ V.

Length of mean turn $L_{mt}=2(L_p+b_p)+4d_f=2(200+120)+4\times25=740$ mm $=0{\cdot}74$ m.

(a) Area of shunt field conductor $a_f=\dfrac{AT_f\,\rho\,L_{mt}}{E_f}=\dfrac{5000\times0{\cdot}02\times0{\cdot}74}{56{\cdot}25}=1{\cdot}32$ mm²

(b) Diameter of bare conductor $d=1{\cdot}3$ mm.

Diameter of insulated conductor $d_1=1{\cdot}30+0{\cdot}1=1{\cdot}4$ mm.

Space factor $S_f=0{\cdot}75\,(d/d_1)^2=0{\cdot}75(1{\cdot}3/1{\cdot}4)^2=0{\cdot}65$

Space for conductors $=0{\cdot}65\times120\times25=1950$ mm².

∴ Number of turns $T_f=1950/1{\cdot}32=1480$

Resistance of each field coil $R_f=\dfrac{1480\times0{\cdot}02\times0{\cdot}74}{1{\cdot}32}=16{\cdot}6\ \Omega$.

Shunt field current $I_f=\dfrac{56{\cdot}25}{16{\cdot}6}=3{\cdot}39$ A.

∴ Mmf provided $AT_f=3{\cdot}39\times1480=5017$ A.

(c) I^2R loss in each field coil $Q_f=I_f^2\,R_f=(3{\cdot}39)^2\times16{\cdot}6=190{\cdot}8$ W.

Length of outer turn $L_0=L_{mt}+2d_f=690+4\times25=790$ mm $=0{\cdot}79$ m.

Cooling area of outer cylindrical surface, and the top and the bottom surfaces

$$S=L_0\,h_f+2\,L_{mt}\,d_f \qquad \text{(See Fig. 3·59 page 112)}$$
$$=0{\cdot}79\times0{\cdot}12+2\times0{\cdot}74\times0{\cdot}025=0{\cdot}132\ \text{m}^2.$$

Dissipation $q_f=Q_f/S=190{\cdot}8/0{\cdot}132=1445$ W/m².

Example 9·20. *A 50 h.p. 4 pole 480 V, 600 r.p.m. shunt motor has a wave wound armature with 770 conductors. The leakage factor for the poles is 1·2. The poles are to be of circular in cross-section, the field coils are 70 mm thick and produce an mmf of 10,000 A per pole. The flux density in the poles is 1·5 Wb/m². Calculate the*

(a) diameter of pole, (b) diameter of field wire, (c) length of field coil,
(d) turns per pole and, (e) field current.

Keep 20% of voltage applied to shunt field coil in reserve for speed control.

Assume that the thickness of insulation over wires is 0·125 mm, permissible loss is 1600 W/m² of external cylindrical surface. Resistivity of copper is 0·02 Ω/m and mm².

Solution. (a) Speed $n=\dfrac{600}{60}=10$ r.p.s.

∴ Useful flux per pole $\Phi=\dfrac{E\times a}{Znp}=\dfrac{480\times2}{770\times10\times4}=31{\cdot}2\times10^{-3}$ Wb.

Flux in pole body Φ_p = leakage factor × useful flux per pole

$$=1{\cdot}2\times31{\cdot}2\times10^{-3}=37{\cdot}4\times10^{-3}\ \text{Wb}.$$

∴ Area of each pole $A_p=\Phi_p/B_p=37{\cdot}4\times10^{-3}/1{\cdot}5=25\times10^{-3}$ m².

or diameter of each pole $D_p=\left(\dfrac{4}{\pi}\times25\times10^{-3}\right)^{1/2}=0{\cdot}18$ m.

(b) Length of mean turn $L_{mt}=\pi(D_p+d_f)=\pi(0.18+0.07)=0.785$ m.

Voltage across each shunt field coil $E_f=0.8\times480/4=96$ V.

$\therefore$ Area of conductor $a_f=\dfrac{AT_f\,\rho\,L_{mt}}{E_f}=\dfrac{10000\times0.02\times0.785}{96}=1.64\text{ mm}^2$.

Diameter of bare conductor $d=1.45$ mm and diameter of insulated conductor $d_1=1.45+0.25=1.70$ mm.

(c) and (d) Outer diameter of the field coil $=0.18+2\times0.07=0.32$ m.

Length of outer turn $L_0=\pi\times0.32\simeq1.0$ m

Outer cylindrical surface $=h_f\,L_0=h_f\times1=h_f$. $\therefore$ Permissible loss $Q_f=1600\,h_f$.

Field current $I_f=Q_f/E_f=1600\,h_f/96=16.7\,h_f$.

Now, field mmf $=I_fT_f=16.66\,h_fT_f=10000$ (given) $\therefore$ $h_fT_f=\dfrac{10000}{16.7}=600$. ...(i)

Space factor $S_f=0.75\,(d/d_1)^2=0.75(1.45/1.7)^2=0.546$

$$\therefore\quad T_f=\frac{S_f\,h_f\,d_f\times10^6}{a_f}=\frac{0.546\times h_f\times0.07\times10^6}{1.64}=23.3\times10^3\,h_f \quad ...(ii)$$

From (i) and (ii), we have : $T_f^2=600\times23.3\times10^3$ or $T_f=3740$

From (i), height of field winding $h_f=600/3740=0.16$ m.

(e) Resistance of field coil $R_f=3740\times\dfrac{0.02\times0.785}{1.64}=35.8\ \Omega$.

Field current $I_f=E_f/R_f=96/35.8=2.68$ A.

Example 9·21. *Calculate the size of conductor and number of turns for the field coil of a 6 pole, 460 V, d.c. shunt motor. The coil is to supply an mmf of 4000 A at working temperature, the length of inside turn is 0·74 m, the length available for winding is 0·13 m, the space factor for the winding is 0·52, and the permissible dissipation from external surface excluding the ends is 1200 W/m². Solution should not be attempted by assuming a numerical value for winding depth. Resistivity of conductors is 0·02 Ω/m and mm².*

Keep 15% of applied voltage as reserve for speed control.

Solution.

Voltage across shunt field winding $=0.85\times460=390$ V.

Voltage across each field coil $E_f=390/6=65$ V. Let the depth of winding be d_f

$\therefore$ Length of mean turn $L_{mt}=L_i+4\,d_f=0.74+4\,d_f$,

and length of outer turn $L_0=L_{mt}+2\,d_f=0.74+8\,d_f$.

External dissipating surface of each coil excluding top and bottom surface

$$S=L_0\,h_f=(0.74+8\,d_f)\times0.13=0.0961+1.04\,d_f \quad ...(i)$$

$\therefore$ Loss dissipated $Q_f=1200(0.0962+1.04\,d_f)$

$$=115+1248\,d_f \quad ...(ii)$$

Also
$$Q_f=I_f^2R_f=I_f^2\frac{T_f\,\rho\,L_{mt}}{a_f}=\frac{I_f(I_fT_f)\,\rho\,L_{mt}}{a_f}$$

$$=\frac{I_f}{a_f}\times4000\times0.02(0.74+4\,d_f)=\frac{I_f}{a_f}(59.2+320\,d_f) \quad ...(iii)$$

where a_f is expressed in mm².

Total area of conductors $=S_f\,h_f\,d_f=0.52\times0.13\,d_f=0.0676\,d_f$ ...(iv)

But, total conductor area $=T_f\,a_f\times10^{-6}$

$\therefore$ Area of each conductor $a_f = \dfrac{0.0676^* d_f \times 10^6}{T_f}$ ...(v)

Substituting this, we have value in (iii)

loss dissipated $Q_f = (59.2 + 320\ d_f)\dfrac{I_f T_f}{0.0676\ d_f \times 10^6} = \dfrac{59.2 + 320\ d_f}{0.0676\ d_f \times 10^6} \times 4000$

$$= \frac{3.5 \times 18.94\ d_f}{d_f} \quad ...(vi)$$

Equating (ii) and (vi), $115 + 1248\ d_f = \dfrac{3.5 + 18.94\ d_f}{d_f}$

$\therefore$ Depth of winding $d_f = 0.0273$ m $= 27.3$ mm.

From (ii) loss dissipated $Q_f = 115 + 1248 \times 0.0273 = 149$ W.

Length of mean turn $L_{mt} = 0.74 + 2\ d_f = 0.74 + 2 \times 0.0273 = 0.795$ m.

Area of field winding conductor $a_f = I_f T_f\ \dfrac{\rho\ L_{mt}}{E_f} = \dfrac{4000 \times 0.02 \times 0.795}{65} = 0.978$ mm^2

From (vi), number of field turns $T_f = \dfrac{0.0676\ d_f \times 10^6}{a_f} = \dfrac{0.0676 \times 0.0273 \times 10^6}{0.978} = 1887.$

Example 9·22. *A rectangular field coil is to produce an mmf of 7500 A when dissipating 220 W at a temperature of 60°C. The inner dimensions of coil are : length=0·24 m, width =0·10 m and height=0·15 m. The heat dissipation is 30 W/m²–°C from the outer surface, neglecting top and bottom surfaces of coil. Temperature of ambient air is 20°C. Calculate the thickness of the coil, the space factor and the current density. Resistivity is 0·02 Ω/m and mm².*

Solution. The winding, instead of having rounded off corners, has sharp corners or in other words, it is a true rectangle.

Length of inner turn $L_i = 2(0.24 + 0.10) = 0.68$ m.

Length of mean turn $= 0.68 + 4\ d_f$. Length of outer turn $L_0 = 0.68 + 8\ d_f$

Temperature rise $\theta = 60 - 20 = 40$°C.

$\therefore$ Loss dissipated per unit surface $q_f = 30 \times 40 = 1200$ W/m^2
based upon outer surface excluding ends.

Permissible loss $Q_f = 220$ W. $\therefore$ Outer surface area required $= 220/1200 = 0.1835$ m^2 ...(i)

Outer surface area excluding ends $= L_0 h_f = (0.68 + 8\ d_f) \times 0.15$ ...(ii)

$\therefore$ From (i) and (ii), we have : $(0.68 + 8\ d_f) \times 0.15 = 0.1835$

or depth of winding $d_f = 0.0682$ m $\simeq$ 68 mm

Length of mean turn $L_{mt} = 0.68 + 4\ d_f = 0.68 + 4 \times 0.068 = 0.952$ m.

Permissible loss $Q_f = I_f^2 R_f = I_f^2\ \dfrac{T_f\ \rho\ L_{mt}}{a_f} = (I_f T_f) . \dfrac{I_f}{a_f} . \rho\ L_{mt}$

Substituting the values for $I_f T_f$, L_{mt} and ρ in the above expression

$$Q_f = 7000 \times \frac{I_f}{a_f} \times 0.02 \times 0.952 = 133\ \frac{I_f}{a_f} \quad ...(iii)$$

But I_f/a_f is the current density in A/mm^2

$\therefore$ Current density $\delta_f = \dfrac{I_f}{a_f} = \dfrac{Q_f}{113} = \dfrac{220}{133} = 1.65$ A/mm^2.

Total area of conductors $= T_f a_f \times 10^{-6}$

$$= T_f\ \frac{I_f}{\delta_f} \times 10^{-6} = \frac{AT_f}{\delta_f} \times 10^{-6} = \frac{7000}{1.65} \times 10^{-6} = 0.00424\ \text{m}^2.$$

Total area for winding $A_w = h_f \times d_f = 0.15 \times 0.068 \approx 0.0102\ m^2$

$\therefore$ Space factor $S_f = \dfrac{\text{total area of conductors}}{\text{total area of winding}} = \dfrac{0.00424}{0.0102} = 0.416.$

Example 9.23. *A 6 pole, 220 V, 200 kW dynamo is to be level compounded. The mmf required per pole is 7500 A at no load and 9000 A at full load. Calculate the number of series turns per pole, and show a suitable arrangement for these turns. The height of winding is 0.15 m, the field coils are 50 mm thick and fit around at square pole of 0.23 m side. Calculate the diameter of shunt field conductor. If insulation increases the diameter by 0.1 mm, calculate also the shunt field current. Resistivity is 0.02 Ω/m and mm².*

Keep 10% of the voltage across the shunt field in reserve.

Solution. The mmf required at no load is developed by the shunt field while the extra mmf required at full load is developed by the series field.

$\therefore$ Mmf of shunt field winding = 7500 A

and mmf of series field winding $AT_s = 9000 - 7500 = 1500$ A.

Line current at full load $I_l = \dfrac{200 \times 1000}{220} = 909$ A.

Assuming the generator to be short shunt.

Current through series field $I_s = I_l = 909$ A.

$\therefore$ Series field turns per pole $T_s = 1500/909 = 1.65$.

This is a mixed number. The number of turns should be an integer.

Divide the series field in three parallel paths.

Current per parallel path $= 909/3 = 303$ A.

$\therefore$ Series field turns per pole $= 1500/303 \simeq 5$.

Thus there are 5 turns connected in 3 parallel paths.

Two parallel paths have two turns each while the third parallel path has one turn. The path with one turn has a resistor connected in series with the turn, the resistor has resistance equal to the resistance of each turn.

Shunt Field

Length mean turn $L_{mt} = 2(0.23 + 0.23) + 4 \times 0.05 = 1.12$ m.

Voltage across shunt field winding $= 0.9 \times 220 = 198$ V.

Voltage across each field coil $E_f = 198/6 = 33$ V.

Area of shunt field conductor $a_f = \dfrac{AT_f\, \rho\, L_{mt}}{E_f} = \dfrac{7500 \times 0.02 \times 1.12}{30} = 5.09\ mm^2$.

Diameter of bare conductor $d = 2.54$ mm

Diameter of insulated conductor $d_1 = 2.54 + 0.1 = 2.64$ mm.

Assume that the winding height taken by shunt field is 80 per cent of the total winding height available, the rest 20 per cent is taken by series field.

$\therefore$ Height of shunt field winding $h_f = 0.8 \times 0.15 = 0.12$ m.

Space factor $S_f = 0.75 \left(\dfrac{2.54}{2.64}\right)^2 = 0.694$

$\therefore$ Number of turns $T_f = \dfrac{0.694 \times 0.15 \times 0.05}{5.09} = 1022$

Now, $R_f = \dfrac{1022 \times 0.02 \times 1.12}{5.09} = 4.5\ \Omega$ and $I_f = \dfrac{33}{4.5} = 7.33$ A.

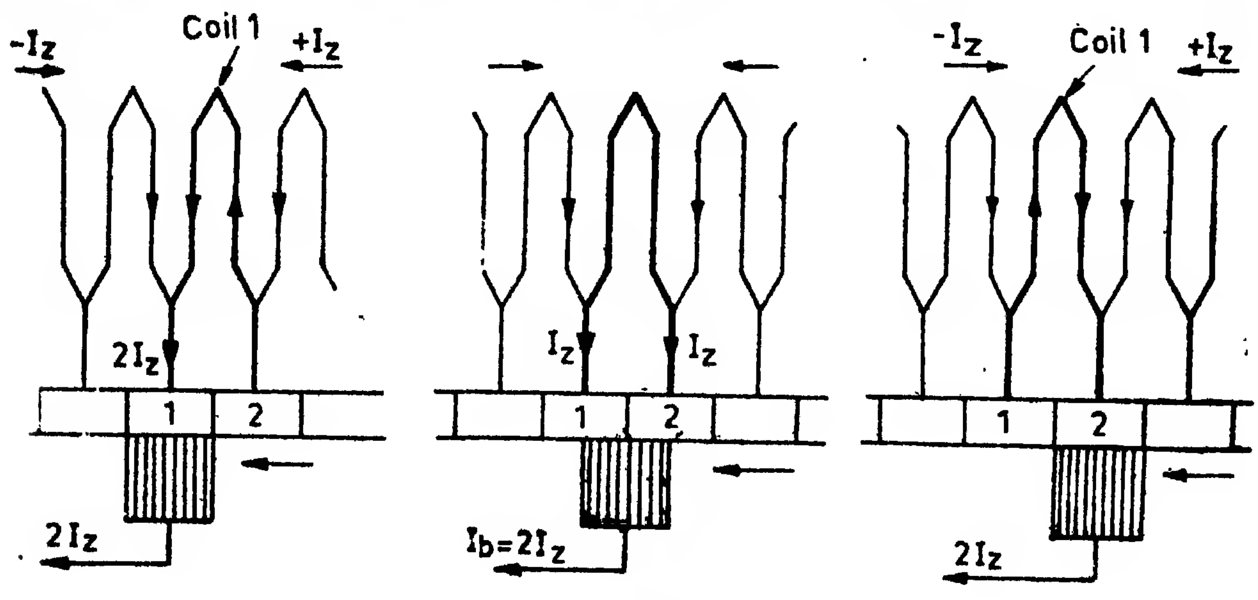
Mmf provided by shunt field $= 7.33 \times 1022 = 7495$ A.

COMMUTATION

9.37. **Commutation phenomenon.** When the commutator segments to which the armature coils are connected pass under the brushes, the armature coils are successively transferred from one parallel path, in which the current has one direction to an adjoining parallel path, in which the current has the opposite direction. During this period, the coils are short-circuited by the brush and the current must be reduced from its original value to zero and then built up to an equal value in the opposite direction. This process is known as *commutation* and the time during which it takes place is called the *time of commutation*.

One of the most important limiting factors on satisfactory operation of a d.c. machine is proper commutation. Good commutation conditions require the transfer of necessary armature current through the brush contact at the commutator without sparking and without excessive local losses and heating of brushes and the commutator. Sparking at the brushes results in destructive blackening, pitting and wear of brushes as well as the commutator. A worsening of these conditions may lead to burning of copper and carbon. The reasons for bad commutation may be both electrical and mechanical in nature. The former conditions are seriously influenced by the armature reaction and the armature leakage flux.

Consider the case of an idealized winding with single-turn full pitch coils and brush width equal to the width of a commutator segment. Fig. 9·41 shows the conditions in a particular coil (*i.e.* coil 1 shown with heavy line) as the commutator segments to which its two



(a) (b) (c)

Fig. 9·41. Coil undergoing commutation.

ends are connected pass under a brush through which a constant current I_b flows.

*Initially *i.e.* just before the commutation of the coil 1 is to begin, the coil current is $+I_z = I_b/2$ as shown in Fig. 9·41 (*a*). (The coil current $I_z = I_a/a$).

*Finally *i.e.* just after the commutation of coil 1 is to end, the current is $-I_z = I_b/2$ as shown in Fig 9·41 (*c*). Fig. 9·41 (*b*) depicts conditions during the process of commutation.

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*The commutation of coil 1 takes place during the time segments 1 and 2 are short-circuited by the brush. The commutation begins when the leading edge of the brush just starts touching segment 2 while the commutation ends when the trailing edge of brush just stops touching segment 1.

Therefore, during the time of commutation the current change from $+I_s$ to $-I_s$ and the total change in current is $2\ I_s = 2\ I_a/a$.

The *time of commutation* t_c in this case, is the interval between the time when the leading edge of the brush just touches segment 2 and the time when the trailing edge of brush just leaves segment 1.

If we neglect the thickness of the mica strip, the time of commutation of one coil is given by : $t_c = t_b/V_c$.

If the thickness of mica strip is considered $t_c = (t_b - t_i)/V_c$

where V_c = peripheral speed of commutator, m/s ;

t_b = thickness of the brush, m ; t_i = thickness of mica strip, m.

If we assume that the thickness t_b of the brush is equal to pitch of a commutator segment, then the brush can short circuit one coil at a time. Just at the beginning of short circuit, the brush covers one complete segment. The average current density in the brush is therefore given by : $\delta_{b(av)} = 2\ I_s/A_b = 2\ I_a/aA_b$
where A_b is the area of the brush.

9·38. Form of current in coil undergoing commutation. During the commutation the current in the coil is not constant but is changing. Let us denote it by i, being positive in the direction shown in Fig. 9·42. The form of the current during commutation will depend upon the resistance of and the emfs induced in the commutation circuit.

The resistance in the circuit consists of :

(*i*) the resistances r_1 and r_2 of the two contact areas between the brush and the two commutator segments,

(*ii*) the resistance R_f of each riser, (*ii*) the resistance R_c of the coil.

The emfs induced in the coil undergoing commutation are the :

(*ii*) *emf of self induction or statically induced emf*. The current in the coil changes from $+I_s$ to $-I_s$ and the coil has a self-inductace owing to slot leakage and overhang leakage flux. Therefore, a voltage is induced in the coil due to change of flux linkages. This is also known as the ***reactance voltage***.

In d.c. machines, there may be several coil sides per slot. The statically induced emfs may be due to self-inductance of a coil side and mutual inductance between coil sides in the same slot.

(*ii*) ***dynamically induced emf or rotational emf***.

The source of production of rotational emfs are explained below :

i) If the brushes are left in the geometric neutral position, the armature reaction produces a field there, when the machine is loaded. The armature coils cut through this field and a rotational emf is produced in them.

(*b*) If the brushes are given a shift, the armature coils actively cut through the field of the pole towards which the brushes are shifted and thus there is a rotational emf produced in them.

In view of the resistances and the emfs listed, the commutation process becomes highly complicated.

9·38·1. Resistance commutation. **Let us first consider the idealized case, wherein the rotational emf and the reactance voltage produced in the coil undergoing commutation are neglected. Under this condition, as is explained below, the brush contact resistance is the main factor helping the current reversal and the commutation in this case is known as *resistance commutation*.**

Referring to Fig. 9·42 which represents the position of the brush at a time t after the commutation begins, we obtain the following relations.

$$i_1=I_s-i \text{ and } i_2=I_s+i.$$

The thicknesses of the portions in contact with the two brushes are :

leading edge : $t_{b1}=t_b \,.\, t/t_c$, and

trailing edge : $t_{b2}=t_b\,(1-t/t_c)$.

Let R_b be the total contact resistance of brush and A_{b1} and A_{b2} be the areas of contact of the brush at the entering and leaving edges respectively.

∴ Resistances of two contact areas are :

$$\text{leading edge : } r_1=R_b\,.\,\frac{A_b}{A_{b1}}=R_b\,.\,\frac{t_b}{t_{b1}}=\frac{R_b}{t/t_c}.$$

$$\text{trailing edge : } r_2=R_b\,.\,\frac{A_b}{A_{b2}}=R_b\,.\,\frac{t_b}{t_{b2}}=\frac{R_b}{1-t/t_c}.$$

On the assumption that no emfs are present in the short circuit paths, the sum of all voltage drops, according to Kirchoff's law, must be equal to zero. Thus,

$$iR_c+i_2(R_r+r_2)-i_1(R_r+r_1)=0$$

where R_c and R_r are resistances of coil and risers respectively.

Substituting the values of i_1, r_1 and r_2 in the above relationship.

$$iR_c+\left(I_s+i\right)\left(R_r+\frac{R_b}{1-t/t_c}\right)-\left(I_s-i\right)\left(R_r+R_b\frac{t_c}{t}\right)=0$$

Solving the above equation we have : $i=\dfrac{I_s\,(1-2\,t/t_c)}{1+K_1[t/t_c-(t/t_c)^2]}$ ...(9·51)

where $K_1=(R_c+2R_r)/R_b$.

However, the values of resistances of coil (R_c) and risers (R_r) are quite small compared with the brush contact resistance R_b and therefore, $K_1\simeq 0$.

Hence, where resistances of coil and risers are negligible and the rotational emf and the reactance voltage neglected, the current in the coil undergoing commutation is given by

$$i=I_s\left(1-\frac{2t}{t_c}\right)=\frac{I_a}{a}\left(1-\frac{2t}{t_c}\right) \quad ...(9\cdot52)$$

Therefore, the curve for i over the time period t_c is linear as shown in Fig. 9·43.

Hence we codclude that if the brush contact resistance is made very large, the current in the coil undergoing commutation follows a linear law or a "*straight line commutation*" results.

Therefore, if brushes of high contact ressitance are used and the emf in the coil undergoing commutation is zero, a straight line commutation is obtained.

Let us examine the value of current density in the brush at any time t.

Current at the leading edge

$$i_1=I_s-i=I_s-I_s\,(1-2t/t_c)=I_s\,.\,2t/t_c.$$

Area of brush carrying this current, $A_{b1}=A_b\,t_{b1}/t_b=A_b\,.\,t/t_c$.

∴ Current density in the leading edge of the brush $\delta_{b1}=\dfrac{I_s\,.\,2t/t_c}{A_b\,.\,t/t_c}=\dfrac{2I_s}{A_b}=\delta_{(av)}$

Current at the trailing edge

$$i_2=I_s+i=I_s+I_s(1-2t/t_c)=2I_s\,(1-t/t_c).$$

Area of brush carrying this current $A_{b_2}=A_b \cdot t_{b_2}/t_b=A_b(1-t/t_c)$

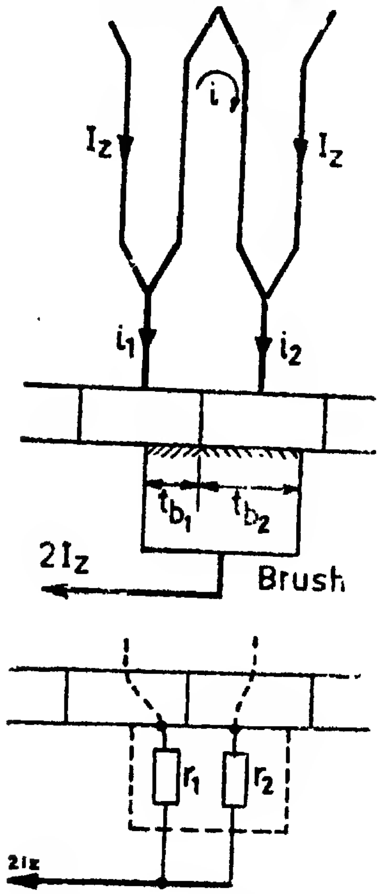


Fig. 9·42. Resistance commutation.

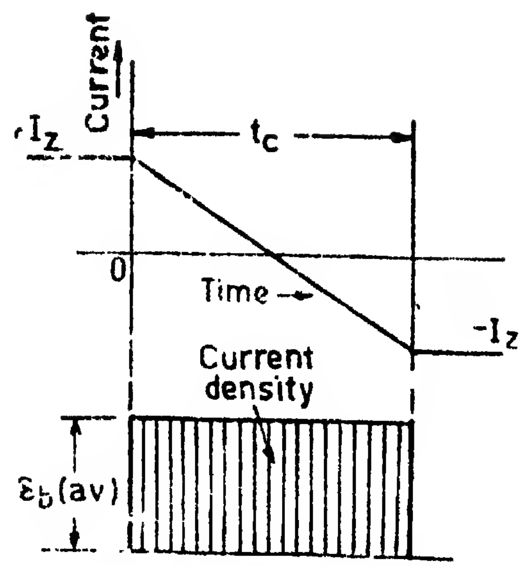


Fig. 9·43. Straight line commutation.

$\therefore$ Current density at the trailing edge $\delta_{b_2}=\dfrac{2 \cdot I_z(1-t/t_c)}{A_b(1-t/t_c)}=\dfrac{2I_z}{A_b}=\delta_{(av)}$.

The current densities at any instant in both leading and trailing edges of the brush are equal to the average current density or in other words every section of the brush is equally loaded as shown in Fig. 9·43.

Resistance commutation is used in small machines designed without interpoles. Some modern small machines employ *face plate commutators* as shown in Fig. 9·44. These commutators use sector shaped commmutators which can be either moulded or made from PCBs (printed circuit boards). These commutators use *carbon fibre* brushes. (The carbon fibre brushes are made up of fine carbon fibres with the flexibility of a paint brush. These brushes produce a high contact drop, 2—3 volt, and therefore, produce resistance commutation conditions in small machines).

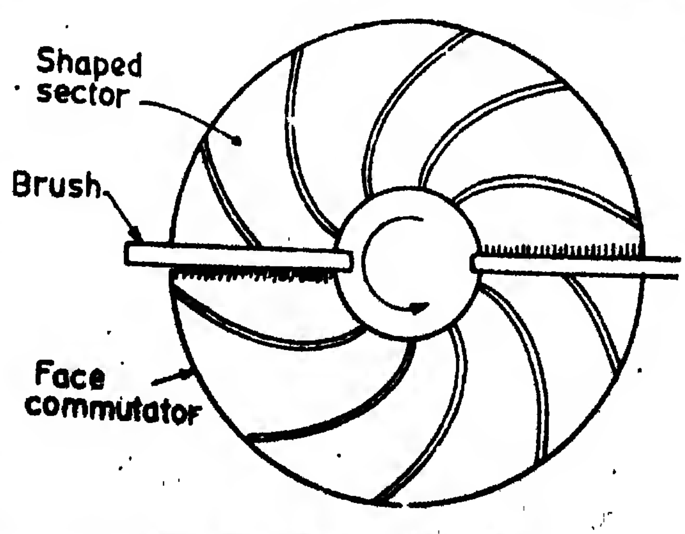


Fig. 9·44. Face plate commutator.

The rate of change of areas of leading and trailing edges are optimized in order to give a uniform current density over the brush.

9·38 2. **Retarded commutation.** The reactance voltage produced in the coil due to the reversal of the current must always be present because of the inductance of coil. The reactance voltage is of such a nature that it opposes the change which produces it. The change which produces the reactance voltage is the reversal of current and therefore, it must oppose the reversal of current. Thus the nature of the reactance voltage is to *retard or delay* commutation.

As discussed in Art. 9·18 page 513 the armature reaction field for a generator has polarity main of pole which the coil has just left behind. Due to rotation of the coil in this field an emf is generated which tries to maintain the current of the type flowing under the pole which it has left. For example, suppose a coil crosse the geometric neutral leaving behind a N pole and comes under a S pole. The armature reaction produces in it an emf corresponding to N pole even beyond the geometrical neutral axis Thus the coil tries to maintain current corresponding to a N pole even when it has crossed the geometrical neutral axis. However, the coil should carry current corresponding to S pole after crossing the geometrical neutral axis. Therefore, the tendency of the armature reaction is *to delay commutation.*

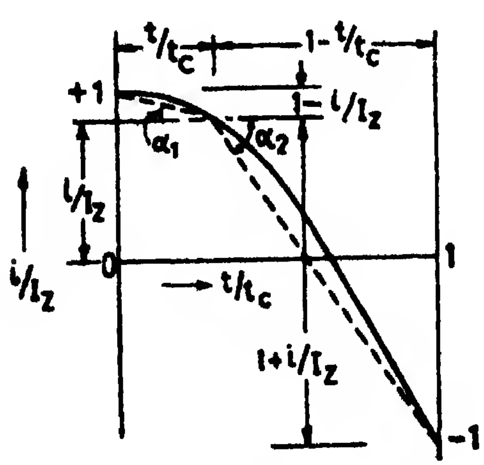


Fig 9·45. Retarded commutation.

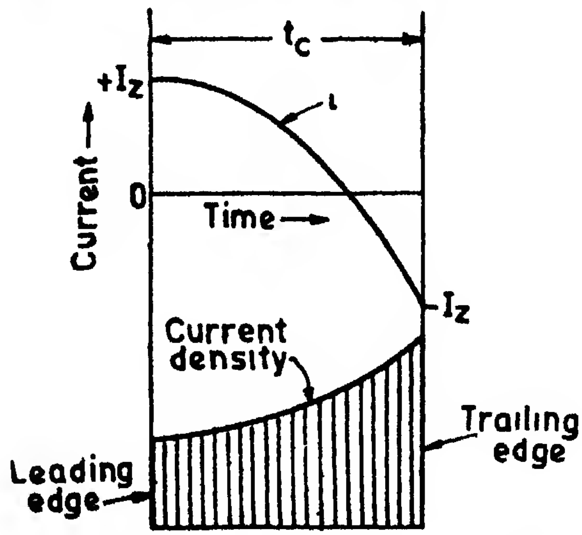


Fig. 9·46. Variation of coil current and current denisty in brush with retarded commutation.

Thus, both the reactance voltage and the rotational emf due to the armature reaction field delay the commutation and lead to a current variation of the type shown in Fig. 9·45.

The current ratio i/I_z is plotted against relative time t/t_c.

Current density at the leading edge of the brush $\delta_{b1} = \dfrac{i_1}{A_{b1}} = \dfrac{i_1}{A_b \,.\, t/t_c}$

or
$$\frac{\delta_{b1}}{\delta_b\,(av)} = \frac{i_1}{A_b \,.\, t/t_c} \times \frac{1}{2I_z/A_b} = \frac{i_1}{2I_z\, t/t_c}$$
$$= \frac{(I_z - i)}{2\, I_z \,.\, t/t_c} = \frac{1 - i/I_z}{2t/t_c} = \frac{1}{2} \tan \alpha_1$$

∴ Current density at the leading edge of brush $\delta_{b1} = \delta_b(av) \times \frac{1}{2} \tan \alpha_1$ ...(9·53)

Similarly, current density at trailing edge of brush $\delta_{b2} = \delta_b\,(av) \times \frac{1}{2} \tan \alpha_2$ ...(9·54)

Examining the curve for retarded commutation, we find that α_2 is greater than α_1. This means that the current density in the trailing edge of brush is higher than that in the

leading edge (Fig. 9·46). This current density may assume a very high value resulting in glowing of the brush, which would lead to high commutator temperatures, rapid deterioration of the brushes and excessive brush contact loss.

The commutation may be delayed to the extent as shown in Fig. 9·47. In such a case the commutation is completed before the current can reach its final value and therefore the current has to jump through air in the form of a *spark*.

Sparking damages the commutator leading to still greater brush wear.

9·38.3. Accelearted commutation. If the brushes of a generator are shifted beyond the magnetic neutral position, the coil undergoing commutation actively cuts through the fringe of the pole towards which it is moving, the dynamic emf produced is of such a nature as to help in the reversal of current or *accelerate commutation*. If excessive brush shift is given to the brushes, accelerated commutation would result. Fig. 9·48 shows the curve for accelerated commutation.

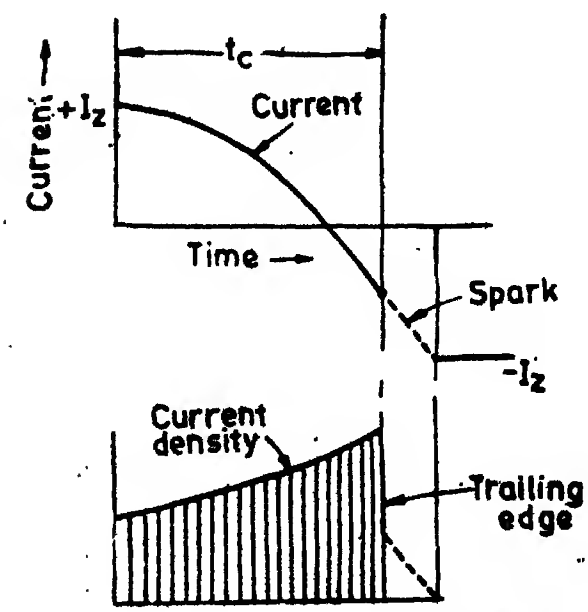


Fig. 9·47. Retarded commutation and sparking.

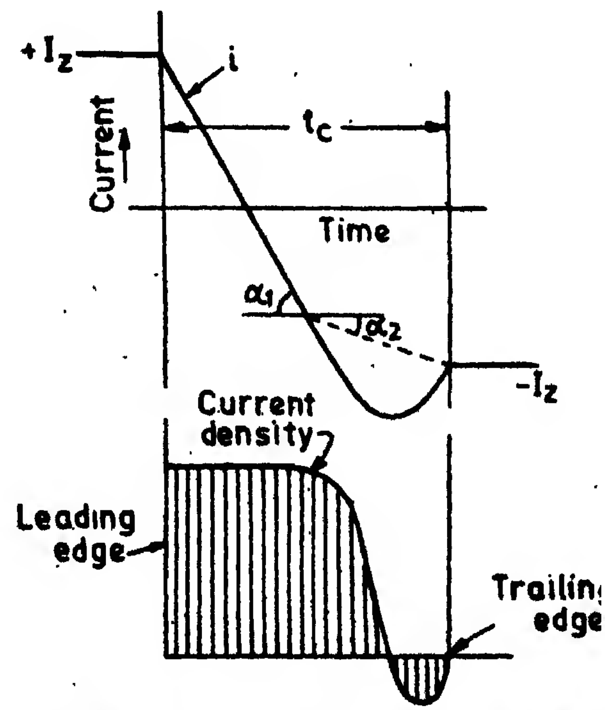


Fig. 9·48. Variation of coil current and current density in brush with accelerated commutation.

It is clear that α_1 is greater than α_2. Therefore, the current density in the leading edge of the brush is greater than that in the trailing edge (see Eqns 9·53 and 9·54). In an acute case the current density in leading edge may become very excessive giving rise to glowing of brush, which would lead to high commutator temperatures, rapid deterioration of brushes and excessive brush contact loss.

When the commutation is highly accelerated the coil current has a value different from I_z even before the commutation starts.

Thus both delayed and accelerated commutation conditions are bad because both of them result in non-uniform current density over the brush with consequent deleterious effects.

9·38·4. Sinusoidal commutation. From the above discussion, we gather that, with *delayed or retarded commutation*, the current density is *excessive at the trailing edge while it is small at the leading edge ; with accelerated commutation, current density is excessive at the leading edge and small at the trailing edge.*

Thus a consideration of the two forms would indicate that it is desirable to retard the commutation during the first moments and to accelerate it at the later stage commutation. This is known as *sinusoidal commutation*. Commutation is thus retarded in the first half of time t_c ; it becomes normal at $t=t_c/2$ and is then accelerated during second half of t_c.

DESIGN OF INTERPOLES

9·39. **Interpoles**. It is clear from above that if the resultant emf in the coil undergoing commutation is zero, and if the resistance of brush contacts is appreciably higher than that of coil and risers, a straight line commutation results which is ideal in d.c. machines.

However, in actual practice, emfs are produced in the short-circuited coil. These emfs may delay or accelerate the commutation. For example, the reactance voltage and the rotational voltage produced on account of cross magnetizing armature tend to retard the commutation. In order to aid the commutation process, another rotational voltage is produced in the short-circuited coil which opposes the reactance voltage. The general principle of producing in the coil undergoing commutation a rotational voltage which is equal to and opposite in sign to reactance voltage, a principle called *voltage commutation* is used in all modern d.c. machines. The appropriate field required at the geometric neutral axis to generate the neutralizing rotational voltage is produced by *interpoles* or *commutating poles*.

The interpoles are the smaller poles placed in between the main poles as shown in Fig 9·48. The polarity of the interpole must be that of the main pole just ahead for a generator and just behind it for a motor. The winding of the interpole must produce an mmf which is sufficient to neutralize the cross magnetizing armature mmf at the interpolar axis and enough more to produce the flux density required to generate rotational voltage in the coil undergoing commutation to cancel the reactance voltage.

The mmf required for interpoles should be correctly estimated. If it is too weak, it leads to retarded commutation ; and if it is too strong, it leads to accelerated commutation.

Since both the armature reaction and reactance voltage are proportional to armature current, the interpole winding should be connected in series with the armature for production of neutralizing effect at all conditions of load. In order to preserve linearity, the interpoles should be so designed that they work at low saturation levels.

We have seen above, that with the use of interpoles, the resultant emf in the coil undergoing commutation is reduced to zero and hence if brushes with high contact resistance are used a straight line commutation is achieved. Therefore, by the use of interpoles or commutating poles, a sparkless commutation is secured over a wide range.

9·40. **Time of commutation.** The time of commutation for a single coil in an *idealised* case where the width of brush is equal to pitch of commutator segments is $t_c=t_b/V_c$ or $t_c=(t_b-t_i)V_c$, if the thickness of insulation, t_i between the sectors is considered. However, in actual practice, the commutation is a much more complicated process and therefore the determination of *effective* time of commutation τ_c, during which induced emfs are produced in a coil as a result of current changes in associated coils is far more involved than that which appears at the first sight on account of the following reasons :

(*i*) The width of brushes is not equal to the width of commutator segments. The brushes normally span 2 to 4 commutator segments (not necessarily a whole number) with the result that commutation takes place in more than one coil simultaneously.

(*ii*) There are many coil sides in a slot belonging to different coils and therefore, in addition to emf of self-induction in the coil undergoing commutation, mutual induction takes place between coils on account of change in other coil-sides placed in the same slot and this causes additional emf.

(iii) The top and the bottom coil sides belonging to a coil do not undergo commutation simultaneously in case the coils are short pitched.

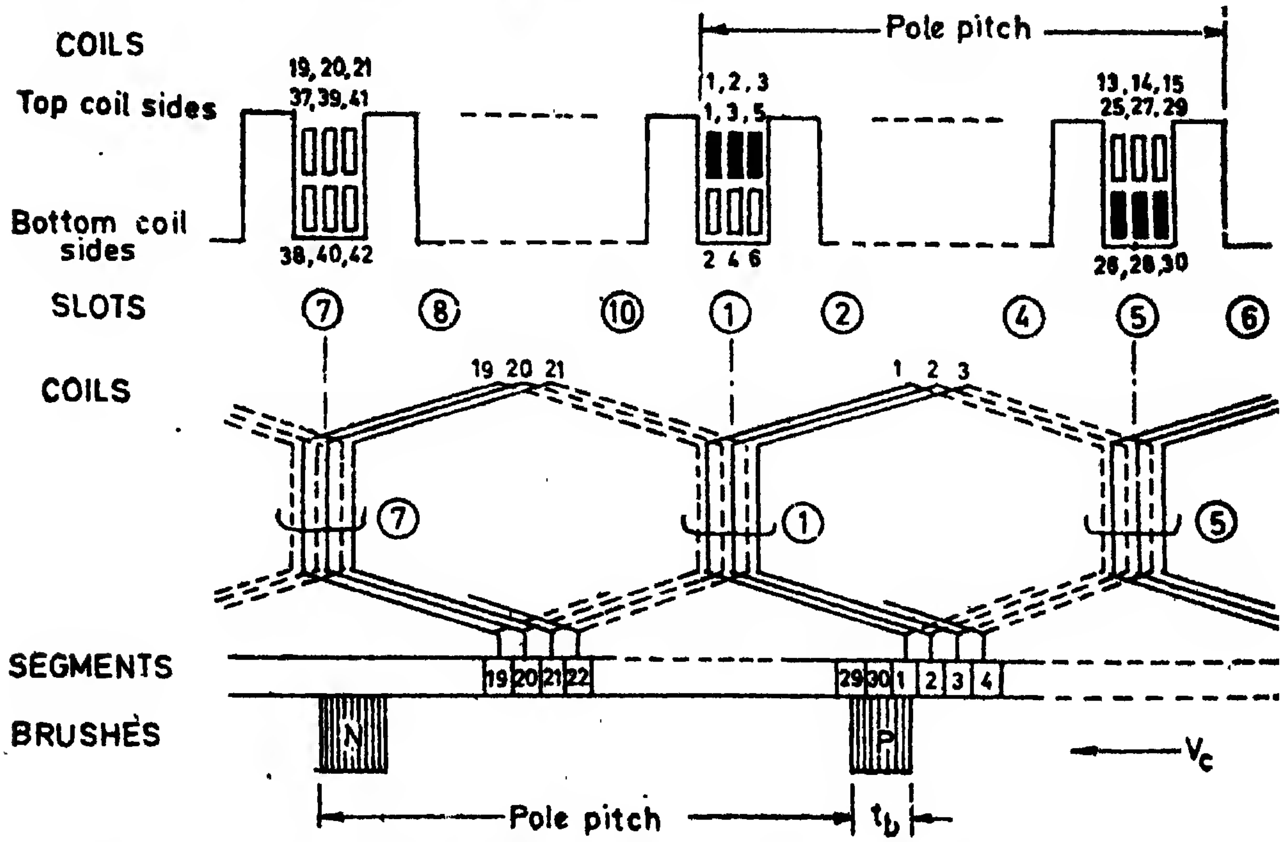


Fig. 9·49. Winding with 2 poles, 10 slots and 6 coil sides per slot.

Lap windings. Let us consider the case of a 10 slot, 2 pole armature having 6 coil sides per slot. The back pitch is 25 and the front pitch 23. Top coil side in slot 1 is connected at the back to coil side 26 in slot 5 and therefore the coils are short pitched by 1 slot (as coil span is 4 slots while the number of slots per pole is 5). The winding is shown in Fig. 9·49.

We examine the commutation process in coils 1, 2 and 3. The top coil sides (t.c.s) are 1, 3 and 5 which are located in slot 1 while their bottom coil sides (b.c.s.) are 26, 28, and 30 which are in slot 5. The b.c.s. sides in slot 1 are 2, 4 and 6 and they belong to coils 19, 20 and 21 respectively. The t.c.s. of coils 19, 20 and 21 are 37, 39 and 41 respectively and are located in slot 7.

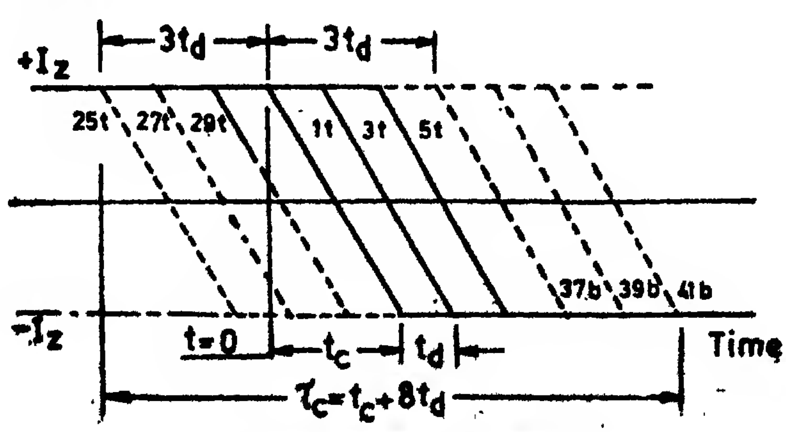


Fig. 9·50. Current/time graph.

The commutation of coil 1 begins as soon as commutator segment 2 touches the leading edge of brush P and the current i, begins to reverse along the current/time line 1 t as shown in Fig 9·50.

The commutation of coil 2 starts when the commutator segment 3 starts making contact with leading edge of brush P The *delay time* is defined as the time that elapses between start of commutation between two adjacent coils. Therefore, the delay time, t_d, is the time taken to travel one commutator segment in this case or $t_d=\beta_c/V_c$ where β_c is the pitch of commutator segments and V_c is the pripheral speed of the commutator. Therefore, current i_s in coil side 3 starts changing its current along line 3t and commutation of coil side 3 is complete after a time interval of t_c+t_d. Similarly, coil side 5 starts changing its current after a further delay time of t_d The commutation of coil side 3 completes its commutation after a time $t_c+2\,t_d$. Therefore, the total time of commutation for the t.c.s in slot 1 is $t_c+2\,t_d$. In general, if there are u sides per slot, the total time of commutation for the t.c.s. is $t_c+(\frac{1}{2}\,u-1)\,t_d$.

In windings, where full pitched coils are used, the commutation of the t.c.s. and the b.c.s. takes place simultaneously (at brush N) and the effective time of commutation is therefore :

$$T_c=t_c+(\tfrac{1}{2}\,u-1)\,t_d \qquad \ldots(9\cdot55)$$

However, in the case of short pitched coils the commutation of t.c.s. and b.c.s. does not take place simultaneously. As stated earlier, the b.c.s. in slot 1 are 2, 4 and 6 and belong to coils 19, 20 and 21. The commutation of b.c.s. 2 in slot 1 does not start until commutator segment 20 strats touching the leading edge of brush N, which occurs after a time 3 t_d as indicated by dotted line 37 b. The commutation of b.c.s 4 and 6 occurs after a time 4 t_d and 5 t_d respectively as shown by dotted line 39 b and 41 b.

The bottom coil side of coil 1 *i.e.*, 26 in slot 5, has emfs induced by current changes in t.c.s. 25, 27 and 29 Coil side 25 is seen to have completed commutation one half a commutator pitch before $t=0$, and also the lines depicting the changes in current *i.e.* 25 t, 27 t and 29 t precede those for t.c.s. 1 t, 3 t and 5 t by a time that corresponds to short pitching of coils.

Therefore, the total effective time of commutation for t.c.s. in slot 1 is :

$$\tau_c=2\times 3\,t_d+(3-1)\,t_d+t_c.$$

In general, for a winding shorts-pitched by α slots and with u coil sides per slot, the time of commutation is :

$$\tau_c=2\alpha\,(u/2)\,t_d+(\tfrac{1}{2}\,u-1)\,t_d+t_c$$

$$=[u(\alpha+\tfrac{1}{2})-1]\,t_d+t_c \qquad \ldots(9\cdot56)$$

$$=\frac{\{[u(\alpha+\frac{1}{2})-1]\,\beta_c+(t_b-t_i)\}}{V_c} \qquad \ldots(9\cdot57)$$

Wave winding. In case of wave windings, the commutation conditions are similar to those obtaining in lap winding if only two brush arms are used for a winding with p poles. The only difference is that $p/2$ series connected coils are commutated simultaneously.

However, in actual practice, the number of brush arms used is equal to the number of poles.

The effctive time of commutation for wave connected machines is :

$$\tau_c=[u/2-a/p]\,t_d+t_c \qquad \ldots(9\cdot58)$$

$$=\frac{\{[u/2-a/p].\,\beta_c+(t_b-t_i)\}}{V_c} \qquad \ldots(9\cdot59)$$

9·41 Width of commutation zone. The width of commutation zone is the portion of armature circumference where one or more coils are short-circuited. The width of commutation zone is equal to the distance moved by the armature during the effective time of commutation, τ_e.

∴ Width of commutation zone is :

Lap winding.

$$w_c = V_a \tau_c = \left\{\left[u(a+\tfrac{1}{2})-1 \right]\beta_c+(t_b-t_i)\right\}\frac{V_a}{V_c} \qquad \ldots(9{\cdot}60)$$

$$=\left\{\left[u(a+\tfrac{1}{2})-1 \right]\beta_c+(t_b-t_i)\right\}\frac{D}{D_c} \qquad \ldots(9{\cdot}61)$$

where D and D_c are respectively the diameters of armature and commutator.

Wave winding

$$w_c=\left\{\left[u/2-a/p \right]\beta_c+(t_b-t_i)\right\}\frac{V_a}{V_c} \qquad \ldots(9{\cdot}62)$$

$$=\left\{\left[u/2-a/p \right]\beta_c+(t_b-t_i)\right\}\frac{D}{D_c} \qquad \ldots(9{\cdot}63)$$

9·42. Width of interpole shoe. If *a straight line commutation* is desired, the width of interpole shoe should not be less than the width of commutating zone as given b Eqns 9·61 and 9·63. An allowance of 1·5 to 2 times the length of air gap under the inte: pole may be made for fringing of flux at the interpole tips.

∴ Width of interpole shoe $W_{ip}=w_c-(1{\cdot}5 \text{ to } 2)\, l_{gi}$

where l_{gi}=length of air under the interpole.

The length of air gap under the interpole l_{gi} must not be so small as to produce large *pulsations of the interpole flux* caused by the armature slots. It must not be too large otherwise the main pole flux will penetrate the interpole air gap. In general : $l_{gi}=(1 \text{ to } 2)\, l_g$.

For machines for which good commutation is difficult to obtain, larger air gap is found more satisfactory.

The interpole width must also be so chosen that the leakage flux is not excessive. To avoid excessive *leakage flux*, the width of interpole should not be greater than one half of the space between adjacent pole tips or $W_{ip} < \frac{1-\psi}{2}\tau$.

The width of the interpole should also be chosen with regard to armature slot pitch. With narrow interpoles, the pulsation of the interpole flux caused by armature slots is large. In general, $W_{ip} \geqq 1{\cdot}5\, y_s$.

For *sinusoidal commutation* the width of interpole should be taken equal to the slot pitch or $W_{ip}=y_s$.

9·43. Calculation of reactance voltage. The reactance voltage can be calculated by calculating the specific permeances for slot, tooth top and overhang leakage.

(*i*) The specific slot permeance λ_s is calculated with reference to Fig. 4·38 and from Eqn. 4·72.*

$$\lambda_s=\mu_0\left[\frac{h_1}{3W_s}+\frac{h_2}{W_s}+\frac{2h_3}{W_s+W_0}+\frac{h_4}{W_0}\right]$$

*This relationship has been derived for single layer windings but can be used for double layer windings as well.

(ii) The tooth top specific permeance λ_t for a machine with interpoles is

$$\lambda_t = \mu_0 \frac{W_{ip}}{6\, l g_i} \qquad \text{(See Eqn. 4·91)}$$

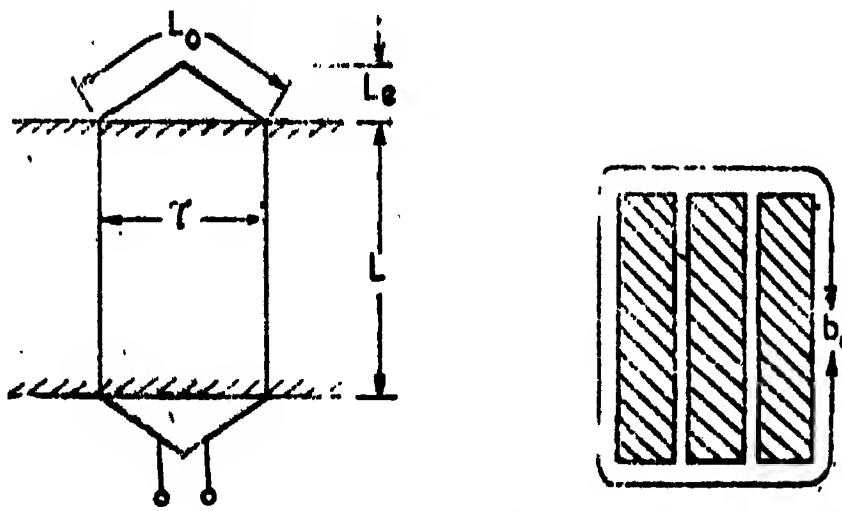
(iii) The overhang leakage flux is calculated from the following empirical relation

$$\lambda_0 = \frac{L_0}{L}\left(0·23 \log \frac{L_0}{b_0} + 0·07\right) \times 10^{-6} \qquad ...(9·64)$$

where L_0 = length of overhang of one coil side

$$\simeq 2\sqrt{\left(\frac{\tau}{2}\right)^2 + L_e^2}$$

where b_0 is the periphery of the all coil sides in one layer and L_e is the length of outhang (See Eqn 9·25). The terms L_0, b_0 and L_e are explained in Fig. 9·51.



(a) (b)

Fig. 9·51. Pertaining to overhang leakage flux.

Total specific permeance, $\lambda = \lambda_s + \lambda_t + \lambda_0$.

Flux = mmf × permeance = mmf × specific permeance × length.

∴ The effective value of leakage flux established for a slot containing Z_s conductors each carrying a current I_s is :

$$= \text{mmf per slot} \times \text{specific permeance} \times \text{length} = I_s Z_s \lambda L \qquad ...(9·65)$$

Since there are two active coil sides per coil, the total flux linked with a coil is

$$\Phi_s = 2\lambda L \times I_s Z_s \qquad ...(9·66)$$

Full pitch windings. For a full pitch winding, the top and bottom coil sides in a slot are commutated at the same time. For such a winding, during the time that the current in the upper layer is commutated, the whole of the current in the slot reverses and flux changes from $+\Phi_s$ to $-\Phi_s$,

or change in flux $= 2\Phi_s = 4\lambda L \times I_s Z_s$.

∴ Average value reactance voltage induced in each turn undergoing commutation is

$$e_{rav} = d\phi/dt = 4\lambda L\, Z_s/\tau_c.$$

If there are T_c turns per coil in the winding, the average reactance voltage per segment,

$$E_{rav} = \frac{4 T_c \lambda L\, I_s Z_s}{\tau_c} \qquad ...(9·67)$$

An empirical formula which can be used for the calculation of approximate value of average reactance voltage is :

$$E_{rav}=\frac{4uT_c I_s (4L+0{\cdot}4L_0)}{\tau_c \times 10^8} \quad ...(9{\cdot}68)$$

Chorded windings. In the case of a lap winding, chorded by one slot pitch, during the time of commutation of one layer, the current in the other layer remains steady. The total flux change that is produced for the complete commutation of one coil group is reduced to one half that for a full pitch winding as far as the slot leakage and tooth top leakage is concerned.

∴ Specific slot permeance with winding chorded by one slot pitch is :

$$\lambda_s=\mu_0\left[\frac{5}{24}\frac{h_1}{W_s}+\frac{h_2}{2W_s}+\frac{h_3}{W_s+W_0}+\frac{h_4}{2W_0}\right] \quad ...(9{\cdot}69)$$

and specific permeance for tooth top leakage, $\lambda_t=\mu_0\frac{W_{ip}}{12\, l_{gt}}$. ...(9·70)

There is no reduction of overhang leakage flux due to chording as this flux is due to simply one layer of winding.

Occasionally lap windings are short-chorded by half slot pitch. In this case it may be taken for all practical purposes that the total specific permeance is 1/2 of the specific permeance obtained with windings chorded by one slot pitch.

With wave windings, the degree of chording of the coil is usually small. If f is the fraction of a slot pitch by which the winding is chorded, the total specific permeance may be taken as

$$\lambda=(1-f/2)\,(\lambda_s+\lambda_t)+\lambda_0. \quad ...(9{\cdot}71)$$

9·44. Pitchelmayer's equation. Let us consider a coil with T_c turns. The inductance of coil is $2\,T_c^2 L\lambda$.

∴ The average reactance voltage in the coil

$E_{rav}=Ldi/dt=2T_c^2 L\lambda\times 2I_s/\tau_c$.

Let the brush thickness be equal to one commutator segment.

$$\therefore \quad \tau_c=t_c=\frac{\beta_c}{V_c}=\frac{\pi D_c}{C}\times\frac{1}{\pi D_c n}=\frac{1}{Cn}.$$

where C is the total number of coils or commutator segments and n is the speed of the machine.

Hence,
$$E_{rav}=2T_c^2 L\lambda\times 2I_s\, Cn=2T_c\, L\lambda\,(I_s Z)n \qquad (\text{as } 2C\,T_c=Z)$$
$$=2\,T_c\, L\lambda\left(I_s\,\frac{Z}{\pi D}\right)\pi\, Dn=2T_c\, ac\, V_a\lambda L \quad ...(9{\cdot}72)$$
$$\text{as } ac=\frac{I_s Z}{\pi D}.$$

Eqn. 9·72 is known as *Pitchelmayer's equation.*

The value of total specific permeance may be taken as

1. Low voltage machines with a winding of round copper wire for $T_c>1$ ...$(6 \text{ to } 8)\times 10^{-6}$
2. Average output machines with bar windings ...$(5 \text{ to } 6)\times 10^{-6}$
3. Low speed large output machines with short cores ...$(6 \text{ to } 8)\times 10^{-6}$
4. Low speed, large output machines with long cores. ...$(3{\cdot}8 \text{ to } 4{\cdot}5)\times 10^{-6}$
5. High speed, large output machines ...$(4 \text{ to } 5)\times 10^{-6}$

9·45. Length of interpole. The interpoles are made of cast steel or punched from sheet steel with no special pole shoe ; that is the length and width of pole body are equal to the length and width of pole shoe.

The length of the interpole is generally chosen from the st nd point of economy, for the shorter the interpole, the shorter will be the length of mean turn of interpole winding and the smaller the copper weight, losses and leakage flux. The length must, of course, be so chosen that the flux density in the interpole will be below the saturation point of the material. This is required in order to obtain proper compensation of reactance voltage at all loads, which is only possible if the flux density under the interpole varies linearly with the load current. Hence there should be no saturation in the iron parts of the interpole magnetic circuit.

For machines designed for large fluctuating loads or for variable speed motors, which employ completely laminated core, the length of interpole is taken equal to that of main pole.

9·46. Flux density under interpole shoe. The mean emf generated in the coils undergoing commutation, by their rotation in the reversing field under the interpole, must be just equal to reactance voltage so that the resultant emf in the coil is zero.

Let B_{gi}=flux density under the interpole and L_{ip}=length of the interpole.

∴ Rotational emf produced in a coil (with T_c turns) by cutting the field under the interpole

$$E_{gi}=2\times T_c\times \text{voltage generated in each conductor}=2\ T_c\ B_g.\ L_{ip}\ V_a.$$

This voltage must balance the reactance emf E_{rav} in each turns as given by Eqn. 9·67.

$$\therefore \qquad \frac{4\ T_c\ \lambda L\ I_z\ Z_s}{T_c}=2\ T_c\ B_{gi}\ l_{gi}\ V_a$$

Hence, flux density under the interpole $B_{gi}=2\ I_z\ Z_s\ \dfrac{L}{L_{ip}}\cdot\dfrac{1}{V_a\ T_c}\ \lambda$ ...(9·73)

For an idealized arrangement where the each brush spans only one commutator segment the value of B_{gi} can be calculated from Pitchelmayer's equation.

From Eqn. 9·72, reactance voltage in coil of T_c turns, $E_{rav}=2T_c\ ac\ V_a\ \lambda L$.

Equating the rotational emf with reactance voltage, we have :

$$2\ T_c\ B_{gi}\ L_{ip}\ V_a=2\ T_c\ ac\ V_a\ \lambda L$$

$$\therefore \qquad B_{gi}=ac\ \lambda\ \frac{L}{L_{ip}}. \qquad \text{...(9 73)}$$

9·47. Design of interpole winding. The length of air gap under the interpole is relatively large, and the iron parts of interpole magnetic circuit are worked much below the saturation region, the reluctance of the iron parts may be neglected.

Let l_{gi}=length of air gap under the interpole, K_{gi}=interpole gap contraction factor,

and AT_a=armature mmf per pole$=I_z\ Z/2p$.

Mmf required to establish a flux density B_{gi} =800000 $B_{gi}\ K_{gi}\ l_{gi}$.

Mmf required to overcome armature reaction, for machines with no compensating winding$=I_z\ Z/2p$.

Mmf required to overcome armature reaction, for machines with compensating winding$=(1-\psi)\ I_z\ Z/2p$.

∴ Mmf required for interpole AT_i=800000 $B_{gi}\ K_{gi}\ l_{gi}+I_z\ Z/2p$ ...(9·74)
for machines with no compensating winding.

$$AT_i = 800000\ B_{gi}\ K_{gi}\ l_{gi} + (1-\psi)\ I_z\ Z/2p \qquad \text{...(9·75)}$$

for machines with compensating winding.

The interpole winding is connected in series with the armature and the total armature current flows through the interpole winding. A single layer coil wound strip on edge, may be used with thin pressboard between the turns. The winding may consist of bare copper conductors which are air spaced if the coil is rigid. For large machines, a conductor having two or more strips in parallel may be used in order to facilitate the bending operation. However, modern large sized machines especially traction motors use a winding arrangement similar to the one which has described for series field.

Number of interpole turns $T_i = AT_i/I_a$ ...(9·76)

By slight adjustment of the gap dimensions, this number of turns can very readily made into a convenient number. With large heavy current machines, the number of turns required is very small, and in some cases it may be convenient to arrange the interpole winding into two parallel circuits, each carrying one half of total current.

The current density in the interpole winding should be between 2·5 to 4 A/mm^2; the higher value is used where ventilation is specially good, and the insulation thin. The same value of current density is used for compensating winding, if employed.

Example 9·25. *A d.c. machine has the following nameplate data and physical properties : 550 V, 275 kW, 900 r.p.m., 6 poles, wave wound armature winding in 180 slots with 8 conductors in each slot, 70% of the armature surface covered by poles.*

(a) *Find the armature mmf per pole at rated armature current.*

(b) *Find the number of conductors that should be placed in each pole face to provide adequate compensating winding.*

(c) *Estimate the number of turns required on each of the 6 commutating poles to provide a flux density across an effective gap length of 5 mm if the machine has no compensating winding.*

(d) *Repeat part (c) if the machine has a compensating winding.*

Solution. Armature current $I_a = \dfrac{275 \times 1000}{550} = 500$ A.

With a wave winding $a=2$, ∴ Current in each conductor $I_z = 500/2 = 250$ A.

Total number of armature conductors $Z = 180 \times 8 = 1440$.

(a) Armature mmf/pole $AT_a = I_z\ Z/2p = 250 \times 1440/2 \times 6 = 30{,}000$ A.

(b) Pole arc $\psi = 0·7$.

∴ Mmf required by compensating winding $AT_c = \psi AT_a = 0·7 \times 30{,}000 = 21000$ A.

Compensating winding turns per pole $= AT_c/I_a = 21000/250 = 42$.

∴ Pole face conductors of compensating winding per pole $= 2 \times 42 = 84$.

(c) Mmf required for each interpole for a machine without compensating winding

$AT_i = 800{,}000\ B_{gi}\ K_g\ l_{gi} + AT_a$

$= 800{,}000 \times 0·5 \times 5 \times 10^{-3} + 30{,}000 = 32000$ A.

Number of turns on each interpole, $T_i = AT_i/I_a = 32000/500 = 64$.

(d) Mmf required for each interpole for a machine with compensating winding

$AT_i = 800{,}000\ B_{gi}\ K_g\ l_{gi} + (1-\psi)\ AT_a$

$= 800{,}000 \times 0·5 \times 5 \times 10^{-3} + (1-0·7) \times 30{,}000 = 11000$ A.

Number of turns on each interpole $T_i = 11000/500 = 22$.

Example 9·26. *Determine the number of turns on each commutating pole of a 6 pole machine if the flux density in the air gap of the commutating pole is 0 5 Wb/m² at full load and the effective length of air gap is 4 mm. The full load current is 500 A and the armature is lap wound with 540 conductors. Assume the mmf required for the remainder of the magnetic circuit to be one-tenth that for the air gap.*

Solution. Armature mmf per pole, $AT_a=I_z\dfrac{Z}{2p}=\dfrac{I_a}{a}\dfrac{Z}{2p}=\dfrac{500}{6}\times\dfrac{540}{2\times6}=3750$ A.

Mmf required for air gap$=800,000\ B_{gi}\ K_{gi}\ l_{gi}=800,000\times0{\cdot}5\times4\times10^{-3}=1600$ A.

Mmf required for iron parts$=0{\cdot}1\times1600=160$ A

$\therefore$ Total mmf per pole on each interpole $AT_i=3750+1600+160=5510$ A.

Number of turns on each interpole, $T_i=5510/500\simeq11$.

Example 9·27. *Estimate the reactance voltage for a machine with following particulars, for straight line and sinusoidal commutation :*

number of segments$=60$, revolutions per second$=10$, brush width in segments$=1{\cdot}5$, co-efficient of self-induction$=0{\cdot}2$ mH, and current per coil$=20$ A.

Solution The time of commutation may be taken equal to the time required by the commutator to travel the width of the brush. (The width here is the thickness of the brush according to our terminology See Fig. 9·52).

$\therefore$ Time of commutation, $T_c=\dfrac{1{\cdot}5}{60}\times\dfrac{1}{10}=2{\cdot}5\times10^{-3}$ s.

Change of current is from $+20$ A to -20 A. Total charge of current$=40$ A.

Average reactance voltage, $E_{rav}=L\dfrac{(2I)}{\tau_c}=0{\cdot}2\times10^{-3}\times\dfrac{40}{2{\cdot}5\times10^{-3}}=3{\cdot}2$ V.

$\therefore$ Reactance voltage with striaght line commutation$=3{\cdot}2$ V.

With sinusoidal commutation, reactance voltage$=\pi/2\times3{\cdot}2=5$ V.

Example 9·28. *A 350 kW, 500 V generator has 8 poles, an armature diameter of 1·3 m and a core length of 0·35 m. A duplex wave winding is accommodated in 114 slots with 6 coil sides per slot The axial length of commutating poles is 0·2 m and the gap length under the commutating poles is 10 mm. Find the necessary mmf for each interpole if the specific permeance is 6×10^{-6}. Find also the number of turns.*

Solution. Armature current $I_a=\dfrac{350\times1000}{500}=700$ A.

For a duplex wave winding number of parallel paths $a=4$.

$\therefore$ Current in each conductor $I_z=700/4=175$ A.

Number of coils $C=\frac{1}{2}\ us=\frac{1}{2}\times6\times114=342$.

Suppose the machine has single turn coils or $T_c=1$.

$\therefore$ Number of armature conductors $Z=2\times1\times342=684$.

Ampere conductors per meter $ac=I_z\dfrac{Z}{\pi D}=175\times\dfrac{684}{\pi\times1{\cdot}3}=29300$.

From Eqn. 9·71, average reactance voltage in a coil, $E_{rav}=2T_c\ ac\ V_c\ \lambda L$

$$=2\times1\times29300\times V_a\times6\times10^{-6}\times0{\cdot}35=0{\cdot}123\ V_a. \quad ...(i)$$

Rotational emf produced in a coil by cutting through the field of interpole

$$E_{gi}=2T_c\ B_{gi}\ L_{ip}\ V_a=2\times1\times B_{gi}\times0{\cdot}2\ V_a=0{\cdot}4\ B_{gi}\ V_a \quad ...(ii)$$

Equating (i) and (ii), we have : $0{\cdot}123\ V_a=0{\cdot}4\ B_{gi}\ V_a$

$\therefore$ Flux density in gap under the interpole, $B_{gi}=0{\cdot}308$ Wb/m².

Armature mmf per pole $AT_a = I_z \dfrac{Z}{2p} = 175 \times \dfrac{684}{2\times 8} = 7480$ A.

Mmf required for each interpole $AT_i = AT_a + 800{,}000\ B_{gi}\ K_{gi}\ l_{gi}$

$= 7480 + 800{,}000 \times 0{\cdot}3075 \times 10 \times 10^{-3} = 9940$ A

Interpole winding turns $T_i = \dfrac{9940}{700} \approx 14$.

Example 9·29. *A 1500 kW, 500 V, 300 r.p.m., 14 pole lap connected d.c. generator has 287 slots and 574 single turn coils. The diameter of armature is 2m and core length 0·28 m.* ***Find :***

(*i*) *number of turns on each interpole,* (*ii*) *commutating flux in the air gap,*

(*iii*) *time of commutation, and* (*iv*) *emf generated in the short-circuited coil.*

Given :

air gap length under interpole = 10 mm ; flux density under interpole = 0·3 Wb/m² ;
mmf required for iron parts = 20% of gap mmf ; width of interpole = 2 slot pitches
width of flux path in commutating zone = 1·2 × width of interpole ;
length of interpole = 0·28 m ; gap contraction factor = 1·1.

Solution. Armature current $I_a = \dfrac{1500\times 1000}{500} = 3000$ A.

Number of parallel paths $a = p = 14$. Current in each conductor $I_z = 3000/14 = 214{\cdot}3$ A.

Number of armature conductors $Z = 2\times 574 = 1148$.

(*i*) Mmf required for gap under each interpole

$= 800{,}000 \times 0{\cdot}3 \times 1{\cdot}1 \times 10 \times 10^{-3} = 2640$ A.

Mmf required for iron parts $= 0{\cdot}2 \times 2640 = 530$ A.

Armature mmf per pole $= 214{\cdot}3 \times \dfrac{1148}{2\times 14} = 8780$ A.

Total mmf required for each interpole $AT_i = 2640 + 530 + 8780 = 11950$ A.

Number of turns on each interpole $T_i = \dfrac{11950}{3000} = 4$.

(*ii*) Slot pitch $= \dfrac{\pi\times 2}{287}$ m $= 21{\cdot}9$ mm. Width of each interpole $= 2\times 21{\cdot}9 = 43{\cdot}8$ mm.

Width of commutating zone $w_c = 1{\cdot}2 \times 43{\cdot}8 = 52{\cdot}6$ mm.

Commutating flux per pole $= B_{gi}\ w_c\ l_{ip} = 0{\cdot}3 \times 52{\cdot}6 \times 10^{-3} \times 0{\cdot}16 = 2{\cdot}52 \times 10^{-3}$ Wb

(*iii*) Speed $n = 300/60 = 5$ r.p.s. Armature peripheral speed, $V_a = \pi \times 2 \times 5 = 31{\cdot}4$ m/s.

Time of commutation $T = \dfrac{w_c}{V_a} = \dfrac{52{\cdot}6 \times 10^{-3}}{31{\cdot}4} = 1{\cdot}67 \times 10^{-3}$ s.

(*iv*) Rotational emf generated in the short-circuited coil

$E_{ci} = 2T_c\ B_{gi}\ L_{ip}\ V_a = 2\times 1\times 0{\cdot}3 \times 0{\cdot}28 \times 31{\cdot}4 = 5{\cdot}3$ V.

Example 9·30. *The armature of a 200 kW, 250 V, 6 pole, 1200 rpm d.c. machine has a diameter of 0·51 m and an axial length of 0·26 m. The armature has a simplex lap winding with 189 single turn coils housed in 63 slots. The diameter of commutator is 0·35 m and brushes short circuit three coils simultaneously. Find the time of commutation, width of commutation zone and the flux density under the interpole to generate a commutating emf of 4V in each coil.*

Solution. Total number of coil sides $=2C=2\times189=378$.

$\therefore$ Coil sides/slot $u=378/63=6$.

Let us figure out the winding arrangement.

Back pitch, $y_b=2C/p\pm K=378/6\pm K=63$.

In case $y_b=63$ is used, $(y_b-1)/u$ is not an integer and hence the winding will have split coils Therefore, in order to avoid split coils, a back pitch of $y_b=61$ is used.

With $y_b=61$, top coil side 1 in slot 1 is connected to bottom coil side 62 in slot 11, giving a coil span of 10 slots The slots per pole are $63/6=10{\cdot}5$ and hence the coils are short pitched by 1/2 slot or $\alpha=\frac{1}{2}$ (See Eqn. 9·56).

Pitch of commutator segments $\beta_c=\dfrac{\pi D_c}{C}=\dfrac{\pi\times0{\cdot}35}{189}=5{\cdot}8\times10^{-3}$ m.

The brushes short circuit three coils simultaneously and, therefore, thickness of each brush is

$$t_b=3\beta_c=3\times5{\cdot}8\times10^{-3}=17{\cdot}4\times10^{-3}\text{ m},$$

$$V_c=\pi D_c n=\pi\times0{\cdot}35\times1200/60=22\text{ m/s},$$

and $$V_a=\pi\times0{\cdot}51\times1200/60=32\text{ m/s}.$$

Delay time $t_d=\beta_c/V_c=5{\cdot}8\times10^{-3}/22=0{\cdot}26\times10^{-3}$ s.

Time required to travel through brush thickness

$$t_c=t_b/V_c=17{\cdot}4\times10^{-3}/22=0{\cdot}79\times10^{-3}\text{ s}.$$

From Eqn. 9·56, time of commutation for a simplex lap winding

$$\tau_c=[u(\alpha+1/2)-1]t_d+t_c=[6(1/2+1/2)-1]\times0{\cdot}26\times10^{-3}+0{\cdot}79\times10^{-3}$$
$$=0{\cdot}79\times10^{-3}\text{ s}.$$

Width of commutation zone

$$w_c=V_a\tau_c=32\times0{\cdot}79\times10^{-3}=25{\cdot}3\times10^{-3}\text{ m}=25{\cdot}3\text{ mm}.$$

Rotation emf in a coil $E_{gt}=2T_c\,B_{gt}\,LV_a$ or $4=2\times1\times B_{gt}\times0{\cdot}26\times32$.

$\therefore$ Flux density under the interpole $B_{gt}=0{\cdot}24$ Wb/m².

DESIGN OF COMMUTATOR AND BRUSHES

9·48. **Number of segments.** The number of segments is equal to the number of coils or segments $C=1/2\,uS$.

The minimum number of segments is that which gives a voltage of 15 V between segments at no load.

$\therefore$ Minimum number of segments $=E\times p/15$.

9·49. **Commutator diameter.** The diameter of commutator generally lies between 0·6 to 0·8 of armature diameter. It varies from 62% of armature diameter for 350/700 V machines, 68% for 200/250 V machines and 75% for 100/125 V machines. The larger diameter being necessary on heavy current (low voltage) machines because of higher mechanical stresses and heating.

The peripheral voltage gradient around the commutator should be limited to about 3 V/mm in order to avoid ionization of the skin of air at the commutator surface. Thus for 600 V machine the minimum peripheral distance between adjacent brush arms should be 600/3=200 mm. The diameter must be chosen with regard to the peripheral speed and the thickness of commutator segment.

Peripheral speed. The commutator peripheral speed is generally kept below 15 m/s. Higher peripheral speeds upto 30 m/s are used but should be avoided wherever possible. The higher commutator peripheral speeds generally lead to commutation difficulties.

Commutator segment pitch. The thickness of the commutator at the commutator surface should not be less than 3 mm. If the thickness of mica is about 0·8 mm, then the minimum segment pitch is approximately 4 mm.

∴ Pitch of segment $\beta_c = \pi D_c/C$ should not be less than 4 mm.

9·50. Length of commutator. The length of the commutator depends upon the space required by the brushes and upon the surface required to dissipate the heat generated by the commutator losses.

Fig. 9·52 shows various terms connected with the brushes.

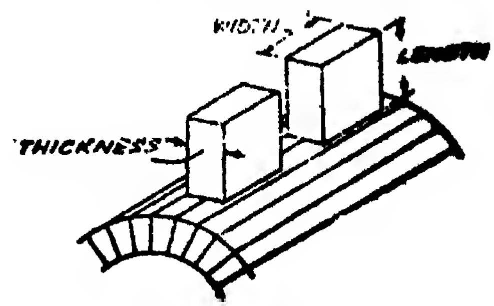


Fig. 9·51. Commutator and brushes.

9.51. Dimensions of brushes. The thickness of brushes has a profound influence on the commutation conditions. This is clear from Eqn. 9·62 where the thickness of the brush t_b and the commutator segment pitch β_c are the factors which determine the width of the commutating zone and the number of coils undergoing commutation at a time. The thickness of brush should be so selected that it covers 2 to 3 commutator segments.

Current carried by each brush spindle is $2I_a/p$ as there are as many brush spindles as the number of poles.

Total brush contact area per spindle $A_b = \dfrac{2I_a}{p\delta_a}$ ...(9·77)

where δ_a is the current density in the brushes which can be taken from Table 9·10.

Each spindle may have many brushes. The area of each individual brush should be taken so that it does not carry more than 70 A. It is better to use a large number of brushes of relatively smaller width than a few very wide brushes.

Table 9·10.

Properties of Brush Materials

Type of Material	*Brush contact drop V*	*Current density A mm²*	*Pressure k N/m²*	*Commutator speed m/s*	*Co-efficient of friction*
Natural graphite	0·7—1·2	0·1	14	50—60	0·1—0·2
Hard carbon	0·7—1·8	0·065—0·085	14—20	20—30	0·15—0·25
Electrographitic	0·7—1·8	0·085—0·11	18—21	30—60	0·1—0·2
Metal graphitic	0·4—0·7	0·1—0·2	18—21	20—30	0·1—0·2

Suppose, n_b = number of brushes per spindle, and w_b and t_b are respectively the width and thickness of each brush respectively.

Contact area of each brush $a_b = w_b t_b$ Total contact area of brushes in one spindle

$$A_b = n_b a_b = n_b w_b t_b$$

∴ Width of each brush $w_b = \dfrac{A_b}{n_b t_b} = \dfrac{2I_a}{p\delta_a n_b t_b}$ ...(9·78)

The number n_b is so selected that each brush does not carry more than 70 A.

Length of commutator $L_c = n_b(w_b + c_b) + C_1 + C_2$ ...(9.79)

where c_b is the clearance between the brushes and depends upon the construction of the brush holder. It is usually 5 mm. C_1 is the clearance allowed for staggering the brushes. This clearance varies with the size of the commutator, but generally lies between 10 mm for small machines to about 30 mm for large machines. C_2 is the clearance for allowing the end play and is usually between 10 to 25 mm.

Eqn. 9.79 gives the minimum length of commutator ; the length required for brushes. If this length gives too small a dissipating surface, so that the temperature rise of commutator exceeds the permissible value, then L_c must be increased to give sufficient surface to dissipate the heat generated by the commutator losses.

9.52. Losses at commutator surface. The losses at the commutator are the brush contact losses and the brush friction losses.

Brush contact loss. The brush contact drop for different materials is given in Table 9.10. However, the brush contact loss also depends upon the condition of the commutator and upon the quality of commutation obtained. It is, therefore, very difficult to predetermine accurately the brush contact losses The brush contact drop v_b is independent of load current. The typical values of brush drop are 1 V per brush arm for carbon/graphite brushes, 0.25 V for metal/graphite, and 0.1 V or less for small machines used for control applications.

Brush friction loss. The brush friction loss depends upon the brush pressure, the peripheral speed of the commutator and the coefficient of friction between brush and the commutator. It may be calculated from the following approximate formula :

Brush friction loss $P_{bf} = \mu P_b p A_b V_c = \mu P_b A_B V_c$ ...(9.78)

where μ = co-efficient of friction, P_b = brush contact pressure on commutator, N/m² ;

$A_B = pA_b$ = total contact area of all brushes, m² ;

and V_c = peripheral speed of commutator, m/s.

The coefficient of friction depends upon the peripheral speed V_c, decreasing at high speeds. It largely depends upon the type of brush used. The co-efficient of friction for different brush materials is given in Table 9.10.

The co-efficient of friction μ normally varies from 0.1 to 0.3 and typical value of brush pressure, P_b for industrial machines is 12.5 kN/m².

Example 9.31. *Prove that the kW output of a d.c. machine with single turn coils is given by :*

$$P = \frac{1}{2} E_c \, ac \frac{V_a}{n} \frac{a}{p} 10^{-3}$$

where a = number of parallel paths, p = number of poles,

E_c = average voltage between adjacent segments, ac = ampere conductors per metre, V_a = peripheral speed of armature, m/s ;

n = speed of machine, rps.

(b) Taking the limiting values of E_c = 15 V, V_a = 50 m/s, ac = 50,000 A/m, find the largest output for a d.c. machine running at (i) 300 rpm (ii) 1000 rpm.

Solution. (i) Let N_c be the number of coils connected between adjacent segments.

∴ Voltage between adjacent segments

E_c = number of coils between adjacent segments × voltage per coil

= number of coils between adjacent segments × turns per coil × voltage per turn

$=$ number of coils between adjacent segments $\times$ turns per coil $\times 2 \times$ voltage per conductor

$=2\, N_c\, T_c\, e_s$

where T_c = turns per coil, and e_s = voltage per conductor.

Let Z be the total number of conductors and C be the total number of coils.

$\therefore \quad T_c=\dfrac{Z}{2C}=1$ (given) or $C=2Z$.

Also voltage per conductor $e_s=\Phi\, p\, n$.

$E_c=2N_c\times 1\times \Phi\, p\, n=2N_c\times \Phi\, p\, n=2N_c\times \dfrac{Ea}{Z}$.

$\therefore$ Generated emf $=\dfrac{Z\, E_c}{2N_c\, a}$

where E = terminal voltage.

kW output $P=E\, I_a\times 10^{-3}=\dfrac{Z\, E_c}{2N_c\, a}I_a \cdot 10^{-3}$

$$=\frac{E_c}{2\, N_c}\left(\frac{I_a}{a}\cdot \frac{Z}{\pi D}\right)\pi D \cdot 10^{-3}=\frac{E_c}{2N_c}=ac\, \pi D\, 10^{-3}$$

$$=\frac{E_c}{2\, N_c}\cdot ac \cdot \frac{\pi D n}{n} \cdot 10^{-3}=\frac{E_c}{2\, N_c}\cdot \frac{ac\, .\, V_a}{n}\cdot 10^{-3}\ \text{kW} \quad \ldots(i)$$

as $\quad ac=\dfrac{I_a}{a}\cdot \dfrac{Z}{\pi D} \quad$ and $V_a=\pi D n$.

For a lap winding, $N_c=1$.

$\therefore$ From Eqn. (*i*)

$$P=\frac{1}{2}\cdot E_c\, ac\frac{V_a}{n}\cdot 10^{-3}\ \text{kW} \quad \ldots(ii)$$

Also for a lap winding, number of parallel paths a = number of poles p.

$\therefore$ We can write (*ii*) as :

$$P=\tfrac{1}{2} \cdot E_c \cdot ac \cdot \frac{V_a}{n}\ \frac{a}{p}\cdot 10^{-3}\ \text{kW}. \quad \ldots(iii)$$

For a *wave winding,* $N_c=p/2$

$\therefore$ From Eqn. (*i*),

$$P=\frac{E_c}{p}\ ac\ \frac{V_a}{p}\cdot 10^{-3}\ \text{kW}. \quad \ldots(iv)$$

But for a wave winding, number of parallel paths $a=2$.

$\therefore$ We can write Eqn. (*iv*) as,

$$P=\tfrac{1}{2}\, E_c \cdot ac\ \frac{V_a}{n}\cdot \frac{a}{p}\cdot 10^{-3}\ \text{kW}. \quad \ldots(v)$$

Eqn. (*iii*) and (*v*) are similar.

$\therefore$ The general equation of output is

$$P=\frac{1}{2}\cdot E_c \cdot ac \cdot \frac{V_a}{n}\cdot \frac{a}{p}\cdot 10^{-3}\ \text{kW}.$$

(*b*) The largest output is obtained when $a=p$.

(Considering only simplex windings).

Largest output $P=\frac{1}{2} \cdot E_c \cdot ac \cdot \frac{V_c}{n} \times 10^{-3}$

$\therefore \quad P=\frac{1}{2} \times 15 \times 50{,}000 \times \frac{50}{n} \times 10^{-3}=\frac{18750}{n}$ kW.

(*i*) Speed $\quad n=\frac{300}{60}=5. \quad \therefore \quad P=\frac{18750}{5}=3750$ kW.

(*ii*) Speed $\quad n=\frac{1000}{60}=16{\cdot}66. \quad \therefore \quad P=\frac{18750}{16{\cdot}66}=1125$ kW.

Example 9·32. *Find the minimum number of poles for a 1200 kW generator if the average voltage between commutator segments is not to exceed 15 and the armature mmf per pole is not to exceed 10,000 A.*

Solution For simplex lap or wave winding, average voltage between adjacent segments $E_c=Ep/C$.

$\therefore$ Voltage of machine $E=CE_c/p$.

Taking single turn coils, total number of coils in the machine

$$C=\frac{Z}{2} \qquad \therefore \qquad E=\frac{ZE_c}{2p}.$$

Output of machine $P=E\,I_a \times 10^{-3}$ kW $=\frac{Z\,E_c}{2\,p} \cdot I_a \times 10^{-3}$

Now, armature mmf per pole $AT_a=\frac{I_a}{a} \cdot \frac{Z}{2p}$.

or $$\frac{I_a Z}{2p}=a\,AT_a.$$

Hence $$P=a\,AT_a\,E_c \times 10^{-3}$$

$\therefore$ Minimum number of parallel paths $a=\frac{P \times 10^3}{AT_a\,E_c}=\frac{1200 \times 10^3}{10{,}000 \times 15}=8.$

These parallel paths can be obtained by using a simplex lap winding with 8 poles.

$\therefore$ Minimum number of poles $=8$.

Example 9·33 *The following are the limiting parameters for a conventional industrial machine :*

average flux density in air gap $B_{av}=0{\cdot}8$ Wb/m²,

peripheral speed of commutator $V_c=40$ m/s,

average emf between adjacent segments $E_c=20$ V,

minimum pitch of commutator segments $\beta_c=4$ mm.

The frequency of flux reversals in the armature should not be less than 40 Hz on account of economic considerations. Calculate the maximum values of armature voltage that can be developed in a machine.

Solution. Minimum number of segments $C=pE/E_c$

where $\qquad E=$ emf generated in the armature.

Minimum pitch of segments

$$\beta_c=\frac{\pi D_c}{C}=\frac{\pi D_c\,n}{Cn}=\frac{V_c}{Cn}=\frac{V_c}{n} \cdot \frac{E_c}{pE}$$

but $f=\frac{pn}{2}. \qquad \therefore \quad \beta_c=\frac{V_c\,E_c}{2fE}$

Hence maximum armature voltage

$$E=\frac{V_a E_s}{2 f \beta_c}=\frac{40\times 20}{2\times 40\times 4\times 10^{-3}}=2500 \text{ V.}$$

Example 9 34. *Determine the total commutator losses for a 800 kW, 400 V, 300 r.p.m., 10 pole generator having the following data :*

commutator diameter, 100 cm ; current density in brushes, 0·075 A/mm² ; brush pressure 14·7 kN/m² ; co-efficient of friction, 0·23 ; total brush contact drop, 2·2 V.

Soluton. Armature current $I_a=\frac{800\times 1000}{400}=2000$ A.

Current per brush arm $=\frac{2\,I_a}{p}=\frac{2\times 2000}{10}=400$ A.

Brush area per brush arm $A_b=\frac{400}{0\cdot075}=5330$ mm².

Total brush area on the commutator $A_B=pA_b=10\times 5330$ mm² $=53\cdot3\times 10^{-3}$ m².

Peripheral speed $V_c=\pi D_c\, n=\pi\times 1\times 300/60=15\cdot7$ m/s.

Brush friction loss $W_{cf}=\mu\, P_b A_B\times V_c=0\cdot23\times 14\cdot7\times 10^3\times 53\cdot3\times 10^{-3}\times 15\cdot7=2830$ W.

Brush contact loss $=I_b\times$ brush constant drop $=2000\times 2\cdot2=4400$ W.

$\therefore$ Total commutator loss $=2830+4400=7230$ W.

Example 9 35. *The commutator of a 50 rpm machine is 0·3 m in diameter. The brush friction loss is 100 W. If at full load the commutator loss is twice the brush friction loss, calculate the length of commutator which will give a final temperature rise of 40°C. Assume that a commutator of this diameter and 75 mm in length running at 700 rpm gives a temperature rise of 40°C with a commutator loss of 300 W.*

The cooling co-efficient is $c=\frac{K}{1+0\cdot1\, V_c}$

where V_c is the peripheral speed of commutation in m/s and K is a constant.

Solution. Considering the commutator of 75 mm length which runs at 700 rpm.

We will be denoting quantities connected with it by suffix 1.

Peripheral speed $V_{c1}=\pi D_{c1}\, n_1=\pi\times 0\cdot3\times 700/60=11$ m/s.

Barrel surface of commutator $S_1=\pi\, D_{c1}\, L_{c1}=\pi\times 0\cdot3\times 75\times 10^{-3}=0\cdot0707$ m².

$\therefore$ Cooling co-efficient $c_1=\frac{K}{1+0\cdot1\, V_{c1}}=\frac{K}{1+0\cdot1\times 11}=0\cdot476\, K$.

From Eqn. 3 52 temperature rise, $\theta=\frac{Q\times c}{S}$.

$$\therefore \qquad 40=\frac{350\times 0\cdot476\, K}{0\cdot0707} \qquad \text{or} \qquad K=0\cdot017..$$

Now consider the commutator whose length is to be calculated. Let us refer its quantities with suffix 2.

Peripheral speed $V_{c2}=\pi D_{c2}\, n_2=\pi\times 0\cdot3\times 500/60=7\cdot85$ m/s.

$\therefore$ Cooling co-efficient $c_2=\frac{0\cdot017}{1+0\cdot1\times 7\cdot85}=0\cdot00952$.

Loss to be dissipated $Q_2=2\times$ brush friction loss $=200$ W.

Barrel surface of commutator $=\pi D_{c2} L_{c2}=\pi\times0.2\times L_{c2}=0.942\ L_{c2}$.

Now the temperature rise is 40°C,

or $$40=\frac{200\times0.00952}{0.942\ L_{c2}}.$$

$\therefore$ Length of commutator $L_{c2}=\dfrac{200\times0.00952}{0.942\times40}$ m $=50.5$ mm.

Example 9·36. *The armature of a 10 pole, 1000 kW, 500 V, 300 rpm, d.c. generator has a diameter of 1·6 m. There are 450 coils. Determine suitable axial length and diameter for the commutator, giving details of brushes, having regard to commutation conditions and temperature rise.*

The design limitations are :

Peripheral speed of commutator $\not>$ 20 m/s, pitch of segments $\not<$ 4 mm, current/brush $\not>$ 70 A, temperature rise $\not>$ 40°C.

The other data given is :

The brushes span three segments approximately ; brush contact drop = 1·5 V, co-efficient of friction = 0·15, brush pressure = 20 kN/m², cooling co-efficient = 0·012/(1+0·1 V_c).

Make suitable assumptions for clearance between brush boxes, staggering of brushes and end play.

Solution. Speed $n=500/60=5$ rps.

The diameter of commutator is assumed as 0·62 of armature diameter.

$\therefore$ Diameter of commutator $D_c=0.62\times1.6\simeq1.0$ m.

Peripheral speed of commutator $V_c=\pi\times1.0\times5=15.7$ m/s (within limits)

Pitch of commutator segments $\beta_c=(\pi\times1)/450$ m $\simeq7$ mm. This is more than the minimum allowable pitch of 4 mm.

Armature current $I_a=\dfrac{1000\times1000}{500}=2000$ A.

$\therefore$ Current/brush arm $=\dfrac{2I_a}{p}=\dfrac{2\times2000}{10}=400$ A.

Minimum number of brushes $n_b=\dfrac{400}{70}\simeq6$ as the current per brush $\not>$ 70 A.

Using 8 brushes per brush arm.

Current carried by each brush $=\dfrac{400}{8}=50$ A.

Area of each brush $a_b=\dfrac{50}{60\times10^{-3}}=0.833\times10^{-3}$ mm².

Each brush covers 3 segments.

$\therefore$ Thickness of each brush $t_b=3\times7=21$ mm.

Width of each brush $w_b=a_b/t_b=0.833\times10^{3}/21\simeq40$ mm.

$\therefore$ Actual area of brush $=21\times40=840$ mm².

Total area of brushes per brush arm $A_b=8\times840$ mm² $=6.72\times10^{-3}$ m².

Assuming : clearance between brushes = 5 mm, allowance for staggering $C_1=20$ mm and allowance for end play $C_2=20$ mm.

$\therefore$ Length of commutator $L_c=8(40+5)+20+20=400$ mm $=0.4$ m.

Assuming 50 mm as the space accupied by risers, length commutator including the space for risers

$$=0{\cdot}4+50\times10^{-3}=0{\cdot}45 \text{ m}.$$

Temperature Rise

Brush contact drop $P_c=1{\cdot}5\times2000=3000$ W.

Brush friction loss $=\mu P_b p A_b V_c=0{\cdot}15\times20\times10^3\times10\times6{\cdot}72\times10^{-3}\times15{\cdot}7$

$=3165$W.

Total loss dissipated from commutator surface$=3000+3165=6165$ W.

Barrel surface of commutator$=\pi\times1\times0{\cdot}4=1{\cdot}256$ m^2.

$$\text{Cooling co-efficient, } c=\frac{0{\cdot}012}{1+0{\cdot}1\times15{\cdot}7}=4{\cdot}67\times10^{-3}.{^\circ}m^2\text{—}{^\circ}C/W.$$

$$\text{Temperature rise of commutator, } \theta=\frac{6165\times4{\cdot}67\times10^{-3}}{1{\cdot}256}=22{\cdot}9C.$$

LOSSES AND EFFICIENCY

9·53. Losses and efficiency. The losses in a d.c. machine can be classified into two general types :

(a) Rotational losses (b) I^2R losses.

9·53·1. Rotational losses. Rotational losses are made up of :

(i) Friction and windage losses (ii) Iron losses.

Friction and windage losses. The friction losses occur in the bearings and at the commutator.

The amount of bearing friction losses depends upon the pressure in the bearing, the peripheral speed of the shaft at the bearing and the co-efficient of friction between the bearing and the shaft.

The friction losses in the commutator due to brush contact can be calculated from Eqn. 9·78.

The windage losses produced by rotation depend upon the peripheral speed of the rotor, the rotor diameter, the core length and largely upon the construction of the machine. The windage loss, in fact, depends upon too many intricate factors and cannot be easily assessed.

Table 9·11 gives the values of windage plus bearing friction losses expressed as percentage of rated output.

Table 9·11. Windage and bearing friction losses.

Peripheral speed m/s	*Losses percentage of output*
10	0·2
20	0·4
30	0·6
40	0·9
50	1·2

Iron Losses.

Iron losses have been dealt with in chapter 4.

The iron loss per kg is given by the following relationships

$$0{\cdot}06\, f\, B_m^2 + 0{\cdot}008\, f^2\, B_m^2\, t^2 \text{ for teeth}$$

and

$$0{\cdot}06\, f\, B_m^2 + 0{\cdot}005\, f^2\, B_m^2\, t^2 \text{ for core.}$$

B_m is expressed in Wb/m^2, f in Hz and t (thickness of laminations) in mm.

The pulsation loss in the pole faces may be taken as 20 to 50 per cent of total iron loss as calculated above.

9·53·2. I²R losses. The I^2R losses occur in :

(*i*) Main series circuit of armature. They include the I^2R losses in

(*a*) the armature winding$=I_a^2 r_a$ (*b*) the interpole winding$=I_a^2 r_{ip}$

(*c*) the series field winding$=I_a^2 r_s$ (*d*) the compensating winding$=I_a^2 r_c$

(*e*) the brush contacts$=P_{bc}$.

The compensating winding is normally not present. The series field winding is present only in compound machines.

(*ii*) Shunt field circuit. These losses include the I^2R losses in the winding itself and also the I^2R losses in the field regulator.

For the purposes of I^2R losses in industrial machines, the resistance of windings should be calculated at 75°C for insulation classes A, E and B and at 115°C for classes F and H.

9·53·3. Stray load losses. These are certain types of losses which cannot be easily determined. They appear when the machine is loaded. This indeterminable losses are called stray load losses and are due to the following reasons :

(*i*) there is a large increase in the iron losses when the machine is loaded due to distortion of field caused by armature reaction,

(*ii*) due to eddy currents in conductors, there is an additional I^2R loss,

(*iii*) when a coil undergoes commutation, it is short-circuited by the brush. This causes a circulating current to flow which produces additional losses.

The stray load losses may be assumed as 0·5 or 1·0% of the *basic output* for machines with and without compensating winding respectively. The basic output is that power which corresponds to maximum current at the highest rated voltage for constant speed machines. The basic speed for variable speed machines depends upon method of speed control.

9·53·4. Efficiency. The percentage efficiency is given by

$$\eta = \frac{\text{output}}{\text{output}+\text{losses}} = 1 - \frac{\text{losses}}{\text{input}}$$

Condition for maximum efficiency. In the case of constant speed constant voltage machines, the losses may be divided into three main categories :

(*i*) *Constant losses* P_0. These losses do not vary if the load current varies. For a shunt machine these losses are : bearing friction and windage, brush friction, shunt excitation losses and no load iron losses.

(*ii*) *Losses proportional to armature current* P_1. These losses vary linearly with load current *i.e.* brush contact loss.

(*ii*) *Losses proportional to square of current* P_2. These losses vary as the square of the armature current and for a shunt machine include : I^2R losses in the armature circuit and stray load losses.

The total losses are $P_L = P_0 + P_1 + P_2$

and to a first approximation, be expressed in the form

$$P_L = k_0 + k_1 I + k_2 I^2$$

where I denotes the total current drawn by the armature of the motor.

In order to simplify the analysis, the total current drawn by the motor is assumed as I. The motor input power is, therefore, VI.

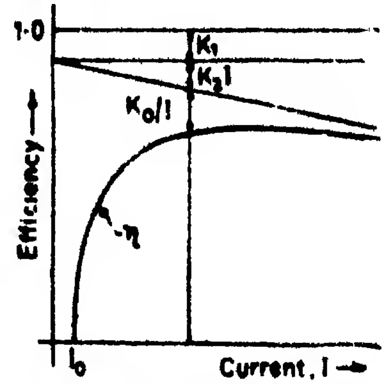


Fig. 9·53. Variation of losses and efficiency with load current.

$$\text{Efficiency, } \eta = 1 - \frac{k_0 + k_1 I + k_2 I^2}{VI}$$

$$= 1 - \left(\frac{K_0}{I} + K_1 + K_2 I\right)$$

where $K_0 = k_0/V$, $K_1 = k_1/V$ and $K_2 = k_2/V$.

The variation of efficiency with load current is shown in Fig. 9·53.

In order to find out the maximum efficiency, η is differentiated with respect to I and $d\eta/dI$ is equated to zero.

$$\frac{d\eta}{dI} = -\frac{K_0}{I^2} + K_2 = 0 \quad \text{or} \quad K_0 = K_2 I^2$$

∴ For maximum efficiency :

constant losses = losses proportional to square of current.

9·54. Temperature rise. The calculation of temperature rise in armature has already been explained in chapter 3 page 111.

Design Problem. *Design a separately excited industrial motor, 440 V, 4 pole, 75 kW continuous rating, duty cycle S_1, 1000 rpm, screen protected, class B insulation. The machine is built into a standard frame with shaft height 400 mm. The motor is supplied power from SCR bridge rectifiers.*

Solution.

Main Dimensions

Rated power output $P = 75$ kW. Speed $n = 1000/60 = 16{\cdot}67$ rps

Assuming an efficiency of 0·92, power developed by armature

$$P_a = \left(\frac{1+2\eta}{3\eta}\right)P = \left(\frac{1+2\times0{\cdot}92}{3\times0{\cdot}92}\right)\times75 = 77{\cdot}2 \text{ kW}$$

The specific loadings used in the machine are :

$B_{av} = 0{\cdot}5$ Wb/m² and $ac = 35{,}000$ A/m.

∴ Output co-efficient, $C_0 = \pi^2 B_{av}\, ac \times 10^{-3}$

$$= \pi^2 \times 0{\cdot}5 \times 35000 \times 10^{-3} = 172{\cdot}7,$$

Product $D^2L = P_a/C_0 n = 77{\cdot}2/(172{\cdot}7 \times 16{\cdot}67) = 26{\cdot}8 \times 10^{-3}$ m³.

The cross section of the pole face should be almost a square in order to have saving in the cost of conductors used for the field coils.

For a square pole face $L=\psi T=\psi\pi D/p$.

The ratio of pole arc to pole pitch, ψ, is assumed as 0·67.

Thus, $L=0{\cdot}67\times\pi\times D/4=0{\cdot}526\ D$

or $0{\cdot}5\ 6D^3=26{\cdot}8\times10^{-3}$ or $D=0{\cdot}37$ m.

The standard armature diameter for the specified diameter is $D=0{\cdot}40$ m. This diameter gives nearly a square cross-section for the pole face.

Length $L=26{\cdot}6\times10^{-3}/(0{\cdot}40)^2=0{\cdot}166$ m.

The main dimensions of the machine are :

$D=0{\cdot}40$ m$=400$ mm and $L=0{\cdot}17$ m$=170$ mm.

The standard frame used can accommodate core lengths upto 0·25 m.

Pole pitch, $\tau=\pi\times0{\cdot}4/4=0{\cdot}314$ m ; pole arc, $b=0{\cdot}67\times0{\cdot}314=0{\cdot}21$ m.

Peripheral speed of armature $V_a=\pi\times0{\cdot}4\times16{\cdot}67=21$ m/s.

The length of core is 0·17 m and, therefore, one radial ventilating duct of 10 mm width is used.

Net iron length $L_i=0{\cdot}9(0{\cdot}17-1\times0{\cdot}01)=0{\cdot}144$ m.

The frequency of flux reversals is $f=4\times16{\cdot}67/2=33{\cdot}34$ Hz.

The thickness of laminations used for the machine is 0·35 mm

ARMATURE DESIGN

Armature Winding

Terminal voltage 240V. Line current $I_l=75\times1000/(0{\cdot}92\times240)=340$ A.

Product $P_a\times rpm=77{\cdot}2\times1000=7{\cdot}72\times10^4$. From Fig. 9·35 corresponding to this product :

Field current $I_f=1{\cdot}2\%=0{\cdot}012\times340=4$A,

and internal voltage drop$=4{\cdot}7\%=0{\cdot}047\times240=11{\cdot}3$ V.

∴ Armature current $I_a=340-4=336$ A and

generated emf $E=240-11{\cdot}3=228{\cdot}7$ V.

Type of winding

Since the armature current is 336 A (*i.e.* less than 400A), a simplex wave winding is used for the machine.

The number of parallel paths is 2 current

∴ Current per parallel path $I_a=336/2=168$ A.

Number of armature conductors

Flux per pole $\Phi=0{\cdot}5\times0{\cdot}314\times0{\cdot}17=26{\cdot}7\times10^{-3}$ Wb.

∴ Number of armature conductors, $Z=\dfrac{228{\cdot}7\times2}{26{\cdot}7\times10^{-3}\times16{\cdot}67\times4}=257.$

Co... machines, the slots

(i) *Constan*...ue of slot pitch lies between 25 mm to 35 mm

shunt machine these ...er of slots lie between.

losses and no load iron ... and $\pi\times400/25=50$.

(ii) *Losses proportiona*... commutation conditions, the number of slots per pole

current *i.e.* brush contact loss. ...the total number of slots range from 36 to 64.

(iii) For a simplex wave winding the number of slots should not be a multiple of pair of poles. The machine has four poles and, therefore, the number of slots should not be a multiple of 2. Thus the number of slots is an odd integer.

(iv) The number of slots per pole arc should be an integer $+1/2$ in order to reduce flux pulsations. With $\psi=0{\cdot}67$, the number of slots per pole arc are nearly an integer $+1/2$ for armature slots $S=39$ and 45.

(v) The number of coil sides per layer should not be a multiple of pair of poles in a simplex wave winding. Therefore, we can use only 1, 3 or 5 coil sides per layer *i.e.* 2, 4 or 6 coil sides per slot.

The use of armature slots $S=45$ and coil sides per slot $u=6$ with single turn coils ($T_c=1$) results in a suitable winding for the machine.

Total number of coils, $C=\frac{1}{2}uS=1/2\times6\times45=135$.

Total number of armature conductors actually used $Z=2\times1\times135=270$.

Thus, there is only a small difference between the number of conductors calculated and actually used.

Number of conductors per slot $Z_s=270/45=6$.

Slot pitch$=\pi\times0{\cdot}4/45=0{\cdot}028$ m$=28$ mm.

Modified value of flux per pole, $\Phi=26{\cdot}7\times10^{-3}\times257/270=25{\cdot}4\times10^{-3}$ Wb.

The values of specific loadings actually used are :

$B_{av}=\Phi/\tau L=25{\cdot}4\times10^{-3}/(0{\cdot}314\times0{\cdot}17)=0{\cdot}476$ Wb/m^2.

$ac=I_zZ/\pi D=168\times270/(\pi\times0{\cdot}4)=36{,}100$ A/m.

Checks

(i) *Slot Loading*. Ampere conductors per slot$=I_zZ_s=168\times6=1008$ A.

This is lower than the maximum permissible value of 1500 A.

(ii) *Pitch of commutator segments*. The diameter of commutator, D_c, is assumed to be 260 mm which is 0·65 times the diameter of armature.

$\therefore$ Pitch of commutator segments $\beta_c=\pi D_c/C=\pi\times260/135=6$ mm.

This is greater than the minimum allowable pitch of 4 mm.

Winding Layout

Back pitch $y_b=2C/p\pm K=2\times135/4\pm K=67, 69$.

We use $y_b=67$ as with this value of back pitch, $(y_b-1)/u$ is an integer and hence split coils are avoided.

For a simplex wave winding, commutator pitch $y_c=2(C\pm1)/p=67, 68$.

Taking $y_c=67$, total winding pitch $Y=2y_c=134$.

$\therefore$ Front pitch $y_f=Y-y_b=67$.

Design of Slot

The current density used in the armature winding is $\delta_a=6$A/mm^2

$\therefore$ Area of each armature conductor$=168/6=28$ mm^2.

Referring to Table 17·1, a rectangular copper conductor $2{\cdot}8\times10$ mm^2 which has an area of 27·5 mm^2 is used.

The dimensions of insulated conductor are $3{\cdot}5\times10{\cdot}7$ mm^2.

The arrangement of conductors in the armature slots is shown in Fig. 9·54.

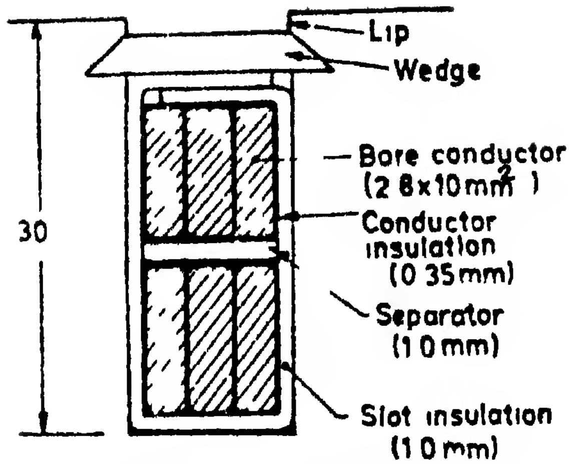


Fig. 9 54. Details of armature slots.

Slot Width

Bare conductor	3×2 8	=8 4 mm
Conductor insulation	6×0·35	=2 1 mm
Slot insulation	2×1 0	=2 0 mm
Slack		=0 5 mm
Total slot width	W_s	=13 0 mm.

Slot Depth

Bare conductor	2×10	=20 0 mm
Conductor insulation	4×0·35	= 1·4 mm
Slot insulation	3×1·0	= 3 0 mm
Separator		= 1 0 mm
Lip and Wedge		= 4 0 mm
Slack		= 0 6 mm
Total slot depth	d_s	=30·0 mm

Check for flux density in teeth

The flux density in teeth at a section 1/3 height of tooth from the root should not exceed 2·1 Wb/m².

Slot pitch at 1/3 height from root

$$y_{s1/3}=\frac{\pi[400-(4/3)\times 30]}{45}=25\cdot 1 \text{ mm.}$$

∴ Width of tooth at 1/3 height from root $W_{t1/3}=25\cdot1-13\cdot0=12\cdot1$ mm.

Flux density $B_{t1/3}=\frac{p\Phi}{\psi S\ L_i\ W_{t1/3}}=\frac{4\times25.4\times10^{-3}}{0.67\times45\times0.144\times12.1\times10^{-3}}$

$=1.93\ \text{Wb/m}^2$.

$\therefore$ Within limits.

Length of air gap

Armature mmf per pole $AT_a=\frac{168\times270}{2\times4}=5670$ A.

The mmf required for air gap is assumed to be 0·6 times the armature mmf.

$\therefore$ Mmf required for air gap, $AT_g=0.6\times5670=3400$ A.

Maximum flux density in air gap $B_g=B_{av}/\psi=0.476/0.67=0.71\ \text{Wb/m}^2$.

Assuming a gap contraction factor $K_g=1.15$,

length of air gap $l_g=\frac{AT_g}{800{,}000\ B_g\ K_g}=\frac{3400}{800{,}000\times0.71\times1.15}=5.2\times10^{-3}$ m.

Length of air gap used $l_g=5.0$ mm.

Armature core

Flux in armature core $\Phi_c=\Phi/2=25.4\times10^{-3}/2=12.7\times10^{-3}\ \text{Wb/m}^2$.

A flux density of 1·2 Wb/m² is used for the armature core.

$\therefore$ Area of armature core $A_c=12.7\times10^{-3}/1.2=10.58\times10^{-3}\ \text{m}^2$.

Depth of armature core $d_c=A_c/L_i=10.58\times10^{-3}/0.144=0.0735$ m.

Depth of armature core actually used $d_c=75$ mm.

Actual area of armature core $A_c=0.144\times0.075=10.8\times10^{-3}\ \text{m}^2$.

Actual flux density in armature core $B_c=12.7\times10^{-3}/(10.8\times10^{-3})=1.18\ \text{Wb/m}^2$.

Internal diameter of armature $D_i=D-2(d_s+d_c)=0.19$ m $=190$ mm.

DESIGN OF FIELD SYSTEM

Pole section

Referring to Table 9·7, leakage co-efficient for the main poles is assumed as $C_l=1.15$.

$\therefore$ Flux in the pole body $\Phi_p=1.15\times25.4\times10^{-3}=29.2\times10^{-3}$ Wb.

The flux density in pole body is assumed to be $B_p=1.5\ \text{Wb/m}^2$.

$\therefore$ Area of pole body $A_p=29.2\times10^{-3}/1.5=19.47\times10^{-3}\ \text{m}^2$.

The axial length of the main pole is the same as that of the armature.

$\therefore$ Axial length of main pole $L_p=L=0.17$ m.

There are no radial ventilating ducts in the pole body.

Net iron length of pole body $L_{pi}=0.9\times0.17=0.153$ m.

$\therefore$ Width of pole body $b_p=A_p/L_{pi}=19.47\times10^{-3}/0.153=0.127$ m $=127$ mm.

Tentative design of field winding

The field mmf at full load is assumed to be 90 armature mmf.

$\therefore$ Field mmf at full load $AT_{fl}=0.9\times5670=5100$ A.

The depth of field winding is assumed as 45 mm and, therefore, $d_f=0.045$ m.

The space factor is assumed to be 0·68 and loss dissipated (based upon cooling surface excluding top and bottom surfaces) can be taken as 700 W/m²

From Eqn. 9·41, mmf per metre of winding height

$$=10^4\times\sqrt{q_f\,S_f\,d_f}=10^4\times\sqrt{700\times0{\cdot}68\times0{\cdot}045}=46{\cdot}3\times10^3\ \text{A/m}.$$

$\therefore$ Height of field winding $h_f=5100/(4{\cdot}63\times10^3)=0{\cdot}11$ m $=110$ mm.

The height of pole shoe is 20 mm and leaving a space of 10 mm for insulation etc., height of pole $h_{pl}=110+20+10=140$ mm $=0{\cdot}14$ m.

Yoke

Flux in yoke $\Phi_y=\Phi/2=12{\cdot}7\times10^{-3}$ Wb.

Assuming a flux density of 1·5 Wb/m², area of yoke $A_y=\Phi_y/B_y=12{\cdot}7\times10^{-3}/1{\cdot}5=8{\cdot}47\times10^{-3}$ m².

Depth of yoke $d_y=A_y/L_{pl}=8{\cdot}47\times10^{-3}/0{\cdot}153=0{\cdot}0554$ m.

The depth of yoke is $d_y=55$ mm. Length of Pole

Outer diameter of yoke $D_y=D+2(l_g+h_{pl}+dy)=0{\cdot}8$ m $=800$ mm.

Fig. 9·55 shows a sketch of the magnetic circuit of the machines.

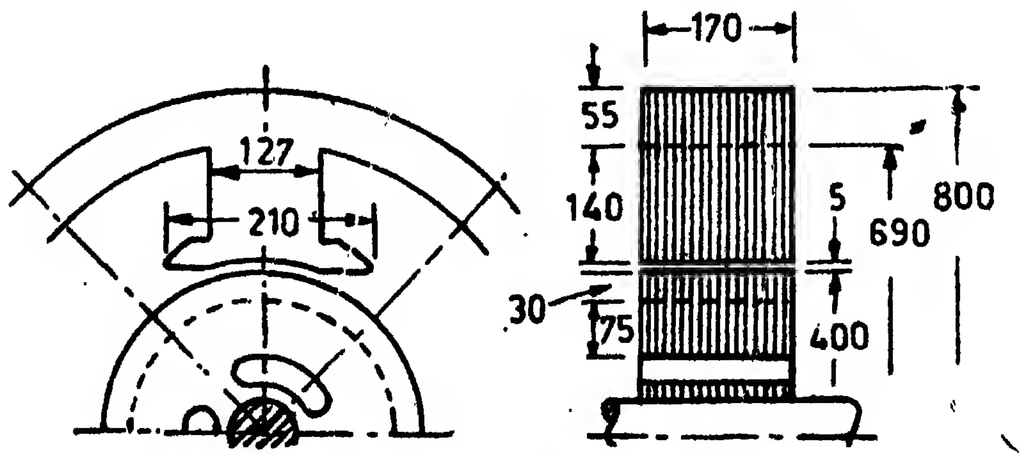


Fig. 9·55. Magnetic circuit of 75 kW, 240 V motor.

Magnetic Circuit

The mmfs required for the different parts of the magnetic circuit are calculated below. The B-'at' curves given in Fig. 4·2 are used for the calculations.

(i) ***Mmf required for air gap.***

Flux density at pole centre $B_g=B_{av}/\psi=0{\cdot}476/0{\cdot}67=0{\cdot}71$ Wb/m².

Ratio $\dfrac{\text{slot opening}}{\text{gap length}}=\dfrac{13}{5}=2{\cdot}6.$

Carter's co-efficient corresponding to this ratio is 0·35 (See Fig. 4·9, for open slots).

Gap contraction factor for slots, K_{gs}

$$=\frac{y_s}{y_s-K_{cs}\,W_s}=\frac{28}{28-0{\cdot}35\times13}=1{\cdot}194.$$

Ratio $\dfrac{\text{duct width}}{\text{gap length}}=\dfrac{10}{5}=2.$

Corresponding to this ratio, co-efficient $K_{cd}=0{\cdot}3.$

$\therefore$ Gap contraction factor for ducts K_{gd}

$$=\frac{L}{L-K_{cd}\,n_d\,W_d}=\frac{170}{170-0\cdot3\times1\times10}=1\cdot018.$$

Gap contraction factor $K_g=1\cdot194\times1\cdot018=1\cdot215$.

$\therefore$ Mmf required for air gap $AT_g=800{,}000\times0\cdot71\times1\cdot215\times5\times10^{-3}=3450$ A.

(ii) Mmf required for teeth

The flux density in teeth at a section 1/3 height from root

$B_{t1/3}=1\cdot93$ Wb/m^2 (calculated earlier).

From Eqn. 4·49, $K_s=\dfrac{L\,y_{s1/3}}{L_i\,W_{t1/3}}=\dfrac{170\times25\cdot1}{144\times12\cdot1}=2\cdot45$.

Following the procedure explained in Example 4·91 (page 138) and using Fig. 4·2, $at_t=16000$ A/m.

$\therefore$ Mmf required for teeth $AT_t=16000\times30\times10^{-3}=480$ A.

(iii) Mmf required for core

Corresponding to a flux density of 1·18 Wb/m^2 in core, $at_c=300$ A/m.

Length of flux path in core $l_c=\dfrac{\pi(0\cdot4-2\times0\cdot03-0\cdot075)}{2\times4}=0\cdot104$ m. no. of poles

$\therefore$ Mmf required for core $AT_c=300\times0\cdot104=30$ A.

(iv) Mmf required for pole body.

Corresponding to a flux density of 1·5 Wb/m^2 in the pole body, $at_p=1000$ A/m.

$\therefore$ Mmf required for pole body $AT_p=1000\times0\cdot14=140$ A.

(v) Mmf required for yoke

The flux density in yoke is 1·5 Wb/m^2 and, therefore, $at_y=2000$ A/m.

*Length of flux path in yoke $l_y=\dfrac{\pi(0\cdot8-0\cdot055)}{2\times4}=0\cdot293$ m.

$\therefore$ Mmf required for yoke $AT_y=1000\times0\cdot293=300$ A.

Total field mmf required at no load

$AT_f=AT_g+AT_t+AT'_c+AT_p+AT_y=4400$ A.

Assuming the full load field mmf to be 1·15 times the no load field mmf,

field mmf required at full load $AT_{fl}=1\cdot15\times4400=5060$ A.

Design of field winding

The voltage across the shunt field winding is 240 V and 20% of this voltage is kept in reserve for speed control. The four field coils are connected in series.

$\therefore$ Voltage across each field coil $E_f=\dfrac{240-0\cdot2\times240}{4}=48$ V.

The data available for the design field coils is :

Mmf of each coil $AT_{fl}=5060$ A.

Height of coil $h_f=108$ mm. Depth of coil $d_f=110$ mm.

Axial length of pole $L_p=1700$ mm. Width of pole body $b_p=127$ mm.

*The yoke has been assumed to be circular for the purposes of magnetic circuit calculations. However, the yoke for this machine is not circular but is of the shape shown in Fig. 9·56.

The mean conductor temperature is assumed 75°C and the resistivity of copper at this temperature is 0·021 Ω/m/mm². $2(L_p + b_p) + 2d_f$

Length of mean turn $L_{mt}=2(0{\cdot}17+0{\cdot}127)+2\times0{\cdot}045)=0{\cdot}774$ m.

Area of conductor $a_f=\dfrac{AT_{fl}\,\rho\,L_{mt}}{V}=\dfrac{5060\times0.021\times0{\cdot}774}{48}=1{\cdot}71$ mm².

The nearest standard conductor has a bare diameter of 1·5 mm and the diameter with synthetic enamel covering is 1·58 mm (See Table 17·7).

Area of conductor used $a=1{\cdot}77$ mm².

Space factor $S_f=0{\cdot}75(d/d_1)^2=0{\cdot}676$.

Number of turns $T_f=\dfrac{0{\cdot}676\times0{\cdot}1\ldots}{1{\cdot}77\times10^{-6}}=1890.$

Resistance of each coil $R_f=\dfrac{1890\times0{\cdot}021\times0{\cdot}774}{1{\cdot}77}=17{\cdot}4\ \Omega.$

Field current $I_f=48/17{\cdot}4=2{\cdot}76$ A.

∴ Field mmf provided $AT_{fl}=2{\cdot}76\times1890=5210$ A.

Loss in each field coil $Q_f=(2{\cdot}76)^2\times17{\cdot}4=132{\cdot}5$ W.

Cooling surface $S=2\times0{\cdot}774(0{\cdot}11+0{\cdot}045)=0{\cdot}24$ m².

Cooling co-efficient $c=\dfrac{0{\cdot}16}{1+0{\cdot}1\,V_a}=\dfrac{0{\cdot}16}{1+0{\cdot}1\times21}=0{\cdot}052.$ (See page 111)

∴ Temperature rise $\theta=132{\cdot}5\times0{\cdot}052/0{\cdot}24=28{\cdot}7$°C

DESIGN OF COMMUTATOR

Diameter of commutator $D_c=0{\cdot}26$ m; peripheral speed $V_c=\pi\times0{\cdot}26\times16{\cdot}67=13{\cdot}6$ m/s; number of segments $C=135$; pitch of segments $\beta_c=6{\cdot}05$ mm.

Brushes

Armature current $I_a=340-2{\cdot}76=337$ A.

Current per brush arm $I_b=2\times333/4=168{\cdot}5$ A.

Electrographitic brushes are used. The current density in the brushes is 100×10^{-3} A/mm².

The current in each brush should not be more than 70A. Using 4 brushes per arm.

current in each brush $=168{\cdot}5/4=42{\cdot}1$ A.

Area of each brush $a_b=\dfrac{42{\cdot}1}{100\times10^{-3}}=421$ mm².

The brushes should cover at least 2½ segments.

∴ Thickness of each brush $t_b=2{\cdot}5\times6{\cdot}05\cong15$ mm.

Width of each brush $W_b=421/15=88$ mm.

Area of each brush used $a_b=15\times21=420$ mm².

Area of brushes in each brush arm $A_b=4\times420\times10^{-6}=1{\cdot}68\times10^{-3}$ m².

Allowing 5 mm for clearance between brushes, 10 mm for staggering and 10 mm for end play,

length of commutator excluding risers $L_c=4(28+5)+10+10=160$ mm $=0{\cdot}16$ m.

Allowing a 20 mm space for risers,

overall length of commutator $=160+20=180$ mm $=0{\cdot}18$ m.

Losses

The brush contact drop is assumed as 1 V per brush.

∴ Brush contact loss $P_{bc}=2\times1\times337=670$ W.

The brush pressure used is 20 kN/m² and the co-efficient of friction is 0·15.

∴ Brush friction loss $P_{bf}=0.15\times20\times10^3\times4\times1.68\times10^{-4}\times13.6=270$ W.

Total loss at commutator$=670+270=940$ W.

Barrel surface of commutator$=\pi\times0.26\times0.16=0.1$

The total heat dissipating surface including half of the surface of risers is estimated to be 0·15 m².

From Table 3·6, cooling co-efficient for commutator

$$c=\frac{0.025}{1+0.1\,V_c}=\frac{0.025}{1+0.1\times13.6}=0.0106.$$

Temperature rise of commutator$=\dfrac{940\times0.0106}{0.15}=66.4$ °C.

DESIGN OF INTERPOLES

Width of commutation zone $w_c=\{[u/2-a/p]\beta_c+(b_b-t_i)\}\dfrac{D}{D_c}$ (See Eqn. 9·63)

$$=\{[6/2-2/4]\times6.05+15\}\times\frac{0.4}{0.26}=46.3 \text{ mm}.$$

The length of air gap under the interpole is generally 1 to 2 times the length of air gap under main pole.

Assuming length of air gap under interpole $l_{gi}=1.2\,l_g=6.0$ mm.

The width of interpole should be greater than 1·5 times the slot pitch

The width of interpole, W_{ip}, is assumed as 50 and mm, therefore, the interpole face will cover the commutation zone and also approximate to two slot pitch (the slot pitch at the gap surface is 28 mm).

Specific slot permeance $\lambda_s=\mu_0\left[\dfrac{h_1}{3W_s}+\dfrac{h_2}{W_s}+\dfrac{2h_3}{W_s+W_o}+\dfrac{h_4}{W_o}\right]$.

$$=4\pi\times10^{-7}\left[\frac{24.8}{3\times13}+\frac{2.7}{13}+\frac{2\times2.5}{17+3}+\frac{1}{13}\right]=1.37\times10^{-6}.$$

Tooth top specific permeance $\lambda_t=\dfrac{\mu_0\,W_{ip}}{6l_{gi}}=\dfrac{4\pi\times10^{-7}\times50}{6\times6}=1.75\times10^{-6}$.

Outhang $L_o=0.3\tau+0.0125\,d_o=0.3\times0.314+0.0125\times0.03=0.095$ m.

Length of overhang of one coil side $L_o=\sqrt{(\tau/2)^2+L_o^2}$ (See Fig. 9·51 page 562)

$$=\sqrt{(0.314/2)^2+(0.095)^2}=0.184 \text{ m}.$$

Periphery of one coil side $b_o=2\times(10.5+10.7)=42.4$ mm$=0.0424$ m.

From Eqn. 9·64, overhang specific permeance

$$\lambda_o=\frac{L_o}{L}\left(0.23\log\frac{L_o}{b_o}+0.07\right)\times10^{-6}=0.22\times10^{-6}.$$

Total specific permeance $\lambda=1.37\times10^{-6}+1.75\times10^{-6}+0.22\times10^{-6}=3.34\times10^{-6}$.

Time of commutation $T_c=\{[\pi/2-a/p]\beta_c+t_b-t_i\}/V_c$ (See Eqn.)
$=2.2$ ms.

From Eqn. 9.67, average reactance voltage.
$E_{rav}=4T_c\lambda LI_sZ_s/T_c=4\times1\times3.34\times10^{-6}\times0.17\times168.5\times6/(2.2\times10^{-3})=1.03$ V.

The maximum reactance voltage may be taken as $E_{rm}=1.3$ V.

$\therefore$ Maximum flux density under the interpole $B_{gim}=E_{rm}/LV_a=1.3/(0.17\times21)$
$=0.364$ Wb/m².

The axial length of interpole is the same as that of the same pole.

The length of air gap under the interpole is greater than that under the main pole herefore, the gap contraction factor for the interpoles, K_{gi}, is smaller than K_g. Taking 1.18.

Mmf required for the gap under the interpole $AT_{gi}=800,000\times0.364\times1.18\times6\times10^{-3}$
$=2060$ A.

Armature mmf per pole $AT_a=168.5\times270/(2\times4)=5690$ A.

Mmf required for the interpole $AT_i=2060+5690=7750$ A.

Number of turns on each interpole $T_i=7750/\underline{337}=23$.

Taking a current density 2.5 A/mm² for the interpoles, area of interpole winding ctor $a_i=337/2.5=134.8$ mm².

Using a strap conductor 3 mm × 50 mm,

area of conductor provided = 148 mm².

The interpole winding consists of two coils, one at the top consisting of 14 turns and her at the bottom having 9 turns.

s and Efficiency

(*i*) ***Friction and windage losses***

The friction losses have already been calculated.

Brush friction loss $P_{bf}=270$ W.

The peripheral speed of armature of the machine is 21 m/s and, therefore, the friction in bearings and the winding losses are 0.4% of the output (See Table 9.11).

$\therefore$ Bearing friction and windage losses $=\frac{0.4}{100}\times75\times1000=300$ W.

Total friction and windage losses $=270+300=570$ W.

(*ii*) ***Iron losses***

Mean width of tooth $=\frac{\pi(D-d_s)}{S}-W_s=\frac{\pi(0.4-0.03)}{45}-0.013=0.0128$ m.

Weight of armature teeth $=45\times0.17\times0.0128\times0.03\times7800=22.9$ kg.

The flux density in teeth at 1/3 height from root is 1.93 Wb/m².

Specific iron loss in teeth $=0.06\,f\,B_m^2+0.008\,f^2\,B_m^2\,t^2$
$=0.06\times33.34\times(1.93^2+0.008)\times(33.34)^2\times(1.93)^2\times(0.35)^2=11.5$ W/kg.

Iron loss in teeth $=22.9\times11.5=260$ W.

Weight of armature core $=\pi(0.4-2\times0.03-0.08)\times0.144\times0.08\times7800=73$ kg.

Specific iron loss in c... $=0.06\,f\,B_m^2+0.005\,f^2\,B_m^2\,t^2$
$=0.06\times33.34\times(1.18)^2+0.005\times(33.34)^2\times(1.18)^2\times(0.35)^2=3.5$ W/kg.

Iron loss in core $=3.5\times73=256$ W. Total iron loss $=260+256=516$ W.

Allowing an additional 20% for loss caused by ripples in the motor supply waveform, total iron loss $=1{\cdot}2\times516\approx620$ W.

(*iii*) *Copper losses*

Length of mean turn of armature $=2L+2{\cdot}3\tau+5d_c=1{\cdot}21$ m.

Resistance of armature $r_a=\dfrac{270}{2}\times\dfrac{1{\cdot}21\times0{\cdot}021}{27{\cdot}5\times(2)^2}=0{\cdot}031\ \Omega$ at 75°C.

Armature copper loss $=(337)^2\times0{\cdot}031=3520$ W.
Copper loss in shunt field $=243\times2{\cdot}76=660$ W.
The length of mean turn of interpole winding is estimated as 0·6 m.
Resistance of interpole winding $r_i=4\times23\times0{\cdot}021\times0{\cdot}6/148=0{\cdot}0078\ \Omega$.
Copper loss in interpole winding $=(337)^2\times0{\cdot}0078=890$ W.
Brush contact loss $=2\times329=660$ W.
The efficiency can be obtained by loss summation.

Armature copper loss	= 3520 W
Field copper loss	= 660 W
Interpole copper loss	= 890 W
Brush contact loss	= 670 W
Iron loss	= 620 W
Friction and windage losses	= 570 W
Total losses =	6930 W

Input $=75000+6830=81930$ W

Efficiency at full load $\eta=\dfrac{75000}{81930}\times100=91{\cdot}5\%$.

Temperature rise of armature

The cooling co-efficients are taken from Table 3·6 Page 111
(*i*) Outside cylindrical surface $=\pi\times0{\cdot}4\times0{\cdot}17=0{\cdot}214$ m².

Cooling co-efficient $=\dfrac{0{\cdot}03}{1+0{\cdot}1\times21}=0{\cdot}0097$.

Loss dissipation $=0{\cdot}214/0{\cdot}0097=22{\cdot}1$ W/°C.
(*ii*) Inside cylindrical surface $=\pi\times0{\cdot}19\times0{\cdot}17=0{\cdot}101$ m².
Peripheral speed $=\pi\times0{\cdot}19\times16{\cdot}67=10$ m/s.

Cooling co-efficient $=\dfrac{0{\cdot}03}{1+0{\cdot}1\times10}=0{\cdot}015$.

Loss dissipation $=\dfrac{0{\cdot}101}{0{\cdot}015}=6{\cdot}7$ W/°C.

(*iii*) Cooling surface of one duct and two end surfaces

$$=3\times\frac{\pi}{4}(0{\cdot}4^2-0{\cdot}19^2)=0{\cdot}292\ \text{m}^2.$$

Velocity of air in ducts $=0.1\times21=2.1$ m/s.

Cooling co-efficient$=\dfrac{0.15}{2.1}=0.071$. Loss dissipation$=\dfrac{0.292}{0.071}=4.1$ W/°C.

Total loss dissipation$=22.1+6.7+4.1=32.9$ W/°C.

Total loss to be dissipated=copper loss in active portion+iron loss

$$=3520\times\frac{2\times0.17}{1.21}+620=1610 \text{ W}.$$

∴ Temperature rise of armature$=\dfrac{1610}{32.9}=49$°C.

The complete assembly of machine is shown in Fig. 9.56.

DESIGN SHEET

Rating—75 kW *Voltage—240 V.*

Speed—1000 r.p.m. *Type—Industrial separately excited*

Main Dimensions

1.	Output		75 kW
2.	Armature power	P_a	77.2 kW
3.	Armature diameter	D	400 mm
4.	Speed	n	16.27 r.p.s.
5.	Armature peripheral speed	V_a	21 m/s.
6.	Output co-efficient	C_o	172.7
7.	Average flux density	B_{av}	0.5 Wb/m²
8.	Ampere conductors per metre	ac	35000
9.	D^2L		26.8×10^{-3} m³
10.	Gross core length	L	170 mm
11.	Number of poles	p	4
12.	Frequency of reversals	f	33.34 Hz
13.	Pole pitch	τ	314 mm
14.	Pole arc	b	210 mm
15.	Number of ducts	n_d	1
16.	Width of each	W_d	10 mm
17.	Gross iron length		160 mm
18.	Net iron length	L_i	144 mm

Winding

1.	Voltage drop (estimated)	$I_a r_m$	11.3 V
2.	Generated emf (estimated)	E	228.7 V.
3.	Air gap flux density (estimated)	B_g	0.71 Wb/m²
4.	Ratio pole arc/pole pitch	ψ	0.67
5.	Flux per pole	Φ	25.4 m Wb
6.	Armature current (estimated)	I_a	336 A
7.	Type of winding		Simplex Wave
8.	Parallel paths	a	2

9. Armature conductors	Z	270
10. Number of slots	S	45
11. Conductors per slot	Z_s	6
12. Coil sides per slot	u	6
13. Turns per coil	T_c	1
14. Coils	C	135
15. Coil sides		270
16. Back pitch	y_b	67
17. Front pitch	y_f	57
18. Slot pitch	y_s	28 mm
19. Conductor cross section		$2{\cdot}8 \times 10$ mm^2
20. Area of conductor	a_a	$27{\cdot}5$ mm^2
21. Slot depth	d_s	30 mm
22. Slot width	W_s	13 mm
23. Length of mean turn	L_{mt}	$1{\cdot}21$ m
24. Hot resistance	r_a	$0{\cdot}031\ \Omega$
25. Armature voltage drop	$I_a r_a$	$10{\cdot}5$ V
26. Armature copper loss		3520 W

Magnetic Circuit

1. Armature mmf per pole	AT_a	5690 A
2. Estimated field mmf		5060 A
3. Mmf per metre of coil height		46300 A
4. Height of pole	h_{pl}	140 mm
5. Air gap density	B_g	$0{\cdot}71$ Wb/m^2
6. Length of gap	l_g	5 mm
7. Gap contraction factor	K_g	$1{\cdot}215$
8. Mmf for gap	AT_g	3450 A
9. Height of teeth	d_t	30 mm
10. Flux density in teeth at $\frac{1}{3}$ height	$B_{t1/3}$	$1{\cdot}93$ Wb/m^2
11. Mmf for teeth	AT_t	480 A
12. Depth of core	d_c	75 mm
13. Area of core	A_c	$10{\cdot}8 \times 10^{-3}$ m^2
14. Flux density in core	B_c	$1{\cdot}18$ Wb/m^2
15. Mmf for core	AT_c	90 A
16. Leakage factor	C_l	$1{\cdot}15$
17. Area of pole	A_p	$19{\cdot}47 \times 10^{-3}$ m^2
18. Width of pole	b_p	127 mm
19. Length of pole	L_p	170 mm
20. Flux density in pole	B_p	$1{\cdot}5$ Wb/m^2
21. Mmf for pole	AT_p	140 A
22. Depth of yoke	d_y	55 mm
23. Length of yoke	L_y	170 mm

24.	Area of yoke	A_y	$8{\cdot}47\times10^{-3}$ m^2
25.	Flux density yoke	B_y	1·5 Wb/m^2
26.	Mmf for yoke	AT_y	300 A
27.	Total mmf at no load		4400 A
28.	Total mmf at full load		5060 A

Main Field Winding

1.	Mmf required		5060 A
2.	Number of turns per pole	T_f	1890
3.	Conductor area	a_f	1·77 mm^2
4.	Conductor diameter (bare)	d	1·5 mm
5.	Conductor diameter (insulated)	d_1	1·58 mm
6.	Depth of winding	d_f	45 mm
7.	Height field winding	h_f	110 mm
8.	Resistance of each coil	R_f	17·4 Ω
9.	Field current	I_f	2·76 A
10.	Mmf provided		5210 A
11.	Copper loss per coil	Q_f	132·5 W
12.	Cooling surface per coil		0·24 m^2
13.	Temperature rise	θ	28·7 °C
14.	Current density	δ_f	1·56 A/mm^2

Interpole

1.	Width of commutating zone	w_c	46·3 mm
2.	Length of interpole air gap	l_{gi}	6 mm
3.	Width of interpole	W_{ip}	50 mm
4.	Length of interpole	L_{ip}	170 mm
5.	Flux density under interpole	B_{gi}	0·364 Wb/m^2
6.	Interpole mmf	AT_i	7750 A
7.	Interpole turns	T_i	23
8.	Conductor dimensions		3×50 mm^2
9.	Conductor area	a_i	148 mm^2
10.	Winding height		110 mm
11.	Resistance of winding	r_i	0·0078 Ω
12.	Voltage drop		2·6 V
13.	Power loss		890 W

Commutator

1.	Commutator diameter	D_c	260 mm
2.	Segments	C	135
3.	Pitch of segments	β_c	6 mm
4.	Thickness of mica separator		0·8 mm
5.	Peripheral speed	V_c	13·6 m/s.
6.	Current density in brushes	δ_b	0·1 A/mm^2

7.	Current per brush arm		168·5 A
8.	Area of brushes per arm		168×10^{-6} m^2
9.	Number of brushes per arm	n_b	4
10.	Area of each brush	a_b	420 mm^2
11.	Thickness of each brush	t_b	15 mm
12.	Width of each brush	w_b	28 mm
13.	Effective length of commutator	L_c	160 mm
14.	Total length of commutator		180 mm
15.	Brush contact loss	P_{bc}	670 W
16.	Brush friction loss	P_{bf}	270 W
17.	Temperature rise		66·4 °C

Losses

1.	Armature copper loss	3520 W
2.	Field copper loss	660 W
3.	Total iron loss	620 W
4.	Friction and windage loss	570 W
5.	Interpole copper loss	890 W
6.	Brush contact loss	670 W
7.	Total loss	6930 W
8.	Efficiency at full load	91·5 per cent
9.	Armature temperature rise	49 °C

UNSOLVED PROBLEMS

1. Find the main dimensions of a 200 kW, 250 V, 6 pole, 1000 r.p.m generator. The maximum value of flux density in the gap is 0·87 Wb/m^2 and the ampere conductors per metre of armature periphery are 31000. The ratio of pole arc to pole pitch is 0·67 and the efficiency is 91 per cent. Assume the ratio of length of core to pole pitch = 0·75. [Ans. $D = 0{\cdot}57$ m ; $L = 0{\cdot}23$ m]

2. Find the main dimensions and the number of poles of a 37 kW, 230 V, 1400 r.p.m. shunt motor so that a square pole face is obtained. The average gap density is 0·5 Wb/m^2 and the ampere conductors per metre are 22000. The ratio of pole arc to pole pitch is 0·7 and the full load efficiency is 90 per cent.

[Ans. poles = 4 ; $D = 0{\cdot}3$ m ; $L = 0{\cdot}165$ m]

3. Determine the diameter and length of armature core for a 55 kW, 110 V, 1000 r.p.m., 4 pole shunt generator, assuming the specific electric and magnetic loadings as 26000 ampere conductors per metre and 0·5 Wb/m^2 respectively. The pole arc should be about 70 per cent of pole pitch and length of core about 1·1 times the pole arc. Allow 10 ampere for the field current and assume a voltage drop of 4 volt for the armature circuit. Specify the winding used and also determine suitable values for the number of armature conductors and the number of slots. [Ans. $D = 0{\cdot}356$ m ; $L = 0{\cdot}212$ m ; lap $Z = 228$; $S = 38$]

4. Choose, with reasons, a suitable number of poles for a 430 kW, 250 V, 250 r.p.m. shunt generator having an armature diameter of 1·2 m and a length of 0·3 m. The maximum gap density is 0·9 Wb/m^2 and the ratio of pole arc to pole pitch is 0·7. [Ans. 10]

5. A 150 kW, 6 pole, d.c. generator has a flux per pole of 0·06 Wb and its speed is 600 r.p.m. Determine the armature demagnetizing and cross-magnetizing mmf per pole if the brushes are given an actual lead of 3°. [Ans. 347 A ; 3125 A]

6. Determine the maximum output that can be obtained from a 375 r.p.m. d.c. generator without exceeding a peripheral speed of 40 m/s, an average emf of 7 V in each conductor, and an electric loading of 45000 ampere conductors per metre. [Ans. 2080 kW]

7. A 4 pole, 25 h.p. 500 V, 600 r.p.m. series crane motor has an efficiency of 82 per cent. The pole faces are square and the ratio of pole arc to pole pitch is 0·67. Assuming an average gap density of 0·55 Wb/m^2 and ampere conductors per metre as 17000, obtain the main dimensions of the core and particulars of a suitable armature winding. [Ans. $D = 0{\cdot}36$ m ; $L = 0{\cdot}174$ m ; $Z = 840$; $C = 105$; $S = 35$]

8. A 4 pole, 400 V, 963 r.p.m shunt motor has an armature 0·3 m in diameter and 0·2 m in length The commutator diameter is 0·22 m. Give full details of a suitable winding stating the number of slots, the number of commutator segments and the number of conductors in each slot-for an average flux density of approximately 0·55 Wb/m^2 in the air gap. Give also a sectional view through a slot, showing the details of insulation and arrangement of conductors. [Ans. Lap. $S=30$, $C=90$, 5 turn coils]

9. The armature of a 550 kW, 500 V, 8 pole generator is 1·2 m in diameter and the specific electric loading is approximately 27500 ampere conductors per metre. Design a symmetrical armature winding. Give particulars of the equilizers and sketch the cross section of a slot with insulated conductors in position, specifying the insulation. [Ans. $S=144$, $C=430$, $T_c=1$, $Y_{eq}=108$, $Y_{gh}=9$]

10. The diameter of armature of a 300 kW, 460 V, six pole d.c. generator is 1 m. Determine the number of conductors, the number of slots and the number of commutator segments to satisfy the conditions of symmetry and to provide tappings for three slip rings for a static balances. Assume provisionally a specific electric loading of 25000 ampere conductors per metre. Calculate the cross-section of conductors using a current density of 4 A/mm^2. [Ans. $Z=738$, $S=123$, $C=369$, $a_a=27{\cdot}2$ mm^2]

11. Calculate the following design data for 37 kW, 220 V, 4 pole 900 r.p.m. d.c. shunt motor :

(*i*) Length and diameter of core. (*ii*) Number of armature conductors and type of winding.

(*iii*) Armature and line currents. (*iv*) Armature mmf per pole.

(*v*) Field mmf per pole. (*vi*) Length of air gap.

Assume : Specific magnetic loading$=$0·7 Wb/m^2 ; specific electric loading$=$26000 ampere conductors per metre ; ratio pole arc to of pole pitch$=$0·7 ; length of machine$=$pole arc ; internal drop in armature circuit$=$10 V ; field current$=$2·5 A ; field mmf$=$1·25$\times$armature mmf ; mmf required for air gap$=$0·5$\times$armature mmf ; gap contraction factor$=$1·15 ; full load efficiency$=$89 per cent.

12. A d.c. machine has the following data and physical properties : 550 V, 275 kW, 900 r.p.m., 6 poles, wave wound armature winding in 180 slots with 8 conductors in each slot, 70 percent of armature surface covered by poles.

(*a*) Find the armature mmf per pole at rated current.

(*b*) Find the number of conductors that should be placed in each pole face to provide an adequate compensating winding.

(*c*) Estimate the number of turns on each of 6 commutating poles to provide a flux density of 0·4 Wb/m^2 across an effective gap length of 5 mm at rated armature current if the machine has no compensating winding.

(*d*) Repeat part, (*c*) if the machine has a compensating winding of part (*d*).

[Ans. (*a*) 30,000 A (*b*) 84 (*c*) 64 (*d*) 22]

13. A rectangular field coil is required to produce an mmf of 8000 A when power dissipated is 250 W, the temperature rise is 40°C and the specific dissipation is 30 W/m^2—°C from the outer surface neglecting top and bottom of the coil. The length of a turn on the innermost layer is 0·68 m and the coil is 1·5 m high. The resisitivity of conductors is 0·02 Ω/m/mm^2. Find the current density in field conductors. [Ans 1·51 A/mm^2]

14. The field coils (cylindrical) of a 4 pole, 460 V, d.c. shunt motor are required to produce an mmf of 5700 A per pole. The length of mean turn is 0·66 m and the winding depth is 40 mm. Heat is dissipated at the rate of 1000 W/m^2 of the outside cylindrical surface of coil. Determine :

(*a*) dimensions of coil, (*b*) the size of conductor, (*c*) the number of turns.

Assume the diameter of insulated conductor to be 0·175 mm greater than the diameter of bare conductor. Resistivity$=$0·02 Ω/m/mm^2. [Ans. Height$=$154·5 mm ; conductor area$=$0·818 mm^2 ; turns$=$4325]

15. Design the shunt field winding of a 6 pole, 440 V, d.c. generator allowing a drop of 15% in the regulator. The data available is :

Mmf per pole$=$7000 A ; length of mean turn$=$1·2 m ; winding depth$=$35 mm ; loss dissipation$=$650 W/m^2 Take the diameter of insulated wire to be 0·4 mm greater than that of bare wire Resistivity$=$0·02 Ω/m/mm^2. [Ans. Height$=$0·19 m ; current$=$5·65 A ; turns$=$1275 ; area$=$2·77 mm^2]

16. The open circuit characteristics of a 4 pole, d.c. shunt generator are :

Mmf/pole : A	0	2500	5000	7500	10,000	12,500
Generated emf : V	0	57	114	168	207	226

The field coils are circular, 30 mm thick and 100 mm long, and fit tightly round a circular pole 75 mm in diameter. Calculate (*a*) the diameter shunt field conductor to give a generated voltage of 220 V. The resistance of the field winding, assuming that the insulation increases the diameter of field conductor by 0·4 mm. Resistivity$=$0·0220 Ω/m/mm^2

Calculate also the open circuit voltage if the resistance of field circuit is increased by (*a*) 3 Ω (*b*) 10 Ω.

[Ans. 0·155 mm ; 30·2 Ω ; 200 V ; will not excite

17. Calculate the reactance voltage induced per coil for a single turn two layer winding with two conductors per slot of a 250 kW, 525 V, 6 pole lap wound d.c. generator driven at 220 r.p.m. The number of armature conductors is 600. The inductance per coil is 0 0057 mH. The brush covers one commutator segment.

If the armature diameter is 1·6 m and core length is 0 3 m, determine the flux density under the interpole. The length of interpole is 0·18 m. [Ans. 1·98 V ; 0·3 Wb/m²

18. Obtain the rating of a machine with a lap wound armature running at 420 r.p.m. The voltage between adjacent segments at no load is 15 V, the specific electric loading is 32,000 ampere conductors per metre and the peripheral speed is 50 m/s. [Ans. 1714 kW

19. Determine the total commutator losses for a 1000 kW, 500 V, 800 r p m , 10 pole generator Given commutator diameter = 1·0 m ; current density at brush contact = 75×10^{-3} A/mm² ; brush pressure = 14·7 kN/m² ; co-efficient of friction = 0·28 ; brush contact drop = 2·2 V. [Ans. 7240 W

20. Design a suitable commutator for a 350 kW, 600 r p.m., 440 V, 6 pole d.c. generator having an armature diameter of 0 75 m The number of coils is 288. Assume suitable values wherever necessary.

[Ans D_c = 0·52 m, L_c = 0·35 m

10

Three Phase Induction Motors

10·1. Introduction

A polyphase induction motor consists essentially of two major parts, the stator and the rotor. The construction of each one is basically a laminated core provided with slots which house windings. When one of the windings is excited with a.c. voltage, a rotating field is set up. This field produces an e.m.f. in the other winding by transformer action which in turn circulates current in the later if it is short circuited. The currents flowing in the second winding interact with the field produced by the first winding thereby producing a torque which is responsible for the rotation of the rotor.

10·2. Stator

This is the stationary part of an induction motor. It is a cylindrical structure, built up of dynamo grade laminations. The laminations are either 0·35 or 0·5 mm thick. Motors having outside diameters of the stator core upto about one metre use one-piece core laminations as shown in Fig. 10·1. The centre circles are used for punching rotor laminations. The stator laminations are welded at several places around the outer cylindrical surface and the stack is later pushed into a frame for assembly. For larger sized motors, the stator cores are made of segmental laminations. This is done in order to avoid wasting of steel from the centre of the rotor and from the outside corners of the stator with the cores made up of segments assembled in ring form. The segments are held together by axial key bars fitting into dovetailed slots in the outer rim of the core. Fig. 10·2 shows two typical methods of securing stator cores with segmental laminations. The peripheral length of one segment, usually between 0·3 to 0·6 m, is chosen to give most economical balance between the cost of dies, the cost of assembly and the amount of scrap left over in cutting the laminations from steel strips. For quick assembly of stator core, maximum chord of segment should not be less than 0·37 m. It is necessary to determine the total number of dovetails for fixing the segments to the frame and also to determine the location and number of dovetails per segment. As a rule, the distance between adjacent dovetails should not be less than 60 mm.

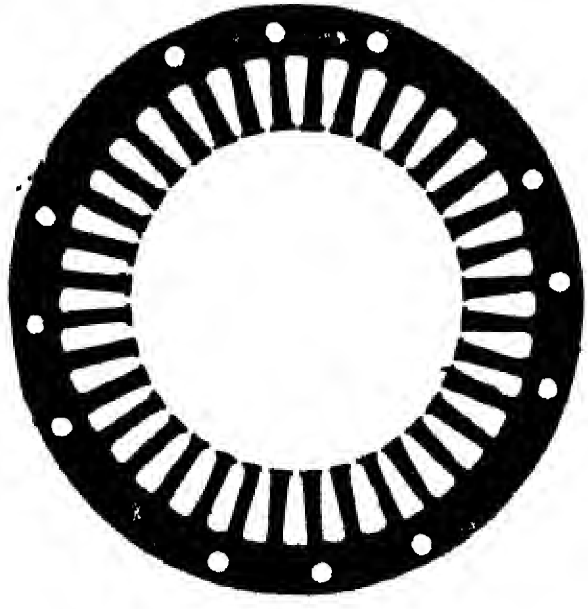


Fig. 10·1. Stator lamination for small machines.

The total number of segments is chosen in such a way as to provide an equal number of joints in the core flux paths of alternating poles. This is because, if the flux leaving the stator core from every south pole encounters a core joint when it turns anticlockwise, and no joint when it enters clockwise, the different reluctances offered to the two paths will result

into a net difference between the core fluxes in two directions of flow, and the resultant flux links with the shaft. This resultant flux produces an alternating voltage between the two ends of the shaft, giving rise to *shaft currents* which in turn may cause damage of bearings, unless the bearings are insulated from the end shields.

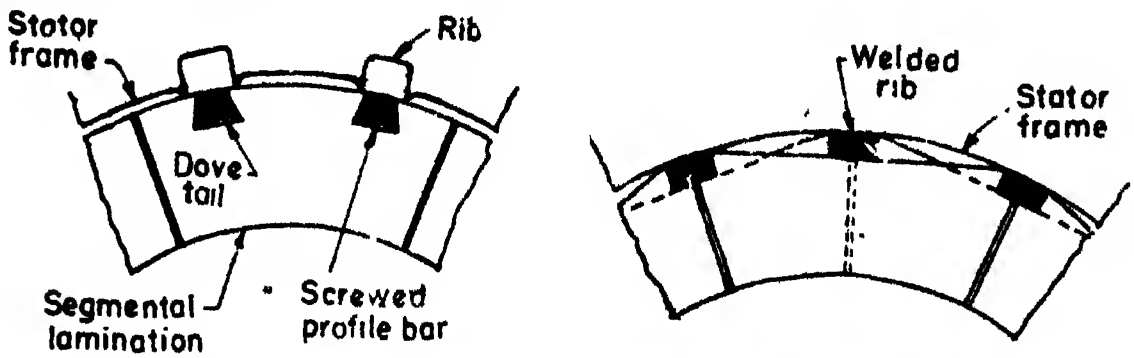


Fig. 10·2. Segmental laminations and methods of securing stator core.

Long cores are divided into a number of stacks with radial ventilating ducts in between in order to facilitate the cooling. The width of a single stack of core should not exceed 50 to 60 mm.

Fig. 10·3 shows a segmental lamination used for large induction motors. These motors have wide stator slots and consequently the teeth are thin in section. Therefore, the teeth

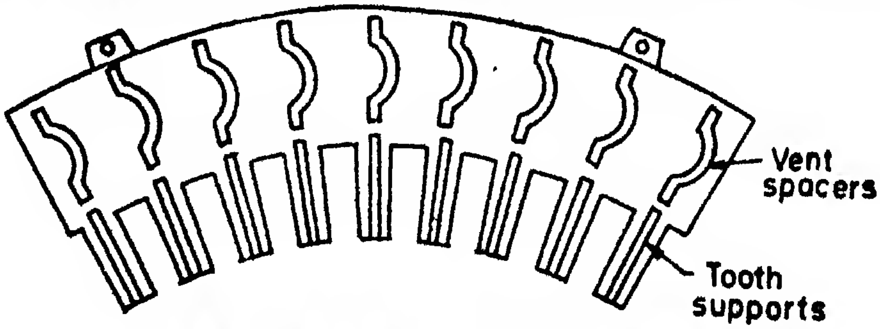


Fig. 10·3. Segmental lamination, tooth supports and vent spacers.

must be supported by tooth supports at the radial ventilating ducts. The supports act as vent spacers and also prevent vibrations. The tooth supports generally consist of a piece of rolled steel, spot welded to the end laminations.

10·3. Stator frames. Frames of electrical machines are structures in which stator core is assembled. They serve four distinct purposes :

(*i*) They enclose the core and windings.

(*ii*) They shield the live and moving machine parts from human contact and from injury caused by intruding objects or weather exposure.

(*iii*) They transmit the torque to the machine supports, and are therefore designed to withstand twisting forces and shocks.

(*iv*) They serve as ventilating housing or means of guiding the coolant into effective channels.

A great variety of designs is employed to meet the above requirements, and to adapt machines to particular service conditions. The design of frames has special significance in the case of induction motors of large dimensions on account of the relatively small air gaps

used in these machines. For induction motors the frame should be strong and rigid both during construction and after assembly of the machine. This is because the length of air gap is very small and if the frame is not rigid, the rotor will not remain concentric with stator giving rise to *unbalanced magnetic pull.*

The frames may be die-cast or fabricated. Machines up to about 50 kW rating usually have their frames die-cast in a strong silicon aluminium alloy and in some cases with the stator core cast in. The process of die-casting has the advantage that it facilitates the use of thicker cross section of frame at places where greater mechanical strength is required. The die-cast frames do not require machining.

The frames of larger sizes of machines are fabricated by welding steel plates. The advantage of fabrication is its adaptability to new designs and modifications.

Frames of small machines are made as a single unit. They are provided with feet by which they are fixed to the base plate. Machines, which have radial ventilating ducts, the stator core is placed inside the frame on axial ribs thereby providing an annular space for air between core and the frame.

The frames of totally enclosed machines are provided with axial fins in order to increase the heat dissipating surface. Fig. 10·4 shows a die-cast frame provided with axial fins.

The stator core laminations in small machines (which are punched as annular rings) are fixed to the frame with the help of clamping rings as shown in Fig. 10·5.

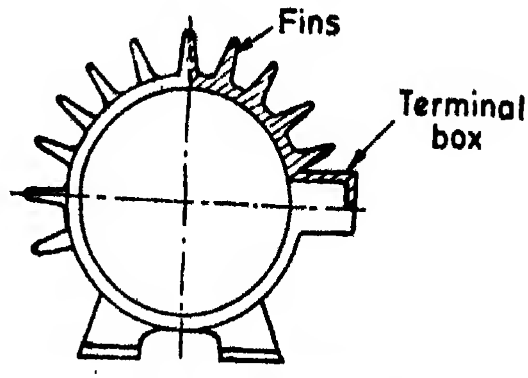


Fig. 10·4. Die-cast frame with axial fins.

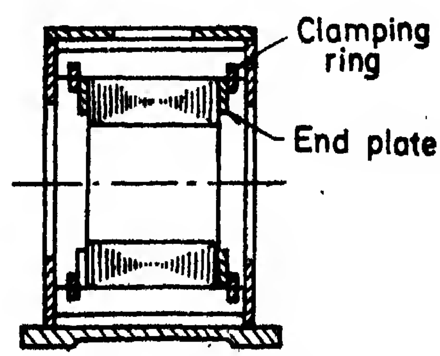


Fig. 10·5. Assembly of stator core using clamping rings.

The frames of medium and large sized machines are fabricated as stated earlier. The frame is a short cylinder, or a box, with end plates and axial ribs. The stator core is inserted into the frame after the inner surfaces of the ribs have been machined and the ends turned where necessary for the end covers.

Medium sized machines (*i.e.* machines whose stator core diameter exceeds 1 m, but is not more than 2·5 to 3·0 m) are provided with radial ventilating ducts. In this case ring chambers with large radial dimensions are required in order to obtain proper rigidity of frame and sufficient cross-section for ducts for air. As stated earlier, the stator core of these machines is assembled from segmental laminations and fixed to dovetails welded or screwed on to inside surface of the frame. The frame is usually made in the form of a box of T-shaped cross-section in order to increase rigidity. In machines of large axial length the box has intermediate walls in addition to the two side walls. These walls are so designed that

the two consecutive walls are not more than 0 5 m apart (see Fig. 10·6). This construction gives additional rigidity to the frame.

The frames of larger horizontal shaft machines (with outside diameter greater than 3·5 to 4 m) are made with joints located in the horizontal plane. Openings are located in the outside surface of frame for providing outlet to cooling air in radial direction.

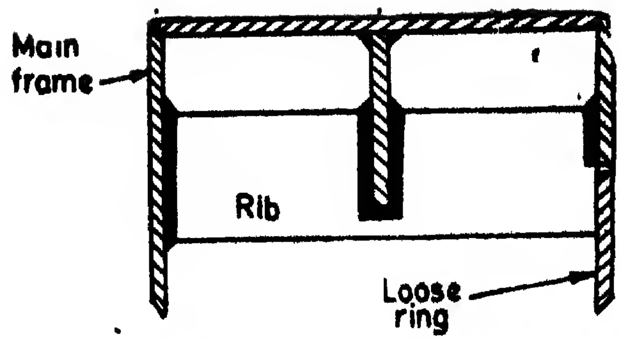


Fig. 10·6. Box-type stator frame

10·4. Rotor. Like stator, the rotor laminations are punched as a single unit in the case of small machines while in larger machines the laminations are segmented. Fig. 10·7 shows a one piece rotor lamination.

Rotor cores of small machines are often put on the shaft directly and keyed to it for transfer of torque. Washers or thrust rings are used for axial clamping (Fig 10 8). The thrust ring is put on the shaft when hot and on cooling the ring grips the shaft. However, this process is costly for mass production. In practice, a thrust ring with a cut (Fig. 10·9) is used. This ring is placed in a ring groove in the shaft.

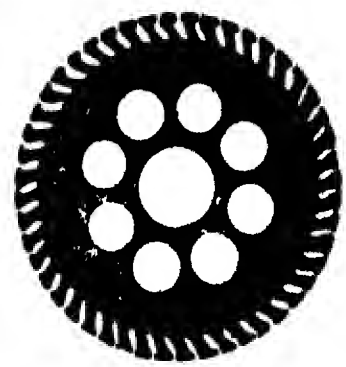


Fig. 10·7. Rotor lamination.

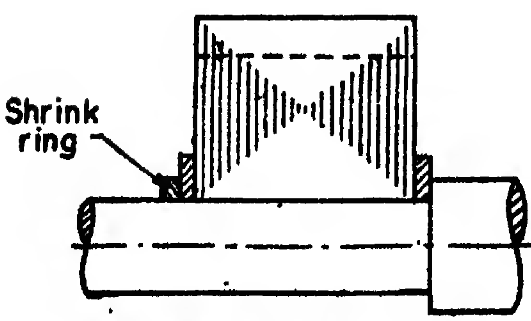


Fig. 10·8. Axial clamping of rotor laminations using shrink-rings.

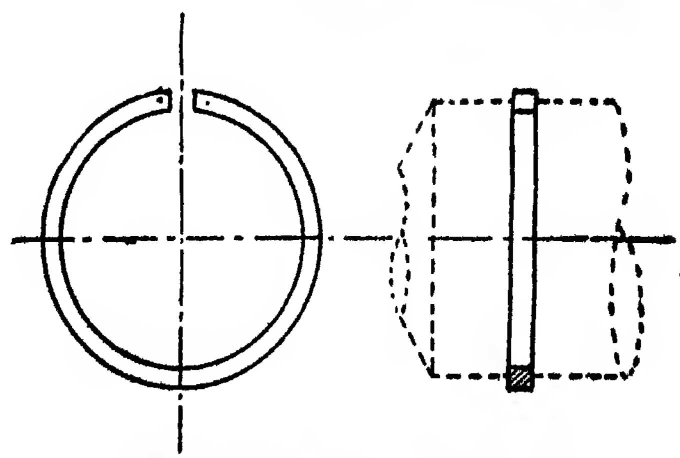


Fig. 10·9. Ring groove and thrust ring with cut for axial clamping of rotor laminations.

In order to provide paths for ventilating air, radial and axial ducts are used. The number of radial ventilating ducts provided in the rotor is equal to that in the stator.

The single unit laminations are fixed on the shaft by means of armature hub (Fig. 10·10) or on ribs (Fig. 10·11).

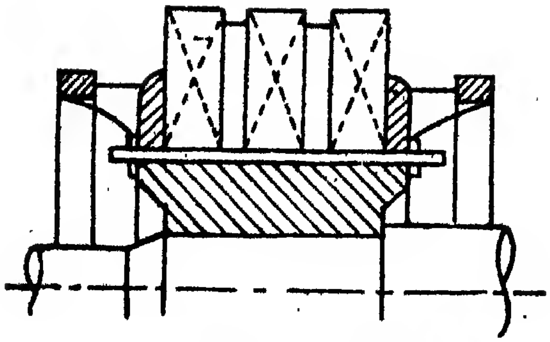


Fig. 10·10. Assembly of rotor laminations using an armature hub.

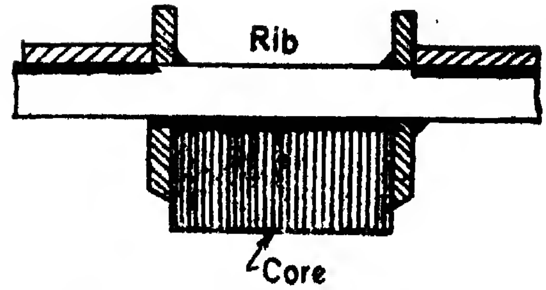


Fig. 10·11. Thickened part of shaft provided with ribs.

In the case cast-steel hub the laminations are pressed together in axial direction with clamping rings (Fig. 10·10). The core is compressed between end plates when ribs are used.

The segmental laminations are fixed to rotor spider. A rotor spider is shown in Fig. 10·12. This comprises of a shaft with arms and stiffners. For machines with large diameters, the rotor spider is built up from a hub, arms and ribs as shown in Fig. 10·13. The ribs are machined to accommodate the rotor core.

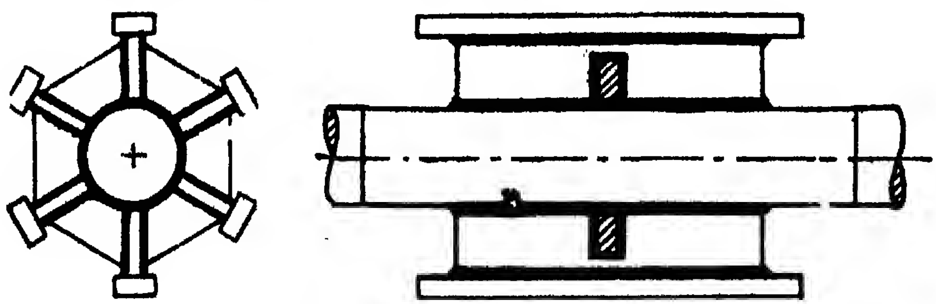


Fig. 10·12. Rotor spider

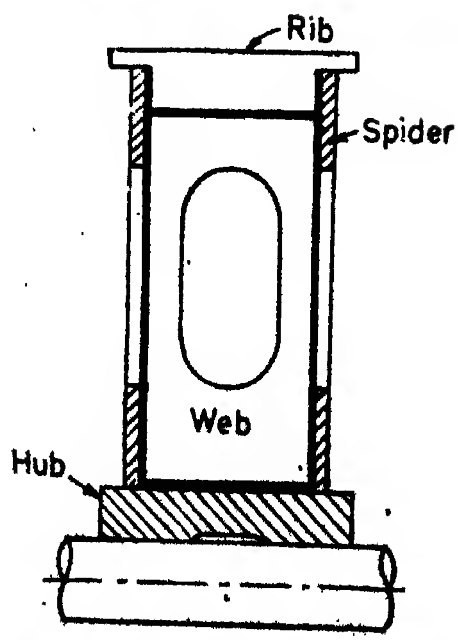


Fig. 10·13. Rotor spider built with hub.

10·5. Rotor windings. Two general types of rotor constructions are employed for induction motors, namely, the *squirrel cage* and the *wound rotor*. The great majority of present day induction motors are manufactured with squirrel cage rotors, consisting of uninsulated bars of aluminium or copper that are joined together at both ends by rings of similar conducting material. A common practice is to employ winding of cast aluminium. In this construction the assembled rotor laminations are placed in a mould after which molten aluminium is forced in, under pressure, to form bars, end rings and cooling fins as extensions of end rings. This is known as die-cast rotor and has become very popular as there are no joints and thus there is no possibility of high contact resistance.

In practice a silicon alloy with 6—12% Si is used because pure aluminium does not cast well.

In other designs, copper or brass bars of round or rectangular shape to fit the rotor slots tightly, are driven into slots, projecting a short distance on each end of the core. These bars are then soldered, welded or brazed to the end rings. Some common types of construction for connection of bars to end rings are shown in Fig 10·14.

It is necessary to keep the bars tightly in the slots because loose bars can be damaged quickly by mechanical vibrations and thermal cycling.

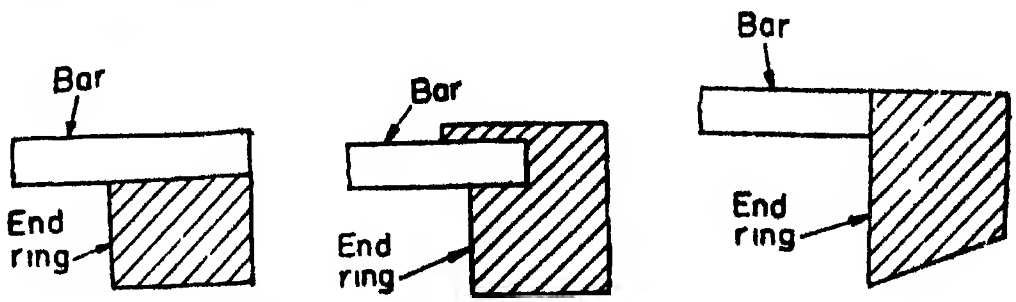


Fig. 10·14. Connection of bars to end rings.

It is a common practice to use deep bar rotors when high starting torque is required. Fig. 10·15 shows the method of joining deep bars to end rings.

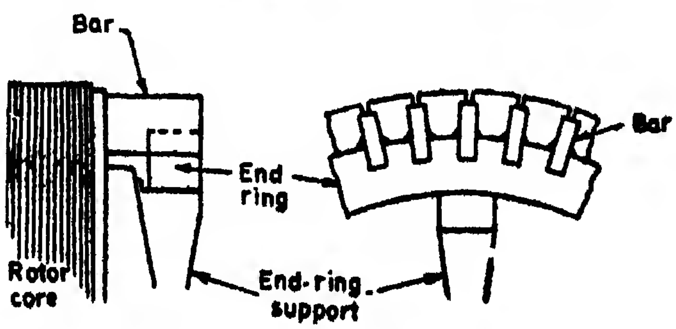


Fig. 10·15. Joining of deep bars with end rings.

For induction motors requiring speed control or extremely high values of starting torque, the wound rotor construction is employed. The wound rotor has completely insulated copper windings very much like the stator windings. *Wave winding* is used as it has the advantage that the number of cross connectors between groups of coils is reduced. Also it gives a compact arrangement of cross-connectors making it possible to have a good mechanical balance of revolving structure. The windings can be connected in star or delta and the three ends are brought out the three slip rings. The current is collected from these slip rings with carbon brushes, from which it is led to resistances for starting purposes. Earlier the brushes were provided with a device for lifting them from the slip rings when the motor

had started up. A collar was provided to short circuit the slip rings. This was done to reduce the wear of brushes. Also it resulted in reduction of frictional losses. However, brush lifting and short circuiting gear is generally not used in modern machines.

10·6. Comparison of squirrel cage and wound rotors. The squirrel cage motor has the following advantages as compared with wound rotor machine.

(*i*) No slip rings, brush gear, short circuiting devices, rotor terminals for starting rheostats are required. The star delta starter is sufficient for starting.

(*ii*) It has a slightly higher efficiency.

(*iii*) It is cheaper and rugged in construction.

(*iv*) It has better space factor for rotor slots, a shorter overhang and consequently a smaller copper loss.

(*v*) It has bare end rings, a larger space for fans and thus the cooling conditions are better.

(*vi*) It has a smaller rotor overhang leakage which gives a better power factor and a greater pull out torque and overload capacity.

The greatest disadvantage of squirrel cage rotor is that it is not possible to insert resistances in the rotor circuit for the purpose of increasing the starting torque. The cage rotor motor has a smaller starting torque and larger starting current as compared with wound rotor motor.

10·7. Slip rings. The slip rings (for wound rotor machines) are made of either brass or phosphor bronze. They may be pressed together on a body of reinforced thermo-setting resin carried on a mild steel hub. The slip rings are located either between the core and the bearing or on the shaft extension. In the latter case the shaft is made hollow to allow the three connections from rotor to slip rings to pass through bearings.

10·8. Shafts and bearings. The air gap of an induction motor is made as small as possible. Therefore the shaft is made short and stiff in order that the rotor may not have any significant deflection, as even a small deflection would create large irregularities in the air gap which would lead to produetion of an *unbalanced magnetic pull* (see page 179). There is also a possibility of rotor and stator fouling with each other.

Ball and roller bearings are generally used as with their use, accurate centring is much simpler than with journal bearings. Also the overall length of the machine is reduced. For small motors, a roller bearing may be used at the driving end and a ball bearing at the non driving end. For large and heavy rotors journal bearings of self aligning spherical seated type are used.

DESIGN

10·9. Output Equation. Reference chapter 8 page 456.

The output equation (Eqn. 8·18) for a.c. machines is :

$$\text{kVA input } Q = C_0 D^2 L n_s \qquad \ldots(10\cdot1)$$

$$\text{Output coefficient } C_0 = 1\cdot1\, K_w B_{av}\, ac \times 10^{-3} \qquad \ldots(10\cdot2)$$

From Eqn. 10·1, $D^2L = \dfrac{Q}{C_0 n_s}$

$$\text{kVA input} = \text{kW}/(\eta \cos\phi) \qquad \ldots(10\cdot3)$$

The rating of an induction motor is sometimes given in horse power and therefore the output equation should be expressed in terms of horse power. The kVA input is :

$$Q = \frac{\text{h.p.} \times 0\cdot746}{\eta \cos\phi} \qquad \ldots(10\cdot4)$$

The horse power, speed, power factor and efficiency of a machine are specified. Therefore, in order to calculate the value of D^2L, we must evaluate the output coefficient. The value of output coefficient depends upon the choice of electric and magnetic loadings *i.e.* values of *ac* and B_{av}.

10·10. Choice of average flux density in air gap

(*i*) *Power factor*. The value of flux density in air gap should be small as otherwise the machine will draw a large magnetising current (See Art. 10·8) giving a poor power factor. However, in induction motors the flux density in the air gap, should be such that there is no saturation in any part of the magnetic circuit.

(*ii*) *Iron loss:* An increased value of gap density results in increased iron loss and decreased efficiency. (See Art. 8·1·7 page 458).

(*iii*) *Overload capacity.* The value of air gap flux density determines the overload capacity. A high value of B_{av} means that the flux per pole is large. Thus for the same voltage, the winding requires less turns per phase and if the number of turns is less, the leakage reactance becomes small. With small leakage reactance the circle diagram of the machine has a large diameter which means that the maximum output, which the machine is capable af giving, is large or in other words the machine has a large *over-load capacity*. Thus, with the assumption of a higher value of B_{av}, we get a higher value of overload capacity.

Most induction motors have an overload capacity of twice in horse power but as the speed gets lower and lower—*i.e.* in machines with large number of poles—it is very difficult to get this capacity and still get a reasonably good power factor. There has to be a compromise between the two.

For 50 Hz machines of normal design the value of B_{av} lies between 0·3 and 0·6 Wb/m². For machines used in cranes, rolling mills etc., where a large overload capacity is required, a value of 0·65 Wb/m² may be used.

10·11. Choice of ampere conductors per metre

(*i*) *Copper loss and temperature rise.* A large value of *ac* means that a greater amount of copper is employed in the machine. This results in higher copper losses and large temperature rise of embedded conductors (See Art. 8·18 page 461).

(*ii*) *Voltage.* A small value of *ac* should be taken for high voltage machines as in their case the space required for insulation is large (See Art. 8·18 page 462).

(*iii*) *Overload capacity.* A large value of ampere conductors would result in large number of turns per phase. This would mean that the leakage reactance of the machine becomes high and the diameter of circle diagram is reduced resulting in reduced value of overload capacity. Therefore, higher the value of *ac*, the lower would be the overload capacity.

Hence the value of ampere conductors per metre depends upon the size of the motor, the voltage of stator winding, the type of ventilation and the overload capacity desired. It varies between 5000 to 45000 ampere conductors per metre depending upon the factors listed above.

10·12. Efficiency and power factor. Table 10·1 gives the usual values of efficiency and power factor for 50 Hz machines.

Table 10 1. Efficiency and power factor

Output kW	Efficiency		Power Factor	
	$p=4$	$p=8$	$p=4$	$p=8$
Squirrel Cage				
0·75	0 72	—	0 75	–
2·20	0 81	0 75	0·82	0 66
3 70	0 83	0·81	0 84	0·69
7·50	0·86	0·82	0·87	0·78
15·00	0·88	0·85	0 89	0 83
37·00	0·90	0·89	0 90	0·85
75·00	0·91	0 90	0 92	0·89
Slip Ring				
7·50	0·84	0·83	0·84	0·70
15·00	0·87	0·85	0·89	0·80
37 00	0 89	0 88	0 90	0 83
75·00	0 91	0 89	0 92	0·89

10·13. Main Dimensions. The product D^2L obtained from Eqn. 10·3 is split up into its two components D and L. The separation of D and L for induction motors is discussed in chapter 8. The ratio of core length to pole pitch (ratio L/τ) for various design features is :

Minimum cost—1·5 to 2

Good power factor—1·0 to 1·25

Good efficiency—1·5

Good overall design—1.

As discussed later in the chapter, for best power factor

$$^*\tau=\sqrt{0{\cdot}18\,L}$$

For small motors, high values of L/τ result in small diameters which may not be able to accommodate even a small number of slots. In such cases the above values are not practicable and so lower values down to 0 6 may be taken. In general, the value of L/τ lies between 0·6 and 2 depending upon the size of machine and the characteristics desired.

Peripheral Speed. Standard constructions can generally be used for peripheral speeds upto 60 m/s. Higher peripheral speeds upto 75 m/s are permissible only with special rotor construction which may involve higher costs. For a normal design, the diameter should be so chosen that the peripheral speed does not exceed about 30 m/s.

*It should be noted that this relationship is dimensionally not correct and is valid for values of τ and L expressed in metre.

Ventilating ducts. The stator is provided with radial ventilating ducts if the core length exceeds 100 to 125 mm. The width of each duct is about 8 to 10 mm (See page 57).

10·14. Stator winding. The windings used for induction motor stators, their choice and layout has been described in the chapter on armature windings. Double layer lap type winding with diamond shaped coils is generally used for stators. Small motors with a small number of slots and having a large number of turns per phase may use single layer mush windings.

The modern insulating materials for diamond coils belong to classes *E*, *B* and *F*. The slot and phase insulation is Polyester foil coated with compressed fibre for class *E* and plastic foil baked with polyamide fibres for class *F*. The insulants in both cases are impregnated with class *F* insulation.

The three phases of the winding can be connected in either star or delta depending upon starting methods employed. The squirrel cage motors are usually started by start delta starters and therefore their stators are designed for delta connection and the six leads are brought out to be connected to the starter. The wound rotor motors are started by putting resistance in the rotor circuit and therefore the stator can be connected either in star or in delta as desired.

10·14·1. Turns per phase. Flux per pole

$$\Phi_m = B_{av}\,\tau L = B_{av} \times (\pi DL)/p$$

Stator voltage per phase $E_s = 4{\cdot}44\, f\, \Phi_m T_s K_{ws}$

where T_s = number of turns per phase in stator

and K_{ws} = stator winding factor.

The winding factor may be initially assumed as 0·955 which is the value of winding factor for infinitely distributed winding with full pitch coils.

$$\therefore \text{ Stator turns per phase } T_s = \frac{E_s}{4{\cdot}44\, f\, \Phi_m K_{ws}}$$

10·14·2. Stator conductors. The current density in the stator windings is usually between 3 to 5 A/mm².

Stator current per phase $I_s = Q/3E_s$

$\therefore$ Area of each stator conductor $a_s = I_s/\delta_s$

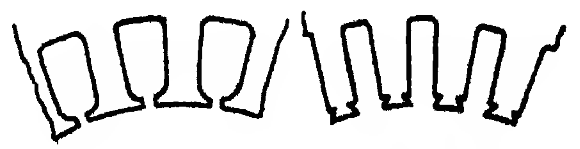
where δ_s = current density in stator conductors.

For lower values of current, round conductors would be most convenient to use while for higher currents bar or strip conductors should be adopted as anything above 2 or 3 mm in diameter is difficult to wind. The use of bar and strip conductors gives a better space factor for the slots.

10·15. Shape of stator slots. The shape of slots has an important effect upon the operating performance of the motor as well as the problem of installing the winding. The slots may be completely open or semi-enclosed as shown in Fig. 10·16. When open slots are used, the winding coils can be formed and insulated completely before they are inserted in the slots. Also the windings are reasonably accessible when individual coils must be replaced or serviced in the field. On the other hand, the coils must be taped and insulated after they are placed in the slots for machines with semi-enclosed slots. Semi-enclosed slots are usually preferred for induction motors because with their use the air gap contraction factor is small giving a small value of magnetizing current. The use of semi-enclosed slots results in low tooth pulsation loss and a much quieter operation as compared with that with open slots.

Therefore, open slots (*i.e.* equal to slot opening equal to width of slot) are used where it is desired to complete the coils outside the armature and drop them into the slots. Incidentally, an advantaage of open slots is that their use avoids excessive slot leakage thereby reducing the leakage reactance.

In small motors where round conductors are used, the tapered slot with parallel sided tooth arrangement [Fig. 10·16 (*a*)] is useful as it gives the maximum slot area for a particular tooth density. In large and medium size machines where strip conductors are preferred, parallel sided slots with tapered teeth are used [Fig. 10·16 (*b*)]



(*a*) (*b*)

Fig. 10·16. Semi-enclosed and open slots.

10·16. Number of stator slots. The designer has no definite rules to guide him in selecting the number of stator slots, however, the following points help to serve as guide-lines in the selection.

(*i*) *Tooth pulsation loss.* In motors with open type slots, the slot openings have a considerable influence on the air gap reluctance. The slots should be so proportioned that minimum variations in the air gap reluctance are produced. The effect of these variations is to produce tooth pulsation losses and noise. (For additional information on tooth pulsation losses see pages 109, 304 and 305).

These effects can be minimised by using a large number of narrow slots.

(*ii*) *Leakage reactance.* If there are larger number of slots, there are larger number of slots to insulate. Therefore the width of insulation becomes more and this means that the leakage flux has a longer path through air which results in its (leakage flux's) reduction. Therefore with larger number of slots, the leakage flux and hence the leakage reactance is reduced. In fact the slot leakage reactance is inversely proportional to the number of slots/pole/phase as explained on page 172. With small values of leakage reactance the diameter of the circle diagram is large and hence the overload capacity increases. Thus, with larger number of slots the machine has a higher overload capacity.

(*iii*) *Ventilation.* The larger the number of slots for a given diameter, the smaller will be the slot pitch. If the slot pitch is small, the tooth width is also small since width of stator slots is generally about one half the slot pitch on the gap circumference. So with larger number of slots, the thickness of the teeth becomes smaller and the teeth may become mechanically weak and they may have to be supported at the radial ventilating ducts by welding *T* or *I* sections as shown in Fig 10·3. This obstructs the flow of air in the ducts thereby impairing the cooling.

(*iv*) *Magnetising current and iron loss.* It has been explained above that the teeth section is reduced. Therefore the use of larger number of slots may result in excessive flux density in teeth giving rise to higher magnetising current and higher iron loss.

(*v*) *Cost.* With larger number of slots there are larger number of coils to wind, insulate and install involving higher costs.

It is good practice to use as many slots as economically possible. However the number of slots per pole per phase q, should not be less than 2 otherwise the leakage reactance becomes high.

In general the number of slots should be selected to give an integral number of slots per pole per phase. The slot pitch at the air gap surface for open type of slots should be between 15 to 25 mm. For semi-enclosed slots the slot pitch may be less than 15 mm.

The stator slot pitch is

$$y_{ss}=\frac{\text{gap surface}}{\text{total number of stator slots}}=\frac{\pi D}{S_s}$$

where S_s is the number of stator slots.

Total number of stator conductors$=3\times 2T_s=6T_s$

$\therefore$ Conductors per stator slot $Z_{ss}=6T_s/S_s$.

The number of conductors per slot must be an even integer for double layer windings because one half of the conductors per slot belong to the top layer and the other half to the bottom layer.

10·17. Area of stator slots. When the number of conductors per slot has been obtained, an approximate area of the slot can be calculated.

$$\text{Approximate area of each slot}=\frac{\text{copper area per slot}}{\text{space factor}}$$

$$=\frac{Z_{ss}\times a_s}{\text{space factor}} \qquad ...(10·5)$$

The space factors ordinarily obtained vary from 0·25 to 0·4. High voltage machines have lower space factors owing to large thicknesses of insulation. After obtaining the area of the slot, the dimensions of the slot should be adjusted. The tooth width and the slot width at the gap surface should be approximately equal. The slot should not be too wide to give a thin tooth. The width of the slot should be so adjusted such that the mean flux density in the tooth lies between 1·3 to 1·7 Wb/m². The width of teeth should not be too large as it results in narrow and deep slots. The deeper slots give a large value of leakage reactance. In general the ratio of slot depth to slot width should be between 3 and 6.

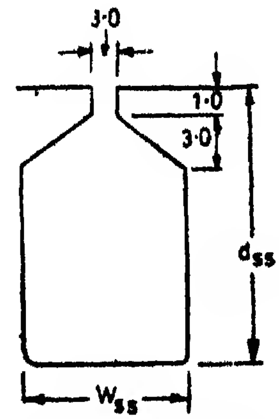


Fig. 10·17. Stator slot.

Fig. 10·17 gives the dimensions for the slot opening and for lip and wedge in the case of semi-enclosed slots (The dimensions are in mm).

10·18. Length of mean turn. The approximate length of mean turn of the winding on induction motor stators for use on voltages upto 650 V may be calculated from the following empirical relationship.

Length of mean turn of stator $L_{mts}=2L+2·3\tau+0·24$ ...(10·6)

with values of L and τ are expressed in m.

10·19. Stator teeth. The dimensions of the slot determine the value of flux density in the teeth. A high value of flux density in the teeth is not desirable, as it leads to a higher iron loss and a greater magnetizing mmf. As stated earlier, the maximum value of B_{ts}, the mean flux density in stator teeth should not exceed 1·7 Wb/m².

$\therefore$ Minimum tooth area per pole$=\Phi_m/1·7$

Tooth area per pole=number of slots per pole × net iron length × width of tooth

$$=(S_s/p)\times L_i\times W_{ts}$$

or minimum width of stator tooth

$$(W_{ts})_{min}=\frac{\Phi_m}{1·7\times(S_s/p)L_i} \qquad ...(10·7)$$

The minimum width of stator teeth is near the gap surface. A check for minimum tooth width using Eq. 10·7 should be applied before finally deciding the dimensions of stator slot.

10·20. Stator core. The flux density in the core should not exceed about 1·5 Wb/m². Generally it lies between 1·2 to 1·4 Wb/m². From Fig. 10·18, it is clear that the flux passing through the stator core is half of the flux per pole.

Flux in the stator core $=\Phi_m/2$.

$$\therefore \quad \text{Area of stator core} = \frac{\text{flux through core}}{\text{flux density in stator core}} = \frac{\Phi_m}{2B_{cs}}$$

Area of stator core $= L_i \times d_{cs}$

where d_{cs} = depth of stator core.

$$\text{Thus} \quad L_i \times d_{cs} = \frac{\Phi_m}{2B_{cs}}$$

$$d_{cs} = \frac{\Phi_m}{2B_{cs} \times L_i} \qquad \ldots(10·8)$$

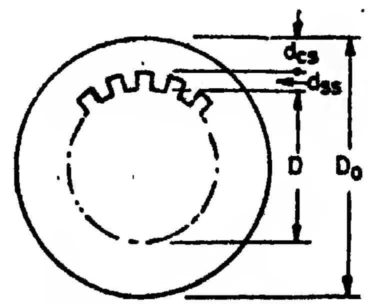


Fig. 10·18. Stator lamination dimensions.

The outside diameter of stator laminations (Fig. 10·18).

$D_o = D + 2$ (depth of stator slots + depth of core)

$$= D + 2d_{ss} + 2d_{cs} \qquad \ldots(10·9)$$

Example 10·1. *Determine the main dimensions, number of radial ventilating ducts, number of stator slots and the number of turns per phase of a 3·7 kW, 400 volt, 3-phase, 4 pole, 5 Hz squirrel cage induction motor to be started by a star delta starter. Work out the winding details.*

Assume :

Average flux density in the gap = 0·45 Wb/m², ampere conductors per metre = 23000, efficiency = 0·85, and power factor = 0·84.

Machines rated at 3·7 kW, 4 pole are sold at a competitive price and therefore choose the main-dimensions to give a cheap design.

Assume :

winding factor = 0·955, stacking factor = 0·9.

Solution.

(i) Main Dimensions

$$\text{kVA input } Q = \frac{\text{kW}}{\eta \cos\phi} = \frac{3·7}{0·85 \times 0·84} = 5·18$$

Output co-efficient $\quad C_0 = 11\, K_w\, B_{av}\, ac\, 10^{-3}$

$$= 11 \times 0·955 \times 23000 \times 10^{-3} = 108·7$$

Synchronous speed $\quad n_s = 2f/p = 2 \times 50/4 = 25$ r.p.s.

$$\therefore \quad \text{Product} \quad D^2L = \frac{Q}{C_0 n_s} = \frac{5·18}{(108·7 \times 25)} = 1·91 \times 10^{-3}\ \text{m}^3.$$

For a cheap design ratio, $L/\tau = 1·5$ to 2. Taking $L/\tau = 1·5$,

we have $$\frac{L}{\pi D/p} = 1·5 \quad \text{or} \quad \frac{L}{D} = \frac{1·5 \times \pi}{p} = \frac{1·5 \times \pi}{4} = 1·178.$$

Now $$D^2L = 1·91 \times 10^{-3}\ \text{m}^3 \quad \text{or} \quad 1·178\, D^3 = 1·91 \times 10^{-3}$$

or $$D=0.117 \text{ m} \simeq 0.12 \text{ m and } L=\frac{1.91\times10^{-3}}{(0.12)^2}\simeq 0.13 \text{ m}.$$

Pole pitch $\tau=\pi D/p=\pi\times0.12/4=0.094$ m.

The length of core is 0.13 m and therefore one radial duct 10 mm wide is provided.

∴ Net iron length $L_i=0.9\ (0.13-0.01)=0.108$ m.

(ii) Turns per phase

Flux per pole $\Phi_m=B_{av}\ L\ \tau=0.45\times0.13\times0.094=5.5\times10^{-3}$ Wb.

As the machine is started by a star delta starter, it is designed for delta connection.

∴ Stator voltage per phase $E_s=400$ V.

Stator turns per phase $$T_s=\frac{E_s}{4.44\ f\ \Phi_m\ K_{ws}}$$

$$=\frac{400}{4.44\times50\times5.5\times10^{-3}\times0.955}=343.$$

(iii) **Number of stator slots.** It is a small sized machine and since semi-enclosed slots are used for this machine the slot pitch can be lower than 15 mm. However, for mechanical reasons the slot pitch should not be below 10 mm.

Taking, slots per pole per phase $q_s=3$.

Total number of stator slots $S_s=3\times4\times3=36$.

Stator slot pitch $=y_{ss}=\pi D/S_s=\pi\times0.12/36$
$=0.01047$ m $=10.47$ mm.

Total number of stator conductors $=6T_s=6\times343=2058$

Conductors per slot $$Z_{ss}=\frac{2058}{36}=57.$$

∴ Actual number of turns per phase $$T_s=\frac{36\times57}{2\times3}=342$$

(iv) **Winding details.** For small machines like this, a single layer mush winding placed in semi-enclosed slots is used. In a single layer winding, each coil occupies two slots and, therefore,

number of coils $=36/2=18$

and number of coils per phase $=18/3=6$.

As there are 3 slots per pole phase or 9 slots per pole, the coil span is 9 slots. Coil side in slot 1 is connected to coil side, $1+9=10$ in slot 10. Actually the coil span in mush windings should always be an odd number in terms of slots spanned. Here it is odd and is equal to slots per pole and therefore the coils are full pitch.

The complete winding is shown in Fig. 6.42.

Example 10.2. *Find the main dimensions of a 15 kW, 3 phase, 400 V, 50 Hz, 2810 r.p.m squirrel cage induction motor having an efficiency of 0.88 and a full load power factor of 0.9. Assume:*

specific magnetic loading = 0.5 Wb/m²; specific electric loading = 25000 A/m.

Take the rotor peripheral speed as approximately 20 m/s at synchronous speed.

Solution.

kVA input $Q=\dfrac{15}{0.88\times0.9}=18.94.$

Output co-efficient $C_0=11\ K_w\ B_{av}\ ac\times10^{-3}$
$=11\times0.955\times0.5\times25000\times10^{-3}=131.3$

The speed of the rotor at full load is 2810 r.p.m. and the nearest synchronous speed corresponding to 50 Hz is 3000 r.p.m.

Synchronous speed $n_s=3000/60=50$ r.p.s.

$\therefore$ Product $D^2L=\dfrac{Q}{C_0n_s}=\dfrac{18.94}{131.3\times50}=2.88\times10^{-3}\ m^3.$

The rotor diameter in an induction motor is almost equal to stator bore.

$\therefore \qquad \pi Dn_s=20$

or $$D=\frac{20}{\pi\times50}=0.1275\ m,$$

and $$L=\frac{2.88\times10^{-3}}{(0.1275)^2}=0.177\ m.$$

Example 10·3. *Determine the main dimensions, turns per phase, number of slots, conductor cross section and slot area of a 250 h.p, 3 phase, 50 Hz, 400 V, 1410 r.p.m. slip ring induction motor. Assume $B_{av}=0.5\ Wb/m^2$, $ac=30000$ A/m, efficiency$=0.9$ and power factor$=0.9$, winding factor$=0.955$, current density$=3.5\ A/mm^2$. The slot space factor is 0·4 and the ratio of core length to pole pitch is 1·2. The machine is delta connected.*

Solution.

(i) Main Dimensions

The speed of motor is 1410 r.p.m. and the nearest synchronous speed corresponding to 50 Hz is 1500 r.p.m.

Synchronous speed $n_s=1500/60=25$ r.p.s.

Number of poles $=2\times50/25=4.$

Output co-efficient $C_0=11\times0.955\times0.5\times30,000\times10^{-3}=157.6$

kVA input $Q=\dfrac{250\times0.746}{0.9\times0.9}=230.2$

$\therefore \qquad D^2L=\dfrac{Q}{C_0n_s}=\dfrac{230.2}{157.5\times25}=58.4\times10^{-3}\ m^3.$

We have $L/\tau=1.2$ or $L/D=1.2\times\pi/4=0.942.$

$\therefore \qquad 0.942\ D^3=58.4\times10^{-3}$

or $D=0.395$ m and $L=0.375$ m

(ii) Winding

Flux per pole $\Phi_m=0.5\times\pi\times0.395\times0.375/4=58.2\times10^{-3}$ Wb

The machine is delta connected.

$\therefore$ Stator voltage per phase $E_s=400$ V.

$\therefore$ Stator turns per phase $T_s=\dfrac{400}{4.44\times50\times58.2\times10^{-3}\times0.955}=32.4$

Total conductors $=6T_s=6\times32=192$.

The slot pitch lies between 15 to 25 mm.

$\therefore$ The number of slots lies between :

$$\frac{\pi\times0{\cdot}395\times10^3}{25}=50 \text{ to } \frac{\pi\times0{\cdot}395\times10^3}{15}=84$$

The machine is large in size and therefore a large number of slots should be chosen.

The value of number of slots per pole per phase and the conductors per slot should be so chosen that there is not much difference in the value of conductors provided and the conductors calculated earlier.

Taking 5 slots per pole per phase,

Total number of stator slots $=3\times4\times5=60$

Providing 3 conductors per slot. Total number of conductors $=3\times60=180$.

Turns per phase $=180/6=30$.

The value of turns per phase calculated earlier is 32·4. Thus there is a decrease of about 7 per cent in the turns provided and therefore the value of flux density would increase by this amount.

Single layer concentric winding with semi-enclosed slots is used. (The number of conductors per slot is odd and therefore double layer winding is not possible).

Stator current per phase $I_s=\dfrac{250\times746}{3\times400\times0{\cdot}9\times0{\cdot}9}=192$ A.

Area of stator conductor $a_s=I_s/\delta_s=192/3{\cdot}5=55$ mm^2.

Total copper area in each slot $=3\times55=165$ mm^2.

Total area of slot $=\dfrac{\text{copper area per slot}}{\text{space factor}}=\dfrac{165}{0{\cdot}4}=412{\cdot}5$ mm^2.

Example 10·4. *Estimate the stator core dimensions, number of stator slots and number of stator conductors per slot for a 100 kW 3300 V, 50 Hz, 12 pole star connected slip ring induction motor. Assume :*

average gap density $=0{\cdot}4$ Wb/m^2,

conductors per metre $=25{,}000$ A/m,

efficiency $=0{\cdot}9$, *power factor* $=0{\cdot}9$ *and winding factor* $=0{\cdot}96$.

Choose main dimensions to give best power factor. The slot loading should not exceed 500 ampere conductors.

Solution.

$$\text{kVA input}=\frac{100}{0{\cdot}9\times0{\cdot}9}=123{\cdot}5.$$

Synchronous speed $n_s=\dfrac{2\times50}{12}=8{\cdot}33$ r.p.s.

Output coefficient $C_0=11\times0{\cdot}96\times0{\cdot}4\times25{,}000\times10^{-3}=105{\cdot}6$.

$$\therefore \quad \text{Product } D^2L=\frac{123{\cdot}5}{105{\cdot}6\times8{\cdot}33}=140{\cdot}4\times10^{-3}\text{ m}^3.$$

For best power factor $\tau=\sqrt{0{\cdot}18\,L}$ (Eqn. 10·63)

or $\quad \pi D/12=\sqrt{0{\cdot}18\,L}$ or $D^2=2{\cdot}63\,L$

Thus, we have $2.63\ L^3 = 140.4 \times 10^{-3}$

$L = 0.23$ m and $D = 0.78$ m.

Flux per pole $\Phi_m = 0.4 \times \dfrac{\pi \times 0.78}{12} \times 0.23 = 18.8 \times 10^{-3}$ Wb.

Stator voltage per phase $E_s = 3300/\sqrt{3} = 1905$ V.

Stator turns per phase $T_s = \dfrac{1905}{4.44 \times 50 \times 18.8 \times 10^{-3} \times 0.96} \simeq 487.$

Total number of stator conductors $= 6 \times 487 = 2922.$

The slot pitch varies between 15 to 25 mm.

$\therefore$ The number of stator slots S_s lies between

$$\frac{\pi \times 0.78 \times 10^3}{25} = 98 \text{ and } \frac{\pi \times 0.78 \times 10^3}{15} = 163.$$

The total number of slots for different number slots per pole per phase is

Slots/pole/phase	q_s	2	3	4	5
Stator slots	S_s	72	108	144	180

Thus we can use either 108 or 144 slots for the stator.

With $S_s = 108$:

Conductor per slot $Z_{ss} = \dfrac{2922}{108} \simeq 27.$

Stator current per phase $I_s = \dfrac{100 \times 10^3}{\sqrt{3} \times 3300 \times 0.9 \times 0.9} = 21.6$ A.

Slot loading $= I_{ss} Z_s = 21.6 \times 27 = 583$ ampere conductors.

This exceeds the maximum specified limit of 500 ampere conductors and, therefore, we cannot use 108 slots.

With $S_s = 144$

Conductors per slot $Z_{ss} = 2922/144 \simeq 20$

$\therefore$ Slot loading $= 21.6 \times 20 = 432$ ampere conductors.

This is below the maximum specified limit.

$\therefore$ We use 144 slots with 20 conductors per slot.

Example 10.5. ***A 15 kW, 440 V, 4 pole, 50 Hz, 3 phase induction motor is built with a stator bore 0.25 m and a core length of 0.16. The specific electric loading is 23000 ampere conductors per metre. Using the data of this machine, determine the core dimensions, number of stator slots and number of stator conductors for a 11 kW, 460 V, 6 pole, 50 Hz motor. Assume a full load efficiency of 84 per cent and power factor of 0.82 for each machine. The winding factor is 0.955.***

Solution.

15 kW Motor :

kVA input $= \dfrac{15}{0.84 \times 0.82} = 21.8.$ Synchronous speed $n_s = 2 \times 50/4 = 25$ r.p.s.

Now, $C_0=\dfrac{Q}{D^2Ln_s}=\dfrac{21.8}{(0.25)^2\times0.16\times.5}=87.2.$

From Eqn. 10.2

$$C_0=11\,K_w\,B_{av}\,ac\times10^{-3}=11\times0.955\times B_{av}\times23000\times10^{-3}$$
$$=241.6\,B_{av}$$

$\therefore$ Average flux density in the air gap $B_{av}=87.2/241.6=0.36$ Wb/m².

Pole pitch $\tau=\pi\times0.25/4=0.196$ m.

$\therefore$ Ratio $L/\tau=0.16/0.196=0.815.$

11kW Motor :

We have to use the same data for the 11 kW machine as is calculated above for 15 kW machine.

$\therefore$ $B_{av}=0.36$ Wb/m² ; $ac=23000$ A/m ;
$L/\tau=0.815$ and $C_0=87.2.$

Synchronous speed $n_s=\dfrac{2f}{p}=\dfrac{2\times50}{6}=16.67$ r.p.s. kVA$=\dfrac{11}{0.84\times0.82}=16.$

$\therefore$ Product $D^2L=\dfrac{16}{87.2\times16.67}=11\times10^{-3}$ m³

and ratio $\dfrac{L}{D}=0.815\times\dfrac{\pi}{6}=0.427$

or $0.427\,D^3=11\times10^{-3}$

$\therefore$ $D=0.30$ m and $L=0.125$ m.

The number of stator slots lies between $\dfrac{\pi\times0.3\times10^3}{25}=37$ and $\dfrac{\pi\times0.3\times10^3}{15}=63.$

Using 3 slots per pole per phase,

Number of stator slots $S_s=3\times6\times3=54.$

Flux per pole $\Phi_m=\dfrac{0.36\times\pi\times0.3\times0.125}{6}=7.07\times10^{-3}$ Wb.

Delta connection is used for the stator winding.

$\therefore$ Stator voltage per phase $E_s=460$ V.

Stator turns per phase $T_s=\dfrac{460}{4.44\times50\times7.07\times10^{-3}\times0.955}=307.$

Total number of stator conductors$=6\times307=1842.$

$\therefore$ Conductors per slot $Z_{ss}=\dfrac{1842}{54}=34.1$

Using 34 conductors per slot, total conductors$=1836.$

$\therefore$ Stator turns per phase $T_s=1836/6=306.$

Example 10.6. *A 3 phase, 440 V, 750 r.p.m., 50 Hz star connected induction motor has a stator with an internal diameter of 0.25 m and an axial length of 0.15 m. It has 48 slots with 24 conductors per slot. Calculate the air gap flux per pole.*

The area of each stator conductor is to be 5 mm². Calculate the width and the depth of the slot to accommodate the stator conductors. The maximum flux density in the teeth is to be 1·7 Wb/m². Conductor insulation is 0·08 mm thick and slot insulation is 0·8 mm thick. Make other suitable assumptions.

Solution

Synchronous speed $n_s=750/60=12.5$ r.p.s. and number of poles $p=2\times60/12.5=8$.

Voltage/phase $E_s=\dfrac{440}{\sqrt{3}}=254$ V.

Turns per phase $T_s=\dfrac{41\times24}{6}=192$.

$\therefore$ Flux per pole $\Phi_m=\dfrac{254}{4.44\times50\times192\times0.955}=6.24\times10^{-3}$ Wb.

Circular conductors are used with a bare diameter of 2·53 mm (area 5 mm²)

Diameter of insulated conductors $=2.53+2\times0.08\simeq2.7$ mm.

There are 24 conductors per slot. Let us assume that there are 3 conductors width wise and 8 conductors depth wise.

$\therefore$ Width of stator slot

$W_{ss}=$ number of conductors widthwise × diameter of insulated conductor + 2 (width of slot insulation) + slack

$=3\times2.7+2\times0.8+0.8=10.5$ mm.

(The slack along width is assumed as 0·8 mm)

Depth of stator slot

$d_{ss}=$ number of conductors depth wise × diameter of insulated conductor + 3 × width of slot insulation + depth of wedge and lip + slack

$=8\times2.7+3\times0.8+4+1.0=29.0$ mm.

(The slack along depth is taken as 1 mm)

Let us check for maximum flux density in teeth. The tooth width is minimum near the air gap surface and therefore flux density in teeth is maximum at this section.

The core length is 0·15 m and therefore one radial duct 10 mm wide is used.

The stacking factor is assumed as 0·9

Net core length $L_i=0.9(0.15-1\times0.01)=0.126$ m.

Slot pitch $Y_{ss}=\pi\times0.25\times10^3/48=16.36$ mm.

Tooth width at the gap surface $W_{ts}=16.36-10.5=5.86$ m.

Maximum flux density in teeth

$$B_{ts\,(max)}=\frac{\Phi_m}{(S_s/p)\times L_i\times W_{ts}}$$

$$=\frac{6.24\times10^{-3}}{(48/8)\times0.126\times5.86\times10^{-3}}=1.41\ \text{Wb/m}^2.$$

This is below the maximum allowable limit of 1·7 Wb/m².

ROTOR DESIGN

10.21. Length of air gap. The folowing factors should be considered when estimating the length of air gap.

(i) Power factor. The mmf required to send the flux through air gap is proportional to the product of flux density and the length of air gap. Even with very small densities, the mmf required for air gap is much more than that for the rest of the magnetic circuit. Therefore, it is the length of air gap that primarily determines the magnetising current drawn by the machine.

Fig 10.19 shows phasor diagrams of an induction motor with two different gap lengths where V_s = stator applied voltage, Φ_m = air gap flux, E_r = rotor induced emf, I_r = rotor current, I_r' = rotor current referred to stator side, I_m = magnetizing current, I_l = loss component of no load current, I_o = no load current, I_s = stator current and ϕ = phase angle between stator applied voltage and stator current. Subscripts 1 and 2 refer to two different cases with phasor diagram of the machine with greater gap length *i.e.* case 2 shown in Fig. 10.19 (*b*). The magnetizing current in case 2 is greater than that in case 1 and therefore the phase angle between stator applied voltage and stator current is greater for case 2 *i.e.* ϕ_2 is greater than ϕ_1 or $\cos\phi_2$ is smaller than $\cos\phi_1$. Hence, the slower factor of machine with a greater gap length is smaller.

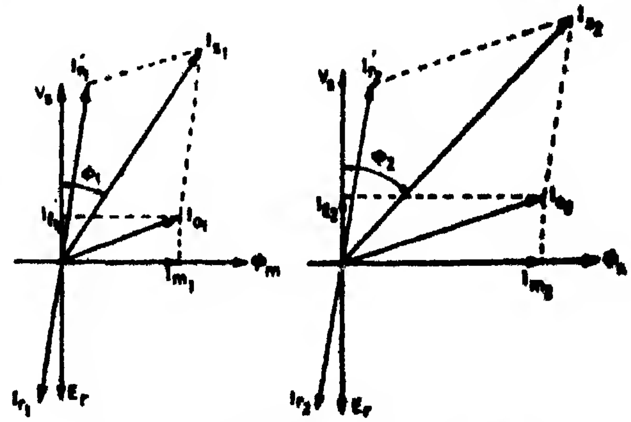


Fig. 10.19. Effect of length of air gap on power factor.

(ii) Over-load capacity. The over load capacity of an induction motor is defined as the ratio of the maximum output to the rated output. The maximum output of induction motor is obtained from its circle diagram as shown in Fig. 10.20. The overload capacity is *MN*/*AL*.

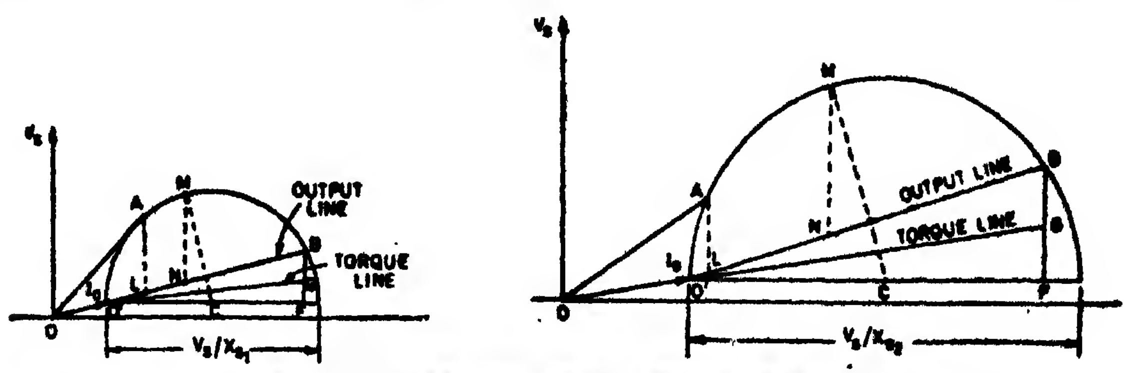


Fig. 10.20. Effect of leakage reactance on overload capacity.

Fig. 10.20 shows the circle diagrams of two induction motors with different values of leakage reactance. The diameter of the circle diagram is V_s/X_s where X_s is the leakage reactance of the motor referred to the stator side. Thus, smaller the leakage reactance greater the diameter of circle diagram. The circle diagram shown in Fig. 10.20 (*b*) is for a machine with a smaller leakage reactance than that of a machine whose circle diagram is shown in Fig. 10.20 (*a*). The diameter of the circle diagram of a machine with smaller leakage reactance is obviously greater. It is clear from Fig. 10.20 (*a*) and (*b*) that the overload capacity (Ratio *MN*/*AL*) is greater for a machine having smaller leakage reactance.

The length of air gap affects the value of *zig zag leakage reactance* which forms a large part of total leakage reactance in the case of induction motors. If the length of air gap is large, the zigzag leakage flux is reduced resulting in a reduced value for leakage reactance. With the decrease in the value of leakage reactance, the diameter of circle diagram increases and hence the overload capacity increases. Therefore, greater is the length of air gap, greater is the overload capacity.

(*iii*) **Pulsation loss** With larger length of air gap, the variation of reluctance due to slotting is small. The tooth pulsation loss, which is produced due to variation in reluctance of the air gap, is reduced accordingly. Therefore, the pulsation loss is less with large air gaps.

(*iv*) **Unbalanced magnetic pull.** If the length of air gap is small, even a small deflection or eccentricity of the shaft would produce a large irregularity in the length of air gap and is responsible for production of large unbalanced magnetic pull which has the tendency to bend the shaft still more at a place where it is already bent resulting in fouling of rotor with stator. If the length of air gap of a machine is large, a small eccentricity would not be able to produce noticeable unbalanced magnetic pull (See Art 4 20 page 179).

(*v*) **Cooling** If the length of air gap is large, the cylindrical surfaces of rotor and stator are separated by a large distance. This would afford better facilities for cooling at the gap surfaces especially when a fan is fitted for the circulation of air.

(*vi*) **Noise.** The principal cause of noise in induction motors is the variation of reluctance of the path of the zigzag leakage flux. To ensure that the noise produced will not be objectionable, it is necessary to make the zigzag leakage as small as possible. This can be done by increasing the length of the gap.

From above, we conclude that the length of air gap in an induction machine should be as small as mechanically possible in order to keep down the magnetizing current and to improve the power factor. This is a major consideration. But if a higher overload capacity, better cooling, reduction in noise or reduction in unbalanced magnetic pull is important, large air gap lengths should be used.

10·21 1. Relations for calculation of length of air gap

(*i*) In order to estimate the length of air gap of small induction motors, the following expression can be used

$$l_g = 0{\cdot}2 + 2\sqrt{DL} \text{ mm} \qquad \ldots(10\cdot10)$$

where D and L are expressed in metre. The air gap is a mere clearance between rotor and stator and is made smaller than the value given by Eqn. 10·10 if roller and ball bearings are used.

(*ii*) Another expression, which can be used for small machines, is

$$l_g = 0{\cdot}125 + 0{\cdot}35\,D + L + 0{\cdot}015\,V_a \text{ mm} \qquad \ldots(10\cdot11)$$

where D and L are expressed in metre and V_a is the peripheral speed in metre per second

(*iii*) The following relation may also be usefully used

$$l_g = 0{\cdot}2 + D \text{ mm} \qquad \ldots(10\cdot12)$$

where D is expressed in metre.

(*iv*) For machines with journal bearings, following expression may be used

$$l_g = 1{\cdot}6\sqrt{D} - 0{\cdot}25 \text{ mm} \qquad \ldots(10\cdot13)$$

where D is expressed in metre.

The following air gaps may be used for 4 pole machines.

Table 10·2

Length of air gap for 4 pole machines

D m	l_g mm	D m	l_g mm
0·15	0·35	0·45	1·3
0·20	0·50	0·55	1·8
0·25	0·60	0·65	2·5
0·30	0·70	0·80	4·0

Rotor diameter D_r=stator bore$-2\times$length of air gap$=D-2l_g$

DESIGN OF SQUIRREL CAGE ROTOR

10·22. Number of rotor slots. The selection of number of rotor slots in squirrel cage motors is very important and a considerable attention should be paid to select a suitable value. This is because with certain numbers of poles and of stator and rotor slots in squirrel cage motors, peculiar and deleterious behaviour may be observed. With certain combination of stator and rotor slots the machine may refuse to start or may crawl at some subsynchronous speed. In some cases, severe vibrations may be set up generating excessive noise.

These effects are produced by harmonic fields. The harmonic fields are due to :

(*i*) windings, (*ii*) slotting, (*iii*) saturation and (*iv*) irregularities in the air gap.

The harmonic fields are superposed upon the fundamental sine wave field and induce emfs in the rotor winding and thus circulate harmonic currents. These harmonic currents, in turn, interact with the harmonic fields to produce harmonic torques. In fact the harmonic fields may be thought of as separate low power motors that are direct coupled to the same shaft as the fundamental. Therefore, the net motor torque is equal to the sum of the torque due to the fundamental and the torques produced by a myriad of harmonic fields.

It should be understood that the space harmonic fields have more poles than the fundamental and therefore have lower synchronous speeds. Some of these fields revolve in the forward direction and some in the backward direction. At motor speeds above their respective values, the forward rotating harmonic fields produce braking torques while the backward rotating harmonic fields produce braking torque at all speeds.

In addition the harmonic fields are responsible for increase in stray load losses and increased motor heating.

The essential difference in behaviour of wound rotor and squirrel cage machines is that the cage rotor, being a multiphase winding, will circulate currents due to any harmonic emf produced by the gap flux except that which has a wavelength equal to the pitch of the bars, the wound rotor machines on the other hand tend to reduce the effect of most harmonics.

The effects of space harmonic fields produced by windings are greatly intensified by slotting, which not only introduces steps in the mmf wave, and produces further harmonics, but also modulates the gap flux. Therefore, the choice of rotor slots is particularly important in the case of squirrel cage machines. Any bad combination of stator and rotor slots may result in awkward behaviour.

The effects of harmonics are explained below in details :

1. **Harmonic induction torques.** It has been explained earlier (page 314) that a 3-phase winding carrying sinusoidal currents produces harmonics of the order $n=6N\pm1$,

where N is an integer. The movement of the harmonics is with or against the direction of rotation depending upon the sign (+ means with the rotation and − means against the rotation). The number of poles for the nth harmonic is n times the number of poles of the fundamental and therefore the synchronous speed of nth harmonic is $1/n$th of the synchronous speed of fundamental.

Now a three phase winding will produce a forward rotating 7th harmonic and a backward rotating 5th harmonic (for $N=1$). The 5th and the 7th harmonic fluxes may be deemed as produced by sets of additional poles superimposed the fundamental poles. They generate rotor emfs, currents and torques of the same general torque/speed shape as that of the fundamental but with synchronous speeds 1/5 (backward) and 1/7 (forward) of the synchronous speed of fundamental as shown in Fig. 10·21.

The 7th harmonic torque reaches its maximum just before 1/7th synchronous speed, but beyond this speed the 7th harmonic torque becomes negative, since the slip in the harmoinc field is negative. The resultant torque-speed curve combined with the fundamental shows marked dips and with certain slot combinations the dip due to 7th harmonic may become very pronounced. Assuming that the mechanical load on the shaft involves a constant load torque, the torque developed may fall below this load torque and, when this occurs, the motor cannot accelerate upon its full speed but continues to run at a speed a little lower than the 1/7th synchronous speed this is called **Crawling**.

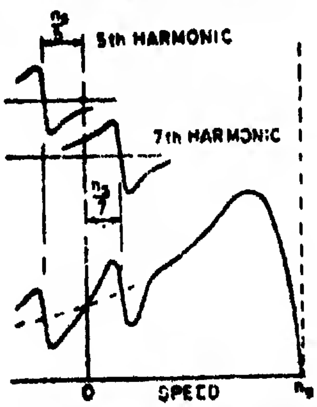


Fig. 10·21. Dips caused by 5th and 7th harmonics in the torque-speed characteristics.

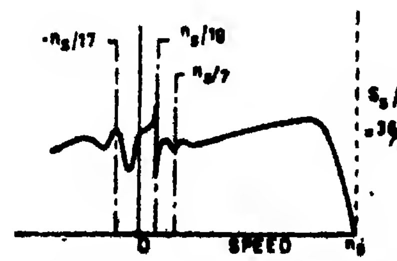


Fig. 10·22. Dips caused by slot harmonics in the torque-speed characteristics.

Slotting produces harmonics of the order $6Aq\pm$ (see page 305) in a 3 phase machine, where A is any integer. Considering a 4 pole 36 slot machine. The slots per pole per phase are $q=3$. Thus $n=18\pm1=19$th and 17th harmonics are produced due to slotting. The 19th harmonic field rotates forward and 17th harmonic field rotates backward. Thus the dips in the torque speed characteristic would be produced at $+1/19$ and $-1/17$ of synchronous speed (Fig. 10·22.) The effect of production of dips may be augmented by rotor slotting. Corresponding to the above 4 pole, 36 slot stator if we choose 76 rotor slots, there would be one rotor bar corresponding to every 19th harmonic pole. Thus the 19th harmonic torques would be very large and the rotor would vibrate considerably as shown in Fig. 10·23. Therefore, it is necessary to avoid values of rotor slots exceeding stator slots by about 15—30%.

2 Harmonic synchronous torques. If the stator and rotor harmonics of the same order i.e. having the same number of poles, are present, the torque will be alternately in opposite directions as they move past each other. But if their speeds happen to coincide, they will lock together, if sufficiently powerful, giving rise to a synchronous torque. In such a case the motor would crawl at constant subsynchronous speed.

The stator produces harmonics (due to slotting) of the order

$$n=6q\pm1=2(S_s/p)\pm1 \text{ (for } A=1).$$

These harmonics revolve at a speed $1/n$ of synchronous speed with respect to stator

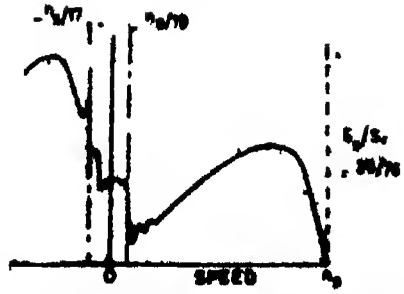


Fig 10·23. Intensification of torque produced by slot harmonics due to improper slot combination.

The rotor slotting produces harmonic fields of the order

$$n'=2\ (S_r/p)\pm1$$

and they revolve at a speed $1/n'$ that of the fundamental. The speeds would be equal if

$$2\ (S_s/p)\pm1=2\ (S_r/p)\pm1.$$

One of the possibilities for this to happen, is when $S_s=S_r$

or when number of stator slots is equal to number of rotor slots

Thus when the number of rotor slots is equal to the number of stator slots, the speeds of all the harmonics produced by stator slotting coincide with the speed of corresponding rotor harmonics Thus harmonics of every order would try to exert synchronous torques at their corresponding synchronous speeds and the machine would refuse to start. This is known as **Cogging**. Therefore, the number of stator slots should never be equal to the number of rotor slots.

An alternative way to explain the phenomenon of cogging is :

The magnetic circuit has always the tendency to align itself in a position of minimum reluctance. Thus if the number of rotor slots is equal to the number of stator, there exists a position of minimum reluctance, when the teeth of rotor and stator are aligned opposite to each other. The radial alignment forces become very strong when the machine is at rest and these may exceed the tangential accelerating force thereby preventing the motor from starting.

The cogging effect will always be present, though in reduced degree, whenever the number of rotor and stator slots have a common factor.

However, when the number of rotor slots is a prime number, the rotor will all the time be chasing a position of minimum reluctance but never to find it. Hence in this case there is no tendency to cog.

The other possibility can be :

$$2\ (S_s/p)+1=2\ (S_r/p)-1 \quad \text{or} \quad S_r-S_s=p$$

Thus the forward rotating harmonic field produced by stator slotting moves synchronously with backward rotating harmonic field due to rotor slotting if the difference of stator and rotor slots is equal to number of poles.

Considering a 4 pole stator with 36 slots. It produces a forward rotating 19th harmonic which revolves at a speed $n_s/19$ with respect to stator. If the rotor slots are 40, it produces a backward rotating 19th harmonic. Its speed with respect to rotor is $(n_s-n_r)/19$ where n_r is the speed of the rotor. The rotor itself revolves forward at a speed n_r and therefore it revolves its 19th harmonic at a speed :

$$(n_s-n_r)/19+n_r$$

with respect to stator. Therefore, in order that the two fields may resolve synchronously

$$n_s/19 = -(n_s - n_r)/19 + n_r \quad \text{or} \quad n_r = n_s/10$$

Thus the machine would crawl synchronously at 1/10 synchronous speed. Fig. 10·24 shows the saddle effect produced by 19th harmonic at 1/10th synchronous speed.

Thus in order to avoid synchronous cusps the difference of stator and rotor slots should not be $\pm p$ or a multiple of p. The difference in harmonic induction and synchronous torques (shown respectively in Figs. 10·20 and 10·24) is that the operating speed changes slightly for the former but is constant for the latter, for a small variation of the shaft load.

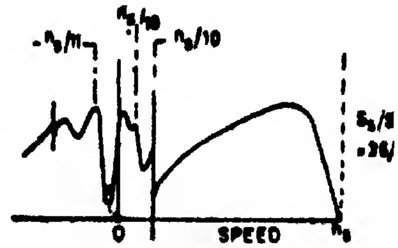


Fig. 10·24. Harmonic synchronous torque and synchronous crawling.

3. **Vibrations and noise.** When the rotor runs, its teeth continuously move with respect to the stator teeth. This results in the rapid variations in flux density in the gap thereby producing rapid changes in forces of attraction between stator and rotar teeth. The teeth, being cantilevers, respond to varying forces and are set into vibrations. The vibrations are large in machines with few poles since the variations in force are more concentrated.

Noise production in squirrel cage induction motor can be strongly supported by unbalanced magnetic pull. When the rotor runs, the unbalanced magnetic pull moves round the air gap at a definite speed and causes the rotor to vibrate. At some speed of the rotor these vibrations may begin to resonate with the natural oscillations of the rotor. If this phenomenon attains further development, it may make the rotor operation impossible.

The varying forces of low frequency are important because of their ability to transfer vibrations to the structure, while forces in the frequency range 10Hz—10kHz cause accoustical effects like humming and whistling etc.

An analysis shows that the vibration torques are produced if $S_s - S_r = \pm 1 \pm p$. Some of the investigations show undesirable slot combinations (as far as noise is concerned) may exist if $S_s - S_r = \pm 2 \pm p$.

Consider the relationship :

$$S_s - S_r = \pm 1 \pm p$$

Now number of stator slots $S_s = 3p\, q_s$. If q_s (*i.e.* slots per pole per phase of stator) is an integer, S_s is an even integer. Therefore, examination of the above relationship will reveal that rotor with an odd number of slots is undersirable from the point of noise and vibrations.

4. **Voltage ripples.** The harmonic fields produced by the stator current induce harmonic currents in the rotor which in turn reflects back additional harmonic fields into the stator. This cause ripples in the terminal voltage and also additional iron losses.

The voltage ripples produce high frequency currents in the supply lines which, in turn, may produce inductive interference with communication circuits.

10·22·1. Rules for selecting rotor slots. The following general rules should be followed concerning the choice of rotor slots for squirrel cage machines.

(*i*) As stated earlier, the number of rotor slots should never be equal to stator slots but must either be large or smaller. Satisfactory results are obtained when the number of rotor slots is 15 to 30 per cent larger or smaller than the number of stator slots.

(*ii*) The difference between stator slots and rotor slots should not be equal to p, $2p$ or $5p$ to avoid synchronous cusps.

(*iii*) The difference between the number of stator and rotor slots should not be equal to $3p$ for 3 phase machines in order to avoid magnetic locking

(*iv*) The difference between number of stator slots and rotor slots should not be equal to, 1 2, $(p\pm1)$ or $(p\pm2)$ to avoid noise and vibrations.

Summarizing,

$S_s - S_r$ should not be equal to

$$0, \pm p, \pm 2p, \pm 3p, \pm 5p$$
$$\pm 1, \pm 2, \pm(p\pm1), \pm(p\pm2).$$

10·22·2. Reduction of harmonic torques

Following are some of the methods used for reduction/elimination of harmonic torques.

(*i*) **Chording.** The simplest way to eliminate the harmonic induction torques is to weaken the stator winding mmf harmonics. In order to achieve this, chorded windings with integral number of slots per pole per phase are used.

(*ii*) **Integral slot windings.** Windings with fractional number of slots per pole per phase create asymmetrical mmf distribution around the air gap and favour the creation of noise in the motor. Therefore, fractional slot windings are not used for induction motor stators and only integral slot windings are used.

(*iii*) **Skewing.** The motor noise and vibrations, cogging and synchronous cusps can be reduced or even entirely eliminated by skewing either the stator or the rotor. The practice generally followed in India is to skew the rotor (See Fig. 10 25). If either stator or rotor slots are skewed, the variations in flux density, magnetic pull, and torque due to the slot openings will be displaced in time phase along the core length, resulting in more uniform torque, less noise, and better voltage waveform. In order to eliminate the effect of any harmonic, the rotor bars should be skewed through an angle so that the bars lie under alternate harmonic poles of the same polarity or in other words, bars must be skewed through two pitches. Suppose it is desired to eliminate a harmonic of the order n in a machine with p poles. The number of nth order harmonic poles is np.

Angle between two adjacent harmonic poles $=360/np$.

$\therefore$ For elimination of nth harmonic by skewing,

angle of skew $\theta=720/n\times p$ degree mechanical ...(10·14)

The electrical angle of skew is :

$$\theta_{sk}=(720/np)\,(p/2)=360/n \text{ degree electrical}$$
$$=2\pi/n \text{ electrical radian}$$

A skewed rotor bar is, in effect, spread over an angle θ_{sk} and its induced emf is reduced in accordance with the distribution factors.

$$K_{d1}=\frac{\sin\theta_{sk}/2}{\theta_{sk}/2} \qquad ...(10·15)$$

and

$$K_{dn}=\frac{\sin n\theta_{sk}/2}{n\theta_{sk}/2} \qquad ...(10·16)$$

for the fundamental and nth harmonic respectively of the gap flux.

It is clear that if $\theta_{sk}=2\pi/n$

$$K_{dn}=\frac{\sin 2\pi/2}{2\pi/2}=0$$

and nth harmonic emf reduces to zero thereby completely eliminating the harmonic.

This is clear from the phasor diagram for the nth harmonic shown in Fig 10 25 The nth harmonic emf is reduced to zero since its phase spread is 2π.

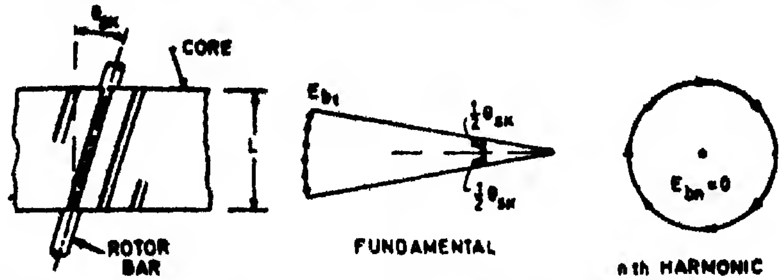


Fig. 10 25. Skewing of rotor bars.

The usual practice is to skew the rotor through one stator slot pitch. Let us examine the effect of skewing the rotor through one stator slot pitch. The angle of skew is, therefore,

$$\theta_{sk}=\pi\, p/S_s \text{ electrical radian.}$$

Order of slot harmonics $n=2(S_s/p)\pm1$.

Distribution factor for the slot harmonics

$$K_{dn}=\frac{\sin n\,\theta_{sk}/2}{n\,\theta_{sk}/2}=\frac{\sin\,[2(S_s/p)\pm1]\,\pi p/2S_s}{[2(S_s/p)\pm1]\,\pi p/2S_s}$$

$$=\frac{\sin\,[\pi\pm\pi\, p/2S_s]}{\pi\pm\pi p/2\,S_s}=\frac{\pm\sin\,\pi\, p/2S_s}{\pi\pm\pi\, p/2S_s}$$

Let us take the case of a machine with 4 poles and 36 stator slots.

$\therefore$ Order of slot harmonics

$$n=2\,S_s/p\pm1=2\times19/4\pm1=19,\ 17.$$

The distribution factors for the slot harmonics are :

$$K_{d19}=\frac{-\sin\,\pi\times4/(2\times36)}{\pi+\pi\times4/(2\times36)}=-0{\cdot}052$$

and

$$K_{d17}=\frac{+\,\sin\,\pi\times4/(2\times36)}{\pi-\pi\times4/(2\times36)}=0\ 058.$$

This shows that the emf induced owing to slot harmonics is drastically reduced. (The magnitude reduces to 5—6%). Thus it is sufficient to skew the rotor through one stator slot pitch.

Fig. 10·26 shows the speed torque curves of a motor with and without slot skewing. It is clear that the harmonic torques are eliminated by skewing and there are no dips in the torque-speed characteristics.

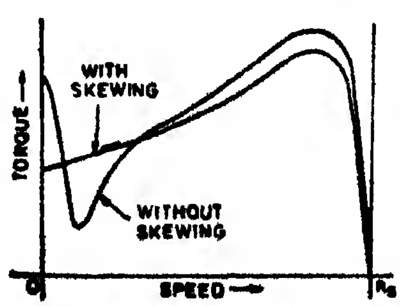


Fig. 10·26. Torque-speed characteristics with and without skewing.

Skewing decreases the winding factor for the fundamental and increases the leakage flux in the motor. Therefore, it lowers the power factor and overload capacity of the machine.

(*iv*) **Increase in air gap length.** An increase in air gap length, decreases the harmonic torques. But an increased gap length leads to an increase in no load current and thus makes the motor power factor poor. Therefore, only in motors of high reliability, for mechanical reasons, the air gap is made larger than the normal size.

Example 10·7. *A 3 phase 4 pole, 50 Hz motor has 24 stator and 44 rotor slots and has a crawling speed of 11·5 r.p.m. What are the precise causes for this abnormal performance? Explain that the machine has large vibrations.*

Solution. Synchronous speed $=120\times 50/4=1500$ r.p.m.

(*i*) Let us first consider harmonic fields produced by windings. A 3 phase winding produces harmonic fields of order

$$n=6N\pm 1 \qquad \text{where } N \text{ is an integer.}$$

For $\qquad N=1, \qquad n=6\times 1\pm 1=7, 5.$

Thus the winding produces a forward rotating 7th harmonic and a backward rotating 5th harmonic field.

If the 7th harmonic is strong, it will make the motor run at approximately 1/7th of synchronous speed.

$\therefore$ 1/7th synchronous speed $=1500/7=214{\cdot}3$ r.p.m.

But the crawling speed is 155 r.p.m. and therefore 7th harmonic produced by winding is not responsible for abnormal behaviour.

For $\qquad N=2, \qquad n=6\times 2+1=13.$

Synchronous speed corresponding to 13th harmonic $=1500/13=115{\cdot}4$ r.p.m.

Therefore, the machine will run at about 115 r.p.m. if 13th harmonic has sufficient strength.

(*ii*) Let us consider the harmonic fields due to slotting.

$$n=\frac{2S_s}{p}\pm 1=\frac{2\times 24}{4}\pm 1=13, 11$$

Thus there is a forward rotating 13th harmonic, the synchronous speed corresponding to which is 115·4 r.p.m. Therefore, the machine crawls at about 115 r.p.m.

Hence the 13th harmonic fields produced by stator winding and slotting are responsible for crawling of machine at 115 r.p.m.

The machine produces 11th harmonic field. The number of 11th harmonic poles is $11\times 4=44$. Thus there is a rotor bar corresponding to every 11th harmonic poles and therefore there will be very large 11th harmonic current flowing in the machine which will cause it to vibrate violently.

Example 10·8. *A 3 phase, 4 pole 50 Hz induction motor has 24 stator and 28 rotor slots. Prove that it has a tendency to run as a synchronous motor at 214·3 r.p.m.*

Solution. Synchronous speed $n_s=2\times 50/4=25$ r.p.s.

Order of forward rotating field produced by stator slotting $n=2\times 24/4+1=13$

and this revolves at a speed $n_s/13$ with respect to stator.

Order of backward rotating field produced by rotor slotting

$$n' = 2\times 28/14 - 1 = 13.$$

Its speed with respect to rotor is $-(n_s - n_r)/13$ where n_r is the speed of the rotor. The rotor revolves forward at a speed n_r and therefore it revolves its 13th harmonic at a speed :

$$-(n_s - n_r)/13 + n_r$$

with respect to stator.

Hence in order that the two fields may revolve synchronously with respect to each order, we have :

$$\frac{n_s}{13} = n_r - \frac{n_s - n_r}{13}$$

$\therefore$ Speed of rotor $n_r = n_s/7 = 25/7$ r.p.s $= 214.3$ r.p.m.

Therefore, the motor has a tendency to crawl at 214.3 r.p.m (fixed) due to harmonic synchronous torque.

Example 10.9. *A 3 phase, 4 pole induction motor has 24 slots. Calculate the order of slot harmonics produced. It is desired to completely eliminate the higher order slot harmonic, find the angle through which the bars must be skewed. Find the effect of skewing on the lower order harmonic.*

Solution.

Order of slot harmonics $n = 2(S_s/p) \pm 1 = 2(24/4) \pm 1 = 13, 11$.

It is desired to completely eliminate the 13th harmonic.

$\therefore$ Angle of skew $\theta_s = 720/(n\times p) = 720/(13\times 4) = 13.85°$ mech.

Electrical angle of skew $\theta_{se} = (4/2)\times 13.85 = 27.70° = 0.483$ radian.

Distribution factor for 11th harmonic

$$K_{d_{11}} = \frac{\sin n\theta_{se}/2}{n\theta_{se}/2} = \frac{\sin 11\times 0.483/2}{11\times 0.483/2} = 0.176.$$

Therefore, the 11th harmonic emf with skewing is reduced to 17.6% of value obtained without skewing.

10.23. Design of rotor bars and slots

10.23.1. Rotor bar current.

From Eqns. 11.46 and 11.47, current in each bar

$$I_b = \frac{2\, m_s\, K_{ws}\, T_s}{S_r} I_s \cos\phi$$

For a three phase machine $m_s = 3$.

$$I_b = \frac{6\, I_s\, T_s}{S_r} K_{ws} \cos\phi \qquad \ldots(10.17)$$

$$= 0.85\times \frac{6\, I_s\, T_s}{S_r}$$

The above relation may be interpreted as that the rotor mmf is about 85 per cent of stator mmf.

10.23.2. Area of rotor bars. The performance of an induction motor is greatly influenced by the resistance of rotor. A motor designed with high rotor resistance has the advantage that it has a high starting torque. However, a rotor with a high resistance has the disadvantage that its I^2R loss is greater and therefore its efficiency is lower under running conditions.

The value of rotor resistance depends upon the current density used for rotor conductors, the higher the current density, the lower is the conductor area and greater the resistance. Therefore, a rotor designed with a high value of current density results in high starting torque and a lower efficiency for the machine.

The rotor resistance is the sum of the resistance of the bars and the end rings. The cross-section of the bars and the end rings must be so selected that a proper value of rotor resistance is obtained *i.e.* a value of rotor resistance which meets both the requirements of starting torque as will as the efficiency.

It is desirable to have a compromise between a high resistance rotor which gives a good starting torque and a low resistance rotor which gives a high value of efficiency under running conditions.

Current density in the rotor bars may be taken between 4 to 7 A/mm².

Area of each bar $a_b = I_b/\delta_b$ mm²

where δ_b is the current density in rotor bars, A/mm².

10·23·3. Shape and size of rotors slots. The rotor slots for squirrel cage rotor may either be closed or semi-enclosed types (Fig. 10·27).

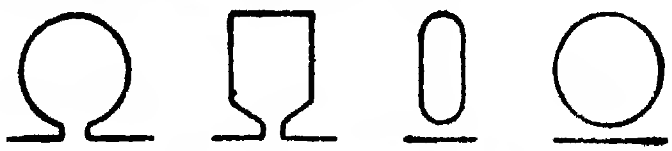


Fig. 10·27. Types of rotor slots.

Closed slots are preferred for small size machines because the reluctance of the air gap is not large owing to absence of slot openings. This gives a reduced value of magnetizing current. As the surface of the rotor is smooth, the operation of the machine is quieter. The biggest advantage is that the leakage reactance with closed slots is large and therefore the current at starting can be limited. This is very useful in the case of machines which are started with direct on-line starters. But the disadvantage is that the increased value of reactance results in reduction of overload capacity. A semi-enclosed slot gives a better overload capacity.

The rectangular shaped bars and slots are generally preferred to circular bars and slots as the higher leakage reactance of the lower part of the rectangular bars, during starting, forces most of the current through the top of the bar. This increases the rotor resistance at starting and improves the starting torque. Deep slots, however, give an increased leakage reactance and a high flux density at the root of the teeth.

10·23·4. Rotor slot insulation. No insulation is used between bars and rotor core. A clearance of 0·15 to 0·4 mm can be left between rotor bars and the core depending upon whether slots are skewed or not. Higher clearances have to be left for the skewed slots.

10·24. Design of end rings

10·24·1. End ring current. The distribution of current in the bars and end rings of a squirrel cage motor is complicated. Fig. 10·28 (*a*) shows a developed cage winding under two pole pitches.

The stator winding is a 3 phase distributed winding and thus produces a revolving field. This field may be considered as sinusoidally distributed in space as the harmonics in most cases are small and produce only secondary effects. This revolving field produces emfs of fundamental frequency in the bars. Fig. 10·28 (*b*) shows the magnitude of emfs in the bars

and if the bars are assumed to be infinitely distributed, the distribution of emfs can be considered as sinusoidal in the bars over a pole pitch. These emfs produced in the bars would circulate currents as shown in Fig. 10 28 (c). If the resistance of end rings is negligible as compared with that of the bars, the resistance coming in each current path is the resistance of two bars. Thus the current which the bars carry would be proportional to their instantaneous emfs which in turn depend upon the position of the bars in the magnetic field. Thus the wave which represents the emf would represent the bar current also. Fig. 10 28 (d) shows the wave representing currents in bars.

It is observed from Fig. 10'28 (c) that at points where the current is maximum in the

(a)

(b)

(c)

(d)

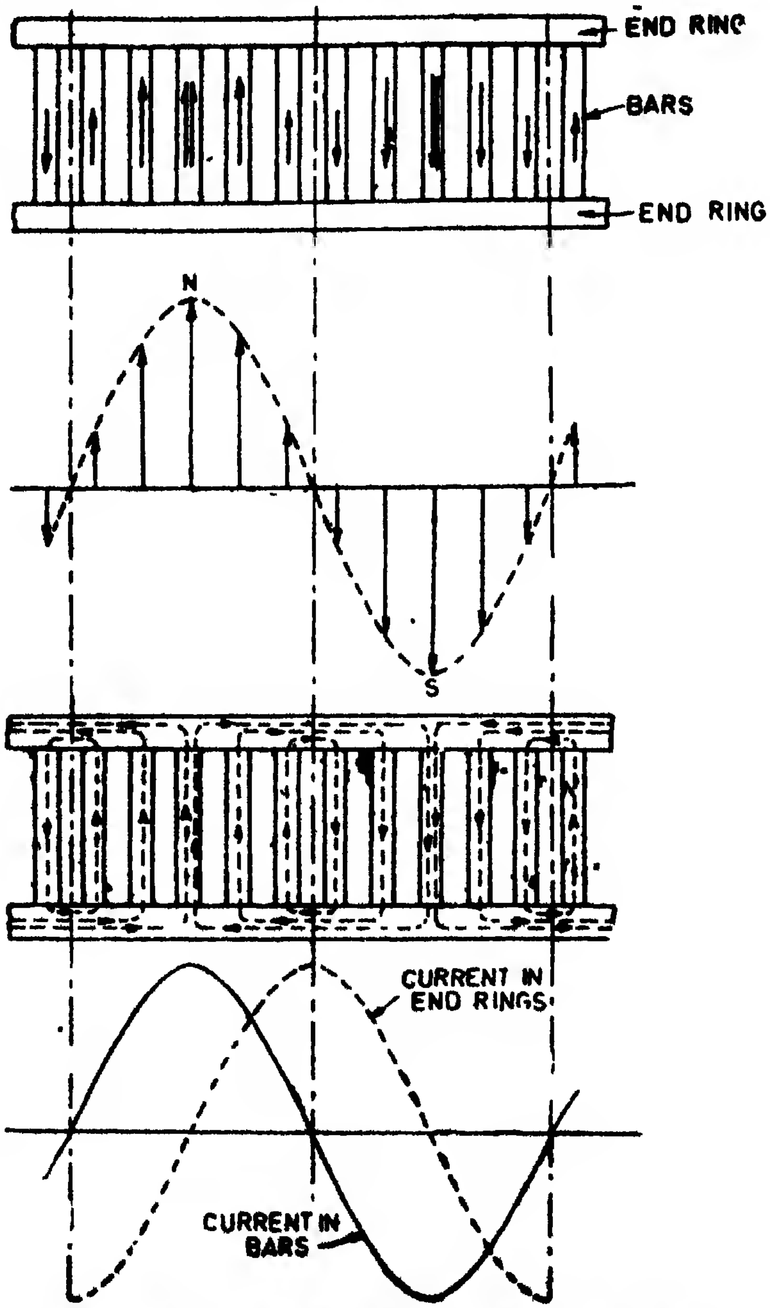


Fig. 10'28. Current distribution in squirrel cage rotors.

bars, current is zero in the end rings but the end ring current is maximum where the current in the bars is zero. The current in the end rings is also sinusoidal, its nature being as indicated in Fig. 10'28 (c) and (d).

It should be understood that the end ring resistance, if not negligible, will tend to distort the bar current distribution from being sinusoidal.

Considering a group of rotor bars under one pole pitch, one half would send current to an end ring in one direction and the other half in the other direction. If the maximum value of the current in each bar is $I_{b(max)}$ and if the current is maximum in all the bars at the same time, then maximum value of the current in the end ring :

$$= \frac{\text{bars per pole}}{2} \times \text{current per bar} = \frac{S_r}{2} I_{b(max)}$$

However, current is not maximum in all the bars under one pole at the same time but varies according to sine law ; hence, the maximum value of the current in the end ring is the average of the current of half the bars under one pole.

∴ Maximum value of end ring current

$$I_{e(max)} = \frac{2}{\pi} \times \frac{S_r}{2p} \times I_{b(max)}.$$

But the bar current varies sinusoidally ∴ $I_{b(max)} = \sqrt{2} I_b$

or
$$I_{e(max)} = \frac{2}{\pi} \cdot \frac{S_r}{2p} \cdot \sqrt{2} I_b$$

The end ring current also varies sinusoidally.

∴ R.m.s value of end ring current

$$I_e = \frac{I_{e(max)}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{2}{\pi} \cdot \frac{S_r}{2p} \sqrt{2}\, I_b = \frac{S_r I_b}{\pi p} \qquad \ldots(10\cdot17)$$

10·24 2. Area of end rings. The value of current density chosen for the end rings should be such that the desired value of rotor resistance is obtained.

The ventilation is generally better for end rings and therefore a slightly higher value of current density than that obtaining in rotor bars can be taken.

Area of each end ring

$$a_e = I_e / \delta_e = S_r I_b / \pi p \, \delta_e \text{ mm}^2$$

where δ_e = current density in end rings, A/mm².

Area of ring a_e = depth of end ring × thickness of end ring.

$$= d_e \times t_e$$

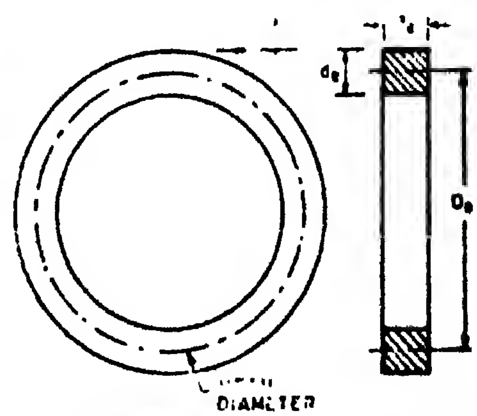


Fig. 10·29. Dimensions of end ring.

10 24 Full load slip The value of slip at full load is determined by the rotor resistance. A reasonable value of rotor resistance to be incorporated in the rotor can be obtained by the knowledge of reasonable values of full load slip. The value of slip, s, is derived from the following relationship

$$\frac{\text{rotor copper loss}}{\text{rotor output}} = \frac{s}{1-s} \qquad \ldots(10\cdot18)$$

where s is the per unit slip.

Table 10·8 gives the usual values of per cent slip at full load for various ratings

Assuming $K_{ws}=K_{wr}$

rotor turns per phase $T_r=\frac{K_{ws}}{K_{wr}}\,\frac{E_r}{E_s}\,T_s=\frac{231}{89}\,63=50.4\simeq 50.$

Rotor conductors per slot $=\frac{6\times 50}{72}\simeq 4.$

Actual number of rotor conductors $=4\times 72=288.$

Actual number of rotor turns per phase $T_r=\frac{288}{6}=48.$

(c) and (d) Actual rotor voltage per phase at standstill $E_r=\frac{48}{63}\times 289=220$ V.

Voltage between slip rings at standstill $=\sqrt{3}\times 220=381$ V.
Coil span = slots per pole $=72/8=9$ slots.

(e) Stator current per phase $I_s=\frac{90\times 10^3}{\sqrt{3}\times 500\times 0.9\times 0.86}=134.3$ A.

Since power factor is 0.86, the rotor mmf may be taken as 0.86 of stator mmf.

$\therefore$ Rotor current per phase $I_r=0.86\,\frac{I_s T_s}{T_r}=0.86\times\frac{134.3\times 63}{48}=151.6$ A.

10.26. Rotor teeth. The width of rotor slot should be such that the flux density in the rotor teeth does not exceed about 1.7 Wb/m². The maximum flux density for rotor teeth occurs at their root as their section is minimum there.

Minimum width of rotor teeth $W_{tr}\text{ (min.)}=\frac{\Phi_m}{1.7\times(S_r/p)\times L_i}$ ...(10.20)

It should be checked that the value of minimum tooth width actually provided in the machine is higher than the value given by Eqn. 10.20.

Minimum width of tooth actually provided

W_{tr} = rotor slot pitch at the root − rotor slot width $=\frac{\pi(D_r-2d_{sr})}{S_r}-W_{sr}$ ...(10.21)

where d_{sr} = depth of rotor slot and W_{sr} = width of rotor slot.

10.27. Rotor core. The flux density in the rotor core is generally equal to stator core density.

Depth of rotor core $d_{cr}=\frac{\Phi_m}{2\times B_{cr}\times L_i}$ ...(10.22)

where B_{cr} = flux density in the rotor core.

Inside diameter of rotor lamination $D_i=D_r-2(d_{sr}+d_{cr})$ ...(10.23)

The flux density in rotor teeth and core can be taken slightly higher than those in the stator teeth and core. This is because the iron losses in the rotor are very small owing to small value of frequency of rotor currents.

10.28. Sizes of induction motor laminations available in the market. The two major firms supplying induction motor, fan and transformer laminations are :

1. M/s Guest Keen Williams Ltd.
Precision Pressings Division
Lal Bahadur Shastri Marg, Bhandup
Bombay 400078.

2. M/s Devidayal Stainless Steel Industries Private Ltd.
Electrical Stampings Division
P.O. 6224, Darukhana, Reay Road,
Bombay.

The various sizes of single phase and three phase induction motor stampings manufactured by Precision Pressings Division of M/s Guest Keen Williams are given in Table 10·4 while those manufactured by Electrical Stampings Division of M/s Devidayal Stainless Steel Industries are given in Table 10·5.

The various slot sizes given in Fig. 10·30 and 0·31 are due to M/s Devidayal Stainless Steel Industries.

Table 10·4

A C. single phase and three phase induction motor laminations (Manufactured by M/s Guest Keen Williams)

Sankey Type No.	STATOR			ROTOR			Approximately pairs per kg (0·5 mm)
	Outside Dia.	Inside Dia	Slots No.	Outside Dia.	Inside Dia	Slots No.	
108 M	3 /4"	2"	30	2"	9/16"	17	47
166 M	4 1/4"	2·1/2"	24	2·1/2"	0·465"	17	43
168 M	4·1/4"	2·1/2"	24	2·1/2"	0·465"	32	43
D 492	4·9/ 6"	2"17/32"	24	2·17/32"	7/16"	23	36
D 922/R	120 mm	2·7/16"	24	2·7/16"	5/8"	17	33
138 M	5·7/16"	3·1/2"	28	3·1/2"	3/4"	20	25
D 655	6"	3·3/8"	24	3 3/8"	1"	30	18
165 M	6"	3·3/4"	24	3·3/4"	7/8"	48	22
164 M	6·3/8"	3·350	24	3·350	1"	22	16
139 M	6·1/2"	3·3/4"	24	3·3/4"	1"	34	17
161 M	7·1/8"	4·300"	36	4·300"	1·1/8"	44	14
140 AM	7·1/2"	4·1/2"	24	4·1/2"	1·3/8"	32	14
141 M	7·1/2"	4·1/2"	36	4·1/2"	1·3/8"	34	14
163 M	7 1/2"	4 1/2"	36	4·1/2"	1"	33	14
162 M	7·99"	5·1/8"	36	5·1/8"	1·1/4"	48	11
142 M	8·3/4"	5·1/2"	36	5·1/2"	1·5/8"	44	10
143 M	10 1/4"	6·1/4"	36	6·1/4"	1·3/4"	34	6
144 M	10·1/4"	7"	36	7"	1·3/4"	40	7
102 M	12"	7·1/2"	36	7·1/2"	2"	40	5
146 M	12"	8·1/2"	36	8·1/2"	2"	40	5
104 M	14"	9"	36	9"	2"	59	4
103 AM	14"	10"	48	10"	2"	59	4

Total rotor copper loss $=457+264=721$ W.

Now, $\dfrac{s}{1-s}=\dfrac{\text{rotor copper loss}}{\text{rated output}}$ or $\dfrac{s}{1-s}=\dfrac{721}{15000}$

$\therefore$ Full load slip, $s=0.0458$.

Synchronous speed $=\dfrac{120\times 50}{6}=1000$ r.p.m.

Rotor speed $=(1-0.0458)\times 1000=954.2$ r.p.m.

This is nearly equal to the specified speed.

10·25. Design of wound rotor

10·25·1. Number of rotor slots. The rotor winding for wound rotor motors are 3 phase windings, and the number of rotor slots should be such that a balanced winding is obtained. Generally windings with an integral number of slots per pole per phase are used for the rotor. Fractional slot windings may also be used. It is preferable to use a number of slots which are a multiple of phases and pair of poles in the case of fractional slot windings.

10·25·2. Number of rotor turns. The rotor represents the secondary of a transformer and the voltage between slip rings is maximum when the rotor is at rest. Therefore, to keep the rotor voltage to an acceptable level the ratio of effective stator to rotor turns must be properly adjusted. The choice of this turns ratio is arbitrary and is controllable by the designer.

The rotor voltage on open circuit between slip rings should not exceed [illegible]00 V for small machines where hand operated starters and switchgear are employed. The voltage is limited to a small value in order to protect persons working the motor if the brush gear is not perfectly protected. More-over with small rotor voltages, it is easier to insulate the rotor windings.

In case of high voltage machines and also in the case of large machines, the rotor voltage should be high because in such cases if the rotor voltage is kept small, the rotor current becomes large involving use of large conductor sections. Large rotor currents complicate the design of slip rings, brush gear and starter contacts. For large size machines, voltages upto 1000–2000 V can be used and there seems to be no objection to rather higher voltages for very large motors where it is worthwhile to completely protect the brush gear.

Let T_s, T_r = number of turns per phase for stator and rotor respectively,

K_{ws}, K_{wr} = winding factor for stator and rotor respectively,

E_s = stator voltage per phase,

E_r = rotor voltage per phase at standstill.

Now $$\frac{E_r}{E_s}=\frac{K_{wr}\,T_r}{K_{ws}\,T_s}$$

$\therefore$ Rotor turns per phase $$T_r=\frac{K_{ws}}{K_{wr}}\cdot\frac{E_r}{E_s}\cdot T_s \qquad \ldots(10·1\ \ )$$

In case of small machines :

E_r should not exceed 500 V and $500/\sqrt{3}=290$ V for delta and star connected machines respectively.

By assuming a suitable value of voltage between slip rings, the rotor turns per phase to be provided can be calculated from Eqn. 10·19.

10·25·3. Area of rotor conductors. The full load rotor mmf is taken as 85 per cent of stator mmf.

$\therefore \quad I_r T_r = 0.85\, I_s T_s \quad$ or $\quad I_r = 0.85\, \dfrac{I_s T_s}{T_r}$

where $\qquad I_r$ = rotor current per phase.

The area of the rotor conductors is found out by assuming a suitable value for current density. In order to avoid excessive rotor copper loss, the current density in the rotor is chosen almost equal to that in the stator.

Round conductors are used for small motors. But for large motors, it becomes necessary to use bar conductors.

10.25.4. Rotor windings. For small induction motors of slip ring type, it is a normal practice to use mush windings for rotor housed in semi-enclosed slots. The coils are roughly formed outside the machine and dropped into the slots through slot opening one by one. It is usual to use several wires in parallel per turn, to keep the conductor small enough to go through the narrow slot opening. The rotor is invariably star connected and three leads are brought through the shaft to the slip rings.

For larger motors, a double layer bar type wave winding is used. This winding has generally two bars per slot. The bars are pushed through partially closed slots and are bent to shape at the other end (See Fig. 6.62 page 280). In motors of outputs of about 750 kW and over, we have to use 4 bars sometimes. The use of 4 bars per slot is made to reduce the current handled by each slip ring. The winding with more than 2 bars per slot is called a barrel winding and is usually wave wound.

Example 10.13. *A 3 phase induction motor has 54 stator slots with 8 conductors per slot and 72 rotor slots with 4 conductors per slot. Find the number of stator and rotor turns. Find the voltage across the rotor slip rings when the rotor is open circuited and at rest. Both stator and rotor are star connected and a voltage of 400 V is applied across the stator terminals.*

Solution.

Stator turns per phase $T_s = \dfrac{54 \times 8}{6} = 72$ Rotor turns per phase $T_r = \dfrac{72 \times 4}{6} = 48$.

Stator voltage per phase $E_s = 400/\sqrt{3} = 231$ V.

Rotor voltage per phase at standstill $E_r = E_s \cdot \dfrac{T_r}{T_s} = 231 \times \dfrac{48}{72} = 154$ V.

$\therefore$ Rotor voltage between slip rings at standstill $= \sqrt{3} \times 154 = 266.7$ V.

Example 10.14. *A 90 kW, 500 V, 50 Hz, 3-phase, 8 pole induction motor has a star connected stator winding accommodated in 63 slots with 6 conductors per slot. If the slip ring voltage on open circuit is to be about 400 V, find a suitable rotor winding, stating :*

(a) number of slots (b) number of conductors per slot (c) coil span (d) slip ring voltage on open circuit (e) approximate full load current per phase in rotor. Assume efficiency = 0.9 ; power factor = 0.86.

Solution. Stator turns per phase $T_s = \dfrac{63 \times 6}{6} = 63$.

Stator voltage per phase $E_s = \dfrac{500}{\sqrt{3}} = 289$ V.

(a) The number of rotor slots should perferably be an integer. Taking 3 slots per pole phase.

Number of rotor slots $S_r = 3 \times 3 \times 8 = 72$.

Using star connection for rotor, rotor voltage at standstill $E_r = 400/\sqrt{3} = 231$ V.

Assuming $K_{ws}=K_{wr}$

rotor turns per phase $T_r=\frac{K_{ws}}{K_{wr}}\,\frac{E_r}{E_s}\,T_s=\frac{231}{89}\,63=50.4\simeq 50.$

Rotor conductors per slot$=\frac{6\times 50}{72}\simeq 4.$

Actual number of rotor conductors$=4\times 72=288.$

Actual number of rotor turns per phase $T_r=\frac{288}{6}=48.$

(c) and (d) Actual rotor voltage per phase at standstill $E_r=\frac{48}{63}\times 289=220$ V.

Voltage between slip rings at standstill$=\sqrt{3}\times 220=381$ V.

Coil span$=$slots per pole$=72/8=9$ slots.

(e) Stator current per phase $I_s=\frac{90\times 10^3}{\sqrt{3}\times 500\times 0.9\times 0.86}=134.3$ A.

Since power factor is 0·86, the rotor mmf may be taken as 0·86 of stator mmf.

$\therefore$ Rotor current per phase $I_r=0.86\,\frac{I_sT_s}{T_r}=0.86\times\frac{134.3\times 63}{48}=151.6$ A.

10·26. Rotor teeth. The width of rotor slot should be such that the flux density in the rotor teeth does not exceed about 1·7 Wb/m². The maximum flux density for rotor teeth occurs at their root as their section is minimum there.

Minimum width of rotor teeth W_{tr} (min.)$=\frac{\Phi_m}{1.7\times(S_r/p)\times L_i}$ ...(10·20)

It should be checked that the value of minimum tooth width actually provided in the machine is higher than the value given by Eqn. 10·20.

Minimum width of tooth actually provided

$W_{tr}=$rotor slot pitch at the root$-$rotor slot width$=\frac{\pi(D_r-2d_{sr})}{S_r}-W_{sr}$ ...(10·21)

where $d_{sr}=$depth of rotor slot and $W_{sr}=$width of rotor slot.

10·27. Rotor core. The flux density in the rotor core is generally equal to stator core density.

Depth of rotor core $d_{cr}=\frac{\Phi_m}{2\times B_{cr}\times L_i}$ ...(10·22)

where $B_{cr}=$flux density in the rotor core.

Inside diameter of rotor lamination $D_i=D_r-2(d_{sr}+d_{cr})$ ...(10·23)

The flux density in rotor teeth and core can be taken slightly higher than those in the stator teeth and core. This is because the iron losses in the rotor are very small owing to small value of frequency of rotor currents.

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Table 10·4
A C. single phase and three phase induction motor laminations
(Manufactured by M/s Guest Keen Williams)

Sankey Type No.	STATOR Outside Dia.	STATOR Inside Dia	STATOR Slots No.	ROTOR Outside Dia.	ROTOR Inside Dia	ROTOR Slots No.	Approximately pairs per kg (0·5 mm)
108 M	3 /4"	2"	30	2"	9/16"	17	47
166 M	4 1/4"	2·1/2"	24	2·1/2"	0·465"	17	43
168 M	4·1/4"	2·1/2"	24	2·1/2"	0 465"	32	43
D 492	4·9/ 6"	2"17/32"	24	2·17/32"	7/16"	23	36
D 922/R	120 mm	2·7/16"	24	2·7/16"	5/8"	17	33
138 M	5·7/16"	3·1/2"	28	3·1/2"	3/4"	20	25
D 655	6"	3·3/8"	24	3·3/8"	1"	30	18
165 M	6"	3·3/4"	24	3·3/4"	7/8"	48	22
164 M	6·3/8"	3·350	24	3·350	1"	22	16
139 M	6·1/2"	3 3/4"	24	3·3/4"	1"	34	17
161 M	7·1/8"	4·300"	36	4·300"	1·1/8"	44	14
140 AM	7·1/2"	4·1/2"	24	4·1/2"	1·3/8"	32	14
141 M	7·1/2"	4·1/2"	36	4·1/2"	1·3/8"	34	14
163 M	7 1/2"	4 1/2"	36	4·1/2"	1"	33	14
162 M	7·99"	5·1/8"	36	5·1/8"	1·1/4"	48	11
142 M	8·3/4"	5·1/2"	36	5·1/2"	1·5/8"	44	10
14 M	10 1/4"	6·1/4"	36	6·1/4"	1·3/4"	34	6
144 M	10·1/4"	7"	36	7"	1·3/4"	40	7
102 M	12"	7·1/2"	36	7·1/2"	2"	40	5
146 M	12"	8·1/2"	36	8·1/2"	2"	40	5
104 M	14"	9"	36	9"	2"	39	4
103 AM	14"	10"	48	10"	2"	59	4

Total rotor copper loss $=457+264=721$ W.

Now, $\dfrac{s}{1-s}=\dfrac{\text{rotor copper loss}}{\text{rated output}}$ or $\dfrac{s}{1-s}=\dfrac{721}{15000}$

$\therefore$ Full load slip, $s=0.0458$.

Synchronous speed $=\dfrac{120\times50}{6}=1\ 00$ r.p.m.

Rotor speed $=(1-0.0\ 58)\times1000=954.2$ r.p.m.

This is nearly equal to the specified speed.

10.25. Design of wound rotor

10.25.1. Number of rotor slots. The rotor winding for wound rotor motors are 3 phase windings, and the number of rotor slots should be such that a balanced winding is obtained. Generally windings with an integral number of slots per pole per phase are used for the rotor. Fractional slot windings may also be used It is preferable to use a number of slots which are a multiple of phases and pair of poles in the case of fractional slot windings.

10.25.2. Number of rotor turns. The rotor represents the secondary of a transformer and the voltage between slip rings is maximum when the rotor is at rest. Therefore, to keep the rotor voltage to an acceptable level the ratio of effective stator to rotor turns must be properly adjusted. The choice of this turns ratio is arbitrary and is controllable by the designer.

The rotor voltage on open circuit between slip rings should not exceed 00 V for small machines where hand operated starters and switchgear are employed. The voltage is limited to a small value in order to protect persons working the motor if the brush gear is not perfectly protected. More-over with small rotor voltages, it is easier to insulate the rotor windings.

In case of high voltage machines and also in the case of large machines, the rotor voltage should be high because in such cases if the rotor voltage is kept small, the rotor current becomes large involving use of large conductor sections Large rotor currents complicate the design of slip rings, brush gear and starter contacts. For large size machines, voltages upto 1000–2000 V can be used and there seems to be no objection to rather higher voltages for very large motors where it is worthwhile to completely protect the brush gear.

Let T_s, T_r = number of turns per phase for stator and rotor respectively,

K_{ws}, K_{wr} = winding factor for stator and rotor respectively,

E_s = stator voltage per phase,

E_r = rotor voltage per phase at standstill.

Now $$\frac{E_r}{E_s}=\frac{K_{wr}\,T_r}{K_{ws}\,T_s}$$

$\therefore$ Rotor turns per phase $T_r=\dfrac{K_{ws}}{K_{wr}}\cdot\dfrac{E_r}{E_s}\cdot T_s$ ...(10.1)

In case of small machines:

E_r should not exceed 500 V and $500/\sqrt{3}=290$ V for delta and star connected machines respectively.

By assuming a suitable value of voltage between slip rings, the rotor turns per phase to be provided can be calculated from Eqn. 10.19.

10.25.3. Area of rotor conductors. The full load rotor mmf is taken as 85 per cent of stator mmf.

$$\therefore \quad I_r T_r = 0.85\, I_s T_s \qquad \text{or} \qquad I_r = 0.85\, \frac{I_s T_s}{T_r}$$

where $\quad I_r$ = rotor current per phase.

The area of the rotor conductors is found out by assuming a suitable value for current density. In order to avoid excessive rotor copper loss, the current density in the rotor is chosen almost equal to that in the stator.

Round conductors are used for small motors. But for large motors, it becomes necessary to use bar conductors.

10.25.4. Rotor windings. For small induction motors of slip ring type, it is a normal practice to use mush windings for rotor housed in semi-enclosed slots. The coils are roughly formed outside the machine and dropped into the slots through slot opening one by one. It is usual to use several wires in parallel per turn, to keep the conductor small enough to go through the narrow slot opening. The rotor is invariably star connected and three leads are brought through the shaft to the slip rings.

For larger motors, a double layer bar type wave winding is used. This winding has generally two bars per slot. The bars are pushed through partially closed slots and are bent to shape at the other end (See Fig. 6.62 page 280). In motors of outputs of about 750 kW and over, we have to use 4 bars sometimes. The use of 4 bars per slot is made to reduce the current handled by each slip ring. The winding with more than 2 bars per slot is called a barrel winding and is usually wave wound.

Example 10.13. *A 3 phase induction motor has 54 stator slots with 8 conductors per slot and 72 rotor slots with 4 conductors per slot. Find the number of stator and rotor turns. Find the voltage across the rotor slip rings when the rotor is open circuited and at rest. Both stator and rotor are star connected and a voltage of 400 V is applied across the stator terminals.*

Solution.

Stator turns per phase $T_s = \dfrac{54 \times 8}{6} = 72$ Rotor turns per phase $T_r = \dfrac{72 \times 4}{6} = 48.$

Stator voltage per phase $E_s = 400/\sqrt{3} = 231$ V.

Rotor voltage per phase at standstill $E_r = E_s \cdot \dfrac{T_r}{T_s} = 231 \times \dfrac{48}{72} = 154$ V.

$\therefore$ Rotor voltage between slip rings at standstill $= \sqrt{3} \times 154 = 266.7$ V.

Example 10.14. *A 90 kW, 500 V, 50 Hz, 3-phase, 8 pole induction motor has a star connected stator winding accommodated in 63 slots with 6 conductors per slot. If the slip ring voltage on open circuit is to be about 400 V, find a suitable rotor winding, stating :*

(a) number of slots (b) number of conductors per slot (c) coil span (d) slip ring voltage on open circuit (e) approximate full load current per phase in rotor. Assume efficiency = 0.9 ; power factor = 0.86.

Solution. Stator turns per phase $T_s = \dfrac{63 \times 6}{6} = 63.$

Stator voltage per phase $E_s = \dfrac{500}{\sqrt{3}} = 289$ V.

(a) The number of rotor slots should perferably be an integer. Taking 3 slots per pole phase.

Number of rotor slots $S_r = 3 \times 3 \times 8 = 72.$

Using star connection for rotor, rotor voltage at standstill $E_r = 400/\sqrt{3} = 231$ V.

Table 10·3.
Full load slip

Output	Per cent slip
0 75	5·0
3 70	4·2
7·50	4·0
18·50	3·7
37·00	3·5
75 00	3 2
150·00	3·0

Example 10·10. *A 11 kW, 3 phase, 6 pole, 50 Hz, 220 V, star connected induction motor has 54 stator slots, each containing 9 conductors. Calculate the values of bar and end ring currents. The number of rotor bars is 64. The machine has an efficiency of 0 86 and a power factor of 0·85. The rotor mmf may be assumed as 85 per cent of stator mmf.*

Also find the bar and the end ring sections if the current density is 5 A/mm².

Solution. Stator current per phase

$$I_s=\frac{11\times1000}{\sqrt{3}\times220\times0{\cdot}86\times0{\cdot}85}=40\ \text{A}.$$

Number of stator conductors $=54\times9=486$.

$\therefore$ Stator turns/phase $T_s=486/6=81$.

Stator mmf $=3\,I_sT_s=3\times40\times81=9720$ A. $\therefore$ Rotor mmf $=0{\cdot}85\times9720=8250$ A.

But rotor mmf $=S_r\,I_b/2=32\,I_b$

$\therefore$ $32\,I_b=8250$ or current in rotor bars $I_b=258$ A.

End ring current $I_e=\dfrac{S_rI_b}{\pi p}=\dfrac{64\times258}{\pi\times6}=883$ A.

$\therefore$ Area of each bar $a_b=258/5=51{\cdot}6$ mm².

and of each end ring $a_e=883/5=176{\cdot}6$ mm².

Example 10·11. *A 3 phase 2 pole, 50 Hz squirrel cage induction motor has a rotor diameter 0 20 m and core length 0·12 m. The peak density in the air gap is 0·55 Wb/m². The rotor has 33 bars, each of resistance 125μΩ and a leakage inductance 2μH. The slip is 6%.*

Calculate (i) the peak value of current in each bar (ii) rotor I²R loss (iii) rotor output and (iv) torque exerted. Neglect the resistance of end rings.

Solution. Synchronous speed $n_s=2\times50/2=50$ r.p.s.

Actual speed of rotor $n_r=(1-s)\,n_s=(1-0{\cdot}06)\times50=47$ r.p.s.

Peripheral speed of stator field $=\pi\times0{\cdot}2\times50=31{\cdot}42$ m/s

Peripheral speed of rotor $=\pi\times0{\cdot}2\times47=29{\cdot}53$ m/s

Relative speed of rotor bars with respect to stator field

$$v=31{\cdot}42-29{\cdot}53=1{\cdot}89\ \text{m/s}.$$

$\therefore$ Maximum e.m.f. in each bar $=B_m\,lv=0{\cdot}55\times0{\cdot}12\times1{\cdot}89=0{\cdot}125$ V.

Slip frequency $= sf = 0{\cdot}06 \times 50 = 3$ Hz

Reactance of each bar $= 2\pi \times 3 \times 2 = 37{\cdot}7\ \mu\Omega$

Impedance of each bar $= \sqrt{(125)^2 + (37{\cdot}7)^2} = 130{\cdot}6\ \mu\Omega$.

Maximum current in each bar $= \dfrac{0{\cdot}125}{130{\cdot}6 \times 10^{-6}} = 957{\cdot}1$ A.

R.m.s. value of current in each bar $I_b = 957{\cdot}1/\sqrt{2} = 676{\cdot}8$ A.

Rotor I^2R loss $= S_r\, I_b^2 r_b = 33 \times (676{\cdot}8)^2 \times 125 \times 10^{-6} = 1890$ W

Output $=$ (rotor I^2R loss) $\times \dfrac{(1-s)}{s} = 1890\ \dfrac{(1-0{\cdot}06)}{0{\cdot}06} = 29{\cdot}61$ kW.

$\therefore$ Torque $= \dfrac{29{\cdot}61 \times 10^3}{2\pi \times 47} = 100{\cdot}3$ Nm.

Example 10·12. *A 15 kW, 3 phase, 6 pole, 50 Hz squirrel cage induction motor has the following data :*

Stator bore diameter $= 0{\cdot}32$ m ; axial length of stator core $= 0{\cdot}125$ m ; number of stator slots $= 54$. number of conductors per stator slot $= 24$; current in each stator conductor $= 17{\cdot}5$ A ; full load power factor $= 0{\cdot}85$ lagging.

Design a suitable cage rotor giving number of rotor slots, section of each bar and section of each ring. The full load speed is to be about 950 r.p.m. approximately. Use copper for the rotor bars and end rings. Resistivity of copper is $0{\cdot}02\ \Omega$/m and mm².

Solution. The number of rotor slots is assumed a pair of poles greater than the number of stator slots.

$\therefore$ Number of rotor slots $S_r = S_s + p/2 = 54 + 6/2 = 57$.

Stator turns per phase $T_s = 54 \times 24/6 = 216$.

Since the full load power factor is 0·85, the rotor mmf may be taken 85% of stator mmf

$\therefore$ Rotor bar current $I_b = \dfrac{0{\cdot}85 \times 6\ I_s T_s}{S_r} = \dfrac{0{\cdot}85 \times 6 \times 17{\cdot}5 \times 216}{57} = 340$ A.

Taking a current density of 7 A/mm², area of each bar $a_b = 340/7 = 49$ mm².

Allowing 45 mm for projection of bar beyond core and skewing, length of each bar $l_b = 0{\cdot}125 + 0{\cdot}045 = 0{\cdot}17$ m.

Copper loss in bars $= S_r \times (I_b)^2\ \dfrac{\rho L_b}{a_b} = 57 \times \dfrac{(340)^2 \times 0{\cdot}02 \times 0{\cdot}17}{49} = 457$ W.

Current in each end ring $I_e = \dfrac{S_r I_b}{\pi p} = \dfrac{57 \times 340}{\pi \times 6} = 1030$ A.

Taking a current density of 7 A/mm² for end rings,

Area of end ring $a_e = 1030/7 = 147$ mm².

Taking the ring section as 15 mm deep and 10 mm thick.

$\therefore$ Area of end ring $a_e = 15 \times 10 = 150$ mm².

We take the outer diameter of end ring to be equal to stator bore diameter. (Actually the ring outer diameter is a little smaller than the stator bore diameter).

Mean diameter of ring $= 0{\cdot}32 - 0{\cdot}015 = 0{\cdot}305$ m.

Copper loss in end rings $= 2 \times (I_e)^2\ \dfrac{\rho \pi D_r}{a_e} = 2 \times (1030)^2 \times \dfrac{0{\cdot}02 \times \pi \times 0{\cdot}305}{150} = 264$ W.

Table 10 5

M ınufactured by M/s Devidayal Stainless Steel

	DIMENSIONS			*SLOTS*			
				Stator		*Rotor*	
Type No	*Stator outer diameter*	*Stator inner diameter = Rotor outer diameter*	*Rotor inner diameter*	*Specifications See Fig 10 29*	*Number of slots*	*Specifications See Fig. 10 30*	*Number of slots*
30 E 20	3 3/4"	2"	9/16"	S—12	30	R—6	17
30 E 26	5 7/16"	3 1/2"	3/.."	S—6	28	R—5	20
30 F 57	6"	3 3/4"	7/8"	S—16	24	R—11	48
30 E 5	6 1/2"	3·3/8'	1 1/2"	S—4	18	R—3	22
30 E 39	6 1/2'	3 3/8"	1·1/2"	S—4	18	R—1	22
30 E 40	6·1/2"	3 3/4"	1"	S—15	36	R—10	34
30 E 27	6 1/2"	3 3/4"	1"	S—9	24	R—5	34
30 E 47	7·1/8"	4 30"	1 1/8"	S—14	36	R—5	44
30 E 53	7 1/8"	4 30"	1 1/8"	S—14	36	R—6	48
30 B 1	7 1/2"	4 1/2"	1 3/8"	S—1	24	R—1	32
30 E [illegible]7	7 1/2"	4 1/2'	1 3/8"	S—10	36	R—1	34
30 E 33	7 1/2"	4 1/2"	1 3/8"	S—11	36	R—1	33
30 B 2	8·3/4"	5·1/2"	1 5/8"	S—2	36	R—1	44
30 E 50	8 3/4"	5 1/2"	1 5/8"	S—13	36	R—9	33
30 D 21	10 1/4"	6 1/4"	1 3/4"	S—5	36	R—4	34
30 D 43	10·1/4"	7"	1 3/4"	S—5	36	R—4	40
30 C 45	12"	7 1/2"	2"	S—17	36	R—4	40
30 C 58	12"	8·1/2"	2"	S—17	36	R—4	44

OPERATING CHARACTERISTICS

10 28. No load current. The no load current I_0 of an induction motor is made up of two components :

(*i*) Magnetizing current I_m, and (*ii*) Loss component of current I_l

The magnetizing current is 90° out of phase with the voltage while the loss component is in phase with the voltage.

10 28 1. Magnetizing current The magnetic circuit of a four pole induction motor is shown in Fig. 10·32. The flux produced by stator mmf turns passes through the following parts :

(*i*) air gap, (*ii*) rotor teeth, (*iii*) rotor core, (*iv*) stator teeth and (*v*) stator core.

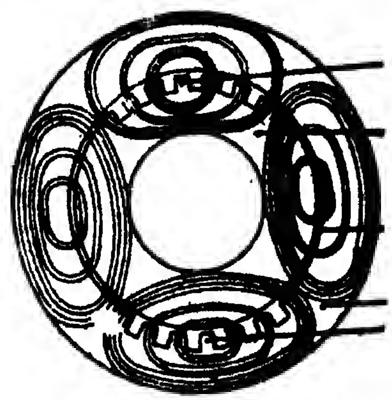


Fig. 10·32. Magnetic circuit of a 4 pole induction motor.

The calculation of magnetizing current of an induction motor follows the same general procedure as the calculation of magnetizing current of a d.c. machine. The main difference is that whereas in a d.c. machine the flux is assumed to be uniform over any cross section and the same mmf for all paths, in an induction motor the flux is distributed approximately sinusoidally and the mmf varies similarly.

If the permeability of iron were constant this would cause no difficulty, and the value of magnetizing current would be obtained accurately by considering the mean mmf and the flux tube where this mean occurs. For sinusoidal distribution the mean is $2/\pi = 0.637 \times$ maximum and occurs at 40° from the interpolar axis.

Owing to variations in the permeability of iron, particularly the stator and rotor teeth, a sinusoidal mmf will produce a flat topped flux density distribution curve as shown in Fig 4·62 page 178. If the value of flux density is calculated for the mean mmf, and a sinusoidal distribution of flux is assumed, the total flux obtained will be larger than true value ; or conversely the calculated magnetizing current for a given sinusoidal flux will be smaller than the true value.

If maximum values are taken instead the opposite result is obtained, *i.e.* flux is too small, or magnetizing current is too large.

Some intermediate position therefor will give a correct value. Though this position may differ some what in different motors, a flux tube crossing the air gap at 60° from the interpolar axis will always give a good approximation.

The reason for this is that the flux density distribution curve can be approximated too closely by a sine-wave with a third harmonic (See Art. 4·18 2 page 177). The value of flux density at 60° from the interpolar avis is the same whether the third harmonic is present or not. Thus the calculation of magnetizing mmf should be based upon the value of flux density at 60° from the interpolar axis as far as the gap and teeth are concerned.

(*i*) **Mmf for air gap.** From Eqn. 4·120, $B_{g60} = 1.36\ B_{av}$

$\therefore$ Mmf for air gap $AT_g = 800{,}000\ B_{g60}\ K_g\ l_g$

(*ii*) **Mmf for stator teeth.** The flux density is uniform in the teeth when they are parallel sided but when parallel sided slots are used, the flux density along the length of teeth is not uniform. The calculation of mmf for tapered teeth is explained in Art. 4·4·3 page 131. The value of mmf for teeth is found out by finding flux density at a section 1/3 height of tooth from narrow end.

Flux density at 1/3 height of tooth from narrow end

$$B_{ts1/3} = \frac{\Phi_m}{(S_s/p) \times L_i \times W_{ts1/3}}$$

where

$W_{ts1/3}$ = width of stator at $\frac{1}{3}$ height from narrow end

$$= \frac{\pi(D + 2d_{ss}/3)}{S_s} - W_{ss}$$

$\therefore$ Magnetizing current

$$I_m = \frac{0.427\, p\, AT_{60}}{K_{ws}\, T_s} = \frac{0.427 \times 6 \times 453}{0.958 \times 96} = 12.7 \text{ A.}$$

10.29. Short circuit (blocked rotor) current. In order to find the value of short circuit (blocked rotor) current, the values of resistance and leakage reactance of the windings have to be evaluated.

10.29.1. Stator resistance. The stator resistance per phase

$$r_s = \rho\, T_s\, L_{mts}/a_s$$

where L_{mts} = length of mean turn of stator, m ;

and a_s = area of stator conductor, mm².

The value of resistivity for copper is 0.021 Ω/m and mm² at 75°C.

10.29.2. Rotor resistance.

Wound Rotor. The resistance of a wound rotor machine is found in a similar manner.

Rotor resistance per phase $r_r = \rho \dfrac{T_r\, L_{mtr}}{a_r}$

The rotor resistance per phase referred to stator, $r'_r = \left(\dfrac{K_{ws}\, T_s}{K_{wr}\, T_r}\right)^2 r_r$.

Cage rotor

Let

ρ = resistivity of material of bars and rings, Ω/m and mm² ;

L_b = length of each bar, m ; a_b = area of each bar, mm² ;

D_e = mean diameter of each ring, m ; and a_e = area of each ring, mm².

Resistance of each bar $r_b = \dfrac{\rho L_b}{a_b}$. Total copper loss in bars $= S_r\, I_b^2\, \rho\, \dfrac{L_b}{a_b}$.

Resistance of each end ring $r_e = \rho\, \dfrac{\pi D_e}{a_e}$

Copper loss in two end rings $= 2\, I_e^2\, \dfrac{\rho\, \pi D_e}{a_e}$

$$= 2 \times \left(\frac{S_r\, I_b}{\pi p}\right)^2 \times \frac{\rho \pi D_e}{a_e} = \frac{2}{\pi}\, \rho\, \frac{I_b^2\, S_r^2\, D_e}{p^2\, a_e}.$$

Total copper loss in rotor

$$= S_r\, I_b^2\, \rho\, \frac{L_b}{a_b} + \frac{\pi}{2}\, \rho\, \frac{I_b^2\, S_r^2\, D_e}{p^2\, a_e}$$

$$= \rho I_b^2\, S_r^2 \left[\frac{L_b}{S_r\, a_b} + \frac{2}{\pi}\, \frac{D_e}{p^2\, a_e}\right] \qquad \ldots(10.31)$$

Hence total rotor resistance

$$= \rho\, S_r^2 \left(\frac{L_b}{S_r\, a_b} + \frac{2}{\pi}\, \frac{D_e}{p^2\, a_e}\right) \qquad \ldots(10.32)$$

The rotor resistance must be referred to stator in order to find out total resistance of motor as viewed from stator.

Let

m_s, m_r = number of phases of stator and rotor respectively ;
T_s, T_r = number of turns of stator and rotor respectively ;
K_{ws}, K_{wr} = winding factor for stator and rotor respectively.

Total resistance of rotor referred to stator

$$=\left(\frac{m_s\,T_s\,K_{ws}}{m_r\,T_r\,K_{wr}}\right)^2\times\rho\,S_r^2\left(\frac{L_b}{S_r\,a_b}+\frac{2}{\pi}\;\frac{D_e}{p^2\,a_e}\right) \qquad ...(10\cdot33)$$

The number of phases in a squirrel cage winding is equal to number of bars per pole or $m_r=S_r/p$.

The number of turns in series per phase for rotor is equal to the number of pole pairs or $T_r=p/2$ and rotor winding factor $K_{wr}=1$.

Substituting the values for m_r, T_r and K_{wr} in Eqn. 10·33.

Total resistance of cage rotor referred to stator

$$=\left[\frac{m_s\,T_s\,K_{ws}}{(S_r/p)\times(p/2)\times 1}\right]^2\rho\,S_r^2\left[\frac{L_b}{S_r\,a_b}+\frac{2}{\pi}\;\frac{D_e}{p^2\,a_e}\right]$$

$$=4\,m_s^2\,T_s^2\,K_{ws}^2\,\rho\left[\frac{L_b}{S\,a_b}+\frac{2}{\pi}\;\frac{D_e}{p^2\,a_e}\right] \qquad ...(10\cdot34)$$

$\therefore$ Resistance of cage rotor, referred to stator, per phase

$$r'_r=4\,m_s\,T_s^2\,K_{ws}^2\,\rho\left[\frac{L_b}{S_r\,a_b}+\frac{2}{\pi}\;\frac{D_e}{p^2\,a_e}\right] \qquad ...(10\cdot35)$$

When the radial width of end ring is large, as often is in small induction motors, the end ring resistance must be multiplied by a constant to take into account the effect of non-uniform current distribution in the end ring. This factor is K_{ring} and is taken from Fig. 10·33.

Modifying Eqn. 10·35 resistance of cage rotor referred to stator, per phase,

$$r'_r=4m_s\,T_s^2\,K_{ws}^2\,\rho$$

$$\left[\frac{L_b}{S_r\,a_b}+\frac{2}{\pi}\;\frac{D_e}{p^2\,a_e}K_{ring}\right] \qquad ...(10\cdot36)$$

For three phase machines $m_s=3$

$$\therefore\quad r_r'=12\,T_s^2\,K_{ws}^2\,\rho$$

$$\left(\frac{L_b}{S_r\,a_b}+\frac{2}{\pi}\;\frac{D_e}{p^2\,a_e}K_{ring}\right) \qquad ...(10\cdot37)$$

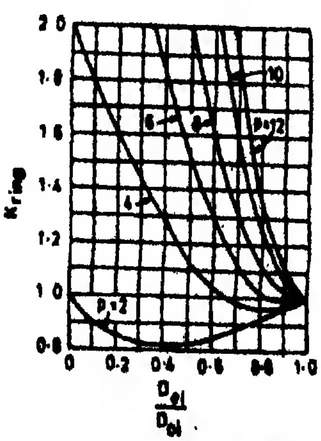


Fig. 10·33. Multiplying factor for non-uniform distribution of current in end rings.

Eqn. 10·37 can be derived in another manner also.

When there are m_s phases in stator with a winding factor K_{ws} and T_s turns in series per phase and a corresponding m_r phase rotor with a winding factor K_{wr} and T_r turns per phase, the rotor current per phase is :

$$I_r=\frac{m_s\,K_{ws}\,T_s}{m_r\,K_{wr}\,T_r}I_r' \qquad ...(10\cdot38)$$

or stator turns per phase, $T_s = \dfrac{0.427\, p\, AT_{60}}{K_{ws}\, I_m}$

$$= \frac{0.427 \times 8 \times 500}{0.95 \times 6.26} = 288.$$

Example 10.16. *A 15 kW, 400 V, 3 phase, 50 Hz, 6 pole induction motor has a diameter of 0.3 m and the length of core 0.12 m. The number of stator slots is 72 with 20 conductors per slot. The stator is delta connected. Calculate the value of magnetizing current per phase if the length of air gap is 0.55 mm. The gap contraction factor is 1.2. Assume the mmf required for the iron parts to be 35 per cent of the air gap mmf. Coil span = 11 slots.*

Solution. Slots per pole per phase $q = 72/(3\times 6) = 4$.

Distribution factor $K_d = \dfrac{\sin 60/2}{4 \sin 60/(2\times 4)} = 0.958.$

Slots per pole $= 72/6 = 12$ and coil span $= 11$ slots.

Therefore, the winding is chorded by 1 slot pitch.

$\therefore$ Angle of chording $\alpha = \dfrac{1}{12} \times 180 = 15°.$

Pitch factor $K_p = \cos \alpha/2 = 0.9914.$

$\therefore$ Stator winding factor $K_{ws} = 0.958 \times 0.991 = 0.95.$

Total stator conductors $= 72 \times 20 = 1440.$

Stator turns per phase $T_s = 1440/(3\times 2) = 240.$

Stator voltage per phase $E_s = 400$ V. (The machine is delta connected)

$\therefore$ Flux per pole $\Phi_m = \dfrac{400}{4.44 \times 50 \times 240 \times 0.95} = 7.9 \times 10^{-3}$ Wb.

Area per pole $= \dfrac{\pi DL}{p} = \dfrac{\pi \times 0.3 \times 0.12}{6} = 18.85$ m².

Average air gap density $B_{av} = \dfrac{7.9 \times 10}{18.85 \times 10^{-3}} = 0.418$ Wb/m².

Gap flux density at 30° from pole axis

$$B_{g60} = 1.36\, B_{av} = 1.36 \times 0.418 = 0.57 \text{ Wb/m}^2.$$

Mmf required for air gap $= 800{,}000 \times 0.57 \times 1.2 \times 0.55 \times 10^{-3} = 301$ A.

Mmf for iron parts $= 0.35 \times 301 = 105$ A. $\therefore$ Total mmf $AT_{60} = 301 + 105 = 406$ A.

Magnetizing current per phase $I_m = \dfrac{0.427 \times p \times AT_{60}}{K_{ws}\, T_s}$

$$= \frac{0.427 \times 6 \times 406}{0.95 \times 240} = 4.56 \text{ A}.$$

Example 10.17. *The magnetic circuit of a 440 V, 6 pole, 3 phase, star connected, 50 Hz, induction motor has the following particulars :*

Core length 0.15 m ; stator teeth length 30 mm ; tooth width at 1/3 height from narrow end 7 mm ; rotor teeth length 15 mm ; rotor tooth width at 1/3 height from narrow end 10.5 mm ; stator bore diameter 0.4 m ; effective air gap length 0.9 mm ; stator and rotor core depth 65 mm ; mean 60° lengths of magnetic circuit per pole pair, in core ; stator 0.25 m, rotor 0.16 m. The stator has 72 slots with 8 conductors per slot. The rotor has 49 slots. The stacking factor is 0.9. Estimate the magnetizing current using the following magnetization curve.

B, Wb/m²	0.5	0.7	1.0	1.2	1.4	1.6
at, A/m	95	110	200	300	600	2500

Solution. Stator slots per pole phase $q=72/(3\times6)=4$.

Distribution factor $K_d=\dfrac{\sin 60/2}{4\sin 60/(2\times4)}=0.958$.

Using full pitch winding, $K_p=1$. $\therefore$ Stator winding factor $K_{ws}=0.958$.

Stator conductors $=8\times72=576$. Stator turns per phase $T_s=576/(3\times2)=96$.

Stator voltage per phase $=440/\sqrt{3}=254$ V.

$\therefore$ Flux per pole $\Phi_m=\dfrac{254}{4.44\times50\times96\times0.958}=12.4\times10^{-3}$ Wb.

Area per pole $=\pi DL/p=\pi\times0.4\times0.15/6=31.4\times10^{-3}$ m²

Average flux density per pole $B_{av}=12.4\times10^{-3}/(31.4\times10^{-3})=0.395$ Wb/m².

B_{60} in air gap $=1.36\times0.395=0.537$ Wb/m².

Mmf for air gap $=800{,}000\times0.537\times0.9\times10^{-3}=387$ A.

Net iron length $L_i=K_iL=0.9\times0.15=0.135$ m.

Flux density in stator teeth at 1/3 height

$$B_{ts1/3}=\frac{\Phi_m}{S_s/p\times L_i\times W_{ts1/3}}=\frac{12.4\times10^{-3}}{(72/6)\times0.135\times7\times10^{-3}}=1.1\ \text{Wb/m}^2.$$

$\therefore$ $B_{ts60}=1.36\times1.1=1.5$ Wb/m².

Corresponding to this flux density, mmf per metre $at_{ts}=1000$ A.

$\therefore$ Mmf required for stator teeth $=1000\times30\times10^{-3}=30$ A.

Flux density in rotor teeth at 1/3 height

$$B_{tr1/3}=\frac{12.4\times10^{-3}}{(49/6)\times0.135\times10.5\times10^{-3}}=1.07\ \text{Wb/m}^2.$$

$B_{tr60}=1.36\times1.07=1.46$ Wb/m².

Corresponding to this flux density, mmf per metre $at_{tr}=700$ A.

$\therefore$ Mmf required for rotor teeth $=700\times15\times10^{-3}=11$ A.

Flux in stator core $=12.4\times10^{-3}/2=6.2\times10^{-3}$ Wb.

Area of stator core $=d_{cs}\times L_i=65\times10^{-3}\times0.135=8.77\times10^{-3}$ m².

$\therefore$ Flux density in stator core $=\dfrac{6.2\times10^{-3}}{8.77\times10^{-3}}=0.707$ Wb/m².

The same flux density is obtained in rotor core as the flux and the area are the same.

Corresponding to this flux density, mmf per metre $=120$ A.

Length of flux path, in stator core, per pole $=0.25/2=0.125$ m

Length of flux path, in rotor core, per pole $=0.16/2=0.08$ m.

Mmf for stator and rotor cores $=120(0.125+0.08)=25$ A.

Total mmf for B_{60}

$AT_{60}=387+30+11+25=453$ A.

The calculation of mmf for stator teeth is based upon B_{ts60}

where $B_{ts60}=1.36\ B_{ts1/3}$

The mmf per metre at_{ts} for stator teeth is found from Fig 4.2. page 120

Mmf required for stator teeth $AT_{ts}=at_{ts}\times d_{ss}$.

(*iii*) **Mmf for rotor teeth**. Flux density in rotor teeth at 1/3 height from narrow end

$$B_{tr1/3}=\frac{\Phi_m}{(S_r/p)\times L_i\times W_{tr1/3}} \quad \text{and} \quad \text{with } W_{tr1/3}=\frac{\pi(D_r-4d_{sr}/3)}{S_r}-W_{sr}$$

where d_{sr} = depth of rotor slot, and W_{sr} = width of rotor slot.

Now $B_{tr60}=1.36\ B_{tr1/3}$

The mmf per metre, at_{tr}, for rotor teeth is found from Fig. 4.2 corresponding B_{tr60}

∴ Mmf required for rotor teeth $AT_{tr}=at_{tr}\times d_{tr}$.

(*iv*) **Mmf for stator core**. Corresponding to flux density in the core, the mmf per metre at_{cs} is found from Fig. 4.2. The length of path through the core can be taken as 1/3 pole pitch at the mean core diameter.

$$\text{Length of path through stator core } l_{cs}=\frac{\pi(D+2d_{ss}+d_{cs})}{3p} \quad \text{...(10.24)}$$

Mmf for stator core $=at_{cs}\times l_{cs}$.

(*v*) **Mmf for rotor core**. Corresponding to flux density in rotor core mmf per metre at_{cr} is found from Fig. 4.2. Length of flux path in rotor core

$$l_{cr}=\frac{\pi(D_r-2d_{sr}-d_{cr})}{3p} \quad \text{...(10.25)}$$

∴ Total mmf for rotor core $AT_{cr}=at_{cr}\times l_{cr}$

∴ Total magnetizing mmf per pole for B_{60}

$$AT_{60}=AT_g+AT_{ts}+AT_{tr}+AT_{cs}+AT_{cr} \quad \text{...(10.26)}$$

From Eqn. 4.121,

$$\text{Magnetizing current per phase } I_m=\frac{0.427\ p\ AT_{60}}{K_{ws}\ T_s} \quad \text{...(10.27)}$$

10.28.2. Loss component. The calculation of loss component of no load current involves the determination of no load loss.

(*i*) **Iron loss.** The iron loss in induction motors consists of hysteresis and eddy current loss in teeth and cores, surface loss in teeth due to variation of air gap density, tooth pulsation loss due to variation of teeth density, loss due to non-uniform flux distribution and loss in end plates.

The iron loss in stator teeth and core is found out by calculating their respective weights. The loss per kg corresponding to the flux densities can be taken from Fig. 4.27 and Fig. 4.28.

The frequency of flux reversals in the rotor is slip times the line frequency. In the case of cage motors, the value of slip is small and, therefore, the iron loss in the rotor is negligible. Wound rotor motors may operate at reduced speed by insertion of resistance in the rotor circuit. Therefore, rotor iron loss must be included while calculating the operating characteristics of a wound rotor machine.

(*ii*) **Friction and windage loss.** Table 10.6 gives the approximate values of friction and windage losses expressed in terms of output.

Table 10·6. Friction and windage loss

Output kW	*F and W loss Percent of output*
0·75	5·5
3·70	3·5
7·50	2·7
37·00	1·5
75·00	1·2
150·00	1·0

Loss component of no load current per phase

$$I_l = \frac{\text{total no load loss}}{3 \times \text{voltage per phase}} \qquad \ldots(10{\cdot}28)$$

No load current I_0 (per phase) $= \sqrt{I_m^2 + I_l^2}$.

The approximate values of no load current expressed as percentage of full load current are given in Table 10·7.

Table 10·7. No load current

Output kW	*No load current Percent of full load current*
0·75	50
3· 0	40
15·0	33
37·00	30
75 and above	27

The no load power factor, $\cos \phi_0 = I_l / I_0$ $\qquad \ldots(10{\cdot}29)$

and $\qquad \phi_0 = \cos^{-1} I_l / I_0 \qquad \ldots(10{\cdot}30)$

Example 10·15. *A 75 kW, 3300 V, 50 Hz, 8 pole, 3 phase star connected induction motor has a magnetizing current which is 35 percent of the full load current. Calculate the value of stator turns per phase if the mmf required for flux density at 30° from pole axis is 600 A.*

Assume winding factor = 0·95, and full load efficiency and power factor 0·94 and 0·86 respectively.

Solution. Full load current $= \dfrac{75 \times 1000}{\sqrt{3} \times 3000 \times 0{\cdot}94 \times 0{\cdot}86} = 17{\cdot}9$ A.

$\therefore$ Magnetizing current $I_m = 0{\cdot}35 \times 17{\cdot}9 = 6{\cdot}26$ A.

From Eqn. 10·27, magnetizing current $I_m = \dfrac{0{\cdot}427\, pAT_{30}}{K_{ws}\, T_s}$

where

I_r' = current in stator m_s phases electrically equivalent to the rotor current in m_r phases.

But $m_r = S_r/p$, $T_r = p/2$, and $K_{wr} = 1$ as stated earlier.

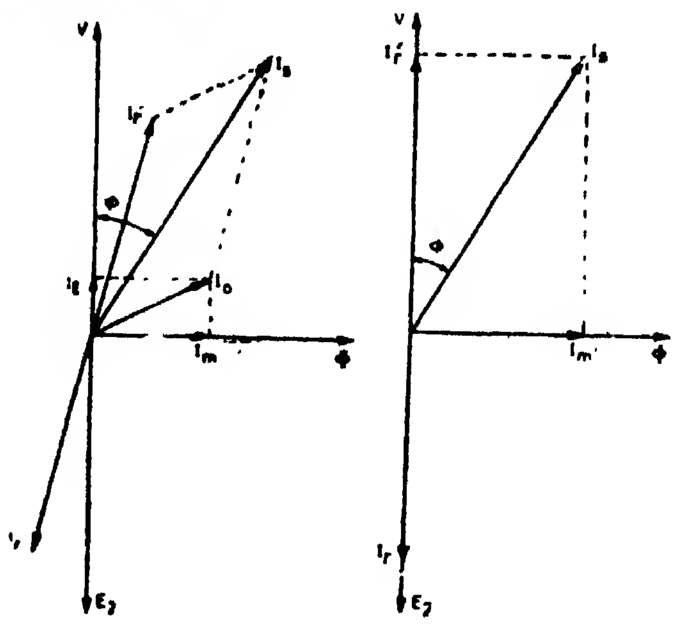
∴ From Eqn. 10·38 current in each bar is

$$I_b = \frac{m_s K_{ws} T_s}{(S_r/p) \times 1 \times p/2)} I_r' = \frac{2\, m_s K_{ws} T_s}{S_r} I_r' \qquad \ldots(10\cdot39)$$

Total loss in rotor cage

$$= \rho\, I_b^2\, S_r^2 \left(\frac{L_b}{S_r\, a_b} + \frac{2}{\pi}\, \frac{D_e}{p^2\, a_e} \right) \qquad \text{(See Eqn. 10·31)}$$

The equivalent loss in stator $= m_s\, I_r'^2\, r_r'$



(a) (b)

Fig. 10·34. Phasor diagrams of induction motors.

∴ Rotor resistance per phase referred to stator

$$r_r' = \frac{\rho\, I_b^2\, S_r^2}{m_s\, I_r'^2} \left(\frac{L_b}{S_r\, a_b} + \frac{2}{\pi}\, \frac{D_e}{p^2\, a_e} \right)$$

$$= \left(\frac{2\, m_s\, K_{ws}\, T_s\, I_r'}{S_r} \right)^2 \frac{\rho\, S_r^2}{m_s\, I_r'^2} \left(\frac{L_b}{S_r\, a_b} + \frac{2}{\pi}\, \frac{D_e}{p^2\, a_e} \right)$$

$$= 4\, m_s\, T_s^2\, K_{ws}^2\, \rho \left[\frac{L_b}{S_r\, a_b} + \frac{2}{\pi}\, \frac{D_e}{p^2\, a_e} \right]$$

Fig 10·34 (a) shows the phasor diagram of an induction motor (for currents only). If we neglect the rotor leakage reactance and the loss component of no load current, the phasor diagram is simplified as shown in Fig. 10·34 (b).

From Fig 10·34 (b), stator current equivalent to rotor current is

$I_r' = I_s \cos\phi$ ...(10·40)

where I_s = stator current and $\cos\phi$ = stator power factor.

Example 10·18. *Calculate the equivalent resistance of rotor per phase referred to stator, from the following data of a 400 V, 3 phase, 4 pole, 50 Hz cage motor.*

Stator slots = 48 with 30 conductors per slot; Rotor slots = 53 with one bar in each slot. The length of each rotor bar is 0.12 m and area 60 mm². The end rings have a mean diameter of 0.18 m and an area of cross-section 150 mm².

Full pitch winding with 60° phase spread is used for the stator.

The material used for bars and end rings has a resistivity of 0.021 Ω/m and mm².

Solution.

Resistance of each bar $=0.021\times\dfrac{0.12}{60}=42\times10^{-6}\ \Omega$.

Loss in 53 bars $=53\times I_b^2\times42\times10^{-6}=2226\times10^{-6}\ I_b^2$ W.

Resistance of each ring $=0.021\ \dfrac{\pi D_e}{a_e}=0.021\times\dfrac{\pi\times0.18}{150}=792\times10^{-6}\ \Omega$.

Loss in two end rings $=2\times79.2\times10^{-6}\ I_e^2=2\times79.2\times10^{-6}\times\left(\dfrac{53}{\pi\times4}\right)^2 I_b^2$

$=2820\times10^{-6}\ I_b^2$ W.

$\therefore$ Total rotor I^2R loss $=(2820+2226)\times10^{-6}\ I_b^2=5046\times10^{-6}\ I_b^2$.

Stator slots/pole/phase $q_s=48/(4\times3)=4$.

Stator distribution factor $K_d=\dfrac{\sin 60/2}{4\sin 60/(2\times4)}=0.958$.

As full pitch coils are used, $K_p=1$. $\therefore$ Stator winding factor $K_{ws}=0.958$.

Stator turns per phase $T_s=48\times30/(2\times3)=240$.

Now neglecting magnetizing current and the rotor leakage reactance, stator mmf is equal to rotor mmf (the power factor is assumed as unity which is strictly not correct)

or $$3T_sI_sK_{ws}=I_b\,\frac{S_r}{2}$$

or current in each bar $$I_b=6\,\frac{T_s\,K_{ws}}{S_r}.\,I_s=\frac{6\times240\times0.958}{53}I_s=26\ I_s.$$

$\therefore$ Rotor I^2R loss expressed in terms of stator current

$=5046\times10^{-6}\times(26I_s)^2=3.42\ I_s^2$

and loss per phase $=(3.42/3)\,I_s^2=1.14\ I_s^2$.

$\therefore$ Rotor resistance referred to stator $r_r'=1.14\ I_s^2/I_s^2=1.14\ \Omega$.

Example 10·19. *A 6 pole, 3 phase squirrel cage induction motor has 72 stator slots with 15 conductors in each slot. There are 55 rotor slots. The coil span is 11 slots and the phase spread is 60°.*

Determine the current in rotor bars and in end rings if the stator current is 24.1A and the power factor is 0.83.

Solution.

Stator slots/pole/phase $=72/(3\times6)=4$.

Distribution factor for 4 slots/pole/phase and 60° phase spread is 0.958 as calculated in the previous example

The number of slots per pole is $72/6=12$ and the coil span is 11 slots. Therefore, the coil are chorded by one slot pitch.

Angle of chording $\alpha=1/12\times180=7.5°$.

Pitch factor $K_p=\cos 7.5/2=0.998$.

$\therefore$ Winding factor $=0.958\times0.998=0.956$.

Stator current equivalent to rotor current

$$I_r'=I_s\cos\phi \quad \text{(See Eqn. 10.40)}$$
$$=24.1\times0.83=20\text{A}.$$

Stator turns per phase $T_s=72\times15/(3\times2)=180$.

From Eqn. 10.39, current in each rotor bar

$$I_b=\frac{2\ m_s\ K_{ws}\ T_s}{S_r}\ I_r'=\frac{2\times3\times0.956\times180}{55}\times20=375.4\text{A}.$$

Current in each end ring $I_e=\dfrac{S_r\ I_b}{\pi p}=\dfrac{55\times375.4}{\pi\times6}=1095\ 3\text{A}.$

Example 10.20. *Calculate the equivalent resistance of rotor per phase in terms of stator, current in each bar and end ring and total rotor I^2R loss for the following :*

4 pole 3 phase, 50 Hz, 400 V cage motor has 48 slots in stator with 35 conductors per slot Each conductor carries a current of 10A. The rotor has 57 slots, each slot has a bar of 0.12 m length and 50 mm² area. The mean diameter of each ring is 0 2 m and area is 175 mm². Resistivity is 0 02Ω/m and mm² and the power factor is 0.8. The stator winding uses full pitched coils with a phase spread of 60°.

Solution.

The winding factor for an infinitely distributed winding with a phase spread of 60° is 0 955.

Equivalent stator current $I_r'=I_s\cos\phi=10\times0.8=8\text{A}$.

tator turns per phase $T_s=48\times35/6=280$.

ur ent in each bar $I_b=\dfrac{2\ m_s\ K_{ws}\ T_s}{S_r}\times I_r'=\dfrac{2\times3\times0.955\times280}{57}\times8=226\text{A}.$

Current in each ring $I_e=\dfrac{S_r\ I_b}{\pi p}=\dfrac{57\times226}{\pi\times4}=1024\text{A}.$

Resistance of each bar $r_b=0.02\times0.12/50=48\times10^{-6}\ \Omega$.

I^2R loss in bars $=57\times(226)^2\times48\times10^{-6}=140$ W.

Resistance of each ring $=0.02\times\pi\times0.2/175=72\times10^{-6}\ \Omega$

I^2R loss in 2 rings $=2(1024)^2\times72\times10^{-6}=151$ W.

Total rotor I^2R loss $=140+151=291$ W.

Total resistance referred to stator per phase

$$r_r'=\frac{\text{total rotor } I^2R \text{ loss}}{m_s\ I_r'^2}=\frac{298}{3\times(8)^2}=1.51\,\Omega.$$

The resistance can be directly calculated by applying Eqn. 10.35

$$r_r'=4\ m_s\ T_s^2\ K_{ws}^2\rho\left[\frac{L_b}{S_r\ a_b}+\frac{2}{\pi}\ \frac{D_e}{p^2\ a_e}\right]$$

$$=4\times3\times(280)^2(0.955)^2\times0.02\left[\frac{0.12}{57\times50}+\frac{2}{\pi}\times\frac{0.5}{4^2\times175}\right]=1.51\ \Omega.$$

10 29.3. Leakage reactance. The calculation of leakage reactance has been explained in Art. 4 15 page 171.

Using Eqn. 4.95, stator slot leakage reactance

$$x_{ss}=8\ \pi\ f\ T_s^2\ L\ (\lambda_{ss}/pq_s) \qquad \ldots(10.41)$$

where λ_{ss} = specific slot permeance for stator and q_s = stator slots/pole/phase.

(*ii*) Rotor specific slots permeance referred to stator

$$\lambda_{sr}'=\left(\frac{K_{ws}}{K_{wr}}\right)^2 \frac{S_s}{S_r}\lambda_{sr} \qquad ...(10.42)$$

where λ_{sr}=specific slot permeance for rotor.

Rotor slot leakage reactance referred to stator

$$x_{sr}'=8\pi f\, T_s^2\, L\, (\lambda_{sr}'/pq_s) \qquad ...(10.43)$$

(*iii*) Using Eqn. 4·96, overhang leakage reactance

$$x_o=8\pi f\, T_s^2\, L_o\, (\lambda_s/pq_s) \qquad ...(10.44)$$

where $L_o\lambda_o=\mu_o \dfrac{L_s\,\tau^2}{\pi\, y_{ss}}$ (See Art. 4·14 page 170)

The value of K_r is taken from Fig. 4·58 page 170.

The specific permeance for overhang leakage may be calculated from

$$\lambda_0=\mu_0 \frac{K\,L_0}{2\sqrt{2}\,y_{ss}} \qquad ...(10.45)$$

The constant K has the following values.

(*i*) Concentric wound stator :

slip ring rotor $K=0.35\times10^{-6}$; cage rotor $K=0.27\times10^{-6}$.

(*ii*) Barrel wound stator :

slip ring rotor $K=0.55\times10^{-6}$; cage rotor $K=0.37\times10^{-6}$.

(*iii*) Zigzag leakage reactance : $x_z=\dfrac{5}{6}\dfrac{X_m}{m_s^2}\left(\dfrac{1}{q_s^2}+\dfrac{1}{q_r^2}\right)$ ...(10.46)

where X_m=magnetising reactance$=E_s/I_m$.

(*iv*) Harmonic or differential leakage reactance : $x_h=X_m(K_{hs}+K_{hr})$ ...(10.47)

where K_{hs} and K_{hr} are constants (for stator and rotor respectively) and are taken from Fig. 10·35.

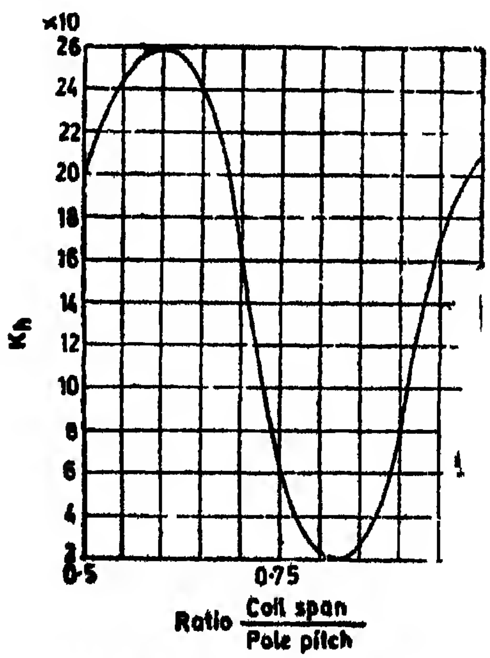


Fig. 10·35. Values of factor K_h.

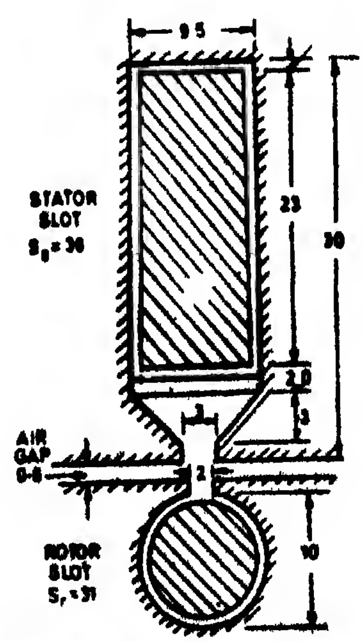


Fig. 10·36. Stator and rotor slot dimensions for example 10·21. All dimensions in mm.

Harmonic leakage reactance is negligible in squirrel cage induction motors.

Total leakage reactance of the machine referred to stator

$$X_s = x_{ss} + x_{sr}' + x_0 + x_z + x_h \quad \text{...(10·48)}$$

Total impedance of motor at standstill $Z_s = \sqrt{R_s^2 + X_s^2}$ ...(10·49)

Short circuit (blocked rotor) current per phase $I_{ss} = E_{ss}/Z_s$ ...(10·50)

Short circuit (blocked rotor) power factor $\cos\phi_{ss} = R_s/Z_s$ ...(10·51)

Phase angle of short circuit (blocked rotor) current $\phi_{ss} = \cos^{-1} R_s/Z_s$ ...(10 5)

Example 10·21. *Find the leakage reactance of a 7 5 kW, 400V, 3 phase, 50 Hz, 4 pole cage type induction motor The stator bore is 0 18 m and the core length is 0 13 m. The stator has 36 slots and the rotor 31 slots The details of the slots are shown in Fig. 10 36. The length of air gap is 0 6 mm and the number of stator turns is 276. The length of overhang on one side is 0 25 m. The stator winding factor may be assumed as 0 955. The stator winding is mush type for which the overhang leakage permeance is given by*

$$\lambda_0 = \mu_0 \times \frac{0{\cdot}37 \times 10^{-6}\, L_0}{2\sqrt{2}\, y_{ss}}$$

where L_0 = length of overhang on one side and y_{ss} = stator slot pitch.

Solution. Stator slots per pole per phase $q_s = 36/(3 \times 4) = 3$.

Stator slot specific permeance $\lambda_{ss} = \mu_0 \left[\frac{h_1}{3\,W_s} + \frac{h_2}{W_s} + \frac{2h_3}{W_s + W_0} + \frac{h_4}{W_0}\right]$

$$= \mu_0 \left[\frac{23}{3 \times 9\,5} + \frac{2}{9{\cdot}5} + \frac{2 \times 2}{9{\cdot}5 + 3} + \frac{1}{3}\right] = 1{\cdot}83\, \mu_0$$

Rotor slot specific permeance (See Eqn. 4·74 page 161)

$$\lambda_{sr} = \mu_0\, [0\,66 + h/W_0] = [0{\cdot}66 + \tfrac{1}{2}] = 1{\cdot}16\, \mu_0.$$

Rotor slot specific permeance referred to stator

$$\lambda_{sr}' = \left(\frac{K_{ws}}{K_{wr}}\right)^2 \times \frac{S_s}{S_r}\, \lambda_{sr} = \left(\frac{0{\cdot}955}{1}\right)^2 \times \frac{36}{31} \times 1{\cdot}123\, \mu_0 = 1{\cdot}17\, \mu_0.$$

Stator slot pitch $y_{ss} = \pi \times 0{\cdot}18 \times 10^3/36 = 15{\cdot}7$ mm.

Width of stator tooth $W_{ts} = 15{\cdot}7 - 3 = 12{\cdot}7$ mm.

Rotor slot pitch $y_{sr} = \pi \times 0{\cdot}18 \times 10^3/31 = 18{\cdot}1$ mm.

Width of rotor tooth $W_{tr} = 18{\cdot}1 - 2 = 16{\cdot}1$ mm.

From Eqn. 4·92 (page 170), specific permeance due to zigzag leakage

$$\lambda_z = \mu_0 \frac{5}{6}\, \frac{W_{ts}\, W_{tr}\, (W_{ts}^2 + W_{tr}^2)}{12\, l_g\, y_{ss}^2\, y_{sr}}$$

$$= \mu_0 \times \frac{12{\cdot}7 \times 16{\cdot}1 (12{\cdot}7^2 + 16{\cdot}1^2)}{12 \times 0{\cdot}6 \times 15{\cdot}7^2 \times 18{\cdot}1} = 2{\cdot}68\, \mu_0.$$

The specific permeance due to slot and zigzag leakage

$$= \mu_0\, (1{\cdot}83 + 1{\cdot}17 + 2{\cdot}68) = 5{\cdot}68\, \mu_0.$$

Leakage reactance due to slot and zigzag leakage

$$x_s + x_z = 8\,\pi\, f\, T_s^2\, L\, \lambda/pq_s$$

$$= 8\pi \times 50 \times (276)^2 \times 0{\cdot}13 \times 5{\cdot}68 \times 4\pi \times 10^{-7}/(4 \times 3) = 7\,4\ \Omega.$$

Specific permeance due to overhang leakage

$$\lambda_0 = \frac{0{\cdot}37 \times 10^{-6} \times 0{\cdot}25}{2\sqrt{2} \times 15{\cdot}7 \times 10^{-3}} = 2{\cdot}08 \times 10^{-6}.$$

Leakage reactance due to overhang leakage $x_o = 8\pi f T_s^2 L_o\lambda_o/pq_s$

$$= 8\pi \times 50 \times (276)^2 \times 0.25 \times 2.08 \times 10^{-7}/(4 \times 3) = 4.2\Omega.$$

Total leakage reactance $= x_s + x_z + x_o = 7.4 + 4.2 = 11.6\ \Omega.$

Example 10·22 *A 75 kW, 3000 V, 8 pole, 50 Hz 3 phase, star connected slip ring induction motor has the following data :*

Stator bore = 0.66 m ; stator core length = 0.50 m , number of stator slots = 96 , number of rotor slots = 72 ; number of stator turns per phase = 286 , total specific permeance due to stator slots = 4.9 μ_o ; no load current per phase = 6.1 A , no load power factor = 0.095 , harmonic leakage reactance per phase = 0.9Ω.

Estimate the total standstill leakage reactance of motor referred to stator. The winding employs full pitch coils

Solution Stator slots per pole per phase $q_s = 96/(3 \times 8) = 4.$

Rotor slots per pole per phase $q_r = 72/(3 \times 8) = 3.$

Slot leakage reactance $x_s = 8\pi \times 50 \times (286)^2 \times 0.5 \times \dfrac{4.9 \times 4\pi \times 10^{-7}}{8 \times 4} = 9.9\Omega.$

No load power factor $\cos\phi_o = 0.095$ $\quad\therefore\quad \sin\phi_o = 0.995$

Magnetizing current $I_m = I_o \sin\phi_o = 6.3 \times 0.995 = 6.27$ A.

Stator voltage per phase $E_s = 3000/\sqrt{3} = 1732$V

Magnetizing reactance $X_m = E_s/I_m = 1732/6.27 = 276.2\Omega.$

From Eqn. 10·46, zigzag leakage reactance

$$x_z = \frac{5}{6}\,\frac{X_m}{m_s^2}\left(\frac{1}{q_s^2}+\frac{1}{q_r^2}\right) = \frac{5}{6} \times \frac{276.2}{(3)^2}\left(\frac{1}{4^2}+\frac{1}{3^2}\right) = 4.4\Omega.$$

Pole pitch $\tau = \pi \times 0.66/8 = 0.26$ m. Stator slot pitch $= \pi \times 0.66/96 = 0.0216$ m

From Eqn. 4·94, overhang permeance

$$L_o\lambda_o = \mu_o \frac{K_s}{\pi}\frac{\tau^2}{y_{ss}} = \mu_o\,\frac{1 \times (0.26)^2}{\pi \times 0.0216} = 0.987\,\mu_o.$$

Overhang leakage reactance $= 8\pi \times 50 \times (286)^2 \times \dfrac{0.987 \times 4\pi \times 10^{-7}}{8 \times 4} = 4.0\Omega.$

Total leakage reactance $= x_s + x_z + x_o + x_h = 9.9 + 4.4 + 4.0 + 0.9 = 19.2\Omega.$

10·38. Circle diagram. It is possible to obtain graphically a considerable range of information from circle diagram. The construction gives estimate of full load current and power factor, maximum power output, pull out torque and the full load efficiency and slip. The circle diagram is constructed from the following design data :

I_m = magnetizing current phase,

I_l = loss component of no load current per phase,

X_s = total standstill leakage reactance per phase referred to stator,

R_s = total resistance per phase referred to stator,

Z_s = total short circuit impedance per phase referred to stator,

E_s = stator voltage per phase.

The procedure for drawing the circle diagram is given below (Refer Fig. 10·37).

1. Draw Oa and Ob perpendicular to each other.

2. Draw $OO' = I_o$, the no load current per phase at an angle ϕ_o with Ob after choosing a suitable current scale.

$$\phi_o = \tan^{-1} I_m/I_l \text{ and } I_o = \sqrt{I_m^2 + I_l^2}.$$

3 Draw $O'D$ passing through O' and parallel to Oa.

4 Draw $OB = I_{sc}$, short circuit current per phase at an angle ϕ_{sc}.

$$\phi_{sc} = \tan^{-1} X_s/R_s \text{ and } I_{sc} = E_s/Z_s.$$

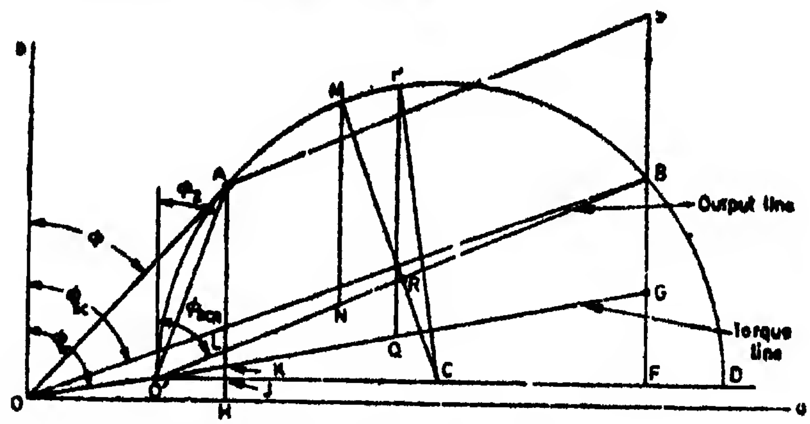


Fig. 10 37. Circle diagram.

5 Join O' with B.

6. Construct the perpendicular bisector of $O'B$ intersecting the line $O'D$ at C. Point C is the centre of circle having radius $O'C$.

7 Draw the circle $O'BD$.

8 Draw BF perpendicular to $O'D$ and divide it at G in such a way that

$$\frac{BG}{GF} = \frac{\text{rotor resistance referred to stator}}{\text{stator resistance}} = \frac{r_r'}{r_s}.$$

9 Join O' to G

The line $O'G$ is known as torque line and line $O'B$ as output line The diagram can be used to determine the characteristics for any current

10 In design, we are mainly interested in characteristics at rated output. The point A corresponding to rated output can be located as given below :

Suppose we have drawn the diagram with a current scale of 1 cm $= x$ ampere

$\therefore$ 1 cm $= x\, E_s$ watt per phase.

(It should be borne in mind that we have drawn the diagram considering phase current and phase voltage and therefore the results are obtained in terms of power per phase In order to get total power we will have to multiply the results for power and torque by 3 for 3 phase machines)

Extend line FB Cut off BS = rated output per phase.

Draw a line SA parallel to output line $O'B$ cutting the circle at A. Then A is the operating point for rated output.

11 Draw AH perpendicular to Oa.

12 Join O' to A This gives : ϕ_2 = rotor current phase angle.

13 Label points JKL

Stator current per phase at full load $I_s = OA$.

Stator power factor at full load $\cos\phi = AH/OA$.

Constant loss $= 3 \times JH$.

Rotor copper loss at full load $= 3 \times LK$.

JH and LK are measured on power scale.

$$\text{Slip } s = \frac{\text{rotor copper loss}}{\text{rotor input}} = \frac{LK}{AK}.$$

$$\text{Efficiency } \eta = \frac{\text{rotor output}}{\text{stator input}} = \frac{AL}{AH}.$$

$$\text{Torque} = 3 \times AK \text{ W (synchronous)}$$

with AK measured on power scale.

The location of point M on circle for maximum power output is done by drawing a perpendicular on the output line from C. Line MN represents maximum output.

Maximum output $= 3 \times MN$.

The location of point P on circle for maximum torque is done by drawing a perpendicular on torque line from C. Line PQ represents the maximum torque

Maximum torque $= 3 \times PQ$ W (synchronous).

Line BG represents starting torque. Starting torque $= 3 \times BG$.

MN, PQ and BG are measured on power scale.

The values of slip, efficiency, and power factor can be more accurately determined by using some additional graphical methods.

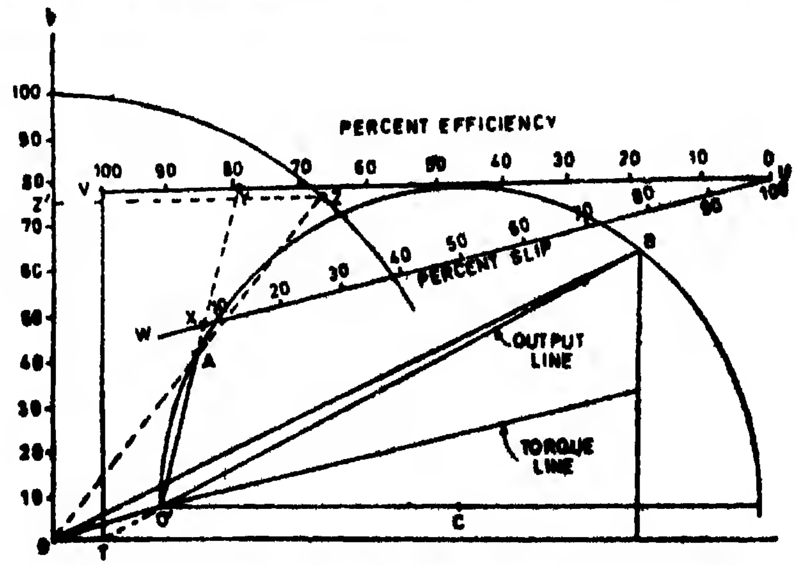


Fig. 10 38. Determination of slip, efficiency and power factor from circle diagram.

Fig. 10 38 shows the circle diagram drawn as per the procedure given above.

Draw the output line $O'B$ backwards to cut Ox at T.

Draw vertical lines TV and $O'W$.

Extend output line $O'B$ to a convenient point U.

From U, draw line UW parallel to torque line and line UV parallel to horizontal axis Ox.

Divide both lines QW and UV into 100 parts.

Extend $O'A$ to cut line UW at X. Point X indicates the value of percentage slip.

Extend $O'A$ to cut line UV at Y. Point Y indicates the percentage efficiency.

The power factor is easily found by drawing a quadrant of a circle with O as centre and a radius 100 arbitrary units.

Extend OA to intersect the quadrant at Z.

Project OZ on vertical axis Qb.

Vertical projection OZ' gives the value of power factor.

10·31. Calculation of maximum output from geometry of circle diagram. Referring to Fig. 10·7.

Maximum output $=3E_s \times MN$ with MN measured on current scale.

We have $MN \sin \phi_{scr} = MR$

or
$$MN = \frac{MR}{\sin \phi_{scr}} = \frac{OM - CR}{\sin \phi_{scr}} = \frac{OM - O'C \cos \phi_{scr}}{\sin \phi_{scr}}.$$

Now $\quad O'B = 2\,OM \sin \phi_{scr} \qquad \therefore \quad OM = \dfrac{O'B}{2 \sin \phi_{scr}}$

Since $\quad OM = O'C$

$$\therefore \quad MN = \frac{O'B\,(1-\cos \phi_{scr})}{2 \sin^2 \phi_{scr}} = \frac{O'B}{2(1+\cos \phi_{scr})}.$$

Now $\quad O'B = OB - OO' = I_{sc} - I_0 \quad$ and $\quad \phi_{scr} = \phi_{sc}$

$$\therefore \quad MN = \frac{I_{sc} - I_0}{2(1+\cos \phi_{sc})}$$

Hence, maximum output $= 3\,E_s\,\dfrac{I_{sc}-I_0}{2(1+\cos \phi_{sc})}$ ...(10·53)

10·32. Dispersion coefficient. There are two major factors which influence the power factor of an induction motor.

(*i*) **Magnetizing current.** If the magnetizing current of a motor is large, its power factor is poor.

(*ii*) **Ideal short circuit current.** Ideal short circuit current is defined as the current drawn by the motor at standstill if its resistance is neglected. If the ideal short circuit current of a machine is large it indicates that its leakage reactance is small. A small value of leakage reactance means that the power factor of the machine is good. Thus a large value of short circuit current (a small value of leakage reactance) indicates a good power factor.

Here we introduce a co-efficient called '*Dispersion co-efficient*'. It is defined as the ratio of magnetizing current to ideal short circuit current. Thus dispersion co-efficient

$$\sigma = \frac{\text{magnetizing current}}{\text{ideal short circuit current}} = \frac{I_m}{I_{sci}} \qquad \text{...(10·54)}$$

Ideal short circuit current $I_{sci} = E_s/X_s$ ...(10·55)

where X_s = total leakage reactance of motor referred to stator.

$$\therefore \qquad \sigma = \frac{I_m}{E_s/X_s} = \frac{I_m X_s}{E_s} \qquad \text{...(10·56)}$$

If I_m and X_s are small, the value of power factor will be good. Thus a small value of dispersion co-efficient indicates a good power factor while a large value of dispersion co-efficient means a poor power factor.

Let us evaluate the dispersion co-efficient.

Now, magnetizing current $I_m = \dfrac{0.427\,p\,AT_{60}}{K_w T_s}$ (See Eqn. 10·27)

$$AT_{60} = 800,000\,B_{g60}\,l_{gs} = 800,000 \times 1.36\,B_{av}\,l_{gs}$$

where $\quad l_{gs}$ = length of effective air gap which consumes the mmf of actual air gap and iron parts.

and $$B_{g60}=1.36\ B_{av}.$$

$\therefore$ $$I_m=\frac{0.427\times 800{,}000\times 1.36\times B_{av}\ p\ l_{gs}}{K_{ws}\ T_s}=\frac{0.465\times 10^6\ B_{av}\ p\ l_{gs}}{K_{ws}\ T_s}\quad \ldots(i)$$

Ideal short circuit current $$I_{sci}=\frac{E_s}{X_s}=\frac{4.44\ f\ \Phi_m\ T_s\ K_{ws}}{8\pi f\ T_s^2\ L(\lambda/pq_s)}$$

$$=\frac{4.44\ B_{av}\ (\pi DL/p)T_s\ K_{ws}}{8\pi f\ T_s^2\ L(\lambda/pq_s)}=\frac{0.555\ B_{av}\ D\ K_{ws}\ q_s}{T_s\ \lambda}\quad \ldots(ii)$$

where λ=effective specific permeance$=\lambda_s+\lambda_o+\lambda_z+\lambda_e$.

$\therefore$ From (i) and (ii), dispersion coefficient

$$\sigma=\frac{I_m}{I_{sci}}=0.838\times 10^6\ \frac{p\ l_{gs}}{D\ K_{ws}^2\ q_s}\ \lambda \quad \ldots(10.57)$$

$$=0.838\times 10^6\ \frac{\pi\ l_{gs}\ \lambda}{\tau\ K_{ws}^2\ q_s} \quad \ldots(10.58)$$

Consider a machine with a given core length and diameter. By examination of Eqn. 10.57 we find that as we increase the number of poles, the dispersion co-efficient increases and an increase in the value of dispersion co-efficient means a poor power factor. Thus induction motors with large number of poles (slow speed machines) have inherently a poor power factor.

Let us examine the same equation in another form (Eqn. 10.58). As the number of poles increases, the pole pitch τ decreases and the number of slots per pole per phase q_s also decreases and hence there is a large increase in the value of dispersion co-efficient with increase in number of poles. Therefore, with increase in number of poles it becomes difficult to get a good power factor.

It is clear from Eqn. 10.57, that the dispersion co-efficient can be decreased and hence power factor increased by using small gap length. But small air gaps give trouble owing to increased zigzag leakage. Therefore, length of air gap should not be unduly decreased.

10.32.1. Effect of dispersion co-efficient on maximum power factor. Fig. 10.39 shows a simplified circle diagram of a polyphase induction motor

where OA=magnetizing current and OB=ideal short circuit current,

$\therefore$ Dispersion co-efficient $\sigma=OA/OB$.

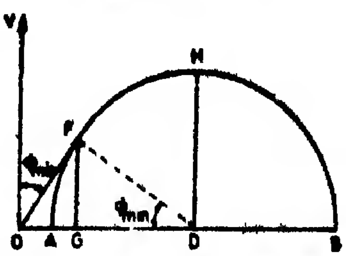


Fig. 10.39. Simplified circle diagram.

Maximum power factor is obtained when line OF is tangent to the circle or $\angle OFD=90°$.

Maximum power factor

$$\cos\phi_{min}=\sin\angle DOF=\frac{FD}{OD}=\frac{AB/2}{OB-AB/2}=\frac{AB}{2OB-AB}$$

$$=\frac{OB-OA}{OB+OA}=\frac{1-\sigma}{1+\sigma}\quad \ldots(10.59)$$

With $\sigma=0.05$, maximum power factor$=\frac{1-0.05}{1+0.05}=0.905$

and with $\sigma=0.1$, maximum power factor$=\frac{1-0.1}{1+0.1}=0.8175$.

Thus the maximum power factor obtainable in a machine reduces drastically with an increase in the value of dispersion co-efficient.

It is possible to obtain $\sigma=0.05$ and hence obtain power factors which are nearly 0.9 and over in machines with 2, 4 or 6 poles. But with larger number of poles σ increases and therefore it becomes difficult to obtain high power factors with machines having large number of poles.

10·32·2. Effect of dispersion co-efficient on overload capacity. If an induction motor is designed to have its maximum power factor at full load, the corresponding output is given by FG (Fig. 10·39), where FG is perpendicular to AB.

The maximum power output which the motor can give occurs when the current is OH. The position of H is vertically above D the centre of the circle. The maximum power output is given by HD. HD in fact represents the maximum power input but if we neglect all losses it represents the maximum power output. Thus the *over load capacity* is :

$$\frac{\text{maximum power output}}{\text{full load output}}=\frac{HD}{FG}=\frac{AD}{FD\sin\phi_{min}}$$

$$=\frac{AD}{AD\sin\phi_{min}}=\frac{1}{\sin\phi_{min}}$$

$$=\frac{1}{\sqrt{1-\left(\frac{1-\sigma}{1+\sigma}\right)^2}}\left(\text{as }\cos\phi_{min}=\frac{1-\sigma}{1+\sigma}\right)$$

$$=\frac{1+\sigma}{2\sqrt{\sigma}} \qquad \ldots(10.60)$$

When the dispersion co-efficient $\sigma=0.05$,

$$\text{ratio }\frac{\text{maximum power output}}{\text{full load output}}=\frac{1+0.05}{2\sqrt{0.05}}=2.35.$$

When the dispersion co-efficient $\sigma=0.1$,

$$\text{ratio }\frac{\text{maximum power output}}{\text{full load output}}=\frac{1+0.1}{2\sqrt{0.1}}=1.74.$$

Thus the overload capacity of induction motors decreases with increase in the value of dispersion co-efficient.

10·32·3. Effect of change of air gap length. The value of magnetising current is directly proportional to the length of air gap. Thus an increase in the length of air gap increases the magnetising current but leaves the ideal short circuit current practically unaffected. An increase in magnetising current means an increase in the value of dispersion co-efficient and therefore an increase in air gap reduces the maximum obtainable power factor. This reduces the output for a given current.

Since there is no change in the value of ideal short circuit current, the diameter of the circle diagram remains the same and therefore the maximum power output and the maximum torque remain unchanged.

10.32.4. Effect of change of number of poles. Let us examine the effect of changing the number of poles for a given frame. Any change in the value of number of poles does not effect the magnetising current since the flux density remains the same.

The flux per pole is inversely proportional to the number of poles. For the same emf the number of winding turns are inversely proportional to flux and therefore are directly proportional to the number of poles. The expression for standstill leakage reactance is :

$$X_s = 8\pi f T_s^2 L\lambda / pq_s.$$

Since $T_s \propto p$ and therefore $X_s \propto p$.

Now ideal short circuit current is $I_{sci} = E_s / X_s \propto 1/p$.

Therefore, the ideal short current varies inversely as the number of poles. Hence we conclude that the diameter of circle diagram varies inversely as the number of poles (See Fig. 10.40), and consequently the maximum power input varies in this way too. The full load input likewise varies approximately inversely as the number of poles necessitating a decrease in their section and therefore in their current carrying capacity.

The torque remains unchanged since it depends upon flux density and rotor current. The speed varies inversely as the number of poles. The output is product of torque and, therefore, it varies inversely as the number of poles.

An increase in number of poles decreases the ideal short circuit current and therefore increases the dispersion co-efficient. Hence an increase in number of poles results in decrease in maximum power factor.

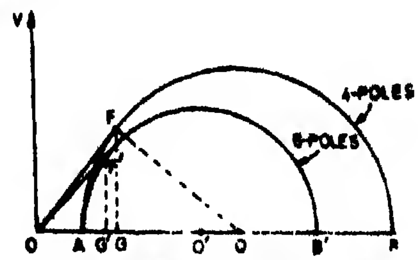


Fig. 10.40. Effect of changing number of poles.

10.32.4. Effect of change of frequency. If the frequency is changed without altering the number of poles it leaves the dispersion co-efficient unchanged. As the speed of rotation of flux varies as the frequency the number of winding conductors must vary inversely as the frequency to produce the same emf with the same flux. The result of lowering the frequency is, therefore, to reduce the output in a manner similar to that caused by an increase in the number of poles. The effect is not so great, however, since the power factor is not lowered *i.e.* if the frequency is reduced from 50 Hz to 40 Hz and the same number of poles are used, the full load input is reduced to 4/5 of the original value.

The copper losses are unaffected, but the friction and windage losses are reduced to 4/5 of original value and the iron losses to less than this. The efficiency is therefore reduced but slightly.

On the other hand, the reduction of frequency may allow the use of fewer poles without causing excessive speed. In this case the dispersion co-efficient is reduced and the power factor improved.

If the number of poles is reduced in the same ratio as the frequency, the speed is unchanged and so is the total number of stator conductors. Hence, the magnetising current is reduced in proportion to the frequency, as there are more turns per pole. The diameter of circle diagram is unchanged and so the effect is the same as changing the air gap length in proportion to frequency.

Example 10.23. *A 4 pole, 3 phase induction motor has its maximum power factor of 0.85 at its normal full load of 10 kW. The efficiency at full load is 88%. Examine the effect of (a) increasing the air gap length by 50 percent (b) rewinding the frame for 6 poles, on (i) maximum power factor (ii) full load output (iii) efficiency.*

Solution. From Eqn. 10.59, maximum power factor

$$\cos\phi_{min} = \frac{1-\sigma}{1+\sigma} \qquad \text{or} \qquad 0.85 = \frac{1-\sigma}{1+\sigma}$$

$\therefore$ Dispersion co-efficient $\sigma = 0.0812$.

From Eqn. 10 57, the dispersion co-efficient σ is directly proportional to air gap length and therefore increasing the gap length by 50 percent increases σ to $1{\cdot}5\times0{\cdot}0812=0{\cdot}122$.

$$\therefore \text{ New maximum power factor}=\frac{1-0{\cdot}122}{1+0{\cdot}122}=0{\cdot}783.$$

$$\text{Original input}=\frac{\text{original output}}{\text{original efficiency}}=\frac{10}{0{\cdot}88}=11{\cdot}36\text{ kW}.$$

$$\text{New input}=\frac{\text{new power factor}}{\text{original power factor}}\times\text{original input}$$

$$=\frac{0{\cdot}783}{0{\cdot}85}\times11{\cdot}36=10{\cdot}46\text{ kW}.$$

The original efficiency is 88 percent and therefore losses are 12 percent of original input. The losses remain the same.

$\therefore$ New output = new output − losses

$=10{\cdot}45-0{\cdot}12\times11{\cdot}36\simeq9{\cdot}1$ kW.

$$\text{New efficiency}=\frac{9{\cdot}1}{10{\cdot}46}\times100=87\text{ percent}.$$

The diameter of circle diagram is unchanged and therefore the maximum power output and the maximum torque remain the same.

(b) An increase in number of poles from 4 to 6, increases the leakage reactance to 1·5 times the original value. Therefore the new diameter of circle diagram is 2/3 times the original diameter.

Thus the full load current must be reduced to 2/3 of its original value. Also the dispersion co-efficient increases to 3/2 times its original value.

New dispersion co-efficient $\sigma=1{\cdot}5\times0{\cdot}0812=0{\cdot}122$.

New maximum power factor $=0{\cdot}783$ [see part (a)]

$$\text{Hence full load input}=\frac{2}{3}\times\frac{\text{new p.f.}}{\text{original p.f.}}\times\text{original input}$$

$$=\frac{2}{3}\times\frac{0{\cdot}783}{0{\cdot}85}=11{\cdot}36=6{\cdot}96\text{ kW}.$$

The losses still remain nearly the same since the stator resistance is increased in the ratio $(3/2)^2$ as there are 50 percent more conductors and the cross-section of each conductor is 2/3 times the original value. The stator current is reduced to 2/3 of original value and as stator copper losses being I^2R, they remain unaltered. The loss in rotor is unchanged since the torque and flux density and hence the rotor current are unaltered. The friction loss is only 2/3 as great as before (due to speed becoming 2/3 of original value), but 1 percent is ample allowance for this reduction. Therefore

New output = new input − (0·12 − 0·01) original input

$=6{\cdot}98-0{\cdot}11\times11{\cdot}36=5{\cdot}73$ kW.

$$\text{New efficiency}=\frac{5{\cdot}73}{6{\cdot}98}\times100=82{\cdot}7\text{ percent}.$$

Since the diameter becomes 2/3 of the original, the new maximum power output is nearly 2/3 times the original maximum power output.

Now torque = output/($2\pi\times$speed).

The maximum power output with 6 poles is 2/3 times that with 4 poles. Also the speed with 6 poles is 2/3 times that with 4 poles and hence we conclude that the maximum torque remains unaltered.

10·33. Relation between D and L for best power factor

The total permeance for leakage flux paths of an induction motor is

$$\Lambda = \Lambda_1 + \Lambda_2$$

where Λ_1, Λ_2 are the permeances of leakage flux path of embedded and overhang portions of windings respectively.

The permeance Λ_1 may be assumed to be directly proportional to the length of the embedded portion of the conductors and inversely proportional to the pole pitch, or

$$\Lambda_1 = AL/\tau \quad \text{where} \quad A = \text{a constant.}$$

The permeance Λ_2 may be assumed to be directly proportional to the pole pitch as an increase in pole pitch will increase the length of the overhang.

$\therefore \quad \Lambda_2 = B\tau$ where B = a constant.

Hence, total permeance of leakage flux paths $= A\dfrac{L}{\tau} + B\tau$

$$= A\frac{pL}{\pi D} + B\frac{\pi D}{p}.$$

We obtain best power factor when the leakage reactance is minimum and for leakage reactance to be minimum the permeance of leakage flux paths should be minimum,

and $\quad \Lambda$ is minimum when $\dfrac{d\Lambda}{dD} = 0$.

Differentiating the expression for total permeance, and equating $\dfrac{d\Lambda}{dD} = 0$, we have

$$-\frac{ApL}{\pi D^2} + \frac{B\pi}{p} = 0 \qquad \text{or} \qquad D^2 = \frac{A}{B}\,\frac{p^2}{\pi^2}\,L$$

or $$D = \frac{p}{\pi}\sqrt{\frac{A}{B}}\,\sqrt{L} \qquad ...(10·61)$$

A study of several normal designs gives an optimum value of 0·18 for $\sqrt{A/B}$.

$\therefore$ $$D = \frac{p}{\pi}\sqrt{0·18}.\sqrt{L} \qquad ...(10·62)$$

and $\quad$ pole pitch $\tau = \sqrt{0·18L} \qquad ...(10·63)$

Eqns. 10·62 and 10·63 give the relationship between D and L for best power factor.

10·34. Methods of improving starting torque

It has been discussed earlier that a high rotor resistance is required for good starting torque and reasonable starting current but a high value of rotor resistance leads to poor efficiency and high speed regulation under load conditions. An overall alteration of the resistance is possible by changes in the cross sectional areas and materials of the bars and the end rings. In the case of plain squirrel cage machines, the rotor resistance is constant, a compromise in design is necessary, and the resulting motor may not be always satisfactory for different applications because of the characteristics that cannot be altered. However, the

simplicity of the cage rotor offers great advantages in manufacture and, therefore, methods have been devised to reduce the disadvantages of fixed resistance. An improvement in the ratio high ratio of starting torque to full load torque is obtained by using the following two types of rotors which employ change of rotor circuit impedance with change in rotor frequency to obtain high starting torque and also high efficiency under load conditions. These are :

(i) double cage rotors (ii) deep bar rotors.

10 34 1. Double cage rotors

The winding consists of two cages which may be electrically linked or independent. The two sets of rotor bars may be short circuited by the same end rings or by separate ones. A die-cast aluminium winding may also be used for double cage rotors. A double cage rotor is shown in Fig. 10 .

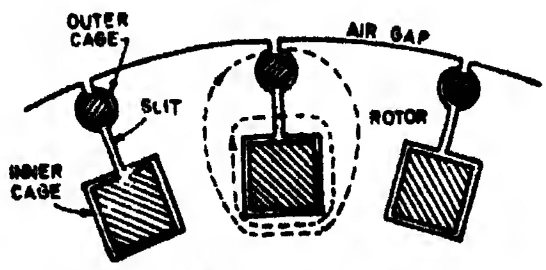


Fig 10·41. Double cage rotor.

The outer cage uses bars of small cross-section and therefore its resistance is high. However, since the outer cage is nearer to the air gap, its leakage reactance is small. On the other hand the inner cage uses bars of large cross-section and therefore its resistance is small. The inner cage is linked with the air gap flux by a long narrow slit, so that it has a high leakage reactance. (It should be noted that slits are necessary to prevent the inner cage from being missed by the air gap flux because if the inner cage bars were buried in iron, the gap flux would link only the outer cage)

The working of the double cage rotor is as follows :

At starting the frequency of the rotor currents is the same as that of the supply and hence the division of current between the outer and the inner cages is determined by impedance of each cage. At starting, the reactances of the cages are much higher than their respective resistances on account of the rotor frequency being equal to the supply frequency and therefore most of the current flows in the cage which has a lower reactance *i.e.* the outer cage, which has the larger resistance.

Under load conditions, the machine runs near about the synchronous speed and therefore the rotor frequency is very small and hence the reactances of the two cages are negligible. Therefore, the current division between the two cages is determined by the resistance, and hence the inner cage which has a small resistance carries most of the current.

Therefore, at starting the resistance of rotor is high giving a high starting torque and a low starting current, and while running the resistance is low, giving good efficiency and low speed regulation. It may be assumed to a first approximation, the two cages produce two separate torques and the total torque of the machine can be obtained by summing the two torques as shown in Fig. 10·41. The resultant speed torque characteristics can be modified and any desired characteristics obtained by modifying the individual cage resistances and leakage reactances. The resistances can be changed by changing the areas of cross-section of bars while the leakage resistances can be changed by changing the width of the slot openings and the depth of the inner cage.

The pull out torque of a double motor is smaller than that of a plain squirrel cage motor because two cages produce their maximum torques at two different speeds. Also, the additional reactance of the inner cage lowers the full load power factor while the high resistance of the outer cage increases the full load I^2R loss thereby lowering the efficiency. However, the energy efficiency of a plain squirrel cage machine designed to give the same starting torque as the double cage machine is lower.

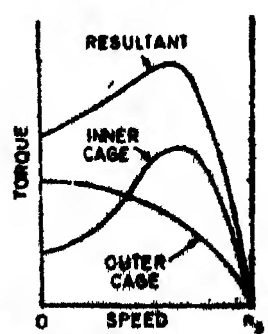


Fig. 10·24. Torque speed characteristics of a double

10·34·2. Deep bar rotor. The production of eddy current loss in conductors has been dealt with in chapter 6, Art. 6·40 on page 322, where it is shown that the loss is proportional to the fourth power of conductor depth. It has been shown that a conductor 4 mm wide in a slot 5 mm wide has a total copper loss of 1·058, 1·91, 5·6 times I^2R loss for conductor heights of 10, 20, 30 mm respectively.

A considerable increase in resistance is thus obtained at starting when the frequency is high (α is large). Fig 10·43 shows some typical shapes of deep bars and slots used. In small motors, large depth cannot be used as adequate space has to be kept for core, shaft and cooling ducts.

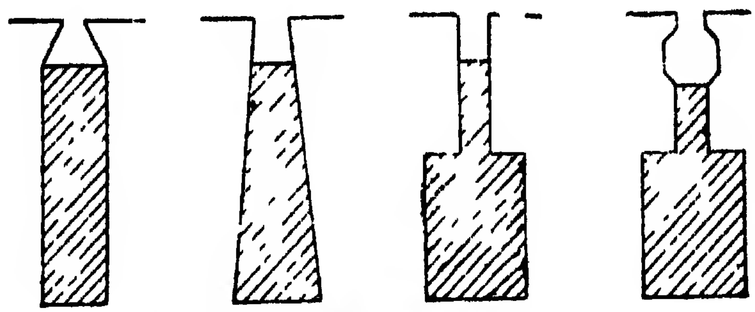


Fig. 10.43. Profiles of deep bar rotors

The increased leakage flux (which is required to produce a high eddy current loss in conductors) affects the power factor adversely, and the efficiency is lower than that in plain cage motors.

The current is mainly confined to the upper portion of the conductor during starting and this part becomes hot. But heat is dissipated to the lower portion, especially where a greater cross section is employed as in T bar.

The connection of the rotor bars to their end rings must be carefully designed because the connection represents the most likely source of weakness.

10·35. Losses and efficiency. The various losses in induction motors are :

1. Stator copper losses. 2. Rotor copper losses. 3. Stator iron losses. 4. Friction and windage losses. 5. Additional losses.

The calculation of losses except additional losses has been explained earlier.

Efficiency at full load $\eta=\dfrac{\text{output}}{\text{output}+\text{losses}}$

10·35·1. Additional losses

The additional losses include

(*i*) additional copper losses (*ii*) additional iron losses.

With a sinusoidal voltage impressed across the terminals of the motor, the additional copper losses are due, in part to the higher order mmf harmonics and in part to skin effect. The additional losses owing to higher order mmf harmonics occur mainly in windings of squirrel cage rotor. These additional copper losses may be decreased by :

(*i*) chording the stator winding,

(*ii*) skewing the rotor,

(*iii*) having a proper slot combination.

The skin effect phenomenon is observed in stator and rotor windings especially in squirrel cage machines. Here the effect may be used for improving starting charact ristics of motors with a cage rotor.

However, during normal operating conditions the frequency of the current in the rotor does not exceed about 3 Hz or so and therefore this skin effect is practically absent.

The additional losses in iron consist of (*i*) pulsation losses and (*ii*) surface losses. The pulsation losses are caused by direct axis pulsation of magnetic flux due to variati n of permeance caused by continuous change in mutual positions of rotor and stator teeth during rotation of rotor.

The additional iron losses in induction motors are about 0·5 percent of the supplied power.

10·36. Temperature rise. The calculation of temperature rise has been explained in Art. 3·36·1 page 111. The cooling co-efficient for the different surfaces are taken from Table 3·6.

DESIGN PROBLEMS

Design Problem 1. *Design a 2·2 kW, 400 V, 3-phase, 50 Hz, 1500 synchronous r.p.m. squirrel cage induction motor. The machine is to be started by a star delta starter. The efficiency is 0·8 and power factor is 0·825 at full load.*

Solution.

Main Dimension. Synchronous speed $n_s=1500/60=25$ r.p.s.

Number of poles $p=2f/n_s=4$.

The machine is meant to be sold in a highly competitive market and therefore its price is the major consideration and its performance characteristics like efficiency and power factor can be sacrificed. In order to design a cheap machine we must use higher values for specific electric and magnetic loadings. Assuming

$$B_{av}=0{\cdot}44\ \text{Wb/m}^2\ ;\ ac=21000\ \text{A/m and}\ K_w=0{\cdot}955.$$

Output coefficient $C_o=11\ K_w\ B_{av}\ ac\ 10^{-3}$

$$=11\times0{\cdot}955\times0{\cdot}44\times21000\times10^{-3}=97.$$

kVA input $\quad Q=\dfrac{2{\cdot}2}{0{\cdot}8\times0{\cdot}825}=3{\cdot}33.$

$\therefore$ Product $D^2L=\dfrac{Q}{C_o\,n_s}=\dfrac{3{\cdot}33}{97\times25}=1{\cdot}375\times10^{-3}\ \text{m}^3.$

For a cheap design, ratio L/τ should be between 1·5 to 2. Assuming $L/\tau=1{\cdot}5$

or $\quad \dfrac{L}{\pi D/4}=1{\cdot}5 \quad$ or $\quad L=1{\cdot}18\,D$

$\therefore \quad 1{\cdot}18D^3=1{\cdot}375\times10^{-3}$

or $\quad D=0{\cdot}105$ m, $\quad L=0{\cdot}125$ m and $\quad \tau=0{\cdot}0825$ m.

As the length of core is 0·125 m and therefore there is no necessity of providing any radial ventilating duct.

$\therefore$ Net iron length $L_i=0{\cdot}9\times0{\cdot}125=0{\cdot}1125$ m.

Lohys 0·5 mm thick laminations are used for the machine.

Stator Design

Winding.

The machine is to be designed for delta connection as it is started by a star delta starter.

$\therefore$ Stator voltage per phase $E_s = 400$ V.

Flux per pole $\Phi_m = B_{av}\,\tau\, L = 0{\cdot}44 \times 0{\cdot}0825 \times 0{\cdot}125 = 4{\cdot}54 \times 10^{-3}$ Wb.

Stator turns per phase $T_s = \dfrac{400}{4{\cdot}44 \times 50 \times 4{\cdot}54 \times 10^{-3} \times 0{\cdot}955} = 416.$

The slot pitch should be between 15 to 25 mm but can be less than 15 mm for small machines using semi-enclosed slots.

Taking slots per pole per phase $q_s = 2$

Total stator slots $S_s = 3 \times 4 \times 2 = 24.$

Stator slot pitch $y_{ss} = \pi \times 0{\cdot}105 \times 10^3/24 = 13{\cdot}75$ mm.

Total stator conductors $= 6T_s = 6 \times 416 = 2496.$

Stator conductors per slot $Z_{ss} = 2496/24 = 104.$

Mush winding in tapered semi-enclosed slots is used for the stator. Mush winding is a single layer winding and the total number of stator coils is equal to half the number of stator slots i.e. 12.

Coil span C_s = slots/poles = 24/4 = 6.

But the coil span should not be an even integer in case of mush windings [illegible] a coil span of 5 slots is used.

Thus the coils are chorded by one slot pitch.

Angle of chording $\alpha = (1/6) \times 180° = 30°.$

$\therefore$ Pitch factor $K_p = \cos \alpha/2 = 0{\cdot}966.$

Distribution factor for 2 slots per pole per phase,

$$K_d = \frac{\sin 60/2}{\sin 60/(2 \times 2)} = 0{\cdot}966.$$

$\therefore$ Stator winding factor $K_{ws} = K_d \times K_p = 0{\cdot}966 \times 0{\cdot}966 = 0{\cdot}933.$

Conductor Size.

Stator current per phase $I_s = \dfrac{2{\cdot}2 \times 10^3}{3 \times 400 \times 0{\cdot}8 \times 0{\cdot}825} = 2{\cdot}77$ A.

Stator line current $= \sqrt{3} \times 2{\cdot}77 = 4{\cdot}8$ A.

Choosing a current density of 4 A/mm^2,

Area of stator conductor required $= 2{\cdot}77/4 = 0{\cdot}6925$ mm^2.

Diameter of conductor (bare) required $= 0{\cdot}94$ mm.

From Table 17·7, the nearest standard conductor has a bare diameter

$d = 0{\cdot}95$ mm.

$\therefore$ Area of stator conductor used $a_s = \pi/4\,(0{\cdot}95)^2 = 0{\cdot}709$ mm^2.

Current density for stator conductors, $\delta_s = \dfrac{2{\cdot}77}{0{\cdot}709} = 3{\cdot}91$ A/mm^2.

Diameter of enamelled conductor $d_i = 1{\cdot}041$ mm. (Using medium covering. See Table 17·7)

Slot Dimensions. Space required for bare conductors in a slot

$$=Z_{ss}\times a_s=104\times 0.709=73.6 \text{ mm}^2.$$

Taking a space factor of 0.4 for the slots

Area of each slot $=73.6/0.4=184$ mm².

Before deciding the slot dimensions to give the above area, the minimum tooth width that would keep the flux density within limits must be found. The maximum allowable flux density is 1.7 Wb/m².

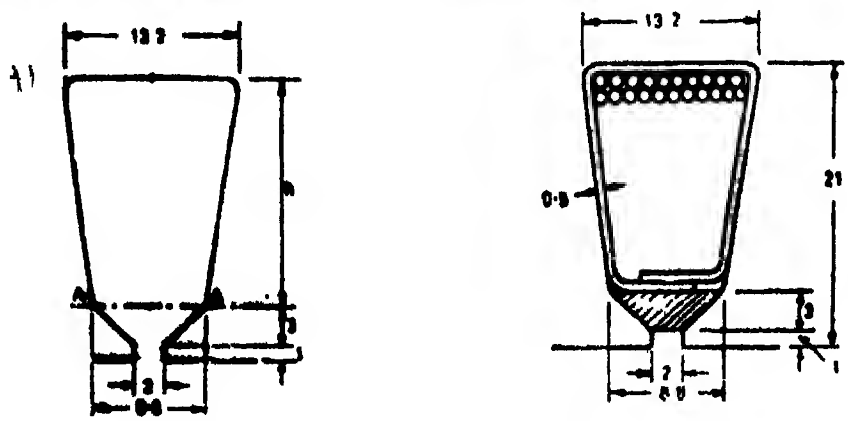
Minimum width of stator teeth

$$(W_{ts})_{min}=\frac{\Phi_m}{1.7\times S_s/p\times L_i}=\frac{4.54\times 10^{-3}}{1.7\times(24/4)\times 0.1125}\text{ m}$$
$$=3.95 \text{ mm}.$$

A tooth of constant width 6.0 mm is taken. The dimensions of the slot are worked as under :

Take lip $=1$ mm and wedge $=3$ mm.

Suppose h be the height in mm as indicated in Fig. 10.44 (*a*).



(*a*) (*b*)

Fig. 10.44. Stator Slot. All Dimensions in mm.

$\therefore$ Slot width at $AA=\dfrac{\pi(105+2\times 4+2h)}{24}-6=8.8$ mm

and width of slot at the bottom

$$=\frac{\pi(105+2\times 4+2h)}{24}-6=8.8+\frac{\pi h}{12}\text{ mm}.$$

Area of conductor portion of slot

$$=\frac{1}{2}\,h\left(8.8+\frac{\pi h}{12}+8.8\right)=184 \text{ mm}^2$$

or $$h=17 \text{ mm}.$$

$\therefore$ Width at the bottom of the slot $=8.8+\dfrac{\pi\times 17}{12}=13.2$ mm

and depth of the slot $d_{ss}=17+4=21$ mm.

The arrangement of conductors and the insulation is shown in Fig. 10.44 (*b*).

Length of mean turn $L_{mts}=2L+2.3\tau+24=0.68$ mm.

Stator Teeth.

$$\text{Flux density in stator teeth} = \frac{\Phi_m}{S_s/p \times W_{ts} \times L_i}$$

$$= \frac{4.54 \times 10^{-3}}{(24/4) \times 6 \times 10^{-3} \times 0.1125} = 1.12 \text{ Wb/m}^2.$$

This is a good figure as there is no saturation and also the teeth are not being worked at a low flux density.

Stator Core. Flux in stator core

$$= (4.54 \times 10^{-3})/2 = 2.27 \times 10^{-3} \text{ Wb.}$$

Assume a flux density of 1.2 Wb/m^2.

$\therefore$ Area of stator core $A_{cs} = \dfrac{2.27 \times 10^{-3}}{1.2} = 1.89 \times 10^{-3}$ m^2.

Depth of stator core $d_{cs} = \dfrac{1.89 \times 10^{-3}}{0.1125} = 16.8 \times 10^{-3}$ m.

Taking the core depth $d_{cs} = 17$ mm.

Flux density in stator core $B_{cs} = (16.8/17) \times 1.2 = 1.185$ Wb/m^2.

Outside diameter of stator laminations $D_o = D + 2d_{ss} + 2d_{cs}$

$$= 105 + 2(21 + 17) = 181 \text{ mm.}$$

Rotor Design.

Air Gap. Length of air gap $l_g = 0.2 + 2\sqrt{\overset{D}{0.105} \times \overset{L}{0.125}} = 0.43$ mm

But this would result in a large magnetizing current and therefore a shorter gap should be used. A gap of 0.3 mm length may be suitable.

Referring to Table 10.2.

$\therefore$ Length of air gap $l_g = 0.3$ mm.

Diameter of rotor $D_r = 105 - 2 \times 0.3 = 104.4$ mm.

Rotor slots. The number of rotor slots is taken as one pole pair (*i.e.* 2 in this case) smaller than the number of stator slots.

The number of stator slots $S_s = 24$ and the number of rotor slots $S_r = 22$.

Rotor slot pitch at the air gap

$$y_{sr} = \pi \times 0.1044/22 = 0.0149 \text{ m} = 149 \text{ m} = 14.9 \text{ mm.}$$

Rotor Bars. From Eqns. 10.39 and 10.40, rotor bar current

$$I_b = \frac{2m_s K_{ws} T_s}{S_r} \times I_s \cos\phi = \frac{2 \times 3 \times 0.933 \times 416}{22} \times 2.77 \times 0.825 = 244 \text{ A.}$$

In order to obtain a good starting torque, a high value of current density should be used for in rotor bars. Taking the rotor bar current density $\delta_b = 6$ A/mm^2.

Area of each rotor bar $a_b = 244/6 = 40.7$ mm^2.

Referring to Table 17.1, the dimensions of conductor used are 7.0×6.3 mm^2 with an area of $a_b = 44.6$ mm^2.

Area of bar used $a_b = 44.6$ mm^2.

The slot used is shown in Fig. 10·45.

Width of rotor slot $W_{sr}=6{\cdot}8$ mm.

Depth of rotor slot $W_{sr}=9{\cdot}3$ mm.

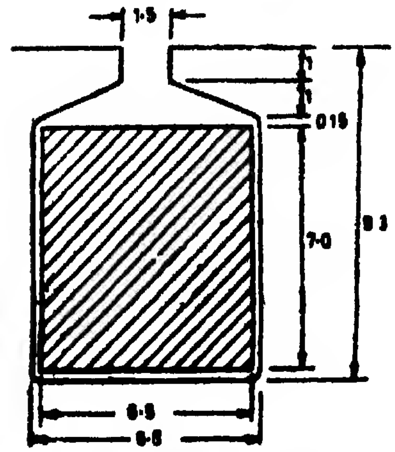


Fig. 10·45. Rotor slot. All dimensions in mm.

Before finally deciding the dimensions of the rotor slots, the flux density in the rotor teeth at the root should be checked.

Slot pitch at the bottom of slots

$$=\frac{\pi(104{\cdot}4-2\times9{\cdot}3)}{22}=12{\cdot}2 \text{ mm}$$

Tooth width at the root

$$W_t=12{\cdot}2-6{\cdot}8=5{\cdot}4 \text{ mm.}$$

Flux density at the root of rotor teeth

$$=\frac{4{\cdot}54\times10^{-3}}{(22/4)\times0{\cdot}1125\times(54\times10^{-3})}=1{\cdot}36 \text{ Wb/m}^2.$$

The flux density in teeth is within limits.

The bars are skewed by one slot pitch. Extending the bars by about 15 mm beyond the core on each side and taking 10 mm as increase in length because of skewing.

Length of each bar $L_b=125+2\times15+10=165$ mm

Resistance of each bar

$$r_b=\frac{0{\cdot}021\times165\times10^{-3}}{44{\cdot}6}=77{\cdot}7\times10^{-6}\Omega.$$

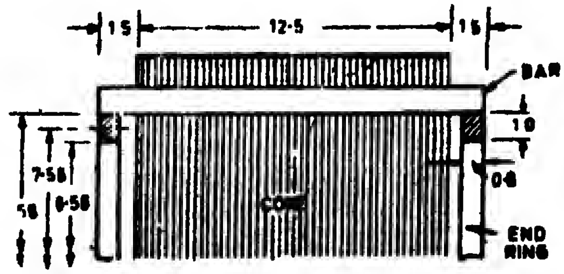


Fig. 10·46. Rotor details. All Dimensions in mm.

Total copper loss in bars

$$=22\times(244)^2\times77{\cdot}7\times10^{-6}=101 \text{ W.}$$

End Rings

End ring current

$$I_e=\frac{S_r I_b}{\pi p}=\frac{22\times244}{\pi/4}=428\text{A}.$$

Taking the current density in end ring $\delta_e=6$ A/mm².

∴ Area of end ring $a_e=428/6=71{\cdot}3$ mm²

Using a ring of 10×8 mm section.

∴ Depth of ring $d_e=10$ mm, thickness of ring $t_e=8$ mm and area of each end ring $a_e=80$ mm².

The ring is brazed to the bars as shown in Fig. 10·46.

(In practice, however, the rotor of this machine may be die cast using aluminium.)

Outer diameter of end ring $=104{\cdot}4-2\times9{\cdot}3=85{\cdot}8$ mm.

Inner diameter of end ring $=85{\cdot}8-2\times10=65{\cdot}8$ mm.

Mean diameter of end ring $D_e=75{\cdot}8$ mm.

Resistance of each ring $r_e = \frac{\rho \pi D_e}{a_e} = \frac{0.021 \times \pi \times 75.8 \times 10^{-3}}{80}$

$= 62.5 \times 10^{-6} \Omega.$

Copper loss in two end rings $= 2 \times (428)^2 \times 62.5 \times 10^{-6} = 23$ W.

Total rotor copper loss $= 101 + 23 = 124$ W.

Now, $\frac{\text{rotor copper loss}}{\text{rotor output}} = \frac{s}{1-s}$

or full load slip $s = 5.3$ per cent.

This is a reasonable figure for a 2.2 kW machine.

Rotor Core

The value of depth of rotor core is taken equal to that of stator core.

$\therefore$ Depth of rotor core $d_{cr} = 17$ mm.

Flux density in rotor core $B_{cr} = 1.185$ Wb/m².

Inner diameter of rotor laminations $D_i = D_r - 2d_{sr} - 2d_{cr}$

$= 104.4 - 2 \times 9.3 - 2 \times 17 = 51.8$ mm.

No Load Current

Magnetizing Current

The mmfs required for various parts of magnetic circuit are calculated below :

(*i*) **Air Gap :**

For stator slots :

ratio (slot opening/gap length) $= 2/0.3 = 6.67$.

From Fig. 4 9 page 124, Carter's co-efficient corresponding to 6.66 for semi-enclosed slots, $K_{cs} = 0.68$.

Gap contraction factor for stator slots .

$$K_{gss} = \frac{y_{ss}}{y_{ss} - K_{cs} W_o} \quad \text{stator slot pitch}$$

$$= \frac{13.75}{13.75 - 0.68 \times 2} = 1.11.$$

in mm

For rotor slots,

ratio (slot opening/gap length) $= 1.5/3 = 5$.

Corresponding to a ratio 5, $K_{cs} = 0.6$.

Gap contraction factor for rotor slots

$$K_{gsr} = \frac{y_{sr}}{y_{sr} - K_{cs} W_o}$$

$$= \frac{14.9}{14.9 - 0.6 \times 1.5} = 1.064.$$

$\therefore$ Gap contraction factor for slots

$$K_{gs} = K_{gss} \times K_{gsr} = 1.11 \times 1.064 = 1.18.$$

As there are no ducts, the gap contraction factor for ducts K_{gd} is unity.

$\therefore$ Gap contraction factor $K_g = K_{gs} \times K_{gd} = 1.18 \times 1 = 1.18$.

Area of air gap A_g

$$=\frac{\pi\times 0.105}{4}\times 0.125=10.3\times 10^{-3}\,m^2.$$

From Eqn. 4·120, $B_{g60}=1.36\ B_{av}=1.36\times 0.44=0.6\ Wb/m^2$.

Effective length of air gap $l_{ge}=1.18\times 0.3=0.354$ mm.

Mmf required for air gap $AT_g=800{,}000\times 0.6\times 1.18\times 0.3\times 10^{-3}=170$ A.

(ii) Stator Teeth :

Area of teeth per pole $=(S_s/p)\times W_{ts}\times L_i$

$$=(24/4)\times 6\times 10^{-3}\times 0.1125=4.05\times 10^{-3}\ m^2.$$

Flux density in stator teeth $B_{ts}=1.12$ Wb/m².

$\therefore$ $B_{ts60}=1.36\times 1.12=1.52$ Wb/m².

Corresponding to this flux density (see Fig. 4·2 page 120 for Lohys steel)

$at_{ts}=1200$ A/m. ← depth of stator slots

$\therefore$ Mmf required for stator teeth $=1200\times 21\times 10^{-3}=25$ A.

(iii) Stator Core :

Area of stator core $A_{cs}=L_i\,d_{cs}=0.1125\times 0.017=1.915\times 10^{-3}\ m^2$.

Flux density in stator core $B_{cs}=1.185$ Wb/m².

Length of magnetic path through stator core

$$l_{cs}=\frac{\pi(D+2d_{ss}+d_{css})}{3p}$$

$$=\frac{\pi(105+2\times 21+17)}{3\times 4}\times 10^{-3}=43\times 10^{-3}\ m$$

Corresponding to a flux density of 1·85 Wb/m², $at_{cs}=280$ A/m.

Mmf required for stator core $AT_{cs}=289\times 43\times 10^{-3}=12$ A.

(iv) Rotor Teeth :

Width of rotor teeth at ⅓ height from narrow end

$$W_{tr1/3}=\frac{\pi(D_r-4d_{sr1/3})}{S_r}-W_{sr}$$

$$=\frac{\pi(104.4-4\times 9.3/3)}{22}-6.8=6.3\ mm.$$

Area of teeth per pole at ⅓ height from narrow end

$$A_{tr}=(22/4)\times 6.3\times 10^{-3}\times 0.1125=3.9\times 10^{-3}\ m^2.$$

Flux density in rotor teeth at ⅓ height $B_{tr1/3}=1.16$ Wb/m²

$\therefore$ $B_{tr60}=1.36\times 1.16=1.575$ Wb/m².

Corresponding to this flux density $at_{tr}=2000$ A/m.

$\therefore$ Mmf required for rotor teeth $A_{tr}=2000\times 9.3\times 10^{-3}=19$ A.

(v) Rotor Core :

Rotor core area $A_{cr}=L_i\,d_{cr}=0.1125\times 17\times 10^{-3}=1.915\times 10^{-3}\ m^2$.

Flux density in rotor core $=1.185$ Wb/m².

Corresponding to this flux density $at_{cr}=280$ A/m.

Length of flux path in rotor core

$$l_{cr}=\frac{\pi(D_r-2d_{sr}-2d_{cr})}{3\times p}=18\times10^{-3}$$

Mmf for rotor core $=AT_{cr}=280\times18\times10^{-3}=5$ A.

The magnetization data is tabulated below :

Part	Area m^2	Length mm	B Wb/m^2	B_{60} Wb/m^2	at	AT
Air Gap	10.3×10^{-3}	0·354	0·44	0·6	48000	170
Stator Teeth	4.05×10^{-3}	21	1·12	1·52	1200	25
Stator Core	1.915×10^{-3}	43	1·185	—	280	12
Rotor Teeth	3.9×10^{-3}	9·3	1·16	1·575	2000	19
Rotor Core	1.915×10^{-3}	1·8	1·185	—	280	5
					Total	$AT_{60}=231$

Magnetizing current per phase $I_m=\dfrac{0.427\,p\,AT_{60}}{K_{ws}T_s}$

$$=\frac{0.427\times4\times231}{0.934\times416}=1.0\text{ A.}$$

Loss Component

Iron loss in stator teeth.

Volume of stator teeth $=0.34\times10^{-3}$ m³. — see

Weight of stator teeth $=0.34\times10^{-3}\times7.6\times10^{3}=2.6$ kg.

Maximum flux density in teeth $=\pi/2\times1.12=1.76$ Wb/m².

Using 0·5 mm thick (Lohys steel) laminations.

Corresponding to this flux density, loss per kg from Fig. 4 28 page 148 is 11·5 W.

∴ Iron loss in stator teeth $=11.5\times2.6=30$ W.

Iron loss in stator core

Volume of stator core $=0.985\times10^{-3}$ m³.

Weight of stator core $=0.985\times10^{-3}\times7.6\times10^{3}=7.5$ kg.

Flux density in stator core $=1.185$ Wb/m².

Corresponding to this flux density, iron loss per kg is 4·9 W.

∴ Iron loss in core $=7.5\times4.9=37$ W.

Total iron loss $=30+37=67$ W.

The actual iron loss will be about 2 times this loss.

∴ Total iron loss $=2\times67=134$ W.

Friction and windage losses From Table 10·6, the friction and windage losses are about 4 per cent of output. But with the use of ball and roller bearings, the loss would be around 2·5 percent of output. o/p

∴ F and W loss $=(1.5/100)\times2200=33$ W.

Total no load losses $=134+33=167$ W

Loss component of no load current per phase $I_l = \frac{167}{3 \times 400} = 0.139$ A.

$\therefore$ No load current $\quad I_0 = \sqrt{1.0^2 + 0.139^2} = 1.0$ A.

No load current expressed as a percentage of full load current

$$= (1.0/2.77) \times 100 = 36\%.$$

This figure is within normal limit (see Table 10.7).

No load power factor $\cos \phi_0 = 0.139/1.0 = 0.139.$

$\therefore$ Phase angle of no load current $\phi_0 = \cos^{-1} 0.139 = 82°6'.$

Short Circuit Current

Leakage reactance

Stator slot leakage :

From Eqn. 4.73, for a tapered slot, specific slot permeance

$$\lambda_{ss} = \mu_0 \left[\frac{2h_1}{3(W_s + W_2)} + \frac{2h_2}{W_1 + W_2} + \frac{2h_3}{W_1 + W_0} + \frac{h_4}{W_0} \right]$$

$$= 4\pi \times 10^{-7} \left[\frac{2 \times 17}{3(13.2 + 8.8)} + \frac{2 \times 3}{8.8 + 2} + \frac{1}{2} \right] = 19.7 \times 10^{-7}.$$

(Considering the height h_2 also to be occupied by conductors).

Rotor Slot Leakage :

From Eqn. 4.72 for a parallel sided slot,

Specific slot permeance

$$\lambda_{sr} = \mu_0 \left[\frac{h_1}{3W_s} + \frac{h_2}{W_s} + \frac{2h_3}{W_s + W_0} + \frac{h_4}{W_0} \right]$$

$$= 4\pi \times 10^{-7} \left[\frac{7}{3 \times 6.8} + \frac{2 \times 1}{6.8 + 1.5} + \frac{1.0}{1.5} \right] = 15.7 \times 10^{-7}.$$

This referred to stator side is $\lambda'_{sr} = \lambda_{sr} \times \frac{K_{ws}^2\, S_s}{K_{wr}^2\, S_r}$

$$= 15.7 \times 10^{-7} \times \frac{0.934^2 \times 24}{1 \times 22} = 14.95 \times 10^{-7}.$$

$\therefore$ Total specific slot permeance $\lambda_s = \lambda_{ss} + \lambda'_{sr} = 34.65 \times 10^{-7}.$

Slot leakage reactance $x_s = 8\pi f\, T_{ph}^2 L(\lambda_s/pq)$

$$= 8\pi \times 50 \times 416^2 \times 0.125 \times \frac{34.65 \times 10^{-7}}{(4 \times 2)} = 11.8\Omega.$$

Overhang leakage : slot pitch

From Eqn. 4.94, $L_0\lambda_0 = \mu_0 K_s \tau^2/\pi y_s$

Now ratio $\frac{\text{coil span}}{\text{pole pitch}} = \frac{5}{6} = 0.833.$

Corresponding to this, the value of K_s from Fig. 4.58 page 170 is 0.875.

$$\therefore \quad L_0\lambda_0 = 4\pi \times 10^{-7} \times \frac{0.875 \times 0.0825^2}{\pi \times 13.75 \times 10^{-3}} = 1.73 \times 10^{-7}.$$

Overhang leakage reactance $x_0 = 8\pi f T_{ph}^2 (L_0\lambda_0/pq)$

$$= 8\pi \times 50 \times 416^2 \times 1.73 \times 10^{-7}/(4 \times 2) = 4.7\ \Omega.$$

Zigzag leakage :

Magnetizing reactance $X_m = E_s/I_m = 400/1{\cdot}0 = 400\Omega$.

From Eqn. 10·46, zigzag leakage reactance per phase

$$x_z = \frac{5}{6}\,\frac{X_m}{m_s^2}\left(\frac{1}{q_s^2}+\frac{1}{q_r^2}\right)$$

$$= \frac{5}{6} \times \frac{400}{3^2}\left[\frac{1}{2^2}+\frac{1}{1{\cdot}835^2}\right] = 20{\cdot}3\ \Omega$$

as $$q_s = 24/(3\times 4) = 2 \text{ and } q_r = 22/(3\times 4) = 1{\cdot}885.$$

The differential leakage reactance can be ignored in the case of squirrel cage induction motors.

∴ Total leakage reactance per phase referred to stator

$$X_s = 11{\cdot}8 + 4{\cdot}7 + 20{\cdot}3 = 36{\cdot}8\Omega.$$

Resistance. Resistance of stator winding per phase

$$r_s = \frac{0{\cdot}021\times 416\times 0{\cdot}68}{0{\cdot}709} = 3{\cdot}37\ \Omega$$

Total stator copper loss $= 3\times 2{\cdot}77^2 \times 8{\cdot}37 = 193$ W.

Total rotor copper loss $= 124$ W. (calculated earlier)

Rotor copper loss per phase $= 124/3 = 41{\cdot}3$ W.

∴ Rotor resistance referred to stator

$$r_r' = \frac{41{\cdot}3}{2{\cdot}77^2} \times \frac{1}{0{\cdot}825^2} = 7{\cdot}83\ \Omega$$

Total resistance referred to stator $R_s = 8{\cdot}37 + 7{\cdot}83 = 16{\cdot}2\ \Omega$.

Impedance

∴ Total impedance of rotor at standstill

$$Z_s = \sqrt{36{\cdot}8^2 + 16{\cdot}2^2} = 40{\cdot}2\ \Omega.$$

Short circuit (blocked rotor) current per phase $I_{ss} = 400/40{\cdot}2 = 9{\cdot}95$ A.

Short circuit power factor $\cos\phi_{ss} = 16{\cdot}2/40{\cdot}2 = 0{\cdot}403$.

Phase angle of short circuit (blocked rotor) current

$$\phi_{ss} = 66^\circ\ 12'.$$

Losses and Efficiency

Stator copper loss	$= 193{\cdot}0$ W.
Rotor copper loss	$= 124{\cdot}0$ W.
Total iron loss	$= 134{\cdot}0$ W.
Friction and windage loss	$= 33{\cdot}0$ W.
Total loss at full load	$= 484$ W.

Output at full load $= 2200$ W. ∴ Input at full load $= 2684$ W.

Efficiency at full load $\eta = 2200/(2684) \times 100 = 81{\cdot}8$ per cent less ½ per cent $= 81{\cdot}3$ per cent

Circle diagram. The circle diagram is shown in Fig. 10·47. From this diagram, the results are :

Power factor at full load $= \cos 34^\circ = 0{\cdot}829$.

Full load phase current $\qquad I_s = OA = 2{\cdot}63$ A.

$$\frac{\text{Starting torque}}{\text{Full load torque}} = \frac{BG}{AK} = 1{\cdot}0.$$

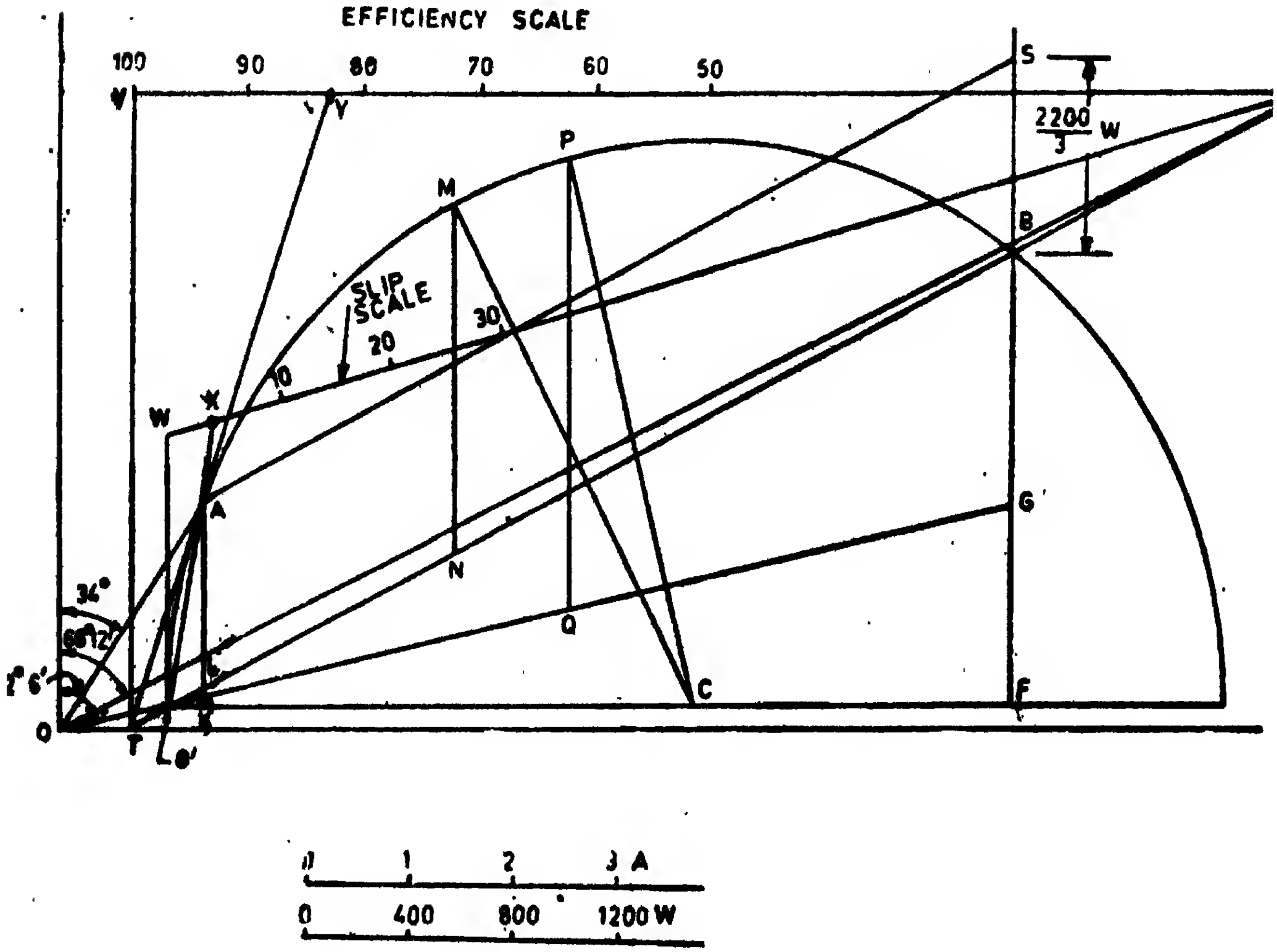


Fig. 10·47. Circle Diagram.

$$\frac{\text{Maximum torque}}{\text{Full load torque}} = \frac{PQ}{AK} = 2$$

and
$$\frac{\text{Maximum output}}{\text{Full load output}} = \frac{MN}{AL} = 1{\cdot}67.$$

Stator Temperature Rise

Iron losses in stator $= 134$ W. Stator copper losses $P_c = 193$ W.

Copper loss in slot portion $= \frac{2L}{L_{mts}} \cdot P_c = 71$ W.

Total loss to be dissipated by stator surface $= 71 + 134 = 205$ W.

Outside cylindrical surface of stator $= \pi \times 0{\cdot}181 \times 0{\cdot}125 = 71{\cdot}2 \times 10^{-3}$ m^2.

Cooling co-efficient $= 0{\cdot}03$ (See Table 3·6 page 111).

Loss dissipated from back of core $= 71{\cdot}2 \times 10^{-3}/0{\cdot}03 = 2{\cdot}38$ W/°C.

Inside cylindrical surface of stator $= 41{\cdot}2 \times 10^{-3}$ m^2.

Peripheral speed $V_a = \pi \times 0.105 \times 25 = 8.25$ m/s.

From Table 3.6, cooling co-efficient

$$= \frac{0.04}{1+0.1\,V_a} = \frac{0.04}{1+0.1\times 8.25} = 0.022.$$

Loss dissipated from inside surface of stator $= 41.2\times 10^{-3}/0.022 = 1.87$ W/°C.

Cooling surface of two ends

$$= 2\times \frac{\pi}{4}(0.181^2 - 0.105)^2 = 44\times 10^{-3}\ \text{m}^2.$$

Velocity of air at end surface $= 0.1\times 8.25 = 0.825$ m/s.

Cooling co-efficient $\quad = 0.15/V_a = 0.182.$

Loss dissipated from end surfaces $= 44\times 10^{-3}/0.182 = 0.242$ W/°C.

$\therefore$ Total loss dissipated $= 2.38 + 1.87 + 0.242 \simeq 4.5$ W/°C.

$\therefore$ Temperature rise $\theta_m = 205/4.5 = 45.6$ °C.

This is within limits.

DESIGN SHEET

KW—2·2	PHASE—3	FREQUENCY—50 Hz
VOLTAGE—400 V	CONNECTION—DELTA	TYPE—CAGE

Rating

1.	Full load output		2·2 KW
2.	Line voltage		400 V
3.	Frequency	f	50 Hz
4.	Phases		3
5.	Efficiency	η	0·8
6.	Power factor	$\cos\phi$	0·825
7.	Number of poles	p	4
8.	Synchronous r.p.s.	n_s	25
9.	kVA input		3·33
10.	Full load line current		4·8 A

Loading

1.	Specific magnetic loading	B_{av}	0·44 Wb/m²
2.	Specific electric loading	ac	21000 A/m
3.	Output coefficient	C_o	97
4.	D^2L		1.375×10^{-3} m³

Main Dimensions

1.	Stator bore	D	105 mm
2.	Gross iron length	L	125 mm
3.	Ducts	n_d	Nil
4.	Gross iron length	L_g	125 mm
5.	Net iron length	L_i	112·5 mm
6.	Pole pitch	τ	82·9 mm

Stator

1.	Type of laminations		0.5 mm thick Lohys
2.	Type of winding		Single layer mush
3.	Connection		Delta
4.	Phase voltage	E_s	400 V
5.	Flux per pole	Φ_m	4.54×10^{-3} Wb
6.	Turns per phase	T_s	416
7.	Number of slots	S_s	24
8.	Slots per pole		6
9.	Slots per pole per phase	q_s	2
10.	Coil span	C_s	5 slots
11.	Distribution factor	K_d	0.966
12.	Pitch factor	K_p	0.9659
13.	Winding factor	K_{ws}	0.934
14.	Slot pitch	y_{ss}	13.75 mm
15.	Conductors per slot	Z_{ss}	104
16.	Conductor : bare diameter		0.95 mm
	insulated diameter		1.041 mm
	area		0.709 mm²
17.	Current density	δ_s	3.91 A/mm²
18.	Length of mean turn	L_{mts}	0.68 m
19.	Phase resistance at 75°C	r_s	8.37 Ω
20.	Copper loss at full load	$3I_s^2r_s$	193 W
21.	Depth of stator core	d_{cs}	17 mm
22.	Outer diameter of stator laminations	D_o	181 mm

Rotor

1.	Length of air gap	l_g	0.3 mm
2.	Diameter of rotor	D_r	104.4 mm
3.	Type of winding		Squirrel cage
4.	Number of slots	S_r	22
5.	Slots per pole per phase	q_r	1.833
6.	Conductors per slot	Z_{sr}	1
7.	Winding factor	K_{wr}	1
8.	Slot pitch	y_{sr}	14.9 mm
9.	Rotor bar current	I_b	244 A
10.	Rotor bar : cross section		7×6.5 mm
	area	a_b	44.6 mm²
	length	L_b	165 mm
	current density	δ_b	5.47 A/mm²
11.	Resistance of each bar	r_b	77.7×10^{-6} Ω
12.	Copper loss in bars	$S_rI_b^2r_b$	101 W
13.	End ring current	I_e	428 A

14.	End ring : cross section		10×8 mm^2
	area	a_e	80 mm^2
	mean diameter	D_e	75·8 mm
	current density	δ_e	5·35 A/mm^2
15.	Resistance of each ring	r_e	$62·8 \times 10^{-6}$ Ω
16.	Copper loss in end rings	$2I_e^2r_e$	23 W
17.	Total rotor copper loss		124 W
18.	Resistance of rotor (referred to stator)	r_r'	7·83 Ω
19.	Depth of rotor core	d_{cr}	17 mm

No load current

1.	Magnetizing mmf per pole		231 A
2.	Phase magnetizing current	I_m	1·0 A
3.	Magnetizing reactance	X_m	400 Ω
4.	Core loss		134 W
5.	Friction and windage loss		33 W
6.	No load loss		167 W
7.	Loss component	I_l	0·139 A
8.	No load current (phase)	I_0	1·0 A
9.	No load current (line)		1·73 A
10.	No load power factor	$\cos\phi_0$	0·139

Short circuit (blocked rotor) current

1.	Slot leakage reactance	x_s	11·8 Ω
2.	Overhang leakage reactance	x_o	4·7 Ω
3.	Zigzag leakage reactance	x_z	20·3 Ω
4.	Total leakage reactance	X_s	36·8 Ω
5.	Total resistance	R_s	16·2 Ω
6.	Short circuit impedance	Z_s	40·2 Ω
7.	Phase short circuit current	I_{sc}	9·95 A
8.	Line short circuit current		17·5 A
9.	Short circuit p.f.	$\cos\phi_{sc}$	0·043

Performance

1.	At full load :	
	Losses	484 W
	Output	2200 W
	Input	2684 W
	Efficiency	81·3 %
	Power factor	0·829
	Slip	5·3 %
2.	Maximum power output/rated output	1·67
3.	Maximum torque/rated torque	2·0

4. Starting torque/rated torque 1·0
5. Temperature rise θ_m 45·6°C.

Design Problem II. *Design a 11 kW, 3 phase, 440 V, 50 Hz, 1000 synchronous r.p.m. squirrel cage induction motor having a full load efficiency of 0·86 and a power factor of 0·86. The temperature rise is not to exceed 50°C.*

Solution.

Main Dimensions

Synchronous speed $n_s=1000/60=16·67$ r.p.s. Poles$=2\times50/16·67=6$.

Assume $B_{av}=0·45$ Wb/m², $ac=22{,}000$ A/m and $K_w=0·955$

KVA input $Q=11/(0·86\times0·86)=14·9$.

Output coefficient $C_o=11\times0·955\times0·45\times22{,}000\times10^{-3}=104$.

∴ Product $D^2L=14·9/(104·6\times16·67)=8·6\times10^{-3}$ m³.

For a good power factor, L/τ should be between 1 to 1·25. Taking $L/\tau=1$ as it gives an overall good electrical design.

$$\therefore \qquad \frac{L}{\pi D/6}=1 \qquad \text{or} \qquad L=0·523\ D$$

or $$0·523\ D^3=0·086\times10^{-3}$$

or $$D=0·254 \text{ m} \qquad \text{and} \qquad L=0·1335.$$

The dimensions used are

$$D=0·25 \text{ m} \qquad L=0·14 \text{ m} \qquad \text{and} \qquad \tau=0·131 \text{ m}.$$

The length of core is 140 mm and therefore one radial ventilating duct 10 mm in width is provided.

Net iron length $L_i=K_i\,(L-n_d\,W_d)=0·9\,(0·14-10\times10^{-3})=0·117$ m.

Laminations of Lohys steel 0·5 mm thick are used.

Stator Design

Winding :

The stator winding is delta connected in order that the machine may be started by a star delta starter.

∴ Stator voltage per phase $E_s=440$ V.

Flux per pole$=B_{av}\,\tau L=0·45\times0·31\times0·14=8·25\times10^{-3}$ Wb.

$$\text{Stator turns per phase } T_s=\frac{440}{4·44\times50\times8·25\times10^{-3}\times0·955}=250.$$

Taking slots per pole per phase for stator, $q_s=3$.

Stator slots $S_s=3\times3\times6=54$. Stator slot pitch $y_{ss}=\pi\times0·25\times10^3/54=14·5$ mm.

Total stator conductors$=6T_s=1500$. Conductors per slot $Z_{ss}=1500/54\approx28$.

Total conductors used$=28\times54=1512$ and stator turns per phase $T_s=1512/6=252$

The number of conductors used is very near to the calculated value and therefore the value of flux density need not be modified.

Mush winding in parallel sided semi-enclosed slots is used.

Coil span C_s=slots per pole$=54/6=9$.

This is an odd number and therefore this allows the use of full pitch coils. With full pitch coils, pitch factor $K_p=1$.

$$\text{Distribution factor } K_d=\frac{\sin \alpha/2}{q \sin \alpha/2q}=\frac{\sin 30°}{3 \sin 10°}=0·96.$$

∴ Stator winding factor $K_{ws}=1\times0·96=0·96$.

Conductor Size :

Stator current per phase $I_s = \dfrac{11 \times 1000}{3 \times 440 \times 0{\cdot}86 \times 0{\cdot}86} = 11{\cdot}25$ A.

Taking a current density of 4 A/mm^2, area of stator conductor $a_s = 11{\cdot}25/4 = 2{\cdot}81$ mm^2.

From Table 17·7, using 2 conductors of diameter 1·4 mm (bare) in parallel.

Diameter of insulated conductor $= 1{\cdot}505$ mm.

Area of conductor provided $a_s = 2 \times (\pi/4) \times 1{\cdot}4^2 = 3{\cdot}08$ mm^2.

Slot Dimensions :

Now each conductor consists of two wires of outside diameter 1·505 mm in parallel. There are 28 conductors per slot and therefore there are 56 wires in a slot.

They are arranged as shown in Fig. 10·48.

Slot width		**Slot depth**	
4 wires 4×1·505	=6·02 mm.	14 wires 14×1·505	=21·07 mm.
Slot lining 2×0·5	=1·00 mm.	Slot lining 3×0·5	=1·50 mm.
Slack etc.	=1·48 mm.	Wedge	=3·00 mm.
		Lip	=1·00 mm.
		Slack etc.	=2·43 mm.
Slot width	W_{ss}=8·50 mm.	Slot depth	d_{ss}=29·00 mm

Before proceeding further the flux density at a section having minimum width of tooth must be checked. The minimum tooth section is at AA (Fig. 10·48).

Slot pitch at $AA = \pi(250+8)/54 = 15$ mm.

Tooth width at AA $W_t = 15{\cdot}0 - 8{\cdot}5 = 6{\cdot}5$ mm.

Flux density in teeth at AA

$$= \frac{8{\cdot}25 \times 10^{-3}}{(54/6) \times 6{\cdot}5 \times 10^{-3} \times 0.117} = 1{\cdot}21 \text{ W/b m}^2.$$

This is within limits.

The dimensions of stator slot are :

Width $W_{ss} = 9{\cdot}5$ mm and depth $d_{ss} = 29{\cdot}0$ mm.

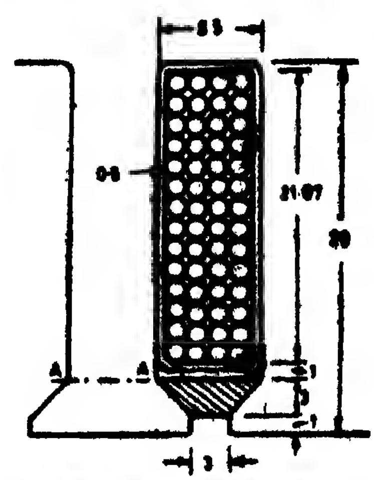


Fig. 10·48. Stator slot. All Dimensions in mm.

Length of mean turn

$$L_{mts} = 2L + 2{\cdot}3\ \tau + 0{\cdot}24 = 2 \times 0{\cdot}14 + 2{\cdot}3 \times 0{\cdot}131 + 0{\cdot}24 = 0{\cdot}82 \text{ m}.$$

Stator Core

Flux in the core $= 8{\cdot}25 \times 10^{-3}/2 = 4{\cdot}125 \times 10^{-3}$ Wb.

Assume a flux density of 1·2 Wb/m^2 for the core.

∴ Area of stator core $A_{cs} = 4{\cdot}25 \times 10^{-3}/1{\cdot}2 = 3{\cdot}54 \times 10^{-3}$ m^2.

Depth of stator core $d_{cs} = \dfrac{A_{cs}}{L_i} = \dfrac{3{\cdot}54 \times 10^{-3}}{0{\cdot}117} = 30{\cdot}3 \times 10^{-3}$ m $= 30{\cdot}3$ mm.

The depth of core used is $d_{cs}=30$ mm.

Outer diameter of stator laminations

$$D_o=D+2\,(d_{ss}+d_{cs})=250+2(29+30)=368 \text{ mm.}$$

Rotor Design

Air Gap

Length of air gap $l_g=0.2+2\sqrt{DL}=0.2+2\sqrt{0.25\times0.14}\simeq0.55$ mm

Diameter of rotor $D_r=D-2l_g=250-2\times0.55=248.9$ mm.

Rotor Slots

The number of rotor slots is chosen such that smooth starting and accelerating conditions are obtained. This is secured by adopting one slot less or one slot more, per pole pair, in the rotor than in the stator.

Taking the rotor slot a pair of poles more than the stator slots.

$\therefore$ Number of rotor slots $S_r=S_s+p/2=54+6/2=57$,

and rotor slot pitch $y_{sr}=\pi\times248.9/57=13.7$ mm.

Rotor Bars

Current in each rotor bar $I_b=\dfrac{2m_s K_{ws} T_s}{S_r}\,I_s\cos\phi$

$$=\frac{2\times3\times0.96\times252}{57}\times11.25\times0.86=250 \text{ A.}$$

Assuming a current density of 5 A/mm^2 in the bars.

Area of each bar $a_b=250/5=50$ mm^2.

A rotor bar 5 mm wide and 10 mm deep is used.

Area of bar used $a_b=10\times5=50$ mm^2.

The complete rotor slot is shown in Fig. 10·49.

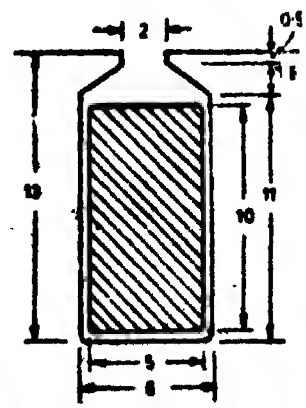


Fig. 10·49. Rotor slot. All Dimensions in mm.

The dimensions of rotor slot are :

width $W_{sr}=6$ mm, depth $d_{sr}=13$ mm.

Before proceeding further, the flux density at the root of the rotor teeth (where the teeth section is minimum) must be checked.

Slot pitch at the root of teeth $=\pi(248.9-2\times13)/57=12.3$ mm.

Slot width at the root of teeth $W_t=12.3-6.0=6.3$ mm.

Flux density at the root of teeth $=\dfrac{8.25\times10^{-3}}{(57/6)\times0.117\times6.3\times10^{-3}}$

$$=1.18 \text{ Wb/m}^2.$$

This is within limits and therefore the mmf required for rotor teeth would not be excessive.

The bars are skewed by one slot pitch. Extending the bars by about 20 mm beyond the core on each side and taking 10 mm as increase in the length because of skewing.

Length of each bar $L_b=140+2\times20+10=190$ mm $=0.19$ m.

Resistance of each bar $r_b=0.021\times0.19/50=80\times10^{-6}\ \Omega$.

Total copper loss in bars $=57\times250^2\times80\times10^{-6}=285$ W.

End Ring

End ring current $I_e = \frac{S_r I_b}{\pi p} = \frac{57 \times 250}{\pi \times 6} = 755$ A.

Taking a current density of 5 A/m², area of each end ring $= a_e = 755/5 = 151$ mm².

Using end ring of 15×10 mm section,

depth of ring $d_e = 15$ mm and thickness of ring $t_e = 10$ mm.

Area of ring provided $a_e = 15 \times 10 = 150$ mm².

The rings are brazed to bars as shown in Fig. 10·46.

Mean diameter of ring $D_e = 209$ mm $= 0·209$ m.

$\therefore$ Resistance of each end ring $r_e = 0·021 \times 0·209/150 = 92 \times 10^{-6}$.

Copper loss in two end rings $= 2 \times 755^2 \times 92 \times 10^{-6} = 105$ W.

Total rotor copper loss $= 285 + 105 = 390$ W.

We have $\frac{\text{rotor copper loss}}{\text{output}} = \frac{s}{1-s}$ or $\frac{390}{11000} = \frac{s}{1-s}$

or slip at full load $s = 3·4\%$.

Rotor Core

Depth of rotor core is taken equal to that of stator core. The area and the flux density in the rotor core are the same as for stator core.

$\therefore$ Depth of rotor core $d_{cr} = 30$ mm.

$\therefore$ Inner diameter of rotor lamination $= D_r - 2d_{sr} - 2d_{cr} = 163$ mm $= 0·163$ m.

The complete assembly drawing of the motor is shown in Fig. 10·50.

No Load Current

Magnetizing Current

The mmf required for various parts of magnetic circuit are calculated below.

(*i*) **Air gap :**

For stator, ratio slot opening/gap length $= 3/0·55 = 5·45$.

Corresponding to this ratio, the Carter's co-efficient is 0·63. (Fig. 4·9 page 124)

Gap contraction factor for stator slots $K_{gss} = 14·5/14·5 - 0·63 \times 3 = 1·15$.

For rotor, ratio slot opening/gap length $= 2/0·55 = 3·64$.

Corresponding to the ratio Carter's co-efficient $= 0·51$.

$\therefore$ Gap contraction factor for rotor slots $K_{gsr} = \frac{13·7}{13·7 - 0·51 \times 2} = 1·08$.

$\therefore$ Gap contraction factor for slots $K_{gs} = 1·15 \times 1·08 = 1·242$.

$$\text{Ratio } \frac{\text{duct width}}{\frac{1}{2}\text{ gap length}} = \frac{10}{\frac{1}{2} \times 0·55} = 36·4.$$

Corresponding to this ratio, the value of K_{cd} is found out from Fig. 4·9 by extrapolating the curve for open slots. We have $K_{cd} = 0·9$.

Gap contraction factor for ducts $K_{gd} = \frac{140}{140 - 1 \times 10 \times 0·9} = 1·07$.

$\therefore$ Total gap contraction factor $K_g = K_{gs} \times K_{gd} = 1·242 \times 1·07 = 1·33$.

Effective length of air gap $l_{gs} = 1·33 \times 0·55 = 0·732$ mm.

Area of air gap $A_g = \tau L = 0·131 \times 0·14 = 18·35 \times 10^{-3}$ m².

Flux density in air gap at 60° from interpolar axis $B_{g60}=1.36\times 45=0.612$ Wb/m².
Mmf for air gap $AT_g=800{,}000\times 0.612\times 1.33\times 0.55=357$ A.

(ii) Stator Teeth :

Width of stator teeth at 1/3 height from narrow end

$$=\frac{\pi(250+2\times 29/3)}{54}-8.5=7.15 \text{ mm.}$$

Area of stator teeth per pole at 1/3 height

$$A_{ts}=(54/6)\times 0.117\times 7.15\times 10^{-3}=7.53\times 10^{-3} \text{ m}^2.$$

Flux density $B_{ts1/3}=\dfrac{8.25\times 10^{-3}}{7.53\times 10^{-3}}=1.095$ Wb/m².

$\therefore$ $B_{ts60}=1.36\times 1.095$ Wb/m².

Corresponding to this flux density $at_{ts}=900$ A/m.

$\therefore$ Mmf required for stator teeth $AT_{ts}=900\times 29\times 10^{-3}=27$ A.

(iii) Stator Core :

Area of stator core $A_{cs}=d_{cs}\times L_i=30\times 10^{-3}\times 0.117=35.1\times 10^{-3}$ m².

Flux density in stator core $B_{cs}=\dfrac{4.125\times 10^{-3}}{3.51\times 10^{-3}}=1.17$ Wb/m².

Corresponding to this flux density $at_{cs}=280$ A.

Length of path through stator core $l_{cs}=\dfrac{\pi(250+2\times 29+30)}{3\times 6}\times 10^{-3}=59\times 10^{-3}$ m.

$\therefore$ Mmf required for stator core $AT_{cs}=280\times 0.059=17$ A.

(iv) Rotor Teeth :

Width of rotor teeth at ⅓ height from narrow end

$$W_{tr1/3}=\frac{\pi(248.9-4/3\times 13)}{57}-6=6.75 \text{ mm.}$$

Area of rotor teeth per pole $=(57/6)\times 0.117\times 6.75\times 10^{-3}=7.5\times 10^{-3}$ m².

Flux density in rotor teeth at ⅓ height $B_{tr1/3}=\dfrac{8.25\times 10^{-3}}{7.5\times 10^{-3}}=1.1$ Wb/m².

$\therefore$ $B_{tr60}=1.36\times 1.1=1.49$ Wb/m².

Corresponding to this flux density $at_{tr}=900$ A/m.

$\therefore$ Mmf required for rotor teeth $AT_{tr}=900\times 13\times 10^{-3}=12$ A.

(v) Rotor Core :

Area of rotor core $A_{cr}=3.51\times 10^{-3}$ m².

Flux density in rotor core $B_{cr}=1.175$ Wb/m² Mmf per meter $at_{cr}=280$ A.

Length of magnetic path in rotor core $=\dfrac{\pi(250-2\times 13-30)}{3\times 6}=34$ mm.

Mmf required for rotor core $AT_{cr}=280\times 3.4\times 10^{-2}=9$ A.

The results are tabulated below :

Part	*Area* m^2	*Length* mm	*B* Wb/m^2	B_{60} Wb/m^2	*at*	*AT*
Air gap	18.35×10^{-3}	0.732	0.45	0.612	48900	357
Stator teeth	7.53×10^{-3}	29	1.095	1.49	900	27
Stator core	3.51×10^{-3}	59	1.17	—	280	17
Rotor teeth	7.50×10^{-3}	13	1.1	1.49	900	12
Rotor core	3.51×10^{-3}	34	1.17	—	280	9
					Total	$AT_{60}=422$

Magnetizing current per phase $I_m=\dfrac{0.427\,p\,AT_{60}}{K_{ws}\,T_s}$

$$=\frac{0.427\times6\times422}{0.96\times252}=4.47\text{ A.}$$

Loss Component. Mean width of stator teeth $=7.75$ mm.

Weight of stator teeth $=54\times29\times10^{-3}\times7.75\times10^{-3}\times0.117\times7.6\times10^{3}=10.8$ kg.

Maximum flux density in teeth at $\frac{1}{3}$ height

$$=(\pi/2)\times1.095=1.72\ \text{Wb/m}^2.$$

Corresponding to above, the specific iron loss is 11 W/kg.

Iron loss in teeth $=10.8\times11=120$ W.

Mean periphery of stator core $=\pi(D_o-d_{cs})=1.06$ m.

Weight of stator core $=1.06\times30\times10^{-3}\times0.117\times7.6\times10^{3}=28.2$ kg.

Flux density in the core is 1.175 Wb/m^2 and the corresponding iron loss is 4.8 W/kg.

Iron loss in stator core is $28.2\times4.8=135$ W and the total iron loss in teeth and core is 255 W.

The actual iron loss is about 2 times the above loss.

$\therefore$ Total iron loss $=510$ W.

Friction and windage loss $=1\%$ of output $=110$ W.

Total no load losses $=510+110=620$ W.

Loss component of no load current per phase

$$I_l=\frac{620}{3\times440}=0.47\text{ A.}$$

$\therefore$ No load current $I_o=\sqrt{4.47^2+0.47^2}=4.5$ A.

No load power factor $\cos\phi_o=0.47/4.47=0.1045.$

Phase angle of no load current $\phi_o=84°$.

Short Circuit Current

Leakage reactance

Stator Slot Leakage :

From Eqn. 4.72, for a parallel side slot,

$$\lambda_{ss}=\mu_0\left[\frac{h_1}{3W_s}+\frac{h_2}{W_s}+\frac{2h_3}{W_s+W_0}+\frac{h_4}{W_0}\right]$$

$$=4\pi\times10^{-7}\left[\frac{21}{3\times8{\cdot}5}+\frac{3{\cdot}5^*}{8{\cdot}5}+\frac{2\times3}{8{\cdot}5+3}+\frac{1}{3}\right]=26{\cdot}3\times10^{-7}.$$

Rotor Slot Leakage :

$$\lambda_{sr}=4\pi\times10^{-7}\left[\frac{10}{3\times6}+\frac{0{\cdot}5}{6}+\frac{2\times1{\cdot}5}{6+2}+\frac{0{\cdot}5}{2}\right]=15{\cdot}9\times10^{-7}.$$

$$\lambda_{sr}'=15{\cdot}9\times10^{-7}\times\frac{(0{\cdot}96)^2\times54}{1\times57}=13{\cdot}9\times10^{-7}.$$

Total specific slot permeance

$$\lambda_s=26{\cdot}3\times10^{-7}+13{\cdot}9\times10^{-7}=40{\cdot}2\times10^{-7}.$$

Total slot leakage reactance $x_s=8\pi\times50\times252^2\times0{\cdot}14\times40{\cdot}2\times10^{-7}/(6\times3)=2{\cdot}5\ \Omega.$

Overhang Leakage :

Corresponding to full pitch coils, $K_s=1$ from Fig. 4·58 page 170

$$L_o\lambda_o=\frac{4\pi\times10^{-7}\times1\times0{\cdot}131^2}{\pi\times14{\cdot}5\times10^{-3}}=4{\cdot}74\times10^{-7}.$$

Overhang leakage reactance

$$x_o=8\pi\times50\times252^2\times4{\cdot}74\times10^{-7}/(6\times3)=2{\cdot}1\ \Omega.$$

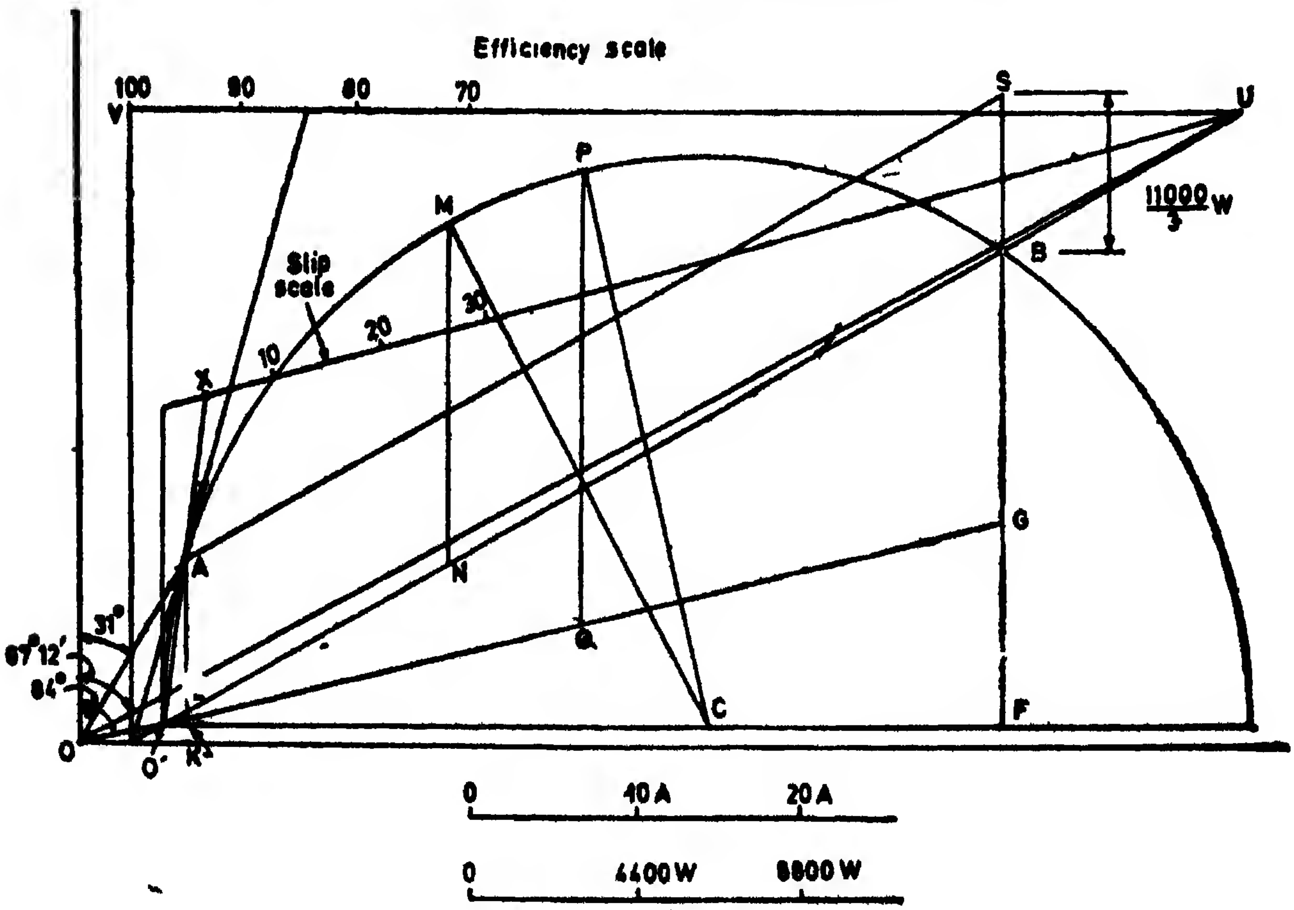


Fig 10·31. Circle diagram.

*The slack is lumped in λ_3.

Zigzag leakage :

Magnetizing reactance $X_m = 440/4{\cdot}47 = 98{\cdot}4\ \Omega$

Zigzag leakage reactance

$$x_z = \frac{5}{6}\frac{X_m}{m^2}\left(\frac{1}{q_s^2}+\frac{1}{q_r^2}\right)$$

$$= \frac{5}{6}\times\frac{98{\cdot}4}{3^2}\left(\frac{1}{3^2}+\frac{1}{3{\cdot}16^2}\right) = 1{\cdot}92\ \Omega$$

where $q_s = 54/(3\times6) = 3$ and $q_r = 57/(3\times6) = 3{\cdot}16$.

Total leakage reactance referred to stator

$\therefore$ $$X_s = 2{\cdot}5 + 2{\cdot}1 + 1{\cdot}92 = 6{\cdot}52\ \Omega.$$

Resistance

Stator resistance per phase

$$= \frac{0{\cdot}021\times252\times0{\cdot}82}{3{\cdot}08} = 1{\cdot}41\ \Omega.$$

Total stator copper loss $= 3\times11{\cdot}25^2\times1{\cdot}41 = 536$ W.

$\therefore$ Rotor resistance referred to stator

$$r_r' = 4\ m_s\ T_s^2\ K_{ws}^2\rho\left(\frac{L_b}{S_r\ a_b}+\frac{2}{\pi}\ \frac{D_e}{p^2\ a_e}\right)$$

$$= 4\times3\times252^2\times0{\cdot}96^2\times0{\cdot}021\left(\frac{0{\cdot}19}{57\times50}+\frac{2}{\pi}\times\frac{0{\cdot}209}{6^2\times150}\right)$$

$$= 1{\cdot}35\ \Omega.$$

Total resistance referred to stator $R_s = 1{\cdot}41 + 1{\cdot}35 = 2{\cdot}76\ \Omega$.

Impedance

Total impedance per phase $Z_s = \sqrt{6{\cdot}52^2+2{\cdot}76^2} = 7{\cdot}1\ \Omega$.

Short circuit current

$$I_{sc} = 440/7{\cdot}1 = 62\ \text{A}.$$

Short circuit power factor

$$\cos\phi_{sc} = 2{\cdot}76/7{\cdot}1 = 0{\cdot}389.$$

$\therefore$ Phase angle short circuit (blocked rotor) current $\phi_{sc} = 67°12'$.

Losses and Efficiency

Output at full load $= 11{,}000$ W.

Input at full load $= 11000 + 1546 = 12546$ W.

$\therefore$ Efficiency at full load

$$\eta = \frac{11000}{12540}\times100 = 87{\cdot}6 = 87{\cdot}1\ \text{percent}.$$

Circle Diagram

The circle diagram is shown in Fig. 10·51.

From this diagram,

Full load power factor $\cos\phi = \cos 31° = 0{\cdot}857$,

Full load stator phase current $I_s = OA = 41{\cdot}3$ A.

$$\frac{\text{Starting torque}}{\text{Full load torque}}=\frac{BQ}{AK}=1{\cdot}4$$

$$\frac{\text{Maximum torque}}{\text{Full load torque}}=\frac{PQ}{AK}=2{\cdot}7 \text{ and } \frac{\text{maximum output}}{\text{full load output}}=\frac{ML}{AL}=2{\cdot}3.$$

Stator Temperature Rise

The cooling co-efficients are taken with reference to Table 3 6.

Outer cylindrical surface of stator core

$$=\pi\times0{\cdot}368\times0{\cdot}14=0{\cdot}162\ \text{m}^2.$$

Cooling co-efficient for outer surface $=0{\cdot}033$.

Loss dissipated from outer surface

$$=0{\cdot}162/0{\cdot}033=4{\cdot}92\ \text{W/°C}.$$

Inner cylindrical surface $=\pi\times0{\cdot}25\times0{\cdot}14=0{\cdot}11\ \text{m}^2$.

Peripheral speed $=\pi\times0{\cdot}25\times16{\cdot}6=13$ m/s.

Cooling co-efficient for inner surface

$$=\frac{0{\cdot}033}{1+0{\cdot}1\,V_a}=\frac{0{\cdot}033}{1+0{\cdot}1\times13}=0{\cdot}0143.$$

Loss dissipated from inner stator surface

$$=0{\cdot}11/0{\cdot}0143=7{\cdot}68\ \text{W/°C}.$$

Area of end surface including ducts

$$=(\pi/4)\,(0{\cdot}368^2-0{\cdot}25^2)\times(2+1)=0{\cdot}225\ \text{m}^2.$$

Air velocity in ducts $=0{\cdot}1\times13=1{\cdot}3$ m/s.

Cooling co-efficient for ducts $=0{\cdot}15/1{\cdot}3=0{\cdot}115$.

Loss dissipated from end surfaces and ducts

$$=0{\cdot}225/0{\cdot}115=1{\cdot}96\ \text{W/°C}.$$

$\therefore$ Total loss dissipation

$$=4{\cdot}92+7{\cdot}68+1{\cdot}96=14{\cdot}56\ \text{W/°C}.$$

Loss dissipated from slot portion of stator

$$=510+536\times2\times0{\cdot}14/0{\cdot}12=693\ \text{W}.$$

$\therefore$ Temperature rise $\theta_m=693/14{\cdot}56=47$°C.

Hence, the temperature rise is within limits.

Design Problem III

Design a 110 kW, 3300 V, 3 phase, 50 Hz, 600 synchronous r.p.m. slip ring induction motor. The full load efficiency is 0 9 and the power factor is 0 86.

Solution.

Main Dimensions

Synchronous speed $n_s=600/10=10$ r.p.s. Number of poles $=2\times50/10=10$.

$$\text{kVA input } Q=\frac{110}{0{\cdot}9\times0{\cdot}86}=143.$$

Assuming $B_{av}=0{\cdot}48$ Wb/m², $ac=28{,}000$ A/m and $K_w=0{\cdot}955$,

Output co-efficient $C_0=11\times0{\cdot}95\times0{\cdot}48\times28000\times10^{-3}=141$.

$\therefore$ Product $$D^2L=\frac{143}{141\times10}=101{\cdot}5\times10^{-3}\ \text{m}^3.$$

For obtaining a good power factor and efficiency, ratio $L/\tau=1.25$

or $L=0.392\,D$, and $0.392\,D^3=101.5\times10^{-3}$

or $D=0.637$ and $L=0.25$ m.

The dimensions used are :

$$D=0.65 \text{ m}, \; L=0.25 \text{ m}, \; \tau=0.204 \text{ m}.$$

Peripheral speed $V_a=\pi\times0.65\times10=20.4$ m/s.

This is within limits.

Providing 2 radial ventilating ducts each 10 mm wide,

Net iron length$=0.9(0.25-2\times10\times10^{-3})=0.207$ m.

Stator Design

Winding

The stator is connected in star.

$\therefore$ Stator voltage per phase $E_s=3300/\sqrt{3}=1910$ V.

Flux per pole $\Phi_m=0.48\times0.204\times0.25=24.5\times10^{-3}$ Wb.

Turns per phase $T_s=\dfrac{1910}{4.44\times50\times24.5\times10^{-3}\times0.955}=368.$

Taking slots per pole per phase $q_s=3$.

$\therefore$ Total number of stator slots $S_s=3\times3\times10=90$.

Stator slot pitch $y_{ss}=\pi\times0.65\times10^3/90=22.7$ mm.

Total stator conductors$=6\times368=2208$.

$\therefore$ Conductors per slot $Z_{ss}=2208/90\simeq24$.

$\therefore$ Stator conductors provided$=90\times24=2160$.

Stator turns per phase $T_s=2160/6=360$.

Modified flux density in air gap

$$B_{av}=\frac{368}{360}\times0.48=0.491 \text{ Wb/m}^2.$$

Modified flux per pole

$$\Phi_m=0.491\times0.204\times0.25=25\times10^{-3} \text{ Wb}.$$

Using a single layer concentric winding with push through coils.

Pitch factor $K_p=1$.

Distribution factor $K_d=\dfrac{\sin 60°/2}{3\sin 60°/2\times3}=0.96.$

Stator winding factor $K_{ws}=1\times0.96=0.96$.

Conductor Size. Stator current per phase

$$I_s=\frac{110\times1000}{3\times1910\times0.9\times0.86}=25 \text{ A}.$$

Taking a current density of $\delta_s=4$ A/mm².

Area of stator conductor $a_s=25/4=6.25$ mm².

Taking a bare conductor 5 mm wide and 1.25 mm deep.

Area of conductor $a_s=5\times1.25=6.25$ mm².

Using 0.25 mm thick covering for the conductor.

Dimensions of insulated conductor are :

depth$=1.75$ mm and width$=5.5$ mm.

Slot Dimensions

Width :		
Insulated conductor	1×5.5	$=5.5$ mm.
Slot insulation	2×1.5	$=3.0$ mm.
Slack etc.		$=0.5$ mm.
Width of slot		$W_{ss}=9.0$ mm.
Depth :		
Insulated conductor	24×1.75	$=42.0$ mm.
Slot insulation	3×1.5	$=4.5$ mm.
Wedge		$=3.0$ mm.
Lip		$=1.0$ mm.
Slack etc.		$=1.5$ mm.
Depth of slot		$d_{ss}=52$ mm.

The slot is shown in Fig. 10·52.

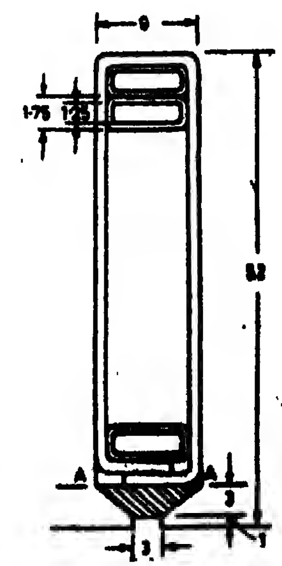


Fig. 10·52. Stator slot. All Dimensions in mm.

Check for flux density. The minimum tooth section is at AA.

Width of tooth at $AA=\pi(650+2\times4)/90-9=14$ mm.

$\therefore$ Flux density in teeth at AA

$$=\frac{25\times10^{-3}}{(90/10)\times0.207\times14\times10^{-3}}=0.96\ \text{Wb/m}^2$$

This is within limits

Length of mean turn of stator

$$L_{mts}=2\times0.25+2.3\times0.204+0.24=1.2\ \text{m}.$$

Resistance of stator

$$r_s=0.021\times360\times1.2/6.25=1.46\ \Omega.$$

Total stator copper loss $=3\times25^2\times1.46=2740$ W.

Stator core. Assume flux density in stator $B_{cs}=1.3$ Wb/m^2.

$\therefore$ Depth of stator core $d_{cs}=\dfrac{26\times10^{-3}}{2\times0.207\times1.3}=0.046$ m $=46$ mm.

Outer diameter of stator laminations

$$D_o=650+2\times52+2\times46=846\ \text{mm}.$$

Rotor Design

Air gap. Length of air gap

$$l_g=0.2+2\sqrt{0.65\times0.25}=1.0\ \text{mm}.$$

Diameter of rotor $\quad D_r=D-2l_g=650-2\times1=648$ mm.

Rotor winding. Assume slots per pole per phase $q_r=4$.

$\therefore$ Total rotor slots $S_r=3\times4\times10=120$

and rotor slot pitch $y_{sr}=\pi\times648/120=17$ mm.

Using a bar type winding with full pitch coils. There are 2 conductors per slot for the rotor.

$\therefore$ Pitch factor $K_p=1$. Distribution factor with $q_r=4$ is 0·958.

Rotor winding factor $K_{wr}=1\times0·958=0·958$.

Rotor turns per phase $T_r=4\times10\times2/2=40$.

Using star connection for the rotor,

Rotor voltage per phase at standstill $E_r=\dfrac{T_rK_{wr}}{T_sK_{ws}}E_s$

$$=\frac{40\times0·958}{360\times0·96}\times1910=212 \text{ V.}$$

Voltage between slip rings $=\sqrt{3}\times212=367$ V.

This is within the allowable limits.

Conductor size. Taking the rotor mmf to be 85 percent of stator mmf.

Rotor current per phase $I_r=\dfrac{0·85\ I_sT_sK_{ws}}{T_rK_{wr}}$

$$=\frac{0·85\times25\times360\times0·96}{40\times0·958}=191·5 \text{ A.}$$

Rotor current density $\delta_r=5$ A/mm².

$\therefore$ Area of rotor conductor $a_r=191·5/5=38·3$ mm².

The frequency in the rotor conductors is equal to the slip frequency which is very small and therefore the eddy current losses in rotor conductors are insignificant even though solid conductors are used.

The dimensions of the rotor conductor are :

depth$=13$ mm and width$=3$ mm.

Area of rotor conductor used $a_r=13\times3=39$ mm².

Slot dimensions. The rotor slot is shown in Fig. 10·53.

Slot Depth :		
Conductors bare	2×13	=26·0 mm
Conductor insulation	$4\times0·5$	=2·0 mm
Slot insulation	$3\times0·3$	=0·9 mm
Separator		=2·0 mm
Wedge		=3·0 mm
Lip		=1·0 mm
Slack etc.		=2·1 mm
Depth		$d_{sr}=37·0$ mm

Slot Width :		
Conductor bare	1×3	$=3.0$ mm
Conductor insulation	2×0.5	$=1.0$ mm
Slot insulation	2×0.3	$=0.6$ mm
Slack		$=0.4$ mm
	Width	$W_{sr}=5.0$ mm

Check for flux density

Width of rotor tooth at the root

$$=\pi(648-2\times37)/120-5=10 \text{ mm}.$$

Flux density at the root of rotor teeth

$$=\frac{25\times10^{-3}}{(120/10)\times0.207\times10\times10^{-3}}$$

$$=1.0 \text{ Wb/m}^2.$$

This is within limits.

Length of mean turn for rotor

$$L_{mtr}=2\times0.25+2.3+0.204+0.24=1.2 \text{ m}.$$

Rotor resistance per phase

$$r_r=40\times0.021\times1.2/39=0.026\Omega.$$

Rotor resistance referred to stator

$$r_r'=0.026\times(360/40)^2=2.1\ \Omega.$$

Total rotor copper loss $=3\times191.5^2\times0.026=3860$ W.

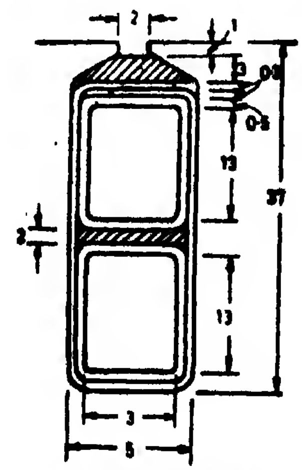


Fig. 10·53. Rotor slot. All dimensions in mm.

Rotor core. Depth of rotor core is taken equal to that of stator core.

$$d_{cr}=d_{cs}=46 \text{ mm}.$$

∴ Inner diameter rotor of laminations

$$=648-2\times37-2\times46=482 \text{ mm}.$$

No Load Current

Magnetizing Current. Let us find out the mmf required for various parts of the magnetic circuit.

(i) Air Gap

For stator slots :

Ratio (slot opening/gap length) $=3/1=3$ ∴ $K_{cs}=0.45$.

Gap contraction factor for stator slots

$$K_{gss}=\frac{7.22}{22.7-0.54\times3}=1.063.$$

For ducts :

Ratio (width of duct/½ gap length) $= \frac{10}{\frac{1}{2}\times 1} = 20$.

$K_{cd} = 0.84$ (the curve for open slots in Fig 4 9 page 124 is extrapolated to find this value).

$\therefore$ Gap contraction factor for ducts $K_{gd} = \frac{250}{250 - 2\times 10 \times 0.84} = 1.07$.

For rotor slots :

Ratio (slot opening/gap length) $= 2/1 = 2$ $\therefore$ $K_{cs} = 0.34$.

Gap contraction factor for rotor slots $K_{gsr} = \frac{17}{17 - 0.34 \times 3} = 1.92$.

$\therefore$ Total gap contraction factor $K_g = 1.063 \times 1.07 \times 1.062 = 1.21$.

Flux density in air gap at 60° from the pole axis

$$B_{g60} = 1.36\ B_{av} = 1.36 \times 0.491 = 0.668\ \text{Wb/m}^2.$$

Mmf required for air gap $AT_g = 800{,}000 \times 0.668 \times 1.21 \times 1 \times 10^{-3} = 646$ A.

(ii) Stator Teeth

Width of stator teeth at ⅓ height from narrow end

$$W_{ts1/3} = \frac{\pi(650 + 2\times 52/3)}{90} - 9 = 14.9 \text{ mm}.$$

Flux density at ⅓ height from narrow end

$$B_{ts1/3} = \frac{25 \times 10^{-3}}{(90/10) \times 0.207 \times 14.9 \times 10^{-3}} = 0.9 \text{ Wb/m}^2.$$

$B_{ts60} = 1.36 \times 0.9 = 1.21$ Wb/m² and $at_{ts} = 300$ A/m (See Fig. 4.2 page 120)

$\therefore$ Mmf required for stator teeth $AT_{ts} = 300 \times 0.052 = 16$ A.

(iii) Stator Core

Flux density in stator core $= 1.3$ Wb/m² and $at_{cs} = 400$ A/m.

Length of flux path in stator core $l_{cs} = \frac{\pi(650 + 2\times 52 + 46)}{8 \times 10} \times 10^{-3} = 84 \times 10^{-3}$ m.

$\therefore$ Mmf required for stator core $AT_{cs} = 400 \times 84 \times 10^{-3} = 34$ A.

(iv) Rotor Teeth

Width of rotor teeth at ⅓ height from narrow end

$$W_{tr1/3} = \frac{\pi(648 - 4 \times 37/3)}{120} - 5 = 10.65 \text{ mm}.$$

Flux density in rotor teeth at ⅓ height from narrow end

$$B_{tr1/3} = \frac{2.5 \times 10^{-3}}{120/10 \times 0.207 \times 0.65 \times 10^{-3}} = 0.954 \text{ Wb/m}^2.$$

$B_{tr60} = 1.36 \times 0.945 = 128$ Wb/m² and $at_{tr} = 380$ A/m.

$\therefore$ Mmf required for rotor teeth $AT_{tr} = 380 \times 39 \times 10^{-3} = 15$ A.

(v) Rotor Core

Flux density in rotor core $B_{cr} = 1.3$ Wb/m² and $at_{cr} = 400$ A/m.

Length of magnetic path in rotor core $l_{cr}=\frac{\pi(648-2\times 39-46)}{3\times 10}\times 10^{-3}=55\times 10^{-3}$

$\therefore$ Mmf required for rotor core $AT_{cr}=400\times 55\times 10^{-3}=2$ A.

Total magnetizing mmf $AT_{60}=646+16+34+15+22=733$ A.

Magnetizing current per phase $I_m=\frac{0{\cdot}427\times 10\times 733}{0{\cdot}96\times 360}=9{\cdot}05$ A.

Loss Component

Weight of stator teeth $=120$ kg.

Maximum flux density at $\frac{1}{3}$ height $=(\pi/2)\times 0{\cdot}9=1{\cdot}41$ Wb/m^2

Iron loss per kg $=7{\cdot}2$ W (Fig. 4·28 page 148).

Iron loss in stator teeth $=120\times 7{\cdot}2=864$ W.

Weight of stator core $=182$ kg.

Flux density in stator core $=1{\cdot}3$ Wb/m^2. Iron loss per kg $=5{\cdot}8$ W.

Iron loss in core $=182\times 5{\cdot}8=1055$ W.

Total iron loss $=1919$ W.

The actual iron loss in the built up laminations is assumed as 3760 W.

Friction and windage loss is taken as 1% of output.

Friction and windage loss $=1100$ W.

Stator copper loss at no load $=3\times 9{\cdot}05^2\times 1{\cdot}46=360$ W.

Total no load losses $=3760+1100+360=5220$ W.

Loss component of no load current $I_l=\frac{5220}{3\times 1910}=0{\cdot}91$ A.

No load current $I_0=\sqrt{9{\cdot}05^2+0{\cdot}91^2}=9{\cdot}1$ A.

No load power factor $\cos\phi_0=0{\cdot}91/9{\cdot}1=0{\cdot}1$.

Phase angle of no load current $\phi_0=84°\ 18'$.

Short Circuit Current

Leakage reactance

For stator slots (see Fig. 10·52)

$h_1=41{\cdot}5,\quad {}^*h_2=4{\cdot}75,\quad h_3=3,\quad h_4=1,\quad W_s=9,\quad W_0=3.$

From Eqn. 4·72,

$$\lambda_{ss}=4\pi\times 10^{-7}\left(\frac{41{\cdot}5}{3\times 9}+\frac{4{\cdot}75}{9}+\frac{3\times 2}{9+3}+\frac{1}{3}\right)=36{\cdot}4\times 10^{-7}.$$

For rotor slots (see Fig. 10·53)

$h_1=29,\quad {}^*h_2=3{\cdot}2,\quad h_3=3,\quad h_4=1,\quad W_s=5,\quad W_0=2,$

$$\lambda_{sr}=4\pi\times 10^{-7}\left[\frac{29{\cdot}0}{3\times 5}+\frac{3{\cdot}2}{5}+\frac{2\times 3}{5+2}+\frac{1}{2}\right]=49\times 10^{-7}.$$

$$\lambda'_{sr}=4\pi\times 10^{-7}\times\frac{0{\cdot}96^2\times 90}{0{\cdot}958^2\times 120}=37\times 10^{-7}.$$

$$\lambda_s=36{\cdot}4\times 10^{-7}+37\times 10^{-7}=73{\cdot}4\times 10^{-7}.$$

$\therefore$ Slot leakage reactance

$$x_s=8\pi f\,T_{ph}^2\,L\left(\frac{\lambda_s}{pq_s}\right)$$

*Includes slack.

$$=8\pi\times50\times360^2\times0{\cdot}25\times\frac{73{\cdot}4\times10^{-7}}{10\times3}=9{\cdot}54\ \Omega$$

For overhang :

$K_s=1$ for full pitch coils. (See Fig. 4·58)

$$L_o\lambda_o=4\pi\times10^{-7}\times\frac{1\times0{\cdot}204^2}{\pi\times22{\cdot}7\times10^{-3}}=7{\cdot}34\times10^{-7}. \text{ (See Eqn. 4·94)}$$

Overhang leakage reactance

$$x_o=8\pi\times50\times360^2\times7{\cdot}34\times10^{-7}/(10\times3)=3{\cdot}96\ \Omega.$$

Magnetizing reactance, $X_m=E_s/I_m=1910/9{\cdot}05=211\ \Omega.$

Zigzag leakage reactance, $x_z=\frac{5}{6}\ \frac{X_m}{m_s^2}\left(\frac{1}{q_s^2}+\frac{1}{q_r^2}\right)$ (See Eqn. 11·10)

$$=\frac{5}{6}\times\frac{211}{3^2}\left(\frac{1}{3^2}+\frac{1}{4^2}\right)=3{\cdot}4\ \Omega.$$

From Eqn. 10·47,

harmonic leakage reactance $x_h=X_m\ (K_{hs}+K_{hr})$

$$=211\ (21{\cdot}6+21{\cdot}6)-10^{-4}=0{\cdot}91\ \Omega.$$

The value of K_{hs} and K_{hr} is taken from Fig. 10·35

K_{hs} and K_{hr} are both equal to $21{\cdot}6\times10^{-4}$.

Total leakage reactance of machine referred to stator,

$$X_s=9{\cdot}54+3{\cdot}96+3{\cdot}4+0{\cdot}91=17{\cdot}81\ \Omega.$$

Resistance

Total resistance of machine referred to stator $R_s=3{\cdot}56\ \Omega$

as $r_s=1{\cdot}46\ \Omega$ and $r_r'=2{\cdot}1\ \Omega.$

Impedance

Total impedance $Z_s=\sqrt{17{\cdot}81^2+3{\cdot}56^2}=18{\cdot}15\ \Omega.$

∴ Short circuit (blocked rotor) current $I_{ss}=1910/18{\cdot}15=105$ A.

Short circuit power factor $\cos\phi_{ss}=3{\cdot}56/18{\cdot}15=0{\cdot}196.$

∴ $\phi_{ss}=78°\ 42'.$

Losses and Efficiency

Stator copper loss	=2740 W.
Rotor copper loss	=2860 W.
Iron losses	=3760 W.
Friction and windage losses	=1100 W.
Total losses at full load	=10460 W.
Output at full load	=110,000 W.
Input at full load	=120,460 W.

$$\text{Efficiency at full load}=\frac{110,000}{120,460}=91{\cdot}3\% \text{ less } \tfrac{1}{2}\%=90{\cdot}8\%.$$

$$\text{Slip at full load } s=\frac{2860}{110,000+2860+1100}\times100=2{\cdot}5\%.$$

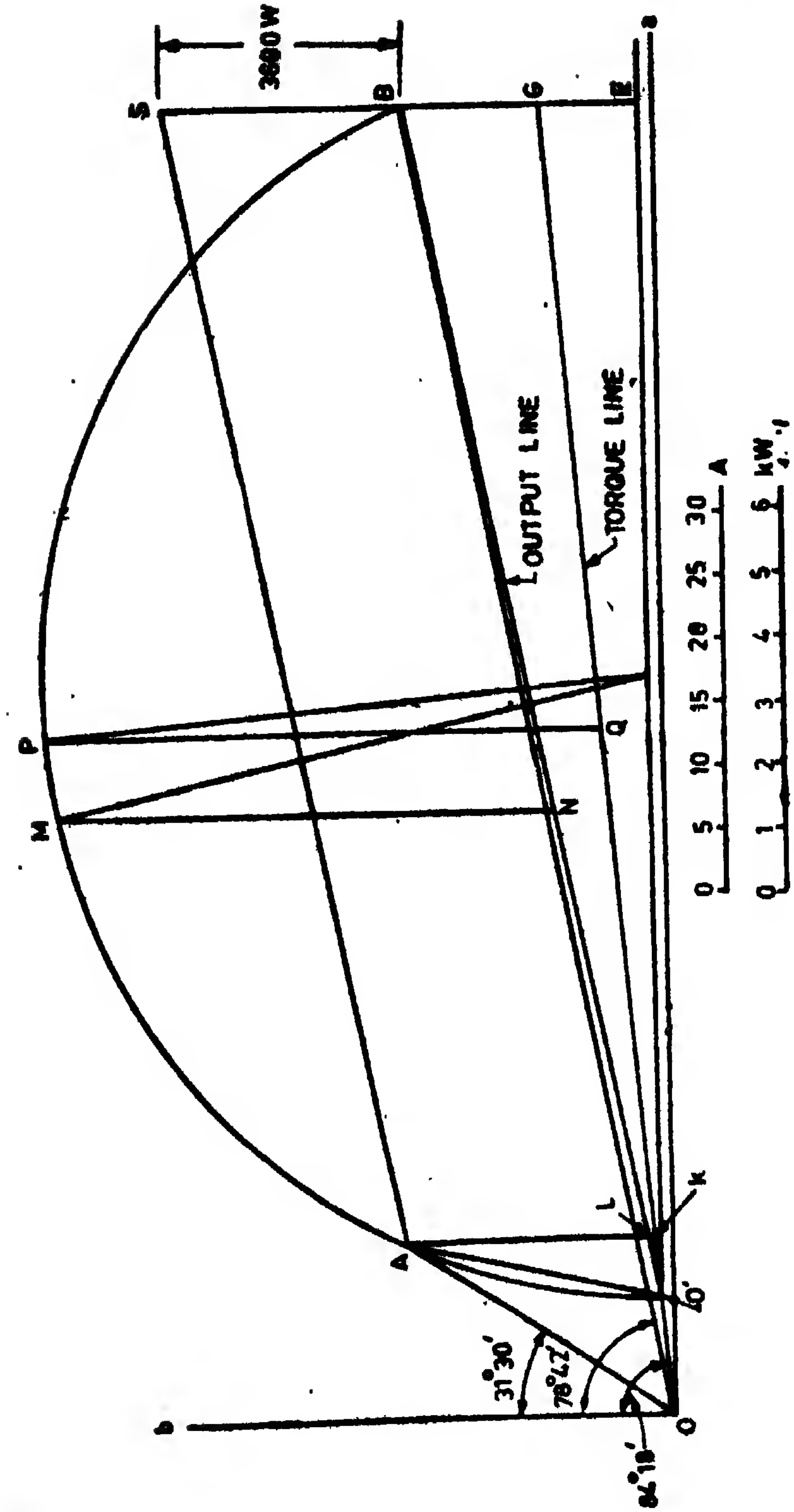


Fig. 10·54. Circle diagram.

Circle Diagram

The circle diagram is shown in Fig. 10·54.

From this diagram,

Stator current at full load $I_s = OA = 25$ A.

Rotor current (referred to stator at full load) $I_r'=O'A=20·5$ A.

Full load power factor $\cos\phi=\cos 31°30'=0·853$.

$$\frac{\text{Starting torque}}{\text{Full load torque}}=\frac{BG}{AK}=0·58.$$

$$\frac{\text{maximum torque}}{\text{full load torque}}=\frac{PQ}{AK}=2·25,\quad \frac{\text{maximum output}}{\text{full load output}}=\frac{MN}{AL}=2·1.$$

UNSOLVED PROBLEMS

1. Determine the approximate diameter and length of the stator core, the number of stator slots and the number of conductors for a 11 kW, 400 V, 3 phase, 4 pole, 1425 r.p.m. delta connected induction motor. Adopt a specific magnetic loading of 0·45 Wb/m² and a specific electric loading of 23,000 A/m. Assume full load efficiency and power factor as 0·85 and 0·88 respectively. The ratio of core length to pole pitch is 1. The stator employs a double layer winding.
[Ans. $D=0·19$ m ; $L=0·15$ m ; slots$=36$; conductors$=1080$]

2. Find the values of diameter and length of stator core of a 7·5 kW, 220 V, 50 Hz, 4 pole, 3 phase induction motor for best power factor. Given : specific magnetic loading$=0·4$ Wb/m² ; specific electric loading$=22000$ A/m ; efficiency$=0·86$; and power factor$=0·87$.

Also find the main dimensions if the ratio of core length to pole pitch is unity.
[Ans. $D=0·18$ m ; $L=0·12$ m ; $D=0·172$ m ; $L=0·136$ m]

3. A 415 V, 3 phase, 50 Hz, 6 pole delta connected induction motor has a specific magnetic loading of 0·5 Wb/m² and a specific electric loading of 24000 A/m. The stator core diameter and length are 0·275 m and 0·15 m respectively. Find the output of the machine if the full load efficiency and power factor are 0·88 and 0·89 respectively.

Determine the number of stator slots, conductors per slot and the length of air gap.
[Ans. 18·75 kW ; slots 54 ; gap$=0·6$ mm]

4. Determine the diameter of stator bore and core length of a 70 h.p., 415 V, 3-phase, 50 Hz star connected, 6 pole induction motor for which the specific electric and magnetic loadings are 32000 A/m and 0·51 Wb/m² respectively. Take the efficiency as 90 per cent and power factor as 0·91. Assume pole pitch equal to core length.

Estimate the number of stator conductors required for a winding in which the conductors are connected in two parallel paths. Choose a suitable number of conductors per slot so that the slot loading does not exceed 750 ampere conductors. [Ans. $D=0·35$ m ; $L=0·1825$ m ; $Z=756$]

5. A 5 h.p., 440 V, 3 phase, 4 pole motor with 375 turns per phase in the stator has the following design data :

Rotor slots$=30$; rotor bar size$=8·5\times6$ mm² ; length of each bar$=0·125$ m ; end ring size$=10\times15$ mm² ; mean diameter of end ring$=0·125$ m.

The bars and end rings are of copper for which the resistivity is 0·021 Ω/m and mm² at the working temperature. Calculate the rotor resistance referred to stator winding. [Ans. 3·55 Ω per phase]

6. Calculate the equivalent resistance of rotor per phase with respect to stator, the current in each bar and end ring and the total rotor copper loss for a 415 V, 50 Hz, 4 pole, 3 phase induction motor having the following data :

Stator :

Slots$=48$; conductors in each slot$=35$; current in each conductor$=10$ A.

Rotor :

Slots$=57$; length of each bar$=0·12$ m, area of each bar$=9·5\times5·5$ mm² ; mean diameter of end ring $=0·2$ m ; area of each end ring$=175$ mm². Resistivity of copper is 0·02 Ω/m and mm².

Full load power factor is 0·85. [Ans. 1·49 Ω per phase ; 241·5 A ; 1095 A ; 323 W]

7. A 30 kW, 400 V, 50 Hz, 6 pole star connected slip ring induction motor has the following design data :

Stator core diameter$=0·4$ m ; efficiency$=0·9$; power factor$=0·8$; flux per pole$=12·4\times10^{-3}$ Wb. Determine (*a*) number of stator slots (*b*) number of rotor slots (*c*) number of rotor conductors (*d*) rotor conductor area and (*e*) rotor slot dimensions.

The rotor voltage at the slip rings at stand-still is to be about 200 V. Make number of rotor slots 3 slots per pole pair less than the number of stator slots. The current density in rotor conductors is 4 A/mm², rotor slot space factor is 0·4 and the ratio of depth of slot to width of slot is about 6. The winding factors may be assumed as 0·96. [Ans. (*a*) 54 (*b*) 45 (*c*) 270 (*d*) 24 mm² (*e*) 45×8 mm²]

8. Calculate the magnetizing current of a 415 V, 4 pole, 3 phase, 50 Hz induction motor having the following data :

Stator slots=36 ; conductors per stator slot=30 ; stator bore=0·13 m ; stator core length=0·13 m ; effective gap length=1 mm. The winding is full pitch and the phase spread is 60°. Assume that iron has infinite permeability. [Ans. 4·64 A]

9. A 150 h.p., 3 phase, 2000 V, 50 Hz, 10 pole star connected induction motor has 3 slots per pole per phase on the stator. The flux per pole is 25×10^{-3} Wb. The air gap area per pole is 48×10^{-3} m² and the effective gap length is 1 mm. If the mmf for iron parts is to be 25 per cent of that required for single gap, calculate the magnetising current for the motor. [Ans. 18·1 A]

10. Calculate the slot leakage permeance per metre of core length for an induction motor having slot of dimensions shown in Fig. 10·55. [Ans. 104·3 H/m]

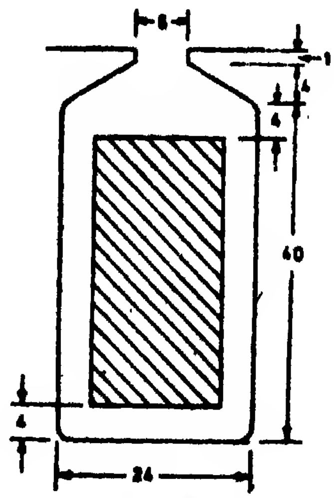


Fig. 10·55. Dimensions of slot of Problem No 10.

11. A 6 pole, 7·5 h.p. motor has its maximum power factor of 0·81 at full load when its efficiency is 85 per cent. Estimate the maximum power factor, the full load and the full load efficiency (*a*) if the air gap length is reduced by 25 percent (*b*) if with the original air gap length the motor is wound with 4 poles.
[Ans. (*a*) 0·85, 7·9 h.p., 0·86. (*b*) 0·865, 12·75 h.p., 0·90]

12. A 6 pole, 10 h.p., 60 Hz motor has its maximum power factor of 0·83 at full load, when its efficiency is 85 percent.

Estimate its maximum power factor, and full load output and efficiency if it is rewound for 40 Hz. (*a*) with 6 poles (*b*) with 4 poles. [Ans. (*a*) 0·83, 6·4 h.p., 0·82, (*b*) 0·88, 10·7 h.p. 0·88]

13. An induction motor frame has a large number of stator slots and a squirrel cage rotor winding. The machine can be wound for various numbers of poles. The maximum flux density, the maximum current density, the supply frequency and the supply voltage are constant.

(*a*) Show that the magnetizing volt-ampere required for the machine are independent of number of poles.

(*b*) Show that the rated torque output available from the machine is inversely proportional to the number of poles.

(*c*) Show that the power conversion capability of machine is inversely proportional to the number of poles.

(*d*) If the power factor at rated load of a 4 pole machine is 0·9 estimate the power factor at rated load if the machine is rewound for 8 poles. [Ans. About 0·78]

14. The following data refer to a 75 kW, 50 Hz, 8 pole, 500 V, slip ring induction motor with 3 phase star connected stator winding :

Turns per phase : stator 64; rotor 35. Resistance per phase : stator, 0·062Ω ; rotor, 0·019Ω. Reactance per phase : stator, 0·21 ; rotor 0·019Ω. Magnetising current, 36 A per phase. Iron loss, 1500 W. Friction and windage loss, 750 W. Draw the circle diagram and deduce therefrom the line current, efficiency, power factor and slip at (*a*) full load and (*b*) half load conditions. Find also (*c*) the maximum output and the pull out torque.

[Ans. (*a*) 104 A, 0·927, 0·89, 0·022 ; (*b*) 60 A, ·92, 0·77, 0·011 ; (*c*) 210 kW, 3100 Nm]

15. Calculate the slip of a 75 kW, 500 V, 8 pole, 3 phase, star connected cage induction motor from the following data :

Stator winding single layer, 4 slots per pole per phase, 4 conductors per slot. Rotor winding : 97 slots, bars 11 mm diameter × 200 mm long, end rings 15×15 mm² cross section, 485 mm mean diameter. Full load efficiency and power factor may be taken as 0·92 and 0·8 respectively. Mechanical losses are 750 W. Take resistivity of copper as 485 Ω/m and mm². [Ans. 2·05 percent]

11

Single Phase Induction Motors

GENERAL INFORMATION AND CONSTRUCTIONAL DETAILS

11·1. Introduction. These motors are in very wide use in industry, especially in the fractional kilowatt range. They are extensively used for electric drive for low power constant speed apparatus such as machine tools, domestic apparatus, and agricultural machinery in circumstances where a three phase supply is not readily available. There is a large demand for single phase induction motors in sizes ranging from a fraction of a kilowatt power up to about 3·7 k.W. Though these machines are useful for small outputs, they are not used for large powers as they suffer from many disadvantages and are never used in cases where three phase machines can be adopted. Chief among the disadvantages are (*a*) their output is only 50 per cent of the three phase motor, for a given frame size and temperature rise ; (*b*) they have lower power factor and (*c*) lower efficiency ; and (*d*) these motors have no inherent starting torque, and some special devices have to be employed to make them self-starting. Although such devices are entirely satisfactory they complicate the construction somewhat and make the motors more expensive than three phase motors of the same output.

11·2. Types of motors. As a single phase induction motor has no inherent starting torque and therefore special means must be used to make it self-starting. The methods in wide use are :

(*i*) Split phase starting.

(*ii*) Shaded pole starting.

(*iii*) Repulsion motor starting.

11·2·1. Split phase starting. Another winding, known as the starting winding or auxiliary winding, is wound on the stator in addition to the running or the main winding The two windings are in space quadrature. The running winding is supplied with current displaced in time from the current in the running winding by as nearly 90° as possible. The requisite phase displacement between the currents in the running and starting windings is obtained by connecting a suitable impedance in series with one of them. If this impedance is a resistor, a "resistor phase split motor" is obtained while if it is a capacitor a "capacitor split phase motor" is obtained. With these split phase motors, the starting winding is cut out of the circuit, usually by a centrifugal switch, after the motor has picked up about 75 per cent of full load speed. In some motors, the capacitor is permanently left in the circuit. Such motors are known as "capacitor motors".

11·2·2. Shaded pole starting. A part of the pole is shaded by a short circuited copper ring. This results in a phase displacement of 20° to 30° between the fluxes in the shaded and the unshaded portions of the pole. The efficiency of such an arrangement is low and therefore this method is suitable only for low outputs usually below 60 watt.

11 2 3. Repulsion-motor starting. This arrangement is used when a high starting torque is desired. The rotor carries a commutator winding in place of an ordinary cage winding and is started as a repulsion motor. In the "repulsion start motor" the motor starts as a repulsion motor with a high starting torque and at a predetermined speed, a centrifugal device short circuits all the commutator segments making the winding equivalent to a cage winding and the motor continues to run as an induction motor.

In the "repulsion induction motor" the rotor carries a commutator winding in addition to a cage winding. At starting the commutator winding predominates and therefore a good starting torque is obtained. When running, the cage winding gives constant speed induction motor characteristics.

Table 11·1 gives the starting characteristics of different types of motors.

Table 11·1, Starting Characteristics

Type	*Range of output watt*	*Starting current (times full load current)*	*Starting Torque (times full load torque)*
Split Phase :			
Capacitor	90 to 750	4 to 6	2 to 3·5
Resistor	7·5 to 370	5 to 7	0·75 to 2·0
Shaded pole	0·37 to 90	1 to 1·5	0·2 to 0·3
Capacitor	90 to 3700	2 to 6	0·25 to .·5
Repulsion start	90 to 3700	2 to 3	2 to 4
Repulsion Induction	370 to 3700	3 to 4	2 to 4

Note This chapter is devoted to the design of split phase motors and therefore the construction and design features of resistance split phase and capacitance split phase motors only will be considered.

11·3. Construction. The construction of single phase split phase induction motors is similar to that of small three phase induction motors. There are many makes of split phase motors in the market and each differs from the others in details of construction. All of them, however, consist essentially of a stator with running and starting windings, a squirrel cage rotor, an automatic switch to disconnect the starting winding when the motor has picked up speed and a body and end shield to which these parts, and also the terminal box, b·arings, etc. are fitted.

11 3·1. Stator. The stator is made up of a block of laminations mounted in a cast iron or die cast aluminium alloy frame. The stator has tapered slots with parallel sided teeth. The slots house the starting and running windings.

11 3·2 Stator windings. Single phase induction motors are generally wound with concentric coils.

Single phase motors have two distinct windings Each is a single layer winding, with th main winding being in the bottom of the slots. In 3-phase motors equal slot

space is devoted to all phases but in single phase motors considerably more slot space is given to main winding and less to auxiliary winding. The most popular kinds of single phase windings are the concentric, progressive and skein.

(*i*) *Concentric windings*. Concentric windings are so named because all the coils for a single pole have a common centre and different pitches for each individual coil *i.e.*, the individual coils in the pole group are concentric coils of varying sizes. Fig 11·3 shows this type of winding.

(*ii*) *Progressive windings*. Progressive windings are a form of single layer diamond coil winding. Fig. 11·1 shows a simple form of this winding.

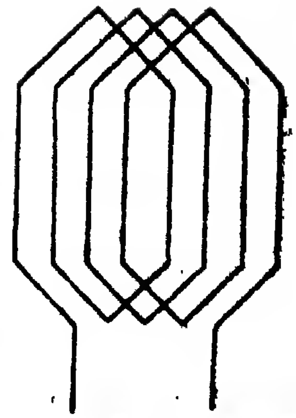


Fig. 11·1. Progressive Winding.

(*iii*) *Skein windings*. Skein winding is used for windings employing small amounts of relatively small size wire. In fractional horse power single phase motors skein winding is used mostly for auxiliary windings. The name skein is derived from the fact that the wire for a number of coils is originally wound in one large coil resembling a skein of yarn in appearance. The entire skein is inserted and passed back and forth to form coils right in the stator. A simple form of this winding is illustrated in Fig. 11·2 where two steps involved in inserting the winding are shown. First the whole skein is inserted in slots 4 and 9, and the skein pulled snugly against the front side of laminations. The open end of skein extends to the rear as shown in (*a*). Second, the skein is twisted as in (*b*), pulled back through the bore with the sides being inserted in slots 3 and 10. This brings the leads on the front side as desired.

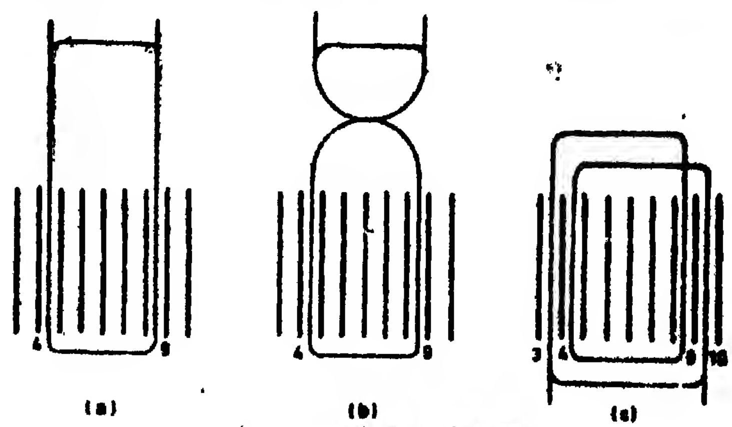


Fig. 11·2. Skein Winding.

Of the three types of windings described, the concentric winding is the most flexible and most widely used for single phase induction motors It allows the greatest choice of distributions. It permits the designer to control the space harmonics in the mmf waves and thus prevents the resultant ill effects. It may be wound using form coils or it can be wound by machine. Concentric type is universal form of winding used for main winding for all single phase motors.

The next most popular form is the skein winding. This has the following advantages :

(*a*) Coils are generally cheaper to wind and insert.

Where skein winding is applicable and where hand-winding is used, skein winding is probably the least expensive method available.

(*b*) It permits some freedom of choice of distribution.

11·3·3. Rotor. The rotor consists of a block of slotted laminations mounted on a shaft. The slots form a series of tunnels when the rotor is assembled. The tunnels are filled with aluminium, poured in the molten state. The bars, end rings and fan blades form one homogeneous casting. In some motors, copper bars and copper end rings are used, the former being brazed into the latter. The rotor slots are skewed so that a quieter operation is obtained.

11·3 4. Starting Switches. These switches are required to cut the auxiliary winding out of circuit when the motor has attained about 75 percent of synchronous speed. Starting switches are centrifugally operated whenever possible. They may be magnetically operated for such applications as hermatically sealed refrigerators where switch contacts inside the refrigerant are not permissible. For most applications, the centrifugal switch is preferable over the magnetic switch because it is not effected by the line voltage, is more reliable and is less expensive

11·3·5. Electrolytic Capacitor. Modern capacitor start motors employ a.c. electrolytic capacitors. An a.c. electrolytic capacitor is formed by winding two sheets of etched aluminium foil, separated by two layers of insulating paper, into a cylindrical shape. The unit is impregnated with an electrolyte, usually ethylene glycolol or a derivative. An anodic film is later produced on each foil by electro-chemical means.

The voltage rating of the capacitor is not necessarily the same as that of the motor, it may be less, the same, or more depending upon how it is used.

Motor starting capacitors are rated by manufacturers on the basis of twenty 3-second periods per hour or an equivalent duty cycle. Actually, the capacitor will be able to withstand severe conditions. Power factor of electrolytic capacitors is generally of the order of 6 to 8 percent

DESIGN

11 4. Output Equation. The output Equation (Eqn. 8·18 page 457) for a.c. machines is

$$\text{kVA input } Q = C_0 D^2 L n_s$$

$$\therefore \quad D^2 L = \frac{Q}{C_0 n_s}$$

where Output co efficient $C_0 = 11\, K_w \cdot B_{av}\, ac \times 10^{-3}$. ...(11·2)

If rating of the machine is given in horse power, we have :

$$Q = \frac{\text{h.p} \times 0{\cdot}746}{\eta \cos\phi} \qquad \text{...(11·3)}$$

and

$$D^2 L = \frac{\text{h.p.} \times 0{\cdot}746}{\eta \cos\phi\, C_0\, n_s} \qquad \text{...(11·4)}$$

The range of full load efficiency and power factor is :

Efficiency—It ranges from 50 percent for a 75 watt to 70 percent for a 750 watt motor.

Power factor—It ranges from 0·55 for a 75 watt to 0 65 for a 750 watt motor.

The smaller values apply for lower rating machines.

Table 11 2 gives the typical values of full load efficiency and power factor for 4 pole single phase induction motors.

Table 11·2. Efficiency and power factor

Output watt	*Efficiency*	*Power factor*
37	0·38	0·46
90	0·48	0·51
180	0·57	0·56
370	0·65	0·62
750	·69	0·65

The horsepower, speed, power factor and efficiency of a machine are specified. In order to find D^2L we must evaluate the output co-efficient. The value of output co-efficient depends upon the choice of specific electric and magnetic loadings.

11 4 1. Choice of specific loadings. The usual values for *average flux density in the air gap* are : $B_{av}=0.35$ to 0.55 Wb/m^2.

The usual values for specific electric loading are :

$$ac=5000 \text{ to } 15000 \text{ A/m}.$$

The values of product ($C_0\,\eta\cos\phi$) may be directly read off from Table 11·3.

Table 11·3. Values of product $C_0\,\eta\cos\phi$

watt/r.p.s.	3·6	7·2	12	18
$C_0\,\eta\cos\phi$ (metric units)	9·5	12	15·5	18

11·5. Main dimensions. The product D^2L is obtained from Eqn. 11·4.

This product is now to be split up into its components D and L.

Core length is generally made equal to the pole pitch but the exact dimensions are governed by manufacturing conditions. The diameter is selected with reference to Tables 10·4 and 10·5 (pages 631 and 632) which give the frame sizes available in the market.

11 6. Relative size of single phase and three phase motors

Single phase motor.

Output co-efficient $C_0=11\,K_w\,B_{av}\,ac.\,10^{-3}$

In a single phase motor, the winding is spread over 180° if all slots are considered to be wound with equal number of conductors.

Distribution factor for a single phase winding

$$K_d=\frac{\sin\sigma/2}{\sigma/2}=\frac{\sin\pi/2}{\pi/2}=\frac{2}{\pi}.$$

Taking full pitch coils, $K_p=1$.

$\therefore$ Winding factor for an infinitely distributed full pitch single phase winding is

$$K_w=2/\pi.$$

$\therefore$ Output co-efficient $C_0=11\times\frac{2}{\pi}\,B_{av}\,ac\times10^{-3}$.

Hence product $$D^2L=\frac{Q}{C_0 n_s}=\frac{Q}{\left(11\times\frac{2}{\pi}\,B_{av}\,ac\times10^{-3}\right)n_s}.$$

Three phase motor.

Output co-efficient $\quad C_0=11\,K_w\,B_{av}\,ac\times10^{-3}$.

In a 3-phase machine, the phase spread is 60° :

Distribution factor for a 3-phase winding

$$K_d=\frac{\sin\pi/6}{\pi/6}=\frac{3}{\pi}.$$

Taking full pitch coils, $K_p=1$.

$\therefore$ Winding factor for an infinitely distributed full pitch three phase winding is

$$K_w = 3/\pi.$$

$\therefore$ Output co-efficient $C_0 = 11 \times \frac{3}{\pi} B_{av}\, ac \times 10^{-3}$.

Hence product $$D^2L = \frac{Q}{\left(11 \times \frac{3}{\pi} B_{av}\, ac \times 10^{-3}\right) n_s}$$

Comparison. Comparing a single phase machine and a three phase machine with similar ratings and designed for the same specific loadings,

$$\frac{D^2L \text{ for three phase motor}}{D^2L \text{ for single phase motor}} = \frac{2/\pi}{3/\pi} = \frac{2}{3}.$$

Thus it is observed that product D^2L for single phase motors is 50 percent greater than that for three phase motors. This is under the condition that all the slots are wound. In other words, the output for a single phase machine is two-thirds of that for a three phase machine for the same D^2L if all the slots are wound in the former machine.

In practice, only two-thirds of the slots are utilized for the running winding and the remaining slots are utilized for starting winding. In this case the output with single phase is only 50 percent of the output with 3 phases for the same D^2L (same frame size).

11.7. Design of stator

11.7.1. Running winding (main winding).

The stator windings of single phase induction motors are concentric type. There are usually 3 or more coils per pole each having same or different number of turns.

The arrangement for winding is governed largely by the necessity of minimizing harmonic fluxes which may otherwise give rise to noise and uneven accelerating torque. Such harmonics are produced owing to non-sinusoidal shape of mmf wave and from the presence of slots. Mmf wave harmonics can be reduced by utilising about 70 percent of the total slots for the running winding as this arrangement gives minimum low order harmonics. The remaining slots (about 30 percent of total slots) are used for accomodating the starting winding.

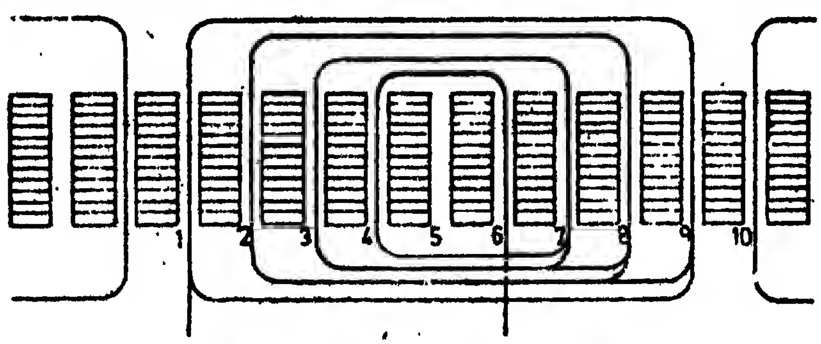


Fig. 11.3. Concentric winding.

In a small single phase motor may be desirable to reduce the harmonics still further by grading the winding i.e., by having different number of conductors in each slot thereby giving an mmf wave which nearly approaches a sine wave.

The winding arrangement shown in Fig. 11·3 is for a stator with 9 slots per pole. There are four coils per pole :

coil (1—9)	—	it spans 8 slots
coil (2—8)	—	it spans 6 slots
coil (3—7)	—	it spans 4 slots
coil (4—6)	—	it spans 2 slots.

The coils can be re-arranged as shown in Fig. 11·4.

There are again 4 coils per pole :

coil (1 —10)	—	it spans 9 slots
coil (2—9)	—	it spans 7 slots
coil (3—8)	—	it spans 5 slots
coil (4—7)	—	it spans 3 slots.

With this arrangement the number of turns in the outside coil (1—10) must be one half of the conductors in slot 1, the other half conductors belong to the outside coil of the adjacent pole.

Suppose, for instance, that a motor with 9 slots per pole is to be wound as shown in Fig. 11·3. It is desired to have a sinusodial distribution.

The turns required in each coil are found as follows :

coil (4—6)—sin ½ coil span=sin (2/9)×90°=0·342

coil (3—7)—sin ½ coil span=sin (4/9)×90°=0·643

coil (2—8)—sin ½ coil span=sin (6/2)×90°=0·866

coil (1—9)—sin ½ coil span=sin (8/9)×90°=0·985

=2·836

Percent turns per pole in coil (4—6)=(0342/2·836))×100=12·10
Percent turns per pole in coil (3—7)=(0·643/2·836)×100=22·70
Percent turns per pole in coil (2—8)=(0·866/2·836)×100=30·60
Percent turns per pole in coil (1—9)=(0·985/2·836)×100=34·60

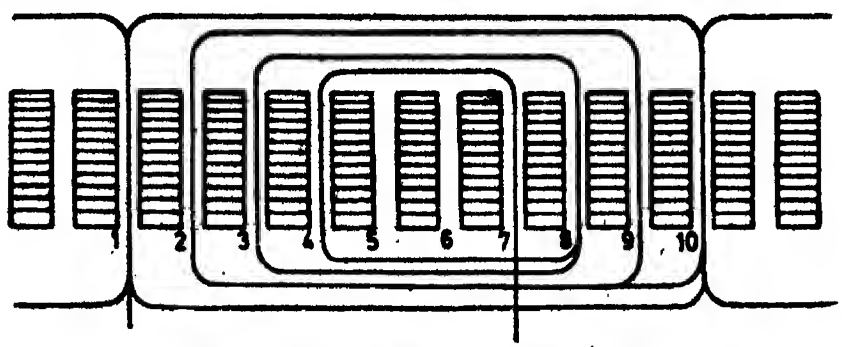


Fig. 11·4. Concentric winding.

The same procedure can be used for arrangement of Fig. 11·4 except in determining the base for percent turn calculations, one half the sine of 90° must be used for coil (1—10).

coil (4—7)—sin ½ coil span=sin (3/9)×90° =0·500

coil (3—8)—sin ½ coil span=sin (5/9)×90° =0·766

coil (2—9)—sin ½ coil span=sin (7/9)×90° =0·940

coil (1—10)—½ sin ½ coil span=½ sin (9/9)×90°=0·500

=2·7

Percent turns per pole in coil (4−7)=(0·5/2·7)×100=18·5

Percent turns per pole in coil (3−8)=(0·766/2·7)×100=28·3

Percent turns per pole in coil (2−9)=(0·94/2·7)×100=34·7

Percent turns per pole in coil(1−10)=(0·5/2·7)×100=18·5

Winding Distribution Factor

The winding distribution factor for concentric type single layer windings is a weighted mean pitch factor and is calculated by multiplying the pitch factor of each coil per pole group by the turns in the coil and dividing the sum of these produced by the total number of turns. For the winding shown in Fig. 11·3.

Pitch factor for coil (4−6)=sin (2/9)×90°=0·342

Pitch factor for coil (3−7)=sin (4/9)×90°=0·643

Pitch factor for coil (2−8)=sin (6/9)×90°=0·866

Pitch factor for coil (1−9)=sin (8/9)×90°=0·985

Suppose turns in coil (4−6), coil (3−7), coil (2−8) and coil (1−9) are respectively T_{4-6}, T_{3-7}, T_{2-8} and T_{1-9}.

∴ Winding factor

$$K_w=\frac{0\cdot342\,T_{4-6}+0\cdot643\,T_{3-7}+0\cdot866\,T_{2-8}+0\cdot985\,T_{1-9}}{T_{4-6}+T_{3-7}+T_{2-8}+T_{1-9}}$$

$$=\frac{0\cdot342\times12\cdot1+0\cdot643\times22\cdot7+0\cdot866\times30\cdot6+0\cdot985\times34\cdot6}{100}=0\cdot8.$$

For usual winding distributions, the value of K_w will be between 0·75 to 0·85.

11·7·2. Number of turns in running winding

The number of turns in the running winding can be calculated as below :

Stator induced voltage $E=4\cdot44\,f\,\Phi_m\,T_m\,K_{wm}$

where T_m=number of turns in the running winding,

K_{wm}=winding factor for the running winding.

∴ Number of turns in the running winding

$$T_m=\frac{E}{4\cdot44\,f\,\Phi_m\,K_{wm}} \qquad ...(11\cdot5)$$

where Φ_m=flux/pole=$B_{av}\times(\pi\,DL/p)$

The value of stator induced voltage E is approximately equal to 95 percent of supply voltage V. The winding factor for the running winding can be assumed between 0·75 to 0·85.

The number of turns in series per pole for the main (running) winding

$$T_{pm}=T_m/p \qquad ...(11\cdot6)$$

11·7·3. Running winding conductors

Current carried by each running winding conductor

$$I=\frac{\text{h.p.}\times746}{V\,\eta\cos\phi} \qquad ...(11\cdot7)$$

Area of running winding conductor $a_m=I_m/\delta_m$ ...(11·8)

where δ_m is the current density for running winding conductors in A/mm².

For open type motors split phase, capacitor and repulsion start, the current density can usually be 3 to 4 A/mm². For enclosed motors much lower values should be used.

Enamelled conductors are used. The exact size of conductor should be taken with reference to Table 17·7.

11·7·4. Number of stator slots. A large number of slots reduces the leakage reactance by reducing the slot and zigzag leakage. This means that we can take out more output from the given frame, and can have better overload capacity and somewhat better efficiency and power factor. A large number of stator slots also reduces troubles due to field harmonics, such as cusps and cogging. It also tends to reduce the intensity of magnetic noise increasing the pitch of the noise components.

However, as the number of slots is increased, the space factor for slots becomes poorer, so that there is always a practical upper limit to the number of stator slots, which in general, will depend upon the size of the lamination.

A small number of slots reduces the cost of winding and gives a better space factor. But very few slots would result in shallow and very wide slots with excessive amount of copper in each.

The number of stator slots per pole is usually between 9 to 12. Actually we should select such a lamination from Tables 10·4 and 10.5 so that not only we get the proper values of D and L but a suitable number of stator slots. For single phase induction motors, the only restriction would be that the number of stator slots should be divisible by the number of poles so that a balanced, regular winding can be used. The laminations given in Tables 10·4 and 10·5 are meant for both single and polyphase machines and therefore our choice regarding the number of slots is restricted.

11·7·5. Size of stator slot. All the stator slots do not have the same number of conductors, and some contain both running winding and starting winding conductors. The starting winding conductor has a small cross sectional area, and its effect upon the size of the slot is small. Generally the running winding coil with the largest number of turns will determine the size of the slot. For semi-enclosed slots, the insulation between core and coils is placed in the slot as slot lining. The slot liner is usually 0·3 to 0·4 mm thick.

The ratio of the insulated conductor area to the slot area should never exceed 0·5. Actually this ratio should not exceed 0·35 if the winding process is to be made easy. The design of single phase motors, in this chapter, will be carried out by using a given frame size. We will have no hold on the slot size and we have only to see whether the slot size used in the lamination is more than the required.

Suppose Z_s is the total number of conductors per slot and d_1 mm is the diameter of insulated conductor.

$\therefore$ Area required for insulated conductors $= Z_s \times (\pi/4)\, d_1^2$.

Minimum slot area required $= (1/0·5)\, Z_s \times (\pi/4)\, d_1^2$ ...(11·9)

The slot area provided in the stamping is calculated by multiplying the mean width by the depth of the slot.

The average slot width $W_{s\,(av)} = \dfrac{\pi(D+d_{ss})}{S_s} - W_{ts}$

where

d_{ss} = depth of stator slot, W_{ts} = width of stator tooth

and S_s = number of stator slots. ...(11·10)

Area of each slot $= W_{s(av)} \times d_{ss}$

The area given by Eqn. 11·10 should be more than the area given by Eqn. 11·9.

11·7·6. Stator teeth We should check the flux density in the stator teeth in order to see that it is not excessive. The stator tooth density B_{ts} can generally be from 1·4 to 1·7 Wb/m². If the low losses and noise are important or if the motor is totally enclosed, then lower densities should be used. For general purpose machines a flux density of 1·45 Wb/m² is taken while for high torque machines it may go up to 1·8 Wb/m².

A stacking factor of 0·95 is taken.

$\therefore$ Net iron length $L_i = 0{\cdot}95\ L$.

Flux density in the stator teeth $B_{ts} = \dfrac{\Phi_m}{(S_s/p)\times L_i \times W_{ts}}$ ...(11·11)

11·7·7. Stator core. The flux density in the stator core should not exceed 1·5 Wb/m². Generally it lies between 0·9 to 1·4 Wb/m².

Flux in stator core $\Phi_c = \Phi_m/2$ ...(11·12)

$\therefore$ Flux density in stator core $B_{cs} = \dfrac{\Phi_m}{2\times L_i \times d_{cs}}$ ...(11·13)

where d_{cs} = depth of stator core.

The flux density should also be checked in rotor teeth and rotor core. The values of flux density in rotor should not exceed the permissible values which are the same as corresponding values for stator. A little higher values are permissible in rotor.

11·7·8. Length of mean turn. The length of mean turn for each of the coils per pole of a concentric winding

$$L_{mt} = \frac{8{\cdot}4\,(D+d_{ss})}{S_s} \times \text{slots spanned} + 2\ L \qquad ...(11{\cdot}14)$$

11·8. Air gap length. The considerations for taking a particular air gap length are same as for three phase induction motors. The following empirical relation gives satisfactory values.

$$\text{Gap length } l_g = \frac{0{\cdot}007\times\text{rotor diameter}}{\sqrt{p}} \qquad ...(11{\cdot}15)$$

11·9. Design of Rotor

11·9·1. Number of rotor slots. The number of rotor slots is so chosen that there is no noise producing combinations. It has been found that a quiet-running motor will result if there are no harmonic fields, of stator and rotor slots, with numbers of poles differing by less than 4.

$$\text{Harmonic poles due to slots} = 2\left(S \pm \frac{p}{2}\right) \qquad ...(11{\cdot}16)$$

where S is the number of slots.

The number of stator slots is usually fixed by winding arrangement, number of poles etc. The number of rotor slots must, therefore, be adjusted to meet the above requirement. Many empirical rules have been suggested for choosing the best ratio of stator to rotor slots in order to give freedom from harmonic fields.

For motors with more than 2 poles, quiet operation can generally be expected, when the number of rotor slots is divisible by the number of pairs of poles and when the number of rotor slots differs from the number of stator slots by more than the number of poles.

Another useful rule states that the number of rotor slots should be equal to the number of stator slots plus twice the number of poles. There are many other combinations which might prove satisfactory, although the number of rotor slots is rarely more than 1·5 times the number of stator slots.

The number of rotor slots must also be selected from the point of view of magnetic locking and cusps, as explained on pages 615 to 619.

11·9·2. Area of rotor bars. The cage rotor winding may be either of copper bars and end rings or of cast aluminium. Also technical advantages lie with copper but manufacture is cheaper with cast aluminium. Also with cast rotors, the joints between bars and end rings are eliminated.

Total stator copper section for main winding $A_m = 2\ T_m\ a_m$ mm² ...(11·17)

A high rotor resistance is desirable from the standpoint of starting torque and current but leads to high slip and poor efficiency. The total rotor copper section is generally 0·5 to 0·8 of total stator copper section.

Total cross-section of rotor bars $A_r = S_r\ a_b$ ...(11·18)

where a_b = area of each bar, mm².

Ratio $A_r/A_m = 0·5$ to 0·8 for copper ...(11·19)

Since aluminium, generally used for this purpose, has a resistivity approximately twice that of copper, total bar area must be two times the area required for copper.

∴ $A_r/A_m = 1$ to 1·6 for aluminium. ...(11·20)

11·9·3. Area of end ring

End ring current $I_e = \dfrac{S_r\ I_b}{\pi \times p}$ (Eqn. 10·17 page 625) ...(11·21)

Area of each end ring $a_e = I_e/\delta_e$ ...(11·22)

Area of each bar $a_b = I_b/\delta_b$ ...(11·23)

or
$$a_e = \frac{I_e}{\delta_e} = \frac{S_r\ I_b}{\pi \times p\ \delta_e} = \frac{S_r\ a_b\ \delta_b}{\pi \times p\ \delta_e}$$

$$= \frac{0·32\ A_r}{p} \cdot \frac{\delta_b}{\delta_e} \qquad ...(11·24)$$

where

I_b = current in each bar $= \delta_b\ a_b$

δ_b, δ_e = current density in bars and end rings respectively.

If we take $\delta_b = \delta_e$,

area of each end ring $a_e = \dfrac{0·32\ A_r}{p}$ ...(11·25)

11·9·4. Rotor resistance. Choice of rotor resistance is very important factor in the design of single phase motors. It should be as low as possible to keep down rotor copper loss and to maintain high efficiency, high full load speed and minimum temperature rise. In the case of single phase motors there is an added advantage that the rotor resistance effects the maximum torque for a given flux and therefore a higher value of pull out torque is obtained with large value of rotor resistance.

The minimum value of rotor resistance is dictated by starting torque requirements, the higher the requirement the higher the value of rotor resistance, to be used. A significant parameter is the ratio r_{rm}'/X_{lm}. Experience has shown that for normal commercial fractional kilowatt machines, the value of r_{rm}'/X_{lm} is approximately :

For split phase motors ...0·45 to 0·55

For capacitor start motors ...0·45 to 0·8

where r_{rm}' = resistance of rotor referred to running winding

X_{lm} = leakage reactance of stator main winding plus rotor in terms of running winding.

The value of r_{ra}'/X_m is usually lower for the larger horse power ratings. These values can be used as a guide in designing the ring or in choosing the best ring from a number of available designs.

11·9·5. Rotor teeth. The rotor tooth and core densities might, from the standpoint of losses, be considerably higher than those for the stator because the rotor frequency at normal operating speed is very low. But if very high densities are taken a large magnetising current would result. This will give rise to poor power factor and therefore, these densities are taken only slightly higher than the corresponding stator densities.

$$\text{Flux density in rotor teeth } B_{tr}=\frac{\Phi_m}{S_r/p\times L_i\times W_{tr}} \qquad ...(11\cdot26)$$

where W_{tr}=width of rotor teeth.

11·9·6. Rotor core. Flux density in rotor core

$$B_{cr}=\frac{\Phi_m}{2\times L_i\times d_{cr}} \qquad ...(11\cdot27)$$

where d_{cr}=depth of rotor core.

OPERATING CHARACTERISTICS

11·10. Mmf for air gap. The flux produced by stator mmf passes through the following parts :

(*i*) air gap (*ii*) stator teeth (*iii*) stator core (*iv*) rotor teeth and (*v*) rotor core.

Due to saturation in teeth, the flux density distribution curve is flat topped as explained on page 177. Therefore, the calculation of mmf should be based upon the value of flux density at 60° from the interpolar axis as far as gap and teeth are concerned.

The value of flux density at 60° from interpolar axis is 1·57 times B_{av} for single phase machines.

$$\therefore \qquad B_{60}=1\cdot57\ B_{av} \qquad ...(11\cdot28)$$

The gap contraction factor can be calculated as explained in Art. 4·4·1 page 122.

$$\text{Mmf required for air gap } AT_{g60}=800{,}000\ B_{g60}\ K_g\ l_g \qquad ...(11\cdot29)$$

11·11. Saturation factor. The saturation factor, which is the ratio of the total mmf required for the magnetic circuit to the mmf required for air gap, is difficult to predetermine from the d.c. magnetization curves. The shape of magnetizing current wave is not sinusoidal because of the non-linear nature of the magnetization curve of the core material. By use of a.c. magnetization curves the magnetization characteristics can be fairly accurately perdetermined.

For single phase induction motors the saturation factor will usually lie between the limits 1·1 to 1·35.

$$\text{Satruation factor } F_s=\frac{\text{total mmf required for magnetic circuit}}{\text{mmf required for air gap}} \qquad ...(11\cdot30)$$

$$=1\cdot1 \text{ to } 1\cdot35 \text{ for single phase induction motors.}$$

When the flux densities in teeth and core are low, the lower values should be used, and when high higher, values should be used.

11·12. Iron loss. The iron loss in stator teeth and core is found by calculating their flux densities and weights. The loss per kg can be calculated using the curves given in Chapter 4.

The total iron loss for induction motors is 1·5 to 2·5 times the sum of stator tooth and core loss due to fundamental frequency flux. The multiplying factor should be obtained

from tests of motors of similar design. When test data is not available, a value of 1·75 to 2·2 may be used.

11·13. Friction and windage loss. The bearing friction and windage loss will depend upon the type of bearing to be used, whether ball bearing or sleeve bearing. For sleeve bearings and a speed 1500 rpm, it is usually from 4·0 to 8·0% of the watt output. The high values apply to small motors below 180 W.

Table 12·4 may be used for finding friction and windage loss.

Table 11·4. Friction and windage loss.

Stator outer diameter mm.	*Journal diameter mm.*	*Loss W*			
		750 r.p.m.	*1000 r.p.m.*	*1500 r.p.m.*	*3000 r.p.m.*
Sleeve Bearings					
120	9·0	0·6	0·8	1·2	2·4
135	12·5	0·4	1·9	2·3	5·6
150	15·0	2·8	3·7	5·5	11·0
190	18·0	4·8	6·3	9·5	19.0
Ball Bearings					
190	—	3	7	11	72
230	—	7	14	29	235
250	—	22	50	62	300

11·14. Parameters

11·14·1. Running winding resistance. Resistance of running winding

$$r_{rm} = 0{\cdot}021 \frac{T_m L_{mtm}}{a_m} \text{ at } 75^\circ\text{C (hot)} \qquad \ldots(11{\cdot}32)$$

$$= 0{\cdot}017 \frac{T_m L_{mtm}}{a_m} \text{ at } 20^\circ\text{C (cold)} \qquad \ldots(11{\cdot}33)$$

where L_{mtm} = length of mean turn of running winding, m

a_m = area of running winding conductor, mm².

11·14·2. Rotor resistance. Eqn. 10.35 page 639 gives the value of rotor resistance referred to stator for a m_s phase stator. We can calculate the val : of rotor resistance referred to stator by putting $m_s = 2$ for single phase machines.

From 10, rotor resistance referred to running winding

$$r_{rm}' = 4 m_s T_s^2 K_{ws}^2 \rho \left[\frac{L_b}{S_r a_b} + \frac{2}{\pi} \frac{D_e}{p^2 a_e} K_{rise} \right]$$

$$= 8\, T_m K_{wm}^2\, \rho \left[\frac{L_b}{S_r a_b} + \frac{2}{\pi} \frac{D_e}{p^2 a_e} K_{rise} \right] \qquad \ldots(11{\cdot}33)$$

The value of K_{rise} is obtained from Fig. 1..33 page 699.

11·14 3. Leakage reactance calculations of single phase motors.

1. *Slot leakage reactance.* The windings of induction motors are concentric type with different number of conductors in each slot. Suppose the number of conductors in different slot are Z_1, Z_2, Z_3,............etc. Therefore, there are $2p$ groups of conductors of Z_1, Z_2, Z_3.........etc. conductors per slot.

Hence the total stator slot leakage reactance

$$x_{ss}=2\pi f T^2 L\lambda$$
$$=2\pi f[Z_1^2+Z_2^2+Z_3^2+\ldots\ldots]L\lambda_{ss}\times 2p$$
$$=4\pi f[Z_1^2+Z_2^2+Z_3^2+\ldots\ldots]p\,L\lambda_{ss} \qquad \ldots(11\cdot34)$$
$$=4\pi f(ZK_{wm})^2\frac{L}{S_s}\lambda_{ss}C_s \qquad \ldots(11\cdot35)$$
$$=16\pi f(T_m K_{wm})^2\frac{L}{S_s}\lambda_{ss}C_s \qquad \ldots(11\cdot36)$$

where C_s=an arbitrary correction factor to be determined that permits the use of $(T_m K_{wm})$ in the equation.

The value of C_s is obtained by equating Eqn. 11·34 with Eqn. 11·35.

We have $$C_s=\frac{(Z_1^2+Z_2^2+Z_3^2+\ldots\ldots)\,p\,S_s}{(Z K_{wm})^2} \qquad \ldots(11\cdot37)$$

The expression for C_s can be put in a more convenient form by noting that

$$Z=(Z_1+Z_2+Z_3+\ldots\ldots)2p$$

and substituting this value of Z in Eqn. 11·37,

$$C_s=\frac{(Z_1^2+Z_2^2+Z_3^2+\ldots\ldots)}{(Z_1+Z_2+Z_3+\ldots\ldots)^2}\times\frac{1}{K_{wm}^2}\times\frac{S_s}{4p} \qquad \ldots(11\cdot38)$$

2. *Rotor slot leakage reactance.* It is seen from Eqn. 11·36 that the slot leakage reactance is proportional to the specific slot permeance divided by the number of slots. Thus the rotor slot leakage reactance, in terms of the stator, may be obtained from this equation by substituting rotor specific permeance λ_{sr} for stator slot specific permeance $\lambda_{ss}C_s$ and S_r for S_s.

Thus rotor slot leakage reactance in terms of stator

$$x_{sr}'=16\pi f(T_m K_{wm})^2\frac{L}{S_r}\lambda_{sr} \qquad \ldots(11\cdot39)$$

∴ Total slot leakage reactance in terms of stator $x_s=x_{ss}+x_{sr}'$

$$=16\pi f(T_m K_{wm})^2\frac{L}{S_s}\left(C_s\lambda_{ss}+\frac{S_s}{S_r}\lambda_{sr}\right) \qquad \ldots(11\cdot40)$$

3. *Zigzag leakage reactance.* Similarly we can write the equation for zigzag leakage reactance

$$x_z=16\pi f(T_m K_{wm})^2\frac{L}{S_s}\lambda_z \qquad \ldots(11\cdot41)$$

λ_z=specific permeance for zigzag leakage which can be taken from Eqn. 4·92 page 170.

4. *Overhang leakage reactance.* A simple empirical formula for the calculation of overhang leakage reactance is :

$$x_o=16\pi f(T_m K_{wm})^2\frac{L_{pt}}{6\cdot4\,S_s\,p}\,[\pi(D+d_{ss})\times\text{average coil span in slots}]. \qquad \ldots(11\cdot42)$$

5. *Skew leakage reactance.* Skew leakage reactance is given by

$$x_{sk} = X_m \frac{\theta^2_s}{12} K_l \quad \text{...(11·43)}$$

wher θ_s = rotor bar skew angle expressed in radian ;

$$= \frac{\pi}{S_r/p} \times \text{(rotor slot pitches through which bars are skewed)} \quad \text{...(11·44)}$$

K_l = stator slot leakage factor = 0·95 ;

and X_m = magnetizing reactance.

6. *Magnetizing reactance.* Magnetizing reactance

$$X_m = 16\pi f (T_m K_{wm})^2 \frac{\mu_0 L \tau}{10\, l_g K_g p F_s} \quad \text{...(11·45)}$$

The symbols used for the above relations are :

T_m = number of turns in the main winding ;

K_{wm} = winding factor for the main winding ;

L = length of stator core, m ; D = diameter of stator core, m ;

τ = pole pitch, m ; l_g = length of air gap, m ;

d_{ss} = depth of stator slot, m ; p = number of poles ;

S_s = number of stator slots ; S_r = number of rotor slots ;

λ_{ss} = specific stator slot permeance ; λ_{sr} = specific rotor slot permeance ;

λ_z = specific permeance for zigzag leakage ;

K_g = gap contraction factor ;

$$F_s = \text{saturation factor} = \frac{\text{total mmf required for magnetic circuit}}{\text{mmf required for air gap}}.$$

7. *Total leakage reactance.*

Total leakage reactance of the stator running (main) winding plus rotor in terms of running winding

$$X_{lm} = x_s + x_z + x_o + x_{sk} \quad \text{...(11·46)}$$

Reactance of the stator running winding with secondary open *i.e.* open circuit reactance.

$$X_{om} = X_m + X_{lm}/2 \quad \text{...(11·47)}$$

The leakage flux factors $K_r = \dfrac{X_{lm} - X_{om}}{X_{om}}$ ...(11·48)

and $$K_l = \sqrt{(X_{om} - X_{lm})/X_{om}} \quad \text{...(11·49)}$$

11·15. Running performance.

The running characteristics are calculated by two methods.

(*i*) Equivalent circuit for double revolving field theory.

(*ii*) Analytical method prepared by Veinott.

11·15·1. Equivalent circuit. The equivalent circuit for the double revolving field theory for any slip s is shown in Fig. 11·5.

In the equivalent circuit, the resistance representing the iron loss is omitted. The iron loss is lumped with the friction and windage loss. The net output is obtained by subtracting iron plus friction and windage loss from the gross output.

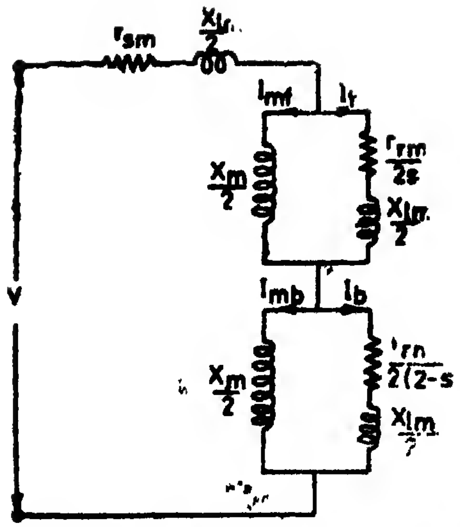


Fig. 11·5. Equivalent circuit for doule revolving field theory.

For any slip s, the value of I_f and I_b can be calculated.

$$\text{Forward torque} = I_f^2 \frac{r_{rm}'}{2s} \text{ synchronous watt} \qquad \ldots(11{\cdot}50)$$

$$\text{Backward torque} = I_b^2 \frac{r_{rm}'}{2(2-s)} \text{ synchronous watt} \qquad \ldots(11{\cdot}51)$$

$$\text{Gross motor torque} = \frac{r_{rm}'}{2}\left(\frac{I_f^2}{s} - \frac{I_b^2}{[2-s]}\right) \text{ synchronous watt} \qquad \ldots(11{\cdot}52)$$

Net motor torque = gross motor torque − iron, friction and windage loss ...(11·53)

$$\text{Net output} = (\text{net motor torque})(1-s) \text{ watt}$$

$$= \frac{(\text{net motor torque})(1-s)}{746} \text{ h.p.} \qquad \ldots(11{\cdot}54)$$

11·15·2. Veinott's Method

The calculations are done in the following steps.

Performance Calculations

H.P. = Voltage = Speed =

Motor constants

Line voltage V	=	$F_1 = (2 - K_t^2)\, r_{rm}'$	=
X_{lm}	=	$F_2 = \frac{(2r_{sm} + r_{rm}')\, r_{rm}'}{X_{om}}$	=
X_{om}	=	$F_3 = (I_m\, r_{rm}')\, r_{rm}'/X_{om}$	=
r_{sm}	=	$F_4 = 2\, I_m\, r_{rm}'$	=
r_{rm}'	=	$F_5 = (I_m\, r_{rm}')K_t$	=
$K_t \sqrt{(X_{om} - X_{lm})/X_{lm}}$	=	$F_6 = [(I_m\, r_{rm}')K_t]^2\, r_{rm}'$	=
$\frac{r_{rm}}{X_{lm}}$	=	$F_7 = VK_t$	=
$\frac{r_{rm}'}{X_{om}}$	=	$F_8 = (VK_t)^2\, r_{rm}'$	=
$I_m = \frac{V}{X_{om}}$	=	$F_9 = \frac{\text{Core loss}}{2V}$	=
		Core loss	=
$I_m\, r_{rm}'$	=	Friction and windage loss	=

*1.	c = r.p.m./s r.p.m.	=
*2.	c^2	=
*3.	$(1-c^2)$	=
*4.	$(1-c^2)\ r'_{rm}$	=
*5.	F_1	=
*6.	$U=(4)+(5)$	=
*7	$(1-c^2)\ X_{lm}$	=
*8.	F_2	=
*9.	$W=(7)-(8)$	=
*10.	$\sqrt{U^2+W^2}$	=
11.	$(1-c^2)E$	=
12.	F_3	=
13.	$M=(11)-(12)$	=
14.	$F_5\ U$	=
15.	$N=(13)+(14)$	=
16	$\sqrt{N^2+F_4^2}$	=
17.	$I_1=(16)/(10)$	=
18.	$(1-c^2)\ F_7$	=
19.	$\sqrt{(18)^2+(F_5)^2}$	=
20.	$I_2=(19)/(10)$	=
21.	$c\ F_5$	=
22.	$I_3=(21)/(10)$	=
*23.	$(1-c^2)F_5$	=
*24.	F_6	=
*25.	$(23)-(24)$	=
26.	Primary copper loss	$=I_1^2\ r_{m}$
27.	Secondary copper loss (m)	$=I_2^2\ r_m'$
28.	Secondary copper loss (s)	$=I_3^2\ r_m'$
29.	Core loss (m) $=\dfrac{\text{core loss}}{2}$	=
*30.	$(25)\times(2)/(10)^2$	=
31.	Input $=(26)+(27)+(28)+(29)+(30)$	=
*32.	$\dfrac{\text{Core loss}}{2}+(F \text{ and } W \text{ loss})$	=
*33.	Output $=(30)-(32)$	=
*34.	R.P.M. $=c\times s$ r.p.m.	=
*35.	Torque $=\dfrac{60}{2\pi}\times(33)/(34)$ Nm	=
36.	Efficiency $=(33)/(31)$	=
37.	Power factor $=(31)/EI_1$	=
38.	Percent full load	=

Only item with (*) mark should be calculated if output and torque are to be calculated.

11·16. Pull-out torque. The speed at pull out torque can be determined from the curve given in Fig. 11·6. This value of speed may be used to calculate the value of maximum torque. The calculation of pull out torque can be done by either of the above two methods.

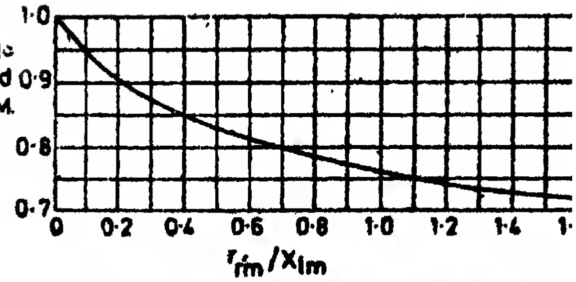


Fig. 11·6. Speed for pull out torque.

An empirical formula for the pull out torque of a single phase induction motor has been developed by C.G. Veinott. It is simple and easy to use, and the results are usually conservative : this is, more often than not the motor develops slightly more torque than given by the equation. The equation is :

Pull out torque

$$T_{po}=1\cdot27\left(\frac{V}{115}\right)^2\times\frac{p}{f}\times\frac{345-9\cdot2(R_m/X_{lm})^2}{R_m+X_{lm}}\;K_r\ \text{Nm} \quad \ldots(11\cdot55)$$

where $R_m=r_{sm}+r_{rm}$.

STARTING (AUXILIARY) WINDING

11·17. Design of starting winding for split phase motors. After a satisfactory main winding has been designed, the next step is to design a suitable starting or auxiliary winding. In order that the starting winding can produce a revolving field the flux set up by it must be out of phase with flux set up by the main winding. The number of turns of the main winding must satisfy the requirements of the core and the size of conductor requirements of the load. This means that the reactance of main winding is high and its resistance is low. The starting winding must have parameters just the reverse of those of main winding if the flux, it produces, is to be appreciably out of phase with that produced by main winding.

It is essential to design the best possible starting winding for the main winding and rotor already designed. For any given main winding, there are almost an unlimited number of possible starting windings. The problem now is to determine which one of these is the best possible so far as meeting the specifications is concerned. Fortunately, it is not necessary to try every possible starting winding blindly, for a number of equations and procedures have been developed to assist the designers in finding the one best suited to his requirements.

With resistance split phase motors the required resistance is usually obtained by using a small section wire i.e. about 25% of that of main winding. The current density at starting may be as high as 100 A/mm². This is permissible as the winding is in service for about 2 seconds. The phase angle between the starting winding current and the line voltage should be about 0·4 of that for the main winding.

12·17·1. Starting torque

Let

R_m=total resistance in terms of main winding$=r_{sm}+r_{rm}'$ ...(11·56)

X_{lm}=leakage reactance in terms of main winding

Z_m=total locked rotor impedance (i.e. impedance at starting) in terms of main winding

$$=\sqrt{R_m^2+X_{lm}^2} \quad \ldots(11\cdot57)$$

I_m=current in the main winding at starting$=V/Z_m$ ...(11·58)

θ_m = phase angle between applied voltage and current in the main winding at starting.

$= \tan^{-1} X_{lm}/Z_m$ (See Fig. 11·..) ...(11·59)

T_m = series turns in the main winding,

K_{wm} = winding factor for the main winding,

T_a = series turns in the starting (auxiliary) winding,

K_{wa} = winding factor for the starting winding,

K = ratio of effective starting winding turns to main winding turns

$$=\left(\frac{T_a K_{wa}}{T_m K_{wm}}\right) \qquad ...(11·60)$$

r_{sa} = resistance of starting winding,

r_{ra}' = resistance of rotor in terms of starting winding,

$$=\left(\frac{T_a K_{wa}}{T_m K_{wm}}\right)^2 r_{rm}' = K^2 r_{rm}' \qquad ...(11·61)$$

R_a = total resistance in terms of starting winding $= r_{sa} + r_{ra}'$ ...(11·62)

$= r_{sa} + K^2 r_{rm}'$ ...(11·63)

X_{la} = total leakage reactance in terms of starting winding $= K^2 X_{lm}$ ...(11·64)

Z_a = total locked rotor impedance (*i.e.*, impedance at starting) in terms of starting winding

$= \sqrt{R_a^2 + X_{la}^2}$ ...(11·65)

I_{sa} = current in the starting winding at starting $= V/Z_a$ ...(11·66)

θ = phase angle between applied voltage and current in the starting winding at starting

$= \tan^{-1} X_{la}/R_a$ ...(11·67)

The starting torque is given by

$$T_s = \frac{1}{2\pi} \frac{p K r_{rm}'}{f} I_{sm} I_{sa} \sin(\theta_m - \theta_a) \qquad ...(11·68)$$

This equation does not take into account the effect of magnetizing current. P.H. Trickey gives the following multiplying factor to take this into account

$$C_r = \frac{K_r}{1 + (r_{rm}'/X_{om})^2} \qquad ...(11·69)$$

The value of C_r is usually equal to K.

$\therefore$ The starting torque is

$$T_s = \frac{1}{2\pi} \cdot \frac{p K C_r r_{rm}'}{f} I_{sm} I_{sa} \sin(\theta_m - \theta_a) \qquad ...(11·70)$$

Eqn. 11·70 can be written as

$$T_s = \frac{1}{2\pi} \cdot \frac{p K C_r r_{rm}'}{f} \cdot \frac{V^2}{Z_m Z_a} [\sin\theta_m \cos\theta_a - \sin\theta_a \cos\theta_m]$$

$$= \frac{1}{2\pi} \cdot \frac{p K C_r r_{rm}'}{f} \cdot V^2 \left[\frac{R_a X_{lm} - R_m X_{la}}{Z_m^2 Z_a^2}\right] \qquad ...(11·71)$$

Assuming that the parameters of the main winding are fixed there are three variables in Eqn. 11·70 which are due to the starting winding *i.e.*, K, I_{sa} and θ_a. Since these three variables are partially independent and partially interrelated a solution, that at once gives

the optimum starting winding, is not self evident. Moreover, there are other conditions to be fulfilled like current density in the starting winding should not be too high, and pull-out torque should not be too low.

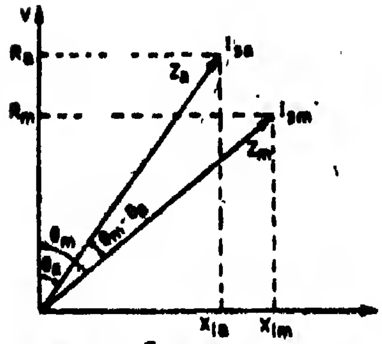


Fig. 11·7. Phasor diagram for split phase induction motor.

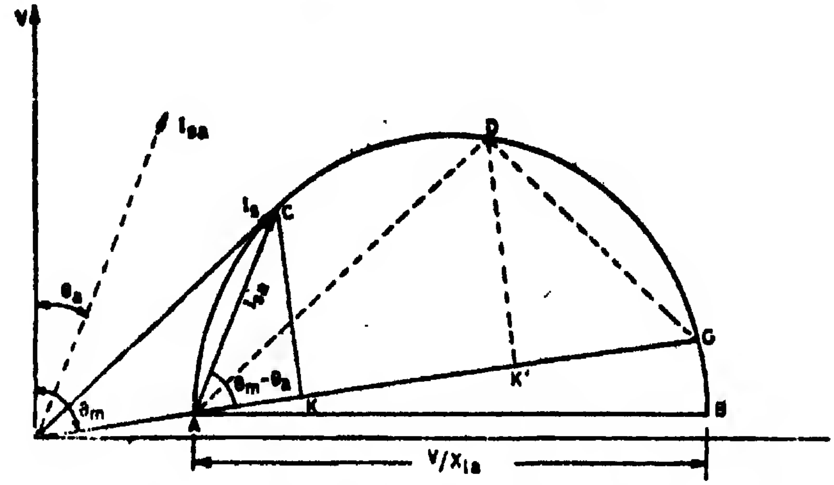


Fig. 11·8. Circle diagram of resistance split phase single phase induction motor and conditions for maximum starting torque.

11·18. Circle diagrams. A circle dirgram is found to be most helpful in visualising an approach to the problem of selection of a proper starting winding.

Let the main winding parameters be fixed. Assume an arbitrary number of turns for starting winding, this fixes the value of K and X_{la}, the reactance of the starting winding. Now suppose the total resistance of starting winding is varied by varying the resistance of the starting winding. The locus of starting winding current I_{sa} $(=AC)$ is a semi-circle on AB $(=V/X_{la})$ as the diameter (See Fig. 11·8). The total starting current I_s is the phasor sum of I_{sm} and I_{sa} and follows the locus ACB. Since I_{sm} is constant, the starting torque is proportional to

$$I_{sa} \sin(\theta_m-\theta_a). \qquad \text{(See Eqn. 11·70)}$$

From the geometry of the Fig. 11·8, we find that

$$CK=I_{sa} \sin(\theta_m-\theta_a).$$

Thus CK is proportional to starting torque for assumed conditions (as all other terms in Eqn. 11·70, are constant).

11·18·1. Maximum starting torque. It is clear from above that the maximum torque that can be obtained by varying R_a and keeping all other parameters fixed occurs when CK is maximum in length. On inspection, it can be seen that this condition is fulfilled by point D, where D is midway between A and G on the arc.

This means $AD=DG$ and $AK'=GK'$.

$\therefore$ It can be shown that $\theta_a=\theta_m/2$

Hence $$\cot\theta_a=\cot\frac{\theta_m}{2}=\frac{1+\cos\theta_m}{\sin\theta_m}$$

or $$\frac{R_a}{X_{la}}=\frac{1+R_m/Z_m}{X_{lm}/Z_m}=\frac{R_m+Z_m}{X_{lm}}$$

$$R_a=\frac{X_{la}}{X_{lm}}\times(R_m+Z_m)=K^2(R_m+Z_m) \qquad \text{..(11·72)}$$

Thus the resistance of starting winding to satisfy Eqn. 11·72 should be

$$r_{sa}=R_a-r_a'.$$

Therefore, we can calculate the value of conductor section to give the above resistance.

Starting winding locked rotor impedance

$$Z_a=\sqrt{R_a^2+X_{la}^2}=\sqrt{[K^2(R_m+Z_m)]^2+(K^2 X_{lm})^2}$$
$$=K^2\sqrt{2Z_m(R_m+Z_m)}$$

∴ Starting winding locked rotor current $I_{sa}=\dfrac{V}{Z_a}=\dfrac{V}{K^2\sqrt{2\,Z_m(R_m+Z_m)}}$

Total locked rotor current for both windings is parallel *i.e.* total starting current is

$$I_s=\frac{I_{sm}\,(Z_m+Z_a)}{Z_a}$$

Therefore, for a given number of turns of the starting winding, the torque and current can thus be calculated for a winding giving maximum torque. Curves have been given by Lloyd and Karr, which assist this procedure.

11·18·2. Maximum torque per ampere. An alternative criterion for starting winding may be the torque per ampere of starting current. Maximum starting torque per ampere (of line current) is obtained when the ratio CK/OC is a maximum. This condition obviously occurs when the line OC is tangent to the semicircle. In Fig. 11·9, C is actually this point. Point F is the centre of the semicircle and triangle OCF is right angled at C.

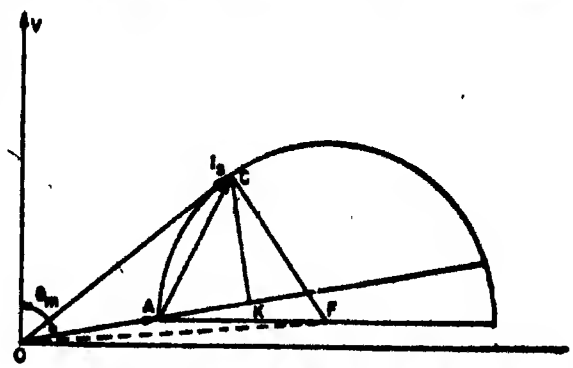


Fig. 11·9. Circle diagram of resistance split phase single phase induction motor and conditions for maximum torque per ampere.

Now

$$I_s^2=OC^2=OF^2-CF^2=OF^2-AF^2$$
$$=OA^2+AF^2+2\,OA\,AF\sin\theta_m-AF^2$$
$$=I_{sm}^2+2I_{sm}\,\frac{V}{2\,X_{la}}\cdot\frac{X_{lm}}{Z_m}$$
$$=I_{sm}^2+I_{sm}^2\frac{X_{lm}}{X_{la}}$$
$$=I_{sm}\;(1+X_{lm}/X_{la})$$

or
$$(I_s/I_{sm})^2=(1+X_{lm}/X_{la})$$

Hence
$$X_{la}=\frac{X_{lm}}{(I_s/I_{sm})^2-1}\qquad\ldots(11\cdot76)$$

From Eqns. 11·75,

$$I=I_{sm}(Z_m+Z_a)/Z_a$$

or
$$\left(\frac{I_s}{I_{sm}}\right)^2=\left(\frac{Z_m+Z_a}{Z_a}\right)^2$$

From Eqn. 11·76 and 11·77

$$R_a=\frac{R_m\,X_{la}+Z_m\,\sqrt{X_{la}(X_m+X_{la})}}{X_{lm}}\qquad\ldots(11\cdot78)$$

Eqn. 11·78 gives the value of total starting winding resistance if the value of starting winding reactance is known. Usually, however, the known quantities are I_{sm} and I_s.

Eqn. 11·76 shows that $X_{ls}=\dfrac{X_{lm}}{(I_s/I_{sm})^2-1}$

It can be derived that

$$R_s=\frac{R_m+Z_m\ (I_s/I_{sm})}{(I_s/I_{sm})^2-1} \quad \text{...(11·79)}$$

These are very useful equations. They completely specify a starting winding that gives maximum torque per ampere of starting current for a fixed value of main winding and a specified line current. Before adopting a starting winding from these equations, the current density in the winding should be checked for locked rotor conditions, as sometimes they may lead to a winding with too high a current density.

The maximum torque per ampere winding results in less weight of copper and works the starting winding at a higher current density than the maximum torque winding. Practical windings are a compromise between the two favouring the maximum torque per ampere winding.

11·19. Design of starting winding for capacitor start motor. Improvements in capacitor design and construction are tending to make the capacitor split phase motor more effective than resistance split phase motor. This motor is a little more expensive but it gives a considerably higher starting torque and a lower starting current.

The design of starting winding for a capacitor start motor generally presents all the problems of designing such a winding for a split phase motor with additional problems caused by the use of capacitor in the circuit. As in the case of split phase motor, the design of main winding is fixed by running performance. Hence the procedure is to find the best possible combination of starting winding and capacitor to work in conjunction with the main winding. If this combination is not good, it may be necessary to alter the main winding.

In this discussion, we will consider the main winding as fixed and shall find out the best starting winding to go with it.

In the design of starting winding, we will not only be concerned with the starting torque and current but a lot of attention will have to be paid to the size of capacitor and the voltage across it. Cost of capacitor depends upon its microfarad rating and its voltage. Actual voltage across the capacitor may be in excess of the line voltage depending upon the design.

Starting torque. From Eqn. 11·70, starting torque

$$T_s=\frac{1}{2\pi}\ \frac{pKC_r\, r_{rm}'}{f}\ I_{sm}\, I_{ss} \sin(\theta_m-\theta_a)$$

$$=\frac{1}{2\pi}\cdot\frac{pKC_r\, r_{rm}'}{f}V^2\left(\frac{R_a\, X_{lm}-R_m\,(X_{la}-X_c)}{(R_m^2+X_{lm}^2)\{R_a^2+(X_{la}-X_c)^2\}}\right) \quad \text{...(11·80)}$$

where X_c=reactance of the capacitor.

11·19·1. Capacitance for maximum torque. The value of capacitance to give maximum starting torque can be found by method similar to that used in finding resistance for the resistance split phase motor. Fig. 11·10 represents the circle diagram of a capacitor start motor. OA represents I_{sm} the locked rotor current of main winding. Assuming a given total starting winding resistance R_s, the locus of starting winding locked rotor current is a circle on AB as a diameter where

$$AB=V/R_s$$

The starting torque is maximum when the perpendicular from the circle on line OA is maximum i.e., when I_{ss} at is a point D such that DK passes through the centre of the circle. This condition is represented in Fig. 11·10.

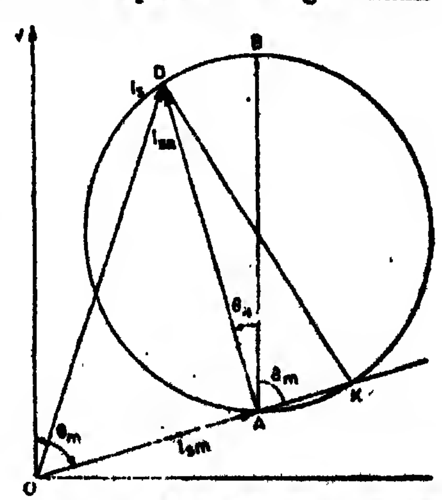


Fig. 11·10. Circle diagram of capacitor start induction motor and condition for maximum torque.

From geometry of the figure : $\theta_a=(90°-\theta_m)/2$

also $$\tan\theta_a=\sqrt{(1-\cos 2\theta_a)/(1+\cos 2\theta_a)}=\sqrt{(1-\sin\theta_m)/(1+\sin\theta_m)}$$
$$=\sqrt{(Z_m-X_{lm})/(Z_m+X_{lm})}=R_m/(Z_m+X_{lm})$$

But $$\tan\theta_a=(X_e-X_{la})/R_a$$

$$\therefore\quad \frac{X_e-X_{la}}{R_a}=\frac{R_m}{Z_m+X_{lm}}$$

or $$X_e=X_{la}+\frac{R_aR_m}{Z_m+X_{lm}} \qquad ...(11{\cdot}81)$$

11·19·2. Capacitance for maximum torque per ampere. Referring to Fig. 11·11, the maximum torque per ampere occurs when CK/OC is a maximum, *i.e.*, when OC, the line starting current is tangent to the circle. Thus

$$I_s^2=OC^2=OF^2-CF^2$$
$$=OF^2-AF^2$$
$$=OA^2+AF^2+2OA.AF\cos\theta_m-AF^2$$
$$=OA^2+2OA.AF\cos\theta_m$$
$$=I_{sm}^2+2I_{sm}\left(\frac{V}{2R_a}\right)\left(\frac{R_m}{Z_m}\right)$$
$$=I_{sm}^2\left(1+\frac{R_m}{R_a}\right)$$

or $$R_a=\frac{R_m}{(I_s/I_{sm})^2-1} \qquad ..(11{\cdot}82)$$

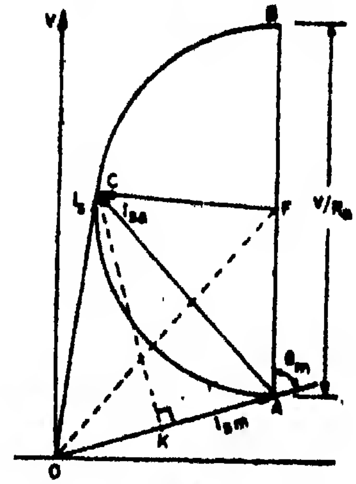


Fig. 11·11. Circle diagram of a capacitor start motor and condition for maximum torque per ampere.

Total starting current

$$I_s=I_{sm}\frac{\sqrt{(R_m+R_a)^2+\{X_{lm}+(X_{la}-X_e)\}^2}}{\sqrt{R_a^2+(X_{la}-X_e)^2}} \qquad ...(11{\cdot}83)$$

Substituting the value of I_s/I_{sm} in the above equation, we have

$$X_{la}-X_e=\frac{X_{lm}R_a-Z_m\sqrt{R_a(R_m+R_a)}}{R_m} \qquad ...(11{\cdot}84)$$

From Eqn. 11·84 we can find the value of X_e.

We can write Eqn. 11·84 as

$$X_{la}-X_e=\frac{X_{lm}-Z_m\sqrt{1+R_m/R_a}}{R_m/R_a} \qquad ...(11{\cdot}85)$$

From Eqn. 11·82, $R_m/R_a=(I_s/I_{sm})^2-1$

Substituting these values in Eqn. 11·85,

$$\therefore\quad X_{la}-X_e=\frac{X_{lm}-Z_m(I_s/I_{sm})}{(I_s/I_{sm})^2-1} \qquad ...(11{\cdot}86)$$

The above relations may be used for the design of starting winding.

The size of capacitor normally required varies from 20 to 30 μF for motors upto 60 W and 60 to 100 μF for a 55 or 90 W motor.

Design Problem. *Design a 370 W, 230 V, 50 Hz, 4 pole single phase capacitor start induction motor. The full load efficiency and power factor should not be less than 0·65*

and 0·82 respectively. The starting torque should be about 300 percent of full load torque with starting current not over 37 A. The motor is to be of open type with temperature rise of windings not above 40°C for continuous operation.

Solution.

Actual speed = 1430 r.p.m. Nearest Synchronous speed = 1500 r.p.m.

Synchronous r.p.s n_s = 1500/60 = 25. poles $p = 2 \times 50/25 = 4$.

Watt output/r.p.s. = 370/25 = 14·8.

From Table 11·3, value of $C_0\,\eta\,\cos\phi = 16·5$.

$\therefore$ Product $$D^2L = \frac{\text{kW}}{C_0\,\eta\,\cos\phi\, n_s}$$

$$= \frac{0·37}{16·5 \times 25} = 0·897 \times 10^{-3}\ \text{m}^3.$$

Taking $L/\tau = 1$, we have $L = 0·785\,D$

Thus $0·785\,D^3 = 0·897 \times 10^{-3}$

or $D = 0·1045$ m $= 4·11''$.

From Table 10·5 on page 632 the lamination size selected is 30 E 47 of M/s Devidayal Stainless Steel Industries.

The equivalent size of M/s Guest Keen Williams is 161 M. The stamping is shown in Fig. 11·12.

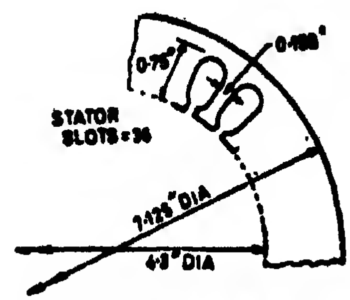


Fig. 11·12. Stator lamination.

We have

Stator bore $D = 4·3'' = 109$ mm $= 0·109$ m.

$\therefore$ Length of core $L = 0·897 \times 10^{-3}/(0·109)^2$

$= 0·075$ m $= 75$ mm.

Pole pitch $\tau = \pi \times 0·109/4 = 0·0855$ m. Net iron length $L_i = 0·95 \times 0·075 = 0·0713$ m.

There are 36 stator slots.

$\therefore$ Stator slot pitch $y_{ss} = \pi \times 109/36 = 9·52$ mm.

Flux per pole

Width of stator tooth $W_{ts} = 0·158'' = 4$ mm $= 4 \times 10^{-3}$ m.

Taking a flux density of 1·1 Wb/m² for stator teeth.

Flux per pole Φ_m = flux density × area of teeth per pole $= B_{ts} \times (S_s/p) \times L_i \times W_{ts}$

$= 1·1 \times (36/4) \times 4 \times 10^{-3} = 2·72 \times 10^{-3}$ Wb.

Check

Outer diameter of stator lamination $D_0 = 7·125'' = 181$ mm $= 0·181$ m

Depth of stator slot $d_{ss} = 0·75'' = 19$ mm

Depth of stator core $$d_{cs} = \frac{D_0 - D - 2\,d_{ss}}{2} = 17\ \text{mm}$$

Flux density in stator core

$$B_{cs} = \frac{\Phi_m}{2L_i\,d_{cs}} = \frac{2·72 \times 10^{-3}}{2 \times 0·0713 \times 17 \times 10^{-3}} = 1·12\ \text{Wb/m}^2.$$

This is within limits.

Stator Winding. The number of stator slots is 36 and the slots per pole are 9. The winding arrangements is shown in Fig. 11·13. The winding factor is calculated in Art. 11·7·1.

Winding factor for main winding $K_{wm}=0\cdot8$.

Induced emf $E=0\cdot95\ V=0\cdot95\times230=219$ V.

Number of series turns in main winding

$$T_m=\frac{E}{4\cdot44\,K_{wm}\,f\,\Phi_m}=\frac{219}{4\cdot44\times0\cdot8\times50\times2\cdot72\times10^{-3}}=457.$$

Turns in series per pole $T_{pm}=457/4=114$.

The turns in each coil are : (Reference Fig. 11·3 and Art. 11·7·1)

coil 4—6	$=0\cdot121\times114=14$
coil 3—7	$=0\cdot027\times114=26$
coil 2—8	$=0\cdot306\times114=35$
coil 1—9	$=0\cdot546\times114=39$
∴ Total	$=114$.

∴ Turns in series per pole $T'_{pm}=114$.

Total turns in series for main winding $T_m=4\times114=456$.

The distribution of turns is shown in Fig. 11·13.

Conductor size. Current in the main winding

$$I_m=\frac{0\cdot37\times10^3}{230\times0\cdot65\times0\cdot62}=4\cdot07\ \text{A}.$$

Assume a current density of 4·8 A/mm².

Area of main winding conductor $=4\cdot07/5=0\cdot815$ mm².

Referring to Table 17·8 the nearest size is 19 SWG.

Area of main winding conductor $a_m=0\cdot811$ mm².

Diameter of bare conductor $=1\cdot02$ mm.

Enamelled conductors are used. From Table 17·9 addition to bare conductor diameter is 0·063 mm.

∴ Diameter of insulated conductor $=1\cdot02+0\cdot063=1\cdot083$ mm.

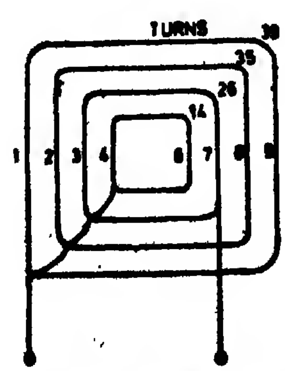


Fig. 11·13. Main winding details.

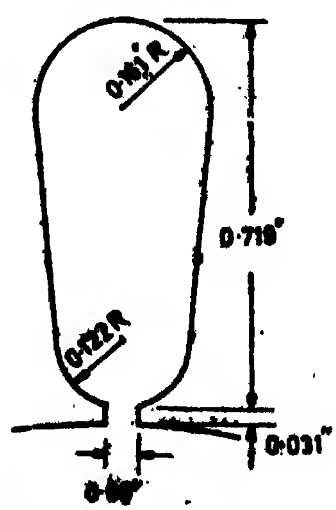


Fig. 11·14. Stator slot.

The stator slot is shown in Fig. 11·14.

The area of slot $=0·189$ square inch $=121·5$ mm^2.

The largest number of turns per coil is 39 and therefore the maximum number of main winding conductors in a slot is 39.

Space occupied by 39 conductors $=39\times(\pi/4)\times(1·083)^2=36$ mm^2.

Ratio of space occupied by conductors to total slot area $=36/121·5=0·296$

This is less than the maximum allowable value of 0·5.

Length of mean turn. From Eqn. 11·14, length of mean turn

$$L_{mt}=\frac{8·4(D+d_{ss})}{S_s}\times\text{slots spanned}+2L.$$

Length of mean turn of coil 4—6 $=\dfrac{8·4(109+19)}{30}\times10^{-3}\times2+2\times75\times10^{-3}$

$=0·03\times2+0·15=0·21$ m.

Length of mean turn of coil 3—7 $=0·03\times4+0·15=0·27$ m.

Length of mean turn of coil 2—8 $=0·03\times6+0·15=0·33$ m.

Length of mean turn of coil 1—9 $=0·03\times8+0·15=0·39$ m.

Length of mean turn of main winding

$$L_{mtm}=\frac{14\times0·21+26\times0·27+35\times0·33+39\times0·39}{114}=0·321 \text{ m.}$$

Rotor Design

Length of air gap. From Eqn. 11·15, length of air gap

$$l_g=\frac{0·007\times\text{rotor diameter}}{\sqrt{p}}$$

$$=\frac{0·007\times109}{2}=0·38 \text{ mm.}$$

We take $l_g=0·32$ mm.

Ratio diameter $D_r=109-2\times0·32=108·36$ mm.

The details of rotor slot are shown in Fig. 11·15.

Number of rotor slots. The rotor has 44 slots. This number is 8 (*i.e.* $2p$) more than the number of stator slots and this slot combination of $S_s=36$ and $S_r=44$ results in quiet operation for the machines.

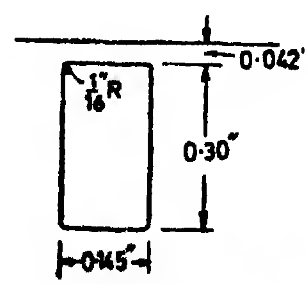


Fig. 11·15. Rotor slot.

Rotor Bars. From Fig. 11·15, area of each rotor bar

$=0·3\times0·145=0·0435$ square inch $=28$ mm^2.

Allowing for rounding of corners and clearances

$a_b=20$ mm^2.

Total area of rotor $A_r=S_r\,a_b=44\times20=880$ mm^2.

Total area of conductors in main winding $A_m=2T_m\,a_m=2\times456\times0·811=740$ mm^2.

Ratio $A_r/A_m=830/740=1·19$

End rings. From Eqn. 11·24, area of each end ring

$$a_e=\frac{0·32\,A_r}{p}\cdot\frac{\delta_b}{\delta_e}=\frac{0·32\times880}{4}=70·5 \text{ mm}^2.$$

The end ring section is depth : $d_e=10$ mm, thickness $t_e=7$ mm,

and area $a_e=10\times7=70$ mm^2.

Outer diameter of ring $D_{ere}=106$ mm

Inner diameter of ring $D_{eri}=106-2d_e=86$ mm

Mean diameter of ring $D_e=96$ mm.

Gap contraction factor. Stator slot opening $W_{os}=0'09''=2'29$ mm.

$$\text{Ratio}\quad\frac{\text{slot opening}}{\text{gap length}}=\frac{2\cdot29}{0\cdot32}=7\cdot15.$$

From Fig. 4'9, Carter's co-efficient for semi-enclosed slots,

Corresponding a ratio 6'02 is $K_{os}=0\cdot7$

$$\text{Gap contraction factor for stator slots}\quad K_{gss}=\frac{9\cdot52}{9\cdot52-0\cdot7\times2\cdot29}=1\cdot2$$

The rotor slots are closed but their leakage permeance cannot be taken as infinite because of saturation. The rotor slots are considered to have an opening of 1 mm,

or rotor slot opening $W_{or}=1$ mm.

$$\text{Ratio}\quad\frac{\text{slot opening}}{\text{gap length}}=\frac{1\cdot0}{0\cdot32}=3\cdot13.$$

Corresponding to ratio 2'63, Carter's co-efficient $K_{rs}=0\cdot47$

$$\text{Rotor slot pitch}\qquad y_{sr}=\frac{\pi\times108\cdot24}{44}=7.73\text{ mm}.$$

$$\text{Gap contraction factor for rotor slots}\ K_{gsr}=\frac{0\cdot773}{0\cdot773-0\cdot47\times0\cdot1}=1\cdot065.$$

Gap contraction factor $K_g=1\cdot2\times1\cdot065=1\cdot28$.

Saturation factor. Saturation factor F_s is assumed as 1'15 as flux densities in iron parts have small values.

Resistance of main winding. Resistance of main winding

$$r_m\text{ (hot)}=0\cdot021\times\frac{456\times0\cdot321}{0\cdot811}=3\cdot06\ \Omega\text{ at }75°\text{C}$$

$$r_m\text{ (cold)}=0\cdot017\times\frac{456\times0\cdot321}{0\cdot811}=2\cdot5\ \Omega\text{ at }20°\text{C}.$$

Resistance of rotor. The slots are skewed through one slot pitch *i.e.*, through 9'52 mm.

$\therefore$ Length of each bar $L_b=\sqrt{(75)^2+(9\cdot52)^2}=75\cdot6$ mm.

Ratio $D_{eri}/D_{ere}=86/106=0\cdot81$

From Fig. 10'33 page 639, $K_{r,m}=0\cdot96$.

From Eqn. 11'33, resistance of rotor referred to main winding

$$r_{rm}'=8T_m^2K_{wm}^2\rho\left(\frac{L_b}{S_ra_b}+\frac{2}{\pi}\frac{D_e}{p^2a_e}K_{ring}\right)$$

$$=8\times(456)^2\times(0\cdot8)^2\times0\cdot021\times\left(\frac{0\cdot0756}{44\times20}+\frac{2}{\pi}\times\frac{0\cdot096}{(4)^2\times70}\times0\cdot96\right)$$

$=3\cdot1\ \Omega$ at 75°C.

$$r_{rm}'=\frac{0.017}{0.021}\times 3.1=2.51\ \Omega \text{ at } 20^\circ\text{C}.$$

REACTANCES

Slot leakage reactance

The stator slot is shown in Fig. 11·14. Referring to Fig. 4·46 on page 163

$a_1=0.244''$; $a_2=0.322''$; $b=0.436''$;

$c=0.122''$; $d=0.031''$; $e=0.09''$.

Note : Portion c is taken as tapered.

Ratio $b/a_2=0.436/0.322=1.35$;

Ratio $a_1/a_2=0.244/0.322=0.727$.

Corresponding to above ratios, value of F from Fig. 4·44 page 162 is $F=0.8$.

From Eqn. 4·76 specific permeance for stator slots

$$\lambda_{ss}=\mu_0\left(F+\frac{d}{e}+\frac{2c}{e+a_1}\right)$$

$$=\mu_0\left(0.8+\frac{0.031}{0.09}+\frac{2\times 0.122}{0.09+0.244}\right)=1.875\ \mu_0.$$

From Eqn 4·72 page 159 specific permeance for rotor slots

$$\lambda_{sr}=\mu_0\left[\frac{h}{3W_s}+\frac{h_4}{W_0}\right]$$

$$=\mu_0\left[\frac{0.3}{3\times 0.145}+\frac{0.042}{1/25.4}\right]=1.78\ \mu_0.$$

From Eqn. 11·38,

$$C_s=\frac{Z_1^2+Z_2^2+Z_3^2+Z_4^2}{(Z_1+Z_2+Z_3+Z_4)^2}\times\frac{1}{K_{wm}^2}\times\frac{S_s}{4p}$$

$$=\frac{(14)^2+(26)^2+(35)^2+(39)^2}{(114)^2}\times\frac{1}{(0.8)^2}\times\frac{36}{4\times 4}=0.96.$$

From Eqn. 11·40, slot leakage reactance in terms of main winding

$$x_s=16\pi f\, T_m^2 K_{wm}^2\frac{L}{S_s}\left(\lambda_{ss}+\frac{S_s}{S_r}\lambda_{sr}\right)C_s$$

$$=16\pi\times 50\times(456)^2\times(0.8)^2\times\frac{0.075}{36}\times 4\pi\times 10^{-7}$$

$$\left(1.875\times\frac{36}{44}\times 1.78\right)\times 0.96=2.8\ \Omega$$

Zigzag leakage reactance : we have

$l_g=0.32$ mm, $y_{ss}=9.52$ mm

$W_{os}=2.29$ mm ; $W_{ts}=9.52-2.29=7.23$ mm

$y_{sr}=7.73$ mm ; $W_{or}=1$ mm ; $W_{tr}=7.73-1.0=6.73$ mm.

From Eqn. 4·92, specific permeance for zigzag leakage

$$\lambda_z=\mu_0\,\frac{W_{ts}\,W_{tr}\,(W_{ts}^2+W_{tr}^2)}{12\,l_g\,y_{ss}^2\,y_{sr}}$$

$$=\frac{\mu_0\times 7.23\times 6.73(7.23^2+6.73^2)}{12\times 0.32\times 9.52^2\times 7.73}=1.86\ \mu_0.$$

From Eqn. 11·41, zigzag leakage reactance

$$x_z=16\pi\times 50\times(456)^2\times(0.8)^2\times\frac{0.075}{36}\times 4\pi\times 10^{-7}\times 1.86=1.63\ \Omega$$

Overhang leakage reactance

From Eqn. 11·42,

$$x_o=16\pi f\ T_m^2\ K_{wm}^2\ \frac{\mu_0}{6.4\ S_s p}\ [\pi(D+d_{ss})\ \text{average coil span}]$$

$$=16\pi\times 50\times(456)^2\times(0.8)^2\times\frac{4\pi\times 10^{-7}}{6.4\times 36\times 4}\ [\pi(109+19)\times 10^{-3}\times 5]$$

$$=0.92\ \Omega.$$

(We have, average coil span$=(2+4+6+8)/4=5$ slots.)

Magnetising reactance

$$X_m=16\pi f\ T_m^2\ K_{wm}^2\ \frac{\mu_0\ L\ \tau}{10\ l_g\ K_g\ p\ F_s}$$

$$=16\pi\times 50\times(456)^2\times(0.8)^2\times\frac{4\pi\times 10^{-7}\times 0.075\times 0.0855}{10\times 0.32\times 10^{-3}\times 1.28\times 4\times 1.15}$$

$$=144\ \Omega.$$

Skew leakage reactance

The bars are skewed through one stator slot pitch.

Angle of skew $\theta_s=\frac{\pi}{44/4}\times 1\times\frac{44}{36}=0.35$ radian.

From Eqn. 11·43, skew leakage reactance

$$x_{sk}=X_m\ \frac{\theta_s^2}{12}\ K_l=144\times\frac{(0.35)^2}{12}\times 0.95=1.4\ \Omega.$$

Total leakage reactance referred to main winding

$X_{lm}=x_s+x_z+x_o+x_{sk}=2.8+1.63+0.92+1.4=6.75\ \Omega.$

Ratio $r_r'{}_m/X_{lm}=3.1/6.75=0.46.$

Open circuit reactance $X_{om}=X_m+X_{lm}/2\approx 147.4\ \Omega.$

Leakage factor $K_r=(X_{om}-X_{lm})/X_{om}=0.953.$

$\therefore\qquad K_l=\sqrt{K_r}=0.976.$

Core loss

Lohys steel of 0·5 mm thickness is used for the laminations.

Weight of stator teeth

$$=36\times 4\times 10^{-3}\times 19\times 10^{-3}\times 0.0713\times 7.6\times 10^3=1\ \text{kg}.$$

Maximum flux density in stator teeth$=(\pi/2)\times 1.1=1.725\ \text{Wb/m}^2.$

From Fig 4·28 page 148, loss per kg$=11.0$ W.

Iron loss in stator teeth$=1.5\times 11=16.5$ W.

Mean diameter of stator core$=D_o-d_{cs}=0.181-0.017=0.164$ m.

Weight of stator core

$$=\pi\times 0.164\times 17\times 10^{-3}\times 0.0713\times 7.6\times 10^{3}=4.73 \text{ kg.}$$

Flux density in core $=1.12$ Wb/m^2.

Corresponding to the flux density in core, loss per kg $=3.6$ W.

Total iron loss in core $=1.73\times 3.6=17.2$ W.

Total iron loss due to fundamental frequency flux $=11+17.2=28.2$ W.

Total core loss is about 2.2 times the above loss as the rotor has closed slots owing to which there is considerable surface loss.

$\therefore$ Total core loss $=2.2\times 28.2=62$ W.

Friction and windage loss

The friction and windage loss is assumed to be 15 W.

PERFORMANCE CALCULATIONS

Motor constants (Veinott's method)

$V=230$	$F_1=(2-K_l^2)\,r_{rm}'=3.25$
$X_{lm}=6.75$	$F_2=(2r_{sm}+r_{rm}')\,\dfrac{r_{rm}'}{X_{om}}=0.193$
$X_{om}=147.4$	$F_3=(I_m' r_{rm}')\,\dfrac{r_{rm}'}{X_{lm}}=0.1015$
$r_{sm}=3.06$	$F_4=2\,I_m\,r_{rm}'=9.68$
$r_{rm}'=3.1$	$F_5=(I_m\,r_{rm}')\,K_l=4.73$
$K_l=0.976$	$F_6=(I_m\,r_{rm}'\,K_l)^2\,r_{rm}'=69.5$
$\dfrac{r_{sm}}{X_{lm}}=0.454$	$F_7=V\,K_l=224$
	$F_8=(VK_l)^2\,r_{rm}'=155{,}000$
$\dfrac{r_{rm}'}{X_{om}}=0.021$	$F_9=\dfrac{\text{core loss}}{2V}=0.135$
$I_m=\dfrac{V}{X_{om}}=1.56$	Core loss $=62$
$I_m\,r_{rm}'=4.84$	F and W loss $=15$

*1	σ	0.984	0.83
*2	σ^2	0.968	0.689
*3	$1-\sigma^2$	0.032	0.311
*4	$(1-\sigma^2)\,r_{rm}'$	0.099	0.965
*5	F_1	3.250	3.25
*6	$U=(4)+(5)$	3.35	4.1
*7	$(1-\sigma^2)\,X_{lm}$	0.216	2.1
*8	F_2	0.193	0.193
*9	$W=(7)-(8)$	0.023	1.9

*10	$\sqrt{U^2+W^2}$	3·35	4·54
11	$(1-c^2)V$	7·35	
12	F_8	0·1015	
13	(11)−(12)	7·25	
14	F_8U	0·45	
15	N=(13+(14)	7·7	
16	$\sqrt{N^2+F_4^2}$	12·35	
17	I_1=(16)/(10)	3·69	
18	$(1-c^2)F_7$	7·[illegible]6	
19	$\sqrt{(18)^2+F_8^2}$	8·58	
20	I_2=(19)/(10)	2·56	
21	cF_6	4·65	
22	I_3=(21)/(10)	1·39	
*23	$(1-c^2)F_3$	4960	48200
*24	F_6	69·5	69·5
*25	(23)−(24)	4[illegible]90·5	481[illegible]0
26	Primary copper loss$=I_1^2 r_{lm}$	41·6	
27	Secondary copper loss $(m)=I_2^2 r_{rm}'$	20·3	
28	Secondary copper loss $(s)=I_3^2 r_{rm}'$	6·0	
29	Core loss (m)=core loss/2	31	
*30	(25)×(2)/(10)²	4[illegible]2	1610
31	Input=(26)+(27)+(28) +(29)+(30)	521	
*32	Core loss/2+F and W loss	46	46
*33	Output=(30)—(32)	376	1564
*34	R.P.M.=c×s r.p.m.	1476	1245
*35	Torque=60/2π×(33/34)	2·44 N-m	12 N-m
36	Efficiency=(33)/(31)	0·72	
37	Power factor=(31)/EI_1	0·62	
38	Per cent full load	101 3	

Pull out torque

Ratio r_{rm}'/X_{lm}=3·1/6·75=0·459.

From Fig. 11·6, the speed corresponding to pull out torque is 0·83 of s.r.p.m.

or c=0·83

The results correspondinр to [illegible]

The pull out torque is 1[illegible] N-m wh[illegible]

$$\text{Ratio } \frac{\text{pull out torque}}{\text{full load torque}}=\frac{12}{2·44}\simeq 5.$$

Design of starting winding

Assume $K=1{\cdot}53$ and winding factor for starting winding $K_{ws}=0{\cdot}85$.

$\therefore$ Number of turns in the starting winding

$$T_s=K\ T_m\ (K_{wm}/K_{ws})=1{\cdot}53\times456\ (0{\cdot}8/0{\cdot}85)=656.$$

The winding arrangement and distribution for this motor are shown in Fig. 11·16.

The number of turns per pole for starting winding $T_{ps}=656/4=164$.

For sinusoidal distribution the turns per coil are calculated as follows :

$$\begin{aligned}\sin (3/9)\times90^\circ&=0{\cdot}500\\ \sin (5/9)\times90^\circ&=0{\cdot}766\\ \sin (7/9)\times90^\circ&=0{\cdot}940\\ \sin (9/9)\times90^\circ&=1{\cdot}000\\ \hline \text{Total}&=3{\cdot}206\end{aligned}$$

Turns in the four coils are :

$$\begin{aligned}(0{\cdot}5/3{\cdot}206)\times164&=26\\ (0{\cdot}766/3{\cdot}206)\times164&=39\\ (0{\cdot}94/3{\cdot}206)\times164&=48\\ (1{\cdot}0/3{\cdot}206)\times164&=50\\ \hline \text{Total}&=163\end{aligned}$$

Therefore, turns in series per pole $T_{ps}=163$.

Number of turns in starting winding

$$T_s=4\times163=652.$$

Winding factor for starting winding is, then

$$K_{ws}=\frac{0{\cdot}5\times26+0{\cdot}766\times39+0{\cdot}94\times48+1{\cdot}0\times50}{164}$$

$$=0{\cdot}847.$$

$$\therefore\ K=\frac{652\times0{\cdot}847}{456\times0{\cdot}8}=1{\cdot}52.$$

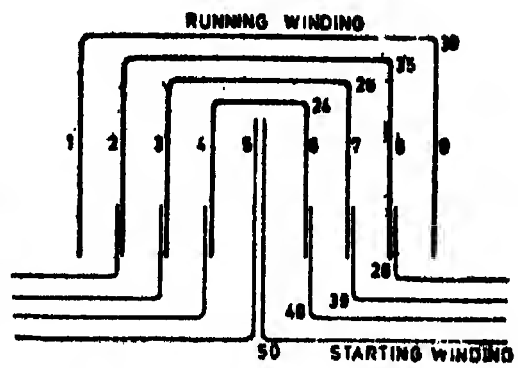


Fig. 11·16. Winding distribution.

Using 22 SWG for the starting winding.

Area of starting winding conductor $a_s=0{\cdot}397\ \text{mm}^2$.

Length of mean turn of secondary winding is calculated below :

$$\left[\frac{8{\cdot}4^*(109+19)}{36}\times10^{-4}\times3+2\times0{\cdot}075\right]26=624$$

$$\begin{aligned}[0{\cdot}03\times5+2\times0{\cdot}075]39&=1170\\ [0{\cdot}03\times7+2\times0{\cdot}075]48&=1728\\ [0{\cdot}03\times9+2\times0{\cdot}075]50&=2100\\ \hline \text{Total}&=5622\end{aligned}$$

* $\frac{8{\cdot}4(109+19)}{36}\times10^{-4}=0{\cdot}03.$

Length of mean turn of starting winding $L_{mts}=56.22/164=0.34$ m.

Resistance of starting winding $r_{ss}=0.017\times\dfrac{652\times0.34}{0.397}=9.65\ \Omega$ at 20°C.

For starting torque calculations the d.c. rotor resistance in terms of the main winding is increased by 20 per cent. Therefore, effective rotor resistance in terms of main winding

$$r_{rm}'=1.2\times2.51=3.0\ \Omega \text{ at } 20°\text{C}.$$

The total resistance in terms of main winding

$$R_{m}=r_{sm}+r_{rm}'=2.5+3.0=5.5\ \Omega \text{ at } 20°\text{C}.$$

Rotor resistance in terms of starting winding

$$r_{rs}'=K^2\ r_{rm}'=(1.52)^2\times3.0=6.95\ \Omega \text{ at } 20°\text{C}.$$

Total resistance in terms of starting winding

$$R_{s}=r_{ss}+r_{rs}'=9.65+6.95=16.6\ \Omega \text{ at } 20°\text{C}.$$

Total leakage reactance in terms of starting winding

$$X_{ls}=K^2X_{lm}=(1.52)^2\times6.75=15.6\ \Omega.$$

Locked impedance of main winding

$$Z_m=\sqrt{(R_m)^2+(X_{lm})^2}=\sqrt{(5.5)^2+(6.75)^2}=8.7\ \Omega.$$

Locked rotor current in main winding $I_{m}=230/8.7=26.5$ A.

Capacitive reactance required for maximum starting torque (Eqn. 11.81).

$$X_c=X_{ls}+\frac{R_s+R_m}{Z_m+X_{lm}}=15.6+\frac{16.6+5.5}{8.7+6.75}=17.03\ \Omega.$$

Capacity $\quad C=\dfrac{10^6}{2\pi\times50\times17.03}=184\ \mu\text{F}.$

We select a capacitor of 100 μF capacity.

$$\therefore \qquad X_c=\frac{10^6}{2\pi\times50\times150}=32\ \Omega.$$

Impedance of starting winding with capacitor in series

$$Z_s=\sqrt{R_s^2+(X_{ls}-X_c)^2}=\sqrt{(16.6)^2+(15.6-32)^2}=25.6\ \Omega.$$

Locked rotor current in starting winding $I_{ss}=230/25.6=8.98$ A.

Current density in starting winding $=8.98/0.397=22.6$ A/mm².

Locked rotor current for both windings in parallel $I_s=I_m\ \dfrac{(Z_m+Z_s)}{Z_s}$

$$=26.5\times\frac{\sqrt{(5.5+16.6)^2+(6.75+16.4)^2}}{25.6}=25.2\ \text{A}.$$

This is below the maximum allowable limit of 27 A (given in the problem).

From Eqn. 11.80, starting torque

$$T_s=\frac{1}{2\pi}\,p\,C_r\,\frac{r'_{rm}V^2}{f}\left[\frac{R_s X_{lm}-R_m(X_{ls}-X_c)}{(R_m^2+X_{lm}^2)\{R_s^2+(X_{ls}-X_c)^2\}}\right]$$

$$=\frac{1}{2\pi}\times4\times0.952\times3\times\frac{(230)^2}{50}\left[\frac{16.6\times6.75-5.5(15.6-32)}{(8.7)^2\times(25.6)^2}\right]=7.9\ \text{N-m.}$$

Full load torque is 2.44 N-m.

Ratio $\quad\dfrac{\text{starting torque}}{\text{full load torque}}=\dfrac{7.9}{2.44}=3.24.$

This ratio is above the minimum requirement of 300 per cent.

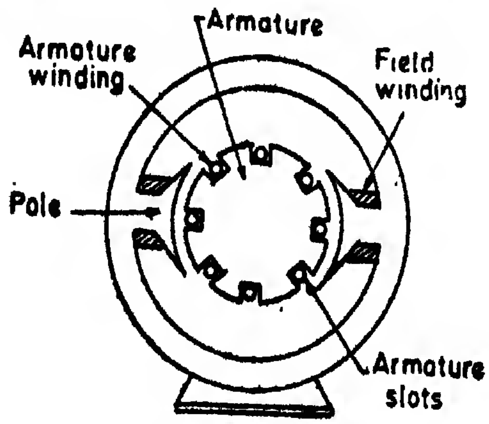
12

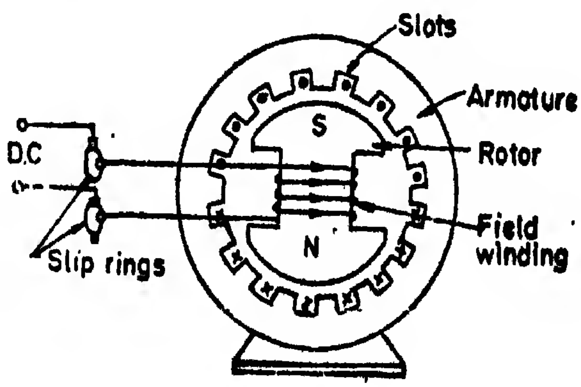
Synchronous Machines

12·1 Type of construction. A synchronous machine consists of two major parts viz (*i*) armature and (*ii*) field system. The arrangement of fundamental parts of a synchronous machine is shown in Fig. 12·1 (*a*). This construction is similar to the one used for d.c machines wherein the armature winding is placed on the rotor and the field system is housed in the stationary stator. This type of construction is used only for low power synchronous machines and is unsuitable for medium and high power machines.

The second type of construction used for synchronous machines is shown in Fig. 12·1 (*b*). This type of construction uses a stationary armature and a revolving field structure.

Long experience in the construction and operation of synchronous machines has shown that most economical and convenient construction is of the second type wherein the field poles, excited by d c supply, are arranged on the rotor while the armature winding is placed on the stator. The use of revolving field system is almost universal because it has





(*a*) (*b*)

Fig. 12·1. Types of synchronous machine construction.

the following advantages :

(*i*) It permits the use of a stationary armature on which the windings can be easily braced (mechanically reinforced) and insulated for high voltages.

(*ii*) The operation of slip rings on account of their sliding contacts is unreliable with large currents at high potential differences. The use of slip rings carrying large currents at high voltages is, therefore, avoided in the stationary armature construction. However, two slip rings are required for d.c. excitation but there is no trouble owing to small excitational power involved. Henceforth whenever we refer to a synchronous machine, it is implied that it is a stationary armature, revolving pole machine unless specified otherwise.

The synchronous machines may be classified as (*i*) salient pole machines and (*ii* cylindrical rotor machines depending upon the type of construction used for the rotor.

Salient pole machines have *salient*, or projecting poles with concentrated field windings as shown in Fig. 12·1 (*b*). The salient pole construction is used for generators driven by hydraulic turbine since these turbines operate at relatively low speeds and a relatively large number of poles are required to produce the desired frequency ; the salient pole construction is b tter adopted mechanically to this situation. **Cylindrical rotor machines** have their field winding *distributed* in slots as shown in Fig 12·18. The distributed field winding produces a sinusoidal flux distribution in the air gap. Cylindrical rotor construction is used for turbo-alternators which are driven by high speed steam or gas turbines.

12 2 Types of synchronous machines Synchronous machines operating on general power supply networks may be divided inio the following categories

1. **Hydro.generators**. The synchronous generators driven by water turbines are known as *hydro generators* or *water wheel generators* They have ratings upto 750 MW and are driven at speeds ranging from 100 to 1000 rpm.

2. **Turbo-alternators**. They are driven by steam turbines. Since the efficiency of steam turbines is high at large speeds, the turbo-alternators are designed for speeds upto 3000 rpm Turbo-alternators have ratings upto 1000 MW.

3 **Engine driven generators**. These generators are driven by different forms of internal combustion engines at speeds upto 1500 rpm and ratings upto 20 MW. Generators driven by gas turbines have higher speeds and also higher power ratings.

4. **Motors**. Synchronous motors may be either plain synchronous machines or synchronous induction machines. The plain synchronous machine with salient poles is commonly used. Synchronous motors have some definite advantages over induction motors and these include constant speed operation, power factor control and high operating efficiency. Also synchronous motors prove to be cheaper than induction motors for high power low speed applications. The applications of synchronous machines include constant speed drives for compressors, blowers, fans, low head pumps.

5. **Compensators**. Synchronous compensators are used for control of reactive power in power supply networks. They are designed for ratings upto 100 MV A_r and speeds upto 3000 rpm.

12·3 Prime movers for synchronous generators. The type of construction used for synchronous generators depends upon the type of prime mover. The following types of prime movers are used :

(*i*) Steam turbines (*ii*) Hydraulic turbines

(*iii*) Diesel engines.

(*i*) Steam Turbines

The efficiency of steam turbines is high at large speeds and therefore synchronous machines driven by steam turbines are high speed machines. The synchronous generators driven by steam turbines are known as "Turbo-alternators". The maximum speed of turbo-alternators is 3000 rpm corresponding to 2 poles and 50 Hz. Such high speeds call for a horizontal shaft for the machines which have to be designed for lower values of armature diameter in order to limit the centrifugal forces which have a profound influence on the design.

The peripheral speed of a machine is given by $V_a = \pi D n$ m/s

where D=diameter of rotor m ; and n=speed of rotor, r.p.s.

Since the peripheral speed increases in direct proportion to diameter and with increase in diameter the centrifugal forces increase, the diameter of the rotor has to kept low. The high speed of the rotor limits the diameter of the 2 pole machine to about 1·2 m giving a peripheral speed of about 175 m/s. Cylindrical rotor construction has to be used as the salient poles are impracticable owing to large mechanical forces.

The 2 pole construction with a speed of 3000 rpm for 50 Hz is usually used for turbo-alternators. The four pole construction with a speed of 1500 rpm is now obsolete.

(ii) Hydraulic Turbines

The synchronous generators driven by water turbines are known as water wheel generators. The hydraulic turbines are of different types. The type of turbine to be used depends upon the water head available. If the water head is high, the speed of the turbine is high and therefore a pelton wheel is used. While for low heads, Francis or Kaplan turbines are used. The use of hydraulic turbines at various heads is listed below :

Water heads 400 m and above—Pelton wheel
Water heads upon 380 m —Francis turbine
Water heads upto 50 m —Kaplan turbine.

As the water heads are not high, the speed of the turbine is low and usually varies from 50 to 500 rpm. Therefore for synchronous machines coupled to hydraulic turbines, salient poles (poles which project out) are used. The number of poles of water wheel generators is 12 upward.

The machines driven by pelton wheels can either be vertical or horizontal shaft type. The use of horizontal shaft being more common But high power, low speed synchronous generators installed in low head hydro-electric plants (coupled with either Francis or Kaplan turbines) are built with a vertical shaft, coupled by a flange to the shaft, of the turbine placed under the generator.

There are two fundamental types of vertical shaft water wheel generators :

(a) the suspended type in which the thrust bearing is arranged in the upper spider or bracket above the alternator rotor ;

(b) the umbrella type in which the bearing is mounted on the lower spider on the turbine cover.

Now-a-days the large size generators are built with umbrella type of construction to reduce generator weight and height of the power station building. This is explained by the fact that when we use the bearing in the upper spider, it has to be made bigger in size in order to withstand the generator and turbine load and also the water reaction. This increase in dimensions of upper spider increases the generator weight and also the height of the building.

(iii) Diesel Engines :

They are used as prime movers for synchronous generators of small ratings. The diesel engines are manufactured as horizontal type and therefore both the diesel engine and the

synchronous generator are mounted horizontally and are connected by a horizontal shaft. The diesel engines are slow speed machines and therefore salient pole type of construction is used for alternators coupled to them. The torque of the diesel engines is not uniform and this makes the synchronous generator sensitive to torque variations

12·4. Run-away Speed. The *run-away speed* is defined as the speed which the prime mover would have, if it is suddenly unloaded when working at its rated load. When the prime mover is working at full load it receives its feed (water, steam or diesel) corresponding to full load conditions and therefore when it is suddenly unloaded it tries to race. This is because there is no load on the prime mover while it is receiving its input corresponding to full load. Steam turbines are equipped with a quick acting overspeed governor set to trip at 1·1 times the rated speed and therefore the operation of the governer is reliable. Thus the synchronous machines driven by steam turbines may be designed for only 1 25 times the rated speed. However in water turbines, following are the values of run-away speeds with full gate operating :

Pelton wheel	—1·8 times rated speed
Francis turbine	—2 to 2·2 times rated speed
Kaplan turbine	—2·5 to 2 8 times rated speed.

Therefore there is a great difference of peripheral speed at normal speeds and run-away speeds in the case of water turbines. The salient pole machines are thus designed to withstand mechanical stresses encountered at run-away speeds. The maximum peripheral speed for which salient pole machines are designed is about 140 m/s while turbo-alternators are designed for a maximum peripheral speed of about 175 m/s.

12 5. Construction of Hydro-generators. The constructional features of hydro-generators are basically dependent upon the mechanical considerations which depend upon the speed of the machine. The hydro-generators are low speed machines, the speed depending upon the available head and the type of turbine used. The low speed demands a multi-polar construction and consequently a large diameter which may present transport problems. Therefore, the design should be such that it permits the machine to transported to the site in sections. The turbine governing, or the transient stability of the network to which the generator is connected is dependent upon the total inertia of turbine and the generator. Most of inertia is provided by the generator. The rotor, therefore, must be designed to give the requisite inertia

It is a normal practice to design the rotor to withstand centrifugal stresses produced at twice the normal operating speed. Also, the design of the rotor is such that the over speed due to run-away is not near the first critical speed.

12·5·1. Stator Core. The stator core is built up of laminations in order to reduce eddy current iron loss. The loss in the laminated core is usually the largest single loss in a hydrogenerator and therefore the design of stator core particularly the choice of type and grade of steel is of utmost importance. Earlier, laminations of hot rolled steel were used. These laminations had many imperfections, chief amongst which was appreciable variation in thickness between individual sheets. The variability in thickness of laminations resulted in unequal lengths of core along different points on the periphery. This presented many practical problems in the assembly of core like selection of sheets according to thickness or fitting and putting of additional laminations at the points on the periphery where the length of core was short. Also to ensure a tight core very high axial clamping pressures had to be used and these high pressures in turn required a high clamping structure with a correspondingly stiff stator frame in order to withstand the reaction. In late forties, the cold rolled oriented (anistropic) sheet steel was introduced. This material has directional properties that give a low specific iron loss when the direction of the flux is parallel to the direction of rolling but a substantially higher iron loss when the flux is across the direction of rolling. Correspondingly the magnetisation is very low along the direction of rolling and very much higher at right angles to this direction (See page 24 for details). Cold rolled

steel is ideal for transformer cores where its use is widespread. However, its efficient use in salient pole synchronous machines is difficult to acheive as is clear from Fig. 12 2. The flux follows two paths, one along the core at the back of the slots and the other along the teeth at right angles to the first path. As shown in the diagram, the flux path is along the direction of rolling in teeth but is at right angles to the direction of rolling in the core. Therefore, low iron loss occurs in the teeth but the iron loss in core becomes much higher. Therefore, only a small reduction in total iron loss can be achieved and that too in machines where it is possible to have all the teeth over a pole pitch practically parallel to each other. This condition is almost achieved in machines with a large diameter and large number of poles. In a typical generator as shown in Fig. 12 2, the reduction may not be achieved as even in teeth the direction of flux is not along the direction of rolling.

The modern synchronous machines use non directional (isotropic) cold rolled steel which has electrical characteristics identical to hot rolled steel but has much improved mechanical characteristics like uniformity of thickness of laminations, smoothness of surface of laminations, higher fatigue strength and lower clamping pressures.

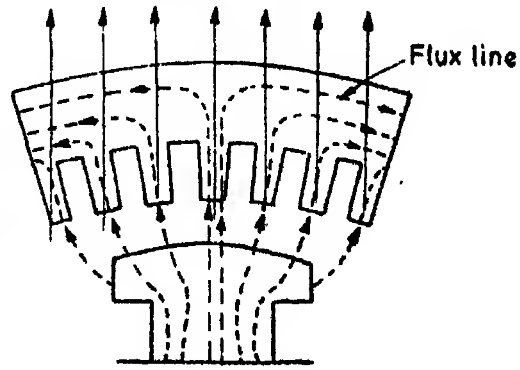


Fig. 12·2. Directions of flux paths in teeth and core, and direction of rolling in salient pole machines.

The commonly used grade for stator laminations is Grade 230, 0·5 mm thick. (See page 22). The laminations are insulated with paper stuck on one side, kaolin clay, or enamel. Paper or clay are rarely used now, the former because not being strictly a class *B* insulant, is likely to shrink in service. Also it does not provide protection for cut edges. Both paper and clay are slightly hydroscopic. Therefore, enamel is used for insulation of laminations.

The stator laminations are either punched as a complete circle or are in the form of segments (Fig. 12·3) in case the diameter is large. The maximum width of standard sheet steel is about one metre so that this represents the maximum diameter of laminations produced as a complete circle. Since the outside core diameters of practically all large hydrogenerators are much greater than one metre the laminations are made as segments. The number of segments per circle varies from 6 to 42 and it depends upon the number of slots and the number of stator sections.

The outside diameter of the stator frame of large hydro-generators usually exceeds 3·5 m and may be as high as 18 m. In order to facilitate handling in the factory and to relieve the problem of transport from factory to site of erection the stator core and frame are divided in two or more equal sections. For simplicity of manufacture and assembly at site all stator sections should be similar and this requires the number of segments per circle be divisible by the number of stator sections. The number of slots per segment should be an even integer as far as possible as an odd integer of slots complicates the core assembly and the keying arrangement.

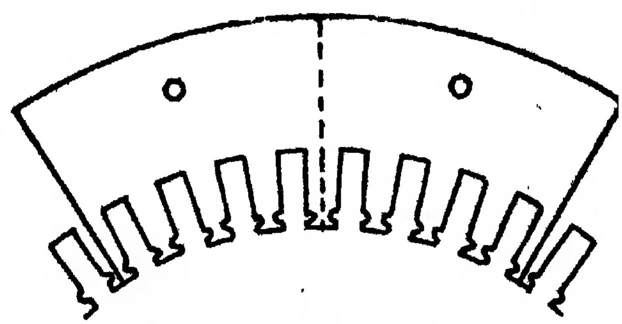


Fig. 12·3. Segmental lamination.

The insulated laminations are assembled in the stator frame. Every segment has two keyways which engage with keybars provided in the stator frame. In building up the core to the required thickness, successive layers are arranged so that the butt joints between adjacent segments of one layer come midway between those of the next.

12 5 2. Stator winding. The stator winding of all synchronous generators is star connected with neutral earthed. This arrangement has the advantage that the winding has to be insulated to earth for the phase voltage and not the line voltage. Star connection also has the advantage that it eliminates all triple frequency harmonics from the line voltage.

The present practice is to use double layer lap or wave windings with 60° phase spread. Fractional slot windings are used to reduce higher order harmonics and pitch of the windings is so selected that 5th and 7th harmonics are greatly reduced.

The windings may have multiturn coils or single turn coils and in the latter case the winding is known as *bar winding* (see page 289) High voltage machines having a large number of poles have a relatively large number of turns and therefore multiturn coils are used in these machines. On the other hand, low voltage machines have smaller number of turns and therefore employ bar winding. The multi-turn coils are machine made while the coils of a bar winding are made by hand.

The capacity of pull out machines used for the making of the coils limits the physical dimensions of the multi-turn coil side to approximately 75×25 mm^2 with a length of slot portion to about 3m and pole pitch not exceeding 0 8 m. Therefore, multiturn coils cannot be used for heavy current machines with current per circuit greater than 1500 A. The choice in this case is a bar winding with either a lap or a wave connection

In case of a bar winding, the stator winding is designed as a Roebel bar winding having two bars in each slot. The Roebel bar consists of a large number of conductors insulated from each other and connected to each other in parallel. The conductors are put in two layers along the width of stator slot. The conductors in a Roebel bar are so twisted that each one of them occupies all possible positions along the height of the slot. This Roebel transposition greatly reduces the eddy current loss in conductors.

The bar winding has many advantages as compared with a winding with multi-turn coils. These are :

(*i*) In a multi-turn coil there are many turns in each coil side while a bar winding has only one turn per coil side. Therefore, in addition to the main insulation to earth the multi-turn coil requires each turn to be insulated with several layers of mica tape in order to provide sufficient dielectric strength to withstand impulse voltages. This inter-turn insulation lowers the space factor.

In the bar winding with two bars per slot the interturn insulation is twice as thick as the main insulation to earth so that no special precautions against impulse voltages are necessary

(*ii*) Special advantages of the bar winding are noticed when the winding is a wave winding. A lap connected winding with p poles and q slots per pole per phase requires $(p-1)$ pole to pole connectors and $p\,(q-1)$ coil to coil connectors while a similar wave winding requires only one reversing jumper extending over one pole pitch and no coil to coil connectors. Therefore, a wave winding avoids the use of a large number of connectors and is particularly useful for machines with a large number of poles.

The advantages of a winding with multi-turn coils are :

(*i*) These windings allow greater flexibility in selecting the value of stator slots to give a required number of turns per phase.

(*ii*) Since the multi-turn coils are machine made, they are cheaper than the hand made coils of a bar winding. It is intersecting to note that many firms in Europe and USA particularly the latter when the labour charges are high prefer the use of multi-turn coils on account of their lower labour content and consequent lower overall cost.

However, in USSR the single turn bar winding with wave connection is used for all hydro-electric generators due to superior technical properties which certainly outweigh the slightly higher cost of this winding.

The use of bar windings is advantageous only when the stator current is high and the voltage low and, therefore, the choice of the machine voltage should be left with the manufacturer. This helps the designer to design the best and the cheapest winding. Since most of the hydro-electric generators these days operate on grids over a unit transformer, it should not be difficult to leave the choice of machine voltage to the designer.

12·5·3. Bracing of stator overhang. Electromagnetic forces are produced in the stator overhang due to the attraction between conductors carrying current in the same direction and due to repulsion between conductors carrying in the opposite direction.

Under normal conditions the electromagnetic forces produced by the current carrying conductors are negligible. But during sudden short circuits at the line terminals, the current in the windings may rise to about 15 times the full load current (or higher depending upon the value of direct axis sub-transient reactance) and the electromagnetic forces, being proportional to square of current, may rise to about 250 times the force under normal full load conditions. These forces may be either tangential or radial. Conductors in the same phase tend to bunch themselves together while conductors in different phases may suffer repulsion. Any movement of overhang tends to pull the slot conductors outwards and may result in cracking of insulation at the core ends. Thus the conductors in the overhang must be braced *i.e.* their mechanical strength be raised.

A method which is still widely used employs one or two circular steel rings to support the overhang against radial forces. These rings are in turn supported by 4–6 steel brackets. The use of steel rings and supporting brackets have the following disadvantages.

(*i*) the steel rings have to be heavily insulated with mica.

(*ii*) there is a large core loss in the steel rings produced by leakage flux especially in large generators with large pole pitches. Due to this loss the rings get overheated damaging the ring insulation.

Fig. 12·4 shows another method of bracing the stator overhang. The advantage of this method is that except the support steel plate, all other winding supports are made of non-magnetic material so that no loss occurs in them.

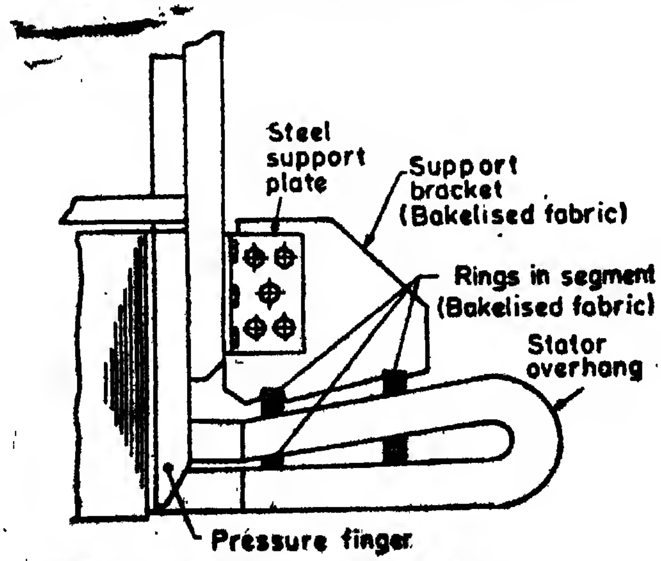


Fig. 12·4. Bracing of stator overhang.

12.5.4. Rotor body. The salient poles are attached to the rotor body. The type of rotor body used depends, in general, on the peripheral speed. The body is

(*i*) machined with its shaft from a forging,

(*ii*) built up from discs shrunk on to a shaft,

(*iii*) fabricated from a cast-steel spider mounted on the shaft and carrying laminar ring of segmental plates.

The forged steel construction is used for high speed machines. The earliest construction, particularly at relatively low outputs (below 40 MVA) consisted of a body and shaft extension made as a single forging (Fig. 12.5).

The forging has a high tensile strength and is useful for high speed machines. However, the cost of the forging and its machining is high and almost prohibitive in machines with large diameters.

Another type of construction (Fig. 12.6) employs thick rolled steel discs, 120—180 mm in length, either shrunk on to the shaft or spigotted to each other and formed into a solid ring by axial through bolts. The ring is attached to stub shaft at each end. This construction is cheaper and is used for generators running at 600 rpm and above and upto a peripheral speed of 190 m/s.

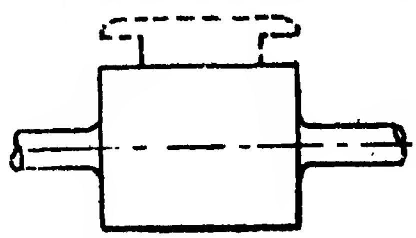


Fig. 12.5. Single forging of rotor body and shaft extention.

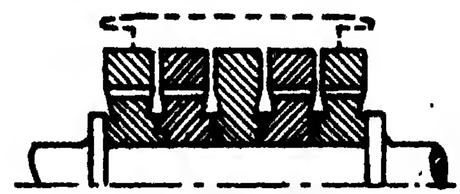


Fig. 12.6. Rotor body formed from separate discs.

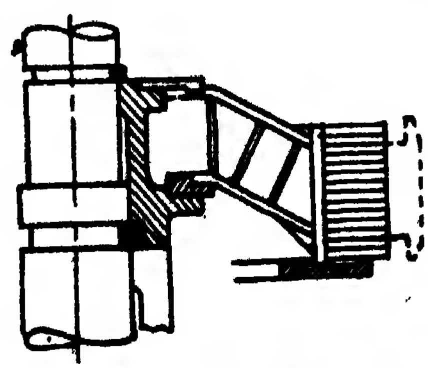


Fig. 12.7. Segmental rim on fabricated spider.

The cheapest form of rotor body construction is the punched segmental rim carried on a fabricated steel spider (Fig 12.7). The laminations are 1.8 mm thick and are in the form of overlapping segments tightly bolted. The rim rests on the spider arms and is driven from them by floating keys. The spider is therefore relieved of all centrifugal forces other than that due to its own mass and is required to transmit torque to the rim. Hence the spider is a light fabricated steel structure consisting of arms attached to a hub fitting on the shaft. This type of construction can be used for peripheral speeds upto 130 m/s. The advantages of a segmental rim are that it is easy to transport and assemble at site.

12·5·5. Poles. The poles are clamped or fixed to the rim in different ways. In the case of generators with peripheral speeds upto 25 m/s the poles are bolted to the yoke. Fig. 12·8 shows the bolted on pole construction. The attachment of poles to the periphery of the wheel is done by studs inserted from the underside of the rim. When studs are employed, the laminations are pierced from side to side by a wrought iron bar to provide something solid in the poles for the studs to grip.

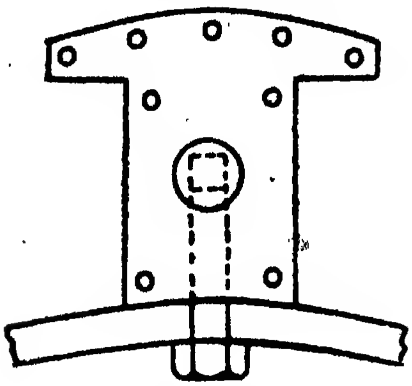


Fig. 12·8. Bolted on pole construction.

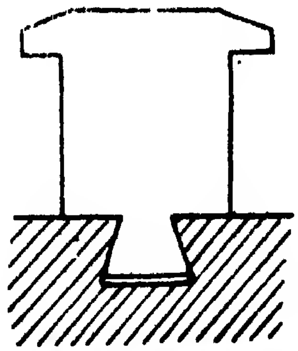


Fig. 12·9. Dove tail construction.

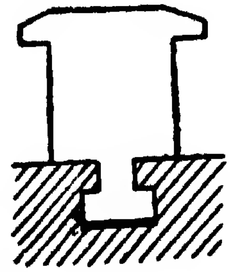


Fig. 12·10. T-head construction.

In the case of water wheel generators having peripheral speeds between 20 to 80 m/s either the dove tail (Fig. 12·9) or the T-head type of construction (Fig. 12·10) is used for fixing the pole to the yoke.

Double dove tail construction can be used in case the poles are very wide. Sometimes a multiple T-head construction as shown in Fig. 12·14 is used.

The cross section of the poles may be reactangular or circular. The circular cross-section can only be used if the poles are to be massive. The circular poles have the following advantages over rectangular poles :

(*i*) The length of mean turn of the winding is smallest with circular poles and therefore cost of copper is reduced. The copper losses are also less.

(*ii*) The whole of the winding surface is uniformly exposed to air and therefore ventilation is better.

(*iii*) The field coils with circular poles are circular in shape and therefore they cannot be easily deformed by centrifugal forces. In the case of rectangular poles, the coils easily bulge out due to centrifugal forces (Fig. 12·11) and in order to keep them in place, distance blocks have to be used (Fig. 12·12).

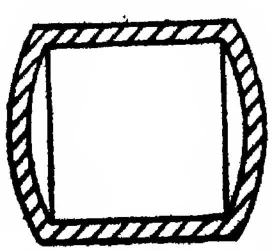


Fig. 11·11. Bulging out of rectangular coils.

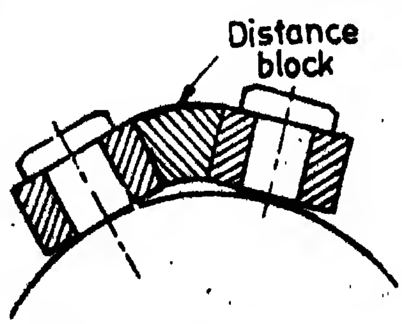


Fig. 12·12. Distance blocks.

Siemens has developed a new type of rotor construction, which is very useful for high speed machines. It is called *comb type* construction. In this alternate packets of

laminations are interleaved with packets in the rim, the poles being secured to the rim by axial through bolts.

12·5·6. Field winding. In hydroelectric generators, the so-called *"strip on edge"* winding is used for the field coils. The field coils are formed from a flat copper strip wound edgewise with interturn insulation on a machine operated former. The coils are then cured and consolidated under a pressure exceeding that due to centrifugal force on the coils at overspeed.

The coils may have a smooth surface or may have some of the turns made of a wider copper strap Fig. (12·13) so that they protect outside and act as cooling fins. The cooling fins substantially increase the external bare copper cooling area thereby lowering the temperature rise.

The coils are supported by Vee blocks in order to withstand centrifugal forces. Alternatively, a canted coil construction (Fig. 12·14) may be used. In this construction, the arrangement of turns is such that it avoids the necessity of side support for the coils.

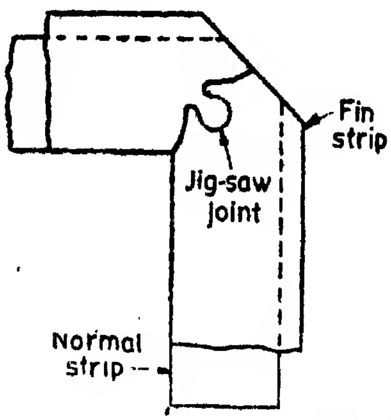


Fig. 12·13. Normal and fin strips for field winding conductor.

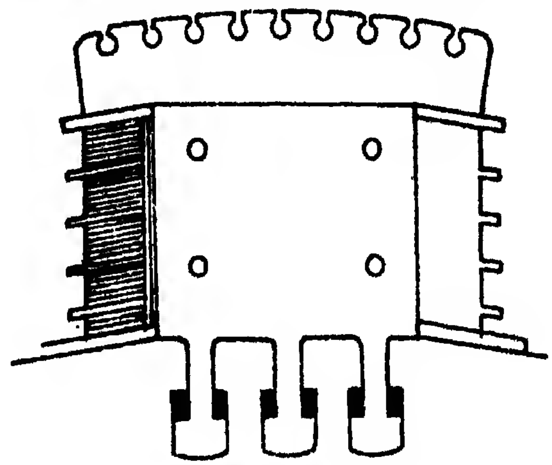


Fig. 12·14. Canted field coil.

12·5·7. Damper winding. In order to damp out the oscillations damper windings are used which are housed in the pole shoes. These damper windings consist of heavy copper rods, one in each slot or hole, riveted at the ends to common bars one at each end so as to form short circuited grid or the rods may be brazed to copper end bars attached to the pole end plates. In most cases copper rods in the pole shoe are adequate without interpolar cage connection (Fig. 12·15). In certain cases (Fig. 12·16), the short circuiting end bars are carried right around the rotor so that they form two short circuiting rings all around the periphery. This converts damper winding into a cage winding.

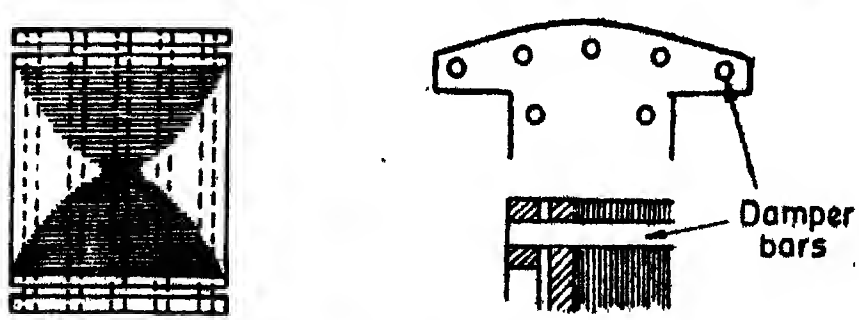


Fig. 12·15. Damper winding without interpolar connections.

When the damper rods are completely burried in iron as in Fig. 12·8, their inductance may be appreciable, causing the damper winding currents to lag. By opening the holes to the air gap, as in Fig. 12·14, the inductance is reduced and the power factor of damper winding current is improved.

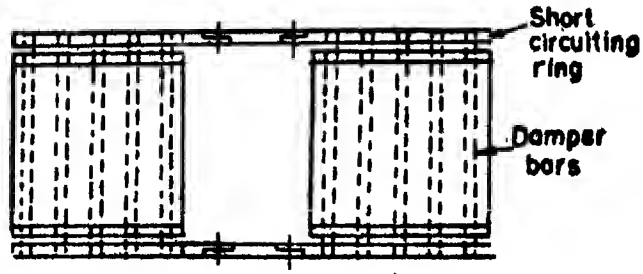


Fig. 12·16. Damper winding with interpolar connections.

The damper bars are subjected to considerable centrifugal forces as the peripheral speed of synchronous machines is high. Special attention must be paid to connecting bars lying in the interpolar region as this portion has no other support except the support of its ends in the pole shoe.

12·5·8. Bearings. The bearings for horizontal shaft hydro-generators are of conventional types. In the case of vertical shaft generators special features have to be incorporated in the bearing set up because of the requirements of the rotor and the turbine runner and the hydraulic thrust which may be twice the dead weight of rotating masses. The thrust bearing may be provided either in the top or the bottom spider depending upon the construction employed. Oil is supplied to the bearings by pumps, and cooled externally. Another arrangement uses direct cooling by circulating water in tubes embedded in the bearing metal.

12·5·9. Brakes and jacks. Large machines may take half an hour to stop, after the prime mover has been shut down, if not braked. In order to bring the rotor quickly to a stop in the case of any disturbance or fault, the generator is provided with brakes so that the machine may not run for longer periods at low speeds which are dangerous for the thrust bearings. The brakes are so designed that they can dissipate the complete energy of the rotating parts and the machine is brought to rest within three minutes. The brakes may be air or oil operated. The brakes have pads made of asbestos interlaced with metal wire. When the brakes are applied the pads come in contact with the rotor ring.

The brakes are also made to work as jacks when erecting and overhauling the machines.

12·5·10. Slip rings. The slip rings are required to supply excitation to the field winding. The slip rings are made of steel and are shrunk over cast iron sleeve with micanite as insulation between the two.

12·6. Construction of Turbo-alternators. All modern turbo-alternators are 2 pole machines and their speed is 3000 r.p.m. corresponding to a frequency of 50 Hz.

Turbo-alternators are characterized by long lengths and short diameters. This is because it is not possible to increase the diameter beyond a certain value (1·2 m) owing to the limitations imposed by mechanical considerations like centrifugal force, deflection of shaft and the critical speed. The diameter being limited, the only way to raise the rating is to increase the length, with the rotor diameter limited to 1·2 m, the active core length must be of the order of 10 mm per MVA Thus a 500 MW generator has a core length of 5 m, a shaft length of 12 m with outside diameter of stator core at about 3 m and that of outer casing 4 m.

With large lengths of core, it is very difficult to cool the machine, especially its central portions. In fact, the cooling of turbo-alternators is one of the most complex engineering problems.

12·6·1. Stator core. The stator core is built up segmental laminations of type as shown in Fig. 12·3 page 726. The use of grain oriented steel laminations with direction of

flux being along the grains in the armature core and perpendicular in teeth (Fig. 12·17) results in lower core loss but it is usual to use cold rolled non-oriented steel.

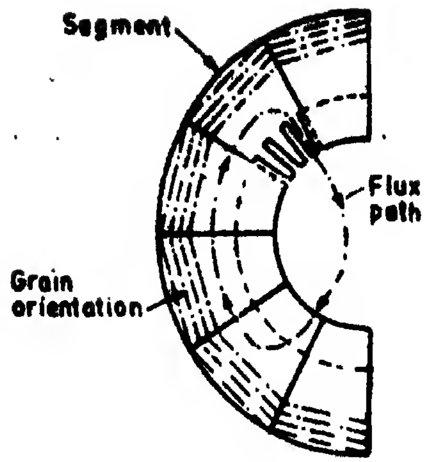


Fig. 12·17. Use of grain oriented steel laminations for stator of turbo-alternators.

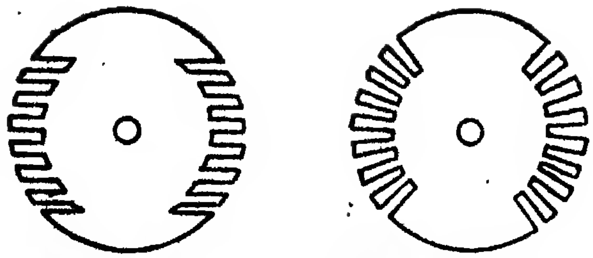
12·6·2. Stator winding. The windings of small turbo-alternators are designed to generate a voltage of some standard system level. For large machines which are permanently connected to a power network through a transformer, the choice of voltage is left to the designer. The choice of generation voltage is important. Both high voltage and low voltage generators have their advantages and disadvantages. A generator designed for a high voltage has a small current but requires the use of thicker slot insulation. On the other hand, a generator designed with a low voltage has high currents which produce large pulsational forces between conductors which may be as high as 80 kN/m of conductor length for a 500 MW machine. Also the presence of large currents requires the use of multi-circuit windings and also the use of laminated and transposed conductors to decrease the eddy current loss.

The stator winding of turbo-alternators is a double layer winding with the pitch of winding so adjusted as to reduce the 5th and 7th harmonics.

A generator designed for high voltage no doubt requires a high level of slot insulation but permits the use of smaller sized conductors with the ensuring advantage of lesser number of parallel circuits, smaller pulsational forces between conductors lesser conductor subdivision and ease in coil formation and installation. The generation voltages normally used are 15 kV for 100—200 MW machines and 20—25 kV for larger machines.

The electromagnetic forces developed in turbo-alternators under short circuit conditions are very high on account of large pole pitch and high armature mmf per pole. The problem of bracing the overhang to withstand electromagnetic forces is more acute in turbo-alternators because the conductors in the overhang are concentrated in only two sections (for 2-pole machines). The situation, in this regards, is comparatively much simpler in hydrogenerators where on account of large number of poles, the overhang is distributed over a large number of sections which eliminates the concentration of conductors in the overhang. Therefore, the overhang has to be highly reinforced in turbo-alternators. Hardwood blocks with glass fibre cord or tape along with non-magnetic metallic brackets are used for bracing the overhang.

12·6·3. Rotor. The cylindrical or non salient pole rotor is adopted for tubro alternators, the field winding being distributed in slots, instead of being wound on salient poles. The rotor is generally made up of chromium nickel-steel or chromium molybdenum steel. The rotor consists of a core and shaft generally forged in one piece except in very large sizes.



(a) Radial Slots (b) Parallel Slots
Fig. 12·18. Cylindrical rotors

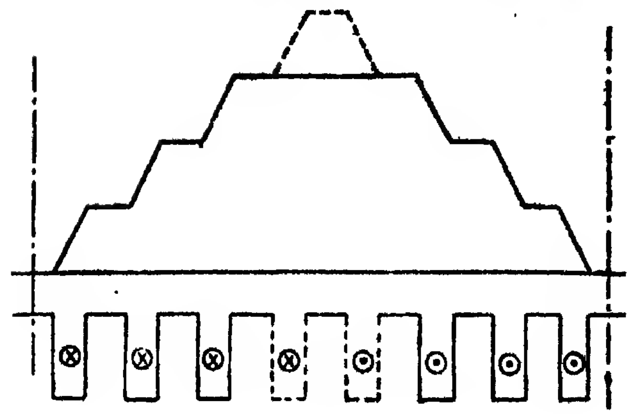


Fig. 12·19. Flux distribution in tubro-alternators

Slots are milled in the rotor for inserting and securing the field windings. Rotors are distinguished as (*i*) radial slot (*ii*) parallel slot rotors as shown in Fig. 12·18 (*a*) and (*b*) respectively. Usually the radial slot construction is used. Generally two thirds of rotor is wound and the rest one third is left without slots. This unwound portion forms the so called large tooth through which the main part of generator flux passes. Consider the developed cylindrical surface of a turbo-alternator as shown in Fig. 12·19. The flux distribution is shown by the heavy lime curve. The total flux per pole is proportional to the area enclosed by the curve formed by the heavy lines. Now if two more slots are added to each pole, the flux per pole is increased by the amount proportional to the area enclosed by the dotted curve. This shows that only a small increase in flux is produced and that too at the cost of considerable increase in expenditure (33% in this case) and rotor resistance and consequent rotor copper loss which is not easy to dissipate. Hence the increase in number of rotor slots is not justified considering the additional cost involved without any significant increase in flux, and also the increased rotor copper loss and consequent increase in temperature rise on account of increased losses and difficulty in dissipation of losses from rotor. The rotor, therefore, is slotted for only two third of its periphery.

The slots have equal width throughout their depth and the rotor teeth are tapered. The teeth have minimnm area of cross-section at the bottom and therefore the mechanical stresses produced there limit the slot dimensions. The slots are parallel sided because rectangular conductors have to be used. Concentric multiturn coils are accommodated in slots which are a multiple of four. The slot pitch being chosen to avoid undesirable harmonics in the field form. The field winding consists of copper strips laid flat in the slots and insulated with moulded micanite or asbestos, thus obtaining a solid winding which will not shrink under the effects of centrifugal stresses and temperature rise. A manganese bronze or steel wedge is driven into the mouth of each slot for the purpose of keeping the winding in place. At the bottom of each slot a ventilating duct or a sub slot may be used for providing a thorough passage for cooling air.

The end connectors (overhang) of the field winding must be rigidly supported by end bells because of the large centrifugal forces due to high speed of rotation, but also because of still larger force to which the field winding is subjected in case of a sudden short circuit of armature. This is because under short circuit conditions, not only large transient currents are produced in the armature but also there are transient unidirectional voltages and currents which are many times the normal values.

The end bells are made of a non-magnetic austenitic steel (18%Mn, 3%Cr, 0·5%C) in order to reduce the leakage flux. These end bells are mechanically strong and have an ultimate strength of 1150 MN/m². A typical end bell and overhang arrangement for an air cooled machine is shown in Fig. 12·20.

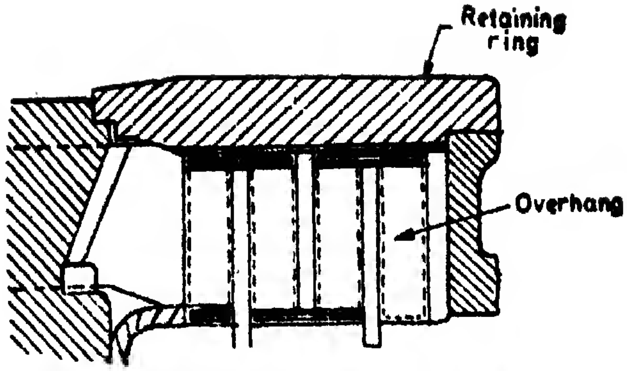


Fig. 12·20. Rotor overhang and retaining ring.

On account of the high electromagnetic forces produced on the conductors under fault conditions, it is obvious that the conductor and slot insulation will not only be designed to have sufficient dielectric strength but also have strength to withstand mechanical stresses as well.

12·7. Industrial Generators. Special-purpose and standby synchronous generators are useful in industry. These units are useful as captive units, and also as units to generate power in case of peak loads and in emergencies.

These generators are driven by internal combustion (IC) engines, gas turbines and steam turbines They are invariably designed with a horizontal rotor with shaft supported on end-shields or padestal bearings. The power ratings of these machines is small and therefore they are built with laminations which are complete rings. The use of complete ring laminations considerably reduces the cost of manufacture and core assembly. Segmental laminations are used in case the diameter is greater than one metre. The stator winding is of conventional type.

Modern industrial generators use a 4-pole construction with brushless excitation. Salient pole construction is used and the poles may be attached to the rotor body using a bolted on pole or a T head construction. Fig. 12·21 shows a typical spider and pole assembly for a 6 pole generator. In generators driven by IC engines it is necessary to include sufficient flywheel effect in the mass of the rotor generator or where this is uneconomic, additionally to attach a flywheel that gives the required inertia. The flywheel effect provided must be sufficient to prevent undue flicker in the voltage or changes in frequency.

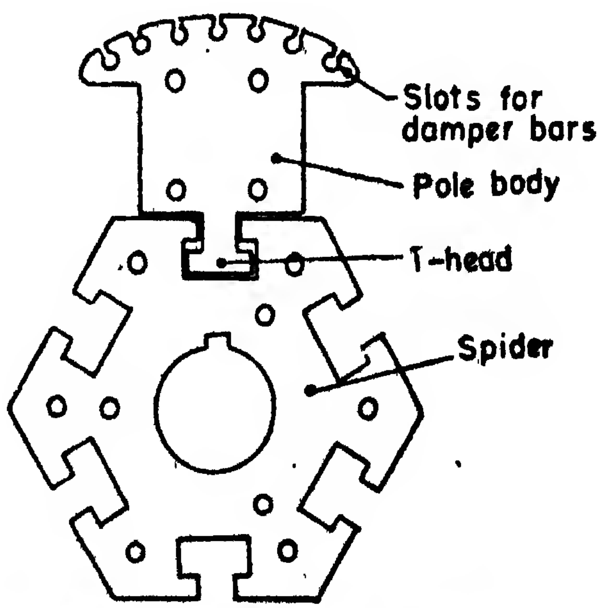


Fig. 12·21. Pole and spider assembly.

DESIGN

12 8. Output Equation. The output equation for polyphase a.c machines have been derived on page 456.

$$\text{kVA output } Q = C_0 D^2 L\, n_s \qquad ...(12\cdot1)$$

where output co-efficient $C_0 = 11\ B_{av}\ ac\ k_w \times 10^{-3}$ ...(12·2)

The output equation in terms of peripheral speed is

$$\text{kVA output } Q = (1\cdot11\ B_{av}\ ac\ k_w \times 10^{-3})\ V_a^2 L/n_s \qquad ...(12\cdot3)$$

12 8·1. Choice of specific magnetic loading. Some of the factors that influence the choice of average gap density have been discussed in Art. 8·1·7 (page 458). These factors along with same additional factors which are specific to the choice of average gap density for synchronous machines are discussed below.

(*i*) *Iron loss.* A high value of flux density in air gap leads to a high value of flux density in stator teeth and core, resulting in high iron loss with consequent decrease in efficiency and increase in temperature rise.

Therefore, a lower value of gap density should be used in order to increase the efficiency and to decrease the temperature rise.

(*ii*) *Voltage.* In case of machines designed for high voltages, the space occupied by insulation becomes greater and smaller space is left for teeth. Therefore, a lower value of gap density should be used in high voltage machines to avoid excessive values of flux density in teeth and core.

(*iii*) *Transient short circuit current.* A high value of gap density results in decrease in the leakage reactance of the machine with consequent increase in initial value of armature under short circuit conditions. Therefore, a low value of gap density should be used to limit the initial electromagnetic forces under short circuit conditions.

(*iv*) *Stability.* The maximum power which a cylindrical rotor machine can deliver under steady state conditions is $P_{max} = EV/X_s$ where E is the excitation voltage, V is the terminal voltage and X_s is the synchronous reactance. Therefore, the maximum power or the steady state stability limit of a machine is inversely proportional to its synchronous reactances. If a high value of gap density is used, the flux per pole is large and therefore a smaller number of turns are required for the armature winding. This results in reduction in the value of synchronous reactance (as synchronous reactance is directly proportional to the square of number of turns). Therefore, the use of a high gap density improves the steady state stability limit.

(*v*) *Parallel operation.* All synchronous generators, except those required to feed isolated loads, are connected in parallel with other synchronous generators. The satisfactory parallel operation of synchronous generators is dependent upon the *synchronising power*, the higher this power, the higher is the capability of the system to keep the machines in synchronism. The synchronising power is inversely proportional to the synchronous reactance and therefore machines designed with high value of gap density operate satisfactorily in parallel.

Following are the normal values of average gap density for the conventially cooled generators :

Salient pole machines — 0·52 to 0·65 Wb/m²

Turbo-alternators — 0·54 to 0·65 Wb/m².

Lower values normally apply to smaller sized machines.

12·8·2. Choice of specific electric loading. Some factors which influence the choice of specific electric loading *ac* have been discussed in Art 8·1·8, page 460. These factors

along with some additional factors relevant to synchronous machines are explained below.

(*i*) *Copper loss and temperature rise*. A high value of *ac* gives higher copper loss resulting in lower efficiency and higher temperature rise. The value of *ac* used depends upon the cooling techniques employed. Higher values of *ac* are used in machines which employ cooling techniques that effectively dissipate the generated heat.

(*ii*) *Voltage*. A higher value of *ac* can be used for low voltage machines since the space required for insulation is small.

(*iii*) *Synchronous reactance*. The value of *ac* affects the leakage reactance and armature reaction in the machine. A high value of *ac* leads to high values of leakage reactance and armature reaction and consequently a high value of synchronous reactance. Therefore, a machine designed with a high value of *ac* will have (*i*) a poor inherent voltage regulation (*ii*) low current under short circuit conditions and therefore many large units especially turbo-alternators are designed with a large value of *ac* in order that they may be able to withstand momentary short circuits without mechanical injury and (*iii*) low value for steady state stability limit and small synchronizing power and consequently leads to instability.

(*iv*) *Stray load loss*. The stray load loss increases steeply with an increase in *ac*.

Following are the usual values for specific electric loadings, used in conventially cooled generators.

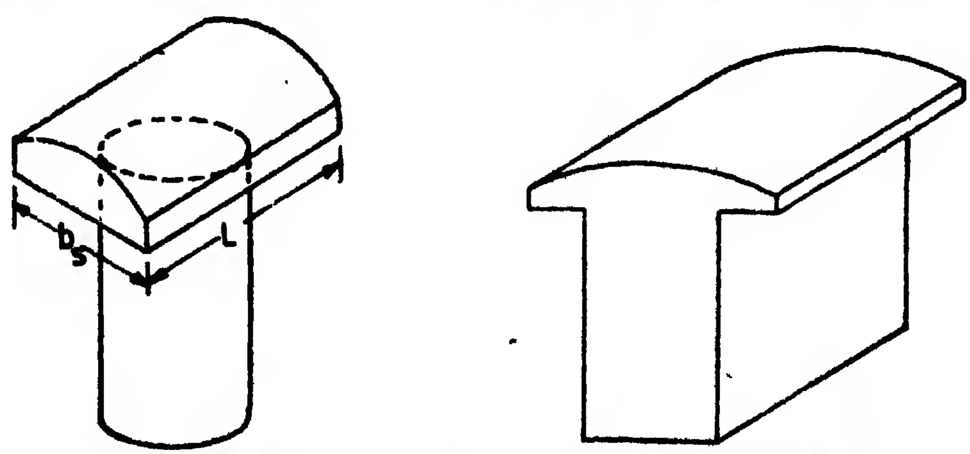
Salient pole machines	—20,000 to 40,000 A/m
Turbo-alternators	—50,000 to 75,000 A/m.

DESIGN OF SALIENT POLE MACHINES

12·9. Main Dimensions. Diameter D is the diameter of stator bore. The outer diameter of rotor D_r is nearly equal to D as the length of air gap is negligible as compared with diameter D.

The selection of diameter D depends upon

(*i*) the type of poles used (*ii*) the permissible peripheral speed.



(*a*) Round pole (*b*) Rectangular pole

Fig. 12·22. Shape of salient poles.

There are two types of poles used for salient pole machines

(*i*) round poles (*ii*) rectangular poles.

(*i*) *Round poles*. (Fig. 12·22 *a*). When round poles are used the ratio of pole arc to pole pitch b_p/τ is between 0·6 to 0·7. Under these conditions it is possible to use round poles with square pole shoes. Therefore, for round poles

$$\text{length of pole} = \text{width of pole shoe} \quad \text{or} \quad L = b_p$$

and $$\text{ratio } L/\tau = 0{\cdot}6 \text{ to } 0{\cdot}7 \qquad \ldots(12{\cdot}4)$$

(*ii*) *Rectangular poles* (Fig. 12·22 *b*). The ratio of pole arc to pole pitch varies between 1 to 5 for rectangular poles. However, this ratio should not exceed 3 for normal machines otherwise the design of field system becomes uneconomical.

Therefore, for rectangular poles :

$$\text{ratio } L/\tau = 1 \text{ to } 5. \qquad \ldots(12{\cdot}5)$$

From Eqns 12·4 and 12·5 it is clear that the diameter with circular poles is comparatively larger than that with rectangular poles.

The deciding factor for the diameter is the peripheral speed. The rotor should be designed to withstand centrifugal forces produced under runaway speeds.

The values of run-away-speeds for different types of turbines have already been given on page 725.

The values of allowable peripheral speeds for different types of pole attachments are :

Bolted on pole construction—50 m/s.

Dovetailed and T head construction—80 m/s.

The diameter of the rotor should be such that sufficient flywheel effect is obtained.

Example 12·1. *Determine the main dimensions for a 1000 kVA, 50 Hz, 3 phase, 375 r.p.m alternator. The average air gap flux density is 0·55 Wb/m² and the ampere conductors per metre are 28,000.*

Use rectangular poles and assume a suitable value for ratio of core length to pole pitch in order that bolted on pole construction is used for which the maximum permissible peripheral speed is 50 m/s. The run away speed is 1·8 times the synchronous speed.

Solution.

Synchronous speed $n_s = 375/60 = 6{\cdot}25$ r.p.s.

Number of poles $p = 2\times50/6{\cdot}25 = 16$.

Assume a winding factor of 0·955.

From Eqn. 12·2 output co-efficient $C_0 = 11\, K_w.\, B_{av}.\, ac.\, 10^{-3}$

$$= 11\times0{\cdot}955\times0{\cdot}55\times28000\times10^{-3} = 162$$

$$\therefore \quad \text{Product } D^2L = \frac{Q}{C_0 n_s} = \frac{1000}{162\times6{\cdot}25} = 0{\cdot}987 \text{ m}^3.$$

Taking $L/\tau = 2$

We have $L = 0{\cdot}393\, D$

$$\therefore \quad 0{\cdot}393\, D^3 = 0{\cdot}987$$

$$D = 1{\cdot}36 \text{ m and } L = 0{\cdot}535 \text{ m}.$$

Peripheral speed $V_a = \pi\, D\, n_s = \pi\times1{\cdot}36\times6{\cdot}25 = 26{\cdot}7$ m/s.

Peripheral speed at run-away speed $= 1{\cdot}8\times26 = 48$ m/s. This is below 50 m/s and therefore a simple bolted on pole construction can be used.

Example 12·2. *Find the main dimensions of a 2500 kVA, 187·5 r.p.m., 50 Hz, 3 phase, 3 kV, salient pole synchronous generator. The generator is to be a vertical, water wheel type. The specific magnetic loading is 0·6 W/bm² and the specific electric loading is 34000 A/m. Use circular poles with ratio of core length to pole pitch=0·65. Specify the type of pole construction used if the run-away speed is about 2 times the normal speed*

Solution.

Synchronous speed $n_s = 187{\cdot}5/60 = 3{\cdot}125$.

Number of poles $p = 2\times 50/3{\cdot}125 = 32$.

Assuming a winding factor of 0·955,

output co-efficient $C_o = 11\times 0{\cdot}955\times 0{\cdot}6\times 34000\times 10^{-3} = 214$.

$$\therefore \quad \text{Product } D^2L = \frac{Q}{C_o\, n_s} = \frac{2500}{214\times 3{\cdot}125} = 3{\cdot}74 \text{ m}^3.$$

We have $L/\tau = 0{\cdot}65$

or $L = 0{\cdot}65\times(\pi/32)\, D = 0{\cdot}0638\, D$.

$\therefore$ $0{\cdot}0638\, D^3 = 3{\cdot}74$

or $D = 3{\cdot}9$ m and $L = 0{\cdot}245$ m.

Peripheral speed $V_a = \pi\, D\, n_s = \pi\times 3{\cdot}9\times 3{\cdot}125 = 38{\cdot}2$ m/s.

The runway peripheral speed will be about 80 m/s and therefore a dove-tail construction is used for attaching the rotor poles to the rim.

Example 12·3. *Find the main dimensions of a 100 MVA, 11 kV, 50 Hz, 150 r.p m., 3 phase water wheel generator. The average gap density is 0·65 Wb/m² and ampere conductors per metre are 40,000. The peripheral speed should not exceed 65 m/s at normal running speed in order to limit the run-away peripheral speed.*

Solution.

Synchronous speed $n_s = 150/60 = 2{\cdot}5$ r.p.s.

Number of poles $p = 2\times 50/2{\cdot}5 = 40$.

Taking winding factor $K_w = 0{\cdot}955$,

output co-efficient $C_o = 11\times 0{\cdot}955\times 0{\cdot}65\times 40{,}000\times 10^{-3} = 274$.

$$\therefore \quad \text{Product } D^2L = \frac{Q}{C_o\, n_s} = \frac{100000}{274\times 2{\cdot}5} = 146 \text{ m}^3.$$

Trying circular poles, we have

$$L/\tau = 0{\cdot}6 \text{ to } 0{\cdot}7.$$

Taking a value $\frac{L}{\tau} = 0{\cdot}65$, we get $L = \frac{\pi D}{40}\times 0{\cdot}65 = 0{\cdot}051\, D$.

$\therefore$ $0{\cdot}051\, D^3 = 146$ or $D = 14{\cdot}2$ m.

Peripheral speed at synchronous speed

$$V_a = \pi\, D\, n_s = \pi\times 14{\cdot}2\times 2{\cdot}5 = 111{\cdot}5 \text{ m/s}.$$

This is above the maximum permissible value of 60 m/s. Actually this is a very high peripheral speed and the peripheral speed with run-away-speed will be still higher (about 200 m/s.) Therefore, circular poles cannot be used for this speed and so rectangular poles are employed.

Ratio $L/\tau = 1$ to 5 for rectangular poles.

The following table gives value of L and D for different values of L/τ ratio

L/τ	D m	L m	V_a m/s
1·0	12·30	0·965	96·5
2·0	9·75	1·53	76·7
3·0	8·50	2·02	66·7
4·0	7·75	2·43	60·1
5·0	7·30	2·75	52·4

The value $L/\tau=4$ and 5 give designs where the rotor peripheral speed is below the maximum allowable value of 65 m/s.

We use $L/\tau=4$ as it results in a cheaper design for the field system. Therefore,

$$D=7{\cdot}75 \text{ m and } L=2{\cdot}43 \text{ m}.$$

12·10. Short circuit ratio. The short circuit ratio (SCR) of a synchronous machine is defined as the ratio of field current required to produce rated voltage on open circuit to field current required to circulate rated current at short circuit. Fig. 12·23 shows the open circuit and short circuit characteristics of a synchronous machine. According to the definition,

$$\text{SCR}=\frac{OF_o}{OF_s}=\frac{CF_o}{bF_s}=\frac{CF_o}{aF_o}=\frac{1}{aF_o/CF_o}$$

$$=\frac{1}{\dfrac{\text{per unit voltage on open circuit}}{\text{corresponding per unit current on short circuit}}}$$

$$=\frac{1}{X_d} \qquad \ldots(12{\cdot}6)$$

Thus the short circuit ratio is the reciprocal of synchronous reactance X_d, if X_d is defined in per unit value for rated voltage and rated current. The value of X_d for a given load is affected by saturation conditions that then exist, while SCR is specific and univalued for a given machine as it is defined at the rated voltage.

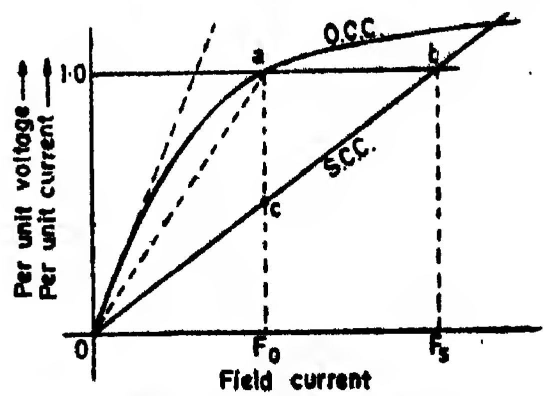


Fig. 12·23. O.C.C. and S.C.C.

For modern turbo-alternators, the SCR is normally between 0·5 to 0·7. This may have to be raised to 1·0–1·5 if the loading is likely to be capacitive (as with connection to long unloaded transmission lines or extensive high voltage cable runs). For salient pole hydro-electric generators SCR varies from 1·0 to 1·5.

12·10·1. Effect of SCR on machine performance

(*i*) **Voltage regulation.** A low value for SCR means that the synchronous reactance has a large value. Synchronous machines with low value of SCR thus have greater changes in voltage under fluctuations of load *i.e.* the inherent voltage regulation of the machine is poor.

(*ii*) **Stability.** A machine with a low value of SCR (and thus high value of X_s) has a lower stability limit as the maximum power output of machine is inversely proportional to X_s.

(*iii*) **Parallel operation.** Machines with a low value of SCR are also difficult to operate in parallel because a high value of X_s gives a small synchronizing power. This power is responsible for keeping the machines in synchronism. The parallel operation of machines with high value of X_s becomes more difficult if they are interconnected through a transmission line. This is because the impedance of the line between generators adds directly to the sum of the impedances of the machines. This increase in impedance acts to reduce the synchronizing power so that they are weakly held in synchronism. They become more sensitive to torque and voltage disturbances. Also the decreased synchronizing power is more likely to lead to disconnections of individual units of apparatus and shut downs from the operation of automatically reclosing type circuit breakers.

(*iv*) **Short circuit current.** A small value of SCR indicates a smaller value of current under short circuit conditions owing to large value of synchronous reactance. But this is not a problem, because the short circuit currents can be limited and thus the synchronous generators need not be designed with large values of synchronous reactance (*i.e.* low values of SCR).

(*v*) **Self excitation.** Machines feeding long transmission lines should not be designed with a small short circuit ratio (high X_s) as this would lead to large voltages on open circuit produced by self excitation owing to large capacitive currents drawn by the transmission lines.

We have noted that a machine with higher value of SCR has a higher stability limit and a low value of inherent regulation. On the other hand a higher value of SCR means a high value of short circuit current. Also a machine designed with a higher value of SCR has a long air gap which means that the mmf required by field is large. Hence a machine having a higher value of SCR is costlier to build. Present trend is to design the machine with a low value of SCR. This is due to the recent advancement in the fast acting control and excitation systems.

12·11. Length of air gap. The length of air gap greatly influences the performance of a synchronous machine. A large air gap offers a large reluctance to the path of flux produced by the armature mmf and thus reduces the effect of armature reaction. This results in a small value of synchronous reactance and a high value of SCR. Thus a machine with a large air gap (and consequently with a small X_s and a high SCR) has

(*i*) a small value of inherent regulation,

(*ii*) a higher value of stability limit,

(*iii*) a higher synchronizing power which makes the machine less sensitive to load variations.

In addition a machine designed with a large air gap has better cooling at the gap surface, lower tooth pulsation loss, lower noise level and a smaller unbalanced magnetic pull.

But with the increase in length of air gap, a larger value of field mmf is required resulting in increase of cost of the machine.

For salient pole machines of normal construction and having open type slots,

$$\frac{\text{length of air gap}}{\text{pole pitch}}=\frac{l_g}{\tau}=0{\cdot}01 \text{ to } 0{\cdot}015$$

where l_g is the length of air gap at the centre of poles.

For turbo-alternators, with massive rotors,

$$\frac{\text{length of air gap}}{\text{pole pitch}} = \frac{l_g}{\tau} = 0{\cdot}02 \text{ to } 0{\cdot}025.$$

For synchronous motors designed with maximum output 1·5 times rated output,

$$\frac{\text{length of air gap}}{\text{pole pitch}} = \frac{l_g}{\tau} = 0{\cdot}02.$$

12·12. Shape of pole face. The ratio of pole arc to pole pitch, ψ, varies between 0·67 and 0·75. If the value of ψ is too large ($\psi > 0{\cdot}75$), the interpolar flux leakage becomes excessive leading to high value of flux density in the pole body and improper flux distribution over the armature. On the other hand too small a value ($\psi < 0{\cdot}67$) will leave insufficient overhang of the pole shoe to support the field coil in the radial direction. A common practice is to use a value of $\psi = 0{\cdot}7$.

In salient pole machines the length of the air gap is not constant over the pole arc but increases from centre outwards in order to produce the required flux distribution. An attempt is usually made to obtain sinusoidal distribution of flux by proper shaping and proportioning of the pole shoe. For an exact sinusoidal flux distribution, length of air gap at a distance x from centre (Fig. 12·24)

$$l_{gx} = l_g / \cos\left(\frac{\pi x}{\tau}\right) \qquad \ldots(12{\cdot}7)$$

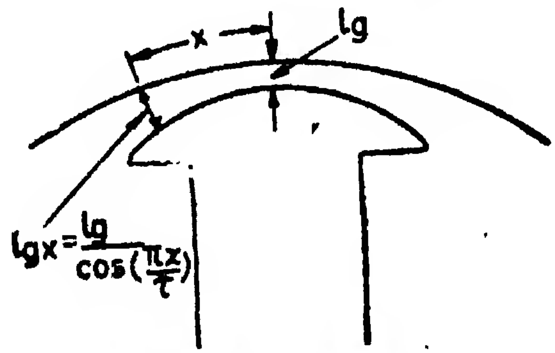


Fig. 12·24. Shape of pole face for sinusoidal flux distribution.

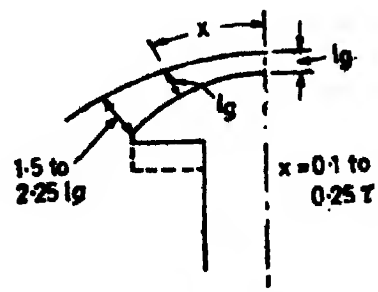


Fig. 12·25 Commonly used pole face profile.

A satisfactory gap flux distribution curve is generally obtained when the pole face is shaped as indicated in Fig. 12·25. For machines requiring no damper windings in the pole faces, the tip of the pole face may be rounded off as shown by full line. For machines with damper windings, a heavier pole face (as indicated by dotted line) is generally required.

12·13. Armature design. The windings used in synchronous machines may be single layer or double layer type. Machines having large values of flux per pole have small number of turns per phase and therefore a double layer bar winding is used for them. However, high voltage machines and machines with small values of flux per pole have a large number of turns per phase. Therefore, multi-turn coils are used for such machines For machines using multi turn coils the choice lies between double layer lap winding and single layer concentric winding. The former is dropped into open type of slots while with the latter type, hair pin coils (Fig. 6·41 page 627) are pushed through semi-enclosed slots.

12·13·1. Comparison between single and double layer windings. Double layer windings in open-slots have the following advantages over single layer windings in semi-enclosed slots :

(i) ease in manufacture of coils and lower cost of winding

(ii) less number of coils are required as spare in the case of winding repairs

(iii) fractional slot windings can be employed

(iv) fractional pitch coils can be used.

The single layer windings have the following advantages :

(i) higher efficiency and quieter operation because of narrow slot openings

(ii) space factor for slots is higher owing to absence of inter layer separator.

A complete description of various types of a.c. windings has already been given in Chapter 6.

Modern practice all over the world favours use of double layer windings Single layer windings are popular only in the Continent.

Where single turn coils are necessary as with turbo alternators and multipolar low voltage machines a double layer bar type lap or wave winding is used. Both types may have the bars pushed through semi-closed slots and be bent to shape at the other end when moderate conductor sections are used But heavy bars used in turbo-alternators must be completely formed before being inserted into open slots. It is advantageous to use chorded bar lap winding as it gives a shorter overhang and is more suited when there are several parallel circuits per phase.

12·13·2. Number of armature slots. The following factors should be considered for the selection of armature slots :

(i) **Balanced windings.** The number of armature slots must be such a number that a balanced winding is obtained. It may be mentioned here that some generators have been designed and built with armature windings having a small amount of unbalance with apparently no ill effects. However, the use of unbalanced windings may lead to overheating of rotor surface due to space harmonics, excessive triple frequency currents flowing in neutral, with ill effects on loads such as delta connected induction motors and due to circulating currents in other generators running in parallel.

(ii) **Cost.** A smaller number of slots leads to a slight saving because there are fewer coils to wind, form insulate, place into slots, and connect.

(iii) **Hot spot temperatures.** A smaller number of slots results in bunching of conductors (*i.e.*, conductors are close to each other) leaving smaller space for the circulation of air. This gives rise to high internal temperatures.

(iv) **Leakage reactance.** When the number of slots is small, leakage flux and therefore, leakage reactance is increased owing to conductors lying near each other.

(v) **Tooth ripples.** The tooth ripples in the field form and the consequent pulsation losses in pole face decrease if a large number of slots are used. Also the waveform of generated voltage is free from ripples.

(vi) **Flux density in iron.** With a larger number of slots a greater space is taken up by the insulation. This results in narrower teeth giving flux densities which may be beyond the acceptable limits. Also the teeth might become mechanically weak and may have to be supported at the ducts.

The value of slot pitch serves as a guide when choosing the number of armature slots.

The value of slot pitch y_s, depends upon the voltage of the machine. For high voltage machines which are normally built in large capacities, it is desirable to use a larger slot pitch. Following are the usual values for slot pitch :

$y_s \leqslant 25$ mm for low voltage machines,

$y_s \leqslant 40$ mm for 6 kV or low voltage machines,

$y_s \leqslant 60$ mm for machines upto 15 kV.

The stator slot pitch for large hydro-electric generators varies between 50 mm and 90 mm.

In salient pole machines, the number of slots per pole per phase is usually between 2 to 4.

It was discussed in Chapter 6 (page 275) that a ***fractional slot winding*** reduces the distribution factor for higher harmonics thus reducing their corresponding generated emfs and making the voltage waveform nearly sinusoidal.

Fractional slot windings are invariably used in synchronous generators.

12·13·3. Coil span. The highest amplitude harmonics in the flux distribution curve of salient pole generators are likely to be 5th and 7th. Therefore, the coil span is so chosen that these harmonics are drastically reduced. The maximum reduction of these harmonics is given by a coil span of 8·33 per cent of pole pitch. Therefore, this coil span is chosen where ratio of number of slots to number of poles does permit it, otherwise the coil span used should be as near to this value as possible.

12·13·4. Turns per phase. Flux per pole $\Phi=B_{av}\,\tau L$.

$\therefore$ Turns per phase $T_{ph}=\dfrac{E_{ph}}{4{\cdot}44\,\Phi f\,K_w}$

where E_{ph}=voltage per phase.

The above relation is applicable when all the turns of a phase are connected in series. But if there are 'a' parallel paths per phase,

$$E_{ph}=4{\cdot}44\,\Phi\,\frac{T_{ph}}{a}\,f\,K_w$$

or
$$T_{ph}=\frac{E_{ph}\times a}{44{\cdot}4\,\Phi\,fK_w} \qquad \ldots(12{\cdot}8)$$

12·13·5. Conductor section. Current in each conductor,

$$I_s=I_{ph}=\text{kVA}\times10^3/(3\,E_{ph})$$

But if there are 'a' parallel paths, the conductor current is $I_s=I_{ph}/a$.

For normally cooled machines, the permissible current density in the armature conductors is assumed to be with 3 to 5 A/mm².

$\therefore$ Area of cross-section of armature conductors $a_s=I_s/\delta_a$

where δ_a=current density in armature conductors, A/mm².

12·14. Armature windings, coils and their insulation. The armature windings of salient pole machines employ two types of coils :

(*i*) single turn bar, (*ii*) multi-turn.

12·14·1. Single turn bar—class B. A single turn bar winding is used in machines when the armature current per circuit exceeds 1500 A. On account of the large current

the conductor cross-section is accordingly large. Therefore, in bar windings the conductors are subdivided into many parts to reduce eddy current loss in them. The subdivision is achieved by laying a number of bare copper strips flat wise in the slot.

There are two conductors in slot if a bar winding is used. Each conductor consists of two vertical stacks of copper laminations (subdivisions of copper conductor) insulated with either treated asbestos or glass rovings. The asbestos insulation has a diametral thickness of 0·38 mm while glass insulation has an average thickness ranging from 0·29 to 0·38 mm. The advantage of glass covering is that it gives a high space factor.

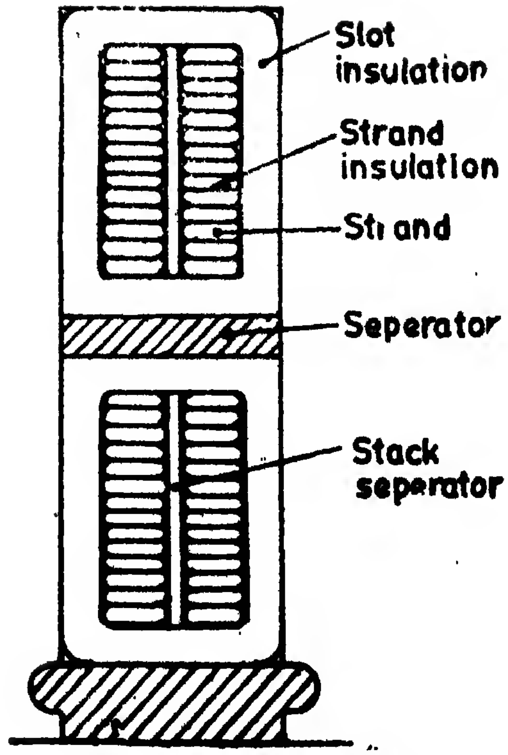


Fig. 12·26. Single turn bar coil

The dimensions of individual laminations (*i.e.* subdivisions) are determined partly by electrical considerations and partly by manufacturing requirements. It is a normal practice to design the stator so that the eddy current loss including the circulating current loss and the strand loss does not exceed about 1/5 of the I^2R (d.c.) loss. The width of individual strands varies between 4 mm to 7 mm. The thickness af strands rarely exceeds 3 mm.

Fig. 12·26 shows a cross-section of slot with a single turn bar coil. In order to achieve reduction in the circulating current loss, it is essential to use some form of transposition of conductor laminations in the slots. In the bar winding, the transposition of the slot portion of conductor laminations is effected by the Roebel transposition (Fig. 12·27) In this transposition each conductor lamination is arranged to move continuously through all positions in the depth of coil side so that the leakage reactance of all conductor laminations is equalized and consequently there is no circulating current between the laminations and hence the circulating current loss is eliminated.

The length required to transpose a strand from one strand to another is normally less than 40 mm.

The width of slot is usually less than 25 mm and with a value of 5 mm for the thickness of main slot insulation, the width available for the bare copper conductor is 2×7 mm. In fact, a width of 7 mm should be regarded as maximum for the bare conductor. On the other hand, the maximum width of copper strand is 4 mm on account of mechanical reasons.

The maximum depth that can be used for an individual strands is determined by the eddy current loss. As is clear from Eqn. 6·94 (page 322), the greater is the depth of strand the greater is the eddy current loss in it. The depth of an individual strand rarely exceeds 3 mm and is normally kept below 2·5 mm.

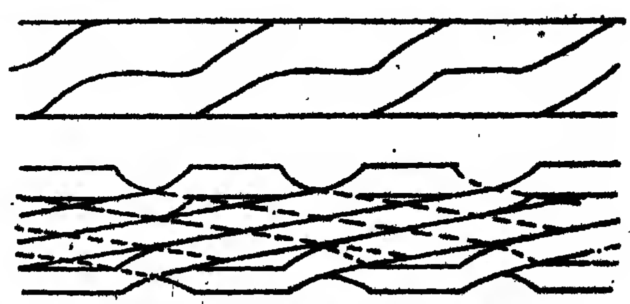


Fig. 12·27. Roebel transposition.

The eddy current loss is greater in the top coil side as compared with that in the bottom coil side. This difference in loss in the two coil sides produces a temperature rise

difference between the two coil sides which may be many degrees Celcius. The temperature rise difference between the top and bottom coil sides is reduced by increasing the number of strands and thereby reducing the thickness of strands in the top coil side. (A similar case of a single layer winding where the depth of individual strands is reduced, as one moves up from the bottom to the top of the slot, in order to equalize the eddy current loss in a conductor is illustrated in Example 6·43 page 328.)

Bitumen Micaflake Insulation System. It is clear from Fig. 12·27 that the stranded and transposed bar has irregular surfaces both at the top and bottom because of strand cross-over. The application of bitumen main insulation leaves voids at every cross-over which leads to corona discharge thereby causing rapid failure of main insulation tube.

This can be avoided by using (*i*) asbestos boards as packers and or (*ii*) applying asbestos or mica putty.

The use of these methods the surface irregularities are filled and a smooth surface is both at the top and bottom of the coil side. The entire stack is saturated with epoxidised phenolic resin and then hot moulded at 160°C. The resin polymerises at this temperature and the dimensions of the bar are consolidated at the desired values.

Till a decade back, bitumen impregnated mica tape applied throughout the length of the bar or bitumen mica folium applied to slot portion of the bar with mica tape on the overhang portion were the most commonly used insulating materials for high voltage generators. The thickness of bitumen-mica main insulation is given in Table 12·1.

Table 12·1. Thickness of bitumen-mica main insulation.

Voltage kV	10	11	12	13	14	15	16	17	18
Thickness mm	3·6	3·8	4·1	4·3	4·6	4·9	5·1	5·4	5·6

The insulation on the overhang portion of coils has a thickness which is 75 per cent of that of main insulation and therefore two adjacent coils in the overhang have a total insulation between them which is 1·5 times slot insulation.

Micafolium applied in a width corresponding to the length of the straight portion of bar and is moulded on to the bar at a temperature of about 160°C in a machine. The mica tape 0·13 mm thick and 20 mm wide is wrapped by hand upto 20 half layers. This process is both time-consuming and expensive.

The wrapped bar is not used in case the thickness of slot tube is greater than 4 mm as above this thickness it may not be possible to get proper consolidation of insulation tube through machine moulding process. Therefore, the use of wrapped bars is limited to machines having voltages upto 12 kV.

Epoxy Novalak Mica Paper Insulation System. The stack of conductor laminations needs to be consolidated into a rigid mass in order to reduce the corona discharge. This is done by using Epoxy Novalak mica paper system wherein the rows of the conductor stack are bound with epoxy based resins. This is done by using two highly loaded epoxy glass separators. The stack is then pressed at 160°C to form a rigid mass. This type of construction does not require the filling of all external voids.

The overhang insulation consists of a number of layers of flexible Isopthalate varnished polyester backed mica flake tapes. The insulation on the slot portion of the conductors consists of a number of half lap layers of Epoxy Novalak bonded glass backed mica paper tape. The slot insulation is hot pressed at 160°C to produce a bar of desired

dimensions. A layer of asbestos is applied over the slot portion of conductors which in turn is coated with colloidal graphite paint. A coat of stress grading paint is applied to the bar in order to distribute the electrical stress evenly.

The thickness of Epoxy Novalak mica paper is 20% less than that of bitumen bonded insulation for the same voltage. Therefore, the thermal conductivity in this insulation system is 20% greater than the corresponding value in bitumen insulation system and so the temperature gradient across the slot insulation is lower. Also, Epoxy Novalak mica paper insulation system permits the machine to be operated at a higher temperature rise. These two factors *i.e.* the lower temperature gradient across the slot insulation and the higher permissible temperature rise results in a higher power output from a given frame size. In addition, the dielectric loss is also smaller

This insulation system has a thermal stability upto about 180°C and therefore can be used for machines designed for class F insulation (155°C).

12·14 2 Multi-turn coil—class B. The multi-turn coils are machine wound as described earlier. In these, in addition to insulation between individual strands, insulation between turns has to be provided. The interturn insulation should be designed to withstand surges of magnitude 1·5 times the line voltage, with a duration of a few microseconds. The inter turn insulation used is mica tape half overlap and asbestos The thickness of mica tape is 0·13 mm and that of asbestos 0·38 mm. Fig. 12·28 shows the slot details of the winding with 2 turns per coil. The insulation thickness used for different test voltage is specified in Table 12·2.

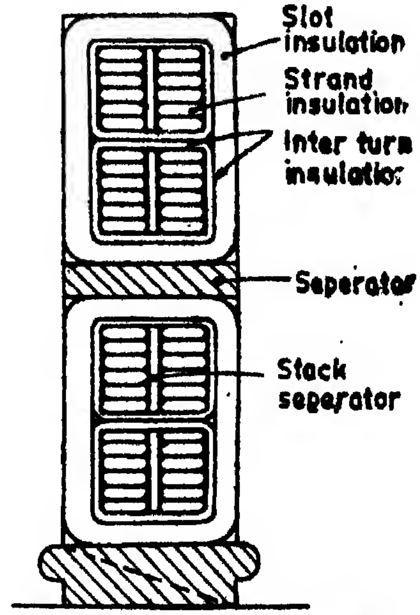


Fig. 12·28. Multi-turn coils.

Multi-turn coils Epoxy Novalak Mica Paper System. The Epoxy Novalak mica paper insulation used is different for slot portion of conductors and the overhang. Novalak mica paper tapes are used for the slot portion while Isopthalate varnished mica flake tapes are used for the overhang. The thickness of interturn insulation is dependent upon individual designs.

Table 12·2. Class B insulation for multi-turn coils.

Test voltage V	*Layers of mica half overlap*	*Layers of bitumen*
2000	1	1
5000	2	1
9000	3	1
12000	4	1

12·14·3. Synthetic resin and mica paper insulation. Till recently, the main insulating materials used for armatures of synchronous machines have mica flakes attached to paper by bitumen varnish. The performance of this insulation system has been satisfactory and cases of insulation failure have been few and in these the failure has resulted on account of bitumen used when used as inter-turn insulating material in multi-turn coils. Therefore, bitumen is being replaced with far more reliable thermo-setting resin systems. In the thermosetting resin insulating systems, the main insulation is composed of mica paper and not mica flakes. There are many systems but the two most commonly used systems are :

(*a*) *Vacuum impregnated process* (*VIP system*). In this system a number of layers of mica paper with glass backing and a binder are applied to the coils till the desired insulation thickness is obtained. The bars are then dried in vacuum and are impregnated with epoxy resin in order to fill the voids. After this the coils are pressed to the desired dimensions at 160°C at which the polymerization is complete.

(*b*) *Resin rich process*. In this system mica paper with a backing of 0·04 m thick glass cloth is used. The mica paper is saturated with Epoxy Novalak resin. Mica provides the desired electrical strength while the required mechanical strength is provided by the glass cloth. The conductors are covered with a number of layers of insulation till the desired insulation thickness is obtained. The coils are then pressed to size and cured at 160°C to complete the polymerization. The advantages of this system are :

(*i*) This insulation has a higher mechanical strength and is better equipped to withstand, without damage, the large forces which are produced during sudden short circuit at the machine terminals.

(*ii*) The loss angle of epoxy insulated coil is about 25% of that of the bitumen insulated coil. As the loss angle is indicative of presence of voids in insulation it is clear that the epoxy insulation is better placed than the bitumen insulation. The presence of voids, over the years, leads to ionization thereby causing insulation failure. Therefore, the epoxy insulation with smaller voids has a longer life because of lesser possibility of insulation failure.

(*iii*) The dielectric strength of epoxy insulation is approximately 40% greater than that of bitumen insulation which results in 20% reduction in thickness of insulation.

(*iv*) The bitumen bonded mica folium and bitumen impregnated mica tape insulation is capable of withstanding only class B temperatures (130°C). However, the synthetic resin mica paper insulation can withstand a temperature of 140°C and therefore it can be classified as class F insulation. In fact, it is a common practice to specify the stator winding insulation to withstand class F temperatures but in actual practice to operate them below allowable temperatures for class B insulation. The operation of the insulation at temperatures lower than the maximum that it can safely withstand results in longer life for the insulation.

12·15. Slot dimensions. The flux density in teeth at no load should not be more than 1·7 to 1·8 Wb/m².

$$\therefore \quad \text{Minimum width of tooth} \quad W_{t\,(min)}=\frac{\Phi}{\psi\,(S/p)\,L_i\times 1{\cdot}8}$$

Parallel sided slots are used. Therefore, the teeth are tapered and their minimum width occurs at the air gap surface.

Hence, maximum permissible width of slot $W_{s\,(max)}=y_s-W_{t\,(min)}$.

The depth of slot preferably should not exceed three times the width. Deeper slots may be used and sometimes slots are deliberately made deeper in order to have a high leakage reactance which limits short circuit currents.

12·16. Length of mean turn

Length of mean turn of armature, $L_{mt}=2L+2{\cdot}5\tau+0{\cdot}06\ \text{kV}+0{\cdot}2$ ...(12·9)

and kV = voltage of machine in kilovolt.

12·17. Stator core. The value of depth of core, d_c, can be calculated by assuming a suitable value of density B_c. The value of flux density in the armature core of salient pole machines lies between 1·0 to 1·2 Wb/m².

$$\text{Depth of armature core} \quad d_c=\frac{\Phi}{2\times L_i\times B_c} \qquad ...(12{\cdot}10)$$

$$\therefore \quad \text{Outer diameter of stator} \quad D_o=D+2(d_s+d_c) \qquad ...(12{\cdot}11)$$

12·13. Elimination of Harmonics The details of harmonics and their elimination have been given earlier in chapter 6 (see pages 290—303). However, some more details are being given here again :

The primary source of harmonics is the non-sinusoidal field form, if this can be made sinusoidal, the harmonics would be eliminated. A field form, which is nearly sinusoidal can be obtained as explained below :

The major reluctance to the path of flux is offered by the air gap. If the air gap is made to vary sinusoidally round the machine, the field form would be sinusoidal (the mmf should be same everywhere). In salient pole machines, the length of air gap is made to vary sinusoidally as described in article 12·12 on page 742. However, it suffices to increase the air gap gradually from centre outwards towards the pole tips. The length of air gap at the pole tips is about 1·5 to 2·25 times that at the pole centre, as shown in Fig. 12·25. An approximation of sinusoidal field form can be obtained by skewing the pole faces.

In cylindrical rotor machines, the length of air gap is constant around the machine and therefore the only way to obtain a sinusoidal field form is to make the mmf of the field winding to vary according to a sine law. This is done by distributing the winding in different slots.

In this connection it may be mentioned here that on account of saturation in iron parts, the field form cannot be sinusoidal in salient pole machines even if the air gap length is varied sinusoidally or in cylindrical rotor machines even if the mmf distribution in space is made sinusoidal. A fair degree of saturation in iron parts is unavoidable but a high degree of saturation should be avoided to obtain an approximately sinusoidal waveform.

An ideal sinusoidal field form is very difficult to achieve and therefore harmonic emfs will be generated in the winding. The redeeming feature is that the harmonics can be easily eliminated from the voltage waveform by properly designing the windings. The different methods for elimination of harmonics from the generated voltage are :

(*i*) **Distribution.** The armature windings are not concentrated but are distributed in different slots. The magnitude of harmonic emfs depends upon their distribution factors. The distribution factor for harmonics being small as compared with that of the fundamental and, therefore, the relative magnitude of harmonic emfs is small. (See page 296)

(*ii*) **Chording.** In short pitched coils, the emfs generated in the two coils do not add algebraically as explained in Art. 6·30 on page 294. The emf generated is proportional to cos ($n\alpha/2$) where α is the angle of chording and n is the order of harmonic. The harmonic emfs can, therefore, be considerably reduced or entirely eliminated by choosing a proper value of α, the angle of chording.

(*iii*) **Skewing.** The slot harmonics can be eliminated by skewing the pole face.

(*iv*) **Fractional slot windings.** The slot harmonic emfs can be drastically reduced and even completely eliminated from the output voltage waveform by using fractional slot windings. In fact, present day synchronous generators invariably use fractional slot windings on account of the fact that these windings give a much smaller distribution factor for harmonics as compared with that for the fundamental (See page 298)

(*v*) **Large length of air gap.** If we use a large air gap length, the reluctance is increased and therefore the magnitude of slot harmonics is reduced.

The flux pulsations can be reduced by having the number of slots per pole arc as an integer plus ½.

Example 12·4. *Determine a suitable number of slots and conductors per slot, for the stator winding of a 3 phase 3300 V, 50 Hz, 300 rpm alternator. The diameter is 2·3 m and the axial length of core is 0·35 m. The maximum flux density in the air gap should be approximately 0·9 Wb/m². Assume sinusoidal flux distribution. Use single layer winding and star connection for stator.*

Solution.

Synchronous speed $n_s = 300/60 = 5$ r.p.s.

Number of poles $p = \dfrac{2 \times 50}{5} = 20$.

Average flux density in air gap $B_{av} = (2/\pi) \times 0.9 = 0.574$ Wb/m².

$\therefore$ Flux per pole $= 0.574 \times \dfrac{\pi \times 2.3 \times 0.35}{20} = 72.5 \times 10^{-3}$ Wb.

Voltage per phase $E_{ph} = \dfrac{3300}{\sqrt{3}} = 1910$ V.

Turns per phase $T_{ph} = \dfrac{1913}{4.44 \times 50 \times 72.5 \times 10^{-3} \times 0.955} = 124$.

The slot pitch should be nearly 40 mm for 3·3 kV machines.

Slots per pole per phase $q = \dfrac{\pi D}{3py_s} = \dfrac{\pi \times 230}{3 \times 20 \times 4} \cong 3$.

As we have to use single layer winding, therefore an integral number of slots per pole per phase has to be employed.

Total number of stator slots $S = 3pq = 3 \times 20 \times 3 = 180$.

Total number of stator conductors $6T_{ph} = 6 \times 124 = 744$.

Conductors per slot $Z_s = 744/180 \cong 4$.

Therefore, total stator conductors used $= 180 \times 4 = 720$.

Turns per phase used $T_{ph} = 720/6 = 120$.

Thus there will be a change of about 3 percent in the flux density.

(The flux density will increase by about 3 percent as the turns per phase used are 120 instead of the calculated value of 124).

Example 12·5. *Two preliminary designs are made for a 3 phase alternator, the two designs differing only in the number and size of stator slots and the dimensions of stator conductors. The first design uses 2 slots per pole per phase and there are 9 conductors per slot, each slot being 75 mm deep and 19 mm wide, and the mean width of stator tooth is 25 mm. The thickness of slot insulation is 2 mm; all other insulations may be neglected. The second design is to have 3 slots per pole per phase. Retaining the same flux density in the teeth and current density in the stator conductors as in the first design, calculate the dimensions of the stator slots for the second design.*

The total height of lip and wedge may be assumed as 5 mm.

Solution.

Ist Design :

Slots per pole per phase $q = 2$.

Total height of conductors $= 75 - 5 - 2 \times 2 = 66$ mm.

Height of each conductor $= 66/9 = 7.33$ mm.

Width of each conductor $= 19 - 2 \times 2 = 15$ mm.

Area of each conductor $= 7.33 \times 15 = 110$ mm².

Slot pitch at mean diameter $= 19 + 25 = 44$ mm.

2nd Design :

The number of slots per pole per phase is 3.

Therefore, the number of stator slots in this design are 3/2 times that in the first design.

∴ Number of conductors per slot in this design are 2/3 times that in the first design.

Number of conductors per slot $=(2/3)\times 9=6$.

Slot pitch at mean diameter $=(2/3)\times 44=29.3$ mm.

Tooth width for same flux density in teeth $=(2/3)\times 25=16.7$ mm.

∴ Slot width $=29.3-16.7=12.6$ mm.

Width of each conductor $=12.6-2\times 2=8.6$ mm.

Height of each conductor $=110/8.6=12.8$ mm.

Total height of conductors $=6\times 12.8=76.8$ mm.

Depth of slot $=76.8+5+2\times 2=85.8$ mm.

∴ The slot dimensions are 85.8×12.6 mm² and the conductor dimensions are 12.8×8.6 mm².

Example 12.6. *A 1000 kVA, 3300 V, 50 Hz, 300 rpm, 3 phase alternator has 180 slots with 5 conductors per slot. Single layer winding with full pitch coils is used. The winding is star connected with one circuit per phase. Determine the specific electric and specific magnetic loadings if the stator bore is 2.0 m and the core length is 0.4 m. Using the same loadings, determine the corresponding data for a 1250 kVA, 3300 V, 50 Hz, 250 rpm, 3 phase star connected alternator having 2 circuits per phase. The machines have 60° phase spread.*

Solution.

1000 kVA Generator :

Total conductors = slots × conductors per slot $=180\times 5=900$.

Turns per phase $T_{ph}=900/(2\times 3)=150$.

Synchronous speed $n_s=300/60=5$ r.p.s.

∴ Number of poles $=2\times 50/5=20$.

Slots per pole per phase $q=180/(3\times 20)=3$.

Distribution factor $K_d=\dfrac{\sin 60/2}{3\sin 60/(2\times 3)}=0.96$.

With full pitch coils, pitch factor $K_p=1$. ∴ Winding factor $K_w=0.96$.

Voltage per phase $E_{ph}=\dfrac{3300}{\sqrt{3}}=1910$ V.

∴ Flux per pole $\Phi=\dfrac{1910}{4.44\times 50\times 150\times 0.96}=59.8\times 10^{-3}$ Wb.

Pole pitch $\tau=\pi\times 2/20=0.314$ m.

∴ Area per pole $A_p=0.314\times 0.4=125.6\times 10^{-3}$ m².

∴ Specific magnetic loading $B_{av}=59.8\times 10^{-3}/(125.6\times 10^{-3})=0.476$ Wb/m².

Current per phase $I_{ph}=\dfrac{1000\times 1000}{3\times 1910}=175$ A.

Since there is only one circuit per phase, current in each conductor I_z = current per phase = 175 A.

$\therefore$ Specific electric loading $ac=\dfrac{6\,I_s\,T_{ph}}{\pi D}$

$$=\frac{6\times175\times150}{\pi\times2}=25000 \text{ A/m.}$$

Peripheral speed $=\pi\times2\times5=31{\cdot}4$ m/s.

1250 kVA Generator :

Synchronous speed $n_s=250/60=4{\cdot}167$ r.p.s.

$\therefore$ Number of poles $p=2\times50/4{\cdot}167=24$.

With 3 slots per phase and full pitch winding $K_w=0{\cdot}96$

output co-efficient $C_o=11\times0{\cdot}96\times0{\cdot}476\times25000\times10^{-3}=126$,

$$\therefore \qquad D^2L=\frac{Q}{C_o\,n_s}=\frac{1250}{126\times4{\cdot}166}=2{\cdot}39 \text{ m}^3.$$

Keeping the same peripheral speed (*i.e.* 31·4 m/s) as in previous case, we have

$$\pi\,D\times4{\cdot}166=31{\cdot}4$$

$$\therefore \qquad D=2{\cdot}4 \text{ m}, \quad \text{and } L=2{\cdot}39/(2{\cdot}4)^2=0{\cdot}414 \text{ m}.$$

Pole pitch $\tau=\pi\times2{\cdot}4/24=0{\cdot}314$ m.

Flux per pole $\Phi=0{\cdot}476\times0{\cdot}314\times0{\cdot}414=62\times10^{-3}$ Wb.

If we have more than one circuit (parallel path) per phase,

Voltage per phase $E_{ph}=4{\cdot}44\,f\,\dfrac{T_{ph}}{a}\,\Phi\,K_{w_1}$.

With 2 parallel paths, $a=2$.

$$\therefore \qquad T_{ph}=\frac{2\times1910}{4{\cdot}44\times50\times0{\cdot}062\times0{\cdot}96}=289.$$

Total conductors $=6\,T_{ph}=6\times289=1734$ and total slots $=3\,pq=3\times24\times3=216$.

Conductors per slot $=1734/216=8{\cdot}03$.

An integral number of conductors per slot has to be used taking 8 conductors per slot.

Total conductors $=8\times216=1728$. $\therefore$ Turns per phase $T_{ph}=1728/6=288$.

Example 12·7. *Determine the main dimensions of a 75000 kVA, 13·8 kV, 50 Hz 62·5 r.p.m., 3-phase, star-connected alternator. Also find the number of stator slots, conductors per slot, conductor area and work out the winding details. The peripheral speed should be about 40 m/s. Assume, average gap density=0·65 Wb/m², ampere conductors per metre =40,000 and current density=4A/mm².*

Solution.

Synchronous speed $n_s=62{\cdot}5/60=1{\cdot}0417$ r.p.s.

Number of poles $p=2\times50/1{\cdot}0417=96$.

Output coefficient $C_o=11\times0{\cdot}955\times0{\cdot}65\times40{,}000\times10^{-3}=273$.

$$\therefore \qquad D^2L=\frac{Q}{C_o\,n_s}=\frac{75000}{273\times1{\cdot}0417}=264 \text{ m}^3.$$

Diameter with a peripheral speed of 40 m/s,

$$D=\frac{40}{\pi\,n_s}=\frac{40}{\pi\times1{\cdot}0417}=12{\cdot}2 \text{ m} \quad \text{and } L=\frac{264}{12{\cdot}2^2}=1{\cdot}77 \text{ m}.$$

Pole pitch $\tau=\pi\times12{\cdot}2/96=0{\cdot}4$ m.

$\therefore$ Flux per pole $\Phi = B_{av}\,\tau\,L = 0{\cdot}65 \times 0{\cdot}4 \times 1{\cdot}77 = 0{\cdot}46$ Wb.

Voltage per phase $E_{ph} = 13800/\sqrt{3} = 7960$ V.

With one circuit per phase,

turns per phase $$T_{ph} = \frac{E_{ph}}{4{\cdot}44\, f \Phi\, K_w} = \frac{7960}{4{\cdot}44 \times 50 \times 0{\cdot}46 \times 0{\cdot}955} = 81{\cdot}8 \simeq 82.$$

As the terminal voltage is 13·8 kV, a slot pitch of about 55 mm should be used.

$$\therefore \text{ Slots per pole per phase } q = \frac{\pi D}{y_s \times 3p} = \frac{\pi \times 12{\cdot}2 \times 100}{55 \times 3 \times 96} = 2{\cdot}42.$$

Fractional slot winding is used and the number of slots is so chosen that the value of turns per phase calculated earlier and the value of turns per phase provided do not differ significantly.

Another factor to be borne in mind is that with fractional slot windings, double layer windings are a must. Therefore, an even number of conductors must be used in each slot.

Assuming slot per pole per phase $q = 2\tfrac{1}{4}$. Total number of slots $= 2\tfrac{1}{4} \times 3 \times 96 = 648$.

The number of conductors $= 6T_{ph} = 6 \times 82 = 492$.

$\therefore$ Conductors per slot $= 492/648 =$ a fraction.

Therefore, a doble layer winding is not possible with one circuit per phase. We should take number of parallel circuits in such a way that an even number of conductors per slot is obtained. The number of parallel circuits should be so chosen that the winding is symmetrical.

Taking number of parallel circuits $a = 8$.

Turns per phase $$T_{ph} = \frac{8 \times 7960}{4{\cdot}44 \times 50 \times 0{\cdot}46 \times 0{\cdot}955} = 654.$$

Total conductors $= 6 \times 654 = 3924$.

Conductors per slot $= 3924/648 \simeq 6$. $\therefore$ Conductors provided $= 6 \times 648 = 3888$.

Turns per phase provided, $T_{ph} = 3888/6 = 648$.

This value is nearly equal to the calculated value and requires only about 1 per cent change in flux density.

Referring to Art. 6·23 page 275 with $q = 2\tfrac{1}{4}$, $d = 4$.

$\therefore$ Number of units $= p/d = 96/4 = 24$.

With 8 parallel paths, 3 units are connected in series in each parallel path. This gives a symmetrical winding.

Current per phase $$I_{ph} = \frac{75000 \times 1000}{3 \times 7960} = 3140 \text{ A}.$$

Current in each conductor $I_s = 3140/8 = 392{\cdot}5$ A.

$\therefore$ Area of each conductor $a_s = 392{\cdot}5/$[illegible] $\simeq 100$ mm².

12·19. Armature parameters.

12·19·1. Armature resistance. The value of L_{mt}, the length of mean turn of the armature is $2L + 2{\cdot}5\tau + 0{\cdot}06$ kV $+ 0{\cdot}2$. Out of this, the length of turn embedded in the slots is $2L$; while the length in the overhang is $2{\cdot}5\,\tau + 0{\cdot}06$ kV $+ 0{\cdot}2$.

[illegible]

ρ = resistivity, Ω/m and mm^2.
= 0·021 for copper at 75°C.

Therefore, the armature I^2R loss $= 3\, I_{ph}^2\, r_{a.c.}$

In order to calculate the total copper losses, we must find out the average eddy current loss factor $K_d(av)$

From Eqn. 6·99 on page 325, $K_d(av) = 1 + (ah')^4 \dfrac{N^2}{9}$.

Now there is significant eddy current copper loss in slot portion of the conductors while it is very small in the overhang and thus eddy current loss in the overhang can be neglected.

∴ Total armature copper loss

$$= 3\, I_{ph}^2 \times \frac{T_{ph}}{a_s} \times \rho \left[L_{mt} + (ah')^4 \frac{N^2}{9} \times 2L \right] \qquad \ldots(12·12)$$

and armature copper loss per phase

$$= I_{ph}^2 \times \frac{T_{ph}}{a_s} \times \rho \left[L_{mt} + (ah')^4 \frac{N^2}{9} \times 2L \right] \qquad \ldots(12·13)$$

Hence, the effective a.c. resistance per phase

$$r_{a.c.} = \frac{T_{ph}}{a_s} \times \rho \left[L_{mt} + (ah')^4 \frac{N^2}{9} \times 2L \right] \qquad \ldots(12·14)$$

Per unit armature resistance $R_{a.c} = I_{ph} \times r_{a.c} / E_{ph}$ ...(12·15)

12·19·2. Armature leakage reactance. In synchronous machines the value of leakage reactance is required for the calculation of the value of regulation and for this purpose an accurate estimate is not necessary. However, an accurate estimate of leakage reactance is very necessary in order to study and predict the behaviour of the machine under sudden short circuit conditions.

For the purpose of calculation of regulation, an approximate assessment is done as below, considering only the leakage from slots and overhang.

The specific slot permeance is calculated by using the relationships given in Chapter 4

From Eqn 4·95 on page 172, stator leakage reactance per phase,

$$x_{ss} = 8\pi f\, T_{ph}^2 L(\lambda_s / pq)$$

From Eqn 4·95 on page 172, overhang leakage reactance per phase,

$$x_o = 8\pi f\, T_{ph}^2\, L_o(\lambda_o / pq)$$

where $L_o\, \lambda_o = \mu_o K_s \tau / \pi y_o$

and K_s is taken from Fig. 4·58, page 170.

The value of overhang permeance may be calculated from the following relation

$$L_o\, \lambda_o = \frac{K L_o^2}{2\sqrt{2}\, y_o}, \qquad \ldots(12·16)$$

where $K = 0·23 \times 10^{-6}$ for concentric windings
$= 0·29 \times 10^{-6}$ for barrel windings

Total stator leakage reactance per phase $x_s = x_{ss} + x_o$

Hence, per unit leakage [illegible] ...(12·17)

Example 12·9. *A 3 phase, [illegible] has parallel sided slots 0·6 m long, 20 mm wide and 100 mm deep. The [illegible] space 60 mm deep and the slot is closed by a non-magnetic wedge 30 mm [illegible] poles with 4 slots per pole per phase and 10 conductors per slot. The coils [illegible] are connected in series. Determine the leakage reactance per phase due to the [illegible]*

Solution.

Height of insulation = depth of slot − height of wedge − height of conductors

$$=100-20-60=20 \text{ mm}.$$

Therefore thickess of insulation is 10 mm. (The insulation is 10 mm at the top and 10 mm at the bottom of conductors with the insulation between conductors being neglected.)

$\therefore$ Specific permeance of slot $\lambda_s=\mu_0\left[\frac{6}{3\times 2}+\frac{1}{2}+\frac{2}{2}\right]=10\pi\times 10^{-7}$.

We have, slots per pole per phase $q=4$. Total slots $=3pq=3\times 8\times 4=96$.

Total conductors $=10\times 96=960$. Turns per phase $T_{ph}=960/6=160$.

From Eqn. 4 95, leakage reactance due to slot leakage

$$x_s=8\pi f\, T_{ph}^2\, L\left(\frac{\lambda_s}{pq}\right)$$

$$=8\pi\times 50\times(160)^2\times 0{\cdot}6\left(\frac{10\pi\times 10^{-7}}{8\times 4}\right)=1{\cdot}89\ \Omega.$$

Example 12·10. *Calculate (a) the size of armature wire (b) the a.c. resistance of each phase for a 3 phase, 50 Hz, 8 pole star connected synchronous generator having the following data :*

Pole pitch = 0·3 m ; line current = 100 A ; slots per pole per phase = 3 ; conductors per slot = 6 ; gross axial length = 0·3 m ; length of active copper = 50 percent of total copper length ; average eddy current loss factor = 1·3 ; current density $\delta=\frac{43{,}000}{ac}+\frac{V_a}{16}$*, where δ is the current density in A/mm² ; ac, ampere conductors per metre and V_a is the peripheral speed in m/s.*

Solution. Armature diameter $D=0{\cdot}3\times 8/\pi=0{\cdot}765$ m.

Synchronous speed $n_s=2\times 50/8=12{\cdot}5$ r.p.s.

Peripheral speed $V_a=\pi\times 0{\cdot}765\times 12{\cdot}5=28{\cdot}8$ m/s.

Total number of slots $=3\times 3\times 8=72$. Total number of conductors $Z=72\times 6=432$.

Turns per phase $T_{ph}=432/6=72$.

Current in each conductor $I_z=100$ A (as all coils in a phase are connected in series)

$$\therefore\quad ac=\frac{I_z Z}{\pi D}=\frac{100\times 432}{\pi\times 0{\cdot}765}=18{,}000 \text{ A/m}.$$

$\therefore$ Current density in armature conductors $\delta=\frac{43{,}000}{18{,}000}+\frac{28{\cdot}8}{16}=4{\cdot}2$ A/mm².

Area of armature conductor $=100/4{\cdot}2=23{\cdot}8$ mm².

Active length of each turn $=2L=2\times 0{\cdot}3=0{\cdot}6$ m.

The total length of a turn is twice the active length,

$\therefore\ L_{mt}=2\times 0{\cdot}6=1{\cdot}2$ m.

D.C. resistance of each phase at 75°C $\quad r_{d.c.}=0{\cdot}021\times\frac{72\times 1{\cdot}2}{23{\cdot}8}=0{\cdot}0762\ \Omega.$

A.C. resistance of each phase $\quad r_{a.c.}=K_{d(av)}\times r_{d.c.}=1{\cdot}3\times 0{\cdot}0762=0{\cdot}099\ \Omega.$

12·20. Estimation of air gap length. No load field mmf per pole is equal to the product of armature mmf per pole and the short circuit ratio.

$$\therefore\quad AT_{fo}=AT_a\times SCR. \qquad \text{...(12·[illegible])}$$

The value of armature mmf per pole $AT_a=2{\cdot}7\,\frac{I_{ph}\,T_{ph}\,K_w}{[illegible]}$ (See [illegible]

$$\therefore \quad AT_{fo} = 2.7 \frac{I_{ph} T_{ph} K_{w1}}{p} \times SCR \qquad \ldots(12.19)$$

Thus the value of no load field mmf can be estimated by assuming a suitable value for short circuit ratio.

The mmf required for during the flux across the air gap is approximately 80% of the no load field mmf and this assumption enables the mmf required for air gap to be estimated.

Mmf required for air gap $= 0.8\ AT_{fo} = 800{,}000\ B_g K_g l_g$

where B_g = maximum flux density in the air gap (at the centre of the pole).

$$\therefore \quad \text{Length of air gap at the centre of the pole } l_g = \frac{0.8\ AT_{fo}}{800{,}000\ B_g K_g}$$

$$= \frac{K_f\ AT_{fo}}{1000{,}000\ B_{av} K_g} \qquad \ldots(12.20)$$

Example 12.11. *A 500 kVA, 3.3 kV, 50 Hz, 600 r.p.m. 3 phase salient pole alternator has 180 turns per phase. Estimate the length of air gap if the average flux density is 0.54 Wb/m²; the ratio pole arc to pole pitch, 0.65; the short circuit ratio, 1.2; the gap contraction factor, 1.15, and the winding factor, 0.955. The mmf required for gap is 80 percent of no load field mmf and the winding factor, 0.955.*

Solution. Synchronous speed $n_s = 600/60 = 10$ r.p.s. $\therefore$ Poles $= 2 \times 50/10 = 10$.

Current per phase $I_{ph} = \dfrac{500 \times 1000}{\sqrt{3} \times 3300} = 87.4$ A.

Armature mmf per pole $AT_a = \dfrac{2.7 \times 87.4 \times 180 \times 0.955}{10} = 4062$ A

No load field mmf per pole $AT_{fo} = SCR \times AT_a = 1.2 \times 4062 = 4875$ A.

Field from factor $K_f = \psi = 0.66$.

Maximum flux density in air gap $B_g = \dfrac{B_{av}}{K_f} = \dfrac{0.54}{0.66} = 0.818$ Wb/m².

Mmf for air gap $= 800{,}000\ B_g\ K_g\ l_g = 0.8\ AT_{fo}$ (given) $= 0.8 \times 4875$.

$\therefore$ Lentgh of air gap $l_g = \dfrac{0.8 \times 4875}{800{,}000 \times 0.818 \times 1.15}$ m $= 5.2$ mm.

Example 12.12. *The following is the design data available for a 1250 kVA, 3 phase, 50 Hz, 3300 V, star connected, 300 r.p.m. alternator of salient pole type:*

Stator bore D = 1.9 m; stator core length L = 0.335 m; pole arc/pole pitch = 0.66; turns per phase = 150; single layer concentric winding with 5 conductors per slot, short circuit ratio = 1.2. Assume that the distribution of gap flux is rectangular under the pole arc with zero values in the interpolar region. Calculate:

(a) *specific magnetic loading,* (b) *armature mmf per pole,*

(c) *gap density over pole arc,* (d) *air gap length.*

Mmf required for air gap is 0.88 of no load field mmf and the gap contraction factor is 1.15.

Solution.

Turns per phase $T_{ph} = 150$. Total stator conductors $= 6 \times 150 = 900$.

Total number of stator slots $= 900/5 = 180$.

Synchronous speed $n_s = 300/60 = 5$ r.p.s. $\therefore$ Number of poles $p = 2 \times 50/5 = 20$.

Slots per pole per phase $q=180/(3\times20)=3$.

Distribution factor $=\dfrac{\sin 60/2}{3\sin 60/(3\times2)}=0.96$

Full pitch coils are used and so $K_p=1$. $\therefore$ Winding factor $K_w=0.96\times1=0.96$.

Voltage per phase $E_{ph}=3300/\sqrt{3}=1910$ V.

Flux per pole $\Phi=\dfrac{1910}{4.44\times50\times150\times0.96}=59.8\times10^{-3}$ Wb.

Area per pole $=\dfrac{\pi DL}{p}=\dfrac{\pi\times1.9\times0.335}{20}=0.1$ m².

Specific magnetic loading $B_{av}=59.8\times10^{-3}/0.1=0.598$ Wb/m²

Gap density over pole arc $B_g=B_{av}/\psi=0.598/0.66=0.907$ Wb/m².

Current per phase $I_{ph}=\dfrac{1250\times1000}{\sqrt{3}\times3300}=219$ A

Armature mmf per pole $AT_a=\dfrac{2.7\times219\times150\times0.96}{20}=4250$ A

No load field mmf $AT_{fo}=SCR\times AT_a=1.2\times4250=5100$ A.

Mmf required for air gap $AT_g=0.88\times5100=4480$ A

$\therefore$ $800{,}000\, B_g K_g\, l_g=4480$

or length of air gap $l_g=\dfrac{4480}{800{,}000\times0.907\times1.15}$ m $=5.38$ mm

Example 12.13. *A 1250 kVA, 3 phase, 50 Hz 3300 V, 300 r.p.m synchronous generator with a concentric winding has the following design data :*

specific magnetic loading $B_{av}=0.58$ Wb/m² ;

specific electric loading $ac=33000$ A/m ;

gap length $=5.5$ mm ;

field turns per pole $=60$;

short circuit ratio $=1.2$.

The effective gap area is 0.6 times the actual area.

Peripheral speed is 30 m/s. Find stator core length, stator bore, turns per phase, mmf for air gap, armature mmf per pole, and field current for no load and rated voltage.

Solution.

Synchronous speed $n_s=5$ r.p.s. Number of poles $p=20$.

Output coefficient $C_0=11\times0.955\times0.58\times33000\times10^{-3}=201$.

Product $D^2L=\dfrac{Q}{C_0 n_s}=\dfrac{1250}{201\times5}=1.245$ m³.

We have $D=\dfrac{V_a}{\pi n_s}=\dfrac{30}{\pi\times5}=1.9$ m and $L=\dfrac{1.245}{(1.9)^2}=0.345$ m

Area per pole $=\pi\times1.9\times0.345/20=0.103$ m².

Flux per pole $=0.58\times0.103=59.7\times10^{-3}$ Wb.

Voltage per phase $E_{ph}=3300/\sqrt{3}=1910$ V.

Turns per phase, $T_{ph}=\dfrac{1910}{4.44\times50\times59.7\times10^{-3}\times0.955}=150$.

Current per phase, $I_{ph}=\dfrac{1250\times1000}{[illegible]}=219$ A.

Armature mmf per pole $AT_a=\dfrac{2.7\times219\times150\times0.955}{20}=4240$ A.

Effective area per pole $=0.6\times0.103=61.8\times10^{-3}$ m².

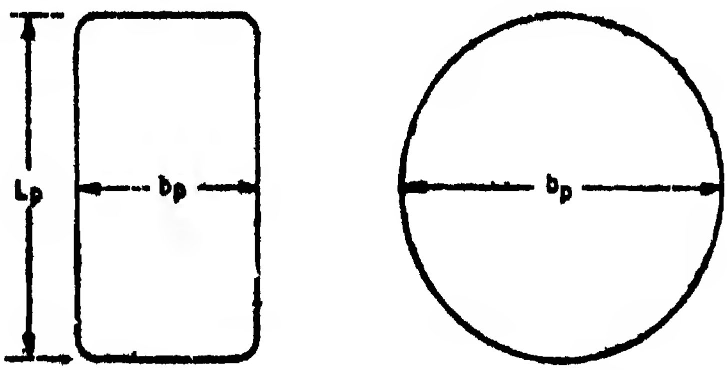
$\therefore$ Effective gap density $K_g B_g=59.7\times10^{-3}/(618\times10^{-3})=0.966$ Wb/m²

Mmf for air gap $=800,000\times0.96\times5.5\times10^{-3}=42.50$ A.

Field mmf per pole at no load $AT_{fo}=1.2\times4250=5100$ A.

Field current at no load $=5100/60=85$ A.

12.21. Design of Rotor



(a) (b)

Fig. 12.29. Rectangular and round shaped pole bodies.

Flux in pole body

$$\Phi_p=\text{leakage coefficient}\times\text{useful flux per pole}=C_l\Phi \qquad \text{...(12.21)}$$

The value of leakage co-efficient C_l lies between 1.15 to 1.2.

Area of cross-section of pole body $A_p=\Phi_p/B_p$.

The flux density in pole body B_p has a permissible value of 1.5 to 1.7 Wb/m².

For rectangular poles shown in Fig. 12.29 (a), $A_p=0.98\ L_p\ b_p$.

The axial length of pole, L_p, is taken equal to gross stator core length L. The stacking factor for pole laminations is taken as 0.98.

For circular poles, shown in Fig. 12.29 (b), $A_p=(\pi/4)\ b_p^2$.

12.21.1. Height of pole. Since the dimensions of the pole, yoke etc. are not completely known at this stage, we can have only approximate estimate of the value of field mmf.

An approximate estimation of full load field mmf can be made by the method given below :

No load field mmf $AT_{fo}=SCR\times AT_a$

and armature mmf per pole $AT_a=2.7\ I_{ph}\ T_{ph}\ K_{w1}/p$.

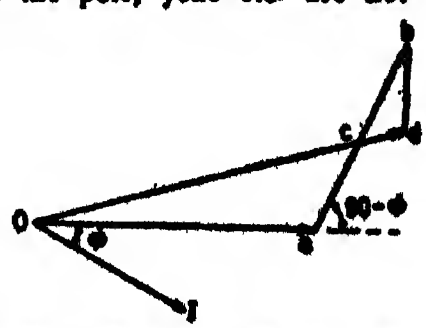


Fig. 12.30. Phasor diagram with resistance and leakage reactance of armature winding neglected.

Referring to Fig. 12.30,

(i) Draw $oa=AT_{fo}$.

(ii) Draw $ab=AT_a$ at angle $(90-\phi)$ to oa, where $\cos\phi$=power factor (lagging).

(*iii*) Cut off *ae* such that $ae/ab=K_r$

where K_r is called the cross reaction co-efficient which depends upon the ratio of pole arc to pole pitch and its value can be taken from Fig. 12·31.

(*iv*) Join *oe* and extend it. Drop a perpendicular from *b* on *oe* extended, cutting it at *d*.

od = field mmf at full load with power factor $\cos\phi$ (lagging).

(This is only an approximate estimate wherein resistance and leakage reactance of the armature winding have been neglected).

The radial length of winding can be approximated by adopting the following procedure.

Copper area of field winding

$$=\frac{\text{full load field mmf}}{\text{current density in the field winding}}=\frac{AT_{fl}}{\delta_f}$$

The value of δ_f may be taken between 3 to 4 A/mm².

Total space required for winding

= copper area/space factor.

The value of space factor for strip on edge winding may be taken about 0·8 to 0·9. (If other types of conductors are used the space factor can be taken as 0·4 for small rounds wires, 0·65 for large round wires and 0·75 for large rectangular conductors). The height of the winding h_f is known by dividing the total space for winding by the depth of the winding.

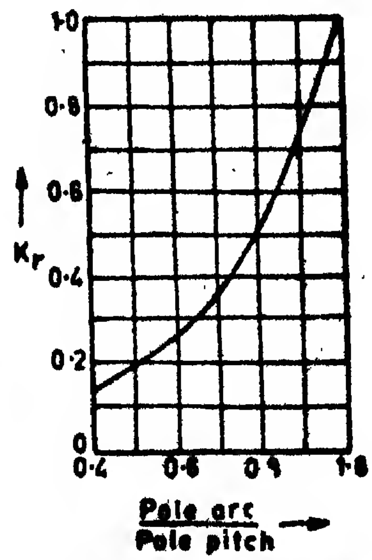


Fig. 12·31. Cross-reaction co-efficient.

Table 12·3 gives the approximate values of depth of winding d_f.

$\therefore$ Height of winding $h_f=\dfrac{\text{total winding area}}{d_f}$.

Height of field winding may be estimated by using Eqn. 9·41 page 541.

Mmf per metre height of field winding $=10^4\times\sqrt{S_f\, d_f\, q_f}$

$$\therefore \text{ Height of field winding } h_f=\frac{AT_{fl}\times 10^{-4}}{\sqrt{S_f\, d_f\, q_f}}$$

where S_f = copper space factor, d_f = depth of winding,

q_f = loss per unit surface, W/m².

The value of q_f may be calculated as under :

Cooling co-efficient for rotating field coils $c=\dfrac{0.08 \text{ to } 0.12}{1+0.1\,V_a}$.

$\therefore$ Specific loss dissipation $\lambda=\dfrac{1}{c}=\dfrac{1+0.1\,V_a}{0.08 \text{ to } 0.12}$ W/m²–C°.

$\therefore$ Loss dissipated per unit surface

$$q_f=\theta_f\left(\frac{1+0.1\,V_a}{0.08 \text{ to } 0.12}\right) \text{ W/m}^2$$

where θ = permissible temperature rise.

The value of permissible temperature rise taken from Table 17.19.

In order to know the value of total radial length of pole, we must add to the winding height, the height taken by flanges etc. (Fig. 12 41), which is usually about 20 mm and also the height of pole shoe h_s. The height of the pole shoe can be taken from the pole drawing.

$\therefore \quad h_{pl}$ = radial length of pole $= h_f + h_s + 0.02$.

Height of pole body $h_p = h_f + 0.02$.

The value of ratio of radial length of pole to pole pitch is generally equal to 0.3 to 1.5. The smaller value is for machines with a smaller number of poles or larger pole pitch. The large value is for machines with large number of poles or smaller pole pitch.

Example 2 14. *A 2500 kVA, 225 r p m., 3 phase, 60 Hz, 2400 V, star connected salient pole alternator has the following design data :*

Stator bore = 2 5 m ; core length = 0 44 m ; slot per pole per phase = 3, conductors per slot = 4 ; circuits per phase = 2 ; leakage factor = 1 2 ; winding factor = 0 95.

The flux density in pole core is 1.5 Wb/m², the winding depth is 30 mm, the ratio of full load field mmf to armature mmf is 2, field winding space factor is 0 84 and the field winding dissipates 1800 W/m² of inner and outer surface without the temperature rise exceeding the permissible limit. Leave 30 mm for insulation, flanges and height of pole shoe along the height of pole.

Find (a) the flux per pole (b) length and width of pole (c) winding height and (d) pole height.

Solution.

Synchronous speed $n_s = 225/60 = 3.75$ r.p.s

$\therefore$ Number of poles $p = 2 \times 60/3.75 = 32$.

Total number of slots $= 3 \times 32 \times 3\frac{1}{2} = 336$.

Total number of conductors $Z = 336 \times 4 = 1344$

Turns per phase $T_{ph} = 1344/6 = 224$.

These turns are connected in two parallel paths

Voltage per phase $E_{ph} = 2400/\sqrt{3} = 1390$ V

We have from Eqn. 12 8, $E_{ph} = 4.44\,\Phi \frac{T_{ph}}{a} f k_w$

$\therefore$ Flux per pole $\quad \Phi = \dfrac{1390 \times 2}{4.44 \times 224 \times 60 \times 0.95} = 49 \times 10^{-3}$ Wb.

Flux in pole body $\quad \Phi_p = C_l\,\Phi = 1.2 \times 49 \times 10^{-3} = 58.8 \times 10^{-3}$ Wb.

Area of pole body $\quad A_p = \Phi_p/B_p = 58.8 \times 10^{-3}/1.5 = 39.2 \times 10^{-3}$ m².

Length of pole body L_p = length of armature core = 0.44 m.

Width of pole body $\quad b_p = 39.2 \times 10^{-3}/0.44 = 0.89$ m.

Current in each phase $\quad I_{ph} = \dfrac{2500 \times 1000}{\sqrt{3} \times 2400} = 600$ A.

$\therefore$ Current in each conductor $I_z = 600/2 = 300$ A,

since there are two circuits per phase.

Armature mmf per pole $AT_a = \dfrac{[illegible]}{32} = 5370$ A.

Field mmf at full load $AT_{fl}=2\times 5370=10740$ A.

From Eqn. 9·41, mmf per metre height of winding

$$=10^4\times\sqrt{S_f\, d_f\, q_f}=10^4\sqrt{0.8\times 0.03\times 1900}=67{,}300\ \text{A}.$$

$\therefore$ Height of field windings $h_f=10740/67300=0.16$ m.

Height of pole-height of winding + height of insulation $=0.16+0.03=0.19$ m.

12·21 2 Design of damper winding. The design of damper winding depends upon the purpose for which it is provided. In synchronous generators, it is provided to suppress the negative sequence field and to damp the oscillations when the machine starts hunting, while in a synchronous motor its function is to provide starting torque and to develop damping power when the machine starts hunting.

The design of damper winding to suppress inverse rotating field is discussed below :

The amplitude of fundamental of mmf AT_1 of one phase of a polyphase winding is obtained from Eqn. 6 78 (page 312) by putting $n=1$, $t=0$ and $x=\pi/2$.

$$\therefore \qquad AT_1=\frac{4}{\pi}AT_m K_{w1}^{*}$$

From Eqn. 6·77,

$$AT_m=q\ Z_s\frac{I_s}{\sqrt{2}}=\sqrt{2}\frac{I_{ph}\ T_{ph}}{p}$$

Current in each conductor $I_s=I_{ph}$ and conductors per slot $Z_s=2\ T_{ph}/qp$.

$$\therefore \qquad AT_1=\frac{4\sqrt{2}}{\pi}\ \frac{I_{ph}\ T_{ph}\ K_{w1}}{p} \qquad \ldots(12{\cdot}22)$$

This pulsating mmf can be resolved into two rotating mmfs, one called the synchronous mmf and the other inverse mmf each having half the magnitude as above. If the damper winding is to suppress the inverse rotating field (negative sequence field), it must develop an equal mmf as that of the inverse field.

$$\therefore \quad \text{Mmf of damper winding}=\frac{4\sqrt{2}}{2\pi}\ \frac{I_{ph}\ T_{ph}\ K_{w1}}{p}.$$

$$\text{Ampere conductors per pole}=ac\ \tau=\frac{6\ I_{ph}\ T_{ph}}{p}$$

$$\therefore \quad \text{Mmf of damper winding}=\frac{4\sqrt{2}}{2\pi}\ \frac{ac\ \tau}{6}K_{w1}$$

$$=\frac{2\sqrt{2}}{\pi}\times\frac{ac\ \tau}{6}\times 0.955$$

$$=0.143\ ac\ \tau \qquad \ldots(12{\cdot}23)$$

Let A_d be the total area of damper bars per pole and δ_d be the current density in the bars

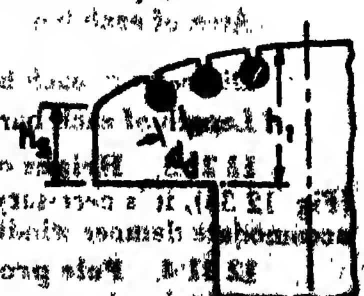
$$\therefore \qquad A_d\delta_d=0.143\ ac\ \tau \quad \text{or} \quad A_d=0.143\ ac\ \tau/\delta_d$$

The area provided for damper windings is greater than this in practice.

Area per pole of damper bar provided

$$A_d=0.2\ ac\ \tau/\delta_d \qquad \ldots(12{\cdot}24)$$

The current density in the damper bars is usually 3 to 4 A/mm².



The total area of damper winding is distributed longitudinally depending upon the number

Fig. 12·34. Damper winding.

*Eqn. 6·78 does not consider the pitch factor but it has to be taken into account here.

of bars used. The damper winding slot pitch is taken approximately equal to stator slot pitch.

The slot pitch of damper windings made different from stator slot pitch by about 20% in order to reduce current induced in damper windings by tooth ripples.

$\therefore$ Pole arc = number of bars per pole $\times y_s \times 0.8$.

The length of each damper bar $L_d = 1.1\ L$ for small machines

$= L + 0.1$ m, for larger machines.

Cross section of each damper bar

$$a_d = \frac{\text{total area of bars per pole}}{\text{number of damper bars per pole}} = \frac{A_d}{N_d}$$

where N_d = number of damper bars per pole.

In case of circular bars $a_d = (\pi/4) d_d^2$

where d_d = diameter of damper bars.

The area of each ring short circuiting the bars

$$A_{ring} = (0.8 \text{ to } 1)\ A_d \qquad \ldots(12.25)$$

Fig. 12.32 shows the position of damper bars.

Example 12.15. *A 1250 kVA, 3 phase, 6600 V, salient pole alternator has the following data :*

Air gap diameter = 1.6 m ; length of core = 0.45 m : number of poles = 20 ; armature ampere conductors per metre = 28000 ; ratio, pole arc : pole pitch = 0.68 ; stator slot pitch = 28 mm ; current density in damper bars = 3A/mm².

Design a suitable damper winding for the machine.

Solution.

Pole pitch $\tau = \pi \times 1.6/0.2 = 0.251$ m.

From Eqn. 12.24, total area of damper bars per pole

$$A_d = \frac{0.2\ ac\ \tau}{\delta_d} = \frac{0.2 \times 28000 \times 0.251}{3} = 473 \text{ mm}^2.$$

The pitch of damper bars is taken as 0.8 times the stator slot pitch.

$\therefore$ Number of bars per pole

$$N_d = \frac{\text{pole arc}}{0.8 \times \text{stator slot pitch}}$$

$$= \frac{0.68 \times 25.1}{0.8 \times 2.8} = 7.6 \simeq 8.$$

Area of each bar $a_d = \dfrac{\text{total area}}{\text{number of bars}} = \dfrac{473}{8} = 59 \text{ mm}^2.$

Diameter of each bar $d_d = 8.7$ mm.

Length of each bar $= 1.1 \times L = 1.1 \times 0.45 \simeq 0.5$ m.

12.21.3. Height of pole shoe. In order to determine height h_1 of the pole shoe (Fig. 12.34), it is necessary to fix up height h_e. The height h_e should be sufficient to accommodate damper winding and therefore, generally, $h_e = 2d_d$.

12.21.4. Pole profile drawing. It is essential to draw the profile of the pole in order to obtain the various dimensions of the pole. The procedure for drawing is as follows :

As it is difficult to draw the whole drawing of pole along with centre of the shaft, only the centre line of the pole and centre line of interpolar space is drawn with the help of simple trigonometry.

Mechanical angle between centre of pole and centre of interpolar space,

$$\theta = 360/2p = 180/p$$

and radius of stator bore $\quad r = D/2.$

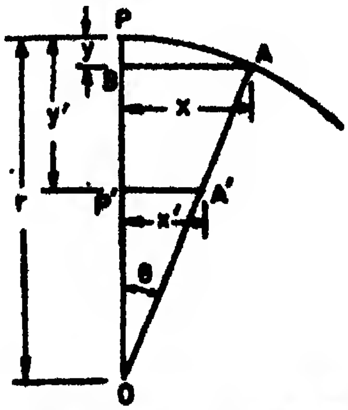


Fig. 12·33. Location of centre lines pole and interpolar space.

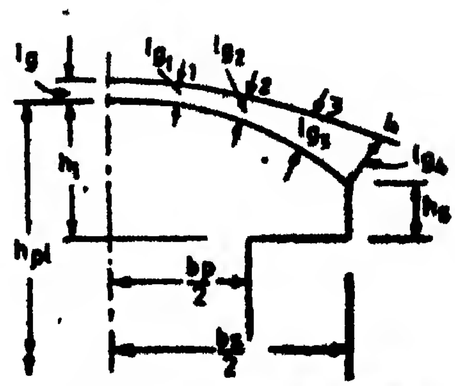


Fig. 12·34. Pole profile drawing.

Let OP be the line corresponding to centre of pole and OA the centre line of interpolar space (Fig. 12·33). Now OA is to be located with reference to OP.

Let $\quad OP = r$

and therefore $\quad x = r \sin \theta$ and $y = r(1 - \cos \theta)$.

Line OP is drawn as vertical and therefore points P and B are located. Point A can be located with reference to P, with the help of above relations *i.e.* values of x and y. In order to fix the direction of OA another point A' is needed. For this,

$$\frac{x'}{r - y'} = \tan \theta.$$

As r, y' and θ are known, x' can be known and thus point A' is located with respect to P'. Thus by drawing a line through AA', line OA is located which represents the centre of interpolar space. This way all the points 1, 2, 3, 4 etc. (Fig. 12·34) on the arc AP can be located. Hence the armature surface is fixed and therefore the pole shoe surface can be drawn by knowing the length of air gap at various places and the type of pole shoe and pole arc.

The pole shoe drawing is completed by fixing the height of pole shoe. This height is fixed by the damper winding as explained in Art. 12·21·3.

12·22. Magnetic circuit. The fundamental relationships for magnetic circuit calculations have already been discussed in Chapter 4. The magnetic circuit for a pair of poles for a salient pole machine is shown in Fig. 12·35.

(i) Mmf for air gap. The calculation of mmf required for air gap is explained in Art. 4·4·1 on page 122.

(ii) Mmf for armature teeth. Parallel sided slots are used in synchronous machines and therefore the teeth are tapered. The mmf for the teeth is found out by finding flux density $B_{t1/3}$ at 1/3 height from the narrow end.

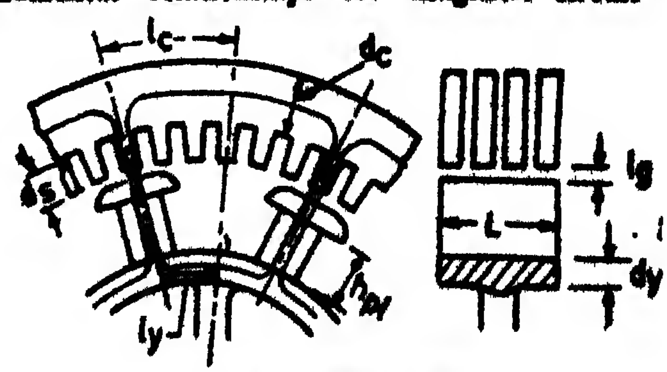


Fig. 12·35. Magnetic circuit.

From the standard B—at curves given in Fig. 12·36 the value of mmf per metre at_t can be found corresponding to $B_{t1/3}$. The length of flux path in the teeth is equal to the depth of the slot d_s.

(*iii*) **Mmf for core.** Corresponding to this flux density B_c, the mmf per metre at_c for the core is found from Fig. 12 36.

Total mmf for the core $AT_c = at_c\, l_c$

and the length of flux path in the core is taken equal to one half of the pole pitch on the mean diameter or

$$l_c = \frac{\pi(D+2d_s+d_c)}{2p} \qquad \ldots(12{\cdot}26)$$

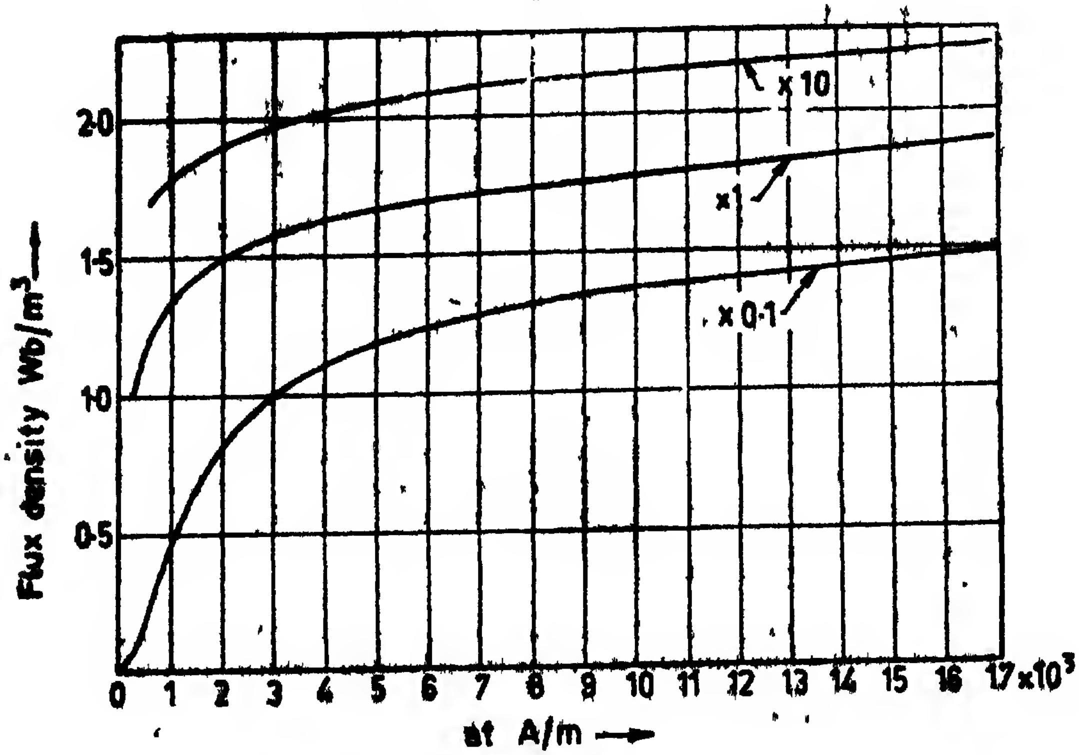


Fig. 12·36. 'B—at' Curve for dynamo sheet steel.

(*iv*) **Mmf for poles.** The flux in the poles is equal to the useful flux which crosses the air gap and enters the armature, plus the leakage flux. Actually the flux in the pole is not uniform throughout its radial length owing to different values of leakage flux from the pole shoes and the pole bodies. It is assumed for calculations that the top 2/3 length of pole carries the useful flux plus the leakage flux between pole shoes while the bottom 1/3 length carries the useful flux plus the leakage flux from both pole shoes and pole bodies. Thus the flux at the pole top is minimum while at the bottom it is maximum

or minimum flux in the poles $\Phi_{p(min)} = \Phi + \Phi_{sl}$ ...(12·27)

and maximum flux in the poles $\Phi_{p(max)} = \Phi + \Phi_{sl} + \Phi_{pl}$ ...(12·28)

The values of Φ_{sl} and Φ_{pl} are given by Eqns. 4·105 and 4·106. (See page 175).

$$\Phi_{sl} = 4\,\mu_0\, AT_f\left[\frac{L_s h_s}{O_s} + 1\cdot 47\, L_s \log_{10}\left(1+\frac{\pi b_s}{2O_s}\right)\right]$$

and
$$\Phi_{pl} = 2\,\mu_0\, AT_f\left[\frac{L_p h_p}{O_p} + 1\cdot 47\, L_p \log_{10}\left(1+\frac{\pi h_p}{2O_p}\right)\right]$$

Axial length of body L_p = axial length of the pole shoe L_s

and
$$AT_f = AT_g + AT_t + AT_c$$

From above it follows that :

Maximum flux density in the pole body $B_{p(max)}=\Phi_{p(max)}/A_p$

and minimum flux density in the pole body $B_{p(min)}=\Phi_{p(min)}/A_p$.

The mmfs, per metre corresponding to $B_{p(max)}$ and $B_{p(min)}$ are found from Fig. 12.37, and let them be $at_{p(max)}$ and $at_{p(min)}$.

Therefore, total mmf for the body is

$$AT_p=at_{p\,(max)}\cdot\frac{h_{pl}}{3}+at_{p(min)}\cdot\frac{2h_{pl}}{3} \quad ...(12.29)$$

(v) Mmf for yoke. Flux in the yoke $\Phi_y=\dfrac{\Phi+\Phi_{sl}+\Phi_{pl}}{2}$ ...(12.30)

Area of yoke, A_y = length of yoke × depth of yoke $=L\,d_y$.

$\therefore$ Flux density in the yoke $B_y=\dfrac{\Phi_y}{A_y}=\dfrac{\Phi+\Phi_{pl}+\Phi_{sl}}{2Ld_y}$ ...(12.31)

Corresponding to this flux density and the material of the yoke, the mmf per metre at_y for the yoke is taken from Fig. 12.36 or Fig. 4.1 depending upon the type of material used

$\therefore$ Mmf for yoke $AT_y=at_y l_y$

where l_y = path of magnetic flux through the yoke which is taken one half of the pole pitch on the mean diameter of the yoke

$$=\frac{\pi(D_r-2h_{pl}-d_y)}{2p} \quad ...(12.32)$$

Total field mmf required at no load

$$AT_{fo}=AT_g+AT_t+AT_c+AT_p+AT_y \quad ...(12.33)$$

12.23. Open circuit characteristics. The open circuit characteristics give the relation between the terminal voltage at no load and the corresponding field mmf per pole. The O.C.C. is shown in Fig. 12.37.

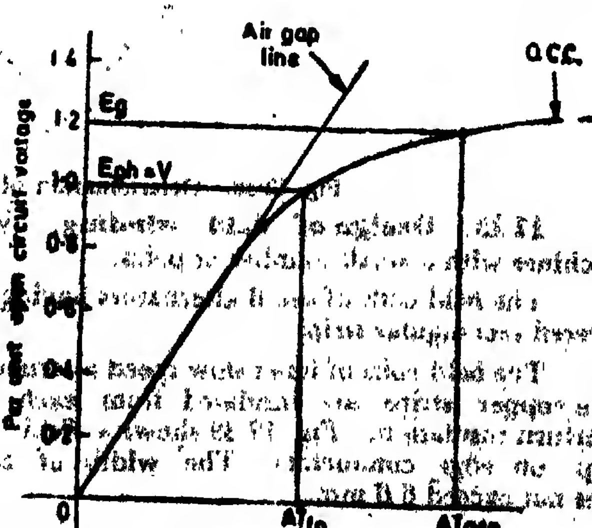


Fig. 12.37. Open circuit characteristics.

12.24. Determination of full load field mmf. The value of full load field mmf AT_{fl} can be calculated as below :

Refer to Fig. 12.38.

1. Select a suitable voltage scale.
2. Draw oa = voltage per phase $E_{ph}=V$.
3. Draw ob = armature current per phase, at an angle φ with respect to oa. (where cos φ = p.f. lagging in this case).
4. Draw [illegible] resistance drop per phase [illegible] parallel to [illegible]
5. Draw bc = leakage reactance drop per phase $I_a X_l$ and perpendicular to [illegible]
6. Join oc. Then oc is the generated voltage E_g.

Corresponding to this voltage E_g, find the field [illegible] from O.C.C. (Fig. 12.37) [illegible] on edge conductor

Let this be equal to AT_{gm}.

8. Plot $od = AT_{gm}$ to some scale.

9. Draw de = field mmf equivalent to armature mmf per pole at full load perpendicular to oI at d.

$$\text{Field mmf equivalent to armature mmf per pole} = \frac{2.7\, I_{ph}\, T_{ph}\, K_w}{p}\, \rho_d.$$

$$\text{The value of} \quad \rho_d = \frac{\alpha + \sin \alpha}{4 \sin \alpha/2} \quad \text{(See Eqn. 6·78 on page 315)}$$

10. Find the value of K_r from Fig. 12·33.)

On line de cut off $df = K_r \times de$.

11. Join of and extend it.

12. Draw a perpendicular from e on of extended cutting it at g.

Then, og = full load field mmf AT_{fl}.

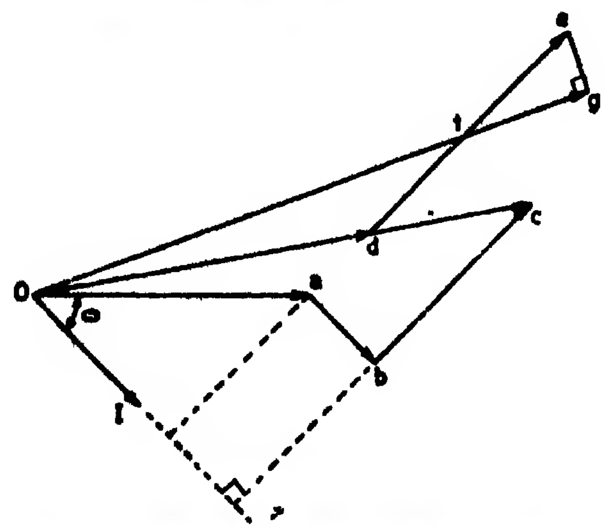


Fig. 12·38. Determination of full load field mmf.

12·25. Design of field winding. Wire wound coils are generally used for machines with a small number of poles.

The field coils of small alternators having large number of poles are wound with glass covered rectangular strips.

The field coils of large slow speed alternators use strip on edge winding wherein the bare copper strips are insulated from each other by interturn insulation. Fig. 12·39 shows a field coil with strip on edge conductors. The width of conductors does not exceed 6·0 mm.

For machines with class B insulation, the interturn insulation consists of 2 layers of treated asbestos paper, each layer having a thickness of 0·18 mm. The total thickness of interturn insulation is therefore 0·36 mm. Paper in the form of straight strips is applied at the sides and at the ends. The paper strips are stuck on with either synthetic resin varnish or shellac. Flanges which are 10 mm thick and made from bakelised asbestos board are then placed in position and coil is pressed and subjected to a pressure which it will encounter due to centrifugal forces at runaway speed. Simultaneously, current is passed through the conductors to raise the temperature of the field coil.

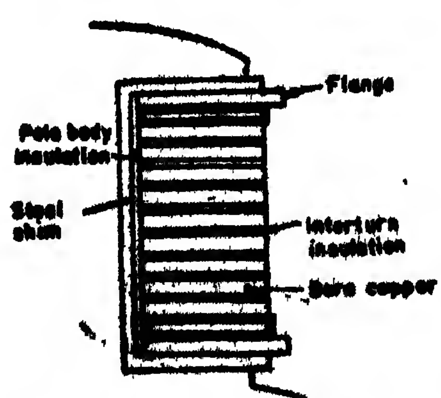


Fig. 12·39. Field coil with strip on edge conductors.

The temperature should be high enough so that polymerisation of interturn insulation is complete. Due to pressing and consolidation the thickness of interturn insulation is reduced from 0·36 mm to 0·26 mm.

For machines designed with class F insulation, the interturn insulation consists of 3 layers of 0·18 mm thick epoxy treated asbestos paper. The paper layers are held by dabbing epoxy adhesive. The flanges are mouldings epoxy resin with glass chopped strand mat.

The pole body insulation is of epoxy glass laminates and is 4 mm thick.

The coil is consoliidated under a pressure which varies 4 to 12 MN/m².

In order to caclulate the length of mean turn of the coils the plan is shown in Fig 12·40.

$$L_{mtf}=2L_m+\pi(b_p+0{\cdot}01+d_f) \qquad \ldots(12{\cdot}34)$$

with $\quad L_m=0{\cdot}9\ L \quad$ and $\quad e=0{\cdot}05\ L.$

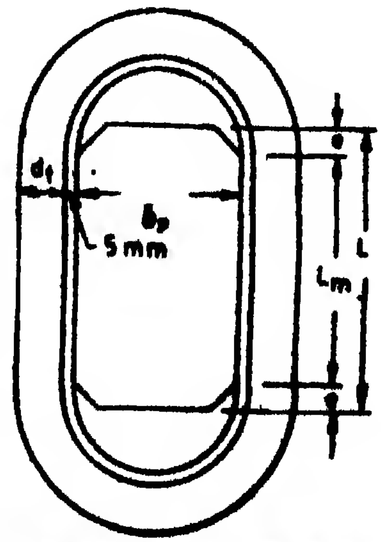


Fig. 12·40. Calculation of length of mean turn.

Procedure

1. In order to design the field winding, the exciter voltage must be known. The exciter voltage varies between 50 V to 400 V. This is usually specified by the customer. However, a voltage of 125 V is used for small and medium size machines while an exciter voltage of 250 V is used for large size machines. The field winding should be designed for a voltage from 15 to 20 per cent less than the exciter voltage to allow for the drop in voltage between field and the exciter and to allow for variations in the reluctance of magnetic field.

Let V_e be the exciter voltage.

$\therefore$ Voltage across each field coil

$$E_f=\frac{(0{\cdot}8 \text{ to } 0{\cdot}85)\ V_e}{p}$$

(there being as many field coils as the number of poles and all of them connected in series).

2. We know h_{pl} the height of the pole From this we subtract the height of shoe h_s and also the thickness of pool, flanges etc. and the space left is the winding height h_f,

or $\qquad h_f=h_{pl}-h_s-$space taken by spool, flanges etc

The space taken up by spool, flanges etc. is approximately 20 mm.

3. Now a suitable depth of windings d_f is assumed and the mean length of turn is evaluated. Table 12·3 may be used as a guide.

Table 12·3 Field winding depth.

Pole pitch m	*Winding depth* mm
0·1	25
0·2	35
0·4	45

Voltage across each field coil

E_f = field current × resistance of each field coil at 75°C

$$= I_f R_f = I_f \frac{T_f \rho L_{mtf}}{a_f}$$

where T_f = number of turns in each field coil.

Now $I_f T_f$ = field mmf per pole at full load = AT_{fl}

$$\therefore \quad E_f = AT_{fl} \frac{\rho L_{mtf}}{a_f}$$

or area of field conductors $a_f = \dfrac{AT_{fl}\, \rho\, L_{mtf}}{E_f}$

Thus the area of the field conductors can be calculated.

5. Now the value of the field current can be calculated by assuming a suitable value of current density in the field conductors.

Current density in field conductors, δ_f, is 3 to 4 A/mm².

∴ Field current I_f = current density × area of conductors $= \delta_f a_f$.

6. Number of field turns $T_f = AT_{fl}/I_f$.

Since the number of field turns is to be whole number, the current density may have to be re adjusted in case a mixed number is obtained.

7. We know the winding space and thus we can easily find out whether it is possible to accommodate the field turns or not. In case the winding space available is less, increase the depth and if the space is more, decrease the depth till the winding fits in.

8. The resistance of the winding is calculated at 75°C.

$$R_f = \frac{T_f \rho L_{mtf}}{a_f}.$$

Copper loss in each field coil at 75°C, $Q_f = I_f^2 R_f = I_f^2 T_f (\rho L_{mtf})/a_f$

This loss is to be dissipated by the field coil and therefore we must check that the temperature rise is within limits.

Dissipating surface of the coil is $S = 2 L_{mt}(h_f + d_f)$.

Referring to Table 3·6 page 111,

Cooling co-efficient to rotating field coils $c_f = \dfrac{0{\cdot}08 \text{ to } 0{\cdot}12}{1 + 0{\cdot}1 V_a}$

A suitable value is taken for c_f.

∴ Temperature rise $\theta = Q_f c_f / S$.

The temperature rise of the coil should be within the specified limits. If it exceeds the limits, increase the depth of winding. This decreases the current density in the windings and thus reduces the copper loss. Also, increase of depth of winding increases the heat dissipating surface of the coil and thus there is a marked decrease in the temperature rise.

9. The final check applied is to note the clearance between adjacent field coils from the pole drawing. The minimum clearance between them should be 15 mm.

Example 12·36. *The field coils of a salient pole alternator are wound with a single layer winding of bare copper strip 30 mm deep, with separating insulation 0·15 mm thick. Determine a suitable winding length, number of turns and thickness of conductor to develop an mmf of 12000 A with a potential difference of 5 V per coil and with a loss of 1200 W/m² of total coil surface. The mean length of turn is 1·2 m. The resistivity of copper is 0·021 Ω/m and mm².*

Solution. Area of field conductor $a_f = \frac{AT_{fl}\, \rho\, L_{mtf}}{E_f}$

$$= \frac{12000 \times 0{\cdot}021 \times 1{\cdot}2}{5} = 60{\cdot}4 \text{ mm}^2.$$

Height of conductor $= \frac{60{\cdot}4}{30} = 2$ mm.

Area of conductor $= 30 \times 2 = 60$ mm².

Total heat dissipating surface $S = 2\, L_{mtf}\,(h_f + d_f)$

$$= 2 \times 1{\cdot}2\,(h_f + 0{\cdot}03) = 2{\cdot}4\, h_f + 0{\cdot}272.$$

Total loss dissipated $Q_f = 1200\,(2{\cdot}4\, h_f + 0{\cdot}072) = 2880\, h_f + 86{\cdot}4.$

$\therefore$ Field current $I_f = \frac{Q_f}{E_f} = \frac{2880\, h_f + 86{\cdot}4}{5} = 5{\cdot}76\ h_f + 17{\cdot}3$

Field mmf $= I_f\, T_f = (576\, h_f + 17{\cdot}3)\, T_f$

$\therefore$ $576\, h_f\, T_f + 17{\cdot}3\, T_f = 12000$ ...(i)

Height occupied by each conductor $= 2 + 0{\cdot}15 = 2{\cdot}15$ mm.

$\therefore$ Height winding $h_f = T_f \times 2{\cdot}15 \times 10^{-3}$.

Substituting the value of h_f in (i)

$$576 \times 2{\cdot}15 \times 10^{-3}\, T_f^2 + 17{\cdot}3\, R_f = 120000$$

or number of turns in the field winding $T_f = 91$.

$\therefore$ Height of field winding $h_f = 2{\cdot}15 \times 91 = 196$ mm.

Example 12·17. *A 3 phase 1000 kVA, 3300 V, 250 r.p.m., 50 Hz star connected alternator has the following open circuit and short circuit characteristics. Using the synchronous reactance method, find the field mmf required at full load, 0·8 power factor lagging.*

Field mmf, A	1000	2000	3000	4000	6000	8000	10,000
Open circuit phase voltage, V	480	960	1400	1720	2110	2300	2480
Short circuit current, A	116	232	—	—	—	—	—

If the field exciter voltage is 110 V, length of mean turn is 1·37 m and the resistivity is 0·02 Ω/m and mm², calculate the area of field conductor. The field coil is a single layer winding of copper strip, 38 mm wide with an insulation of 0·25 mm between turns. The external periphery is 1·49 m and if the loss dissipated is 5000 W/m² of external surface of the coil, calculate the axial length of coil.

Solution. Number of poles $p = 120 \times 50/250 = 24$.

Rated voltage per phase $V = 3300/\sqrt{3} = 1910$ V.

The short circuit characteristics is a straight line.

$\therefore$ Short circuit current corresponding to rated voltage $I_{sc} = \frac{1910}{960} \times 232 = 462$ A.

$\therefore$ Synchronous reactance $X_s = V/I_{sc} = 1910/462 = 4{\cdot}13\ \Omega$.

Rated full load current $I = \frac{1000 \times 1000}{\sqrt{3} \times 3300} = 175$ A.

Per unit synchronous reactance $=IX_s/V=175\times4{\cdot}13/1910=0{\cdot}378$

Per unit no load voltage corresponding to excitation at full load and 0·8 power factor lagging

$$\mathbf{E}_0=1+(0{\cdot}8-j\,0{\cdot}6)\,j\,0{\cdot}37=1{\cdot}227+j\,0{\cdot}302 \text{ or } E_0=1{\cdot}23;$$

No load voltage $E_0=1{\cdot}23\times1910=2350$ V.

The field mmf corresponding to a voltage of 2350 V is 8500A.

$\therefore$ Field mmf required at full load $AT_{fl}=8500$A.

Voltage across each field coil $E_f=110/24=4{\cdot}58$.

Area of field conductor $a_f=\dfrac{8500\times0{\cdot}023\times1{\cdot}37}{4{\cdot}58}=58{\cdot}5$ mm².

Height of each conductor $=58{\cdot}5/38=1{\cdot}54$ mm.

Suppose there are T_f turns in each coil, field current is I_f and the height of field coil is h_f.

$\therefore$ Space occupied by each turn along height $=1{\cdot}54+0{\cdot}25=1{\cdot}79$ mm.

$\therefore$ Number of turns $T_f=\dfrac{h_f}{1{\cdot}79\times10^{-3}}$ ...(i)

Heat dissipated $=5000\times1{\cdot}49\,h_f=7450\,h_f$.

Field current $I_f=7450\,h_f/4{\cdot}58=16{\cdot}25\,h_f$ ...(ii)

From (i) and (ii), we have

$$I_fT_f=\frac{1625\,h_f^2}{1{\cdot}79\times10^{-3}}=908\times10^3\,h_f^2.$$

But $\quad I_fT_f=8500 \quad \therefore \quad h_f^2=\dfrac{8500}{908\times10^3}=9{\cdot}36\times10^{-3}$.

Hence $\quad h_f=0{\cdot}097$ m $=97$ mm.

12·26 Determination of direct and quadrature axis synchronous reactances. The magnetising reactance per phase of a cylindrical rotor machine with uniform air gap and with three phase winding is :

$$x_m=\frac{7{\cdot}54\,f\,T_{ph}^2\,K_w^2\,DL}{p^2\,l_g\,K_g}\times10^{-6} \quad ...(12{\cdot}36)$$

Per unit magnetising reactance $X_m=I_{ph}\,x_m/E_{ph}$ ...(12·37)

To obtain the armature reactance in the direct and quadrature axis for salient pole machines, this expression must be multiplied by flux distribution coefficients.

Per unit direct axis armature reaction reactance $X_{ad}=A_{d1}\,X_m$ ...(12·38)

where $\quad Ad_1=$ flux distribution co efficient for direct axis

$=\rho_d\times A_1$ ...(12·39)

$\rho_d=$ reduction factor for direct axis armature mmf

$=\dfrac{\alpha+\sin\alpha}{4\sin\alpha/2}$ (See Eqn. 6·87 on page 315)

and $\quad A_1=B_{m1}/B_g$ ...(12·40)

Per unit quadrature axis armature reaction reactance $X_{aq}=A_{q1}\,X_m$ ...(12·41)

where $\quad A_{q1}=$ flux distribution co-efficient for quadrature axis

$=\dfrac{4\psi+1}{5}-\dfrac{\sin\psi\pi}{\pi}$ ...(12·42)

where $\quad \psi=$ ratio of pole arc to pole pitch.

Per unit values of unsaturated synchronous reactances for the two axes are :

Direct axis synchronous reactance $X_d = X_l + X_{ad}$ ...(12·43)

Quadrature axis synchronous reactance $X_q = X_l + X_{aq}$ ...(12·44)

where X_l is the per unit leakage reactance.

The phasor diagram based upon two reaction theory for a generator is given in Fig. 12·41.

V = terminal voltage per phase = E_{ph}

I = armature current per phase = I_{ph}

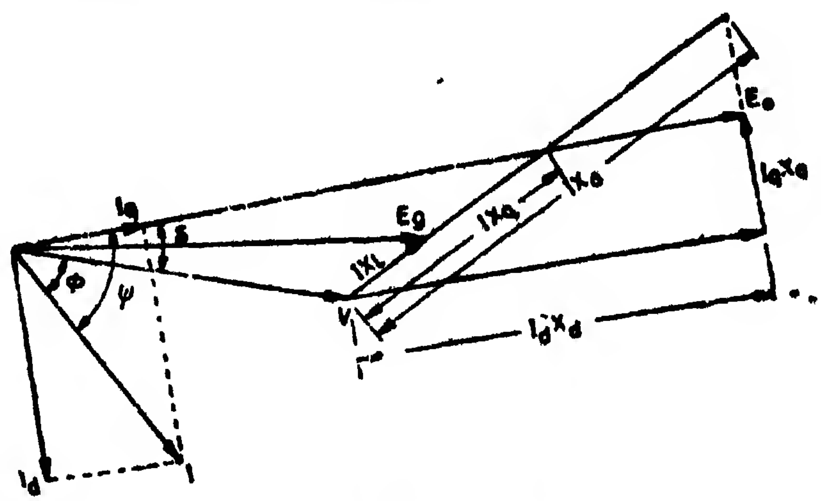


Fig. 12·41. Phasor diagram of a salient pole generator.

cos ϕ = power factor, lagging in this case

E_g = generated voltage per phase ; E_o = no load voltage per phase

δ = power angle ; $\psi = \phi + \delta$ for lagging power factor

I_d = direct axis current = $I \sin \psi$

I_q = quadrature axis current = $I \cos \psi$.

The phasor diagram for a cylindrical rotor machine is shown in Fig. 12·42.

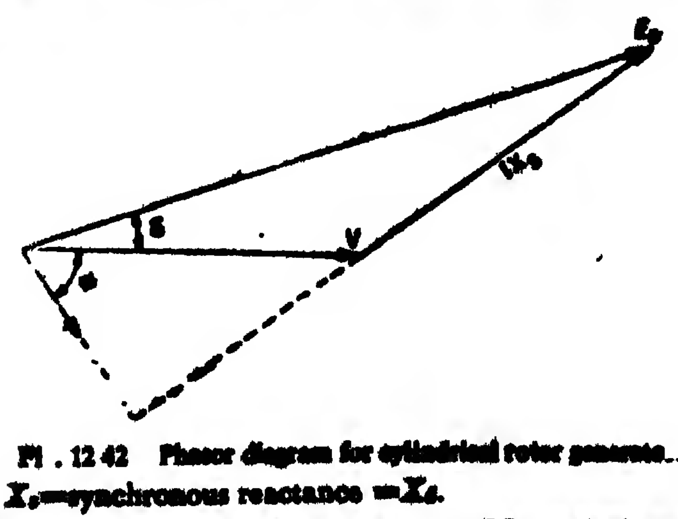


Fig. 12·42 Phasor diagram for cylindrical rotor generator.

X_s = synchronous reactance = X_d.

The reduction factor for direct axis armature mmf for cylindrical rotor machines

$$k_d = \frac{[illegible]}{8 \sin ([illegible]/2)} \qquad \text{...(12·[illegible])}$$

Example 12·18. *A 2500 kVA, 32 pole, 3 phase, 60 Hz, 2400 V, star connected salient pole alternator has the following design data :*

Stator bore = 2 5 m ; core length = 0·44 m ; turns per phase = 224 ; winding factor = 0 95 ; air gap length = 10 m ; air gap contraction factor = 1·11 ; ratio of pole arc to pole pitch = 0 69 ; ratio of amplitude of fundamental of gap flux density to maximum gap density = 1·068 ; per unit leakage reactance = 0·14.

Determine the direct and quadrature axis synchronous reactances.

Solution. Magnetising reactance per phase

$$x_m=\frac{7\cdot54\, f\, T_{ph}^2\, K_w^2\, DL}{p^2\, l_g\, K_g}\times 10^{-6}$$

$$=\frac{7\cdot54\times60\times(224)^2\times(0\cdot95)^2\times2\cdot5\times0\ 44}{(32)^2\times(10\times10^{-3})\times1\cdot11}\times10^{-6}=1\cdot98\ \Omega.$$

Voltage per phase $V=E_{ph}=2400/\sqrt{3}=1390$ V.

Current per phase $I=I_{ph}=\dfrac{2500\times1000}{\sqrt{3}\times2400}=600$ A.

Per unit magnetising reactance $X_m=600\times1\cdot98/1390=0\cdot855$.

Angle embraced by pole arc $\alpha=\psi\pi=0.69\ \pi=2\cdot17$ rad $=124°$.

Reduction factor for direct axis armature mmf (Eqn. 6·87)

$$\rho_d=\frac{\alpha+\sin\alpha}{4\sin\alpha/2}=\frac{2\cdot17+\sin 124°}{4\sin 124°/2}=0\ 85.$$

Flux distribution factor for direct axis (Eqn. 12·39)

$A_{d1}=\rho_d\times A_1=0\cdot85\times1\cdot068=0\cdot91.$

Per unit direct axis armature reaction reactance (Eqn. 12·38)

$X_{ad}=A_{d1}\ X_m=0\cdot91\times0\cdot855=0\cdot776$

From Eqn. 12·42, flux distribution co-efficient for quadrature axis

$$A_{q1}=\frac{4\psi+1}{5}-\frac{\sin\psi\pi}{\pi}=\frac{4\times0\cdot69+1}{5}-\frac{\sin 0\cdot69\ \pi}{\pi}=0\cdot46$$

The per unit quadrature axis armature reaction reactance

$X_{aq}=A_{q1}X_m=0\cdot40\times0\cdot855=0\ 393.$

Per unit direct axis synchronous reactance $X_d=X_l+X_{ad}=0\ 916$.

Per unit quadrature axis synchronous reactance $X_q=X_l+X_{aq}=0\ 533$.

Example 12·19. *A 2500 kVA, 2400 V, 3 phase star connected synchronous generator has the following data :*

$X_d=0\ 916$ p.u. ; $X_q=0\cdot533$ p u Calculate the no load voltage and power angle when the machine is delivering its rated current at rated voltage and 0·8 power factor lagging.

Solution. A graphical solution is done for this problem Choose a proper scale and draw

oa = 1 representing the terminal voltage V. (See Fig. 12·43)

Draw phasor *oI* at angle $\cos^{-1}0\cdot8=37°$ to *oa* representing current *I*.

At *a*, draw *ac* perpendicular to *ob* and cut off

(*i*) $ac=I\times X_d=1\times0\ 916=0\ 916$

and (*ii*) $ad=I\times X_q=1\times0\cdot533=0\cdot533.$

Join *od* and extend. Drop a perpendicular from *c* on *od* extended to cut at *e*.

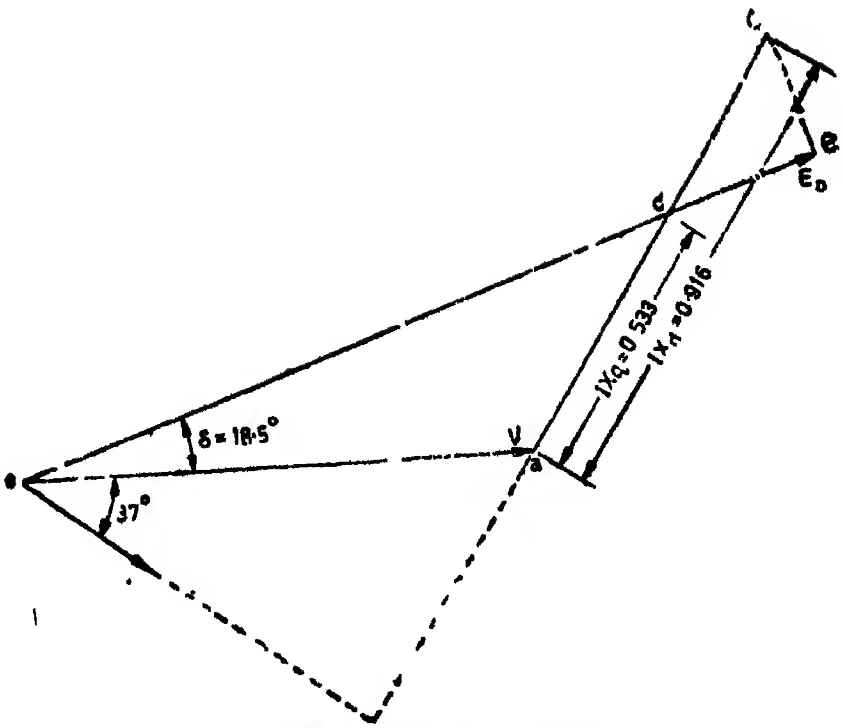


Fig. 12·43. Phasor diagram for Example 12·19.

Then oe = no load voltoge E_0

$= 1{\cdot}67$ p.u. $= 4000$ V

and power angle $\delta = 18{\cdot}5°$.

12·27. Short circuit characteristics. The relationship between the field mmf and the armature current when the armature is short circuited is known as the short circuit characteristics (S.C.C.). The armature short circuit current is proportional to the field mmf over a wide range and so the S.C.C. is a straight line.

For the short circuit conditions, the armature current is practically in quadrature with the voltage, and thus the armature mmf has a demagnetising effect on the field. The voltage induced in the armature when short circuited is nearly equal to IX_l, the leakage reactance drop. The field mmf required to generate this voltage is found from O.C.C. and is equal to OB (See Fig. 12·44). The armature when short circuited has a demagnetizing effect upon the field. The field mmf equivalent to armature mmf $= \rho_d \times AT_a$.

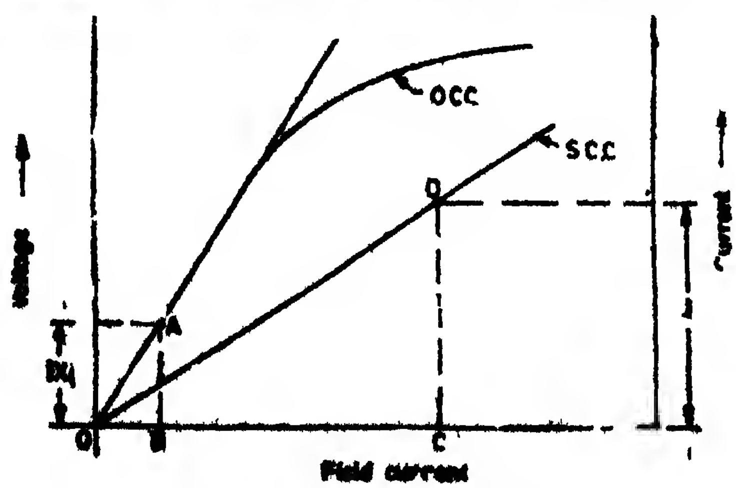


Fig. 12·44. O.C.C. and S.C.C.

In Fig 12·45, BO is cut equal to $\mu_s \times AT_a$. If CD equals the rated armature current, I_s then D is a point on the S.C.C. for rated value of load current. OO equals the field mmf per pole to circulate the rated armature current under short circuit conditions.

12·28 Losses The various losses in synchronous machines can be classified as :

(*i*) iron loss due to main field (*ii*) iron loss due to parasitic fields

(*iii*) I^2R loss in the armature winding (*iv*) eddy current loss in armature conductors

(*v*) stray load loss (*vi*) loss in field winding

(*vii*) friction and windage loss

(*i*) **Iron loss.** The iron losses due to the main field are the hysteresis and eddy current losses. The weights of armature core and teeth are calculated and the loss per kg can be taken from Eqn 4·58, page 149. The total iron loss can thus be evaluated.

(*ii*) **Pole face loss** A ripple is superimposed on the flux density wave owing to presence of slots on the armature surface. The ripple moves with respect to poles and induces eddy currents in the pole faces. The pole face loss depends upon the slot opening, air gap length, number of slots and the speed of the machine.

The poles should be laminated in order to reduce the pole face loss. In practice, either the poles are laminated or when made of solid iron, the pole shoes are laminated. In turbo-generators the rotor is solid but owing to a large air gap, the pole face losses are small.

In synchronous machines pole face loss varies between 25 to 70 per cent of total iron loss.

(*iii*) **Copper Loss.** I^2R loss per phase$=I_{ph}^2 r_{d.c.}$ Total I^2R loss$=3\ I_{ph}^2 r_{d.c.}$

(*iv*) **Eddy current loss in conductors.** Eddy current loss in conductors have been explained in Art 6·40 on page 320.

Total copper loss$=3\ K_{d(av)}\ I_{ph}^2\ r_{d.c.}$

where $K_d(_{av})$ is obtained from Eqn. 6·99 on page 325.

(*v*) **Stray load loss.** Stray load loss occurs due to stray fields which appear when the machine is loaded. For example, the armature mmf wave contains higher harmonics of the order 5, 7, 11, 13. These higher harmonic mmfs cause higher harmonic fields which travel over the pole face and induce eddy currents there and thus increase the iron loss at load.

Additional losses at load may be caused by armature leakage flux in the overhang region surrounding the iron portions and the end plates.

Actually stray field loss may appear in the pole faces, core and overhang and end plates. This loss is very difficult to calculate and yet they may be comparable with whole of the stator I^2R loss.

(*vi*) **Excitation loss.** Field copper loss$=I_f^2\times$resistance field winding.

Besides I^2R loss there is brush contact losses on the slip rings. The following voltage drops may be assumed in brushes :

for carbon and graphite brushes—1 V and for brushes containing metal—0·3 V.

If the pilot and the main exciters are driven from the same shaft, the losses in the exciters must be taken into account.

(*vii*) **Friction and windage loss.** This loss consists of bearing friction and rotor windage loss. This loss depends upon the type of construction, speed and rating of the machine and varies between 0·2 to 0·8 per cent of kVA rating. The higher values are for high speed and high rating machines. With hydrogen cooling the total friction loss reduces to 0·3 to 0·4 per cent of kVA rating.

12·29. Temperature rise. The calculation of temperature rise of stators of rotating electrical machines is explained in Art. 3·26·1 on page 111.

DESIGN OF TURBO-ALTERNATORS

12.30. Main dimensions. The values of specific loadings for conventionally cooled generators are :

$B_{av}=0.54$ to 0.65 W/bm^2,

$ac=50{,}000$ to $75{,}000$ A/m.

The specific loadings now used in large water cooled generators are :

$B_{av}=0.54$ to 0.62 Wb/m^2,

$ac=180{,}000$ to $200{,}000$ A/m.

The value of stator bore D is limited by the peripheral speed. The maximum peripheral speed is 175 m/s Normally, a peripheral speed of about 120 m/s is used. When peripheral speed is specified, the relationship

$$Q=1.11\ K_w\ B_{av}\ ac\ L\ \frac{V_a^2}{n_s}\times 10^{-3}$$

is used for calculation of core length L.

Example 12.20. *A 3000 r.p.m., 50 Hz, 3 phase turbo-alternator has a core length of 0.94 m The average gap density is 0.45 Wb/m² and the ampere conductors per metre are 25000. The peripheral speed of rotor is 100 m/s and the length of air gap is 20 mm. Find the kVA output of the machine when the coils are (i) full pitch (ii) chorded by ⅓ pole pitch. The winding can be taken as infinitely distributed with a phase spread of 60°.*

Solution.

Synchronous speed $n_s=3000/60=50$ r.p.s.

Peripheral speed $V_a=\pi D_r n_s=100$ (given).

$\therefore$ Diameter of rotor $D_r=100/\pi\times 50=0.637$ m.

Stator bore $D=D_r+2l_g=0.637+2\times 0.02=0.677$ m.

With infinite distribution and 60° phase spread, distribution factor $K_d=0.955$.

(*i*) With full pitch coils, pitch factor $K_p=1$.

$\therefore$ Winding factor $K_w=0.955\times 1=0.955$.

Output, $Q=11\ K_w\ B_{av}\ ac\ D^2L\ n_s\ 10^{-3}$

$=11\times 0.955\times 0.45\times 25000\times(0.667)^2\times 0.94\times 50\times 10^{-3}=2480$ kVA.

(*ii*) Angle of chording $\alpha=180/3=60°$ $\therefore$ Pitch factor $K_p=\cos\alpha/2=0.866$.

$\therefore$ Winding factor $K_w=0.955\times 0.866=0.827$.

Hence, output $Q=2480\times(0.827/0.955)=2147$ kVA

as specific loadings, diameter, length and speed remaining the same, the output of the machine is directly proportional to the winding factor.

12.31. Length of air gap. The length of air gap can be approximated by the method given below :

Approximate value of armature ampere conductors per pole$=ac\ \tau$.

Armature mmf per pole $AT_a=ac\ \tau/2$ (approximate)

$\therefore$ No load field mmf $AT_{fo}=SCR\times AT_a=SCR\times ac\ \tau/2$.

The short circuit ratio of modern turbo-alternators is about 0.5 to 0.7.

Assuming 80 per cent of no load mmf to be lost in the air gap,

mmf required for the gap$=0.8\ SCR\times ac\ \tau/2$ ...(i)

But the mmf required for the air gap$=800{,}000\ K_g\ B_g\ l_g$ ...(ii)

$\therefore$ From (*i*) and (*ii*), we have

length of air gap $l_g = \frac{0.5\, ac\, \tau}{K_g\, B_g} \times 10^{-6}$ ...(12.46)

Taking a sinusoidal distribution of flux density in the air gap, $B_g = (\pi/2)\, B_{av}$.

For all practical purposes B_g is taken as $1.5\, B_{av}$ and K_g as 1.1.

12.32. Stator design. The number of stator slots per pole per phase lies between 2 to 4 but in the case of turbo-alternators 8 or 9 slots per pole per phase may be used. The slot pitch is normally about 25 to 60 mm but in the case of large turbo-alternators it may even be 75 to 90 mm.

Single layer concentric or two layer short pitched windings may be used. The advantage of single layer winding is that it can be easily clamped but it gives a higher stray load loss owing to overhang running parallel to the end plates. Also with single layer winding, it is not possible to chord and therefore flux harmonics have full effect. Two layer winding chorded by about 1/6 pole pitch is more common as it practically eliminates 5th and 7th as well as 17th and 19th harmonics.

The stator conductors must be subdivided and transposed to reduce eddy current losses.

In windings of large modern turbo-alternators it is common practice to assemble two conductors per slot. There are two parallel circuits per phase. The current density in the stator windings of modern water-cooled generators is usually between 8 to 9.5 A/mm² as compared to about 4A/mm² in the case of conventially cooled machines.

The stator winding of turbo-alternators is deliberately put in deep slots in order to increase the leakage reactance. This is done to reduce the forces under short circuit conditions. This has the incidental advantage of spacing the overhang away from rotor end rings.

Example 12.21. *Estimate the diameter, core length, size and number of conductors, number of slots for stator of a 15 MVA, 11 kVA, 50 Hz, 2 pole star connected turbo-alternator with 60° phase spread. Assume :*

$B_{av} = 0.55$ Wb/m² ; $ac = 36,000$ A/m ; current density = 5A/mm², peripheral speed = 160 m/s.

The winding should be arranged to eliminate 5th harmonic.

Solution. Synchronous speed $n_s = 2 \times 50/2 = 50$ r.p.s.

Peripheral speed $V_a = \pi D n_s = 160$ (given). $\therefore$ Diameter $D = \frac{160}{\pi \times 50} \simeq 1$ m.

Distribution factor for 60° phase spread, $K_d = 0.955$.

In order to eliminate 5th harmonic, the coils should be chorded by an angle

$$\alpha = 180/5 = 36°.$$

$\therefore$ Pitch factor $K_p = \cos \alpha/2 = 0.951$ and winding factor $K_w = 0.955 \times 0.951 = 0.908$.

Output co-efficient $C_0 = 11 \times 0.55 \times 36000 \times 0.908 \times 10^{-3} = 198$.

$\therefore$ Product $D^2 L = \frac{Q}{C_0\, n_s} = \frac{15,000}{198 \times 50} = 1.51$ m³.

Hence, core length $L = 1.51/(1.0)^2 = 1.51$ m and pole pitch $\tau = \pi \times 1.0/2 = 1.57$ m.

Flux per pole $\Phi = B_{av}\, \tau L = 0.55 \times 1.57 \times 1.51 = 1.3$ Wb.

Voltage per phase $E_{ph} = 11000/\sqrt{3} = 6360$ V.

Turns per phase $T_{ph} = \frac{6360}{4.44 \times 50 \times 1.3 \times 0.908} = 24.3 \simeq 24$.

Total number of stator conductors $= 6 \times 24 = 144$.

The number of slots should be so selected that it suits the winding as regards the number of conductors. Since the fifth harmonic is to be eliminated, the number of slots should be such that chording by 1/5 pole pitch is possible.

In order that the winding should be chorded, requires the use of a double layer winding and chording by 1/5 of the pole pitch can only be obtained if the number of slots is a multiple of 5.

Using 5 slots per pole per phase,

total number of slots $=3\times2\times5=30$.

$\therefore$ Conductors per slot $=144/30\simeq5$.

This is an odd integer and therefore double layer winding is not possible. Double layer winding is only possible with either 4 or 6 conductors per slot and this would require the use of either 120 or 180 conductors resulting an excessive increase or decrease in the value of gap density.

If two parallel circuits are used the number of conductors required is $144\times2=288$ with 5 slots per pole per phase, the total number of slots is 30 with 10 conductors per slot, which permits the use of a double layer winding. The total number of conductors is 300, an increase of only 4%.

Thus the winding used has the following details :

slots per pole per phase $=5$, total number of stator slots $=30$,

conductors per slot $=10$, turns per phase $T_{ph}=50$.

Example 12.32. *A 588 MVA, 22000 V, 50 Hz, 2 pole, 3 phase star connected direct watercooled generator has a stator bore of 1·3 m and a stator core length of 6·0 m. If the stator winding has 2 conductors per slot and there are two circuits per phase, calculate (a) the number of stator slots and (b) the average flux density in the air gap.*

The specific electric loading is 200,000 ampere conductors per metre. Assume a winding factor of 0·92.

Solution.

Speed $n_s=50$ r.p.s.

Voltage per phase $E_{ph}=22000/\sqrt{3}=12750$ V.

Current per phase $I_{ph}=\dfrac{588\times10^6}{\sqrt{3}\times22000}=15{,}400$ A.

Current in each conductor $I_z=15400/2=7700$ A.

Now, specific electric loading 'ac' $=\dfrac{I_zZ}{\pi D}$.

$\therefore$ Total number of armature conductors $Z=\dfrac{\pi Dac}{I_z}$

$$=\frac{\pi\times1\cdot3\times200{,}000}{7700}=106.$$

The number of conductors has to be modified so that a suitable number of slots can be used. We should choose this number in such a way that the number of turns per phase T_{ph} and the number of slots per pole per phase are integers.

For a three phase machine, turns per phase $T_{ph}=Z/6=106/6\simeq18$.

$\therefore$ Actual number of conductors used $Z=6\times18=108$.

Number of slots $S=\dfrac{\text{conductors}}{\text{conductors per slot}}=\dfrac{108}{2}=54$.

The number of slots $S=54$ are suited to the winding.

For a winding with two circuits per phase, $E_{ph}=\dfrac{4.44\, f\Phi\, T_{ph}\, K_w}{2}$.

$\therefore$ Flux per pole $\Phi=\dfrac{2\times 12750}{4.44\times 50\times 18\times 0.92}=6.92$ Wb.

Pole pitch $\tau=\pi\times 1.3/2=2.04$ m.

Average flux density $B_{av}=\dfrac{\Phi}{\tau L}=\dfrac{6.92}{2.04\times 6}=0.565$ Wb/m².

12.33. Rotor design. The rotor wnding is not concentrated but is distributed in slots. Concentric multi turn coils are used as shown in Fig. 12.45. The number of wound slots should be an integer which is a multiple of four (e.g., 16, 20, 24 ....). The slot pitch is so chosen that undesirable harmonics are not introduced in the flux density wave.

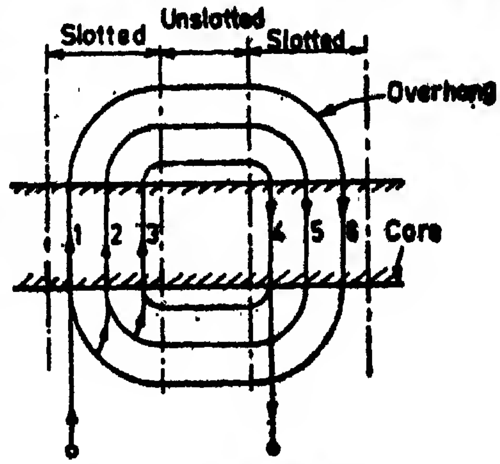


Fig. 12.45. Rotor winding.

The width of rotor slots is limited by stresses at the root of the teeth and by hoop stress in the end retaining rings.

In large modern turbo-alternators Epoxy glass and absestos moulded resin glass and or synthetic rubberised glass are used as insulation. The insulation thickness varies from 0.25 to 0.33 mm per 100 V across the winding.

Rotor current density may be about 2.5 A/mm² for conventially cooled machines. However, in modern direct cooled generators the rotor current densities may be as high as 9.5—14 A/mm².

Procedure for rotor winding design

1. Full load field mmf can be taken as twice the armature mmf.

$\therefore$ Full load field $AT_{fl}=2AT_a$

where $$AT_a=2.7\, I_{ph}\, T_{ph}\, K_w/p.$$

2. A standard exciter voltage may be taken. About 15 to 20 per cent of this voltage is kept in reserve.

Let V_e be the exciter voltage.

$\therefore$ Voltage across each field coil $E_f=\dfrac{(0.8\text{ to }0.85)\,V_e}{p}$.

4. The length of mean turn of field winding is approximated from,

$$L_{mtf}=2L+2.3\tau+0.24 \qquad \ldots(12.47)$$

where τ = effective span of coils.

Voltage across each field coil $E_f=I_f R_f$

$$=I_f\frac{T_f\,\rho\, L_{mtf}}{a_f}=\frac{AT_{fl}\,\rho\, L_{mtf}}{a_f}.$$

$\therefore$ $$a_f=\frac{AT_{fl}\,\rho\, L_{mtf}}{E_f}$$

where I_f = field current, A ; T_f = number of turns in each field coil ;

a_f = area of field conductor, mm² ;

L_{mtf} = length of mean turn, m ;

and ρ = resistivity, Ω/m and mm².

5. Assume a suitable current density δ_f for the field winding

$\therefore$ Total area of field conductors $=\dfrac{2p\,AT_{fl}}{\delta_f}$ ...(12·48)

$\therefore$ Number of field conductors $=\dfrac{2p\,AT_{fl}}{\delta_f\,a_f}$ ...(12·49)

and conductors per slot $=\dfrac{2p\,AT_{fl}}{\delta_f\,a_f\,S_r}$ ...(12·50)

where S_r = number of wound slots in the rotor.

DESIGN PROBLEMS

Design problem 1. *Design a water wheel generator with the following specifications :*

KVA	=	*3000*	*Voltage*	=	*6600 V*
Phase	=	*3*	*Frequency*	=	*50 Hz*
RPM	=	*187·5*	*Connection*	=	*Star*

Power factor = 0·8 lagging.

Solution.

Main Dimensions :

Synchronous speed $n_s=187{\cdot}5/60=3{\cdot}125$ r.p.s.

$\therefore$ Number of poles $p=2\times50/3{\cdot}145=32$.

Assume $B_{av}=0{\cdot}6$ Wb/m², $ac=34000$ A/m and $K_w=0{\cdot}955$.

Output co-efficient $C_0=11\,K_w\,B_{av}\,ac\,10^{-3}$

$=11\times0{\cdot}955\times0{\cdot}6\times34000\times10^{-3}=214.$

$\therefore$ Product $D^2L=\dfrac{Q}{C_0 n_s}=\dfrac{3000}{214\times3{\cdot}125}=4{\cdot}48$ m³.

Using rectangular poles and taking $L/\tau=1{\cdot}5$, we have

$L=1{\cdot}5\pi\,D/32\times\cancel{1{\cdot}5}=0{\cdot}1475\,D$

or $0{\cdot}1475\,D^3=4{\cdot}48$

$\therefore$ $D=3{\cdot}12$ m and $L=0{\cdot}1475\times3{\cdot}12=0{\cdot}46$ m.

Selecting $D=3{\cdot}2$ m and $L=0{\cdot}44$ m.

D^2L provided $=3{\cdot}2^2\times0{\cdot}44=4{\cdot}5$ m³.

Peripheral speed at synchronous speed

$V_a=\pi\times3{\cdot}2\times3{\cdot}125=31{\cdot}4$ m/s.

The peripheral speed at run-away speed will be nearly 60 m/s and therefor dovetail type of construction is used for attachment of poles to the yoke.

Pole pitch $\tau=\pi\times3{\cdot}2/32=0{\cdot}314$ m.

Taking the ratio $\psi=\dfrac{\text{pole arc}}{\text{pole pitch}}=0{\cdot}74.$ = p

$\therefore$ Pole arc $=\psi\tau=0{\cdot}74\times0{\cdot}314=0{\cdot}2325$ m.

Using 5 ducts of width 10 mm each.

Gross iron length $L_i = L - n_d W_d = 0.44 - 5 \times 10 \times 10^{-3} = 0.39$ m $= 390$ mm.

Therefore, the stator core is divided into 6 parts each 65 mm wide.

Net iron length $L_i = 0.9 \times 0.39 = 0.351$ m.

STATOR

Stator Winding

The stator winding is star connected with line voltage $= 6600$ V.

Phase voltage $E_{ph} = 6600/\sqrt{3} = 3820$ V.

Flux per pole $\Phi = B_{av}\, \tau L = 0.6 \times 0.314 \times 0.44 = 82.8 \times 10^{-3}$ Wb.

Turns per phase $$T_{ph} = \frac{E_{ph}}{4.44\, f\, \Phi\, K_w} = \frac{3820}{4.44 \times 50 \times 82.8 \times 10^{-3} \times 0.956} = 218.$$

(This value may have to be modified when considering the number of slots.)

Number of Slots

Taking a slot pitch of about 32 mm.

$\therefore$ Slots per pole per phase

$$q = \frac{\pi D}{3\, p\, y_s} = \frac{\pi \times 3.2 \times 10^3}{3 \times 32 \times 32} = 3.27 \simeq 3\tfrac{1}{4}.$$

A fractional slot winding with $q = 3\frac{1}{4}$ is used. This number is chosen as the number of poles, 32, is divisible by the denominator 4.

$\therefore$ Total number of armature slots $S = 3 \times 3\frac{1}{4} \times 32 = 312$.

Total number of armature conductors $Z = 6T_{ph} = 6 \times 218 = 1308$.

$\therefore$ Conductors per slot $= 1308/312 = 4.2$.

The number of conductors per slot should be an even integer in order that a fractional slot winding is used (with a fractional number of slots per pole per phase, we have to use a double layer winding which requires that the conductors per slot should be even). The nearest number is 4 (actually the number of slots should be so chosen that the number of conductors already calculated has not to be modified very much).

Now conductors per slot $= Z_s = 4$.

$\therefore$ Total conductors $Z = 312 \times 4 = 1248$

and turns per phase $T_{ph} = 1248/6 = 208$.

$\therefore$ Modified average flux density in the gap

$$B_{av} = 0.6 \times 218/208 = 0.628 \text{ Wb/m}^2.$$

Modified flux per pole $\Phi = 0.628 \times 0.314 \times 0.44 = 86.9 \times 10^{-3}$ Wb.

Stator slot pitch $y_s = \pi D/S = \pi \times 3.2 \times 10^3/312 = 32.2$ mm.

Current per phase $$I_{ph} = \frac{3000 \times 1000}{\sqrt{3} \times 6600} = 262 \text{ A}.$$

Slot loading $= I_{ph}\, Z_s = 262 \times 4 = 1048$ A.

This is within limits.

Conductor Size. Taking a current density of 4 A/mm².

Area of cross section of armature conductors.

$$=262/14=65{\cdot}5 \text{ mm}^2.$$

Each conductor is divided into 2 subdivisions

Refer to Table 17·1 for size of copper strip.

Using 2 strips of 6×6 mm² cross-section in parallel.

Area of conductor used $=2\times35{\cdot}1=70{\cdot}2$ mm².

Current density used for armature conductors

$$\delta_a=262/70{\cdot}2=3{\cdot}70 \text{ A/mm}^2.$$

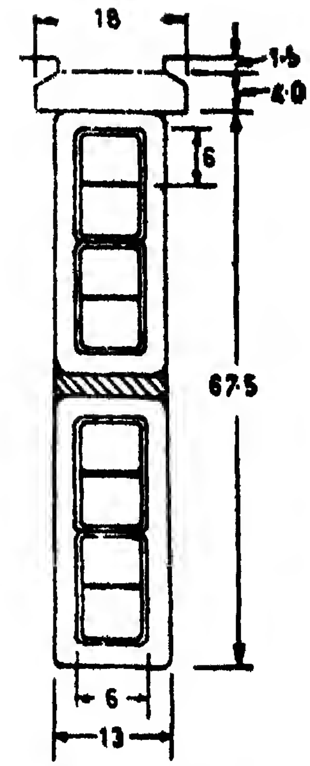


Fig. 12·46. Stator slot. (All Dimensions in mm)

Double layer lap winding with diamond coils is used. There are 4 conductors per slot, 2 in each layer and therefore there are 2 turns per coil.

Slot Dimensions. Using 0·25 mm thick strips between strands and 0·5 mm thick conductor insulation. The thickness of main wall insulation is 2·5 mm.

Referring to Fig. 12·46.

Slot Width :

Bare conductor	1×6	6·0 mm
Conductor insulation	2×0·5	1·0 mm
Main slot insulation	2×2·5	5·0 mm
Slack		1·0 mm
Total slot width		$W_s=13{\cdot}0$ mm

Slot Depth :

Bare conductor	8×6	48·0 mm
Insulation between strands	4×0·25	1·0 mm
Conductor insulation	8×0·5	4·0 mm
Main slot insulation	4×2·5	10·0 mm
Separator	1×2·5	2·5 mm
Tooth lip		1·5 mm
Wedge		4·0 mm
Slack		2·0 mm
Total slot depth		$d_s=73{\cdot}0$ mm

Checks

1. We have to see that the average eddy current loss factor is not high.

Now $\alpha=100\sqrt{b/W_s}=100\sqrt{6/3}=68$

and $h'=\text{height of strand}=6\times10^{-3}$ m

$\therefore$ $\alpha h'=68\times6\times10^{-3}=0.408.$

Since $\alpha h'$ is less than 0.7 and therefore all the equations derived in chapter 6 for eddy current loss in conductors, can be applied here.

Number of conductor layers in the slot $N=8$.

From Eqn. 6.99 on page 325, average eddy current loss factor for slot portion of conductors

$$K_{e(av)}=1+(\alpha h')^4\frac{N^2}{9}=1+(0.408)^4\times\frac{8^2}{9}=1.2.$$

$\therefore$ The eddy current loss in conductors is 20% of the copper loss due to the main current. This is within limits.

2. We have to see that the flux density in the teeth at the air gap surface is not excessive (the teeth have minimum cross-section near the air gap surface).

Width of teeth at the gap surface $W_t=32.2-13=19.2$ mm.

Flux density in the teeth at the gap surface

$$=\frac{\text{flux per pole}}{\text{teeth per pole arc}\times\text{net iron length}\times\text{tooth width}}$$

$$=\frac{86.9\times10^{-3}}{0.74\times(312/32)\times0.351\times19.2\times10^{-3}}=1.785\ \text{Wb/m}^2.$$

Within limits.

Length of mean turn

Length of mean turn of stator winding

$L_{mts}=2L+2.5\,1+6\,\text{kV}+$ [illegible] m.

Out of this a length of 0.88 m is buried in the slots while the remaining 1.38 m length is in the overhang.

Stator resistance

The stator d.c. resistance per phase at 75°C

$$r_{d.c.}=0.021\frac{208\times2.26}{70.2}=0.141\ \Omega.$$

Effective stator resistance

D.C. resistance of conductors embedded in slots per phase

$$=0.021\times\frac{208\times0.88}{70.2}=0.0546\ \Omega.$$

Average eddy current loss factor $K_{e(av)}=1.2$.

$\therefore$ Copper losses per phase in slot portion of conductors

$$=K_{e(av)}\,I_s^2\ (\text{d.c. resistance of slot portion})$$

$$=1.2\times262^2\times0.0546=4.5\ \text{kW}$$

D.C. resistance per phase of conductors in the overhang

$$=0.021\frac{208\times1.38}{70.2}=\text{[illegible]}\ \Omega.$$

Copper losses per phase in overhang

$$=I_{ph}^2 \text{ (d.c. resistance of overhang portion)}$$
$$=262^2 \times 0.086 = 5.9 \text{ kW.}$$

$\therefore$ Total copper loss per phase $=4.5+5.9=10.4$ kW.

$\therefore$ A.C. resistance per phase $r_{a.c.}=\dfrac{10.4\times1000}{262^2}=0.151\ \Omega.$

P.U. resistance $R_{a.c.}=\dfrac{I_{ph}\, r_{a.c.}}{E_{ph}}=\dfrac{262\times0.151}{3820}=0.01035.$

Stator core

Flux in stator core $\Phi_c=\dfrac{86.9\times10^{-3}}{2}=43.45\times10^{-3}$ Wb.

Assuming a flux density of 1.1 Wb/m² in the core.

Depth of core $d_c=\dfrac{\Phi_c}{B_c\times L_i}=\dfrac{43.45\times10^{-3}}{1.1\times0.351}=0.112$ m $=112$ mm.

Outer diameter of stator laminations $D_o=D+2(d_s+d_c)$

$$=3.2+2(0.073+0.112)=3.57 \text{ m.}$$

Length of air gap

Armature mmf per pole

$$AT_a=\frac{2.7\times262\times208\times0.955}{32}=4400 \text{ A.}$$

Assuming a short circuit ratio of 1.3.

$\therefore$ No load field mmf per pole

$$AT_{fo}=1.3\times4400=5720 \text{ A.}$$

Taking 80 per cent of this mmf to be consumed in the air gap.

$\therefore$ Mmf for air gap $AT_g=0.8\times5720=4566$ A.

Maximum flux density in the air gap

$$B_g=\frac{B_{av}}{K_f}=\frac{0.628}{0.74}=0.85 \text{ Wb/m}^2.$$

Assuming the gap contraction factor $K_g=1.15$.

Mmf required for air gap $AT_g=800{,}000\ B_g\ K_g\ l_g$

or $800{,}000\times0.85\times1.15\, l_g=4566$

$\therefore$ $l_g=5.84$ mm.

Air gap length provided $l_g=5.5$ mm.

$\therefore$ Diameter of rotor $D_r=D-2l_g=3.2-2\times5.5\times10^{-3}=3.189$ mm.

Poles

A leakage factor of 1.2 is assumed for the field poles.

$\therefore$ Flux in pole body $\Phi_p=1.2\times86.9\times10^{-3}=104.5\times10^{-3}$ Wb.

Taking a flux density of 1.6 Wb/m² in poles.

Area of pole body $A_p=\dfrac{\Phi_p}{B_p}=\dfrac{104.5\times10^{-3}}{1.6}=65.3\times10^{-3}$ m².

Taking 0.98 as the stacking factor for pole laminations,

width of pole $b_p=\dfrac{A_p}{0.98\ L}=\dfrac{65.3\times10^{-3}}{0.98\times0.44}=0.15$ m $=150$ mm.

Height of pole

In order to determine the height (radial length) of pole we require an approximate estimate of full load field mmf. The method has been explained in details in Art. 12·2·1·1.

Approximate value of no load field mmf

AT_{fo}=5720 A (calculated above).

Armature mmf per pole AT_a=4400 A.

Power factor cos ϕ=0·8 lagging ∴ ϕ=36° 52′.

From Fig. 12·31 for ψ=0·74 the value of cross reaction factor K_r=0·45. See

Referring to Fig. 12·47.

1. Draw oa=5720.
2. Draw ab=4400 at an angle (90°−36° 52′)≅53° to oa.
3. Cut off ac=0.45×4400=1980.
4. Join oc and extend. Drop perpendicular on ac extended to cut it at d.
5. Full load field mmf per pole,

od=8930 A.

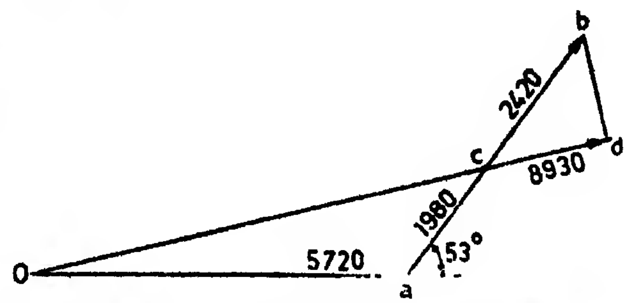


Fig. 12·47. Determination of approximate value of AT_{fl}.

Assume a current density of 2·5 A/mm² in the field winding

Area of copper in the field winding

=8930/2·5=3570 mm².

Strip on edge conductors are used for field winding. Assume a space factor of 0·83.

Area of field winding=3570/0·83=4300 mm².

Assuming depth of field winding d_f=35 mm.

∴ Height of field winding=4300/35=123 mm.

To this we must add about 17 mm which is taken up by flanges etc

∴ Height of pole body h_p=123+17=140 mm.

The height of pole shoe at the centre h_1=25 mm.

(See pole drawing Fig. 12·50)

∴ Total height (radial length of pole) h_{pl}=140+25=165 mm

Damper windings

From Eqn. 12·24, total area of damper bars per pole

$$A_d=0·2\times34000\times0·314/3=713 \text{ mm}^2.$$

if current density is taken as 3A/mm² for damper winding.

Taking the pitch of damper bars to be 80 percent of stator slot pitch.

$$\therefore \text{ Number of damper bars } N_d=\frac{\text{pole arc}}{0·8\times\text{stator slot pitch}}$$

$$=\frac{232·5}{0·8\times32·2}\cong9.$$

Area of each bar a_d=713/9=79 mm².

∴ Diameter of each bar d_d=10 mm.

∴ Height of pole shoe at the tips h_e=2d_d=20 mm.

Yoke design

Flux in yoke $\Phi_y = \Phi/2 = 52{\cdot}25 \times 10^{-3}$ Wb.

Taking a flux density of 1·2 Wb/m².

Area of yoke $A_y = 52{\cdot}25 \times 10^{-3}/1{\cdot}2 = 43{\cdot}5 \times 10^{-3}$ m².

$\therefore$ Depth of yoke $d_y = \dfrac{\text{yoke area}}{0{\cdot}98 \times \text{core length}} = \dfrac{43{\cdot}5 \times 10^{-3}}{0{\cdot}98 \times 0{\cdot}41} = 0{\cdot}1$ m $= 100$ mm.

Estimation of flux per pole

Pole drawing

In order to estimate the flux per pole, the pole drawing is constructed as explained in Art. 12·21·4 with the help of the following data :

Angle (mechanical) between pole axis and the interpolar axis $\theta = 360/2p = 5{\cdot}625°$.

Length of air gap at the centre of pole $l_g = 5{\cdot}5$ mm.

The distance over which the air gap remains constant is $\frac{1}{3}\,\tau$.

Length of air gap at pole tips $= 1{\cdot}8\ l_g = 1{\cdot}8 \times 5{\cdot}5 \approx 10$ mm.

Height of pole shoes at pole tips $h_e = 20$ mm

The rough sketch of pole profile is shown in Fig. 4·72 (page 193).

The pole drawing is shown in Fig. 12·48.

From this drawing, we get

Height of pole shoe at the centre $h_1 = 25$ mm and width of pole shoe $b_s = 230$ mm.

Flux plot

The polar horn and the stator surface at the air gap are drawn in Fig. 4·73 with the help of the pole drawing techniques. The flux plot and the flux distribution curve of this machine are given in Example 4·17 on page 193 (see Figs. 4·73 and 4·74)

The flux distribution curve is

$$B_\theta = 108{\cdot}06 \sin\theta + 3{\cdot}36 \sin 3\theta - 5{\cdot}25 \sin 5\theta - 0{\cdot}75\ 7\theta + \ldots$$

and the field form factor (calculated on page 193) is

$$K_f = B_{av}/B_g = 0{\cdot}702.$$

Voltage per phase $E_{ph} = 6600/\sqrt{3} = 3820$ V.

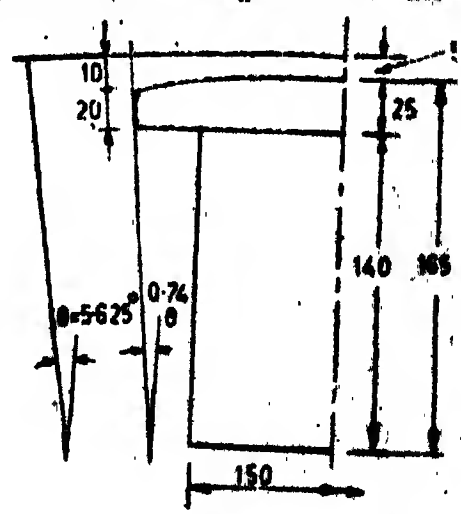


Fig. 12·48. Pole drawing (Dimensions in mm.)

The line voltage of a star connected alternator does not contain any third harmonic emf. Therefore, the value 3820 V of the phase voltage does not take into account the third harmonic voltage which is actually present in the phase voltage. Therefore, when we take the phase voltage equal to $1/\sqrt{3}$ of line voltage, that value does not include the third harmonic voltage. Hence, the third harmonic would be excluded from the calculations. From Eqn 6·72 (page [illegible]) the distribution factor for fractional slot windings of phase spread of 60°, is

$$K_{dm} = \frac{\sin m\pi/6}{N \sin m\pi/6N}$$

We have, $q=3\frac{1}{4}=13/4 \quad \therefore \quad M=13.$

Thus,
$$K_{d_1}=\frac{\sin 30^\circ}{13 \sin 30^\circ/13}=0{\cdot}96,\quad K_{d_5}=\frac{\sin 150^\circ}{13 \sin 150^\circ/13}=0{\cdot}193,$$
$$K_{d_7}=\frac{\sin 210^\circ}{13 \sin 210^\circ/13}=-0{\cdot}139.$$

There are $3\frac{1}{4}$ slots per pole per phase. In order that the coils be full pitch, the coil span should be $3\frac{1}{4}\times 3=9\frac{3}{4}$ slots. The coil span is actually 9 slots.

$\therefore$ Electrical angle spanned by coils $=\dfrac{9}{9\frac{3}{4}}\times 180=166^\circ$.

$\therefore$ Angle through which coils are chorded $\alpha=180^\circ-166^\circ=14^\circ$.

Pitch factor for nth harmonic is $K_{pn}=\cos n\alpha/2$

$\therefore$
$$K_{p1}=\cos 1\times 14^\circ/2=0{\cdot}9925,\quad K_{p5}=\cos 5\times 14^\circ/2=0{\cdot}819$$
$$K_{p7}=\cos 7\times 14^\circ/2=0{\cdot}656.$$

The winding factors are :

$K_{w1}=0{\cdot}96\times 0{\cdot}9925=0{\cdot}952,\quad K_{w5}=0{\cdot}193\times 0{\cdot}819=0{\cdot}158,$

$K_{w7}=-0{\cdot}139\times 0{\cdot}656=-0{\cdot}0913.$

From Eqn. 6·65 page 295, voltage per phase

$$E_{ph}=4{\cdot}44\, T_{ph}\, f\, \Phi_1\, K_{w1}\sqrt{1+(B_{m3}K_{w3}/B_{m1}K_{w1})^2+\ldots\ldots+(B_{mn}K_{wn}/B_{m1}K_{w1})^2+\ldots}$$

As explained earlier, we have not to take into account the third harmonic component.

$$\therefore \quad E_{ph}=4{\cdot}44\, T_{ph}\, f\, \Phi_1\, K_{w1}\sqrt{1+(B_{m5}K_{w5}/B_{m1}K_{w1})^2+\ldots\ldots\ldots+(B_{mn}K_{wn}/B_{m1}K_{w1})^2+\ldots}$$

or
$$[illegible]=4{\cdot}44\times 208\times 50\times \Phi_1\times 0{\cdot}952\left[1+\left(\frac{5{\cdot}25\times 0{\cdot}158}{108{\cdot}06\times 0{\cdot}952}\right)^2+\left(\frac{0{\cdot}75\times 0{\cdot}0913}{108{\cdot}06\times 0{\cdot}952}\right)^2\right]^{1/2}$$

or fundamental component of flux per pole $\Phi_1=87\times 10^{-3}$ Wb.

Average value of fundamental component of flux density

$$B_{av1}=\frac{\Phi_1}{\tau L}=\frac{0{\cdot}087}{0{\cdot}314\times 0{\cdot}44}=0{\cdot}629 \text{ Wb/m}^2.$$

Maximum value of fundamental component of flux density

$B_{m1}=\pi/2\, B_{av1}=\pi/2\times 0{\cdot}629=0{\cdot}987$ Wb/m².

But $\quad B_{m1}=1{\cdot}08\, B_g \qquad \therefore \quad B_g=0{\cdot}987/1{\cdot}08=0{\cdot}914$ Wb/m².

Average flux density over the pole pitch, $B_{av}=0{\cdot}702\times 0{\cdot}914=0{\cdot}642$ Wb/m².

$\therefore$ Total flux per pole $\Phi=0{\cdot}642\times 0{\cdot}314\times 0{\cdot}44=88{\cdot}6\times 10^{-3}$ Wb.

Magnetic circuit. The mmf required for various parts of magnetic circuit are calculated below. Dynamo sheet steel is used for both rotor and stator. The magnetization curve for the laminations used is given in Fig 12·36.

(i) Mmf for air gap.

Ratio $\qquad \dfrac{\text{slot width}}{\text{gap length}}=\dfrac{1{\cdot}3}{0{\cdot}55}=2{\cdot}36.$

Corresponding to this ratio, the Carter's co-efficient K_{cs} is 0·3 (See Fig. 4·9 on page 134 for open slots).

Gap contraction factor for slots $K_{gs}=\dfrac{y_s}{y_s-K_{cs}W_s}$

$$=\frac{3{\cdot}22}{3{\cdot}22-0{\cdot}3\times 1{\cdot}3}=1{\cdot}135.$$

Ratio $\dfrac{\text{duct width}}{\text{gap length}}=\dfrac{1{\cdot}0}{0{\cdot}55}=1{\cdot}82.$

Corresponding to this ratio, the Carter's co-efficient K_{cd} is 0·25 (See Fig 4·9 for open slots).

Gap contraction factor for ducts $K_{gd}=\dfrac{L}{L-n_d\,W_d\,K_{cd}}$

$$=\frac{44}{44-5\times1\times0{\cdot}25}=1{\cdot}03.$$

∴ Gap contraction factor $K_g=K_{gs}\times K_{gd}=1{\cdot}135\times1{\cdot}03=1{\cdot}17.$

Mmf required for air gap $AT_g=800{,}000\times0{\cdot}914\times1{\cdot}17\times5{\cdot}5\times10^{-3}=4700$ A.

(ii) **Mmf for teeth.**

Width of teeth at 1/3 height from narrow end $=\dfrac{\pi(D+2/3d_s)}{S}-W_s$

$$=\frac{\pi[3{\cdot}2+(2/3)\times73\times10^{-3}]}{312}-13\times10^{-3}\text{ m}=19{\cdot}7\text{ mm}.$$

Flux density in teeth at 1/3 height from narrow end

$$B_{t1/3}=\frac{86{\cdot}6\times10^{-3}}{0{\cdot}74\times(312/32)\times0{\cdot}351\times19{\cdot}7\times10^{-3}}=1{\cdot}77\text{ Wb/m}^2.$$

Corresponding to this flux density, mmf per metre $at_t=9500$ A.

Total mmf required for teeth $AT_t=9500\times73\times10^{-3}=694$ A.

(iii) **Mmf for core**

Area of core $A_c=d_c\times L_i=112\times10^{-3}\times0{\cdot}351=40\times10^{-3}$ m^2.

Flux density in core $B_c=\Phi/2A_c=88{\cdot}6\times10^{-3}/(2\times40\times10^{-3})=1{\cdot}11$ Wb/m^2.

Mmf per metre $at_c=420$ A.

Length of flux path in core $l_c=\dfrac{\pi(D+2d_s+d_c)}{2p}$

$$=\frac{\pi(3{\cdot}2+2\times73\times10^{-3}+112\times10^{-3})}{2\times32}=0{\cdot}169\text{ m}.$$

Mmf for core $AT_c=420\times0{\cdot}169=71$ A.

∴ $AT_1=AT_g+AT_t+AT_c=4700+694+71=5465$ A.

(iv) **Mmf for poles.** Width of pole shoe $b_s=230$ mm$=0{\cdot}23$ m (from pole arc)

Distance between adjacent pole shoes,

$$O_s=\frac{\pi(D_r-h_s)}{p}-b_s=\frac{\pi(3{\cdot}189-25\times10^{-3})}{32}-0{\cdot}23=0{\cdot}081\text{m}.$$

Height of pole shoe at pole tip $h_s=20$ mm$=0{\cdot}02$ m.

Width of pole body $b_p=150$ mm$=0{\cdot}15$ m.

Distance between bodies of adjacent poles

$$O_p=\frac{\pi(D_r-h_s-h_p)}{p}-b_p=\frac{\pi(3{\cdot}189-25\times10^{-3}-140\times10^{-3})}{32}-150\times1$$

$$=0{\cdot}147\text{ m}.$$

From Eqn. 4·105 (page 175), leakage flux from pole shoes

$$\Phi_{ls}=4\mu_0 L_p AT_f\left[\frac{h_s}{O_s}+1{\cdot}47\log_{10}\left(1+\frac{\pi}{4}\cdot\frac{b_s}{O_s}\right)\right]$$

$$=4\times4\pi\times10^{-7}\,AT_l\left[\frac{0\cdot44\times0\cdot62}{0\cdot081}+1\cdot47\times0\cdot02\log_{10}\left(1+\frac{\pi}{2}\times\frac{0\cdot23}{0\cdot081}\right)\right]$$

$$=0\cdot664\times10^{-6}\,AT_l.$$

From Eqn. 4 106, leakage flux between pole bodies

$$\Phi_{pl}=2\mu_0\,AT_l\left[\frac{L_p h_p}{C_p}+1\cdot47\,h_p\log_{10}\left(1+\frac{\pi}{2}\,\frac{b_p}{C_p}\right)\right]$$

$$=2\times4\pi\times10^{-7}\,AT_l\left[\frac{0\cdot44\times0\cdot14}{0\cdot147}+1\cdot47\times0\cdot14\log_{10}\left(1+\frac{\pi}{2}\times\frac{0\cdot15}{0\cdot147}\right)\right]$$

$$=1\cdot265\times10^{-6}\,AT_l.$$

$\therefore$ $\Phi_{sl}=0\cdot664\times10^{-6}\times5465=3\cdot62\times10^{-3}$ Wb.

$\Phi_{pl}=1\cdot265\times10^{-6}\times5465=6\cdot92\times10^{-3}$ Wb.

$\therefore$ Minimum flux in pole $\Phi_{p(min)}=\Phi+\Phi_{sl}=(88\cdot6+3\cdot62)\times10^{-3}=92\cdot22\times10^{-3}$ Wb.

Maximum flux in pole $\Phi_{p(max)}=\Phi+\Phi_{pl}+\Phi_{sl}=99\cdot14\times10^{-3}$ Wb.

$\therefore$ Minimum flux density in pole $B_{p(min)}=\dfrac{92\cdot22\times10^{-3}}{65\cdot3\times10^{-3}}=1\cdot41$ Wb/m².

Maximum flux density in pole $B_{p(max)}=\dfrac{99\cdot14\times10^{-3}}{65\cdot3\times10^{-3}}=1\cdot52$ Wb/m².

Corresponding to these, mmfs per metre are

$at_{p(min)}=1200$ A ; $at_{p(max)}=2000$ A.

Total mmf required for poles

$$AT_p=at_{p(min)}\times\frac{2h_{pl}}{3}+at_{p(max)}\,\frac{h_{pl}}{3}$$

$$=1200\times2\times\frac{0\cdot165}{3}+2000\times\frac{0\cdot165}{3}=232\text{ A.}$$

(v) Mmf for yoke.

Flux in yoke $\Phi_y=\dfrac{\Phi+\Phi_{sl}+\Phi_{pl}}{2}=49\cdot6\times10^{-3}$ Wb.

Area of yoke $A_y=0\cdot98\times0\cdot1\times0\cdot44=43\cdot1\times10^{-3}$ m².

Flux density in yoke $=49\cdot6\times10^{-3}/43\cdot1\times10^{-3}=1\cdot15$ Wb/m².

Mmf per metre $at_y=470$ A.

Length of flux path in yoke $l_y=\dfrac{\pi(D_r-2h_{pl}-d_y)}{2p}$

$$=\frac{\pi(3\cdot18-2\times0\cdot165-0\cdot1)}{2\times32}=0\cdot135\text{ m.}$$

Mmf for yoke $AT_y=at_y\times l_y=470\times0\cdot135=64$ A.

Total mmf required at no load for normal flux density

$$AT_{fo}=AT_g+AT_t+AT_c+AT_p+AT_y=4700+694+71+232+64=5761\text{ A.}$$

Open circuit characteristics. For drawing the open circuit characteristics, circulations are done for flux densities 1·1 and 1·2 times the flux density for normal voltage. The results are tabulated on next page. The O.C.C is shown in Fig 12·49.

Armature leakage reactance. From Fig. 12·46

$h_1=59\cdot5$ mm, $h_2=5\cdot0$ mm, $h_3=4\cdot0$ mm,

$h_4=1\cdot5$ mm, $W_s=13\cdot0$ mm, $W_t=18\cdot0$ mm.

(The slack of 2·0 mm has been lumped in h_1.)

Open circuit characteristics

Per Unit normal voltage			1·0				1·1				1·2			
Emf, per phase, E_{ph}			3830				4200				45 80			
Flux Φ, mWb			88·6				97·46				106 32			
Part	Area m^2	Length m	Φ	B	at	AT	Φ	B	at	AT	Φ	B	at	AT
Gap	0·097	0·0055	88 6	0·914	855,000	4700	97·46	1·0	940,000	5170	106·32	1·1	1020,000	5640
Teeth 1/3	0·05	0·075	88·6	1·77	9.500	694	97·46	1·945	23,000	1680	106·32	2·125	85,000	6200
Core	2×0·04	0·169	88·6	1·11	420	71	97·46	1·22	570	97	106·32	1 33	850	144
		AT_1				5645				6947				11884
Pole (min)	0·0653	0·165	92·22	1·41	1,200	232	102·06	1·56	2,300	586	114·21	1·75	8,500	
Pole (max)			90·14	1·52	2,000		110 83	1·7	6,000		129 26	1·98	2",000	2420
Yoke	2×0·0431	0·135	90·14	1·15	470	64	110 83	1·28	700	95	129·26	1·5	1,500	203
Total no load mmf, A						5761				7628				14507

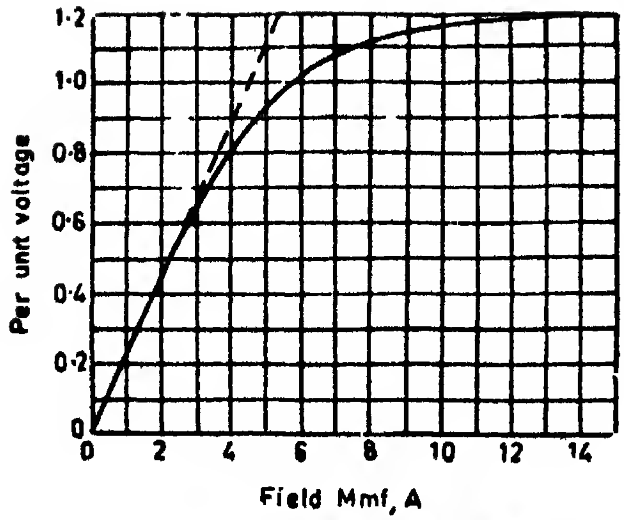


Fig. 12·49. Open circuit characteristics.

Specific slot permeance $\lambda_s = \mu_0 \left[\frac{h_1}{3W_s} + \frac{h_2}{W_s} + \frac{2h_3}{W_s + W_1} + \frac{h_4}{W_s}\right]$

$$= \mu_0 \left[\frac{59{\cdot}5}{3\times 13} \times \frac{5}{13} + \frac{2\times 4}{13+18} + \frac{1{\cdot}5}{13}\right] = 2{\cdot}28\ \mu_0.$$

From Eqn. 4·95 (page 172), slot leakage reactance

$$x_{sl} = 8\times \pi \times 50 \times (208)^2 \times 0{\cdot}44 \left(\frac{4\pi \times 10^{-7} \times 2{\cdot}28}{32\times 3\frac{1}{2}}\right) = 0{\cdot}659\ \Omega$$

Ratio $\dfrac{\text{coil span}}{\text{pole pitch}} = \dfrac{166°}{180°} = 0{\cdot}923.$

From Fig 4·58 (page 170),

corresponding to this ratio of 0·923, the value of K_s is 0·94.

From Eqn. 4·94

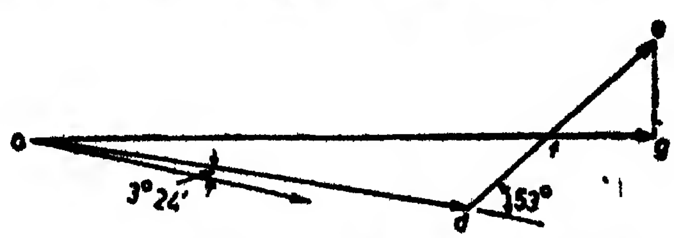
$$L_o\lambda_o = \frac{\mu_0 K_s \tau^2}{\pi y_s} = \frac{\mu_0 \times 0{\cdot}94 \times (0{\cdot}314)^2}{\pi \times 0{\cdot}0322} = 0{\cdot}946\ \mu_0.$$

From Eqn. 4·96, overhang leakage reactance

$$x_o = 8\pi \times 50 \times (208)^2 \times \frac{4\pi \times 10^{-7} \times 0{\cdot}946}{32 \times 3\frac{1}{2}} = 0{\cdot}622\ \Omega.$$

Total stator leakage reactance $x_l = x_{sl} + x_o = 0{\cdot}659 + 0{\cdot}622 = 1{\cdot}28\ \Omega.$

P.U. leakage reactance $X_l = \dfrac{I_{ph}\, x_l}{E_{ph}} = \dfrac{262 \times 1{\cdot}28}{3820} = 0{\cdot}0877.$



$od = 6900$ A, $de = 3660$ A, $df = 1510$ A, $fo = 2150$ A.

Fig. 12·50. Estimation of full load field mmf.

Full load field mmf.

Generated voltage $\mathbf{E}_g=\mathbf{E}_{ph}+\mathbf{I}_{ph}(R_{ac}+jX_l)$

$$=1+j_0+1(0.8-j0.6)(0.01035+j0.0877)=1.053\angle 3°24' \text{ p.u.}$$

$\therefore$ $\quad \mathbf{E}_g=4025$ V.

The field excitation corresponding to this voltage from O C C. (Fig. 12·49) is

$$AT_{gen}=6500 \text{ A}$$

Angle spanned by pole arc $\alpha=\psi\pi=0.74\,\pi=2.32$ radian$=133°$.

Armature reaction factor

$$\rho_d=\frac{\alpha+\sin\alpha}{4\sin\alpha/2}=\frac{2.32+\sin 133°}{4\sin 66.5°}=0.833.$$

Reduction factor for direct axis armature mmf

$$=\rho_d\, AT_a=0.833\times4400=3660 \text{ A}.$$

For the calculation of full load field mmf, the construction given in Fig. 12·38 is followed. We have :

$od=6500$

$de=3660$

$df=K_r\times3660=0.45\times3360=1510$

$fe=3660-1510=2150.$

From construction shown in Fig. 12·50, full load field mmf $AT_{fl}=og=9200$ A.

Field winding design

1. Assume an exciter voltage of 125 V with 25 V in reserve and 100 V across the field coils.

Voltage across each field coil $E_f=100/32=3.125$ V.

2. The field winding consists of conductors wound on edge. Using a sheet metal spool 2·5 mm thick and bakelised paper flanges 6 mm thick.

Height of winding $h_f=h_{pl}-h_1-$height occupied by insulation and spool

$$=165-25-2\times(6+2.5)=123 \text{ mm}.$$

3. The depth of winding, d_f, is assumed to be 35 mm. Referring to Fig. 12·40,

$L_m=0.9L=0.9\times0.44=0.396$ m, $b_p=0.15$ m

and $\quad L_{mtf}=2L_m+\pi(b_p+0.01+d_f)=1.4$ m (See Eqn. 12·34).

4. Area of field winding conductor $a_f=\dfrac{AT_{fl}\,\rho L_{mt}}{e_f}$

$$=\frac{9200\times0.021\times1.4}{3.125}=86.6 \text{ mm}^2.$$

6. Assuming current density δ_f of 2·6 A/mm², for the field winding, field current

$I_f=86.6\times2.6=225$ A.

7. Field winding turns $T_f=AT_{fl}/I_f=9200/225=41$.

8. Area of each conductor $a_f=86.6$ mm² and width of conductor$=35$ mm.

Depth of conductor$=86.6/35=2.5$ mm.

Area of conductor provided $a_f=35\times2.5=87.5$ mm².

Using an insulation of 0·3 mm between conductors

Height occupied by conductors$=41(2.5+0.3)=115$ mm.

The balance space of $123-115=8$ mm is taken up by buckling of conductors.

$\therefore$ Height of winding $h_f = 123$ mm $= 0.123$ m.

Resistance of each field coil $R = 41 \times \dfrac{0.021 \times 1.4}{87.5} = 0.0138\ \Omega$.

Actual value of field current $I_f = 3.125/0.0138 = 226$ A.

Mmf provided $AT_{fl} = I_f T_f = 226 \times 41 = 9270$ A.

Loss in each field coil $Q_f = (226)^2 \times 0.0138 = 705$ W.

Area of dissipating surface $S = 2\ L_{mf}\ (h_f + d_f) = 2 \times 1.4(0.123 + 0.035) = 0.442\ m^2$.

From Table 3.6 (page 111), cooling co-efficient

$$c_f = \frac{0.12}{1+0.1V_a} = \frac{0.12}{1+0.1 \times 31.4} = 0.029.$$

Temperature rise $\theta = \dfrac{Q_f\, c_f}{S} = \dfrac{705 \times 0.029}{0.442} = 46.2°C$.

This is within limits.

Losses

(i) Copper loss

The total copper loss including eddy current loss in each phase is 10.4 kW (This has already been calculated when calculating the effective stator resistance).

$\therefore$ Total copper loss $= 3 \times 10.4 = 31.2$ kW.

(ii) Stray load loss

The stray load loss is taken as 20 per cent of total copper loss.

$\therefore$ Stray load loss $= 0.2 \times 31.2 \cong 6.2$ kW.

(iii) Iron loss

(a) *Stator teeth* :

Diameter of armature at the middle of teeth $= 3.273$ m.

Slot pitch at the middle of teeth $= \pi \times 3.273/312$ m $= 32.9$ mm.

Tooth width at the middle $W_t = 32.9 - W_s = 32.9 - 13 = 19.9$ mm.

$\therefore$ Weight of stator teeth $= S \times L_i \times d_s \times W_t \times 7.8 \times 10^3$

$= 312 \times 0.351 \times 73 \times 10^{-3} \times 19.9 \times 10^{-3} \times 7.8 \times 10^3 = 1240$ kg.

The flux density in the teeth at 1/3 height from narrow end is $1.77\ Wb/m^2$. From Eqn. 4.58 (page 149), with $a = 6.5$,

Specific iron loss in teeth $= aB_m^2 = 6.5 \times (1.77)^2 = 21$ W/kg.

$\therefore$ Iron loss in teeth $= 1240 \times 21$ W $= 26$ kW.

(b) *Core* :

Mean core diameter $= 3.458$ m.

Weight of stator core $= \pi \times 3.458 \times 0.35 \times 114 \times 10^{-3} \times 7.8 \times 10^3 = 3380$ kg.

Flux density in the core $= 1.11\ Wb/m^2$.

From Eqn. 4.58, specific iron loss in core $= 4.7 \times (1.11)^2 = 5.9$ W.

$\therefore$ Iron loss in core $= 3380 \times 5.9$ W $= 20$ kW.

$\therefore$ Total iron loss $= 26 + 20 = 46$ kW.

(iv) Friction and windage loss is taken 0.7 percent of kVA rating.

$\therefore$ Friction and windage loss $= 0.7/100 \times 3000 = 21$ kW.

(v) Field copper loss $= pI_f^2\ R_f = 32 \times 226^2 \times 0.0138 = 22.5$ kW.

Taking a drop of 1 volt at each brush.

Brush contact loss $=2\times1\times226=552$ watt $\cong 0.6$ kW.

$\therefore$ Exciter input $=22.5+0.6=23.1$ kW.

Taking an exciter efficiency of 88 percent,
total exciter output $=23.1/0.88=26.3$ kW.

$\therefore$ Excitation losses $=26.3$ kW.

Total loss

1.	Copper loss including eddy current	$=31.2$ kW
2.	Stray load loss	$=6.2$ kW
3.	Iron loss	$=46.0$ kW
4.	Friction and windage loss	$=21.0$ kW
5.	Excitation loss	$=26.3$ kW
	Total loss	$\cong 131.0$ kW

Rated output $=$ kVA $\times$ p.f. $=3000\times0.8=2400$ kW.

Input $=2400+131=2531$ kW.

$\therefore$ Efficiency at full load $=\dfrac{2400}{2531}\times100=94.7$ percent.

Stator temperature rise

The cooling co-efficients are taken from Table 3.6.

Outer cylindrical surface of core $=\pi D_o L=\pi\times3.57\times0.44=4.92$ m^2.

Cooling co-efficient $=0.03$.

$\therefore$ Loss dissipated from back of stator core $=4.92/0.03=164$ W/°C.

Inner cylindrical surface of stator $=\pi DL=\pi\times3.2\times0.44=4.42$ m^2.

$$\text{Cooling co-efficient}=\frac{0.03}{1+0.1\,V_a}=\frac{0.03}{1+0.1\times31.4}=7.25\times10^{-3}.$$

Loss dissipated from inner surface $=4.42/(7.25\times10^{-3})=610$ W/°C

$$\text{Area of end surfaces including ducts}=\frac{\pi}{4}(D_o^2-D^2)(n_d+2)$$

$$=\frac{\pi}{4}(3.56^2-3.2^2)(5+2)=13.35\text{ m}^2.$$

$$\text{Cooling co-efficient}=\frac{0.1}{V}=\frac{0.1}{0.1\times31.4}=0.0317.$$

(The velocity for ducts $V=0.1\,V_a$.)

Loss dissipated from ducts $=3.35/(3.17\times10^{-2})=442$ W/°C.

Total loss dissipated $=164+610+442=1216$ W/°C.

Copper loss in slot portion of conductors $=3\times4.5=13.5$ kW.

Iron loss in stator $=46$ kW.

$\therefore$ Total loss to be dissipated from stator surface $=13.5+46=59.5$ kW.

$\therefore$ Stator temperature rise $\theta=\dfrac{59.3\times10^3}{1216}=49$°C.

This is within limits

Regulation

From O.C.C. corresponding to full load mmf of 9200 A, the voltage

$E_o=1.16$ per unit.

$\therefore$ **Regulation $=16$ per cent.**

Short circuit characteristics

Leakage reactance=0·0877 p.u.

Mmf corresponding to leakage reactance drop from O.C.C.=410 A.

Field mmf equivalent to armature mmf=3660 A.

∴ Field mmf required to circulate full load current in armature at short circuit
$=410+3660=4070$ A.

Field mmf required to generate rated voltage at no load=5761 A.

∴ Short circuit ratio, $SCR=\frac{5761}{4070}=1·41$.

The SCR was assumed 1·3 and the actual value of SCR is 1·41.

Design Sheet

kVA—3000	Voltage—6600 V	Phase—3
Frequency—50 Hz	Connection—star	Current—262 A.
RPM—187·5	Power Factor—0·8 lagging.	

Rating

1.	Full load kVA	Q	3000
2.	Full load power, kW	P	2400
3.	Line voltage		6600 V
4.	Phase voltage	E_{ph}	3820 V
5.	Power factor	$\cos\phi$	0·8 lagging
6.	Frequency	f	50 Hz
7.	Speed	n_s	3·125 r.p.s.
8.	Number of poles	p	32

Main Dimensions

1.	Specific magnetic loading	B_{av}	0·6 Wb/m²
2.	Specific electric loading	ac	34000 A/m
3.	Stator bore	D	3·20 m
4.	Gross core length	L	0·44 m
5.	Radial ducts : No.	n_d	5
	width	W_d	10 mm
6.	Gross iron length	L_s	0·39 m
7.	Net iron length	L_i	0·351 m
8.	Pole pitch	τ	0·341 m
9.	Current per phase	I_{ph}	262 A.

Stator

1.	Winding		Double layer lap Diamond coils
2.	Number of parallel paths	a	1
3.	Number of slots	S	312
4.	Slots per pole per phase	q	$3\frac{1}{4}$
5.	Conductors per slot	Z_s	4

6.	Coil span	C_s	9 slots, 1—10
7.	Coil span/pole pitch		0·923
8.	Conductor : size		2×(6×6) mm²
	area	a_s	70·2 mm²
9.	Current density	δ_s	3·73 A/mm²
10.	Length of mean turn	L_{mt}	2·26 m
11.	Slot size : width	W_s	13 mm
	depth	d_s	73 mm
12.	Core depth	d_c	112 mm
13.	Outer diameter of stator	D_o	3·57 m
14.	Resistance per phase	r_{ac}	0·151 Ω
15.	Total copper loss		31·2 kW

Rotor

1.	Type of pole		Salient, laminated, dovetail
2.	Damper winding :		
	number of bars	N_d	9
	diameter of each bar	d_d	10 mm
3.	Pole arc	$b=\psi\tau$	0·2325 m
4.	Pole section		0·15×0·44 m²
5.	Radial length of pole	h_{pl}	0·165 m
6.	Turns per pole	T_f	41
7.	Conductor : size		35×2·5 mm²
	area	a_f	87·5 mm²
8.	Full load current	I_f	226 A
9.	Current density	δ_f	2·6 A/mm²
10.	Length of mean turn		1·40 m
11.	Voltage drop	L_{mt}	100 V
12.	Field copper loss		32×705 W
13.	Peripheral speed	V_a	31·4 m/s

Magnetization

1.	Flux per pole	Φ	88·6 m Wb
2.	Core : area	A_c	2×0·034 m²
	density	B_c	1·11 Wb/m²
	mmf	A_{tc}	71 A
3.	Teeth : area (⅓)		
	per pole arc	A_t	0·05 m²
	density	$B_{t1/3}$	1·77 Wb/m²
	mmf	AT_t	694 A
4.	Gap : area	A_g	0·097 Wb/m²
	contraction factor	K_g	1·117
	density	B_g	0·914 Wb/m²
	mmf	AT_g	4700 A

5.	Pole : area	A_p	0·0653 m²
	density	$B_{p\,(mean)}$	1·52 Wb/m²
		$B_{p\,(min)}$	1·41 Wb/m²
	mmf	AT_p	232 A
6	Yoke : area	A_y	2×0·431 m²
	density	B_y	1·15 Wb/m²
	mmf	AT_y	64 A
7.	Total mmf per pole		
	no load	AT_{fo}	5761 A
	full load	AT_{fl}	9200 A
8.	Armature mmf per pole	AT_a	4400 A
9.	Field mmf equivalent to armature mmf per pole		3660 A

Efficiency

1.	Core loss	46·0 kW
2.	Copper loss	31·2 kW
3.	Stray load loss	6 2 kW
4.	Excitation loss	26 3 kW
5.	Friction and windage loss	21 0 kW
6.	Total loss	131·0 kW
7.	Efficiency	94·7 per cent

Temperature Rise

1.	Temperature rise of field coils	46·2°C
2.	Temperature rise of armature	49°C

Design Problem II. *Design a 30,000 kVA, 11 kV, 3000 rpm, 50 Hz, 3 phase air cooled turbo-alternator. The load power factor is 0 8 lagging.*

Solution.

Main Dimensions

Synchronous speed n_s=3000/60=50 r.p.s. ∴ Number of poles p=2×50/50=2.

Assuming the specific loadings as

B_{av}=0·55 Wb/m², ac=54,000 ampere conductors per metre,

and winding factor K_w=0·955.

Output co-efficient C_0=11×0·955×0·55×54000×10⁻³=312.

$$\therefore \quad \text{Product } D^2L=\frac{Q}{C_0\,n_s}=\frac{30000}{312\times 50}=1{\cdot}925 \text{ m}^3.$$

Peripheral speed is a deciding factor for the diameter. A peripheral speed of about 130 m/s is used.

∴ $\pi D\, n_s$=130 ∴ D=0·828 m.

(This is not strictly true as the peripheral speed is $\pi D_r n_s$ and not $\pi D n_s$. But all the same it is on the safe side).

Taking stator bore diameter $D=0{\cdot}83$ m, pole pitch $\tau=\pi\times1{\cdot}83/2=1{\cdot}3$ m and length of core $L=1{\cdot}925/(0{\cdot}83)^2=2{\cdot}80$ m.

Length of air gap

Armature mmf per pole$=\dfrac{ac\,\tau}{2}=\dfrac{54000\times1{\cdot}3}{2}=35600$ A.

Assuming a short circuit ratio of 0 55,

no load field mmf $AT_{fo}=\text{SCR}\times AT_a=0\ 55\times35600=19600$ A.

Taking mmf for the air gap to be 80 per cent of no load field mmf.

$\therefore$ Mmf for air gap $AT_g=0{\cdot}8\times19600=15700$ A.

Maximum flux density in the gap $B_g=1{\cdot}5\ B_{av}=1{\cdot}5\times0{\cdot}55=0{\cdot}825$ Wb/m^2.

Taking the air gap contraction factor to be 1·1.

Length of air gap $l_g=\dfrac{15700}{800{,}000\times0{\cdot}825\times1{\cdot}1}\times10^3=21{\cdot}3$ mm.

Taking the length of air gap $l_g=20$ mm.

Diameter of rotor $D_r=D-2l_g=0{\cdot}83-2\times20\times10^{-3}=0{\cdot}79$ m.

Peripheral speed $V_a=\pi\times0{\cdot}79\times50=124$ m/s.

Stator

Stator winding :

Flux per pole $\Phi=B_{av}\,\tau\,L=0{\cdot}55\times1{\cdot}3\times2{\cdot}8=2$ Wb.

Voltage per phase $E_{ph}=\dfrac{11000}{\sqrt{3}}=6350$ V.

$\therefore$ Turns per phase $T_{ph}=\dfrac{6350}{4{\cdot}44\times50\times2\times0{\cdot}955}=15.$

Conductors per phase$=2\times15=30$. Total armature conductors$=3\times30=90$.

Number of slots :

The number of slots should be so chosen that there is not much modification in the number of conductors and hence in the gap density.

Assume number of armature slots $S_s=30$.

Slots per pole per phase $q=\dfrac{30}{2\times3}=5$. Conductors per slot$=\dfrac{90}{30}=3$

Stator slot pitch $y_{ss}=\pi\times0{\cdot}83/30$ m$=87$ mm.

A single layer concentric winding with 3 plane overhang is used. Push through coils in semi-enclosed slots are employed.

Magnetic circuit :

Combined radial and axial ventilation is used. With this system of ventilation, the stator core length is divided into packets of about 50—100 in length.

Using 28 radial ducts each 10 mm wide, the number of packets is 29. The ducts are more closely spaced in the middle. Therefore, the central packets are shorter in length. (This arrangement is used as the central portions of the core are difficult to ventilate). The core length is made up as given below :—

6 packets of 100 mm each	$6\times100\times10^{-3}$	$=0{\cdot}6$ mm
4 packets of 90 mm each	$4\times90\times10^{-3}$	$=0{\cdot}36$ m
3 packets of 70 mm each	$3\times70\times10^{-3}$	$=0{\cdot}21$ m

3 packets of 60 mm each	$3\times60\times10^{-3}$	$=0.18$ m
3 packets of 70 mm each	$3\times70\times10^{-3}$	$=0.21$ m
4 packets of 90 mm each	$4\times90\times10^{-3}$	$=0.36$ m
6 packets of 100 mm each	$6\times100\times10^{-3}$	$=0.60$ m
29 packets of total length		$L_s=2.52$ m
28 ducts each 10 mm wide		$=0.28$ m
	Total core length	$L=2.8$ m

Gross iron length $L_s=2.80-28\times10\times10^{-3}=2.52$ m and

net iron length$=0.9\times2.52=2.268$ m.

Stator core

Flux in core $\Phi_c=\Phi/2=1$ Wb.

Using a flux density of 1 Wb/m^2 in the core, area of stator core $A_{cs}=1.0/1.0=1$ m^2.

$\therefore$ Depth of core $d_{cs}=1.0/2.268$ m$=0.423$ m.

Axial ventilating ducts are provided in the core and due to this about 20 per cent of core area is lost. Therefore, core depth should be raised by about 20 per cent.

$\therefore$ Depth of core $d_{cs}=1.2\times42.3\cong0.5$ m.

Ventilating holes.

Assuming an efficiency of 96 per cent at the rated output.

Rated output$=30,000\times0.8=24000$ kW and losses$=0.04\times2400=960$ kW.

Taking a temperature rise of 25°C in stator, and air inlet temperature$=20$°C.

From Eqn. 3.39 (page 71)

$$\text{Air required for cooling the machine}=0.78\times\frac{Q}{\theta}\cdot\frac{\theta_i+273}{273}\cdot\frac{760}{h}$$

$$=0.78\times\frac{960}{25}\times\frac{20+273}{273}\times\frac{760}{760}=32.2\ \text{m}^3/\text{s}.$$

In combined radial and axial system used here, air enters the machine from both ends.

$\therefore$ Air entering the machine from each end$=32.2/2=16.1$ m^3/s.

Assume that 70 per cent of the air enters the stator axial ventilating ducts and the rest 30 per cent enters the machine via air gap and rotor axial ducts.

$\therefore$ Air entering the stator axial ducts from each end$=0.7\times16.1=11.27$ m^3/s.

Assume an air velocity of 25 m/s.

$\therefore$ Area of axial holes required in stator$=11.27/25=0.451$ m^2.

Providing

300 holes each of 40 mm diameter, area$=0.3768$ m^2.

100 holes each of 30 mm diameter, area$=0.0707$ m^2.

Total area of holes$=0.4475$ m^2.

Slot dimensions

It has been mentioned earlier that due to bunching of conductors in a turbo-alternator, the short circuit forces are very large. In order to reduce the initial value of this force, which is mainly determined by the leakage reactance and the moment of short circuit, the machine should be designed to have a large value of leakage reactance.

To obtain a high leakage reactance, we must use deep slots. In order to do this, a space known as reactance space is left at the top of the slots. The reactance space including wedge is taken as 30 mm.

In order to estimate the slot dimensions, we must find the minimum width of tooth which keeps the flux density within 1·8 Wb/m². The minimum width of tooth occurs where the reactance space ends. (see Fig. 12·51).

Diameter of rotor at this point $=0·83+2\times30\times10^{-3}=0·89$ m

Slot pitch $=\pi\times0·89/30=0·093$ m.

In order that flux density should not go beyond 1·8 Wb/m²,

$$\text{minimum width of tooth}=\frac{\Phi}{S_s/p\times L_i\times1·8}$$

$$=\frac{2}{30/2\times2·268\times1·8}\text{ m}=32·7\text{ mm.}$$

$\therefore$ Maximum allowable slot width $=93-32·7=60·3$ mm.

Armature current per phase $I_{ph}=\dfrac{30,000\times1000}{2\times6350}=1575$ A.

Taking a current density of 3·0 A/mm²,

area of stator conductor $a_s=1575/3=525$ mm².

Using a conductor of 30 mm width.

$\therefore$ Height of conductor $=525/30=17·5$ mm.

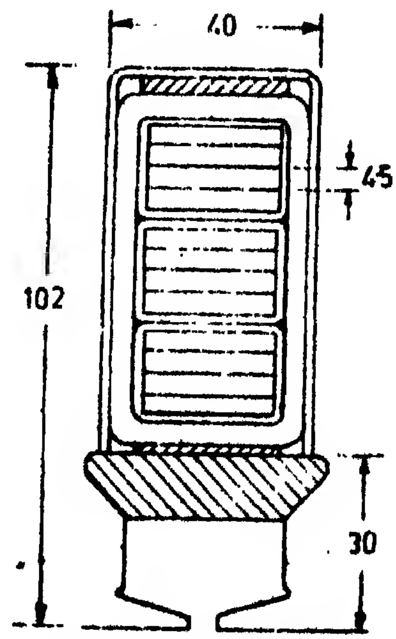


Fig. 12·51. Stator slot. All dimensions in mm.

If a solid conductor $30\times17·5$ mm² is used the eddy current loss in the conductors would be excessive. The conductor should be subdivided such that the average eddy current loss factor is less than 1·2.

Subdividing the conductor into six parts each 3 mm high.

Area of each subdivision $=30\times3=90$ mm².

Area of stator conductor $a_s=6\times90=540$ mm².

Insulation :

Insulation between subdivisions $=0·25$ mm.

Conductor insulation $=0·75$ mm.

Coil insulation $=4$ mm.

Referring to Fig. 12·51.

Slot Width

Conductor width	$1\times30=30·00$ mm
Conductor insulation	$2\times0·75=1·50$ mm
Slot insulation	$2\times4=8·00$ mm
Slack	$=0·50$ mm
Total slot width	$W_{ss}=40·00$ mm.

Slot Depth

Conductor height	$18\times3=54{\cdot}00$ mm
Subdivision insulation	$15\times0{\cdot}25=3{\cdot}75$ mm
Conductor insulation	$6\times0{\cdot}75=4{\cdot}50$ mm
Slot insulation	$2\times4=8{\cdot}00$ mm
Slack	$=1{\cdot}75$ mm
Total conductor height	$=72{\cdot}00$ mm

Reactance space and wedge $= 0$ mm

Total slot depth $d_{ss}=102$ mm.

Before we proceed further, we must check for eddy current loss in conductors.

From page 32[illegible], $\alpha=100\sqrt{30/40}=86{\cdot}6$.

We have, h'=height of subdivision$=3$ mm.

$\therefore$ $\alpha h'=86{\cdot}6\times3\times10^{-3}=0{\cdot}26$ and $(\alpha h')^4=4{\cdot}57\times10^{-3}$.

Number of layers $N=18$.

$\therefore$ Average eddy current loss factor (Eqn. 6 9[illegible])

$$K_{e(av)}=1+(\alpha h')^4 N^2/9=1+4{\cdot}57\times10^{-3}\times(18)^2/9=1{\cdot}16$$

The loss factor is within limits

Rotor Design

Armature mmf per pole $AT_a=2{\cdot}7\times1575\times15\times0{\cdot}955/2=30{,}500$ A.

The field mmf on full load can be taken approximately twice the armature mmf per pole.

$\therefore$ Full load field mmf $AT_{fl}=2\times30500=61000$ A.

Taking a current density of $2{\cdot}5$ A/mm² in the field winding.

$\therefore$ The area of rotor conductors per pole $=\dfrac{2\times AT_{fl}}{\delta_f}$

$$=\frac{2\times61000}{2{\cdot}5}=48{\cdot}8\times10^{3}\ \text{mm}^2.$$

Rotor slots. The rotor slot pitch should not have any common factor with stator slot pitch and for 2 polar designs, the number of wound rotor slots should be a multiple of 4 as otherwise the number of rotor slots per pole would not be an even integer and this means that the number of coils per pole would not be an integer.

The number of rotor slots is taken as 23 as the number has no common factor with 30 (number of stator slots). The number of wound slots is about 70 per cent of total rotor slots.

$\therefore$ Number of wound slots$=0{\cdot}7\times23\cong16$.

Rotor slot pitch $y_{sr}=\pi\times0{\cdot}79/23=108$ mm.

Rotor winding. We take exciter voltage$=220$ V.

Keeping 20 per cent of exciter voltage in reserve the voltage across the field coils is 175 V.

Voltage across each field coil $E_f=175/2=87{\cdot}5$ V.

The length of mean turn of field coil is approximated as :

$$L_{mtf}=2L+2{\cdot}3\,\tau+0{\cdot}24$$

where τ=mean span of coils$=0{\cdot}81$ (estimated).

$\therefore$ $L_{mtf}=2\times2{\cdot}8+2{\cdot}3\times0{\cdot}81+0{\cdot}24=7{\cdot}6$ m.

Area of each field conductor $a_f = \frac{AT_{fl}\ \rho\ L_{mtf}}{E_f}$

$$= \frac{61000 \times 0.021 \times 7.6}{87.5} = 110\ mm^2$$

$\therefore$ Number of field conductors per pole $= 48.8 \times 10^3/110 = 440$.

Wound slots per pole $= 8$. $\therefore$ Conductors per slot $= 440/8 = 55$.

Total field turns per pole $T_f = 440/2 = 220$.

Number of coils per pole $=$ slots per pole$/2 = 8/2 = 4$. Turns per coil $= 220/4 = 55$.

Resistance of each field winding $R_f = T_f\ \frac{\rho\ L_{mtf}}{a_f}$

$$= 220 \times \frac{0.021 \times 7.6}{110} = 0.32\ \Omega.$$

Current in field winding $I_f = 87.5/0.32 = 273$ A.

$\therefore$ Mmf provided per pole $= 273 \times 220 = 60{,}200$ A

Slot dimensions

Choosing a conductor 40 mm wide and 2·75 mm deep. The insulation of the slot must be designed to withstand great mechanical stresses and also the forces originated owing to expansion of slot contents which have different thermal expansion co-efficients. The coil is enclosed in a 0·5 mm steel cell. In order to support the winding at the ventilating ducts, a 2 mm thick spring steel plate is provided at the bottom of slots. The turns are insulated from each other by 0·3 mm mica separators. The coils wrapped first in a 0·5 mm hard mica cell and then in a 2 mm flexible mica cell.

Slot width

Copper conductor	1×40	$= 40.00$ mm.
Hard mica	2×0.5	$= 1.00$ mm.
Flexible mica	2×2	$= 4.00$ mm.
Steel cell	2×0.5	$= 1.00$ mm.
Slack		$= 2.00$ mm.
Total slot width	W_{sr}	$= 48.00$ mm.

Slot depth

Copper conductors	55×2.75	$= 151.25$ mm.
Wedge		$= 30.30$ mm.
Copper strip under wedge		$= 3.00$ mm.
Hard mica	3×0.5	$= 1.50$ mm.
Flexible mica	3×2.0	$= 6.00$ mm.
Mica Separators	55×0.3	$= 16.50$ mm.
Mica bottom strip		$= 1.50$ mm.
Spring steel plate		$= 2.00$ mm.
Steel cell	2×0.5	$= 1.00$ mm.
Slack		$= 2.25$ mm.
Total slot depth	d_{sr}	$= 215.00$ mm.

Rotor Ventilation

Total air entering the machine from each end $=16.1$ m³/s.

Total air entering through stator axial holes from each end $=11.27$ m³/s.

$\therefore$ Air entering the gap and axial holes in rotor from each end

$$=16.1-11.27=4.83 \text{ m}^3/\text{s}.$$

Area required for gap and axial holes in rotor $=4.83/25=0.193$ m²

$$\text{Area to the path of air in the air gap} = \frac{\pi}{4}(D^2-D_r^2)$$

$$=\frac{\pi}{4}(0.83^2-0.79^2)=0.0534 \text{ m}^2.$$

$\therefore$ Area of ventilating holes required in rotor $=0.193-4.0534=0.1396$ m².

Providing

160 holes of 50 mm diameter (under each slot), area $=0.0314$ m².

160 holes of 30 mm diameter, area $=0.1130$ m².

Total area of holes in rotor $=0.1444$ m².

UNSOLVED PROBLEMS

1. Determine suitable stator dimensions for a 500 kVA, 50 Hz, 3 phase alternator to run at 375 r.p.m. Take mean gap density over the pole pitch as 0·55 Wb/m², the specific electric loading as 25,000 A/m. The peripheral speed should not exceed 35 m/s. [Ans. $D=1.75$ m ; $L=0.18$ m]

2. Determine for a 500 kVA, 6600 V, 12 pole, 500 r.p.m., 3 phase alternator, suitable values for (*i*) the diameter at air gap, (*ii*) the core length, (*iii*) the number of stator conductors, (*iv*) the number of stator slots. Assume a star connected stator winding, a specific magnetic loading 0·6 Wb/m² and a specific electric loading of 30,000 A/m. Assume ratio length : pole pitch $=1.5$. Sketch the shape of slot and the arrangement of conductors, and specify the insulation. [Ans. $D=0.94$ m ; $L=0.37$ m ; $S=81$; $Z=1944$]

3. Determine for a 250 kVA, 1100 V, 12 pole, 500 r.p.m.. three phase alternator (*i*) air gap diameter (*ii*) core length, (*iii*) number of stator conductors, (*iv*) number of stator slots, (*v*) cross-section of stator conductors assuming average air gap density as 0·6 Wb/m², ampere conductors pe metre as 28500, and the current density to be 3·5 A/m². [Ans. $D=0.78$ m ; $L=0.30$ m ; $Z=540$; $S=90$; $a_s=37.5$ mm²]

4. Derive from first principles the output co-efficient for a 1500 kVA, 2200 V, 3 phase, 10 pole, 50 Hz, star connected alternator with sinusoidal flux density distribution. The winding has 60° phase spread and full pitch coils The specific electric loading is 30,000 ampere conductors per metre of periphery and the specific magnetic loading is 0·6 Wb/m².

If the peripheral speed of rotor must not exceed 100 m/s and the ratio pole pitch/core length is to be between 0·6 and 1·0, find suitable values for stator bore and core length. Assume an air gap length of 6 mm.

Find also the approximate number the stator conductors. [Ans. $D=1.20$ m ; $L=0.54$ m ; $Z=2910$]

5. A 3 phase altern tor having a full load rating of 11000 kVA at 0·8 power factor, 2200 V, 50 Hz, 300 r.p.m. has a stator diameter of 1·9 m, core length 0·3 m and 180 slots. Using the information from this machine, with suitable modifications where required, determine the stator diameter core length, number of slots and conductors per slot for a 3 phase machine to give 2000 kVA at 0·8 power factor, 6600 V, 50 Hz, 600 r.p m. [Ans. $D=1.44$ m ; $L=0.45$ m ; $S=132$; $Z_s=14$]

6. A 3 phase alternator has a stator bore of 1·70 m and a core length of 0·35 m. The average gap density is approximately 0·55 Wb/m². Determine a suitable number of slots and conductors per slot for a terminal voltage of 6600 V, 50 Hz and 375 r.p.m. Use star connection. [Ans. $S=144$; $Z_s=10$]

7. A 3 phase, 6600 V, 50 Hz, star connected alternator is to run at 750 r.p.m. There are to be 36 conductors per slot, the flux per pole is 42×10^{-3} Wb and the distribution factor is 0·97. The winding is concentric with overhang arranged in two planes. Calculate the number of slots required and draw the winding diagram for 4 pole pitches.

8. An alternator has parallel sided slots 25 mm wide and 80 mm deep. The cross-section of conductors measures 50×20 mm² and they are covered with the coil side insulation of uniform thickness and placed at the bottom of the slot which is closed by a non-magnetic wedge. Determine the leakage reactance due to slot flux only, if the alternator has a single layer winding having 12 turns per coil and 40 coils in series per phase. The core legth is 0·8 m and the frequency 50 Hz. [Ans. 6·2 Ω]

9. A 500 MW, 3 phase 2 pole, 50 Hz direct water cooled generator has a diameter of 1·3 m and a core length of 5·9 m. The number of turns per phase is 11 and there are two parallel circuits per phase. The average gap density is 0·575 Wb/m² and the winding factor is 0·92.

Determine the terminal voltage of the machine. Find also the specific electric loading if the power factor is 0·85.

If the mmf required for air gap is 80 per cent of no load mmf, short circuit ratio 0·55, air gap contraction factor 1·1, determine the length of air gap. [Ans. 27,000 V, 200,000, A/m 95 mm]

10. Design the field winding for the following low speed alternator : 16 poles ; excitation voltage, 110 V : maximum mmf per coil, 16,000 A, approximately ; full load mmf per coil, 12 000 A. Permissible loss at full load, per square metre of the total coil surface, 1800 W. The field coil is rectangular with rounded corners, the internal dimensions being 0 30 m × 0·18 m with corners of 40 mm radius. The total height of coil is 0·17 m. [Ans. Depth = 33 mm ; Turns = 84]

11. Obtain the main dimensions of the rotor of a 50 MVA, 2 pole, 50 Hz synchronous generator. The peripheral speed is limited to approximately 160 m/s. Take an electric loading of 65,000 A/m and a mean gap density of 0 575 Wb/m². Assume a gap length of 25 mm. [Ans $D = 1·0$ m ; $L = 2·3$ m]

12. A two pole, 50 Hz turbo-alternator has a core length of 1·5 m. The mean flux density over the pole pitch is 0·5 Wb/m², the stator ampere conductors per metre are 26,000 and the peripheral speed 100 m/s. The average span of coils is one pole pitch. Determine the output which can be obtained from the machine. [Ans. 4·1 MVA]

13. A salient pole synchronous machine has a direct axis synchronous reactance of 1·0 per unit and a quadrature axis synchronous reactance of 0·6 per unit. The machine is operated as a generator with a terminal voltage 1·0 per unit supplying a current of 1·0 per unit to a load having a lagging power factor of 0·8.

(a) Determine the power angle for the above condition.

(b) Determine the field current required for this load in per unit of the field current required to produce 1·0 per unit terminal voltage at no load. [Ans. 19 ; 1·775]

14. A 500 kVA, 4 pole 3 phase, 60 Hz, 0·8 power factor, 208 V, salient pole generator has 72 slots, 2 conductors per slot, and each conductor having 16 parallel strands. The area of each strand is 8·35 mm². The length of mean turn is 2·0 m. The winding is connected in star with 4 circuits per phase. The field current at full load is 27·2 A, and the field resistance is 4·55 Ω at 75°C. The friction and windage loss is 4·1 kW, the total iron loss 6·2 kW and the stray load loss 1·5 kW. Determine the efficiency of the generator at full load. Also find the current density in the stator winding. [Ans. 95·5%, 2 57 A/mm²]

13

Starters and Field Regulators

13·1. Motor starters. The function of motor starters is to prevent the flow of excessive currents at starting and thus prevent undue large mechanical stresses which would otherwise act on machine parts. At the same time, the starter should allow a current high enough to produce a good starting torque so that the motor starts up against the specified load.

The starter may take up the form of a liquid rheostat whose resistance can be gradually varied or it may be a metallic resistance starter where the resistance is varied in steps. In the case of starters with resistance steps, the current is taken to fluctuate between fixed upper and lower current limits. The method of calculation of resistance steps is given below.

13·2. Calculation of resistance steps

Referring to Fig. 13·1.

Let I_1, I_2 = upper and lower limits of current respectively

$\alpha = I_2/I_1$

Φ_1, Φ_2 = useful flux per pole due to currents I_1 and I_2 respectively

$\beta = \Phi_1/\Phi_2$

$$\delta = \alpha\beta = \frac{\Phi_1 I_2}{\Phi_2 I_1}$$

n = number of resistance sections or elements

$n+1$ = number of studs

$R_1, R_2, R_3 \ldots\ldots R_{n+1}$ = total resistance in circuit, on 1st, 2nd, 3rd......$(n+1)$th stud

$r_1, r_2, r_3 \ldots\ldots = (R_1 - R_2), (R_2 - R_3), (R_3 - R_4)\ldots\ldots$ = resistance of sections or elements

r_m = motor resistance

V = motor or line voltage, E = motor emf

s = slip, and X = reactance.

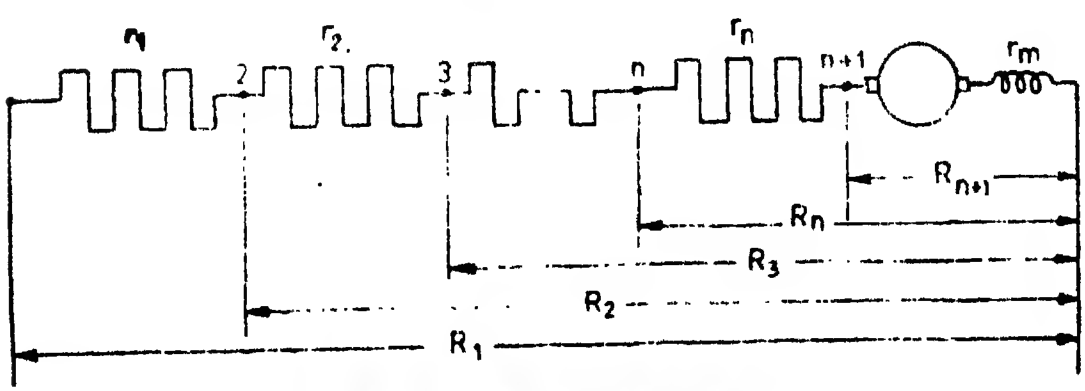


Fig. 13·1. Resistance steps of starter.

13 2 1. **Starters for d.c. shunt motors.** Shunt motors are constant flux machines and therefore $\Phi_1=\Phi_2$ and $\beta=1$.

To start with, the contact arm is put at stud 1.

To limit the starting current to I_1, the resistance

$$R_1=V/I_1 \quad ...(13\cdot1)$$

as there is no back emf. The motor starts rotating and builds up back emf and therefore the current drops to its minimum value I_2. Let the back emf at stud 1 be E_1.

$$\therefore \quad I_2=\frac{V-E_1}{R_1} \quad ...(13\cdot2)$$

When the current has reached a value I_2, the resistance r_1 of the first section is cut out and the contact arm is moved to stud 2. As soon as the resistance is cut out, the current rises to its maximum value, I_1. Let the back emf at stud 2 be E_2.

$$\therefore \quad I_1=\frac{V-E_2}{R_2} \quad ...(13\cdot3)$$

The back emf is proportional to product of flux and speed. Flux remains constant in a shunt machine and it is assumed that during the notching operation (the process of cutting out a resistance), the speed remains constant. Therefore the back emf $E_1=E_2$

So, $$I_1R_2=I_2R_1$$

or $$\frac{R_2}{R_1}=\left(\frac{I_2}{I_1}\right)=\alpha. \quad ...(13\cdot4)$$

Similarly,

$$\frac{R_3}{R_2}=\frac{R_4}{R_3}=\ldots\ldots\ldots\ldots=\frac{R_n}{R_{n-1}}=\frac{R_{n+1}}{R_n}.$$

$\therefore$ We can write,

$$\frac{I_2}{I_1}=\frac{R_2}{R_1}=\frac{R_3}{R_2}=\frac{R_4}{R_3}=\ldots\ldots\ldots\ldots=\frac{R_n}{R_{n-1}}=\frac{R_{n+1}}{R_n}=\alpha \quad ...(13\cdot5)$$

From Eqn. 13·5,

$$\alpha^n=\frac{R_2}{R_1}\times\frac{R_3}{R_2}\times\frac{R_4}{R_3}\times\ldots\ldots\ldots\ldots\times\frac{R_n}{R_{n-1}}\times\frac{R_{n+1}}{R_n}=\frac{R_{n+1}}{R_1}$$

$$=\frac{r_m}{R_1} \quad ...(13\cdot6)$$

as $R_{n+1}=r_m$.

Hence $$\alpha=\left(\frac{r_m}{R_1}\right)^{1/n} \quad ...(13\cdot7)$$

Eqn. 13·7 immediately gives us the value of ratio α if resistances R_1 and r_m are known.

From Eqn. 13·5,

$R_2=\alpha R_1$

$R_3=\alpha R_2=\alpha^2R_1$

$R_4=\alpha R_3=\alpha^3R_1$

....

$R_n=\alpha R_{n-1}=\alpha^{n-1}R_1$

$r_m=\alpha R_n=\alpha^nR_1$.

$\therefore$ The resistance of different sections are :

$r_1=R_1-R_2=(1-\alpha)R_1$

$r_2 = R_2 - R_3 = \alpha(1-\alpha)R_1 = \alpha r_1$

$r_3 = R_3 - R_4 = \alpha^2(1-\alpha)R_1 = \alpha^2 r_1$

....

$r_n = R_n - R_{n+1} = \alpha^{n-1}(1-\alpha)R_1 = \alpha^{n-1} r_1.$

When the upper current limit is fixed, 13·7 may be written as :

$$\alpha = \left(\frac{r_m}{R_1}\right)^{\frac{1}{n}} = \left(\frac{r_m}{V/I_1}\right)^{\frac{1}{n}} = \left(\frac{I_1 r_m}{V}\right)^{\frac{1}{n}} \qquad \ldots(13{\cdot}8)$$

When the lower current limit is fixed Equ. 13·8 is written as :

$$\alpha = \left(\frac{I_1 r_m}{V}\right)^{\frac{1}{n}} = \left(\frac{r_m}{V}\cdot\frac{I_2}{\alpha}\right)^{\frac{1}{n}} = \left(\frac{r_m I_2}{V}\right)^{\frac{1}{n+1}} \qquad \ldots(13{\cdot}9)$$

Example 13·1 ***A 250 V, 370 kW, d.c. shunt motor has to exert a maximum torque of 150 per cent of full load torque during the starting period. The resistance of armature circuit is 0·2 Ω and the full load efficiency is 84 per cent. The number of studs is 8. Determine***

(i) the upper and lower limits of current during starting

(ii) the resistance of each section.

Solution.

(i) Full load current$=\dfrac{37\times10^3}{250\times0{\cdot}84}=176$ A.

Torque is proportional to ΦI and the flux is constant, therefore, the torque is proportional to I.

Now maximum torque$=1{\cdot}5\times$full load torque.

∴ Maximum current $I_1=1{\cdot}5\times$full load current$=264$ A.

Number of studs, $n+1=8$. ∴ Number of sections $n=7$.

From Equ. 13·8,

$$\alpha = \left(\frac{r_m I_1}{V}\right)^{\frac{1}{n}} = \left(\frac{0{\cdot}2\times264}{250}\right)^{\frac{1}{7}} = 0{\cdot}8.$$

Lower limit of current $I_2=\alpha I_1=0{\cdot}8\times264=211$ A.

∴ The current fluctuates between 211 and 264 A during starting.

(ii) Total resistances at starting $R_1=V/I_1=250/264=0{\cdot}947\ \Omega$.

The resistance of 7 sections are :

$r_1=(1-\alpha)R_1$	$=(1-0{\cdot}8)\times0{\cdot}947$	$=0{\cdot}189\ \Omega$.
$r_2=\alpha r_1$	$=0{\cdot}8\times0{\cdot}189$	$=0{\cdot}151\ \Omega$.
$r_3=\alpha r_2$	$=0{\cdot}8\times0{\cdot}151$	$=0{\cdot}121\ \Omega$.
$r_4=\alpha r_3$	$=0{\cdot}8\times0{\cdot}121$	$=0{\cdot}097\ \Omega$.
$r_5=\alpha r_4$	$=0{\cdot}8\times0{\cdot}097$	$=0{\cdot}077\ \Omega$.
$r_6=\alpha r_5$	$=0{\cdot}8\times0{\cdot}077$	$=0{\cdot}062\ \Omega$.
$r_7=\alpha r_6$	$=0{\cdot}8\times0{\cdot}062$	$=0{\cdot}050\ \Omega$.

Total resistance of elements $=0{\cdot}747\ \Omega$.

Motor resistance $r_m=0{\cdot}200\ \Omega$.

Total resistance at starting $R_1=0{\cdot}947\ \Omega$

Example 13·2. *Grade the resistance of a 5 section starter for a 7·5 kW., 250 V, 750 r.p.m. d.c. shunt motor from the following data :*

Maximum torque during starting period=1·5 times full load torque ; full load efficiency=85·5 per cent. Armature circuit copper loss is 50 per cent of total loss. Field current=2·6 A.

Find the speed at each stud when the notching takes place.

Solution.

Full load line current $=\dfrac{7\cdot5\times10^3}{0\cdot855\times250}=35\cdot1$ A.

Full load armature current $=35\cdot1-2\cdot6=32\cdot5$ A.

Maximum armature current $I_1=1\cdot5\times32\cdot5=48\cdot8$ A.

Total input $=7\cdot5\times10^{3}/0\cdot855=8770$ W.

Total losses at full load $=8760-7500=1270$.

Copper loss at full load $=\frac{1}{2}\times1270=635$ W.

$\therefore$ Armature circuit resistance $r_m=635/(32\cdot5)^2=0\cdot601\ \Omega$.

Now number of sections $n=5$.

$$\therefore\quad \alpha=\left(\frac{r_mI_1}{V}\right)^{\frac{1}{n}}=\left(\frac{0\cdot602\times48\cdot8}{250}\right)^{\frac{1}{5}}=0\cdot652.$$

Total resistances at different studs are :

R_1	$=\dfrac{V}{I_1}=\dfrac{250}{48\cdot8}$	$=5\cdot12\ \Omega$.
R_2	$=\alpha R_1=0\cdot652\times5\cdot12$	$=3\cdot33\ \Omega$.
R_3	$=\alpha^2R_1=0\cdot652^2\times5\cdot12$	$=2\cdot18\ \Omega$.
R_4	$=\alpha^3R_1=0\cdot652^3\times5\cdot12$	$=1\cdot42\ \Omega$
R_5	$=\alpha^4R_1=0\cdot652^4\times5\cdot12$	$=0\cdot94\ \Omega$.
R_6	$=r_m$	$=0\cdot601\ \Omega$.

The resistances of different sections are :

$r_1=R_1-R_2=5\cdot12-3\cdot33=1\cdot79\ \Omega$.

$r_2=R_2-R_3=3\cdot33-2\cdot18=1\cdot15\ \Omega$.

$r_3=R_3-R_4=2\cdot18-1\cdot42=0\cdot76\ \Omega$.

$r_4=R_4-R_5=1\cdot42-0\cdot94=0\cdot48\ \Omega$.

$r_5=R_5-R_6=0\cdot94-0\cdot601=3\cdot34\ \Omega$.

Minimum value of current $I_2=\alpha I_1=0\cdot652\times48\cdot8=31\cdot8$ A.

When notching takes place, the current is at its minimum value.

$\therefore$ Back emf at stud 1 when notching takes place

$$E_1=V-I_2R_1=250-31\cdot8\times5\cdot12=87\cdot2\ \text{V}.$$

Similarly
$$E_2=250-31\cdot8\times3\cdot33=144\cdot1\ \text{V}.$$
$$E_3=250-31\cdot8\times2\cdot18=180\cdot6\ \text{V}.$$
$$E_4=250-31\cdot8\times1\cdot42=204\cdot8\ \text{V}.$$
$$E_5=250-31\cdot8\times0\cdot94=220\cdot1\ \text{V}.$$

When the motor is running at full load the speed is 750 r.p.m and the back emf,

$$E=250-32\cdot8\times0\cdot601=230\cdot3\ \text{V}.$$

The back emf is directly proportional to speed. Therefore, the speed at various studs is :

Stud 1, Speed $=750\times E_1/E=750\times 87{\cdot}2/230{\cdot}2=284$ r.p.m.

Stud 2, Speed $=750\times E_2/E=750\times 144{\cdot}1/230{\cdot}2=469$ r.p.m.

Stud 3, Speed $=750\times 180{\cdot}6/230{\cdot}2=588$ r.p.m.

Stud 4, Speed $=750\times 204{\cdot}8/230{\cdot}2=667$ r.p.m.

Stud 5, Speed $=750\times 220{\cdot}1/230{\cdot}2=717$ r.p.m.

13·2·2. Starters for d.c. series motors

Referring to Fig. 13·1.

On switching at stud 1, there is no back emf and therefore the current is limited by resistance R_1. This current is the maximum current and is given by

$$I_1=V/R_1$$

The motor now picks up speed and the back emf is developed. Due to the back emf the current falls and its value comes down to I_2 before notching Just before notching at stud 1, let us say that the back emf, is E_1'.

$$\therefore \qquad I_2=\frac{V-E_1'}{R_1}$$

The current is I_2 and therefore the flux is Φ_2. The back emf E_1' is proportional to product of speed and flux Φ_2. Let the speed at that instant be N_1.

$$\therefore \qquad E_1'=K\Phi_2N_1 \text{ where } K \text{ is a constant.}$$

$$\therefore \qquad I_2=\frac{V-K\Phi_2N_1}{R_1} \qquad \ldots(13{\cdot}10)$$

The resistance r_1 of the first section is cut out and the current rises to I_1 and the flux changes to Φ_1 Let the back emf be E_2'.

$$\therefore \qquad E_2'=K\Phi_1N_1.$$

(Assume that the speed remains constant during the notching operation.)

The resistance in the circuit is now R_2.

$$\therefore \qquad I_1=\frac{V-E_2'}{R_2}=\frac{V-K\Phi_1N_1}{R_2} \qquad \ldots(13{\cdot}11)$$

From Eqn. 13·10, we have,

$$KN_1=\frac{V-I_2R_1}{\Phi_2}$$

Substituting this in Eqn. 13·11,

$$I_1=\frac{V-\dfrac{\Phi_1}{\Phi_2}(V-I_2R)}{R_2}$$

or

$$R_2=\frac{I_2}{I_1}\frac{\Phi_1}{\Phi_2}R_1-\frac{V}{I_1}\left(\frac{\Phi_1}{\Phi_2}-1\right)=\alpha\beta R_1-\frac{V}{I_1}(\beta-1).$$

In general, for any step R_n,

$$R_n=\alpha\beta R_{n-1}-\frac{V}{I_1}(\beta-1) \qquad \ldots(13{\cdot}12)$$

Writing $\delta=\alpha\beta$, we have

$$R_2=\delta R_1-\frac{V}{I_1}(\beta-1)$$

and $$R_n=\delta R_{n-1}-\frac{V}{I_1}(\beta-1).$$

The relation between r_m and R_1 is :

$$r_m=R_1-\left[R_1(1-\delta)+\frac{V}{I_1}(\beta-1)\right]\left(\frac{1-\delta^n}{1-\delta}\right) \quad ...(13\cdot13)$$

But $V/I_1=R_1$, so that

$$r_m=R_1\left[1-\left(\frac{1-\delta^n}{1-\delta}\right)(\beta-\delta)\right] \quad ...(13\cdot14)$$

The resistances of sections are :

$$r_1=R_1-R_2=R_1\beta(1-\alpha)=R_1(\beta-\delta)$$
$$r_2=R_2-R_3=\alpha\beta(R_1-R_2)=\delta r_1$$
$$r_3=R_3-R_4=\alpha\beta(R_2-R_3)=\delta^2 r_1$$
....
$$r_n=R_n-R_{n+1}=\alpha\beta(R_{n-1}-R_n)=\delta^{n-1}r_1.$$

Example 13·3. *Estimate the number of resistance sections and the resistance of each section for the starter of a 7·5 kW, 460 V, d.c. series motor. The starting current varies from 1·5 to 2 times full load current.*

The efficiency is 80 per cent and the resistance of machine measured between terminals is 1·8 Ω. Assume that the flux increases by 10 per cent as the current rises from 1·5 to 2 times the rated full load current.

Solution.

Full load current $=\frac{7500}{460\times0\cdot8}=20\cdot3$ A.

$\therefore$ $I_1=2\times20\cdot3=40\cdot6$ A and $I_2=1\cdot5\times20\cdot3=30\cdot4$ A.

$$\alpha=I_2/I_1=30\cdot4/40.6=0\cdot75$$

and $$\beta=\Phi_1/\Phi_2=1.1.$$

$\therefore$ $$\delta=\alpha\beta=0\cdot75\times1\cdot1=0\cdot825$$

and $$R_1=V/I_1=460/40\cdot6=11\cdot3\ \Omega.$$

The resistances of various sections are :

$r_1=R_1(\beta-\delta)=11\cdot3(1\cdot1-0\cdot825)=3\cdot10\ \Omega.$

$r_2=\delta r_1 \quad =0\cdot825\times3\cdot1 \quad =2\cdot56\ \Omega.$

$r_3=\delta^2 r_1 \quad =0\cdot825^2\times3\cdot1 \quad =2\cdot11\ \Omega.$

$r_4=\delta^3 r_1 \quad =0\cdot825^3\times3\cdot1 \quad =1\cdot74\ \Omega.$

Adding these resistances, we have the total equal to 9·51 Ω. To this if we add the value of motor resistance, *i.e.* 1·8 Ω, we get a total resistance of 11·31 Ω. This value is equal to R_1.

$\therefore$ We should not increase the resistance steps beyond 4 otherwise the total resistance would become higher than the required.

$\therefore$ Number of sections = 4 and number of studs = 5.

Example 13·4. *Find the resistance of each of the four sections of a starter for a 500 V series crane motor having a resistance of 0·3 Ω to give a maximum current of 100 A during starting. Assume the magnetization curve to be a straight line passing through origin.*

Solution. Total resistance at starting

$$R_1 = V/I_1 = 500/100 = 5\Omega.$$

Now $\alpha = I_2/I_1$ and $\beta = \Phi_1/\Phi_2$.

But $\Phi_1/\Phi_2 = I_1/I_2$ as the magnetization curve is linear.

$\therefore$ $\beta = \Phi_1/\Phi_2 = I_1/I_2$

Whence $\delta = \alpha\beta = (I_2/I_1)\times(I_1/I_2) = 1.$

The resistances of the four sections are :

$r_1 = (\beta - \delta)R_1 = (\beta - 1)R_1$

$r_2 = \delta r_1 \qquad = r_1$

$r_3 = \delta^2 r_1 \qquad = r_1$

$r_4 = \delta^3 r_1 \qquad = r_1.$

Now,

$r_1 + r_2 + r_3 + r_4 + r_m = R_1 = 5\cdot0\ \Omega$

$\therefore\ r_1 + r_2 + r_3 + r_4 = 5\cdot0 - 0\cdot3 = 4\cdot7\ \Omega$

But $r_1 = r_2 = r_3 = r_4.$

$\therefore$ $r_1 = r_2 = r_3 = r_4 = 4\cdot7/4 = 1\cdot175\ \Omega.$

14 2 3. Starters for three phase slip ring induction motors. Let us neglect the resistance and leakage reactance of the stator winding. Let X be the standstill reactance and E be the standstill emf of the rotor w nding.

Stud 1. Reference to Fig. 13·1.

The motor is switched on to the supply. The machine draws the maximum current I_1. Let the slip be s_1.

$$\therefore \qquad I_1 = \frac{s_1 E}{\sqrt{(R_1^2 + s_1^2 X^2)}}.$$

The machine picks up speed and the slip falls and so does the current. The value of current just before notching is I_2. Let the slip be s_2 at this instant.

$$\therefore \qquad I_2 = \frac{s_2 E}{\sqrt{(R_1^2 + s_2^2 X^2)}}$$

Stud 2. The resistance r_1 is cut out and the resistance in the circuit is R_2.

The current rises to its maximum value I_1. Let us consider that there is no change in speed during notching operation.

$\therefore$ The slip just after notching is s_2.

$$\text{Hence,} \qquad I_1 = \frac{s_2 E}{\sqrt{(R_2^2 + s_2^2 X^2)}}$$

Just before notching at stud 2, the current falls to I_2 and slip to s_3.

$$\therefore \qquad I_2 = \frac{s_3 E}{\sqrt{(R_2^2 + s_3^2 X^2)}}$$

Thus, from above, we have :

$$I_1 = \frac{s_1 E}{\sqrt{(R_1^2 + s_1^2 X^2)}} = \frac{s_2 E}{\sqrt{(R_2^2 + s_2^2 X^2)}} = \ldots$$

$$= \frac{s_{n+1}{}^2 E}{\sqrt{(r_m^2 + s_{n+1}{}^2 X^2)}} \qquad \ldots(13\cdot15)$$

From Eqn. 13·15,

$$\frac{R_1}{s_1}=\frac{R_2}{s_2}=\ldots\ldots\ldots\ldots=\frac{r_m}{s_{n+1}} \quad \ldots(13·16)$$

and

$$I_2=\frac{s_1E}{\sqrt{(R_1^2+s_1^2X^2)}}=\frac{s_2E}{\sqrt{(R_2^2+s_2^2X^2)}}=\ldots\ldots$$

$$=\frac{s_{n+1}E}{\sqrt{(R_n^2+s_{n+1}^2X^2)}} \quad \ldots(13·17)$$

From Eqn. 13·17

$$\frac{R_1}{s_2}=\frac{R_2}{s_3}=\ldots\ldots\ldots=\frac{R_n}{s_{n+1}} \quad \ldots(13·18)$$

Combining Eqn. 13·16 and 13·18

$$\frac{s_2}{s_1}=\frac{s_3}{s_2}=\ldots\ldots\ldots=\frac{s_{n+1}}{s_n}=\frac{R_2}{R_1}=\frac{R_3}{R_2}=\ldots\ldots\ldots=\frac{r_m}{R_n}=\alpha. \quad \ldots(13·19)$$

Now, slip at starting $s_1=1$ and slip at $(n+1)$th stud, s_{n+1}=slip at full load$=s_m$.
(This is true provided the upper limit of current is the full load current).

$$\therefore \quad \frac{s_2}{s_1}=\frac{s_3}{s_2}=\ldots\ldots\ldots=\frac{s_m}{s_n}=\frac{R_2}{R_1}=\frac{R_3}{R_2}=\ldots\ldots\ldots=\frac{r_m}{R_n}=\alpha$$

or

$$s_m=(\alpha)^n.$$

$$\therefore \quad \alpha=(s_m)^{\frac{1}{n}} \quad \ldots(13·20)$$

Also $\quad r_m/R_1=(\alpha)^n$

or

$$r_m=R_1(\alpha)^n \quad \ldots(13·21)$$

Further,

$$R_2=\alpha R_1$$
$$R_3=\alpha R_2=\alpha^2R_1$$
$$R_4=\alpha R_3=\alpha^3R_1$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$R_n=\alpha R_{n-1}=\alpha^{n-1}R_1.$$

The sections are :

$$r_1=R_1-R_2=R_1(1-\alpha)$$
$$r_2=R_2-R_3=R_1(\alpha-\alpha^2)=\alpha r_1$$
$$r_3=R_3-R_4=R_1(\alpha^2-\alpha^3)=\alpha^2r_1$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$r_n=R_n-R_{n+1}=R_1(\alpha^{n-1}-\alpha^n)=\alpha^{n-1}r_1.$$

From Eqns. 13·15 and 13·17

$$\left(\frac{I_2}{I_1}\right)^2=\frac{s_2^2(R_1^2+s_1^2X^2)}{s_1^2(R_1^2+s_2^2X^2)}=\frac{s_2^2(R_1^2+X^2)}{(R_1^2+s_2^2X^2)}$$

$$=\frac{R_1^2+X^2}{(R_1/\alpha)^2+X^2}\cong\alpha^2 \text{ for normal motors}$$

or

$$\frac{I_2}{I_1}\cong\alpha \text{ for normal motors} \quad \ldots(13·22)$$

The ratio of the rotor current limits is, therefore, approximately equal to α and this is also the approximate ratio of stator currents.

Example 13·5. *Design the sections of a rotor starter for a 75 kW, 3 phase induction motor, using 7 notches. Rotor resistance per phase = 0·018 Ω. The upper current limit is to be full load current, for which the slip is 2 per cent.*

Solution. Notches or studs = 7.

$\therefore$ Number of resistance elements $n=6$.

From Eqn. 13·20,

$$\alpha=(s_m)^{\frac{1}{n}}=(0{\cdot}02)^{\frac{1}{6}}=0{\cdot}521.$$

From Eqns. 13·20 and 13·21,

$$r_m=R_1(\alpha)^n=R_1 s_m.$$

$$\therefore \qquad R_1=\frac{r_m}{s_m}=\frac{0{\cdot}018}{0{\cdot}02}=0{\cdot}9$$

The resistances of elements are :

$r_1=R_1(1-\alpha)=0{\cdot}9(1-0{\cdot}521)=0{\cdot}432\ \Omega.$
$r_2=\alpha r_1 \quad =0{\cdot}521\times0{\cdot}432=0{\cdot}225\ \Omega.$
$r_3=\alpha^2 r_1 \quad =0{\cdot}521^2\times0{\cdot}432=0{\cdot}108\ \Omega.$
$r_4=\alpha^3 r_1 \quad =0{\cdot}521^3\times0{\cdot}432=0{\cdot}062\ \Omega.$
$r_5=\alpha^4 r_1 \quad =0{\cdot}521^4\times0{\cdot}432=0{\cdot}032\ \Omega.$
$r_6=\alpha^5 r_1 \quad =0{\cdot}521^5\times0{\cdot}432=0{\cdot}017\ \Omega.$
$r_m \quad =0{\cdot}018\ \Omega.$

Total resistance of rotor $=0{\cdot}894\ \Omega.$

This resistance is nearly equal to R_1.

Example 13·6. *Design the 6 sections of a 7 stud rotor starter for a 3 phase wound rotor induction motor. The slip at full load current is 2% and the maximum starting current is 1·5 times full load current. The resistance of rotor per phase is 0·02 Ω.*

Solution.

$$\text{Rotor current}=\frac{E}{\sqrt{(R/s)^2+X^2}}$$

$$\simeq\frac{sE}{R}\ \text{if rotor reactance is neglected.}$$

This means that slip changes directly with rotor current. The full load slip is 2%.

$\therefore$ Slip at 1·5 times full load current $=1{\cdot}5\times2=3\%$.

From Eqn. 13·20,

$$\alpha=(s_m)^{\frac{1}{n}}=(0{\cdot}03)^{\frac{1}{6}}=0{\cdot}557.$$

$$R_1=\frac{r_m}{s_m}=\frac{0{\cdot}02}{0{\cdot}03}=0{\cdot}666\ \Omega.$$

The resistances of the sections are :

$r_1=(1-\alpha)R_1=1-(0{\cdot}557)\times0{\cdot}666=0{\cdot}295\Omega.$
$r_2=\alpha r_1 \quad =0{\cdot}557\times0{\cdot}295 \quad =0{\cdot}165\ \Omega.$
$r_3=\alpha^2 r_1 \quad =0{\cdot}557^2\times0{\cdot}295 \quad =0{\cdot}092\ \Omega.$
$r_4=\alpha^3 r_1 \quad =0{\cdot}557^3\times0{\cdot}295 \quad =0{\cdot}051\ \Omega.$
$r_5=\alpha^4 r_1 \quad =0{\cdot}557^4\times0{\cdot}295 \quad =0{\cdot}028\ \Omega.$
$r_6=\alpha^5 r_1 \quad =0{\cdot}557^5\times0{\cdot}295 \quad =0{\cdot}016\ \Omega.$
$r_m \quad =0{\cdot}020\ \Omega.$

Total resistance of rotor $=0{\cdot}667\ \Omega.$

This value is almost equal to R_1.

Example 13.7. *A 37.5 kW, 400 V, 50 Hz, 3 phase slip ring induction motor has a copper loss of 1200 W per phase at full load and a friction and windage loss of 500 W. The rotor resistance per phase is 0.15 Ω.*

Assuming that starting current is not to exceed 1.25 times full load current, work out the steps of a 4 section starter.

Solution.

Gross rotor output $=37.5$ kW $=37500$ W.

Rotor input $=37500+1200+500=39200$ W.

Now
$$\text{slip}=\frac{\text{rotor copper loss}}{\text{rotor input}}$$

∴ Slip at full load $s_m=\frac{1200}{39200}=0.0306.$

Slip at 1.25 times full load current $=1.25\times0.0306=0.03825$

Now,
$$a=(s_m)^{\frac{1}{n}}=(0.03825)^{\frac{1}{4}}$$

and
$$R_1=\left(\frac{r_m}{s_m}\right)=\frac{0.15}{0.03825}=3.92\ \Omega.$$

The resistances of sections are :

$$r_1=R_1(1-a)=3.92(1-0.442)=2.190\ \Omega$$
$$r_2=ar_1\qquad=0.442\times2.19\quad=0.968\ \Omega$$
$$r_3=a^2r_1\qquad=(0.444)^2\times2.19=0.428\ \Omega.$$
$$r_4=a^3r_1\qquad=(0.442)^3\times2.19=0.189\ \Omega.$$
$$r_m\qquad=0.150\ \Omega.$$

Total resistance of rotor $=3.925\ \Omega$.

This is almost equal to R_1.

13.4. Field regulators.

1. *Shunt generators.* The method of calculation of steps of field regulator of a shunt generator is explained in the following example.

Example 13.8. *Find the section resistance of a 7 stud field regulator for a generator to give the limits of 500 and 560 V in equal steps. The magnetization curve is given in Fig 13.2. The field is 934 Ω.*

Solution. There are 7 studs and therefore the number of resistance sections is 6. The range 500 to 560 V is divided into 6 sections of 10 V each. The method for calculation of resistances is shown as under :

Take the step at 550 V. The field current corresponding to 550 V is 0.55 A.

∴ Field circuit resistance
$$=\frac{550}{0.55}=1000.$$

∴ Resistance to be inserted
$$=1000-934=66\ \Omega.$$

Resistance of section $=66\ \Omega$.

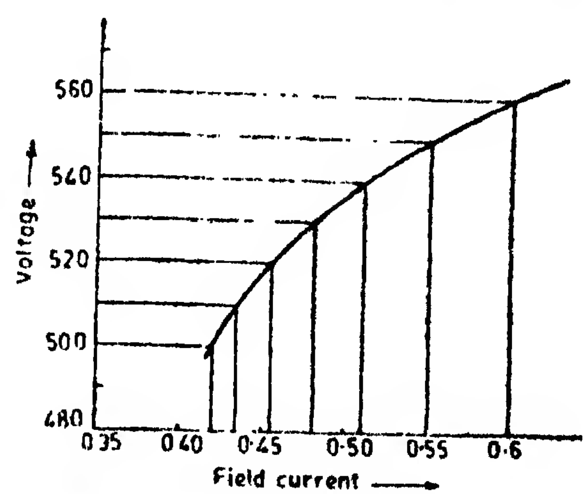


Fig. 13.2. Magnetizing curve of Example 13.8.

Take the step at 540 V. The field current is 0 51 A.

$\therefore$ Total field circuit resistance$=\dfrac{540}{0{\cdot}51}=1058\ \Omega$.

Total resistance to be inserted$=1058-934=124\ \Omega$.
Resistance of section $=124-66=58\ \Omega$.

The value of resistances of other sections can be similarly calculated. The results are tabulated below :

Stud No.	*Emf.* *V*	*Field Current* *A*	*Total resistance* *Ω*	*Section resistance* *Ω*
1	560	0·60	934	—
2	550	0 55	1000	66
3	540	0·51	1058	58
4	530	0·48	1102	44
5	520	0 455	1142	40
6	510	0·435	1172	30
7	500	0·4175	1195	23

2. ***Shunt motor*** The method for calculation of resistance elements of a field regulator for shunt motor is illustrated in the following example :

Example 13·9. *Find the suitable shunt field regulator resistance elements for a speed range of 750 to 1350 r.p.m in increments of 150 r.p.m. for a 250 V shunt motor. Field resistance=100 Ω. Fig. 13·3 shows the O.C C. for 750 r.p.m.*

Solution.

Speed range $=750$ to 1350 r.p.m.

Number of sections$=\dfrac{1350-750}{150}=4$.

Stud 1. Speed$=750$ r.p.m.

Voltage$=250$ V.

Corresponding to 250 volts, field current$=2{\cdot}5$ A.

Shunt field circuit resistance$=\dfrac{250}{2{\cdot}5}=100\ \Omega$.

Shunt field resistance $=100\ \Omega$.

$\therefore$ External resistance $=0$.

Stud 2. Speed 900 r.p.m.

In order to get this speed the flux must be reduced to $\dfrac{750}{900}=0{\cdot}833$ of the flux with 750 r.p.m. or the voltage on *O.C.C.* should be $250\times0{\cdot}833=208$ V.

Corresponding to 208 V, field current $=1{\cdot}8$ A.

$\therefore$ Shunt field circuit resistance $=\dfrac{250}{1{\cdot}8}=139\ \Omega$.

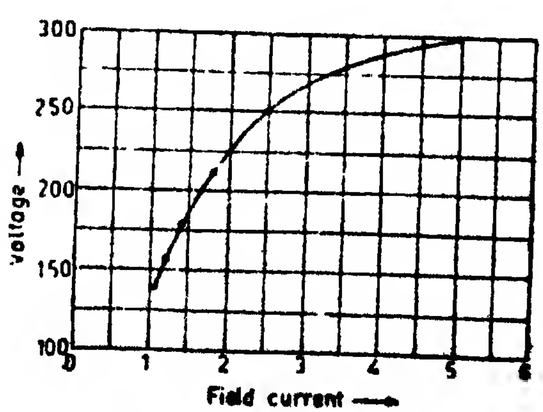


Fig. 13·3. Magnetization curve of Example 13·9.

$\therefore$ External resistance $= 139 - 100 = 39\ \Omega$.

Resistance of step $= 39\ \Omega$.

The rest of the calculations are tabulated below :

N Speed r.p.m.	Emf. $= \dfrac{250\times750}{N}$ V	Field current I_f from O.C.C. A	Field circuit resistance $R_1 = 250/I_f$ Ω	Rheostat resistance $R = R_1 - 100$ Ω	Section resistance Ω
750	250	2·5	100	0	...
900	208	1·8	139	39	39
1050	178	1·4	179	79	40
1200	156	1·2	208	108	29
1350	139	1·05	238	138	30

3. *Potentiometer regulators*. These regulators are used when a wide variation in voltage or speed control is desired. This is very beneficial if a gradual control is desired or when the field current is small.

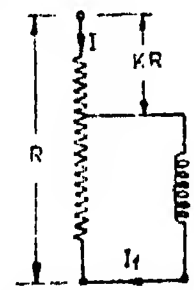


Fig. 13·4. Potentiometer regulator.

Let V = voltage applied to potentiometer,

R = total resistance of potentiometer,

KR = resistance of untapped portion of potentiometer,

R_f = field resistance, $m = R/R_f$,

I = total current and I_f = field current.

Total resistance between terminals is

$$= KR + \frac{R_f\times(1-K)R}{R_f+(1-K)R} = \frac{(K-K^2)R^2+R_fR}{R_f+(1-K)R}$$

$$= \frac{(K-K^2)m^2R_f^2+mR_f^2}{R_f+mR_f(1-K)} = R_f\left[\frac{(K-K^2)m^2+m}{1+m(1-K)}\right]$$

Total current flowing $I = \dfrac{V}{R_f}\left[\dfrac{(1-K)m+1}{(K-K^2)m^2+m}\right]$ ...(13·23)

Field current $I_f = \dfrac{V}{R_f}\left[\dfrac{(1-K)}{(K-K^2)m+1}\right]$ ...(13·24)

The maximum field current is V/R_f.

The value of total current I and the field current I_f can be found for any setting of the potentiometer (*i.e* for any value of K) if the ratio $m = R/R_f$ is specified.

Note. The design of resistance elements is given in Art. 14·1, 14·2, 14·3 and 14·4.

UNSOLVED PROBLEMS

1. Design a suitable 8 section starter for a 20 h.p. 250 V, 1000 r.p.m. d.c. shunt motor from the following data :

Maximum torque = full load torque ; armature resistance = 0·4 Ω ; efficiency = 0·85.

Find also the speeds at which the notching takes place.

[Ans. (*a*) 0·85, 0·65, 0·49, 0·38, 0·28, 0·22, 0·16, 0·13 Ω .
(*b*) 269, 474, 630, 749, 840, 908, 962, 1000 r.p.m.]

2. Grade the five resistance sections of a 6 stud starter for a 15 h.p., 230 V, 1100 r.p.m. d.c. shunt motor. Maximum current is to be limited to 1·25 times full load current during starting period. The full load efficiency of the motor may be taken as 86 per cent. Armature copper loss constitutes half of the total loss. Field current=2A. Armature resistance=0·5 Ω.

Calculate also the speed at which 1st and 2nd notchings take place.

[Ans. (*a*) 0·917, 0·670, 0·486, 0·354, 0·257, 0·189 Ω (*b*) 336, 579 r.p.m.]

3. The starter for a 460 V d.c. series motor has 5 resistance sections and the current limits during starting are 120 A and 156 A. The resistance of the machine is 0·19 Ω, and between these current limits the flux changes by 10%. Find the resistance of each section. [Ans. 0·75, 0·63, 0·537, 0·449, 0394 Ω]

4. A 15kW, 500 V d.c. series motor has a total resistance of 1Ω. The current during starting is allowed to vary between 70 A and 53 A, the corresponding flux are in the ratio 1·22 : 1.

Determine the number of steps in the starter and the resistance in each step.

[Ans. 4 steps ; 1·93, 1·66, 1·40, 1·16 Ω]

5. Find the resistance of each resistance section of a rotor starter of a slip ring induction motor having a rotor resistance of 0·02 Ω per phase and a full load slip of 4 per cent. Use 9 notches.

Determine also the slip at various notches.

[Ans. Resistance : 0·165, 0·11, 0·074, 0·05, 0·0335, 0·0225, 0·015, 0·01 Ω.
Slip : 1·0, 0·67, 0·45, 0·3, 0·2, 0·134, 0·09, 0·06, 0·04]

6. Calculate the steps in a 4 step starter for a 3 phase slip ring induction motor, from the following data : maximum starting current=full load current ; full load slip=2·5 per cent, rotor resistance per phase =0·02 Ω. [Ans. 0·48, 0·19, 0·08, 0·03 Ω]

7. A 250 V, d.c. shunt motor with a normal speed of 600 r.p.m. takes a shunt field current of 2 A when running light. If the speed of the motor is to be raised to 1200 r.p.m. in increments of 100 r.p.m.; find the resistance sections of a shunt field regulator. The O.C.C. of the motor at 600 r.p.m. is as under :

Field current, *A*	0·4	0·8	1·2	1·5	2·0	2·4
Emf, *V*	70	137·5	187·5	225	250	270

[Ans. 48·6, 34·7, 36·8, 39·0 ,32·3, 26·1 Ω]

14

Design of Electrical Apparatus

14·1. Design of resistance elements of field regulators. The design of field regulators for both motors and generators follows the same procedure. The resistances of different sections are calculated as given in Art. 13 3.

14·1·1. Material for resistance elements. The material usually used is constantan which has a resistivity, 0·46 to 0·53 Ω/m and mm² a resistance temperature co-efficient, 0·00002/°C (virtually zero) ; and a maximum operating temperature, 500°C.

Nichrome can also be used. It has a resistivity of 1·1 to 1·27 Ω/m and mm² and a maximum operating temperature of 900—1000°C.

14·1·2. Size of wire. The field rheostats of generators and motors are designed to for continuous duty They are mostly wire wound units ranging in capacity from a few watt a few kilowatt.

The size of wire for a particular current carrying capacity can be calculated as under :

Let

I=current carrying capacity of wire, A ;

d=diameter of wire, mm ; l=length of wire, m ;

ρ=resistivity of wire material, Ω/m and mm² ;

λ=specific heat dissipation, W/m²—°C ;

and θ=maximum permissible temperature rise, °C.

$$\therefore \text{ Loss to be dissipated}=I^2R=I^2\rho\ \frac{l}{(\pi/4)\ d^2} \qquad \ldots(i)$$

Dissipating surface of wire $S=\pi dl\times 10^{-3}$

$$\text{Loss dissipated} \qquad =S\lambda\theta=(\pi dl\times 10^{-3})\lambda\theta \qquad \ldots(ii)$$

Equating (*i*) and (*ii*), we have,

$$I=\left(\frac{\pi^2}{4}\ \frac{\lambda\theta}{\rho}\times 10^{-3}\right)^{1/2} d^{1\cdot5} \qquad \ldots(14\cdot1)$$

$$=K\ d^{1\cdot5} \qquad \ldots(14\cdot2)$$

where $$K=\left(\frac{\pi^2}{4}\ \frac{\lambda\theta}{\rho}\times 10^{-3}\right)^{1/2} \qquad \ldots(14\cdot3)$$

The temperature rise θ may be taken as between 100 to 150°C and specific heat dissipation, λ, as 20 W/m²—°C.

For constantan ρ=0·5 Ω/m and mm². Taking θ=100°C, we have :

$$K=\sqrt{\frac{\pi^2}{4}\times\frac{20\times 100}{0\cdot5}\times 10^{-3}}=3\cdot2.$$

Hence $I=3{\cdot}2\ d^{1{\cdot}5}$ ...(14·4)

For $\theta=150°C$, we have $I=3{\cdot}9\ d^{1{\cdot}5}$ ...(14·5)

For nichrome, $\rho=1{\cdot}2\ \Omega/\text{m and mm}^2$. For $\theta=100°C$, we have

$$K=\sqrt{\frac{\pi^2}{4}\times\frac{20\times100}{1{\cdot}2}\times10^{-3}}\simeq2.$$

Hence $I=2\ d^{1{\cdot}5}$ ...(14·6)

For $\theta=150°C$, we have $I=2{\cdot}5\ d^{1{\cdot}5}$ ...(14·7)

The above relationships are based upon a specific heat dissipation of 20 W/m²—°C.

From Eqns. 14·4 to 14·7, the diameter of wire can be found out. The standard wire size is selected by referring to Table 17·8 (British standard sizes of wires).

The resistance per unit length can be worked out and the length of wire to be used for each resistance section can be calculated.

14 1 3. Resistance box. Regulator and starter rheostats having heavy gauge wire (20 SWG and under or diameter about 1 mm and over) can be arranged in the form of

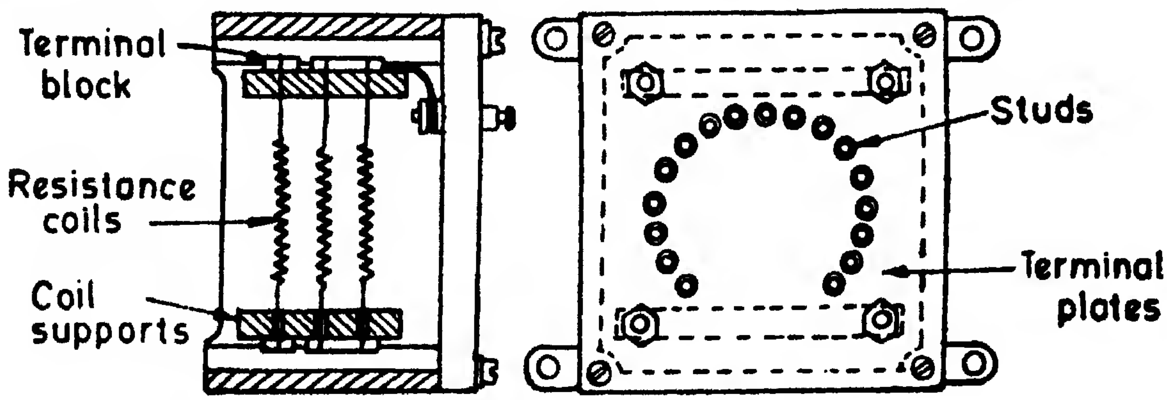


Fig. 14·1. Resistance box of field regulator.

a spiral supported at the ends by porcelain cleats or studs on an iron plate. Smaller gauges must be wound on supporting ceramic tubes. The rheostat frame work is enclosed in a ventilated cast iron box. Fig. 14 1 shows the design of a box suitable for arranging the wires in open spirals.

Example 14·1. *Design the resistance elements of field regulator of the generator of example 13·8. The resistance of the 6 sections are 66, 58, 44, 40, 30, 23Ω. The maximum current is 0·6 A.*

Solution. The field regulator is designed for the maximum current *i.e.* 0·6 A.

(*i*) Using constantan wire, a temperature rise of 150°C and a dissipation of 20 W/m²—°C, we have :

$I=3{\cdot}9\ d^{1{\cdot}5}$ or $0{\cdot}6=3{\cdot}9\ d^{1{\cdot}5}$.

$\therefore\ d=0{\cdot}288$ mm.

From Table 17·8, the nearest standard conductor available is 31 SWG and it has a diameter of 0·2946 mm and an area of 0·06819 mm².

Resistance per meter length $=\rho/a=0{\cdot}5/0{\cdot}06819=7{\cdot}35\ \Omega$.

Length of wire in section No. 1 $=66/7{\cdot}35=8{\cdot}96$ m.

Length of wire in section No. 2 $=58/7{\cdot}35=7{\cdot}88$ m.

Length of wire in section No. 3 $=44/7{\cdot}35=5{\cdot}97$ m.

Length of wire in section No. 4=40/7·35=5·34 m.

Length of wire in section No. 5=30/7·35=4·07 m.

Length of wire in section No. 6=23/7·35=3·12 m.

14·2. Design of resistances for starters for shunt motors. Starters have a discontinuous service and are therefore intermittently rated. Thus the rheostat design for a starter is considerably different from that of a continuously loaded field regulator.

Method 1. In this method the design is based upon the continuous ratings for resistance materials, and multiplying constants are then used to estimate their intermittent capacity.

One rule which is favoured by manufacturers is to group the rheostat sections into three groups and to allow 4 times the continuous rating for first group, 3 times for second group, and twice for 3rd group. Materials used are constantan or nichrome wires for smaller currents and cast iron grids for heavy currents.

An example, given below, illustrates this method.

Example 14·2. *The resistances of a 5 section starter are :*

$r_1=1·74\ \Omega,\ r_2=1·14\ \Omega,\ r_3=0·75\ \Omega,\ r_4=0·49\ \Omega,\ r_5=0·32\ \Omega.$

The maximum and minimum currents during starting are 60 A and 40 A respectively. The full load current is 36 A.

Find areas and lengths of wires used for different resistance sections.

Solution. The 5 sections are divided into three groups.

Group I	—	$r_1=1·74\ \Omega$,	
Group II	—	$r_2=1·14\ \Omega$,	$r_3=0·75\ \Omega$
Group III	—	$r_4=0·49\ \Omega$,	$r_5=0·32\ \Omega$.

(i) Constantan wire

Group I

It is designed with 36/4=9 A continuous rating.

Using a temperature rise of 150°C.

From Eqn. 14·4, $I=3·9\ d^{1·5}$ or $9=3·9\ d^{1·5}$

$\therefore$ $d=1·75$ mm.

The nearest standard size is 15 SWG (See Table 17·8).

It has a diameter of 1·829 mm and an area of 2·627 mm².

$\therefore$ Length of wire required for 1st section $l_1=\dfrac{1·74\times2·627}{0·5}=9·15$ m

Group II

It is designed for 36/3=12 A continuous rating.

A $12=3·9\ d^{1·5}$ or $d=2·12$ mm.

The nearest size is 14 SWG. Its diameter is 2·032 mm and area 3·243 mm².

Length of wire for 2nd section $l_2=\dfrac{1·14\times3·243}{0·5}=7·4$ m.

Length of wire for 3rd section $l_3=\dfrac{0·75\times3·243}{0·5}=4·85$ m.

Group III

It is designed with 36/2=18 A continuous rating.

$\therefore$ $18=3{\cdot}9\,d^{1\cdot5}$ or $d=2{\cdot}78$ mm.

The nearest size is 12 SWG. It has a diameter of 2·642 mm and area of 5·48 mm².

$\therefore$ Length of wire required for 4th section $l_4=\dfrac{0{\cdot}49\times5{\cdot}48}{0{\cdot}5}=5{\cdot}3$ m.

Length of wire required for 5th section $l_5=\dfrac{0{\cdot}32\times5{\cdot}48}{0{\cdot}5}=3{\cdot}5$ m.

(*ii*) Nichrome wire.

Group I

From Eqn. 14·6, $I_{max}=2{\cdot}5\,d^{1\cdot5}$ or $9=2{\cdot}5\,d^{1\cdot5}$.

$\therefore$ $d=2{\cdot}36$ mm.

The nearest standard size is 13 SWG. Its diameter is 2·34 mm and area 4·29 mm².

Length of wire for 1st section $l_1=\dfrac{1{\cdot}74\times4{\cdot}29}{1{\cdot}2}=6{\cdot}22$ m.

Similar calculations can be done for other steps.

It will be noted that the size (area) of nichrome wire is larger than that of corresponding constantan wire but lengths of wire of former (nichrome) are smaller.

Method II

This method is based upon estimation of heat capacity of the wire starting from cold. If this is done for the full starting period with an allowance for consecutive starting, it will give the correct size for last few steps. Reduction constants can then be used for the earlier groups.

Let

a=area of cross section of wire, mm² ; ρ=resistivity, Ω/m and mm².

h=specific heat, J/kg—°C ; g=density of wire, kg/m³ ;

t=starting time, s ; θ=temperature rise, °C.

Proceeding as in Art. 3·34 (page 109), we have

$$a=I\times\sqrt{\frac{\rho t\times10^6}{gh\theta}} \qquad \text{...(14·8)}$$

If we use constantan wire

$\rho=0{\cdot}5$ Ω/m and mm² ; $h=420$ J/kg—°C ;

$g=8860$ kg/m³ ; $\theta=150$°C.

Putting these values in Eqn. 14·8, we get

$$a=0{\cdot}03\,I\sqrt{t} \text{ for constantan} \qquad \text{...(14·9)}$$

For nichrome

$$a=0{\cdot}047\,I\sqrt{t} \qquad \text{...(14·10)}$$

Example 14·3. *Using the data of example 14·2, find the area and length of wire for the last section of the starter if the starting period is 10 seconds. Use constantan wire.*

Solution. The maximum current is 50 A and the minimum current is 36 A. The effective current for heating may be taken as 43 A. The starting time for this starter is 10 s but it will be unsafe to design resistances for this period only, as there may be several consecutive starts without time for cooling. In order to allow for ample margin, a starting time of 30 s is used,

From Eqn. 14·9, area of constantan wire required,

$$a = 0{\cdot}03 I\sqrt{t} = 0{\cdot}03 \times 43 \times \sqrt{30} = 7{\cdot}05 \text{ mm}^2.$$

The nearest standard size is 11 SWG and it has an area of 6·818 mm².

$$\therefore \text{ Length of wire required} = \frac{0{\cdot}32 \times 6{\cdot}818}{0{\cdot}5} = 4{\cdot}35 \text{ m}.$$

14·3. Design of loading rheostat. The coils used in loading rheostats are made up of iron constantan or nichrome wire. The wire is wound over a mandrel in a lathe and then the coils are mounted vertically in a frame.

The size of wire can be calculated from Eqns. 14·2 and 14·3. The usual value of temperature rise and specific heat disipation are :

temperature rise $\theta = 100°$ to $150°C$ and specific heat dissipation $\lambda = 20$ W/m²—°C.

Table 14·1 gives the size of wire coils for rheostats.

Table 14·1. Dimensions of wire coils

Size SWG	*Mandrel diameter mm*	*Length of turn mm*	*Turns per metre*	*Maximum length mm*
8—10	30·0	110	160	450
11—13	25·0	90	190	300
14—16	20·0	73	275	300
17—19	12·5	48	360	300
20—22	6·0	23	550	150

The spacing between coils should be about 3 times the diameter of mandrel.

Example 14·4 *Design a constantan wire rheostat to carry 10 A and to dissipate 1000 W continuously.*

Solution. Constantan wire having a resistivity of 0·5 Ω/m and mm² is used. Taking a temperature rise of 150°C and a dissipation of 20 W/m²—°C, we have from Equ.14·5,

$$10 = 3{\cdot}9\, d^{1{\cdot}5} \qquad \therefore \qquad d = 1{\cdot}875 \text{ mm}.$$

The nearest standard size is 15 SWG. It has a diameter of 1·83 mm and an area of 2·63 mm².

Resistance required $R = P/I^2 = 1000/(10)^2 = 10\ \Omega$.

$\therefore$ Length of wire required $= 10 \times 2{\cdot}63/0{\cdot}5 = 52{\cdot}6$ m.

Referring to Table 14·1, we have :

diameter of mandrel = 20 mm, maximum length of coil = 300 mm, turns/metre = 275 and length of each turn = 73 mm.

Length of wire in each coil

$=300\times10^{-3}\,(275\times73\times10^{-3})$

$=6{\cdot}02$ m.

Number of coils required

$=52{\cdot}6/6{\cdot}02=9.$

The spacing between the coils is kept approximately three times the diameter of the mandrel.

$\therefore$ Spacing between coils

$=3\times20=60$ mm.

The arrangement of the coils is shown in Fig. 14·2.

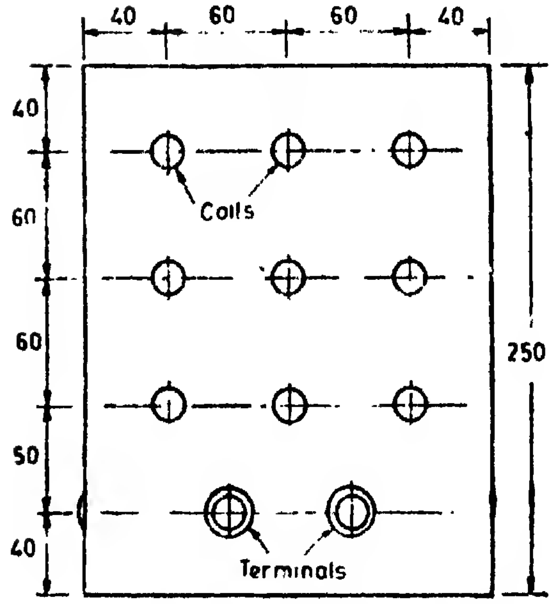


Fig. 14·2. Resistance box (arrangement of coils). The height of un-supported coils is 300 mm and the height of the resistance box is 400 mm.

Total external surface of box excluding bottom $=2(250+200)\times400\times10^{-6}$ $+250\times200\times10^{-6}=0{\cdot}41\,\text{m}^2$.

Surface per watt dissipated $=0{\cdot}41/1000=0{\cdot}41\times10^{-3}\,\text{m}^2/\text{W}$.

This figure serves as a check on the design, because it is found that the temperature rise will not be excessive with this type of resistance box provided the external surface of the box is not less than $0{\cdot}325\,\text{m}^2/\text{W}$ dissipated.

14·4. Design of grid resistances. Resistor grids made up of cast iron are among the oldest and most widely used types of resistors. Individual grids are cast in a shape approximating a square wave, with lugs in the end and in the middle to facilitate mounting. The resistance of each grid varies approximately from 0·01 Ω to 0·18 Ω. A typical grid resistance is shown in Fig. 14·3. Grids are comparatively low in cost but their impact strength is low. Individual grids are stacked on insulated tie rods supported by sheet metal end frames. The assembly is called a box. Cast grids are used primarily where low resistance and high current carrying capacity is desired. Their principal field is starting and intermittent speed control of medium and large size motors.

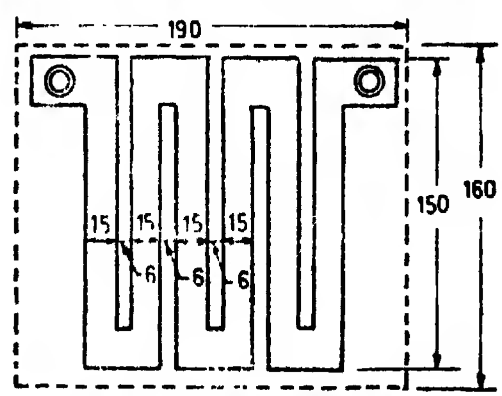


Fig. 14·3. Resistance grids.

Punched steel grid resistors are similar in construction and appearance to cast grids, except that the individual grids are not cast but punched out of alloy sheet steel.

Compared with cast grids, the punched steel grids are less subject to damage by impact and weigh less. But punched steel grids, cost more and because of their lower mass, they reach their final temperature faster and thus have lower ability to absorb momentary overloads.

The design of grid resistances is complicated as the loss that can be dissipated from the surface of the grids depends not only upon the surface area and the temperature rise, but also upon the distance between the grids, the distance between the grid boxes and upon the means provided for the free passage of air. The values for specific heat dissipation given in Table 14·2 are based upon experiments conducted with punched sheet grids similar to the one shown in Fig. 14·3 with 6 mm spacing between grids.

Table 14·2

Specific heat dissipation from punched sheet resistance grids

Temperature rise °C	*Specific heat dissipation W/m²—°C*	
	Enclosed	*Open Type*
100	3·0	4·2
200	4·5	6·5
300	6·0	8·0
400	7·0	9·7
500	8·5	11·8

Heat dissipation from box surface. It may be mentioned here that the temperature rise of the grid units depends largely upon the overall dimensions of the assembled resistance and the total loss dissipated, apart from the actual arrangement of the grids. An approximate relationship for capacity of a grid resistance in terms of external surface is

$$Q = S\lambda\theta$$

where Q = toal loss dissipated continuously, W ;

S = external surface of perforated metal box (not including the two ends supporting the perforating rods meant for assembling grid boxes), m² ;

θ = average temperature rise of grid metal, °C ;

λ = specific loss dissipation in W/m²—°C

= 16 for θ = 200 °C

= 32 for θ = 500 °C.

Example 14·5. *Using the grid of Fig. 14·3, design a grid resistance to dissipate 2000 W continuously with an average temperature rise of 300°C. The grid are made up of a nickel iron alloy of resistivity 0·2 Ω/m and mm² at 0°C and resistance temperature co-efficient of $4·5\times10^{-3}$/°C. Assume :*

Thickness of metal = 2 mm, and specific heat dissipation from grid surface = 6 W/m²—°C.

The resistance is to be enclosed in a perforated metal box and the specific heat dissipation from box surface should not be more than 25 W/m²—°C.

Solution.

Length of current path = 0·9 m (approximately).

Area of each grid = 2×15 = 30 mm².

Resistance at 0°C = $0·2\times0·9/30 = 6\times10^{-3}$ Ω.

Resistance (hot) with 300°C rise in temperature

$$R=6\times10^{-3}(1+4.5\times10^{-3}\times300)=14.1\times10^{-3}\ \Omega.$$

The cooling surface of one grid (considering front and back)

$$=2\times0.9\times15\times10^{-3}=27\times10^{-3}\ \text{m}^2.$$

$\therefore$ Heat dissipated by each grid $=27\times10^{-3}\times6\times300=48.5$ W.

But heat dissipated is also equal to I^2R.

$\therefore$ $\quad I^2=48.5/(14.1\times10^{-3})$ or current $I=57.5$ A.

Current density $=57.5/30\simeq1.9$ A/mm².

This current density is rather high but seems reasonable with a high temperature rise of 300°C.

Number of grid boxes (stracks) $=2000/48.5\simeq41$

The distance between centres of adjacent grid boxes is kept about 10 mm.

Length of stack $=10\times40+\text{clearance}=420$ mm $=0.42$ m.

External surface of box $=0.42\times2(0.16+0.19)=0.294$ m².

$$\text{Specific heat dissipation}=\frac{2000}{300\times0.294}=22.2\ \text{W/m}^2\text{—°C}$$

This is below the maximum allowable limit.

14.5. Design of heating elements. The materials used for heating elements have been discussed in chapter 2. Nichrome is the most commonly used material.

Design of round wire elements. In high temperature furnaces, the heat produced in the elements is mainly dissipated by radiation as the temperatures are very high.

Let $\quad P$ = electrical input, W ; V = voltage applied, V ;

R = resistance of element, Ω ;

ρ = resistivity of element, Ω/m and mm²

d = diameter of wire, mm ; l = length of element, m.

$$\text{Heat input}\quad P=\frac{V^2}{R}=\frac{\pi}{4}\cdot\frac{V^2d^2}{\rho L}$$

Dissipating surface of wire $=\pi\,dl\times10^{-3}$ m².

Heat input of wire surface

$$=\frac{\pi}{4}\cdot\frac{V^2d^2}{\rho L}\cdot\frac{1}{\pi dl\times10^{-3}}=\frac{V^2d\times10^3}{4\rho l^2}\ \text{W/m}^2 \qquad \text{...(i)}$$

Now $\quad q_{rad}$ = heat radiated per unit surface of wire

(See Eqn. 3.9 on page 38)

$$=5.7\times10^{-8}\,e\eta\,(T_1^4-T_2^4)\ \text{W/m}^2 \qquad \text{...(ii)}$$

Here the term η has been included to take into consideration the effective value of emissivity.

We have $\quad \eta$ = radiating efficiency

= 1 for single elements and the value may go down to 0.5 for multiple element units

and $\quad e=0.9$ for heating elements.

From (i) and (ii), we have

$$\frac{d}{l^2}=\frac{4\,\rho q_{rad}}{V^2}\times10^{-3} \qquad \text{...(14.11)}$$

$$\text{But}\quad \frac{V^2}{P}=R=\frac{4\rho L}{\pi d^2}$$

or $$\frac{l}{d^3}=\frac{\pi}{4}\cdot\frac{V^2}{\rho P} \qquad ...(14\cdot12)$$

Using Eqns. 14·11 and 14·12, the length and diameter of wire can be calculated.

Design of ribbon elements. Ribbon wound resistance wires can be similarly designed, a typical example has already been given in Chapter 3. (See Example 3·4 page 40).

Example 14 6. *A 250 V, 1 0 kW single element electric furnace is to employ a nichrome resistance wire operating at 1000°C. Estimate a suitable diameter and length of wire. Take radiating efficiency =1, emissivity=0·9 and the resistivity of wire =0·424 Ω/m and mm² at 1000°C.*

The ambient temperature is 20°C.

Solution. Heat radiated $q_{rad}=5\cdot7\times10^{-8}\ e\eta\ (T_1^4-T_2^4)$

$=5\cdot7\times10^{-8}\times0\cdot9\times1\ [(1000+273)^4-(20+273)^4]$

$=135\times10^3$ W/m².

Total heat radiated$=q_{rad}\ (\pi dl\times10^{-6})=135\times10^{-3}\ (\pi\times dl\times10^{-3})$

$=424\ dl$

$=1000$ (given)

or $\qquad dl=2\cdot31 \qquad ...(i)$

Resistance $R \quad =V^2/P=(250)^2/1000=62\cdot5\ \Omega$.

But $$R=\frac{\rho l}{(\pi/4)d^2}=\frac{0\cdot424\,l}{(\pi/4)d^2}=0\cdot54\ \frac{l}{d^2}$$

$=62\cdot5\ \Omega$ (calculated above)

$\therefore \qquad l/d^2=115 \qquad ...(ii)$

Solving (*i*) and (*ii*), we have,

diameter of wire $d=0\cdot272$ mm and length of wire $l=8\cdot5$ m.

DESIGN OF CHOKES

14·6. Design procedure. The design of chokes is similar to design of small transformers as given in chapter 7.

Let $\qquad V=$voltage applied across the choke, V ;

$I=$current through the choke, A :

$Q=$volt ampere rating of choke$=VI$; $T_v=$turns per volt ;

$A_i=$area of core, m² ; and $A_w=$window area, m².

The value of turns per volt T_v is taken from Table 7·7.

It should be noted that the volume of copper in a choke is half of that of a corresponding transformer and therefore a higher value of T_v than that given in Table 7·7 can be assumed. Actually the choke can be thought of as a transformer with 1 : 1 turns ratio and with an output of $Q/2$. Thus the value of T_v corresponding to a volt-ampere rating of $Q/2$ should be taken from Table 7·3.

Flux $\qquad \Phi_m=1/4\cdot44\,f\,T_v$.

The value of maximum flux density is taken as about 1 Wb/m² for the core.

Net area of core $\qquad A_i=\Phi_m/B_m$.

Assuming a stacking factor of 0·9, gross core area,

$A_{gi}=A_i/0\cdot9=1\cdot11\ A_i$.

E—I, T—U or J type laminations are usually used.

A square section is normally used for the central limb as a square former is easy to manufacture and a square section gives a smaller length of mean turn as compared to the one given by rectangular section.

$\therefore$ Approximate width of central limb $A=\sqrt{A_{gi}}$.

Winding

Number of turns $T=V\times T_e$.

Area of wire $a=I/\delta$ where δ=current density.

The value of current density is about 2 5 A/mm^2.

Diameter of bare wire $d=\sqrt{(4/\pi)\,a}$.

Referring to Tables in Chapter 17, the nearest standard size of wire is selected Usually enamelled wire is used but for larger sizes cotton covered wire is used.

Let d_1=diameter of insulated wire.

$\therefore$ Space required in the window for the winding$=Ta/S_f$.

where S_f=space factor$=0{\cdot}8\,(d/d_1)^2$.

In order to accommodate former etc. this area is increased by about 20 percent.

$\therefore$ Total window area $A_w=1{\cdot}2\;Ta/S_f$.

Choice of Lamination

Referring to Tables 7·4, 7·5, 7·6, a standard size of laminations which gives the required width of lamination and the area of window is selected.

Impedance of Choke

Impedance of choke $Z=V/I=T/(T_eI)$ ...(14·13)

Now
$$I=\frac{\text{mmf required for iron parts}+\text{mmf required for air gaps}}{T}$$

$\therefore$
$$I=\frac{at_il_i+n_g\,at_g\,l_g}{T} \quad ...(14{\cdot}14)$$

where at_i=mmf (r.m.s.) per metre required for iron parts corresponding to flux density B_m.

(The mmf per metre corresponding to B_m can be read off from the relevant B—'at' curves given in chapter 4. This mmf should be divided by $\sqrt{2}$ to get the value of at_i)

$$at_g=800{,}000\;B_m/\sqrt{2} \quad ...(14{\cdot}15)$$

l_g=length of each air gap

n_g=number of air gaps in series in the flux path.

Hence
$$Z=\frac{T^2}{(at_i\,l_i+n_g\,at_g\,l_g)\,T_e} \quad ...(14{\cdot}16)$$

Usually the value of mmf required for iron parts is very small as compared with mmf required for air gaps and, therefore we can neglect $at_i\,l_i$, for practical purposes.

The value of length of l_g can be adjusted to get the required value of impedance.

When E—I, T—U or J type laminations are used, there are two air gaps in the magnetic path.

Example 14·7. *Design an iron cored choke to be connected across a 230 V, 50 Hz supply and to carry a current of 4 A.*

Solution.

Core

Apparent power $Q=VI=230\times4=920$ VA.

The choke may be thought of as a 1 : 1 transformer having a rating of $Q/2$ volt ampere.

Corresponding to $920/2=460$ VA, turns per volt $T_v=2$ (from Table 7·3).

Flux in the core $\quad \Phi_m=\dfrac{1}{4{\cdot}44\times50\times2}=2{\cdot}21\times10^{-3}$ Wb.

The maximum flux density in the core is assumed as 1·0 Wb/m².

$\therefore$ Net area of core $A_i=2{\cdot}21\times10^{-3}/1{\cdot}0\ \text{m}^2=2210\ \text{mm}^2$.

Gross area of core $\quad A_{gi}=2210/0{\cdot}9=2500\ \text{mm}^2$.

Taking a square section for the central limb, width of central limb, $A=50$ mm.

Winding

Number of turns $\quad T=V\times T_v=230\times2=460$.

Taking a current density of 2·3 A/mm² for the winding.

Area of conductor required $a=4/2{\cdot}3=1{\cdot}74\ \text{mm}^2$.

Required diameter of bare conductor $=1{\cdot}49$ mm.

Referring to Table 17·7 (Synthetic enamelled copper conductors) the nearest standard conductor has :

bare diameter $\quad d=1{\cdot}5$ mm,

diameter with covering $d_1=1{\cdot}605$ mm (medium covering).

Area of conductor provided $a=1{\cdot}76\ \text{mm}^2$.

Space factor $\quad S_f=0{\cdot}8\left(\dfrac{1{\cdot}5}{1{\cdot}605}\right)^2=0{\cdot}68.$

Window area required $A_w=1{\cdot}2\dfrac{T_a}{S_f}=\dfrac{1{\cdot}2\times460\times1{\cdot}76}{0{\cdot}68}=1420\ \text{mm}^2.$

Lamination Size

Using E—I laminations and referring to Table 7·4, we have to select a lamination size which has width of central limb 50 mm and a window area of 1420 mm² (minimum).

Using lamination No. 43 (See Fig. 7·4),

we have, $A=2''=50$ mm,

width of window $W_w=\dfrac{B-A-2D}{2}=\dfrac{6-2-2\times1}{2}=1''$,

height of window $H_w=C-2E=5-2\times1=3''$.

$\therefore$ Area window provided $A_w=3\times1=3$ square inches $=1925\ \text{mm}^2$.

Thus the lamination size selected suits the choke under consideration.

Impedance of Choke

From Eqn. 15·14, for $B_m=1$ Wb/m², we have,

$$at_g=\frac{800{,}000}{\sqrt{2}}B_m=\frac{800{,}000}{\sqrt{2}}\times1$$
$$=563{,}000\ \text{A/m}.$$

Number of air gaps in series in the flux path $n_g=2$.

∴ From Eqn. 14·16, impedance of choke

$$Z=\frac{T^2}{T_e\,(at_i\,l_i+n_g\,at_g\,l_g)}$$

Now $Z=230/4=57{\cdot}5\ \Omega$, $T=460$ and $T_e=2$.

Neglecting mmf required for iron parts, we have,

$$57{\cdot}5=\frac{(460)^2}{2(2\times563000\,l_g)}$$

Hence length of air gap $l_g=1{\cdot}61$ mm.

The impedance of coil is approximately equal to its reactance.

∴ Inductance of choke coil

$$L=\frac{X}{2\pi f}=\frac{57{\cdot}5}{2\pi\times50}\,\mathrm{H}=183\ \mathrm{mH}.$$

DESIGN OF WELDING TRANSFORMERS

14·7. Characteristics of welding transformers. Welding transformers may have different designs depending upon the nature of welding operation—arc, butt, seam or spot welding. The design of only arc welding transformers is being discussed in this text.

A welding transformer steps down the mains voltage to the value necessary for striking an arc, *i.e.* 55 to 65 V.

Welding transformers should satisfy the following requirements :

(*i*) They should have drooping voltage current characteristics.

(*ii*) The surge current during a short circuit (when striking an arc the transformer is short-circuited) should be limited to the least possible percentage above the rated current.

(*iii*) The open circuit voltage should not exceed about 80 V. This is required for the safety of the operator.

(*iv*) The current should be controllable continuously over a wide range.

14·7·1. Types of welding transformers. The dropping characteristics are obtained by the use of reactance in series with the arc.

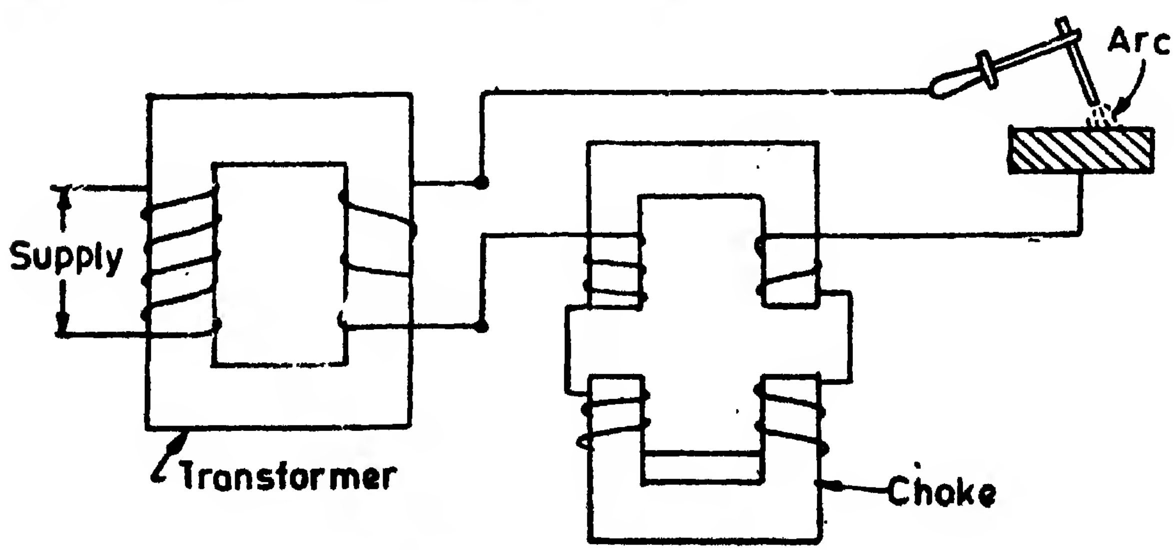


Fig. 14·4. Welding transformer.

The series reactance may be obtained in either of the two ways :

(*i*) A variable reactor may be inserted in the secondary circuit as shown in Fig. 14·4.

(*ii*) The transformer itself may be designed to have a high value of leakage reactance. This can be done by using a very wide window, and putting the primary and secondary windings on separate limbs. In order to increase the value of leakage flux, an additional central limb may be provided with an air gap.

The power factor of arc welding sets tends to be low owing to large value of reactance and therefore to correct the low power factor, shunt capacitors may be installed across the terminals of primary winding.

14·7·2. Electric arc. An electric arc is an electric discharge in gases, accompanied by high heat and a bright glow. For an arc to strike, a higher voltage is necessary than to maintain it. The voltage required to maintain an arc is given by :

$$V=C+Dl \qquad \text{...(14·17)}$$

where C=15 to 20 V,

D=2 to 3 V/mm of arc length,

l=length of arc in mm and is usually about 2 to 4 mm.

Thus the value of voltage to maintain an arc is usually about 30 V.

14·7·3. Series reactance

E_s=voltage at no load at the secondary terminals,

V=voltage across the arc,

I_s=rated welding current,

and X=reactance of secondary circuit.

Welding current I is in phase with voltage, V, across the arc

or $$I_sX=\sqrt{E_s^2-V^2}.$$

∴ Reactance required in the secondary circuit $$X=\frac{\sqrt{E_s^2-V^2}}{I_s} \qquad \text{...(14·18)}$$

This reactance may be obtained by incorporating it in the transformer itself or by using a series external reactor.

Example 14·8. *Design a 30 kVA, 230/60 V, 50 Hz, single phase welding transformer. Design its series reactor also.*

Solution.

Transformer design

Core

Core type construction is used,

Voltage per turn $E_t=K\sqrt{Q}=0·55\sqrt{30}\cong3$ V. (Taking $K=0·55$)

Flux $\Phi_m=\dfrac{E_t}{4·44\,f}=\dfrac{3}{4·44\times50}=13·51\times10^{-3}$ Wb.

Assume $B_m=1·3$ Wb/m²

Net iron area $A_i=\dfrac{13·51\times10^{-3}}{1·3}$ m²=10400 mm².

Gross core area A_{gi}=10400/0·9=11555 mm².

Taking a square section for the core.

Width of lamination $a=110$ mm and depth of core $D_y=110$ mm.

Radius of circumscribing circle $d=\sqrt{2}\times110=156$ mm.

Window Dimensions

The average current density for the two windings is taken as 2·6 A/mm².

From Eqn. 7·29, window space factor

$$K_w=\frac{10}{30+kV}=\frac{10}{30+0{\cdot}23}=0{\cdot}33.$$

From Eqn. 7·6, area of window

$$A_w=\frac{Q}{2{\cdot}22\,f\,B_m\,K_w\,\delta\,A_i\times10^{-3}}$$

$$=\frac{30}{2{\cdot}22\times50\times1{\cdot}3\times0{\cdot}33\times2{\cdot}6\times10^6\times10{\cdot}4\times10^{-3}\times10^{-3}}\ \text{m}^2=23300\ \text{mm}^2.$$

Taking height of window to be 3 times its width or $H_w=3W_w$.

Now $H_w\times W_w=22300$ or $3W_w^2=23300$.

$\therefore$ Width of window $W_w=90$ mm.

Height of window $H_w=23300/90\simeq260$ mm.

Windings

Primary winding

Number of turns in the primary winding $T_p=E_p/E_t=230/3=77$.

Current in primary winding $I_p=\dfrac{30\times1000}{230}=130$ A.

A current density of 2·2 A/mm² is taken for the primary winding as it is on the inside (near to the core).

Area of primary conductor $a_p=130/2{\cdot}2=59{\cdot}2$ mm².

The primary winding is helical type having two layers with a duct in between. The conductor section is chosen with reference to Table 17·1.

Secondary winding

Number of turns in the secondary winding $T_s=E_s/E_t=60/3=20$.

Current in the secondary winding $I_s=\dfrac{30\times1000}{60}=500$ A.

The secondary winding is on the outside and is a bare copper strip and therefore a current density of 3·2 A/mm² is used.

Area of secondary conductor $a_s=500/3{\cdot}2=141$ mm².

The size of the conductor can be chosen with reference to Table 17·1.

Reactor Design. The voltage across the arc is assumed to be 30V. Therefore, voltage drop in the reactance in the secondary circuit

$$I_s X=\sqrt{E_s^2-V^2}=\sqrt{60^2-30^2}=15{\cdot}7\ \text{V}.$$

Taking a drop of 1·7 V in the leakage reactance of transformer itself.

Voltage across the reactor$=51{\cdot}7-1{\cdot}7=50$ V.

$\therefore$ Rating of reactor$=50\times500\times10^{-3}=25$ kVA.

The value E_t for reactor is assumed to be 3·57 V.

$\therefore$ Flux in the reactor core $\Phi_m = \dfrac{3.57}{4.44 \times 50} = 16.1 \times 10^{-3}$ Wb.

The maximum value of flux density in the core is assumed as 1 2 Wb/m^2.

$\therefore$ Net iron area of core $A_i = 16.1 \times 10^{-3}/1.2$ m^2 $= 13450$ mm^2.

Gross core area $A_i = 13450/0.9 = 14400$ mm^2.

Taking a square section for the core :

$\therefore$ Width of core $= 120$ mm and depth of core $= 120$ mm.

Number of turns $= \dfrac{\text{voltage}}{\text{voltage per turn}} = \dfrac{50}{3.57} = 14.$

A core type of construction is used for the choke with 7 turns on each limb.

A current density of 3.5 A/mm^2 is used for the winding.

$\therefore$ Area of conductor $= 500/3.5 = 143$ mm^2.

The conductor used is bare copper strip. A proper size of conductor can be selected with reference to Table 17.1.

Window space factor is taken as 0.33.

$\therefore$ Area of window $A_w = 14 \times 143/0.33 = 6000$ mm^2.

The window dimensions are ·

Height $H_w = 110$ mm and width $W_w = 55$ mm.

Impedance of choke $= 50/500 = 0.1\ \Omega$,

From Eqn. 14.16, impedance of choke $Z = \dfrac{T^2}{T_e(n_g\, at_g\, l_g)}$

if mmf for iron parts is neglected.

or $$Z = \frac{T^2 E_t}{n_g\, at_g\, l_g} \quad \text{as} \quad T_e = \frac{1}{E_t}.$$

$\therefore$ Length of air gap

$$l_g = \frac{T^2 E}{n_g\, at_g\, Z} = \frac{14^2 \times 3.57}{2 \times 800{,}000 \times (1.2/\sqrt{2}) \times 0.1}\ \text{m} = 5.1\ \text{mm}.$$

DESIGN OF CURRENT TRANSFORMERS

14.8. Introduction. A current transformer is defined as a transformer for use with electrical measuring instruments and/or electrical protective devices for the transformation of current and in which the current in the secondary winding, in normal conditions of use, is substantially proportional to the current in the primary winding and differing from it by an angle which is approximately zero for an appropriate direction of connections.

Current transformers are subdivided into two main categories from consideration of their duty requirements, *viz* :

(*i*) *Measuring current transformers.* These transformers are used in measuring and indicating circuits.

(*ii*) *Protective current transformers.* These transformers are used in association with protection equipment like trip coils, relays etc.

There are considerable differences between the duty requirements of these two types and consequently the design approach relevant to one type cannot be used for the other type.

14.9. Errors. The ideal current transformer may be defined as one in which any primary condition is reproduced in the secondary circuit in exact ratio and phase relation.

ship. An alternate definition is that the ideal current transformer is one in which the primary mmf is exactly equal in magnitude to the secondary mmf, and, furthermore, is in precise phase opposition to it. The phasor diagram for such a transformer is shown in Fig. 14·5.

For an ideal transformer

$$I_p T_p = I_s T_s{}^*$$

or

$$\frac{I_p}{I_s} = \frac{T_s}{T_p}$$

Therefore, the ratio of primary and secondary winding currents is exactly equal to the turns ratio. Also, the primary and secondary winding currents are exactly 180° apart in phase or the phase difference between the secondary current reflected back on the primary side, and the primary current is zero.

However, in an actual transformer the windings have resistance and reactance and also the transformer has magnetizing and loss component of current to maintain the flux. Therefore, in an actual transformer the ratio of currents is not equal to the turns ratio and also there is a phase difference between the primary current and the secondary current reflected back on the primary side and consequently we have *ratio error* and *phase angle error*.

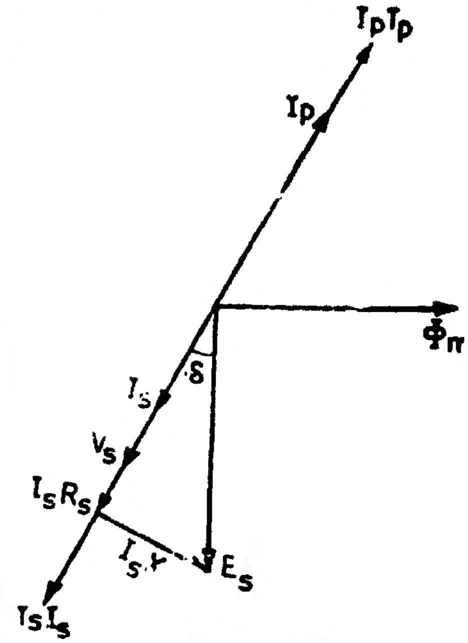


Fig. 14·5. Phasor diagram of an ideal current transformer.

Fig. 14·6 represents the general phasor diagram of an actual current transformer.

Let

K_n = turns ratio

$= \dfrac{\text{number of secondary winding turns}}{\text{number of primary winding turns}}$

= **nominal ratio,

r_s, x_s = resistance and reactance respectively of the secondary winding,

r_e, x_e = resistance and reactance respectively of external burden *i.e.*, resistance of meters current coils etc. including leads,

E_p, E_s = primary and secondary winding induced voltages respectively,

T_p, T_s = number of primary and secondary winding turns respectively,

V_s = voltage at the secondary winding terminals,

I_p, I_s = primary and secondary winding currents respectively,

θ = phase angle of transformer,

Φ_m = working flux of the transformer,

δ = angle between secondary induced voltage and secondary current,

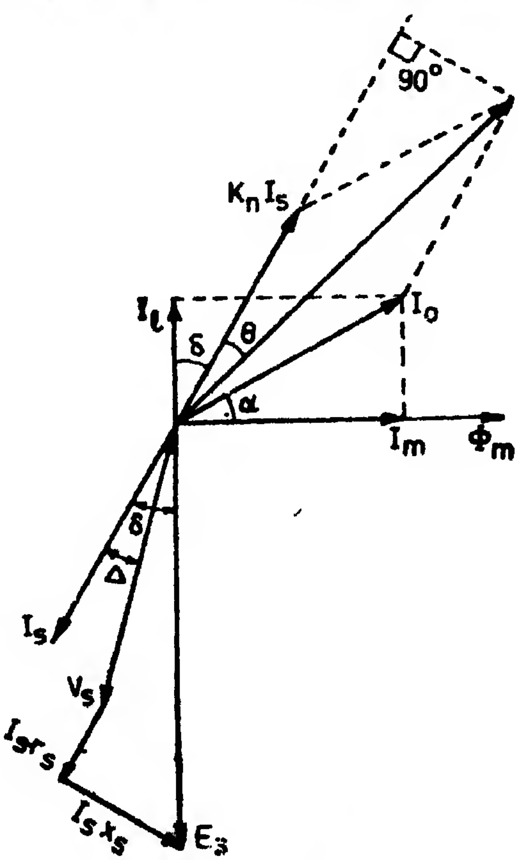


Fig. 14·6. Phasor diagram of an actual current transformer.

*The terms are defined below.

**The nominal ratio is equal to the turns ratio in transformers designed without turns compensation.

$=$phase angle of total burden including impedance of secondary winding

$=\tan^{-1}\left(\dfrac{x_s+x_e}{r_s+r_e}\right)$,

$\Delta=$phase angle of secondary load circuit *i.e.*, of external burden

$=\tan^{-1}\dfrac{x_e}{r_e}$

$I_0=$exciting current,

$I_m=$magnetising component of exciting current,

$I_l=$loss component of exciting current,

$\alpha=$angle between exciting current I_0 and working flux Φ_m.

It can be shown that :

Actual transformation ratio

$$R=\frac{I_p}{I_s}=K_n+\frac{I_l\cos\delta+I_m\sin\delta}{K_n I_s} \qquad \ldots(14.19)$$

$$\text{Phase angle } \theta=\frac{180}{\pi}\left[\frac{I_m\cos\delta-I_l\sin\delta}{K_n I_s}\right] \text{ degrees (electrical)} \qquad \ldots(14.20)$$

$$\text{Ratio error}=\left(\frac{K_n I_s-I_p}{I_p}\right)\times 100 \text{ per cent}$$

$$=\left(\frac{K_n-R}{R}\right)\times 100 \text{ per cent} \qquad \ldots(14.21)$$

The phase angle error is given by Eqn. 14.19.

14.10. Construction. The current transformers may be classified as :

(i) ***Wound Type.*** A current transformer having a primary winding of more than one full turn wound on core.

(ii) ***Bar Type.*** A current transformer in which the primary winding consists of a bar of suitable size and material forming an integral part of transformer.

Figs. 14.10 and 14.11 show wound type and bar type transformers respectively.

The simplest form any current transformer can take is the ring type or window type, examples of which are given in Fig. 14.12 which shows three commonly used shapes *i.e.*, stadium, circular and rectangular orifices. The core, if of a nickel-iron alloy or an oriented electrical steel is almost certainly of the continuously wound type. But current transformers using hot rolled steels consist of stack of ring laminations. Before putting secondary winding on the core, the latter is insulated by means of end collars and circumferential wraps of elephantide or presspahn. These pressboards, in addition to acting as insulating medium, must also protect the secondary winding conductor from mechanical damage due to sharp corners. The secondary winding conductor is put on the core by a toroidal winding machine although hand winding is still frequently adopted if the number of secondary winding turns is small.

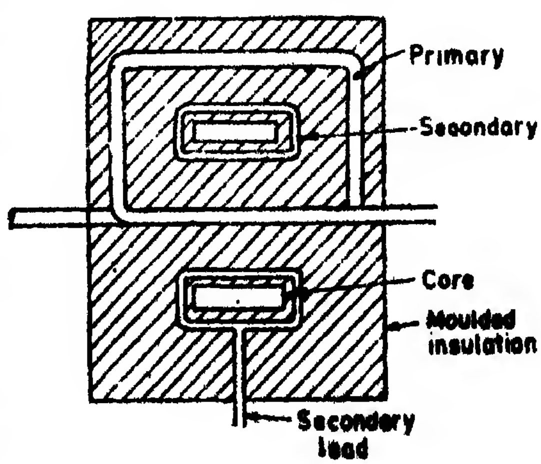


Fig. 14.10. Wound type current transformer.

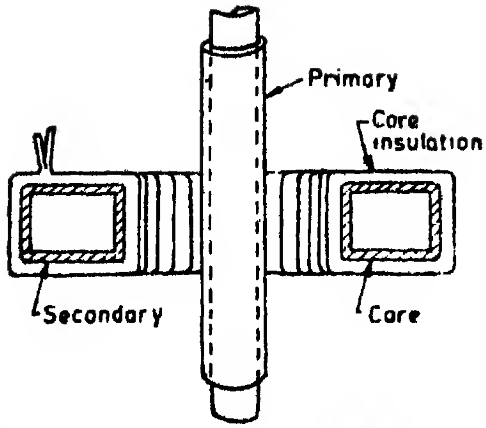
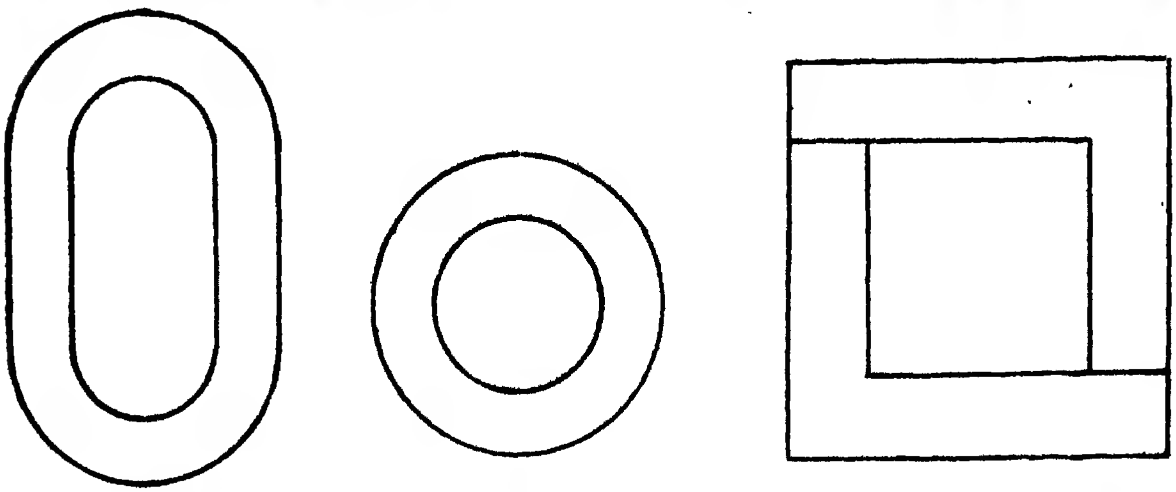


Fig. 14·11. Bar type current transformer.

After the secondary winding has been placed on the core, the ring type transformer is completed by exterior taping with or without first applying circumferential insulating wraps.



(a) Stadium (b) Ring (c) Rectangular

Fig. 14·12. Laminations for window type current transformers.

A near relative to the ring type current transformer is the so called *"bushing type"* transformer. This is, in fact, indistinguishable from the ordinary ring type but the term is used when the current transformer fits over a fully insulated primary conductor such as over the oil end of a terminal bushing of a power transformer or an oil circuit breaker.

In a split core current transformer the core is split, each half having two finely ground or lapped gap faces. These current transformers are assembled on to the primary conductor "on site" for either permanent or temporary duty.

In a bar type current transformer, the core and secondary windings are the same as in a ring type transformer but the fully insulated bar conductor constituting the single turn primary is now an integral part of the current transformer. The insulation on the primary conductor may be a bakelized paper tube or a resin directly moulded on the bar.

In a low voltage wound type current transformer the secondary winding is wound on a bakelite former or bobbin and the heavy primary conductor is either wound directly on top of secondary with suitable insulation being first applied over the secondary winding or the primary is wound entirely separately, taped with suitable insulating material and then assembled with the secondary winding on the core.

In the manufacture of current transformers the assembly on lamination stacks demands somewhat greater care than ordinary transformers in order to keep down the reluctance of the interleaved corners as low as possible and hence to minimize the magnetizing current. Sometimes cut cores are used.

Whenever possible secondary windings should utilize the whole available winding length on the core, the secondary turns being suitably spaced to accomplish this and the insulation between secondary winding and core earth must be capable of withstanding the high peak voltages caused if the secondary winding is open circuited when primary current is flowing. In the case of a large number of secondary turns, requiring more than one winding layer, the frequently adopted technique is to sectionalize the secondary winding to considerably reduce the peak voltage between layers.

With wound primary current transformers this particular problem is rarely met but it is of importance to try to obtain good relative positioning of primary and secondary coils, thus minimizing the axial forces on both coils caused by primary short circuit currents.

14·11. Design principles

14·11·1. Core

In order to minimize the error, the exciting mmf be kept to a low value. This means that the core must have a low reluctance and low iron loss. A protective current transformer may well operate over a working range of flux densities extending from the ankle point to the knee point (See Fig. 14 13) while the measuring current transformer frequently has a flux density in the region of ankle point only (not greater than 0·1 Wb/m²). Present day magnetic alloys used in current transformers are conveniently divided into three categories :

(*i*) hot rolled silicon steel

(*ii*) cold rolled grain oriented silicon steel

(*iii*) nickel-iron alloys.

Mumetal cores are now commonly used when it is essential that transformer errors shall be small. Mumetal has the properties of high permeability, low loss and small retentivity, all of which are advantageous in current transformer work. But it has the disadvantage that if its maximum relative permeability (90,000) occurs with a flux density of only 0·35 Wb/m² as compared with maximum relative permeability or silicon steel (4500) occuring at a flux density of about 0·5 Wb/m². Mumetal is very costly also. Permender has a very high saturation point of 2—4 Wb/m² compared with 0·7 to 0·8 Wb/m² of other high permeability alloys.

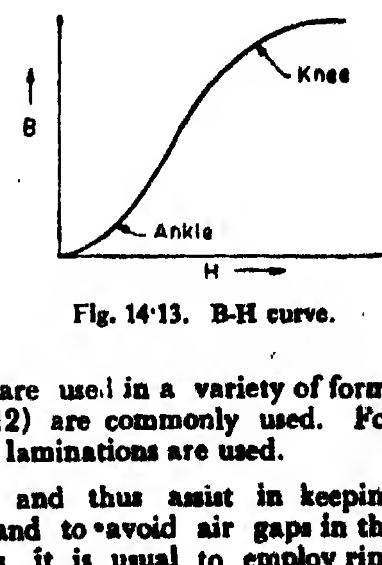


Fig. 14·13. B-H curve.

In current transformer practice hot rolled silicon steels are used in a variety of forms for ring type current transformers 'ring' stampings (Fig. 14 12) are commonly used. For wound-primary type current transformers T—U, L or E and I laminations are used.

To minimize secondary winding leakage reactance and thus assist in keeping secondary circuit impedance to as low levels as possible, and to avoid air gaps in the magnetic circuit which would lead to higher exciting currents, it is usual to employ ring shaped cores around which toroidal secondary windings of one or more layers are uniformly distributed. Cores generally consist of either a cylindrical stack of laminations as shown in Fig. 14·14 or they are made up of strips wound in spiral form, like a clock spring as shown in Fig. 14·15. The latter method is much to be preferred when grain oriented magnetic materials are being used, as it ensures that the flux is able to follow the path of minimum possible reluctance.

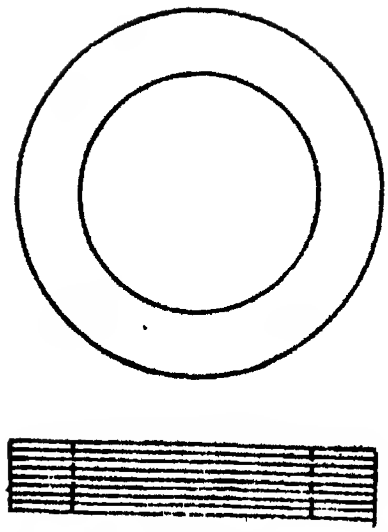


Fig. 14·14. Cylindrical stack.

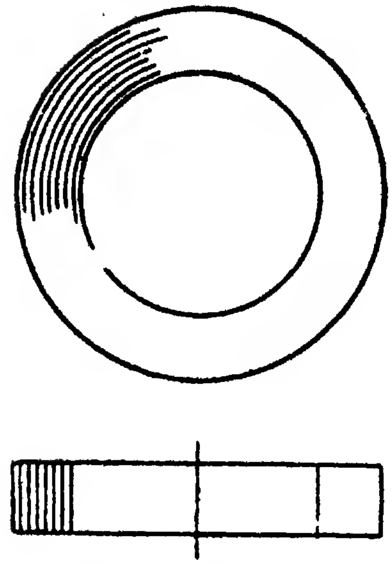


Fig. 14·15. Spiral core.

The ring—core type of construction is relatively expensive and consequently when a high standard of performance is not required, cores made up of rectangular strips or C—shaped sections are sometimes employed. The length of magnetic path in core should be as small as is consistent with good mechanical construction and with insulation requirements, and in order to reduce core reluctance. For the same reason, joints in the core should be avoided as far as possible, as such joints as do exist should be rendered efficient as possible by careful assembly.

The ring type of construction is a robust construction and has the further advantage of jointless core.

14·11·2 Secondary current rating.

The value of rated secondary current is 5 A. The secondary current ratings of 2 and 1 A may also be used in some cases, as for example :

(a) if the number of secondary turns is so low that the ratio cannot be adjusted within the requisite limits by addition or removal of one turn,

(b) if the length of secondary connecting lead is such that the burden due to them at higher secondary current would be excessive.

The disadvantage of making transformers with lower secondary current ratings are that they produce much higher voltages if they are ever accidentally left with their secondary windings open circuited and they are more costly to produce because of extra time involved in winding the increased secondary winding turns. For these reasons, it is better to adopt 5 A rating for those applications where the interconnecting leads are short. This is normally the situation in power installations used in laboratory, or in relatively low current transformers where relays and instruments are often mounted on the switchgear.

14·11·3. Primary current rating.

Whatever equipment a current transformer is feeding, it is desirable that the ratio of excitation mmf to primary mmf should be low. It is difficult to achieve this condition if the latter quantity is small and an improvement in performance is always obtained at a given current level if the primary mmf is increased. Satisfactory results can usually be achieved if the primary mmf at rated current of transformer is at least 500 A, and thus transformers

with a rated current of 500 A or more almost invariably have a single turn primary winding. Transformers for ratings below 500 A are, where possible, provided with multiturn primary windings, if this enables the core size needed for particular duty to be significantly decreased.

The rated primary currents (IS 2705—1964) are given in Table 14·3.

Table 14·3. Rated primary current

A	*A*	*A*	*A*
0·5	0 0	100	1000
1·0	12·5	125	1250
2·5	15·0	150	1500
5·0	20·0	200	2000
	25·0	250	2500
	30·0	300	3000
	40·0	400	4 00
	50·0	500	5000
	60·0	600	6000
	75·0	750	7500
		800	10,000

14·11·4. Windings.

The windings should be close together to reduce the secondary leakage reactance as this increases the ratio error. Round copper wire of about 3 mm² is frequently used for the secondary windings rated at 5 A. Copper strip is used for primary winding, the dimensions of which depend upon the primary current.

The current density in windings is about 1 to 2 A/mm². When using bar primary, the external diameter of the tube must be large enough to keep the electric stress, in the dielectric at its surface, to an acceptable value.

The windings must be designed with a view to withstand, without damage, the large short circuit forces that are caused when a short circuit takes place on the system in which the current transformer is connected.

The windings are separately wound, and are insulated by tape and varnish for small line voltages. For voltages above 7 kV the transformers are oil immersed or compound filled.

14·12. Behaviour of transformer under system short circuit.

When a short circuit occurs on the system, the initial value of current flowing through the primary rises to a value

$$i=2\sqrt{2}\,\frac{V_{ph}}{\epsilon_s}I_p$$

where V_{ph}=system voltage per phase,

ϵ_s=p.u. impedance of system,

I_p=rated primary current.

If the associated circuit breaker is rated for a breaking capacity of x ampere r.m.s. short circuit current, the current transformers must be designed to withstand the effects of a current

$$i=2\sqrt{2}\,x\,\frac{V_{ph}}{\epsilon_s}I_p.$$

There are three aspects which are to be considered when a short circuit occurs on the system :

(*i*) *Temperature rise*

The temperature rise under short circuit conditions is given by Eqn. 7·101 on page 432

$$\theta = at\left[\frac{2\,T_1+at}{2\,T_1}+\frac{620\,K_e}{2T_1+at}\right]°C$$

The temperature rise should not be more than 200°C.

(*ii*) *Current density*

The current density in the primary winding should not exceed the values given in Table 14·4.

Table 14·4. Current density under short circuit conditions

Rated time s	*Current density* A/mm²
0·5	235
1·0	165
2·0	115
3·0	95

(*iii*) *Mechanical forces*

The short circuit forces in a wound primary current transformer are :

(*a*) radial force on primary coil tending to burst it or in other words if the primary winding is not of circular shape, the forces try to make it circular,

(*b*) radial force on secondary winding tending to compress it,

(*c*) axial forces on both primary and secondary winding and these are compressive.

The effect of (*a*) and (*b*) can be minimized by making the primary and secondary coils as circular in shape as possible. The primary conductor should be wound strip on edge.

The dissymmetry between primary and secondary windings, both in axial and radial directions, should be avoided.

The radial force is given by Eqn. 7·83 on page 414.

$$F_r = \frac{\mu_0}{2}\,(iT)^2\,\frac{L_{mt}}{L_c}\ \text{N.}$$

This gives rise to a peripheral force $F_p = \dfrac{F_r}{2\pi}$

$$\therefore\quad F_p = \frac{\mu_0}{4\pi}\,(iT)^2\frac{L_{mt}}{L_c}\ \text{N}$$

Hoop stress

$$P_p = \frac{F_p}{S_f\,b_p\,L_c} = \frac{\mu_0}{4\pi}\,(iT)^2\frac{L_{mt}}{S_f\,b_p\,L_c^2}\ \text{N/m}^2$$

where S_f = primary space factor.

The value of hoop stress P_p must be less than the elastic limit of primary conductor.

14 13. Turns compensation

Turns compensation is used in current transformers to reduce ratio error. If the phase angle of secondary δ is zero, the actual ratio

$$R = K_n + \frac{I_l}{I_s} \text{(See Eqn. 14·19 for } \delta=0)$$

The effect of reducing the number of secondary turns, by say 1 percent, will obviously be to reduce actual transformation ratio by an equal percentage. By this means, partial compensation for the effect of I_l in increasing the ratio, can be obtained. Usually the best number of secondary turns is 1 or 2 less than the number which will make K_n equal to the nominal current ratio of the transformer. For example in a 1000/5, current transformer with bar winding the number of secondary turns would be either 198 or 199 instead of 200. The phase angle error is not significantly effected by a small change in secondary turns.

Example 14·9. *Design a 200/5 A, 50 Hz current transformer for a watt hour meter. The primary is connected to a 6600 V network. The other specifications are :*

External Burden VA=5 at 0·85 power factor.

The permissible errors are (IS 2705 Pt II—1964)

(for any value of secondary burden between 25 percent to 100 percent.)

$$\textit{Current}\ldots\ldots\ldots\ldots\begin{bmatrix}\textit{Ratio}-\pm 0{\cdot}5 \textit{ percent}\\ \textit{Phase angle}-30 \textit{ minutes}\end{bmatrix}$$

Short circuit current--50 times rated current.

Duration of short circuit=1 s.

Solution.

Winding turns

The primary mmf $I_p T_p$ is assumed as 1000 A.

$\therefore$ Primary turns $T_p = \frac{I_p F_p}{I_p} = \frac{1000}{200} = 5.$

Secondary current $T_s = 5$ A

$\therefore$ Secondary turns $T_s = \frac{I_p}{I_s} T_p = \frac{200}{5} \times 5 = 200.$

Core

The total flux in the core will depend upon the total voltage to be generated in the secondary winding and not only on the external burden. We assume a total burden (including secondary winding and connecting leads) to be 15 VA.

$\therefore$ $E_s I_s = 15$

or secondary induced voltage $E_s = 15/5 = 3$V.

Flux in the core $\Phi_m = \frac{3}{4{\cdot}44 \times 50 \times 200} = 67{\cdot}5 \times 10^{-6}$ Wb.

Using a flux density of 0·05 Wb/m².

Net area of core $A_i = 67{\cdot}5 \times 10^{-6}/0{\cdot}05$ m² $= 1350$ mm².

Gross core area $A_{gi} = 1350/0{\cdot}9 = 1500$ mm².

Taking a square section for the core with 40 mm side.

Gross core area provided $A_{gi} = 1600$ mm² and net core area provided $A_i = 1440$ mm².

Flux density in core $B_m = \frac{67{\cdot}5 \times 10^{-6}}{1440 \times 10^{-6}} = 0{\cdot}047$ Wb/m².

Window dimensions

From Eqn. 7 29, window space factor

$$K_w = \frac{8}{30+kV} = \frac{8}{30+6\,6} = 0\,22.$$

Taking a current density 2 A/mm²

$$\text{Area of window } A_w = \frac{2\,a_p\,T_p}{K_w} = \frac{2\,(I_p/\delta_p)\,T_p}{K_w}$$

$$= 2\times\frac{200}{2\times10^6}\times\frac{5}{0{\cdot}22}\,\text{m}^2 = 4540\ \text{mm}^2.$$

Taking ratio $H_w/W_w = 2$, we have

∴ width of window $W_w = 50$ mm and height of window $= 10$ mm.

Windings

Both primary and secondary windings are put on the same limb as shown in Fig 14·6.

Secondary winding

Taking a current density of 1·7 A/mm².

Area of secondary winding conductor $a_s = 5/1{\cdot}7 = 2{\cdot}94$ mm².

Diameter of bare conductor required $= 1{\cdot}93$ mm.

From Table 17·5, the nearest standard conductor has :

	bare diameter	$= 1{\cdot}9$ mm
and	insulated diameter	$= 2{\cdot}045$ mm.

Area of secondary conductor used $a_s = 2{\cdot}83$ mm².

Current density in secondary winding $\delta_s = 5/2{\cdot}83 = 1{\cdot}94$ A/mm².

There are 200 secondary winding turns. Arranging the winding in 5 layers with 40 turns per layer.

Height of secondary winding $L_{ws} = 40\times2{\cdot}045 \cong 82$ mm.

This leaves a space of 9 mm on each side between core and secondary winding.

Using 0·125 mm thick paper between layers.

Width of secondary winding $b_s = 5\times2{\cdot}045+4\times0{\cdot}125 = 10{\cdot}72$ mm.

Allowing for bulging the width of secondary winding $b_s = 12{\cdot}0$ mm.

Primary winding

Taking a current density of 2 A/mm².

Area of each conductor $a_p = 200/2 = 100$ mm².

As we have to insulate primary winding for 6600 V, we shall assume a winding height of about 70 mm leaving 15 mm on each side between winding and core.

Height taken up by each turn $= 75/5 = 15$ mm

Using 2 strips 13×4 mm in parallel.

From Table 17·1, area of 13×4 mm strip is 5·14 mm²

Area of primary conductor used $a_p = 2\times51{\cdot}4 = 102{\cdot}8$ mm².

Current density in primary winding $\delta_p = 200/102{\cdot}8 = 1{\cdot}94$ A/mm².

The conductor is covered with paper

The insulated conductor is 13·5×4·5 mm in size.

Height of primary coil $= 5\times13{\cdot}5 = 67{\cdot}5$ mm :

with allowance for bulging etc. $L_{sp}=70$ mm.
Width of primary winding conductor$=2\times4{\cdot}5=9$ mm,
with allowance for bulging $b_p=10$ mm.

Check on window dimensions

As a check on width of window, we calculate the width of coils.

Insulation on core and clearance	=3·0 mm
Width of secondary winding	=12·0 mm
Insulation between primary and secondary windings	=4·0 mm
Width of primary winding	=10·0 mm
Insulation between primary and core	=5·0 mm
Total	=34·0 mm.

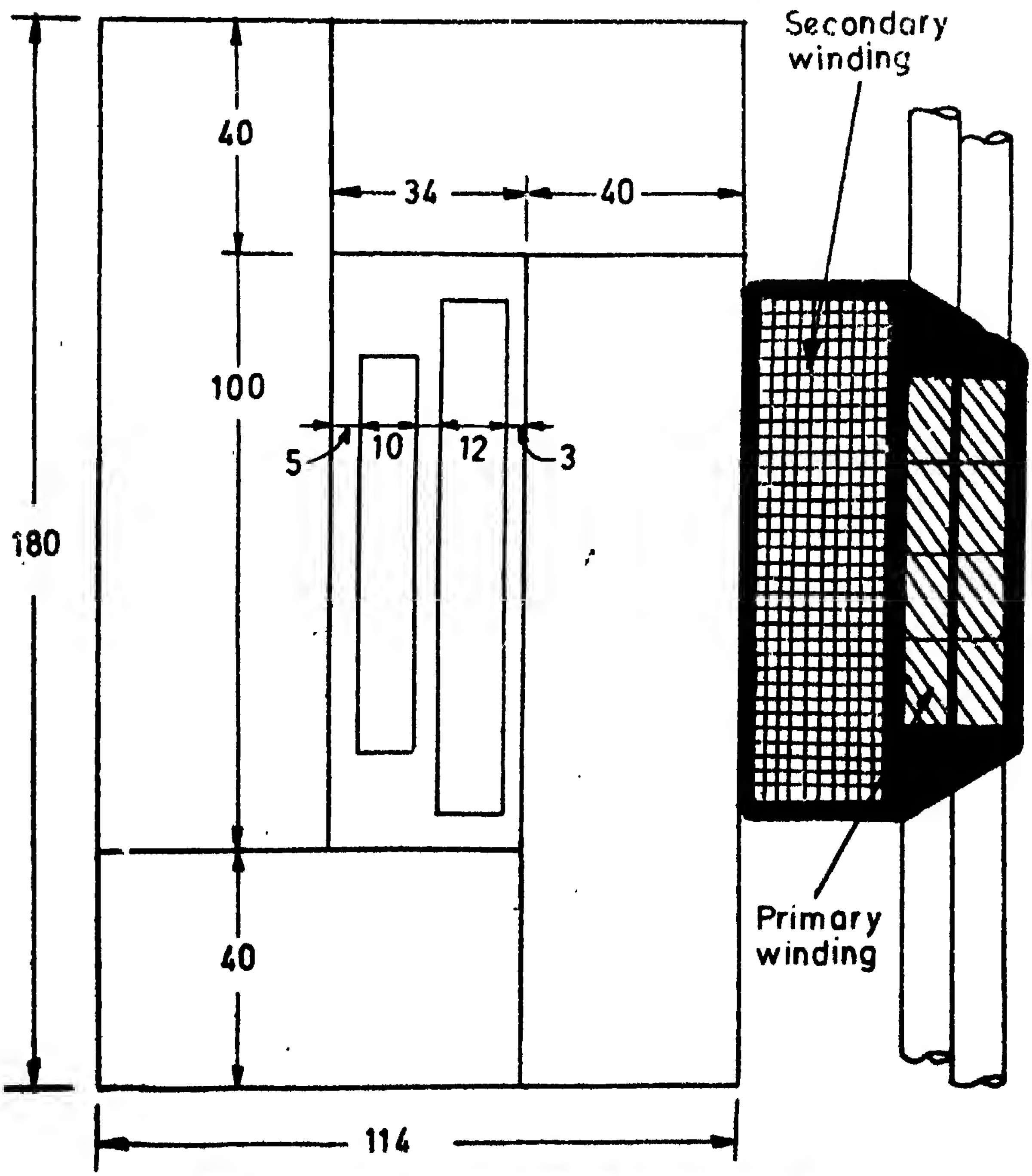


Fig. 14·16. Details of current transformer core and windings.

With width of window equal to 34 mm, there is enough space for accomodation of windings. The reason why this dimension is appreciably less than the estimated width of 50 mm is that the windings are all on one limb instead of being on both limbs as in power transformers and also solid insulation has been used and there are no ducts for oil circulation.

Secondary winding resistance

Length of mean turn of secondary winding

$$L_{mts}=4\times[40+2\times3+12]=232 \text{ mm}.$$

Resistance of secondary winding at 75°C

$$r_s=\frac{200\times0{\cdot}021\times0{\cdot}232}{2{\cdot}83}=0{\cdot}344\ \Omega.$$

Resistive drop in secondary winding $=I_s r_s=5\times0{\cdot}342=1{\cdot}71$ V.

The total internal IR drop of secondary winding plus leads etc. may be taken as 1·9 V.

Total burden

In order to calculate the total burden (internal and external), we calculate the burden of instrument.

VA rating of instrument $=5$

$\therefore$ IZ drop in instrument $=VA/I_s=5/5=1$ V.

IR drop in instrument $=(IZ)\times$ p.f. $=1\times0{\cdot}85=0{\cdot}85$ V.

IX drop in instrument $=1\times\sqrt{1-(0{\cdot}85)^2}=0{\cdot}527$ V

If we neglect the leakage reactance of the secondary winding,

total IR drop $=IR$ drop in (secondary winding+leads+instrument)
$=1{\cdot}9+0{\cdot}85=2{\cdot}75$ V

total IX drop $=0{\cdot}527$ V.

From Fig. 14·16,

$$\tan\delta=\frac{\text{total } IX \text{ drop}}{\text{total } IR \text{ drop}}=\frac{0{\cdot}527}{2{\cdot}75}=0{\cdot}192.$$

$\therefore$ $\delta=10°\ 54'$ with $\cos\delta=0{\cdot}982$ and $\sin\delta=0{\cdot}189$.

$\therefore$ Total emf of secondary winding

$$E_s=\frac{\text{total } IR \text{ drop}}{\cos\delta}=\frac{2{\cdot}75}{0{\cdot}982}=2{\cdot}8 \text{ V}.$$

$\therefore$ Actual secondary burden $=2{\cdot}8\times5=14$ VA.

Actual flux density in core $B_m=\dfrac{2{\cdot}8}{4{\cdot}44\times50\times14{\cdot}4\times10^{-4}\times200}=0{\cdot}0438$ Wb/m^2.

Magnetizing Current

Mean length of flux path $=2(100+34+40)=348$ mm.

Using 92 Grade steel, (See Fig. 4·3) mmf $=7$A/m (estimated)

$\therefore$ Magnetizing mmf $=7\times0{\cdot}348=2{\cdot}44$ A

and magnetizing current $I_m=\dfrac{2{\cdot}44}{\sqrt{2}\times5}=0{\cdot}345$ A.

Loss Component

Weight of core $=348\times40\times40\times10^{-9}\times7{\cdot}8\times10^3=4{\cdot}35$ kg.

The specific iron loss can be estimated from Fig. 4·29 by assuming that the loss is proportional to B_m^2.

From Fig. 4·29, loss corresponding to $B_m=1$ Wb/m^2 is 1·2 W/kg.

$\therefore$ Loss per kg for $B_m=0{\cdot}0438$ Wb/m^2 is $1{\cdot}2\times(0{\cdot}0438)^2$ W.

$\therefore$ Total core loss$=4{\cdot}35\times1{\cdot}2\times(0{\cdot}0438)^2=0{\cdot}01$ W.

$$\text{Loss component of current } I_l=\frac{T_s}{T_p}\times\frac{\text{iron loss}}{E_s}$$

$$=\frac{200}{5}\times\frac{0{\cdot}01}{2{\cdot}8}=0{\cdot}143 \text{ A.}$$

Errors at rated Burden :

From Eqn. 14 19, actual current ratio,

$$R=K_n+\frac{I_l\cos\delta+I_m\sin\delta}{I_s}$$

$$=40+\frac{0{\cdot}143\times0{\cdot}982+0{\cdot}345\times0{\cdot}189}{5}$$

$$=40{\cdot}041.$$

From Eqn. 14·21,

$$\text{Ratio error}=\left(\frac{K_n-R}{R}\right)\times100=0{\cdot}1025\%$$

which is well within the limits.

From Eqn. 14·20,

$$\text{phase angle}=\frac{180}{\pi}\left[\frac{I_m\cos\delta-I_l\sin\delta}{K_n I_s}\right]\text{ degrees}$$

$$=\frac{180}{\pi}\left[\frac{0{\cdot}345\times0{\cdot}982-0{\cdot}143\times0{\cdot}189}{40\times5}\right]\times60\text{ minutes}$$

$$=5{\cdot}33 \text{ minutes.}$$

This is well within the limits.

Similar calculations can be done for 25 percent burden.

Flux density at 25 percent burden$=0{\cdot}0438/4=0{\cdot}01095$ Wb/m^2.

For this flux density, at $=3$ A/m

$\therefore$ Magnetising mmf $=3\times0{\cdot}348=1{\cdot}044$ A

and $$I_m=\frac{1{\cdot}024}{\sqrt{2}\times5}=0{\cdot}148 \text{ A.}$$

Total core loss$=(1/4)^2\times0{\cdot}01=0{\cdot}625\times10^{-3}$ W.

$$\text{Loss component } I_l=\frac{200}{5}\times\frac{0{\cdot}625\times10^{-3}}{2{\cdot}8/4}=0{\cdot}036 \text{ A.}$$

$$\therefore\quad R=40+\frac{0{\cdot}036\times0{\cdot}982+0{\cdot}148\times0{\cdot}189}{5/4}=40{\cdot}004$$

$$\text{Ratio error}=\frac{40-40{\cdot}004}{40{\cdot}004}\times100=0{\cdot}01 \text{ percent (within limits)}$$

$$\text{Phase angle } \theta=\frac{180}{\pi}\left[\frac{0{\cdot}148\times0{\cdot}982-0{\cdot}036\times0{\cdot}189}{40\times5/4}\right]\times60$$

$$=7{\cdot}4 \text{ minutes (within limits).}$$

Temperature rise under short circuit :

The short circuit current of system on which this current transformer is connected is 50 times the rated value and it continues for 1 s

∴ Short circuit current$=50\times200=10{,}000$ A.

Current density in primary winding conductor

$$=\frac{10{,}000}{102\cdot8}=97\cdot3\ \text{A/mm}^2.$$

This is below the allowable value of 165 A/mm².

From Eqn. 7·101 on page 432

Temperature rise under short circuit conditions

$$\theta=at\left[\frac{2T_1+at}{2T_1}+\frac{620\ K_e}{2T_1+at}\right]$$

$$=61\cdot5\times1.\left[\frac{2\times340+61\cdot5\times1}{2\times340}+\frac{620\times1\cdot3}{2\times340+61\cdot5\times1}\right]$$

$$\cong140^\circ\text{C}.$$

This is below the allowable limit of 200°C. For the above relationship, we have,

$t=1$ s,

$T_1=\theta_1+235=105+235=340^\circ$C,

$K_e=1\cdot3$ (assumed),

$a=1\cdot9\ \delta^2\ T_1\times10^{-5}=1\cdot9\times(97\cdot5)^2\times3{\cdot}0\times10^{-5}=61\cdot5$

DESIGN OF PERMANENT MAGNETS

14·14. Permanent magnet materials The materials used for the manufacture of permanent magnets are hard magnetic material *i.e*, materials with a broad hysteresis loop.

Two general groups of permanent magnet materials are the "old type" such as carbon steel ond other steel alloys that contain chromium, tungsten and cobalt and the "new type" such as aluminum nickel cobalt alloys called Alnico which are extremely hard and must be produced by sintering and the ceramic magnets of which barium ferrite is the chief constituent.

Table 14·5 gives the data for some materials used for permanent magnets.

Table 14·5. Permanent magnet materials.

Material	*Remanence Wb/m²*	*Co-ercive force A/m*	*Value of B for $(BH)_{max}$*	*Value of H for $(BH)_{max}$*	$(BH)_{max}$
1% Carbon Steel	0·9	4000	0·6	2600	1560
6% Tungten Steel	1·05	5200	0·7	3750	2650
35% Cobalt Steel	0·9	20000	0·6	13000	7800
Chromium Steel	0·96	6500	—	—	—
Alnico	0·8	40000	0·56	24000	13500
Allni	0·55	46000	0·36	28000	10000
Alcomax III	1·25	54000	0·95	40000	38000
Alcomax IV	1·1	60000	0·8	43000	34400

14.15. Design procedure. The design of a permanent magnet to give minimum volume is given below.

Let Φ=flux in air gap, Wb ; $\mathbf{R}$=reluctance of air gap=$l_g/(\mu_0 A_g)$

l_g=length of air gap, m ; A_g=area of air gap, m^2 ;

l_p=length of magnet, m ; A_p=cross section of magnet m^2 ;

C_l=leakage co-efficient.

Flux in the magnet=$C_l\ \Phi$

The value of leakage co-efficient C_l is likely to be large and ranges from 1·5 to 10. The value cannot be easily calculated and therefore it has to be approximated and later on confirmed by practical tests.

Let B_p be the value of working flux density in the magnet

$\therefore$ Area of magnet $A_p=C_l\ \Phi/B_p$ ...(14·22)

The mmf required for air gap is $\Phi\mathbf{R}$ and the total mmf required will be larger than this as some mmf is consumed in the iron parts. This additional mmf is usually 10 to 30 percent of mmf required for air gap.

$\therefore$ Mmf produced by magnet=(1·1 to 1·3) $\Phi\mathbf{R}$.

Let H_p be the mmf produced by magnet per metre.

$$\text{Length of magnet } l_p=\frac{(1\cdot1 \text{ to } 1\cdot3)\ \Phi\mathbf{R}}{H_p} \quad ...(14\cdot23)$$

$$\text{Volume of magnet}=A_p\ l_p=\frac{(1\cdot1 \text{ to } 1\cdot3)\ C_l\ \Phi\mathbf{R}}{B_p\ H_p} \quad ...(14\cdot24)$$

The volume will be minimum when the product $B_pH_p=(BH)_{max}$ is a maximum. Table 14·5 gives the values of $(BH)_{max}$ and the corresponding values of B_p and H_p for different magnet materials.

Example 14.10. *Design a permanent magnet to have minimum volume. The magnet is to produce a flux density of 0·3 Wb/m², in a parallel sided air gap 60 mm square and 6 mm long. The demagnetization curve of the magnet material is given below :*

B Wb/m^2	0	0·2	0·4	0·6	0·8	1·0	1·1
H A/m	60,000	58,000	55,500	51,000	43,000	24,000	0

Solution. The demagnetization curve is plotted in Fig. 14·17. The BH product is calculated from the data given and is plotted in Fig. 14·17.

From Fig. 14·17, we find that $(BH)_{max}$ occurs with

$$B=0\cdot8\ \text{Wb/m}^2 \text{ and } H=43{,}000\ \text{A}.$$

The leakage coefficient is assumed as 2 and the mmf for iron parts is taken as 20 percent of air gap mmf.

Air gap flux $\Phi=B_gA_g=0\cdot3\times(3\cdot05)^2=0\cdot75\times10^{-3}$ Wb.

$$\text{Area of magnet } A_p=\frac{C_l\ \Phi}{B_p}=\frac{2\times0\cdot75\times10^{-3}}{0\cdot8}\ \text{m}=1875\ \text{mm}^2.$$

Taking a square section, each side of magnet=43 mm.

Mmf required for air gap=800,000 B_gl_g=800,000×0·3×6×10^{-3}

=1440 A.

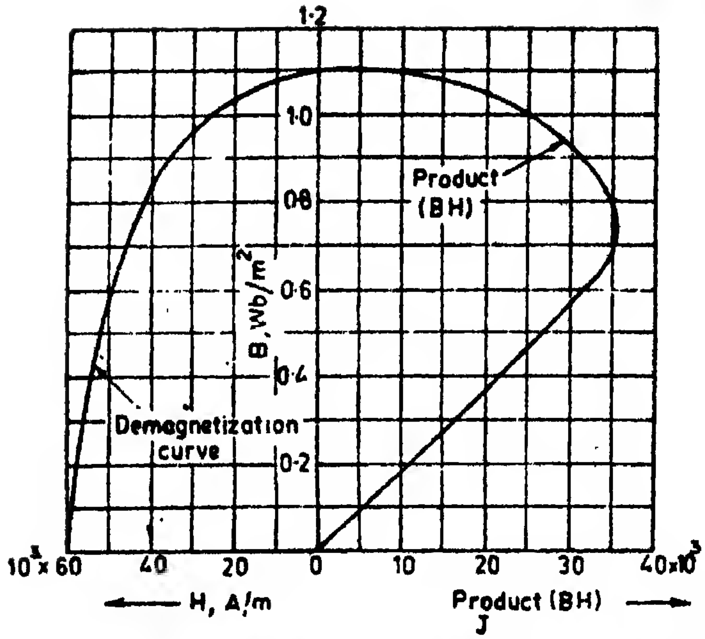


Fig. 14·17. Demagnetizatoin curve and product BH

Total mmf required $= 1{\cdot}2 \times 1440 = 1728$ A.

Length of magnet $l_p = \dfrac{\text{total mmf}}{H_p} = \dfrac{1728}{43{,}000}$ m $= 40$ mm.

The magnet is roughly a cube.

15

Design of Mechanical Parts

15·1. Design of shaft. The diameter of shaft for an electrical machine is determined by considerations of stiffness *i.e.* ability to resist deflection due to weight of rotor or unbalanced magnetic pull rather than strength to transmit power.

The shaft of an electrical machine designed properly has to satisfy the following requirements :

(*i*) the shaft must be strong enough throughout its section to withstand all loads without causing residual strain,

(*ii*) the shaft must have enough rigidity in order that the deflection of shaft under operation of machine does not reach such a dangerous value as to cause the rotor to touch the stator,

(*iii*) critical speeds of rotation should be different from running speeds of machine.

A convenient formula for diameter of horizontal shafts under armatures is :

$$\text{Diameter of shaft } d = 5{\cdot}5 \times \left(\frac{\text{output in watt}}{\text{r.p.s.}}\right)^{1/3} \text{mm.} \qquad \ldots(15{\cdot}1)$$

The diameter of the shaft in the bearings is less than the diameter under the armature. When the shaft diameter under armature is 150 mm or more a good rule is to make the shaft diameter in bearings 50 mm smaller than the maximum diameter. In the case of small shafts the diameter in bearings should be about 2/3 of the maximum diameter.

15·2. Bearings. Plain bearings are used for horizontal shaft machines and thrust bearings are used for vertical shaft machines.

Plain bearings can be either sleeve bearings or anti-friction bearings (*i.e.* ball or roller bearings). In horizontal shaft machines, the forces acting in a radial direction are most prominent. In vertical shaft machines the axial load acting downwards is taken up by the thrust bearings. Radial loads in this case can be caused either by the dynamic unbalance of rotor or by unbalanced magnetic pull of rotor towards stator. A simple thrust bearing alone cannot take up radial loads and therefore an additional bearing called guide bearing is provided. Usually 2 or 3 guide bearings are used in vertical shaft machines depending upon the load.

15·2·1. Sleeve bearings

Bearings for horizontal shaft machines. Bearings for horizontal shaft machines can either be outside the machine or can be built in the end shields of the machine. Bearings put outside the machine are called padestal bearings (see Fig. 15·1) and are used in medium and large capacity machines.

Bearings built inside end shields are called shield bearings and are used for machines with small and medium outputs (see Fig. 15·2).

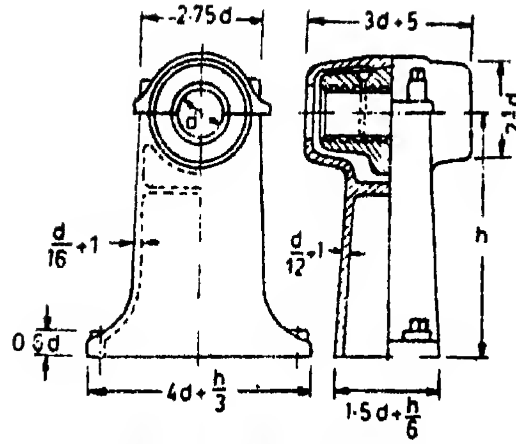
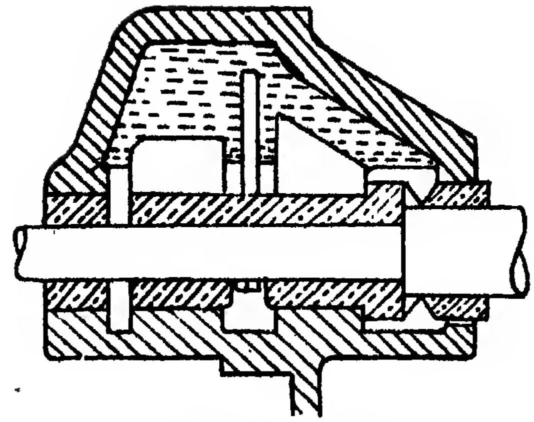


Fig. 15·1 Padestal bearing.



Fi . 15·2 Shield bearing.

Bearing dimensions. Phosphor bronze sleeve bearings are used for small electrical machines with journal diameters of 50–60 mm.

The ratio of length of sleeve to its diameter varies from 1·5 to 2.

Fig. 15·1 shows the usual proportions for a padestal bearing. (Dimensions in mm).

15·2 2. Anti-friction bearings. Anti-friction bearings may be ball or rollor type. Ball bearings (Fig. 15·3) are mainly intended for taking up radial loads. However, they can withstand considerable axial loads also. Roller bearings (Fig. 15·4) are used in cases of heavier loads than are permissible for ball bearings. Anti friction bearings may be fixed to the shaft in many ways, one of the methods is shown in Fig. 15 5. Anti-friction bearings are lubricated by greese.

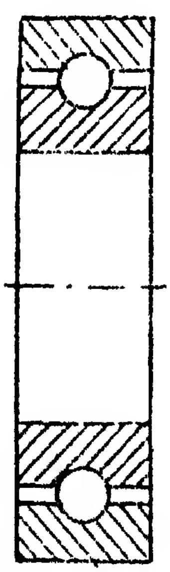


Fig. 15·3. Ball bearing.

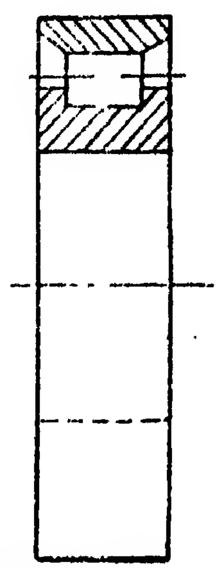


Fig. 15·4. Roller bearing.

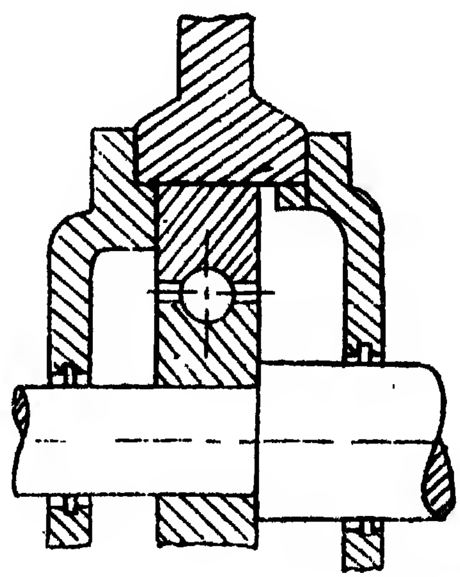


Fig. 15·5. Fixing of anti-friction bearing.

At present anti-friction bearings are being used in electrical machines with power outputs upto 500 kW and some times even more.

15·3. Shaft couplings. Fig. 15·6 shows the usual proportions (dimensions in mm) of a cast iron shaft coupling.

Number and diameter of bolts

The number of bolts in flange couplings are :

Shaft diameter	Number of bolts
50—75 mm	4
75—200 mm	6
200 mm and above	8

$$\text{Diameter of bolt} = \frac{4}{7}\frac{d}{\sqrt{\text{number of bolts}}} \quad \ldots(15.2)$$

where d is the diameter of shaft.

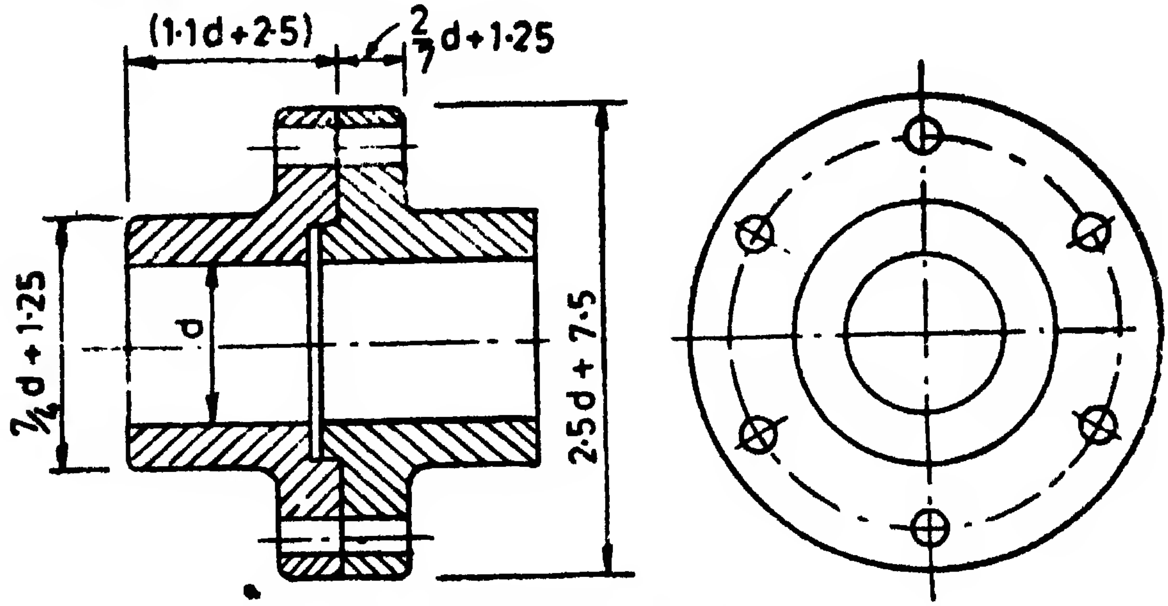


Fig. 15.6. Shaft coupling.

15·4. Frames for d.c. machines. The frame (yoke) of d.c. machines carries the flux also and therefore it must be of sufficient cross-section to carry flux. Fig. 15·7 (*a*) shows a cross-section which is sufficient for small machines The length of yoke L_y is usually made larger than the pole cores so as to cover and protect the field windings. The depth of yoke $d_y = h$ is then calculated to give the required cross-section for the magnetic circuit.

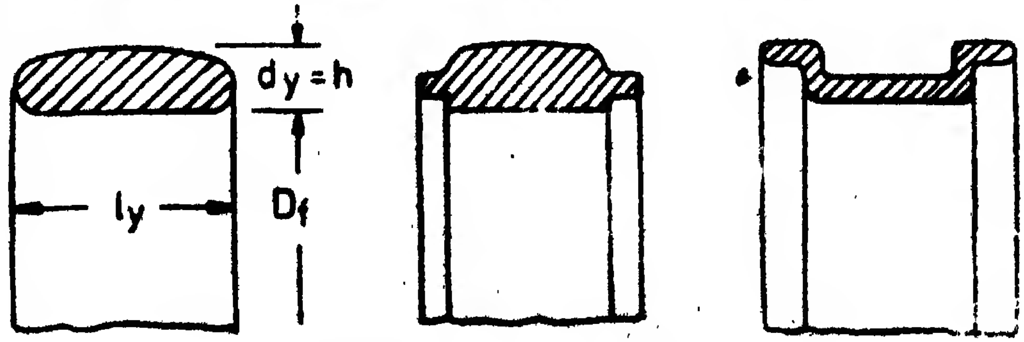


Fig. 15·7. Stator frames.

In large machines, the thickness h may be very small relative to diameter D_f. In that case the frame is not sufficiently rigid and the required rigidity has to be obtained by putting metal as shown in Fig. 15.7 (b) and (c).

In order to check the rigidity, the following relation should be satisfied

$$J > \frac{G_f R^2}{225} \times 10^{-6} \text{ m}^4 \qquad \ldots(15.3)$$

where J=moment of inertia ;

R=radius at the centre of gravity of cross-section of magnetic frame, m ;

G_f=weight of magnetic frame in kg.

15.5. Frames for a.c. machines. Fig. 15.8 shows box type frame for a.c. machines and it combines light weight with high rigidity.

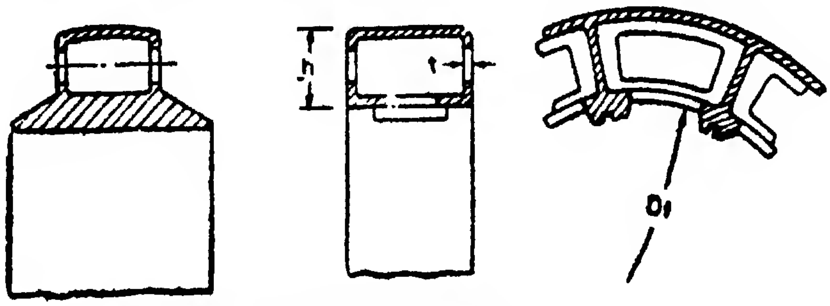


Fig. 15.8. Box type stator frame.

The following empirical relationships may be used for the estimation of dimensions of a box type frame :

$$h = 40 + \frac{D_f}{12} \text{ mm}$$

$$t = 6 + 0.01\, D_f \text{ mm}$$

where D_f, the inner diameter of frame is expressed in mm.

In order to check rigidity, we should apply the following formulae

$$J > \frac{G_f R^2}{225} \times 10^{-6} \text{ m}^4 \qquad \ldots(15.4)$$

where G_f=weight of stator frame without active iron, kg.

For induction machines :

$$J > \frac{RL}{90} \times 10^{-6} \text{ m}^4 \qquad \ldots(15.5)$$

where L=length of stator core, m.

For preliminary determination of moment of inertia of the stator frame of a.c. machines, R is taken as

$$R = \frac{D_o^{1.5}}{6.3} \text{ mm} \qquad \ldots(15.6)$$

where D_o=outer diameter of stator core, expressed in mm.

15.6. Centrifugal force

Let G=weight of revolving body, kg ;

n=speed, r.p.s. ;

R=radius of circular path, m.

$\therefore$ Centrifugal force C.F. $= G\omega^2 R$

$$= 4\pi^2 G n^2 R \text{ N} \qquad \ldots(15.7)$$

15·6. Bracing of rotor windings

15·6 1. Wire bands of rotors. Bands used on rotors of electrical machines are intended for bracing the rotor windings against their shift in the radial direction under action of centrifugal forces

Wire bands are placed on both active and inactive (overhang) portions of rotor conductors (see Fig 15·9).

The sizes of bands placed on active portions of conductors are influenced by the length of air gap and the method of cooling of armatures. In general, the wire bands along the active length of winding are only placed in d.c. machines. In induction machines the use of such bands is not possible owing to very short lengths of air gap.

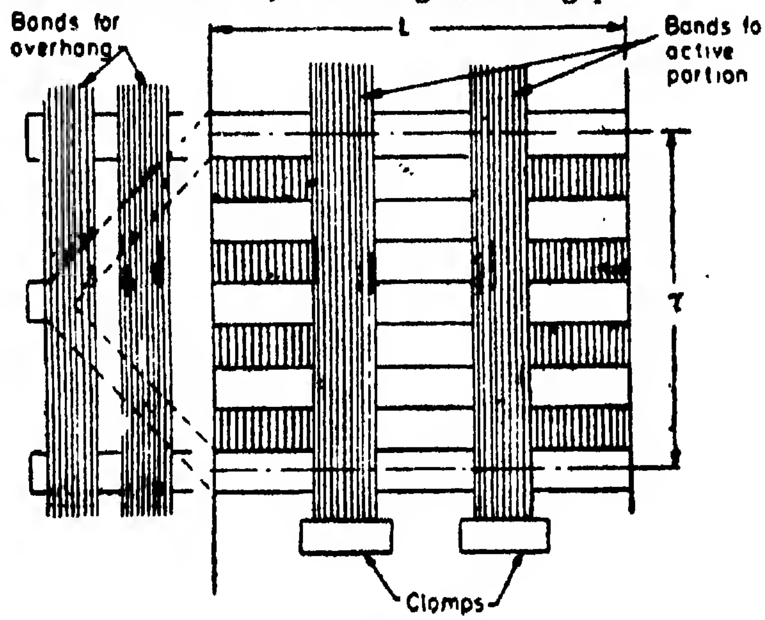


Fig. 15·9. Wire bands or rotor.

Bands placed along the active length of winding are housed in ring slots and width of each band should not exceed 15 to 20 mm. The total width of such bands should not be more than 25 to 35 percent of axial length of armature core.

Breadth of ring slot $O=(N_t+1)d_b+2K$ mm ...(15·8)

where

N_t=number of turns in a band ; d_b=diameter of band wire, mm ;

K=1·0 mm for $d_b<1·5$ mm

=1·5 mm for $d_b>1·5$ mm.

The maximum width of bands placed on the end windings (overhang) of induction machines and high speed d.c. machines is about 40 mm. If calculations warrant the use of bands broader than 40 mm, the bandings should be subdivided.

Wire bands are made of tin, steel or bronze wire upto a diameter of 3 mm.

For the design of banding wires, it is necessary to differentiate between two cases :

(*i*) active part of winding braced by wedges or located in semi-enclosed slots

(*ii*) active part of winding braced by bands.

In case (*i*), the bands are placed on overhang only and the bands are designed to withstand centrifugal forces due to the weight of overhang.

In case (*ii*), the bands are distributed along the axial length of armature and they withstand centrifugal forces due to weight of both the active and inactive parts of armatur-

From Eqn. 15·7, total centrifugal force,

$$C.F.=4\pi^2 G n^2 R=2\pi^2 G n^2 D_m \quad \ldots(15\cdot9)$$

where D_m = mean diameter at the position of centre of gravity $=D-d_s$

Total tension at the rim $T=C.F./2\pi=\pi G n^2 D_m$

Let N_t be the total number of banding wires, the diameter of each wire being d_b mm.

Tensile stress in banding material due to weight enclosed

$$f_1=\frac{T}{\text{area of bands}}=\frac{\pi G n^2 D_m}{N_t(\pi/4)\,d_b^2}\times 10^6$$

$$=\frac{4Gn^2 D_m}{N_t\, d_b^2}\times 10^6 \text{ N/m}^2 \quad \ldots(15\cdot10)$$

It should be noted that d_b is expressed in mm.

$\therefore$ Total number of wires $N_t=\dfrac{4Gn^2 D_m}{f_1 d_b^2}\times 10^6 \quad \ldots(15\cdot11)$

In high speed electrical machines, it is necessary to take into account the additional tensile stress under the influence of centrifugal forces due to weight of the band itself.

Let G_b = weight of banding wires.

$\therefore$ Tensile stress in banding wires due to their own weight

$$f_0=\frac{4G_b n^2 D_m}{N_t d_b^2}\times 10^6 \text{ N/m}^2 \quad \ldots(15\cdot12)$$

But

$$G_b=g_b\times N_t\times \pi D_m\times\frac{\pi}{4}\times d_b^2\times 10^{-6}=\frac{\pi^2 g_b N_t D_m d_b^2}{4}\times 10^{-6}\text{ kg}$$

where g_b = specific gravity of banding wires, kg/m^3.

$$\therefore \quad f_0=\pi^2 g_b n^2 D_m^2 \text{ N/m}^2 \quad \ldots(15\cdot13)$$

Total stress on banding wires $f_t=f_1+f_0 \quad \ldots(15\cdot14)$

This stress should be less than the maximum allowable stress for the material.

The banding wires are stretched a little before they are placed on rotor. The stretch is such that

$$f_1+f_0=(0\cdot7 \text{ to } 0\cdot8)f_2 \quad \ldots(15\cdot15)$$

where $f_2=(0\cdot4 \text{ to } 0\cdot45)\,f \quad \ldots(15\cdot16)$

and f = permissible stress for material of wire.

Usually $f_1=(0\cdot4 \text{ to } 0\cdot45)\,f-f_0 \quad \ldots(15\cdot17)$

For bronze wire :

$d_b=1$ mm, $f=350$ MN/m^2

$d_b=1\cdot5$ mm, $f=300$ MN/m^2

For steel wires :

$d_b=0\cdot5$ to $1\cdot2$ mm, $f=600$ MN/m^2

$d_b=1\cdot5$ to $2\cdot0$ mm, $f=570$ MN/m^2.

15·6·2. Solid bands (retaining rings). Bands (rings) of forged or solid cast material are used for

(*i*) turbo-alternators

(*ii*) high speed d.c. machines

(*iii*) induction machines used for traction purposes.

Construction of solid bands (retaining ring) for rotor overhang is shown in Fig. 12·20 on page 735. There is a spigot on which the retaining ring is centered. The conductors are sloped conically inward to provide sufficient space for thickness of the ring and are strongly braced to fit closely within the ring without movement. Non-magnetic magnesium and magnesium nickel steels are used for the retaining rings. The non-magnetic property is required in order to avoid excessive magnetic leakage and stray load loss.

The stress produced in retaining rings of overhang of turbo-alternators is the summation of stresses caused by the centrifugal forces produced by their own mass and the mass of windings they retain.

Suppose G=weight of mass retained, kg ;

G_r=weight of ring, kg ;

D_m=mean diameter of mass retained, m ;

D_{mr}=mean diameter of ring, m ;

n=speed, r.p.s. ;

A_r=area of retaining ring$=t\times b$, m^2 ;

t=thickness of ring, m ;

b=width of ring, m.

Total centrifugal force C.F.$=2\pi^2Gn^2D_m+2\pi^2G_rn^2D_{mr}$

$=2\pi^2n^2\,(GD_m+G_rD_{mr})$

Total tension of the ring $T=$C.F.$/2\pi=\pi n^2\,(GD_m+G_rD_{mr})$

Tensile stress in retaining ring

$$f_t=\frac{T}{A_r}=\frac{\pi n^2}{A_r}(GD_m+G_rD_{mr})$$

Total weight of ring $G_r=g_r\times\pi D_{mr}\times tb$

where g_r=density of ring material, kg/m^3

$$\therefore\quad f_t=\frac{\pi n^2GD_m}{tb}+\pi^2n^2g_rD_{mr}^2\ \text{N/m}^2 \qquad \ldots(15\cdot18)$$

Example 15 1. *Design wire bands for fixing the armature winding of a 2400 rpm d.c. machine having :*

diameter of armature $=$*245 mm*

length of armature $=$*80 mm*

depth of slot $=$*35 mm*

weight of copper in embedded portion $=$*3 kg*

and weight of copper in overhang $=$*10 kg.*

Use steel wires of 1·2 mm diameter.

Permissible stress f=600 MN/m². The working stress should not exceed about 0·4 of permissible stress.

Solution.

Bands over overhang

Speed $=2400/60=40$ r.p.s.

Mean diameter $D_m=D-d_s=245-35=210$ mm$=0\cdot21$ m.

Permissible tensile stress $f_1=0\cdot4\times600=240$ MN/m^2.

$\therefore$ Number of wires in two sides of overhang

$$N_t=\frac{4Gn^2\,D_m}{f_1d_s^2}\times10^6 \quad \text{(See Eqn. 15·11)}$$

$$= \frac{4\times 10\times (40)^2\times 0.21}{240\times 10^6\times (102)^2}\times 10^6 \simeq 40.$$

$\therefore$ Number of turns on each end of overhang $n_t = 40/2 = 20$.

From Eqn. 15.8 width of each ring

$$C = (N_t+1)\, d_b + 2K$$
$$= (20+1)\times 1.2 + 2\times 1 \simeq 27 \text{ mm}.$$

Bands on active portion

Total number of wires $N_t = \dfrac{4\times 3\times (40)^2\times 0.21}{240\times 10^6\times (1.2)^2}\times 10^6 \simeq 12.$

Using 2 bands, number of turns in each band $= 6$.

Width of each band (ring) $C = (6+1)\times 1.2 + 2\times 1 \simeq 11$ mm.

Example 15.2. *The rotor of a 3000 r.p.m. turbo-alternator has a diameter of 1 m. It has a retaining ring of 350×45 mm² of mean radius 0.49 m. It encloses a copper winding of 3000 kg and radius 0.35 m. Find the hoop stress on the ring. The ring material has a density of 7800 kg/m³.*

Solution.

Speed $n = 3000/60 = 50$ r.p.s.

Mean diameter of overhang $D_m = 2\times 0.35 = 0.7$ m.

Mean diameter of ring $D_{mr} = 2\times 0.49 = 0.98$ m.

From Eqn. 15.18, stress in ring

$$f_t = \frac{\pi n^2 G D_m}{tb} + \pi^2 n^2 g_r D_{mr}^2$$

$$= \frac{\pi (50)^2\times 300\times 0.7}{350\times 45\times 10^{-6}} + \pi^2 (50)^2\times 7800\times (0.98)^2$$

$$= 104.7\times 10^6 + 184.8\times 10^6$$

$$= 289.5 \text{ MN/m}^2.$$

It is clear from above that the total stress in the ring is 289.5 MN/m² out of which a stress of 184.8 MN/m² (nearly 2/3 of the total stress) is caused by the weight of ring itself.

15.6.3. Wedges. If the peripheral speed of rotor is about 30 m/s or above, the bracing of active parts of conductor is done by wedges. Wedges have the cross section shown in Fig. 15.10. They are normally made up of wood or bakelite.

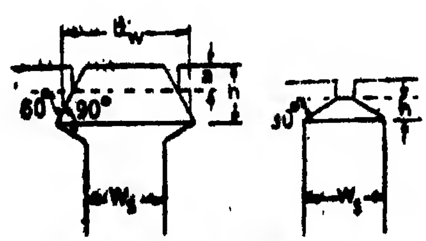


Fig. 15.10. Slot wedges

The centrifugal force on the wedge can be calculated by Eqn. 15.9. If G_w is the weight of copper and insulation of a slot.

$$\text{C.F.} = 2\pi^2 G_w n^2 D.$$

The wedge can be considered as a simple supported beam of length b_w having a concentrated load at the centre. The maximum bending stress will be at the centre of wedge and its value is

$$f_b = 1.5\frac{\text{C.F.}\times b_w}{h^2 L}\ \text{N/m}^2 \qquad \ldots(15.19)$$

The value of shearing stress is checked by

$$f_s = 0{\cdot}5\,\frac{\text{C.F.}}{(h-a)\,L}\ \text{N/m}^2 \qquad \text{...(15·20)}$$

Permissible bending stresses are :

Wood—8 MN/m²

Bakelite—25 MN/m²

Permissible shearing stresses are :

Wood—2 MN/m²

Bakelite—15 MN/m².

The height of wedge is

$$h > 1{\cdot}22\sqrt{\frac{C.F.\times b_w}{Lf_b}} \qquad \text{...(15·21)}$$

The ratio of height of wedge to width of slot is greater than 0 25.

Example 15 3. *A 350 kW, 3600 r p.m , 500 V motor has the following data. Calculate the size of wedges for slots for fixing active part of the winding in slots. Use bakelite wedges with width (at the middle) of wedge 1·2 times the slot width.*

Diameter of armature = 0 5 m, length of armature = 0·35 m, number of armature slots = 50, slot dimensions = 30 mm × 16 mm, conductor size = 8 × 3 8 mm², conductors per slot = 6.

The weight of each conductor with insulation 0·3 kg/m.

For bakelite: Maximum bending and shearing stresses are respectively 25 MN/m² and 15 MN/m².

Solution.

1. **Wedges.** Weight of active copper

$$G_w = 6\times0{\cdot}3\times0{\cdot}35 = 0{\cdot}63 \text{ kg.}$$

Speed $n = 3600/60 = 60$ r.p.s.

Mean diameter $D_m = D - d_s = 0{\cdot}50 - 0\;03 = 0{\cdot}47$ m.

Centrifugal force $\text{C.F.} = 2\pi^2\times0{\cdot}63\times(60)^2\times0\;47 = 21\times10^3$N.

Width of wedge $b_w = 1\;2\times16 \simeq 20$ mm.

From Eqn. 15·21, height of wedge

$$h \geqslant 1{\cdot}22\sqrt{\frac{C.F.\times b_w}{Lb_f}} \geqslant 1{\cdot}22\sqrt{\frac{21\times10^3\times20\times10^{-3}}{0{\cdot}35\times25\times10^6}}$$

$$\geqslant 8{\cdot}45\times10^{-3}\text{ m.}$$

The height of wedge used is $h = 9$ mm. The lip has a height of 1 mm.

Checking stresses at the middle of wedge.

Bending stress $f_b = 1{\cdot}5\,\dfrac{\text{C.F.}\times b_w}{Lh^2}$ (Eqn. 15·19)

$$= 1{\cdot}5\times\frac{21\times10^3\times20\times10^{-3}}{0{\cdot}35\times(9\times10^{-3})^2} = 22{\cdot}2\text{ MN/m}^2.$$

This is below the maximum permissible value.

Shearing stress at the middle of wedge

$$f_s = \frac{0{\cdot}5\,C.F.}{(h-a)L}\ \text{(Eqn. 15·20)}$$

$$= 0{\cdot}5\times\frac{21\times10^3}{(9\times10^{-3}-1\times10^{-3})\times0{\cdot}35} = 3{\cdot}74\text{ MN/m}^2.$$

This is below the maximum permissible value.

15·7. Stresses in poles bolted on to rims. This type of construction is used for poles in salient pole synchronous machines.

Centrifugal force of a pole. C.F. $2\pi^2 G_p n^2 D_p$ ...(15·22)

where n = maximum rotational speed, r.p.s. ;
G_p = weight of pole and field coil, kg ;
D_p = diameter at the centre of gravity of poles and field coils, m.

The tensile stress in bolts of a pole is determined as the ratio of twice the centrifugal force to the total area of cross section of all the bolts in a pole.

$$f = \frac{2\ \text{C.F.}}{N_b\, a_b} = \frac{4\pi^2 G_p n^2 D_p \times 10^6}{N_b\, a_b}\ \text{N/m}^2 \qquad \text{...(15·23)}$$

where N_b = number of bolts per pole,

and a_b = cross section of each bolt, mm².

The cross section of the bolt is determined by using the internal diameter of bolt thread. The maximum number of bolts per pole is 2. The value of f obtained from Eqn. 15·23 should not exceed 0·7 of the elastic limit for the material of the bolts.

15·8. Stresses in dove-tailed poles. The centrifugal force acting radially outwards is given by Eqn 15·22.

The calculation of the stress over cross section de (Fig. 15·11) is not so easily made. It is desired to calculate force $P=AB$ due to centrifugal force of the poles on each side of the cross-section de.

Referring to Fig. 15·11, we have,

DE = centrifugal force (C.F.) on each pole body

$AB = P$ = resultant force on section de

θ = dovetail angle, p = number of poles.

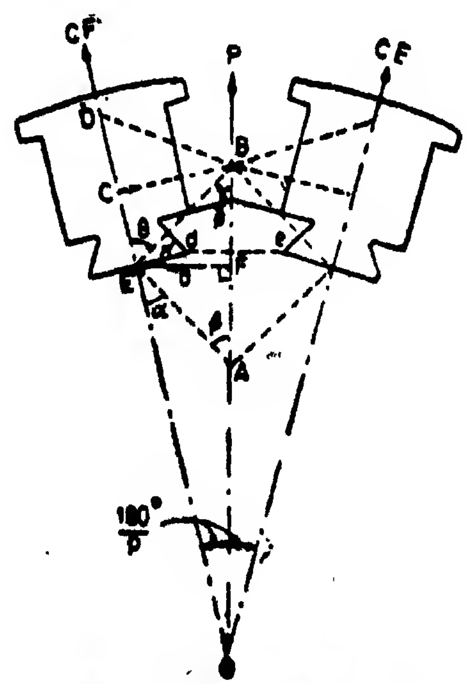


Fig. 15·11. Stresses on dove-tailed poles.

From the geometry of diagram, we have,

$$\theta+\alpha+\angle BEA = 2\beta+\angle BEA$$

$$\therefore \quad \theta+\alpha = 2\beta \text{ or } \alpha = 2\beta-\theta$$

Now $\beta = \alpha+180/p$

$$= (2\beta-\theta)+180/p$$

$$\therefore \quad \beta = \theta-180/p$$

We can write $AB = 2\ AF = 2\ BF$.

Now $BF = BE\cos\beta = BE\cos(\theta-180/p)$.

But $BE\cos\theta = CE = \frac{1}{2}\ DE$

$$\therefore \quad BE = \frac{DE}{2\cos\theta}.$$

Thus $$BP = \frac{DE}{2\cos\theta}\cdot\cos(\theta-180/p)$$

and $$AB=2BF=DE\ \frac{\cos(\theta-180/p)}{\cos\theta}$$

$\therefore$ Force on section de = C F. $\times \dfrac{\cos(\theta-180/p)}{\cos\theta}$ ...(15·24)

15·9. Stresses in turbo-alternator rotors. The electrical and mechanical design of a turbo-rotor must be carried out together, because the space available for field copper cannot be determined until the steel below the rotor coils required for mechanical strength has been fixed.

15·9·1. Stresses at the bottom of teeth. Assume that every tooth carries the centrifugal force due to its own weight and that of contents of one slot. It is also assumed for simplification that the total weight of copper, insulation and wedge in a slot is the same as that of an equal volume of steel.

Let g_s = density of steel, kg/m^3 ;

n = speed of rotor, r p s. ;

α = angle subtended by each slot (See Fig. 15·12)

D_{r_1} = outer diameter of rotor, m ;

D_{r_2} = diameter of rotor at the bottom of teeth, m ;

L = axial length of machine, m ;

t = thickness of tooth at the bottom, m.

Let us consider an elementary strip of width dx at a radius x.

Weight of steel plus copper, insulation etc. over slot pitch in the elementary strip

ΔG_s = density $\times$ volume of strip

$$=g_s\times L\times\left(\frac{\alpha}{360}\cdot 2\pi x\right)dx$$

$$=2\pi g_s L\ \frac{\alpha}{360}x\,dx$$

Centrifugal force due to this weight

$$\Delta\text{C.F.}=4\pi^2\Delta G_s n^2 x$$

$$=8\pi^3 g_s n^2 L\frac{\alpha}{360}\ x^2 dx$$

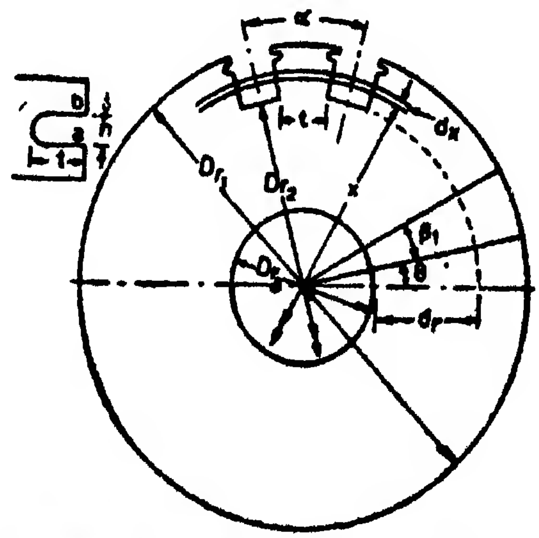


Fig. 15·12. Turbo-alternator rotor.

$\therefore$ Total centrifugal force over a slot pitch

$$\text{C.F.}=\sum_{D_{r_2}/2}^{D_{r_1}/2}\Delta\text{C.F.}$$

$$=8\pi^3 g_s n^2 L\frac{\alpha}{360}\int_{D_{r_2}/2}^{D_{r_1}/2} x^2 dx$$

$$=\frac{\pi^3}{3}g_s n^2 L\frac{\alpha}{360}(D_{r_1}{}^3-D_{r_2}{}^3)\qquad ...(15·25)$$

Stress at the bottom of tooth

$$f = \frac{\text{C.F.}}{\text{area of tooth}} = \frac{\text{C.F.}}{Lt}$$

$$= \frac{\pi^2}{3t} g_i n^2 \frac{\alpha}{360} (D_{r_1}^3 - D_{r_2}^3) \text{ N/m}^2 \quad \text{...(15·26)}$$

Example 15·4. *A 2 pole, 3000 r p m. turbo alternator has the following data :*

Diameter of rotor = 1·15 m ; number of rotor slot = 39 (for spacing) ; depth of rotor slot = 140 mm ; width of rotor slot = 45 mm

Find (a) the tensile stress at the root of a rotor tooth when the speed is 3000 r.p.m. (b) the factor of safety if the rotor speed rises to 20 percent above normal operating value The yield stress of rotor steel is 520 MN/m². Steel weighs 7800 kg/m³.

Solution. Outer diameter of rotor $D_{r_1} = 1\ 15$ m.

Diameter of rotor at the bottom of slots

$$D_{r_2} = 1\ 15 - 2 \times 140 \times 10^{-3} = 0\ 87 \text{ m.}$$

Width of tooth at the bottom of slot

$$t = \frac{\pi \times 0\ 87 \times 10^3}{39} - 45 = 25 \text{ mm.}$$

Angle subtended by each slot $\alpha = 360/39 = 9·23°$.

(a) From Eqn. 15 26, tensile stress at the root of the teeth at normal operating speed,

$$f = \frac{\pi^2}{3t} g_i n^2 \frac{\alpha}{360} (D_{r_1}^3 - D_{r_2}^3)$$

$$= \frac{\pi^2}{3 \times 25 \times 10^{-3}} \times 7800 \times (50)^2 \times \frac{9\ 23}{360} (1·15^3 - 0·87^3)$$

$$= 178 \text{ MN/m}^2.$$

(b) The centrifugal force is proportional of square of speed.

∴ Tensile stress at 20 percent overspeed

$$= (1·2)^2 \times 178 = 256·3 \text{ MN/m}^2.$$

∴ Factor of safety at 20% over speed

$$= \frac{520}{256·3} = 2·03.$$

15 9·2. Stresses in rotor disc. Suppose D_{r_3} is the internal diameter of rotor disc.

Referring to Fig. 15·12 and using Eqn. 15 25, the centrifugal force on a section of the rotor disc enclosed by an angle β is

$$= \frac{\pi^2}{3} g_i n^2 L \frac{\beta}{360} (D_{r_2}^3 - D_{r_3}^3)$$

Vertical component of this force

$$= \frac{\pi^2}{3} g_i n^2 L \frac{\beta}{360} (D_{r_2}^3 - D_{r_3}^3) \sin \theta$$

Total vertical component due to half of rotor (for β = 180°)

$$= \frac{\pi^2}{3} g_i n^2 L \frac{\beta}{360} (D_{r_2}^3 - D_{r_3}^3) \times \text{average of } \sin \theta \text{ over } 180°$$

$$=\frac{\pi^2}{3}g_s n^2 L\frac{\beta}{360}\left(D_{r_2}^3-D_{r_1}^3\right)\times\frac{2}{\pi}$$

$$=\frac{2}{3}\pi^2 g_s n^2 L\frac{\beta}{360}\left(D_{r_2}^3-D_{r_1}^3\right) \qquad \ldots(15\cdot27)$$

Stress in the section of rotor disc having a width d_r

$$f=\frac{2}{3}\pi^2 g_s n^2\,\frac{L}{Ld_r}\,\frac{\beta}{360}\left(D_{r_2}^3-D_{r_1}^3\right) \qquad \ldots(15\cdot28)$$

$$=\frac{2\pi^2}{3d_r}g_s n^2\,\frac{\beta}{360}\left(D_{r_2}^3-D_{r_1}^3\right)\ \mathrm{N/m^2}$$

When no shaft is used $d_r=D_{r_2}/2$ and $D_{r_1}=0$

The stress in such a case is

$$f=\frac{16\pi^2}{3}g_s n^2\frac{\beta}{360}d_r^2 \qquad \ldots(15\cdot29)$$

15·10. Critical speed. The rotor has a natural frequency of vibration as it is a structure with mass and elasticity. Taking the problem in its most elementary form, the shaft can be considered in two bearings carrying a disc of mass m.

If the mass centre has an eccentricity 'e' from the centre of rotation, then the centrifugal force exerted on the mass is '$m\omega_r^2 e$' where ω_r is the angular velocity corresponding to speed of rotation. This causes a deflection δ of the shaft increasing the centrifugal force to $m\omega_r^2(e+\delta)$. The deflection is resisted by an elastic force developed by the shaft which may be taken as proportional to deflection, say $K\delta$. Under equilibrium these two forces are equal and opposite

or $$K\delta=m\omega_r^2(e+\delta)$$

or $$\delta=\frac{m\,\omega_r^2\,e}{K-m\omega_r^2}.$$

At some speed ω_c the denominator becomes zero and the deflection thus is infinite. This speed is known as the *critical speed*. However, the deflection δ does not become infinite in practical cases because it is limited by frictional and deflection losses. Nevertheless the deflection tends to be very large at critical speed. Thus considerable vibrations are set up at critical speed. In small machines with relatively small shafts the critical speeds are always in excess of normal running speeds but with large turbo-alternators having long rotors, the critical speed may be near the running speed unless special precautions are taken. In such cases the rotor is deliberately designed to have critical speed that is much lower than the normal running speed. This critical speed should not be within 20 percent of the running speed, as otherwise the vibrations set up may be dangerous. When starting up, considerable vibrations are experienced as rotor passes through its critical speed. This can be avoided by quickly running through the critical speed region.

15·11. Inertia constant. The inertia of rotating parts of a synchronous machine affects its stored kinetic energy when running. The greater the inertia the greater is the kinetic energy. A high inertia machine owing to its large stored energy does not allow excessive changes in its speed when sudden changes in load occur. Thus a high inertia machine has a higher transient stability limit. Salient pole machines operate at slow speed and in order that they have sufficient kinetic energy to avert large changes in load, they should be designed to have a large inertia.

The magnitude of the disturbance to which a generator may be subjected is a function of the power of a source and receptivity of the generator. Therefore, a machine designed to operate on a grid system can be subjected to higher power surges and the risk of instability is reduced when the machine is designed to have a large inertia.

However, a machine having a large inertia has a large time constant and therefore, its response is sluggish. The disadvantage of sluggish response is that the generator will not allow rapid corrections in its performance after sudden disturbances

The requisite flywheel effect of the rotor is expressed in terms of an inertia constant H defined as

$$H=\frac{\text{energy stored in joule}}{\text{rating of machine in volt ampere}}$$

$$=\frac{1}{2}\,\frac{J\omega_r^2}{Q\times10^3}\ \text{s} \qquad \ldots(15\cdot30)$$

where J = moment of inertia of rotor, kg—m²
ω_r = angular velocity of rotor, rad/s ;
Q = kVA rating of machine.

Moment of inertia $J=Wk^2$ ...(15·31)

where W = mass of rotor, kg ; and k = radius of gyration, m.

The inertia constant of a machine is a design criterian which indicates the quality of the inherent stability of the machine. The value of inertia constant normally required is between 2 to 9s

Example 15·5. *The moment of inertia of rotating parts of a 500 MW, 2 pole, 50 Hz, 3 phase synchronous generator is 50×10^3 kg—m². Find the inertia constant of the generator if it is designed for operation at 0·85 power factor lagging.*

Solution.

Speed $=2\times50/2=50$ r.p.s. and angular speed $=2\pi\times50=314$ rad/s

kVA rating $=\dfrac{500\times10^3}{0\cdot85}=588\times10^3$.

From Eqn. 15·30, inertia constant $H=\dfrac{1}{2}\,\dfrac{J\omega^2}{Q\times10^3}$

$$=\frac{1}{2}\times\frac{50\times10^3\times(314)^2}{588\times10^3}=4\cdot2\ \text{s.}$$

15·12. Mechanical design of commutators. Refer to Fig. 15·13.

(i) Length of commutator should make allowance for brush boxes which should have enough clearance to provide for good sliding fit for brushes. There should be additional lengths C_1 and C_2 at the ends of commutator.

$$l''_c \geqslant a+5+\frac{V}{100}\ \text{mm.} \qquad \ldots(15\cdot32)$$

where a = axial play in mm and V = voltage of machine.

(ii) $e=1\cdot2+2\times$(thickness of brush box) mm. ....(15·33)

(iii) The length l_1 is taken 2—3 mm more than the width of riser.

(iv) Total length of commutator
$l_c=L_c+l_c''+l_1$ ...(15·34)

(v) Height of bar for special machines $h_c=14\cdot7\,(D_c)^{0\cdot25}-10$ mm ...(15·35)

Height of bar for general purpose machines $h_c=11\cdot55\,(D_c)^{0\cdot25}-10$ mm ...(15·36)

D_c is expressed in mm

The height of tail portion for $V_c=40$ m/s

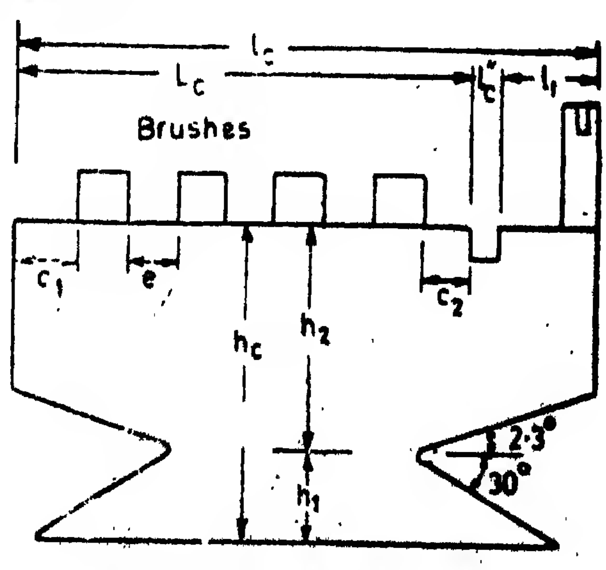


Fig. 15·13. Commutator segment.

$h_1 = 0.45\ h_c$ for short commutators ...(15.37)

$= 0.5\ h_c$ for commutators with $L_c/h_c =$ about 4. ...(15.38)

15.13. Design of fan. The fundamental relationship for the design of a ventilation system for an electrical machine is :

$$H = ZV^2$$

where H = head of air inside the machine, mm of water ;

Z = hydrodynamic resistance,

V = volume of air passing, m³/s.

In a ventilating circuit there are some sharp or projecting inlet edges, corners and variations in cross sections of the air paths. There are losses of head in such cases. Therefore, the total head produced by the fan has to overcome these losses.

Total head $$H = \sum_{i=1}^{i=n} h_i \frac{V^2}{a_i^2} \quad ...(15.40)$$

$$= \sum_{i=1}^{i=n} h_i v_i^2 \quad ...(15.41)$$

where

h_i = co-efficients of hydrodynamic resistances, values of which are shown in Fig. 14.14 (*a*) and (*b*) and Table 15.1.

$v_i = V/a_i$ = speeds of air in corresponding parts, m/s ;

V = volume of air passing per second, m³/s ;

a_i = area of cross section of parts, m².

Table 15.1

Condition of Inlet	h_i
Protruding edges at inlet	$(40-60)\times 10^{-2}$
Rectangular edges at inlet	30×10^{-2}
Rounded edges at inlet	$(12-0)$

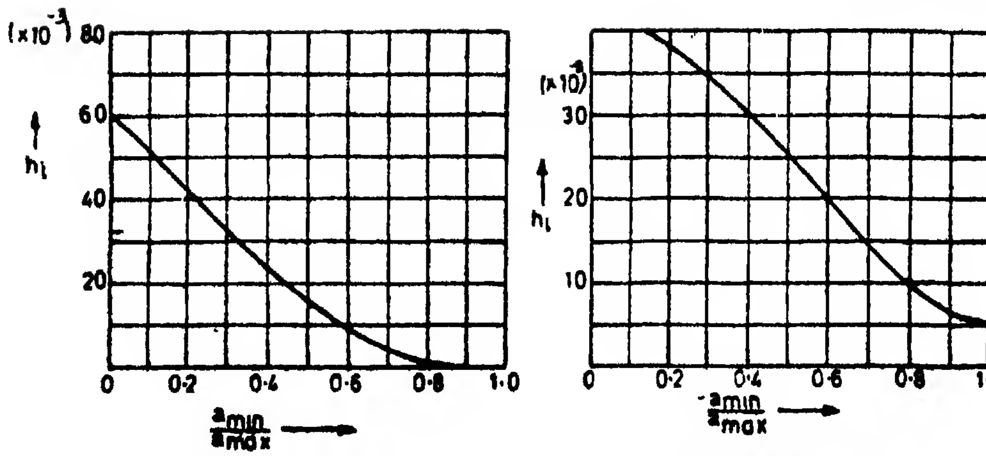
Eqn. 15.40 can be written as

$$H = \left[\sum_{i=1}^{i=n} \frac{h_i}{a_i^2}\right] V^2 \quad ...(15.42)$$

Comparing Eqn. 15.39 with Eqn. 15.42, we have

$$Z = \sum_{i=1}^{i=n} \frac{h_i}{a_i^2} \quad ...(15.43)$$

Thus in order to evaluate hydrodynamic resistance we must know the areas of cross-sections and the hydrodynamic co-efficients



(a) For enlargement (b) For contraction

Fig. 15·14. Co-efficients of hydrodynamic resistance.

Total hydrodynamic resistance of sections joined in series is the summation of individual resistances

$$Z=Z_1+Z_2+\ldots\ldots$$

Total hydrodynamic resistance of sections joined in parallel is

$$Z=\frac{Z_1 Z_2}{(\sqrt{Z_1}+\sqrt{Z_2})^2}$$

The data required for the design of a fan is :

(1) Outside diameter of fan D_2

(2) Volume of air $V=0{\cdot}9\times\frac{\text{losses in kW}}{\theta}$ ...(15·44)

where

losses = total losses in machine minus loss in bearings, kW,

$\theta=12$ to 16°C = difference of air temperature at inlet and outlet.

(3) Hydrodynamic resistance Z.

Steps in Designing a Fan

Refering to Fig. 15·15,

(i) Maximum air passing per second for fan operation at maximum efficiency

$$V_m=2V \qquad \ldots(15{\cdot}45)$$

(ii) Peripheral speed of fan at outside diameter

$$v_2=\pi D_2 n \qquad \ldots(15{\cdot}46)$$

Area of outlet opening $A_2=\frac{V_m}{0{\cdot}42\,v_2}$ ...(15·47)

(iii) Width of fan $b=\frac{A_2}{0{\cdot}92\,\pi D_2 K}$ ...(15·48)

where K = co-efficient of utilization.

(iv) Static head of fan under rated duty

$H=ZV^2$ mm of water. ...(15·49)

Static of fan under no load

$H_0=1{\cdot}33\,H$ mm of water ...(15·50)

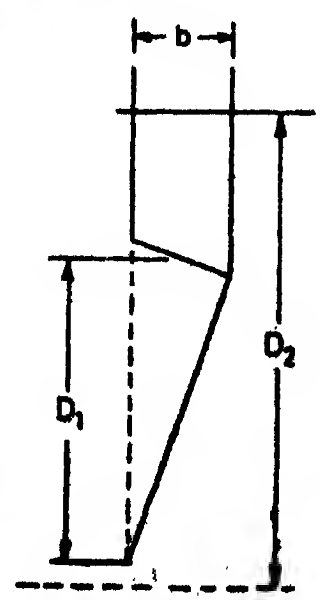


Fig. 15·15. Cooling fan blade.

(v) Inside diameter of fan

$$D_1 = \frac{v_1}{\pi n} \qquad \ldots(15.51)$$

where v_1 = peripheral speed at inside diameter

$$= \sqrt{v_2^2 - \frac{H_o}{0.075}} \text{ m/s} \qquad \ldots(15.52)$$

(*iv*) Number of blades $N_b = \dfrac{\pi D_2}{(1.25 \text{ to } 1.5)b}$ ...(15.53)

(*vii*) Power input of fan $P_f = 9.81 \dfrac{HV}{\eta_f}$ ...(15.54)

where η_f = efficiency of fan and its values are :

0.15 to 0.20 for radial blades,

0.25 to 0.3 for backward bent blades,

0.3 to 0.4 for forward bent blades.

Example 15.6 *Design a built in centrifugal fan with radial blades for cooling a d.c. machine of 50 kW and 1000 r.p.m. having a hydrodynamic resistance Z=350. The efficiency is 90 percent and friction loss in bearings is 5 percent of total loss. The outside diameter of fan is 0.55 m. Outlet opening for air in the end shield has been made by an arc subtended by 180° at the centre.*

Solution. We have efficiency $\eta = \dfrac{\text{output}}{\text{output}+\text{losses}}$

$\therefore$ losses $= \dfrac{0.1 \times 50}{.9} = 5.55$ kW.

Losses excluding bearing losses $= 0.95 \times 5.55 = 5.28$ kW.

From Eqn. 15.44, volume of air required,

$$V = \frac{0.9 \text{ losses}}{\theta} = \frac{0.9 \times 5.28}{16} = 0.3 \text{ m}^3/\text{s (assuming temperature rise } \theta = 16°\text{C).}$$

Data for design is :

$D_2 = 0.55$ m, $V = 0.3$ m³/s, $n = 1000/60 = 16.67$ r.p.s. and $Z = 350$.

(*i*) Maximum volume of air passing $V_m = 2V = 2 \times 0.3 = 0.6$ m³/s (See Eqn. 15.45)

(*ii*) Peripheral speed at outer diameter $v_2 = \pi D_2 n = \pi \times 0.55 \times 16.67 = 28.9$ m/s.

Area of outlet at opening

$$A_2 = \frac{V_m}{0.42 v_2} \text{ (See Eqn. 15.47)}$$

$$= \frac{0.6}{0.42 \times 28.9} = 49.4 \times 10^3 \text{ mm}^2.$$

(*iii*) Width of fan (Eqn. 15.48)

$$b = \frac{A_2}{0.92 \pi D_2 K} = \frac{0.0495}{0.92 \times \pi \times 0.55 \times 0.5} = 0.0625 \text{ m} = 62.5 \text{ mm.}$$

(K is assumed as 0.5 as the opening subtends an angle of 180°)

(*iv*) Static head of fan under full load $H = ZV^2$

$= 350 \times (0.3)^2 = 31.5$ mm of water.

Static head of fan under no load $H_o = 1.33 H$ (Eqn. 15.50)

$= 1.33 \times 31.5 = 42$ mm of water.

(*v*) Peripheral speed at inside diameter $v_1=\sqrt{v_2^2-\frac{H_s}{0.075}}$ (Eqn. 15.52)

$$=\sqrt{28.9^2-\frac{42}{0.075}}=16.6 \text{ m/s.}$$

Inner diameter of fan $D_1=\frac{v_1}{\pi n}=\frac{16.6}{\pi\times16.66}=0.317 \text{ m}=317 \text{ mm.}$

(*vi*) Number of blades $N_b=\frac{\pi D_1}{(1.25 \text{ to } 1.5)b}$ (Eqn. 15.53)

$$=\frac{\pi\times0.317}{1.3\times0.0625}=12.$$

(*vii*) From Eqn. 15.54, power input of fan $P_f=9.81\,\frac{HV}{\eta_f}$

$$=\frac{9.81\times31.5\times0.3}{0.2}=464 \text{ W.}$$

16

Computer Aided Design

16·1. Introduction. The design of electrical machines is both a science and an art. A *science* because it follows established and universally accepted physical and mathematical principles which have been verified by experimental methods and an *art* in that knowledge of these principles is often insufficient to produce correct and economic design. This can only be achieved by correct decisions based upon judgment and intution and through thorough understanding of the subject.

The design of electrical machines consists essentially of the solution of many complex and diverse engineering problems and normally these problems are loosely interrelated to a greater or a lesser degree. In fact the design of electrical machines presents a mathematically indeterminate problem with many solutions as the number of unknowns is greater than the number of equations.

The process of design of a single machine may be divided into three major design problems :

(*i*) electromagnetic design, (*ii*) mechanical design and (*iii*) thermal design.

These problems may be solved separately and the results combined later on. Additionally, each of these problems may be further subdivided into simple but loosely related elements. Each element may then be considered as a separate problem and this procedure may involve the solution of some of elements many times over to arrive at an acceptable solution.

The other aspect of the present day design of electrical machines is designing a set of machines, all of which form part of a single system. The different machines connected in such a system react upon each other, sometimes considerably and on occasions disasterously. Therefore, the machines require to operate on a system cannot be designed in isolation and thus the designs of all such machines have to be completed concurrently since the design of one machine is closely related to the design of other machines. The problem of design of electrical machines in this case, is that of optimization of the system performance.

It is often desired to design a series of machines having different ratings to fit into a given *frame size*. In this case the machines over a range of ratings use the same lamination (same diameter D) but different core lengths (L) to achieve different outputs. In this case the finished designs of machines must be produced in groups, where all the designs within a group are interdependent. This is again an optimization problem because a frame size has to be optimum giving due weightage to the designs of all the machines within a group or a series.

It takes many iterations to arrive at an optimal solution. The iterations require changes in values of variables till both the cost and performance constraints are satisfied.

It is clear from above that the design of electrical machines is an iterative process wherein the assumed data may have to be varied many times to arrive at the desired design. The evolution of design to meet specified optimum criteria is matter of long and tedious

...ions and this is the biggest factor which has led to the wide spread use of digital computers for the design of electrical machines.

16·2. Advantages of digital computers. The digital computer has completely revolutionalized the field of design of electrical machines. The computer aided design eliminates the tedious and time consuming hand calculations thereby releasing the designer from numerical drudgery to enable him time to grapple with physical and logical ideas thereby accelerating the design process.

The use of computer makes possible more *trial* designs, and enables sophisticated calculations to be made without intolerable tedium and excessive time.

The advantages of use of a digital computer for the design of electrical machines may be summed up as :

(*i*) It has capabilities to store amount of data, count integers, round off results down to integers and refer to tables, graphs and other data in advance.

(*ii*) It makes it possible to select an optimized design with a reduction in cost and improvement in performance.

(*iii*) A large number of loops can be incorporated in the design programme and therefore it makes it easier to compare different designs out of which the best suited can be selected.

(*iv*) It performs all simple arithmetic operations at a high speed and makes it possible to produce designs in a short time.

(*v*) It is capable of automatic operation, going from one step to another without the attention of operator.

(*vi*) It reduces the probability of error with the result highly accurate and reliable results are obtained.

(*vii*) Larger maufacturing savings can be obtained by optimization of design. This optimization is economically feasible only through the use of digital computers.

(*viii*) It is capable of taking logical decisions by itself if programmed into thereby saving the man hour of the design engineers which can be utilized for other gainful work.

The high rate of performing calculations at reasonable cost and the ability to carry out logical decisions are the most important qualities of the present generation digital computers. For precisely these two predominating reasons digital computers have been extensively used in the past two decades for the design of electrical machinery. In fact digital computers have been responsible for bringing about a complete revolution in the field of electrical machine design.

16·3. Computer aided design—different approaches. The application of digital computers to the problems of electrical machine design was first introduced in 1950.

In the early stages, the use of digital computer for design of electrical machinery was limited to transformers only. In the year 1956, Moore[1] et al and Williams[2] et al discussed the applications of digital computers to the design of transformers. The advantages of digital computers described by them for design of transformers equally hold good for the design of rotating machines.

In the same year, Veinott[3] published a paper on application of digital computers for the design of induction motors. A flow chart was developed giving basic procedure for the design of polyphase induction motors. The input data consisted of machine specifications along with complete dimensions of stator and rotor and the output data gave performance details like resistance, magnetizing mmf, core losses, breakdown torque and costs of iron and copper. The paper in fact developed the ***analysis method*** for the design of polyphase induction motors.

The concept of optimization in electrical machine design was introduced by Godwin[4] in 1959. His program for optimization of design of squirrel cage induction motors has been presented in this paper. The program eliminates a large number of non-useful combinations

of design parameters. The useful parameters were retained and a few variables varied to give optimized design in the region of satisfactory performance.

In 1959 itself, Heroz[5] et al, through their paper, introduced the concept of two commonly acceptable approaches to machine design, namely

(*i*) Analysis method and (*ii*) Synthesis method.

16·3 1 Analysis method. The analysis method of design is depicted in Fig 16·1. In this method the choice of dimensions, materials, and types of construction are made by the designer and these are presented to the computer as input data. The performance is calculated by the computer and is returned to the designer for him to examine. The designer examines the performance and makes another choice of input, if necessary, and the performance is recalculated. This procedure is repeated over and over again till the performance requirements are satisfied.

The use of term ***analysis method*** means the use of computer only for the purposes of analysis leaving all exercises of judgement to the designer.

The analysis method of design is an excellent starting point for one beginning with the use of application of digital computers for the design of electrical machines and in fact most of the designers initially started off with this method.

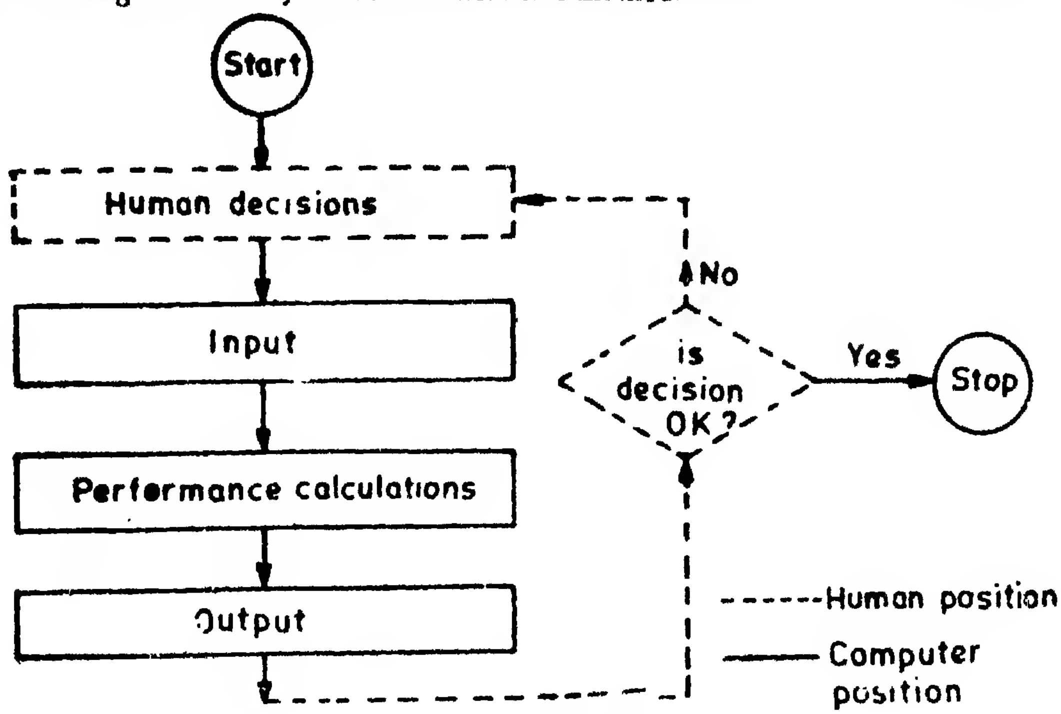


Fig. 16·1. Analysis method of design.

The advantages of analysis method are :

(*i*) It is fairly easy to program, to use and to understand.

(*ii*) It results in considerable time saving thereby giving quick returns on the investments made.

(*iii*) The programs based upon analysis methods are simple but they become the foundations for later day larger and sophisticated programs.

(*iv*) The results of analysis method are highly acceptable by designers.

16·3·2. Synthesis method. This method is depicted in Fig. 16·2. The desired performance is given as input to the computer. The logical decisions required to modify the values of variables to arrive at the desired performance are incorporated in the program as a set of instructions. Unlike in the case of Analysis method, the program or computer run is therefore not interrupted for the designer to take logical decisions.

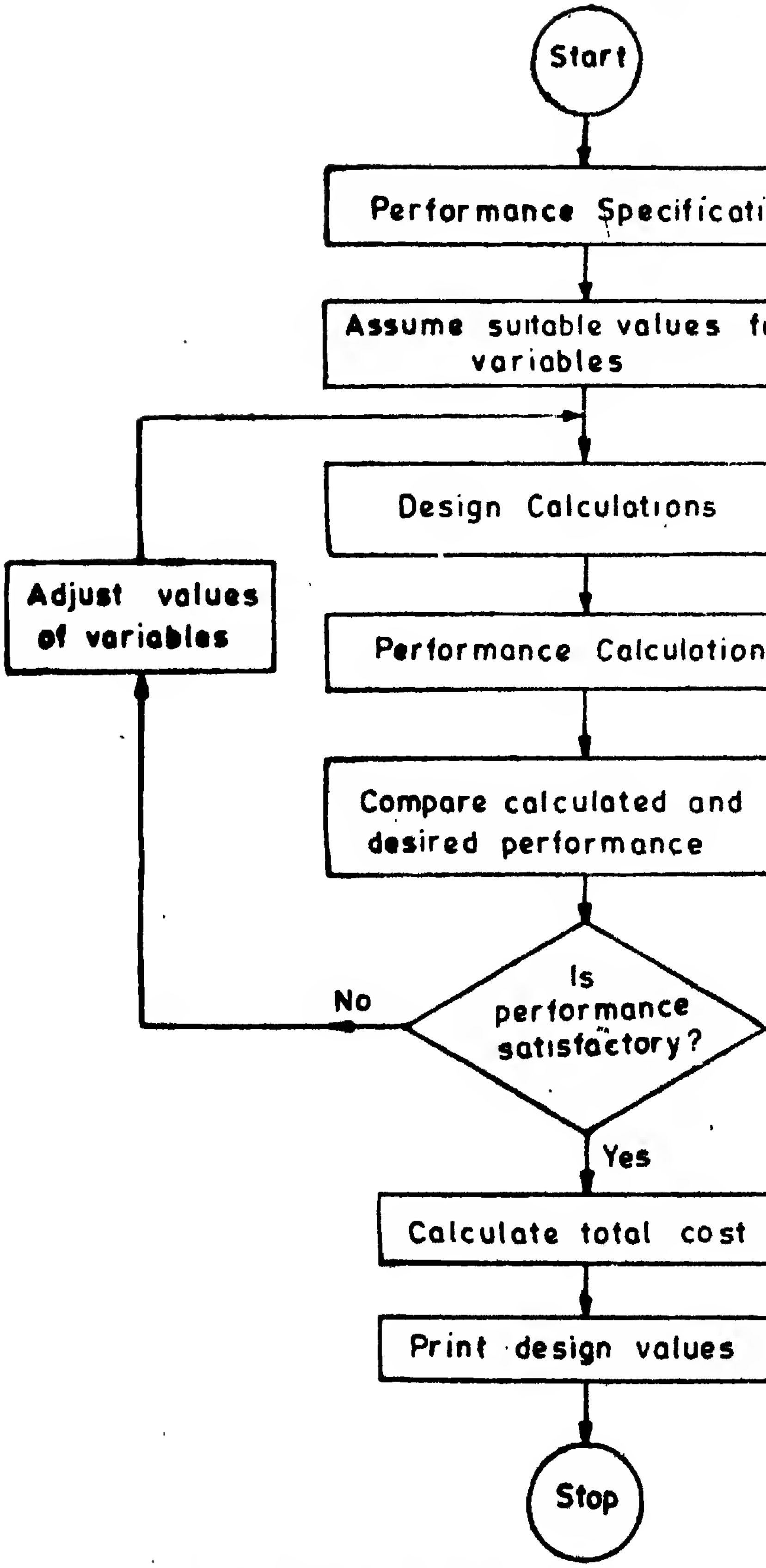


Fig. 16·2. Synthesis method of design.

Synthesis method of design implies designing a machine which satisfies a set of specifications or performance indices. Given a set of performance indices, an infinite number of designs can be produced to satisfy them. Therefore, the synthesis design should be such that it produces an optimum design because the object of any good synthesis program is just that only. Synthesis without optimization has no meaning.

The greatest advantage of synthesis method is the savings in time in lapsed time and in engineering man hours on account of the decision making left to the computer itself.

The synthesis method, however, suffers from a number of serious disadvantages. They are :

1. The synthesis method involves too much of logic since the logical decisions are left for the computer to take. Now, these logical decisions have to be incorporated in the program and before they are incorporated the team of design engineers have to agree upon them.

Firstly, the logical decisions to arrive at a optimum design are too many and then there are too many people with too many ways to suggest to produce an optimum design and it becomes really hard to formulate a logic that really produces an optimum design, a design which every contributor believes is optimum.

2. The formulation of a synthesis program taking into account the factors listed above would make it too complex. The complex program so formulated at high cost would require the use of a large computer and also large running time involving huge expenditure.

3. The synthesis program formulated at high cost cannot be used for ever. This program has to be changed and up dated keeping in view the changes in materials, manufacturing techniques, performance standards, relative costs of materials, market conditions and above all the change in the design logic itself. Therefore, a synthesis program requires both additional effort and cost in order that it is kept upto date.

16·3·3. Hybrid method. This method incorporates both the analysis as well as the synshesis methods in the program. Since the synthesis methods involve greater cost, the major part of the program is based upon analysis with a limited portion of the program being based upon synthesis.

16·4. Optimization. It has been pointed out earlier in Art. 1 4 on page 4 that the design of electrical machines consists essentially of the solution of many complex and diverse engineering problems and normally these problems are loosely interrelated. The design of electrical machines in fact presents a mathematically indeterminate problem with many solutions as the number of equations is less than the number of unknowns.

The aim of optimization in the design of electrical machines is to choose the best solution for a given problem from the multitude of possible solutions.

The optimization process, therefore, involves the choice of various variables in such a manner that the design in regard to a particular feature is the best, and at the same time satisfies all limitations or restraints imposed on its performance. Hence, optimization is the collective process of finding a set of conditions required to achieve the best results from a given situation.

In order to achieve the best possible solution, it is necessary to define the *objective* of study. The objective may vary from one problem to another, but for industrial applications it may be either economic or technical. Such economic aims as maximum profit or minimum cost are common, while possible technical objectives might include the largest yield of a particular product from a plant. Normally most industrial optimization has to be carried out within an economic frame work.

A characteristic feature of optimization in design of electrical machines is the presence of conflicting or opposing influences. For example, the cost of active materials in induction motors can be reduced by using high values of specific electric and magnetic loadings but

these high values of specific loadings will result in unsatisfactory performance like high temperature rise and poor power factor. The choice of low values of specific loadings has the opposite effect *i.e.* high cost and better performance. The best design will be obtained by the compromise of two main factors *i.e.* cost and performance, the two exerting opposing influences.

16 4 1. General procedure for optimization. The general objective in optimization is to choose a set of values of the independent variables, subject to various restrictions which will produce the desired optimum response for the particular problem under examination. A general approach or procedure can be listed below :

(*i*) Define a suitable objective for the problem under examination.

(*ii*) Examine the restrictions imposed upon the problem by external agencies.

(*iii*) Choose a system or systems for study.

(*iv*) Examine the structure of each system and the inter-relationship of the system elements and streams.

(*v*) Construct a model for the system. This is the technical design stage which allows the objective to be defined in terms of the system variables.

(*vi*) Examine and define the internal restriction placed on the system variables.

(*vii*) Carry out the simulation by expressing the objective in terms of the system variables, using the system model. This is the objective function.

(*viii*) Analyse the problem and reduce it to its essential features. This reduction is necessary in many cases to allow optimization to be attempted.

(*ix*) Verify that the proposed model in fact represents the system being studied.

(*x*) Determine the optimum solution for the system and discuss the nature of the optimum conditions.

(*xi*) Using the information thus obtained, repeat this procedure until a satisfactory result is found.

16 4 2. Variables and constraints. The numerical quantities for which values are to be choosen in producing a design are called 'design variables'. The objective function 'Y' is expressed in terms of the independent variables $\overline{v}$, where $\overline{v}$ represents all the variables

$v_1, v_2, \ldots\ldots, v_n$, as

$$Y(\overline{v}) = Y(v_1, v_2, \ldots\ldots, v_n)$$

subject to m restrictions, generally termed as constraints, of the form

$$g_i(v) = 0$$

or $$g_i(v) < 0$$

the problem is independent of actual application. Here $g_i(v)$ can be a variable or a function. The sole considerations are now mathematical, and the optimization techniques to be used are determined by the mathematical structure of the objective function and the associated restrictions.

16 5. Computer aided design of three phase induction motors. The conventional design of 3 phase induction motors is given in Chapter 10 The computer program for the design using *synthesis* approach can be formulated with the help of the flow chart and arithmetic statements given below. The arithmetic statements are based upon the design equations given in chapter.

LIST OF SYMBOLS USED

AKW—Winding factor.

AMEW—Permeability of free space.

AC—Ampere conductors per metre.

AX—Stator slot depth to width ratio.

ASS—Area of stator slot, mm^2.

ATG—Air gap mmf.

AST—Area of stator tooth at 1/3 rd height from narrow end, m^2.

ATST—Total mmf. for slator teeth.

ATSC—Total mmf. for stator core.

ART—Area of rotor tooth at 1/3 rd height from narrow end, m^2.

ATRT—Total mmf for rotor teeth.

ATRC—Total mmf for rotor core.

AGI—Air gap length, mm.

BCLOS—Rotor bar copper losses, W.

BTSS—Stator tooth flux density considering saturation effect, Wb/m^2.

BAV—Average air gap flux density, Wb/m^2.

BCS—Flux density in stator core, Wb/m^2.

BARA—Area of rotor bar, mm^2.

BTR—Maximum flux density in rotor teeth, Wb/m^2.

BLTH—Length of rotor bar, m.

BTRS—Flux density in rotor teeth considering sturation Wb/m^2.

BTSM—Maximum flux density in stator teeth at 1/3 rd height from narrow end, Wb/m^2.

BIMP—Impedance of rotor bar at starting.

BTS—Maximum flux density in stator teeth, Wb/m^2.

BRS—Bar resistance at starting.

CO—Output coefficient.

CI1—Current per phase in stator conductors.

CONA—Area of stator conductor, mm^2.

CIB—Rotor bar current.

CIE—End ring current.

CGCS—Carter's coefficient for stator slots.

CGCR—Carter's coefficient for rotor slots.

CCCD—Carter's co-efficient for ventilating ducts.

CIM—Magnetizing component of no load current per phase.

CIL—Loss component of no load current per phase.

CINL—No load current per phase.

CI—Effect of magnetising branch on torque of motor.

CIFL—Full load current.

CTST—Starting current.

CI—Cost of iron per kg.

CC—Cost of stator copper per kg.

CR—Cost of rotor copper per kg.

DIA—Bore diameter, m.

DELTA—Current density in stator conductor, A/mm^2.

DSS—Depth of stator slot, mm.
DSC—Depth of stator core, m.
DELB—Current density in rotor bars and end rings, A/mm
DRB—Depth of rotor bar, mm.
DRS—Depth of rotor slot, mm.
DE—Depth of end ring, mm.
DEO—Outer diameter of end rings, m.
DEI—Inner diameter of end rings, m.
DEM—Mean diameter of end rings, m.
EFF—Rated efficiency.
ERAR—Area of end ring, mm^2.
EAGL—Effective air gap length, mm.
ECLOS—Copper losses in end rings, W.
ES—Rated voltage per phase.
FLUX—Flux per pole.
FWL—Friction and windage losses, W.
GCFS—Gap contraction factor for stator slots.
GCFD—Gap contraction factor for ventilating ducts.
GCFT—Total gap contraction factor.
LIST—Total iron loss in stator teeth, W.
LIC—Total iron loss in stator core, W.
ND—Number of ventillating ducts.
NLL—Total no load losses, W.
OD—Outer diameter of stator lamination, m.
OP—Permeance of overhang portion.
OLR—Overhang leakage reactance.
OHL—Overhang length.
POL—No. of poles.
PF—Rated power factor.
PFNL—Power factor at no load.
PSS—Specific slot permeance for stator slots.
PRS—Specific slot permeance for rotor slots.
PFFL—Full load power factor.
QS—Stator slots per pole per phase.
QR—Rotor slots per pole per phase.
RKVA—kVA rating of machine.
RKD—kW rating of machine.
RDIA—Rotor outer diameter, m.
RSID—Inner diameter of rotor lamination, m.
RGSI—Reluctance of air gap with slotted armature.
RGS—Reluctance of air gap with smooth armature.

RCPATH—Length of flux path through rotor core, m.
RM—Resistance due to core losses.
RS—Stator resistance per phase.
RB—Resistance of each rotor bar.
RE—Resistance of each end ring.
RCLOS—Total rotor copper losses, W.
RROT—Total rotor resistance.
RSR—Stator referred rotor resistance per phase.
RPRS—Rotor slot specific permeance referred to stator side
RSLR—Stator referred, rotor slot leakage reactance.
RSO—Outside cylindrical surface of rotor, m^2.
RCO—Cooling coefficient for outside rotor surface.
RSP—Relative peripherical speed of rotor surface.
RSD—Surface of ventilating ducts, m^2.
RCD—Cooling coefficient for ventilating ducts.
RTRISE—Rotor temperature rise.
SLTH—Stack length of machine.
SYN—Synchronous speed, r.p.s.
SS—Number of stator slots.
SR—Number of rotor slots.
SPRB—Specific resistivity of rotor bars.
SLTNI—Net iron length, m.
STKF—Stacking factor for laminations.
SLF—Slot factor.
SATST—Mmf per metre for stator teeth.
SATSC—Mmf per metre for stator and rotor core.
SCPATH—Length of flux path through stator core, m.
SATRT—Mmf per metre for rotor teeth.
SPLST—Loss per kg in stator teeth, W.
SPLC—Loss per kg in core, W.
SCML—Mean length of stator conductor, m.
SCLP—Length of conductor per phase, m.
SCLOS—Stator copper losses, W.
SLIP—Slip at rated speed.
SSLR—Stator leakage reactance.
SRROT—Total rotor resistance at starting,
SRSR—Stator referred starting rotor resistance per phase.
SCR—Slip corresponding to maximum torque.
STCR—p.u. starting current.
SSO—Outside cylindrical surface of stator, m^2.
SSI—Inside cylindrical surface of stator, m^2.
SCO—Cooling co-efficient for outside stator surface.

SPS—Relative peripherical speed of stator surface.
SCI—Cooling co-efficient for inner stator surface.
SSD—Surface of ventilating ducts, m^2.
SSD—Cooling co-efficient for ventilating ducts.
SLOS—Total stator power loss, W.
STRISE—Stator temperature rise.
TS—Number of turns per phase.
TWS—Stator tooth width, m.
TWR—Rotor tooth width, mm.
TAT—Total magnetizing mmf.
TWM—Mean width of stator tooth, m.
TIL—Total iron losses, W.
TFL—Full load torque.
TST—Starting torque.
TRT1—p.u. starting torque.
TMAX—Pull out torque.
TRT2—p.u. pull out torque.
TIC—Total cost of iron in rupees.
TCW—Total cost of winding in rupees.
TC—Total cost of active material in rupees.
TE—Thickness of end ring, m.
TCLOS—Total copper losses, W.
TFR—Transformation ratio.
WD—Width of ventillating ducts, m.
WSS—Width of stator slot, mm.
WSO—Width of slot opening, mm.
WTS—Width of stator teeth at 1/3rd height from narrow end, m.
WRT—Width of rotor tooth at 1/3rd height from narrow end, m.
WTST—Weight of stator teeth, kg.
WTCI—Weight of iron in stator core, kg.
WTRI—Weight of iron in rotor, kg.
WSTW—Weight of stator windings, kg.
WTRW—Weight of rotor winding, kg.
WRS—Width of rotor slot, mm.
XM—Magnetizing reactance.
XZ—Zig-zag leakage reactance.
XL—Total leakage reactance per phase.
XS—Total stator leakage reactance per phase.
XR—Total rotor leakage reactance referred to stator side per phase.
YSS—Stator slot pitch, m.
XRS—Rotor slot pitch, m.
YSS1—Contracted slot pitch, m.
ZSS—Stator conductors per slot.

ZS—Stator circuit impedance per phase.

ZR—Rotor circuit impedance per phase.

ZM—Magnetizing branch impedance $G1+jG2$.

ZRM—Impedance of rotor and magnetising circuit—$G3+jG4$.

ZI—Total series impedence referred to stator, per phase.

16·5 2. General design procedure. The flow chart, using synthesis approach of design is developed in Fig. 16·3. The description of the flow chart, blockwise, is as under :

B.1. In this block the specifications of motor like kW and voltage ratings, efficiency, power factor, number of poles and constraints values like pull out torque, starting torque, permissible temperature rise and flux densities in stator and rotor teeth are fed. Initial values of variables and the values of assigned parameters are set here.

B.2. In this block the following equations are fed to calculate the values of bore diameter and stack length of the machine.

The output coefficient

CO=·011*AKW*BAV*AC ...(16·1)

SYN=2*50/POL ...(16·2)

RKVA=RKW/(PF*EFF) ...(16·3)

DIA=(RKVA*POL/(CO*RYN*PR*3·14)**(1/3) ...(16·4)

SLTH=PR*3 14*DIA/POL ...(16·5)

After calculating the stack length, the net iron length is calculated taking into account the space occoupied by cooling ducts and stacking of laminations.

SLTNI=STKF*(SLTH−ND*WD) ...(16·6)

B.3. In this block the following equations are fed to calculate stator parameters :

FLUX=BAV*3·14 DIA*SLTH/POL ...(16·7)

TS=AS/(4·44*50*FLUX*AKW) ...(16 8)

Here the number of slots per pole per phase QS, has to be decided by designer. The value of QS is generally kept between 2 to 5.

Total number of stator slots

SS=3*POL*QS ...(16·9)

Stator slot pitch

YSS=3·14*DIA/SS ...(16·10)

Stator conductors per slot

ZSS=6*TS/SS ...(16·11)

Stator current per phase

CI1=RKVA*1000/3*ES ...(16·12)

At this point the current density DELTA in stator conductors has to be selected by the designer. DELTA is expressed in A/mm^2.

Area of stator conductor

CONA=CI1/DELTA ...(16·13)

Taking a suitable value of slot fullness factor SLF. (The value of SLF depends upon insulation needed).

ASS=ZSS*CONA/SLF ...(16·14)

AX is the ratio stator slot depth to width which is fixed by designer at an initial value. Hence depth of stator slot

DSS=SQRT (ASS*AX) ...(16·15)

and WSS=ASS/SS ...(16·16)

ASS, DSS and WSS are in mm.

Stator tooth width at the gap surface

TWS=YSS-·001*WSS ...(16·17)

At this stage the flux density in the stator teeth is checked.

BTS=FLUX/(SS*TWS*SLTNI/POL) ...(16·18)

This flux density should not exceed 1·7 Wb/m².

Depth of stator core

DCS=(OD*DIA*·001*DSS)/2 ...(16·19)

where OD is outer diameter, which is fixed for a standard frame size.

Flux density in stator core

BSC=FLUX/DCS*SLTNX ...(16·20)

B.4. In this block the number of rotor slots are so selected that smooth starting and accelerating conditions are obtained as discussed in Art. 10·22 on page 615. For this purpose the initial value of number of rotor slots is taken equal to 1·25 time SS, and this is modified until the following conditions are satisfied

SS-SR≠0, 1, 2, POL, POL±1, POL±2, 2*POL, 3* POLand 5*POL ...(16·21)

The minimum value of number of slots should not go below 0·8SS.

B.5. In this block the air gap length and rotor parameters are calculated with the help of following.

Air gap length in mm

AGL=·3+SQRT (DIA*SLTH)*2 ...(16·22)

where DIA and SLTH are in m.

Rotor diameter

RDIA=DIA-·002*AGL ...(16·23)

Rotor slot pitch

YRS=3·14*RDIA/SR ...(16·24)

Rotor bar current

CIB=0·85*6*CI 1*TE/SR ...(16·25)

Area of each bar

BARA=CIB/DELB ...(16·26)

The width of rotor teeth is taken equal to the width of rotor slots.

WRS=TWR=1000*YRS/2 ...(16·27)

In the rotor slot, 1 mm clearance is left in slot width and 3 mm in slot depth to determine the bar area, hence depth of rotor bar

DRB=BARA/(WRS-1) ...(16·28)

and depth of rotor slot is

DRS=DRB*3 ...(16·29)

The flux density in the rotor teeth is then checked.

BTR=FLUX/(SR*TWR*0·001*SLTNI/POL) ...(16·30)

The rotor bars are skewed. Extending the bars by about 20 mm beyond the core on each side and taking 10 mm as increase in length because of skewing, length of rotor bar

BLTH=SLTH*·05 ...(16·31)

The end ring current is given by following relationship

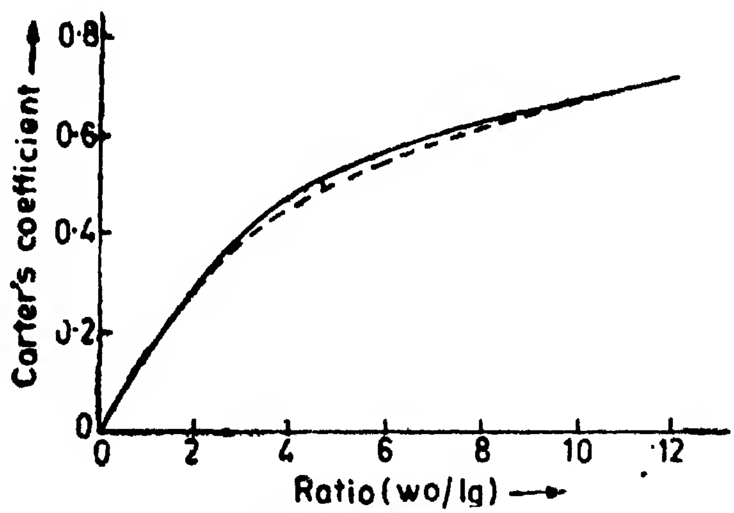
CIE=SR*CIB/3·14*POL ...(16·32)

The current density in end rings is taken same as that in rotor bars. So end ring area

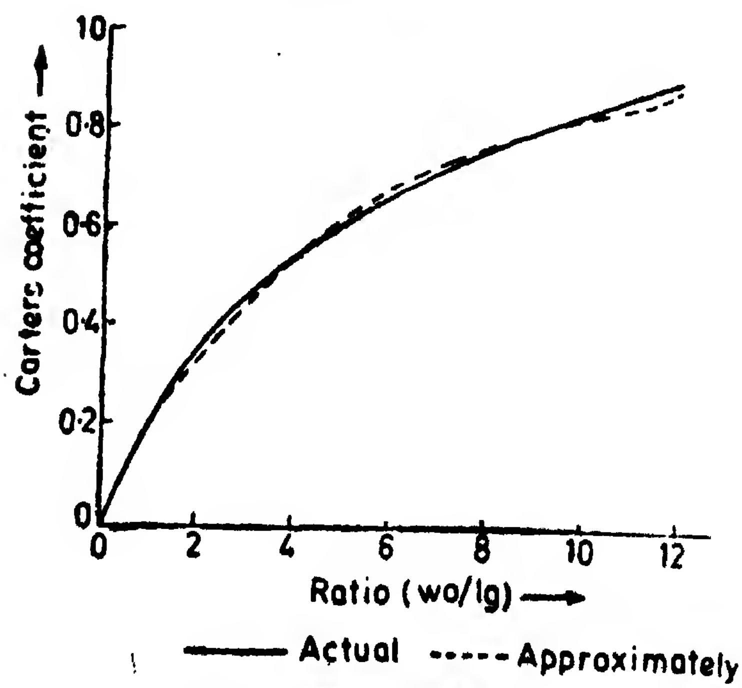
ERAR=CIE/DELB ...(16·33)

The end ring strip of depth DE and thickness TE is used. The depth and thickness ratio is fixed as

DE=TE*3 ...(16·34)



(a) Open slots.



(b) Semi-enclosed slots

Fig. 16·4. Air gap co-efficient.

Outer diameter of end ring

DEO=RDIA−·002*DRS ...(16·35)

Inner diameter of end ring

DEI=DEO−·002*DE ...(16·36)

Mean diameter of end ring is taken as average of outer and inner diameters.

Depth of rotor core is taken equal to depth of stator core.

Allowing for an axial ventiating duct of 70 mm width in rotor to allow air to circulate through rotor, the inner dia of rotor laminations is

RSID=RDIA−·002*DRS−2·*DCS−·07 ...(16·37)

B.6. In this block the performance of the designed machine is evaluated. This involves the calculation of no load current, no load power factor, losses and efficiency.

No load current

(*a*) **Magnetising current**. In order to calculate magnetising component of no load current, the m.m.f. required for various parts of magnetic circuit of machine are calculated as below :

(*i*) *Air gap*. The effective or contracted slot pitch is given as

YSS1=YSS−CGCS*WSO

where CGCS is Carter's coefficient.

The curves giving the values of Carter's co-efficient are shown in Fig. 4·9 on page 124. These curves are approximated by line segments as shown in Fig. 16·4 (*a*) and 16·4 (*b*).

For the purposes of digital simulation the curve can be approximated by the following relationship.

CGCS=1/(1+3·5*AGL/WSO*·001) ...(16·38)

Reluctance of air gap with slotted armature

RGSI=AGL/(AMEW*YSS1*SLTH)

and the reluctance of air gap with smooth armature

RGS=AGL/(AMEW*YLS*SLTH)

Gap contraction factor for stator slots

GCFS=YSS/(YSS−CGCS*WSO) ...(16·39)

Similarly Carter's coefficient and gap contraction factor for rotor slot are given as

CGCR=1/(1+3·5*AGL/WRS*·001) ...(16·40)

GCFR=YRS/(YRS−CGCR*WRS*·001) ...(16·41)

The ventilating ducts are treated as open slots for the purposes of calculation of Carter's co-efficient and the gap contraction factor. For ventilating ducts we may assume half the gap length on stator side and half on rotor side. So the Carter's gap coefficient and gap contraction factor for ducts are

CGCD=1/(1+3·5*AGL/2*WD) ...(16·42)

GCFD=SLTH/(SLTH+COCN*ND*WD) ...(16·43)

Total gap contraction factor is given as

GCFT=GCFS*GCFR*GCFD ...(16·44)

Effective air gap length, taking into account, the gap contraction factor

EAGL=GCFT*AGL ...(16·45)

The flux density in the air gap is taken as 1·36 times the average flux density, BAV to take into account the saturation. Hence air gap m.m.f.

ATG=800*1·36*BAV*EAGL ...(16·46)

(*ii*) *Stator teeth.* Width of stator teeth at 1/3rd height from narrow end

WTS=3·14*(DIA*·002*DSS/3)/SS−WSS*·001 ...(16·47)

Area of stator teeth per pole at 1/3rd height

AST=SS*WTS*SLTNI/POL ...(16·48)

Flux density considering effect of saturation

BTSS=1·36*FLUX/AST ...(16·49)

Corresponding to this the m.m.f. per metre SATST is found from the curve in figure Fig. 16·5. For the purpose of computer solution the curve is approximated by equations as given in Table 16·1.

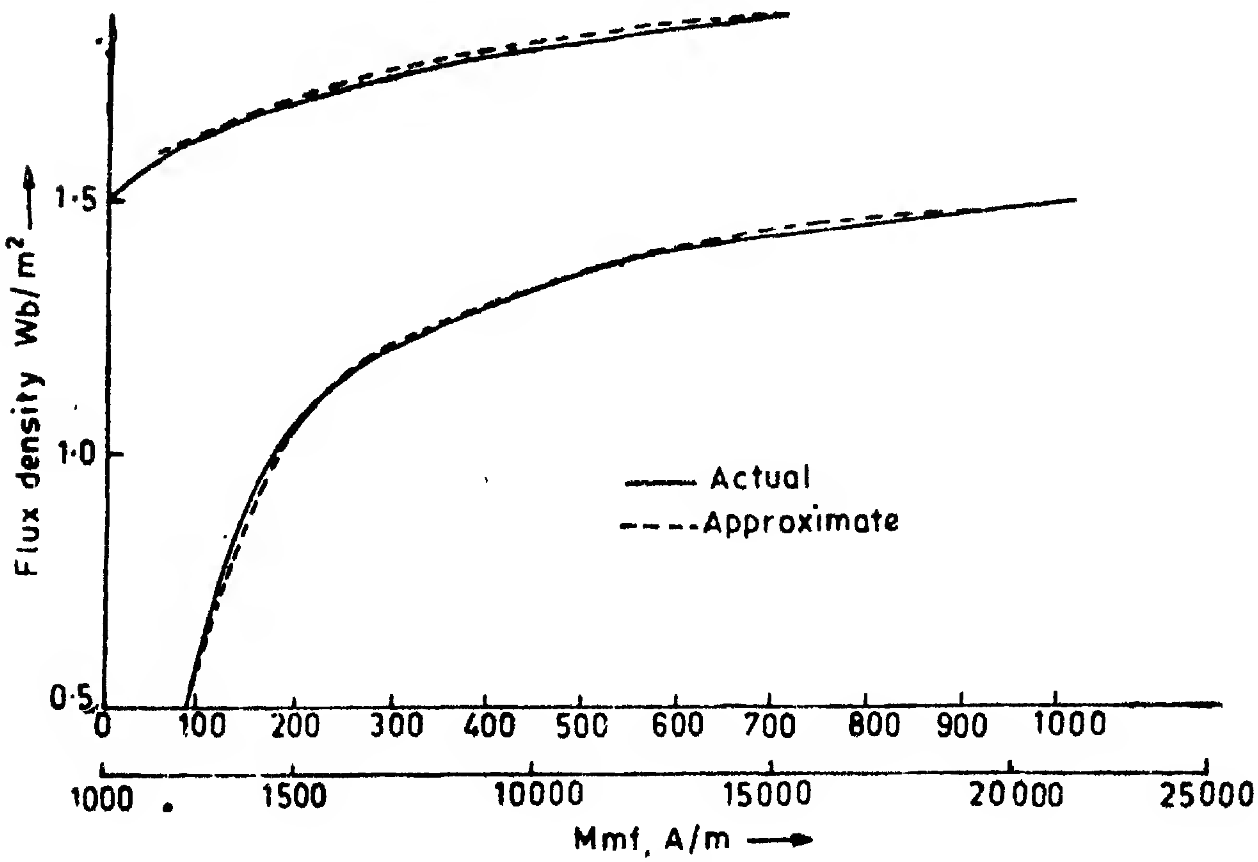


Fig. 16·5. Magnetization curves.

Hence the total m.m.f. required for stator teeth is

ATST=SATST*·001*DSS ...(16·50)

(*iii*) *Stator core.* The flux density in stator core is BCS (Eqn 16·12). Corresponding to this flux density the m.m.f. per metre SATSC is calculated as per Table 16·1. This m.m.f. is multiplied by length of flux path through stator core which is

SCPATH=3·14*(DIA+·002*DSS*DCS)/(3·*POL) ...(16·51)

Hence m.m.f. required for stator core

ATSC=SATSC*SCPATH ...(16·52)

(*iv*) *Rotor teeth.* Width of rotor teeth at 1/3rd height from narrow and

WRT=3·14*(RDIA−·004*DSS/3)/SR−·001*WRS ...(16·53)

Area of rotor teeth per pole at 1/3rd height

ART=SR*WRT*SLTNI/POL ...(16·54)

Flux density in rotor teeth considering effect of saturation

BTHS=1·36*FLUX/ART ...(16·55)

Corresponding to this flux density the m.m.f. per metre SATRT is calculated as per Table 16·1. Total m.m.f. required for rotor teeth

ATRT=SATRT*DRS*·001 ...(16·56)

(v) *Rotor core.* Flux density in rotor core is taken to be same as that in stator core *i.e.* BCS and hence m.m.f. per metre is also same as SATRC. This m.m.f. has to be multiplied by length of flux path through rotor core which is

RCPATH=3·14*(RDIA−·001*DRS−DCS)/3*POL ...(16·57)

M.m.f. required for rotor core

ATRC=SATSC*RCPATH ...(16·58)

Total m.m.f. required

TAT=ATC+ATST+ATSC+ATRT+ATRC ...(16·59)

Magnetising current per phase

CIM=0·427*POL*TAT/(AKW*TS) ...(16·60)

Table 16·1

Range of flux density, FD Wb/m²	*Corresponding equation*
0 to 0·6	AT=91·8*FD+45·0
0·6 to 1·0	AT=200·0*FD−20·0
1·0 to 1·45	AT=6·444* EXP(3·2*FD)
1·45 to 1·7	AT=0·0052*EXP (8·1*FD)
1·7 to 2·0	AT=0·687*EXP (5·23*FD)

(b) **Loss component.** The loss component of the current is calculated as under.

Mean width of stator teeth

TWM=3·14*(DIA+·001*DSS)/(SS−·001*WSS) ...(16·61)

Weight of stator teeth

WTST=DENI*SS*TWM*SLTNT*DSS*·001 ...(16·62)

Corresponding to maximum flux density the specific iron loss SPLST, is determined by the curve in Fig. 16·6. The curve is approximated by square rule upto a flux density of 1·6 Wb/m² and by cube rule beyond that value, for computer solution. Hence iron loss in stator teeth

LIST=SPLST*WTST ...(16·63)

Mean periphery of stator core=3·14*(OD−DCS)

Weight of iron in stator core

WTCI=DENI*3·14*(OD−DCS)*DCS*SLTNI ...(16·64)

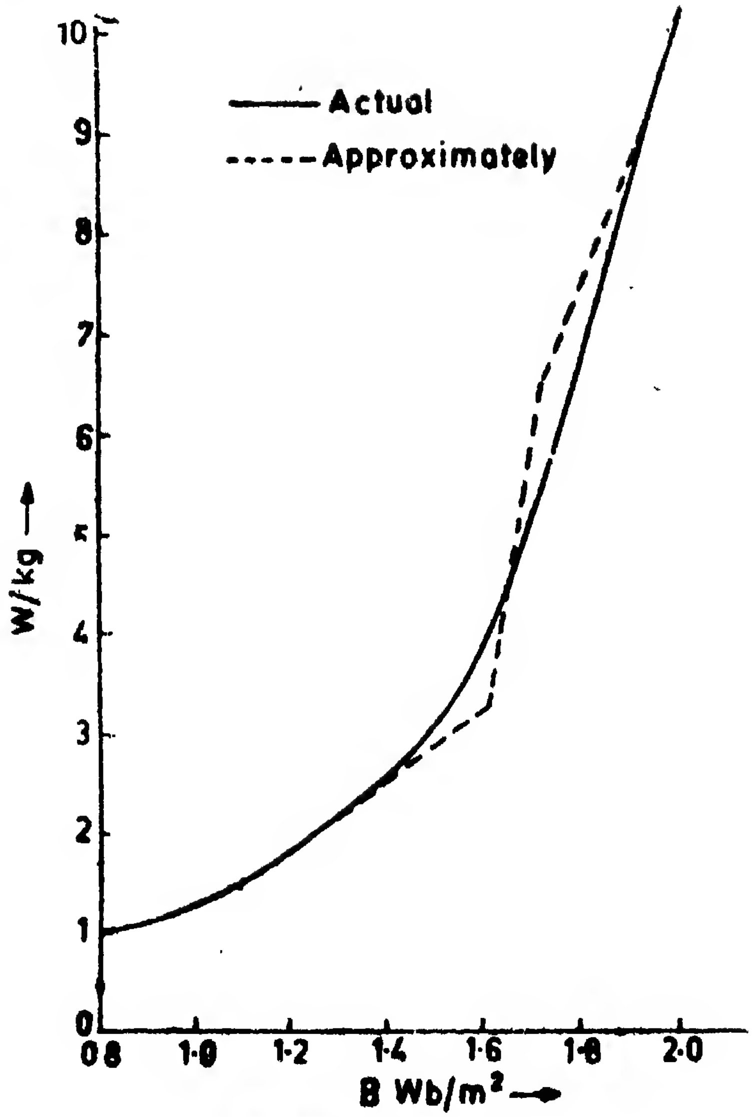


Fig. 16·6. Loss curves.

Flux density in stator core is BCS (Eqn. 16·19) corresponding to this flux density the specific iron loss SPLC is determined by square rule.

Iron loss in stator core

LIC=SPLC*WTCI ...(16·65)

Allowing for the pulsation losses the total iron losses are taken double the combined iron losses in stator teeth and core. Therefore

TIL=2*(LIST+LIC) ...(16·66)

Friction and windage losses are taken as 1% of output power. Hence

FWL=10*RKW ...(16·67)

Total no load loss

NLL=TIL+FWL ...16·68)

Loss component of no load current, per phase

CIL=NLL/(3*RS) ...(16·69)

No load current

CINL=SQRT(CIM**2+CIL**2) ...(16·70)

No load power factor

PFNL=CIL/CINL ...(16·71)

The magnetising reactance

XM=ES/CIM ...(16·72)

The resistance due to core losses

PM=ES/CIL ...(16·73)

Copper losses

Mean length of stator conductor

SCML=SLTH+1·15*3·14*DIA/POL+·012 ...(16·74)

Length of conductor per phase

SCLP=2*SCML*TS ...(16·75)

Stator resistance per phase

RS=·021*SCLP/CONA ...(16·76)

where 0·021 is specific resistivity of copper in Ω/m and mm^2.

Total stator copper loss

SCLOS=3*RS*CI1**2 ...(16·77)

Resistance of each rotor bar

RB=0·021*BLTH/BARA ...(16·78)

Copper losses in bars

BCLOS=SS*RB*CIB**2 ...(16·79)

Resistance of each and ring

RE=0·021*3·14*DEM/ERAR ...(16·80)

Copper loss in two end rings

ECLOS=2*RE*CIM**2 ...(16·81)

Rotor copper loss

RCLOS=BCLOS+ECLOS ...(16·82)

Total copper loss

TCLOS=SCLOS+RCLOS ...(16·83)

Efficiency EFF=RKW*1000/(RKW*1000+TCLOS+NLL) ...(16·84)

and SLIP=RCLOS/(RKW*1000+RCLOS+FWL) ...(16·85)

B.7. In this block the equivalent circuit parameters and full load power factor are calculated. The equivalent circuit of cage induction motor is shown in Fig. 16·7. The parameters RS, RM, XM, and slip are already calculated in block B-6. The remaining are RSR, XS and XR.

The total rotor resistance

RROT=SR*RB+2*RE ...(16·86)

In a cage rotor the transformation ratio can be determined as

TFR=6*TS*AKW/SR ...(16·87)

Sta r referred rotor resistance per phase

RSR=RROT*TFR**2/3.

To calculate reactance the height occupied by conductor portion in the slot is H1 (as shown in Fig. 16·8). HI may be given as

HI=SQRT(ZSS*CONA*AX) ...(16·88)

H3 and H4 may be taken as 3·5 and 1 mm respectively. This gives

H2=DSS−H1−H3−H4 ...(16·89)

Specific slot permeance for stator slot is given by

PSS=AMEW*(H1/(3*WSS)+H2/WSS+2H3/(WSS+WSO)
+H4/WSO) ...(16·90)

The stator slot leaakage reactance now may be calculated as

SSLR=8*3·14*50*TS**2*SLTH*PSS/(POL*QS) ...(16·91)

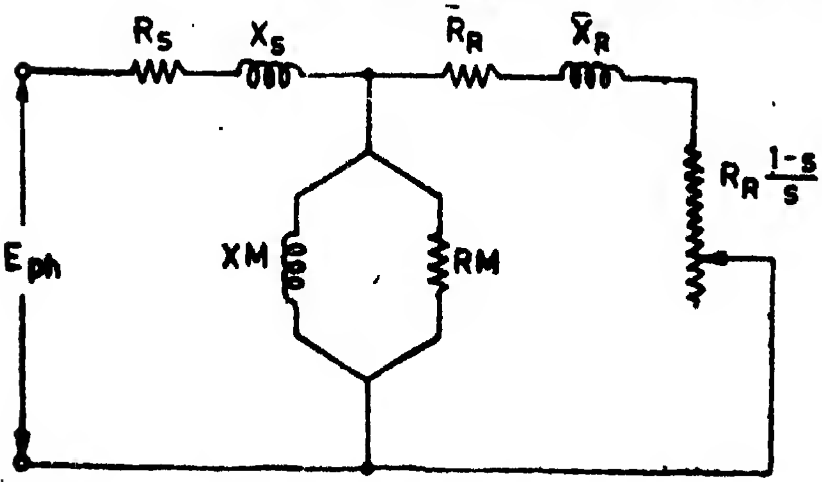


Fig. 16·7. Equivalent circuit of induction motor.

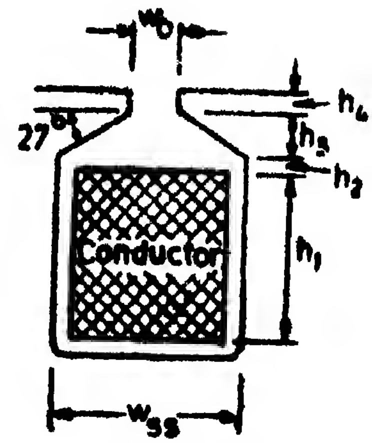


Fig. 16·8. Stator slot.

Similarly for calculating the rotor slot leakage reactance HR1 may be assumed to be equal to the depth of rotor bar, while HR 2 and HR 3 and HR 4 are assumed to be 0·75 mm, 1·75 mm and 0·5 mm respectively.

Specific slot permeance for rotor slot is :

PRS=AMEW*[HR1/(3*WRS)+HR2/WRS+HR3/WRS+HR4/WRS]
...(16·92)

The rotor slot specific permeance referred to stator side is :

RPRS=PRS*AKW**2*SS/SR ...(16·93)

Rotor slot leakage reactance referred to stator side

RSLR=8*3·14*50*TS**2*SLTH*RPRS/(POL*QS) ...(16·94)

Permeance of overhang

OP=AMEW (3·14**DIA/POL)**2/(3·14*YSS) ...(16·95)

∴ Overhang leakage reactance

OLR=8*3 14*50*TS**2*OP/(POL*QS) ...(16·96)

Zig-zag leakage reactance is given by the following relationship

XZ=5*XM/(6*3**2)*{1/(QS**2)+1/(QR**2)} ...(16·97)

Here the number of rotor slots per pole per phase may be calculated as

QR=SR/(3*POL) ...(16·98)

The differential leakage reactance is ignored for squirrel cage induction motor.

Total leakage reactance of machine

XL=SSLR+RSLR+OLR+XZ ...(16·99)

Total stator leakage rectance

XS=SSLR+·5*OLR+XZ ...(16·100)

And total rotor leakage reactance, referred to stator side

XR=RSLR+·5*OLR ...(16·101)

Hence all the equivalent circuit parameters are determined.

Now the equivalent circuit may be solved by dividing it into three parts in order to determine the full load power factor.

(*i*) Stator circuit impedance ZS=RS+*j*XS

(*ii*) Rotor circuit impedance ZR=RSR/SLIP+*j*XR

(*iii*) Magnetic circuit impedance

PH=*j*RM*XM/(RM+*j*XM)

=(RM*XM**2+*j*(RM**2)*XM)/((RM**2)+(XM**2))

This may be written as

PM=G1+*j*G2

where G1=RM*XM**2/((RM**2)+XM**2)) ...(16 102)

G2=RM**2*XM/((RM**2)+(XM**2)) ...(16 103)

Now solving the parallel combination of rotor and magnetic circuit, the impedance comes to be

ZRM=G3+*j*G4

where G3=(RSR*G1/SLIP−XR*G2)*(RSR/SLIP+G1)

+(RSS*G2/SLIP+XR*G1)*(XR+G2)/((RSS/SLIP+G1)**2)

+(XR+G2)**2) ...(16·104)

and G4=(RSR/SLIP+G1)*(RSR*G2/SLIP+XR*G1)

−(XR+G2)*(RSR*G1/SLIP−XR*G2)/((RSR/SLIP*G1)**2

+(XR+G2)**2) ...(16·105)

The total series impedance referred to stator side can now be expressed as

S1=RS+*j*XS+G3+*j*G4

=(RS+G3)+*j*(XS+G4)

Full load power factor may now be calculated as

PFFL=SQRT(RS+G3) ((RS+G3)**2+(XS+G4)**2)) ...(16·106)

B.8. In this block the starting resistance of rotor is calculated, taking into consideration the deep bar effect. The bar impedance may be calculated as under

BIMP=SQRT (BLTH/(WSS)*3·14*50*AMEW*0·021*(1+*j*)*

(SINH(2*THETA)−*j*SIN (2*THETA))/(COSH

(2*THETA)−COS(2*THETA))) ...(16·107)

where THETA=0·001*DRB*SQRT (3·14*50*AMEW*0·021) ...(16·108)

Now separating real and imaginary terms the bar resistance at the time of starting

BRS=REAL (BIMP) ...(16·109)

Total rotor resistance at the time of starting

SSROT=R2*BRS+2*RK ...(16·110)

Transforming it to stator side, the stator referred rotor resistance per phase at the time of starting

SRSR=SRROT*(TFR)**2/3 ...(16·111)

B.9. In this block the full load torque, pull out torque, and starting torque are calculated. Let C1 be the the effect of magnetising branch on torque of motor. This effect is approximated as

C1=1+RS/RM+XS/XM ...(16·112)

The full load torque is given by

TFL=3*(ES**2)*RSP/(SLIP)/((RS+C1**RSR/(SLIP))**2

+(XS+C1*XR)**2) ...(16·113)

At the time of starting the SLIP is unity and the rotor resistance is replaced by its value at the time of starting. Thus, starting torque is

TST=3*(ES**2)*SRSR/((RS+C1+SRSR)**2
+(XS+C1***R)**2) (16·114)

The p.u. starting torque is given by the ratio of starting torque to full load torque. Thus,

TRT1=TST/TFL ...(16·115)

The slip corresponding to maximum torque is given by

SCR=RSR/(SQRT(RS**2+(XS+XR)**2))

Substituting this value in equation 16·113, the maximum or pull out torque may be calculated.

TMAX=3*ES**2/(2*C1*RS/(RS**2+(XS+C1*XR)**2)) ...(16·116)

The ratio maximum torque to full load torque is called p.u. maximum torque

TRT2=TMAX/TFL ...(16·117)

B.10. In this block the ratio starting current to full load current is calculated. The full load current is given by relationship

CIFL=ES/(SQRT(RS+C1*RSR/SLIP)**2)
+(XS+C1*XR)**2) ...(16·118)

The starting current may be calculated by replacing RSR by SRSR and putting SLIP=1,

CIST=ES/(SQRT (RS+CI*SRSR)**2
+(XS+CI*XP)**2)) ...(16·119)

The ratio starting current to full load current is

STCB=CIST/CIFL ...(16·120)

B.11. In this block the stator and rotor temperature rises are calculated. For determining the temperature rise the cooling coefficients for various portions of machine are taken from Table 3·6 on page 111.

Stator temperature rise

Outside cylindrical surface of stator

SSO=3·14*OD*SLTH ...(16·121)

Cooling coefficient for outside surface CO=0·033

Inside cylindrical surface of stator

SSI=3·14*DIA*OHL ...(16·122)

where overhang length

OHL=SLTH+·0254*(0 001*ES+3·0+YSS/4) ...(16·123)

Relative peripherial speed of inner surface

RPS=3·14*DIA*SYN ...(16·124)

Cooling coefficient for inner surface

SCI=·033/(1+0·1*SPS) ...(16·125)

Surface of ventilating ducts

SSD=3·14*(OD**2−DIA**2)*(2+ND)/4 ...(16·126)

Cooling coefficient for ventilating ducts

SCD=0·15/(0·1*SPS) ...(16·127)

Total stator loss

SLOS=SCLOS*SLTH/(SCML)+TIL ...(16·128)

Stator temperature rise

STRISE=SLOS/(SSO/SCO+SSI/SCI+SSD/SCD) ...(16·129)

Rotor temperature rise

Outside cylindrical surface of rotor

RSO=3·14*RDIA*SLTR ...(16·130)

Cooling coefficient of outside rotor surface

RCO=·033/(1+0·1*SPS) ...(16·131)

Surface of ventilating ducts

RSD=3·14*(RDIA**2−RSID**2)*(2+ND)/4 ...(16·132)

Cooling coefficient for ventilating ducts

RCD=0·15/(0·1*SPS) ...(16·133)

Rotor temperature rise

RTRISE=(RCLOS+FWL)/(RSO/RCO+RSD/RCD) ...(16·134)

B.12. The output is printed.

In blocks C—1 to C—8, the various constraints are checked as given in blocks.

Cost of active materials

Weight of iron in stator teeth and core is already calculated in preceding sections.

Weight of iron in rotor

WTRI=DENI*SLTNI*(3·14*(RDIA**2−RSID**2)/4
−(SR*DRS*WRS*E−6)) ...(16·135)

Total cost of iron

TIC=CI*(WTST+WTCI+WTRI) ...(16·136)

where CI is the specific cost of iron.

Weight of stator winding

WTSW=TS*SCML*CONA*DENC*E−6 ...(16·137)

Weight of rotor winding

WTRW=(SR*BARA*BLTH+2*3·14*ERAR*DEM)
*DENC*E−6 ...(16·138)

Total cost of winding

TCW=CC*WTSW+CR+WTRW ...(16·139)

where CC is specific cost of winding material.

Total cost of active materials

TC=TIC+TCW ...(16·140)

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17

Standard Specifications

CONDUCTORS

17·1. Indian Standard Specifications for copper conductors used in electrical machines and apparatus

1. **Copper strip** (IS 1897—1962). The dimensions of bare copper strip used for windings of transformers and rotating machines are given in Table 17·1.

2. **Paper covered rectangular copper conductors for transformer windings** (IS 1656—1961). The sizes of bare rectangular copper conductors are given in Table 17·1. The increase in dimensions due to paper covering over the dimensions given in Table 17·1 can be taken as :

(*a*) For double paper covered—minimum increase 0·25 mm

(*b*) Multiple paper covered—the increase depends upon the number of layers applied.

3. **Cotton covered rectangular copper conductors** (IS 2068—1962). The sizes of bare copper rectangular conductors are given in Table 17·1. For cotton covered conductors width should not be more than 13 mm and thickness should lie between 0·8 and 6·5 mm.

The increase in dimensions due to cotton covering is

Double cotton covering (ordinary)—0·46 to 0·51 mm.

Double cotton covering (fine) —0·36 to 0·43 mm.

4. **Rectangular enamelled copper conductors** (IS 3855—1966).

The sizes of bare strip are given in Table 17·1. The following additional sizes are available.

Table 17·2. Additional sizes of rectangular enamelled conductors.

Dimensions mm	*Area* mm^2	*Dimensions* mm	*Area* mm^2
0·9×0·9	0·67	2·2×2·2	4·58
1·0×1·0	0·86	2·5×2·5	5·94
1·2×1·2	1·30	2·8×2·8	7·28
1·4×1·4	1·75	3·0×3·0	8·44
1·6×1·6	2·35	3·2×3·2	9·86
1·8×1·8	2·93	3·5×3·5	11·69
2·0×2·0	3·69	3·8×3·8	13·38

The following may be taken as increase in dimensions due to enamel (synthetic) covering.

Table 17·3.

Grade of covering	*Minimum increase mm*	*Maximum increase mm*
Fine	0·035	0·060
Medium	0·060	0·100
Thick	0·100	0·150

5. **Paper covered round copper conductors** (IS 3454—1966). Table 17·4 gives the sizes of these conductors.

Table 17·4. Diameter of paper—covered round copper conductors.
(IS : 3454—1966)

Conductor diameter (Nominal) mm	*Overall diameter—maximum*	
	Ordinary covering mm	*Fine covering mm*
0·250	0·500	0·425
0·280	0·530	0·455
0·315	0·565	0·490
0·355	0·605	0·530
0·400	0·650	0·575
0·450	0·700	0·625
0·500	0·750	0·675
0·560	0·810	0·735
0·630	0 880	0·805
0·710	0·985	0·885
0·730	1·025	0·925
0·800	1·075	0·975
0·850	1·125	1·025
0·900	1·175	1·075
0 950	1·225	1·125
1·000	1·275	1·200
1·060	1·335	1·260
1·120	1·395	1·320
1·180	1·455	1·380
1·250	1·525	1·450
1·320	1·595	1·520
1·400	1·700	1·575
1·500	1·800	1·675
1·600	1·900	1·775
1·700	2·000	1·873
1·800	2·100	1·975
1·900	2·200	2·075
2 000	2·350	2·250
2·120	2·470	2·370
2·240	2·590	2·490
2·360	2·710	2·610
2·500	2·850	2·725
2·650	3·000	2·875
2·800	3·150	3·025
3 000	3·350	3·225
3·150	3·500	3·375
3·350	3·700	3·575
3·550	3·900	3·775
3·750	4·100	3·975
4·000	4·350	4·300
4 250	4 600	4·550
4·500	4·850	4·800
4·750	5·100	5·050
5·000	5·350	5·800

6. **Cotton covered round copper conductors** (IS 450—1964). The sizes are given in Table 17·5.

Table 17·5. Cotton covered round copper conductors. (IS : 450—1964)

Conductor diameter	*Overall diameter—maximum*			
	Single cotton		*Double cotton*	
Nominal	*Ordinary*	*Fine*	*Ordinary*	*Fine*
mm	*mm*	*mm*	*mm*	*mm*
0·140	0·244	0·229	0·371	0·295
0·160	0·264	0·249	0·391	0·315
0·180	0·284	0·269	0·411	0·335
0·200	0·304	0·289	0·431	0·355
0·224	0·354	0·326	0·481	0·404
0·250	0·380	0·352	0·507	0·430
0·280	0·410	0·382	0·537	0·461
0·315	0·445	0·417	0·572	0·496
0·355	0·486	0·458	0·613	0·531
0·400	0·531	0·503	0·658	0·582
0·450	0·582	0·554	0·709	0·632
0·500	0·632	0·604	0·759	0·683
0·560	0·693	0·665	0·820	0·744
0·630	0·763	0·736	0·891	0·814
0·710	0·869	0·842	0·997	0·895
0·750	0·910	0·882	1·037	0·935
0·800	0·960	0·933	1·087	0·986
0·850	1·011	0·983	1·138	1·036
0·900	1·061	1·034	1·188	1·087
0·950	1·112	1·084	1·239	1·137
1·000	1·162	1·135	1·290	1·215
1·060	1·225	1·195	1·350	1·275
1·120	1·285	1·[illegible]55	1·410	1·335
1·180	1·345	1·315	1·470	1·395
1·250	1·415	1·385	1·540	1·465
1·320	1·485	1·460	1·615	1·535
1·400	1·590	1·565	1·720	1·645
1·500	1·695	1·665	1·820	1·745
1·600	1·795	1·765	1·920	1·845
1·700	1·895	1·865	2·020	1·945
1·800	1·995	1·970	2·125	2·045
1·900	2·095	2·070	2·225	2·150
2·000	2·225	2·195	2·375	2·275
2·120	2·345	2·315	2·495	2·395
2·240	2·465	2·440	2·620	2·515
2·360	2·585	2·560	2·740	2·635
2·500	2·730	2·700	2·880	2·780
2·650	2·880	2·850	3·035	2·930
2·800	3·030	3·005	3·185	3·080
3·000	3·235	3·205	3·385	3·285
3·150	3·385	3·355	3·540	3·435
3·350	3·585	3·560	3·740	3·635
3·550	3·790	3·760	3·940	3·840
3·750	3·990	3·965	4·145	4·040
4·000	4·245	4·215	4·395	4·295
4·250	4·495	4·470	4·650	4·595
4·500	4·750	4·720	4·900	4·800
4·750	5·000	4·975	5·155	5·050
5·000	5·255	5·225	5·405	5·305

7. **Enamelled round copper conductors** (Oleo resinous enamel) (IS 449—1962). The sizes are given in Table 17·6.

Table 17·6. Diameters of enamelled round copper wire (Oleo—resinous enamel) (IS : 449—1962)

Nominal conductor diameter	*Overall diameter—maximum*	
	Normal covering	*Thick covering*
mm	*mm*	*mm*
0·025	0·035	—
0·030	0·040	—
0·036	0·045	—
0·040	0.050	—
0·045	0·058	—
0·050	0·063	0·068
0·060	0·073	0·080
0·071	0·086	0·094
0·080	0·095	0·105
0·090	0·108	0·115
0·100	0·118	0·128
0·112	0·132	0·142
0·125	0·145	0·155
0·132	0·152	0·162
0·140	0·163	0·173
0·150	0·173	0·186
0·160	0·183	0·196
0·170	0·195	0·218
0·180	0·205	0·218
0·195	0·220	0·236
0·200	0·225	0·241
0·212	0·240	0·255
0·224	0·254	0·270
0·236	0·266	0·282
0·250	0·283	0·298
0·258	0·291	0·306
0·265	0·298	0·316
0·280	0·316	0·331
0·300	0·336	0·354
0·307	0·343	0·361
0·315	0·354	0·372
0·335	0·374	0·394
0·355	0·397	0·417
0·375	0·417	0·437
0·400	0·445	0·465
0·425	0·473	0·495
0·462	0·513	0·535
0·475	0·526	0·548
0·500	0·553	0·576
0·530	0·584	0·609
0·560	0·614	0·642
0·600	0·657	0·681
0·630	0·687	0·713
0·670	0·728	0·756
0·710	0·768	—
0·730	0·791	—
0·750	0·811	—
0·800	0·861	—
0·850	0·914	—
0·925	0·980	—
0·950	1·015	—

Table 17·6 (*Contd.*)

Nominal conductor diameter	*Overall diameter—maximum*	
	Normal covering	*Thick covering*
mm	*mm*	*mm*
1·000	1·065	—
1·060	1·130	—
1·120	1·195	—
1·180	1·255	—
1·250	1·325	—
1·320	1·395	—
1·400	1·480	—
1·500	1·580	—
1·600	1·685	—
1·700	1·785	—
1·800	1·890	—
1·900	1·995	—
2·060	2·160	—
2·120	2·220	—
2·240	2·345	—
2·360	2·465	—
2·500	2·610	—
2·650	2·765	—
2·800	2·920	—
2·900	3·020	—
3·000	3·125	—
3·150	3·280	—
3·250	3·385	—
3·350	3·485	—
3·450	3·590	—
3·550	3·695	—
3·650	3·795	—
3·750	3·895	—
4·000	4·155	—

8. **Enamelled round copper conductors** (Synthetic enamel) (IS 1595—1960). The sizes are given in Table 17·7.

Table 17·7. Round copper wire (Synthetic enamel).
(IS : 1925—1960)

Nominal conductor diameter	*Overall diameter*			
	Fine covering	*Medium covering*	*Thick covering*	*Extra thick covering*
mm	*mm*	*mm*	*mm*	*mm*
0·050	0·060	0·065	—	—
0·060	0·073	0·078	—	—
0·071	0·084	0·092	—	—
0·080	0·095	0·105	0·118	—
0·090	0·105	0·115	0·128	—
0·100	0·118	0·128	0·141	—
0·112	0·132	0·143	0·156	—
0·125	0·146	0·159	0·172	—
0·132	0·155	0·168	0·181	—
0·140	0·163	0·176	0·191	—
0·150	0·173	0·186	0·201	—
0·160	0·185	0·198	0·213	—
0·170	0·198	0·211	0·226	—
0·180	0·208	0·223	0·238	—
0·195	0·224	0·239	0·257	—
0·200	0·230	0·246	0·264	—
0·212	0·242	0·258	0·276	—
0·224	0·254	0·272	0·290	—
0·236	0·266	0·284	0·302	—
0·250	0·283	0·301	0·319	0·344
0·258	0·291	0·309	0·327	0·352
0·265	0·299	0·317	0·335	0·362
0·280	0·316	0·334	0·351	0·379
0·300	0·336	0·354	0·372	0·400
0·307	0·345	0·362	0·380	0·408
0·315	0·354	0·372	0·389	0·417
0·335	0·375	0·393	0·410	0·441
0·355	0·397	0·415	0·432	0·465
0·375	0·418	0·438	0·455	0·489
0·400	0·445	0·465	0·483	0·518
0·425	0·472	0·493	0·513	0·549
0·462	0·511	0·531	0·551	0·588
0·475	0·526	0·546	0·566	0·603
0·500	0·551	0·571	0·591	0·630
0·530	0·581	0·602	0·623	0·662
0·560	0·612	0·635	0·655	0·696
0·600	0·654	0·677	0·697	0·738
0·630	0·684	0·707	0·728	0·768
0·670	0·727	0·750	0·771	0·812
0·710	0·768	0·791	0·814	0·852
0·730	0·788	0·811	0·834	0·874
0·750	0·808	0·831	0·854	0·895
0·800	0·861	0·884	0·907	0·950
0·850	0·912	0·935	0·958	1·001
0·925	0·990	1·016	1·039	1·085
0·950	1·015	1·041	1·064	1·110
1·000	1·070	1·095	1·120	1·165
1·060	1·130	1·155	1·180	1·225
1·120	1·190	1·215	1·240	1·287
1·180	1·253	1·278	1·303	1·353
1·250	1·325	1·350	1·375	1·425
1·320	1·395	1·420	1·447	1·500
1·400	1·480	1·505	1·535	1·585

Table 17·7 (*Contd.*)

Nominal conductor diameter	*Overall diameter*			
	Fine covering	*Medium covering*	*Thick covering*	*Extra thick covering*
mm	*mm*	*mm*	*mm*	*mm*
1·500	1·580	1·605	1·635	1·685
1·600	1·680	1·710	1·740	1·790
1·700	1·785	1·810	1·840	1·890
1·800	1·885	1·915	1·940	1·995
1·900	1·990	2·015	2·045	2·095
2·060	2·150	2·180	2·210	2·265
2·120	2·211	2·241	2·271	2·327
2·240	2·335	2·365	2·347	2·455
2·360	2·486	2·488	2·020	2·575
2·500	2·600	2·630	2·665	2·720
2·650	2·752	2·785	2·815	2·872
2·800	2·905	2·935	2·970	3·025
2·900	3·010	3·040	3·070	3·125
3·000	3·110	3·140	3·175	3·230
3·150	3·262	3·295	3·325	3·382
3·250	3·365	3·395	3·427	3·485
3·350	3·465	3·497	3·530	3·585
3·450	3·567	3·600	3·630	3·690
3·550	3·670	3·700	3·732	3·795
3·650	3·770	3·800	3·835	3·895
3·750	3·872	3·902	3·937	3·997
4·000	4·125	4·155	4·190	4·255

17·2 British Standard Specifications. Some manufacturers still manufacture round copper conductors according to B.S S. Therefore, following tables are being given for the selection of size of standard conductors if they confirm to B.S.S.

Tabe 17·8. Standard wire gauge.

SWG	*Diameter mm*	*Area mm²*	*SWG*	*Diameter mm*	*Area mm²*
1	7·62	45·6	21	0·813	0·519
2	7·01	38·6	22	0·711	0·397
3	6·40	32·2	23	0·610	0·292
4	5·89	27·3	24	0·559	0·245
5	5·38	22·8	25	0·508	0·203
6	4·88	18·7	26	0·457	0·164
7	4·47	15·7	27	0·417	0·136
8	4·06	13·0	28	0·376	0·111
9	3·66	10·5	29	0·345	0·0937
10	3·25	8·3	30	0·315	0·0779
11	2·95	6·82	31	0·295	0·0682
12	2·64	5·48	32	0·274	0·0591
13	2·34	4·29	33	0·254	0·0507
14	2·03	3·24	34	0·234	0·0429
15	1·83	2·63	35	0·213	0·0357
16	1·63	2·07	36	0·193	0·0293
17	1·42	1·59	37	0·173	0·0234
18	1·22	1·17	38	0·152	0·0182
19	1·32	0·811	39	0·132	0·0137
20	0·914	0·657	40	0·122	0·0117

Adition to wire diameters for insulation

Table 17·9. Enamel covering.

SWG	*Addition to diameter mm*	*SWG*	*Addition to diameter mm*
16—18	·075	27—29	0·033
19—20	0·063	30—33	0·025
21—22	0·050	34—31	0·018
23—26	0·038	39—45	0·013

Table 17·10. Cotton covering.

SWG	Addition to diameter mm			
	S.C.C.	D.C.C.	Fine S.C.C.	Fine D.C.C.
10—14	0·20	0·35	0·18	0·25
15—17	0·18	0·30	0·15	0·23
18—22	0·15	0·25	0·13	0·18
23—34	0·13	0·23	0·10	0·15
35—40	0·10	0·20	0·08	0·13

TRANSFORMERS

17·3. General. Indian Standards Institution has published the following standards on transformers :

(*i*) IS : 1180-1964 : Specifications for outdoor type three phase distribution transformers upto and including 100 kVA and voltage ratings upto 11 kV.

(*ii*) IS : 2026-1962 : Specifications for power transformers beyond 100 kVA.

17·4. Outdoor type distribution transformers IS : 1180.1966

Ratings. According to this the standard ratings for transformers (distribution type) are :

16 kVA	63 kVA
25 kVA	80 kVA
40 kVA	100 kVA
50 kVA	

The no load voltage ratios are :

3300/433 V, 6600/433 V, 11000/433 V.

Tappings. The tapping shall be provided on h.v. side and shall be in 5 steps. The range shall be ±2·5% and ±5%. Off load tap changers are used.

Connections. The primary *i.e.* h.v. winding shall be delta connected, while the secondary winding, *i.e.* l.v. shall be star-connected. The neutral is brought out.

Transformer Oil. The transformer tank shall be filled with transformer oil which shall comply with the requirements of IS : 335-1963 (Specifications for Insulating oil for transformers and switchgear).

Accessories. Each transformer shall be fitted with the following accessories :

(*i*) Two earthing terminals. (*ii*) Oil gauge. (*iii*) Lifting lugs. (*iv*) Rating and terminal marking plate. (*v*) Plain breathing device of weather proof type. (*vi*) Drain valve with plug on the transformer with conservators. (*vii*) Thermometer pocket (not provided on transformers of 25 kVA and below).

The conservator tank shall be provided on the transformers of capacity 50 kVA and above. In case of transformers with conservator tank, the h.v. bushing shall be provided on the side.

Limits of temperature rise. The following temperature rises shall be adhered to, over the ambient temperature of 45°C.

(*i*) Temperature rise in winding measured by resistance method —55°C.

(*ii*) Temperature rise in oil measured by thermometer in the top oil —45°C.

The above temperature rises are for *ON*, *OB*, and *OW* type of cooling.

Bushing clearance. The minimum phase to phase external clearance of 75 mm for l.v. (upto 1·1 kV) bushing and 255 mm for h.v. bushing (3·3 kV and above) shall be kept with bushings complete with arcing horns mounted on the transformers.

Iron and copper losses. The iron and copper losses shall be lower than the following values for the various rating on transformers.

Table 17·11. Permissible losses.

kVA Rating	*Iron Loss W*		*Copper losses at 75°C W*
	Hot rolled stampings	*Colled rolled stampings*	
16	155	120	500
25	195	155	700
40	260	200	975
50	295	225	1180
63	350	260	1400
75	385	260	1600
80	400	300	1650
100	450	355	2000

The above losses are subject to variation of ±10%.

Impedances. All distribution transformers mentioned above shall have an impedance of 4·5 per cent. The above impedance shall be subject to a tolerance of ±10%.

Rating and terminal marking plates. Each transformer shall be fitted with three kinds of plates, namely :

(*i*) Rating plate. (*ii*) Terminal marking plate.

(*iii*) Connection diagram.

Rating plate. The rating plate shall give the following information.

(*i*) kVA rating. (*ii*) Voltage on no load on h.v. and l.v. sides. (*iii*) Current in h.v. and l.v. sides (*iv*) Phases on h.v. and l.v. sides. (*v*) Type of cooling. (*vi*) Frequency. (*vii*) Impedance voltage. (*viii*) Vector group. (*ix*) Weight of core and windings in kg. (*x*) Weight of oil in kg (*xi*) Volume of oil in litres. (*xii*) Total weight of transformer. (*xiii*) Year of manufacture. (*xiv*) Customer's reference number. (*xv*) Maximum guaranteed temperature of oil in °C. (*xvi*) Maximum guaranteed temperature of windings in °C.

Terminal marking. It shall indicate the marking of terminals of both h.v. and l.v. windings. It shall give the number of turns on h.v. side expressed in percentage at each off load tap changer position.

Diagram plate. It shall give the single line diagram of the connections made to various taps within the transformer.

Arcing horns Each transformer shall be fitted with arcing horns, whose function is to ensure that any over voltage reaching the transformer is limited to a value not exceeding 80 per cent of the impulse withstand level of the winding.

The arcing horns settings are :

Nominal system voltage kV (r m.s.)	*Arcing horn gap*
3·3	Single gap — 35 mm
6·6	Double gap — 16 mm
11	Double gap — 20 mm

17·5. Power transformers (IS : 2026-1962). This standard covers oil immersed transformers with class A insulation rated 1 kVA and above for single phase and 25 kVA and above for polyphase operation.

Ratings.

Standard kVA ratings. These are given in Table 17·12.

Table 17·12

Standard kVA ratings for three-phase transformers

—	100	1000	10000
—	125	1250	12500
—	160	1600	16000
—	200	2000	20000
25	250	2500	25000
—	315	3150	31500
40	400	4000	40000
—	500	5000	50000
63	630	6300	63000
—	800	8000	80000

Standard kVA ratings for single-phase transformers

1	5	16
2	10	25

Above 25 kVA, the standard ratings for single-phase transformers shall be one-third of the value given for 3-phase transformers.

Tappings. Unless otherwise specified, adjustment of tappings, if required, shall be such as to provide for a voltage adjustment on the high voltage side of $\pm 2\frac{1}{2}$ per cent, and ± 5 per cent of the rated voltage, the tappings being located on the higher voltage winding.

Tap-changing shall be affected by means of an externally operated off-circuit switch capable of being locked in positions. If specified, the tap-changing may also be carried out by means of links under oil, arranged to select terminals to which the tapping leads have been brought.

The transformer may, if required, be equipped with on-load tap changer.

Test voltages. The insulation levels employed on transformers complying with this standard are influenced by several factors. The requirements of this section are based on the highest system voltages .

For transformers designed for operation in electrically non-exposed installation, only power frequency voltage tests are required, the appropriate test voltages shall be as given in Table 17·13.

Insulation to earth. Uniform insulation to earth shall be provided for all delta connected windings and for star or interconnected star windings where the neutral end of the windings is not specified for connection to earth.

Graded insulation may be provided for star and interconnected star windings if the neutral end of the windings is specified for connection to earth.

Table 17·13. Test voltages for transformers not designed for impulse voltage testing.

Nominal system voltage kV r.m.s.	*Highest system voltage kV r.m s.*	*Power-frequency test voltage kV r.m s.*
Less than 1·0	Less than 1·1	2·5
1·0	1·1	3·2
3·3	3·6	8·2
6·6	7·2	15·4
11	12·0	25·0
15	17·5	36·0

Table 17·14. Test voltages for transformer designed for impulse voltage tests

Nominal system voltage	*Highest system voltage*	*Impulse test voltage*		*Power frequency test voltage*	
		Standard 1	*Standard 2*	*Standard 1*	*Standard 2*
kV rms	kV rms	kV peak	kV peak	kV rms	kV rms
3·3	3·6	45	45	16	16
6·6	7·2	60	60	22	22
11	12	75	75	28	28
15	17·5	95	95	38	38
22	24	125	125	50	50
33	36	170	170	70	70
47	52	250	250	95	95
66	72·5	325	325	140	140
88	100	450	380	185	150
110	123	550	450	230	185
132	145	650	550	275	230
220	245	1050	900/825	460	395/360

Transformers shall be capable of withstanding the relevant power frequency test voltages given in Table 17·13 and where appropriate, the relevant power frequency and impulse test voltage given in Table 17·14.

Limits of temperature rise. The temperature rise of transformer windings, oil and cores shall not exceed the limits prescribed in Table 17·15.

Table 17 15. Temperature-rise limits for oil-immersed type transformers.

Part	*Cooling classification*	*Temperature rise, °C*
Windings (measured by resistance)	ON, OB, OW	55
	OFN, OFB	60
	OFW	65
Oil (measured by the thermometer in top oil)	All	45
Cores	Cores shall be so designed that the temperature rise on any part of the external surface does not exceed that of the windings.	

Performance under external short-circuit conditions. Transformers shall be designed to be capable of withstanding, without injury, the thermal and mechanical effects of short-circuits at the terminals of any winding for the periods given in Table 17·6.

The current density in any winding based on the rms value of the initial symmetrical component of the through-fault current shall not exceed the appropriate value given in Table 17·16.

Table 17·16. Impedance voltage and short-circuit duration.

Impedance voltage	*Symmetrical short circuit current, rms value, to be withstood, expressed as a multiple of rated current*	*Duration of short circuit*	*Maximum current density*
Percent		s	*A/mm²*
4 or less	25	2	93
5	20	3	77·5
6	16·5	4	69·7
7 and above	14 or less	5	62

Electrical performance.

Efficiency and Regulation. When statements of efficiency and regulation are required they shall be based on loading at the rated kVA and at unity power factor (and other power factors, if agreed). Recommended load and no load losses for certain ratings of transformers are given in Table 17·18.

The recommended percentage impedances at 75°C are 4·5 per cent for transformers of rating upto and including 100 kVA, 11 kV and 4·75 per cent for transformers of rating above 100 kVA including 1000 kVA, 11 kV.

Tolerances. Table 17·17 gives the various values of tolerances.

Table 17 17. Tolerances on electrical performance.

Item	*Tolerance*
1. Voltage ratio on no-load	= % of the declared ratio or a percentage equal to 10% of the actual percentage impedance at rated load, whichever is smaller.
2. Impedance voltage	On principal tappings (*a*) Two winding transformer=10% (*b*) Multi-winding transformer=15%
3. Load loss	Plus 10% of the guaranteed loss.
4 No-load loss	Plus 10% of the guaranteed loss.
5. Efficiency	In accordance with the tolerance on losses.
6. Regulation	In accordance with tolerances on impedance voltage and load losses.

Note. No load current is not considered to be a subject for guarantee and therefore no tolerance is specified.

Tests. Tests shall be made at the manufacturer's works at room temperature and with all those external components and fittings in place which are likely to affect the performance.

Test of transformers

(*i*) ***Routine tests.*** All transformers shall be subjected to routine tests at the manufacturer's works. The tests shall comprise :

(*a*) Measurement of winding resistance
(*b*) Ratio, polarity and phase relationships
(*c*) Impedance voltage
(*d*) Load losses
(*e*) No load losses and no load current
(*f*) Insulation resistance
(*g*) Induced over voltage withstand
(*h*) Separate source voltage withstand.

(*ii*) ***Type tests.*** In addition to the routine tests the following type tests may be made by mutual agreement between the purchaser and the manufacturer :

(*a*) Impulse-voltage withstand test, and
(*b*) Temperature rise test.

If records of type tests on a transformer which, in essential details, is representative of the one being purchased, are furnished, the purchaser may accept these as evidence of type tests instead of actual tests.

(*iii*) ***Supplementary test.*** Zero phase sequence impedance measurement is a supplementary test.

Table 17·18. Recommended losses of transformers upto and including 1000 kVA and 11 kV.

kVA rating	*No-load loss at normal voltage and frequency W*	*Load loss at fulll oad at 75°C° W*
100	500	2000
125	570	2350
160	670	2840
200	800	3400
250	950	4000
315	1150	4770
400	1380	5700
500	1660	69:0
860	1980	8260
800	2400	9980
1000	2800	11880

ROTATING MACHINERY

INTERNATIONAL ELECTROTECHNICAL COMMISSION PUBLICATION 34-1

17·6. Recommendations for rotating electrical machinery (excluding machines for traction vehicles)

Definitions

Rating. The rating of an electrical machine is a statement of the operating limitations assigned to it by the maker, and comprises output, speed, voltage, current, frequency, power factor, etc., as marked on the rating plate.

Continuous Rating (*CR*). The rating corresponding to the load which can be sustained continuously when all the appropriate requirements of these specifications are specified.

Short-time Rating (*STR*). The rating corresponding to the load which can be sustained for a specific time when all the appropriate requirements of these specifications are satisfied.

Rated-Output. It is usual to speak of a machine by its rated output. It is necessary to note that the term "Rated Output" should be used in the following way :

(*a*) For direct current generators, the electrical power at the terminals, expressed in watt (W), kilowatt (kW), or megawatt (MW).

(*b*) For alternating current generators the apparent electric power at the terminals expressed in volt-ampere (VA), kilovolt-ampere (kVA), or megavolt-ampere (MVA).

(*c*) For motors, the mechanical power available at the shaft, expressed in watt (W), kilowatt (kW) or megawatt (MW).

Duty. The schedule of the loads on a machine taking account of their duration and sequence.

Periodic duty. Operation in a series of identical cycles, each composed of a period of operation at rated load followed by a rest period during which the machine is completely stopped and all input power whether electrical or mechanical is cut off. The periods are insufficient for thermal equilibrium to be attained, either during the periods of operation, or during the rest periods.

Continuous Duty with Intermittent Load (*CIR*). Operation in a series of identical cycles, each composed of a period of operation at rated load followed by a period of operation at no load where no load operation means disconnection of the load only. The periods are insufficient for thermal equilibrium to be attained, either during the heating periods, or during the cooling periods.

Note. Thermal equilibrium is a state in reached when the temperature of the several parts under observations do not vary by more than 10 Centigrade degrees per hour.

Ratings

Four classes of ratings are recognized.

Continuous rating.

Short-time rating. Standard periods for this class of rating are 10, 30, 60 and 90 minute.

Rating for periodic duty. Standard values of load factor for this duty are either 15, 25, 40 or 60 per cent.

Rating for continuous duty with intermittent load. Standard values of load factor are 25, 40, 60 per cent.

Note. The load factor is the ratio of time of operation at rated load to the duration of the cycle.

Service conditions. These recommendations are intended for machines working under the following conditions :

(*i*) Altitude does not exceed 1000 metre (3300 ft).

(*ii*) Temperature of cooling air or gas does not exceed 40°C.

Correction factors are applied for machines working under other conditions.

Methods of measurement of temperature rise

The methods of measurement of temperature rise are given in Art. 3·37 on page 113.

Limits of temperature rise. Table 17·19 gives the limits of permissible temperature rise above the cooling air or gas temperature, for machines, other than hydrogen-cooled turbine-type generators, intended to operate with cooling air or gas temperature not exceeding 40°C and insulated with materials in Classes *A*, *E*, *B*, *F* and *H*.

Dielectric tests. The machine should be able to withstand the following test voltages as per procedure laid down in the recommendations.

Table 17·20. Dielectric tests.

No.	*Machine or Part*	*Test voltage* (*r.m.s*)
1.	Insulated parts of machine of size less than 1 kW or 1 kVA.	500 V+twice the rated voltage.
2.	Rotating machines of size 1 kW or kVA to less than 1 0000 kW or kVA.	1 000 V+twice the rated voltage with a minimum of 1 500 V.

3.	Rotating machines of size 10 000 kW or kVA or more.	
	Rated voltage :	
	V up to 2 000 V	1 000 +2V
	V above 2 000 V to 6 000 V.	2·5 times V
	V above 6 000 V to 16 500 V.	3 000 +2V
	V above 16 500 V.	Subject to special agreement.
4.	Separately-excited field winding of d.c. machines.	1 000 +twice the maximum rated circuit voltage with a minimum of 1 500 volt.
5.	Field winding of synchronous generators.	Ten times the rated excitation voltage with a minimum of 1 500 V and a maximum of 3 500 V.
6.	Field windings of synchronous motors, synchronous condensers and synchronous converters.	
	(*a*) When intended to be started with the field windings short-circuited or connected across an exciter armature, or to be started with the a.c. windings idle.	1 000 +twice the maximum rated excitation voltage with a minimum of 1 500 V.
	(*b*) When intended to be started either with a resistance connected in series with the field windings, or with the field windings on open circuit with or without a field-dividing switch.	1 000 +twice the maximum value of the r.m.s. voltage ; which can occur under the specified starting conditions, between the terminals of the field, or in the case of a sectionalized field, winding between the terminals of any section, with a minimum of 1 500 V.
7.	Secondary (usually rotor) windings of induction motors or synchronous induction motors if not permanently short circuited (*e.g.*, if intended for rheostatic starting).	
	(*a*) For non-reversing motors or motors reversible from standstill only.	1 000 +twice the open circuit standstill voltage as measured between slip rings or secondary terminals with rated voltage applied to primary windings.
	(*b*) For motors to be reversed or braked by reversing the primary supply while the motor is running.	1 000 +four times the open circuit standstill secondary voltage.
8.	Exciters (except as below)	As for the windings to which they are connected
	Exception 1.—Exciters of synchronous motors (including synchronous induction motors) if connected to earth or disconnected from the field windings during starting.	1 000 +twice the rated exciter voltage, with a minimum of 1 500 V.
	Exception 2.—Separately excited field windings of exciters.	
9.	Assembled group of machines and apparatus.	When the test is made on an assembled group of several pieces of new apparatus, each one of which has previously passed its high-voltage test, the test voltage to be applied to such assembled group shall not exceed 85% of the lower test voltage appropriate for any part of the group.

Excess current and excess torque

Momentary excess current for generators. A generator rated in accordance with the specifications shall be capable of withstanding for 15 seconds a current 50% in excess of its rated current, the voltage being maintained as near the rated value as possible consistent with the maximum capacity of the prime mover. The exact value of the voltage is not important.

Momentary excess torque for motors.

(*a*) *D.C. Motor* :—A direct current motor, irrespective of the class of rating shall be capable of withstanding for 15 seconds a torque 50% in excess of that corresponding to its rating, the voltage, being maintained at rated value..

(*b*) *Polyphase Synchronous Motor* :—A polyphase synchronous motor irrespective of the class of rating shall, unless otherwise agreed, be capable of withstanding an excess torque as specified below for 15 seconds without falling out of synchronism, the excitation being maintained at the value corresponding to rated load.

Synchronous (wound rotor) induction motors—35% excess torque.

Synchronous (salient pole) motors—50% excess torque.

(*c*) *Polyphase Induction Motors* :—A polyphase induction motor, irrespective of the class of rating, shall be capable of withstanding for 15 seconds, without stalling or abrupt change in speed (under gradual increase of torque) a maximum torque as specified below, the voltage and frequency being maintained at their rated values.

(*i*) For induction motors of the normal type (*e.g.*, wound rotor or ordinary squirrel-cage motors), the maximum torque shall exceed the torque corresponding to the rating by the following net amounts, no tolerances being permitted.

Motors with a continuous short-time rating, at least 60% excess torque.

Motors with a rating for periodic duty or for continuous duty with intermittent load at least 100% excess torque.

(*ii*) In the case of induction motors for which the field of application is specified in the order, and in the case of induction motors of special type (*e.g.*, motors with eddy current rotors or double cage rotors of the Boucherot type) with special inherent starting properties the value of excess torque shall be a matter of agreement between the manufacturer and the purchaser.

(*d*) *Single-phase Motors* :—The momentary excess torque of single phase motors shall be a matter of agreement between manufacturer and purchaser.

Commutation test

Commutation test for direct-current machines. A direct-current machine shall work with fixed brush-setting, from no load up to the momentary excess current or torque specified earlier without injurious sparking or injury to the commutator or brushes. The commutation test should be applied on the conclusion of the temperature test of the machine.

Tolerances .

Schedule of tolerances on quantities involved in the rating to electrical machinery is given in Table 17·21.

Table 17·21. Schedule of tolerances.

No.	*Item*	*Tolerance*
1.	Efficiency	
	(*a*) By summation of losses	-10% of $(1-\eta)$
	(*b*) By (input—output) test	-15% of $(1-\eta)$ Maximum 0·7%.
2.	Total losses	$+10\%$ of the total losses.
3.	Power factor	$\pm 1/6$ of $(1-\cos\phi)$
		Minimum 0·02.
		Maximum 0·07.

4.	(a) Speed of d.c. shunt motors (at full load at working temperature).	kW per 1000 r.p.m. Not less than 0·67 but below 2·5 ±10%. Not less than 2·5 but below 10±7·5% 10 and upwards ±5%.
	(b) Speed of d.c. series motors (at full load at working temperature).	Not less than 0·67 but below 2·5 ±15%. Not less than 2·5 but below 10 ±10%. 10 and upwards ±7·5%.
5.	Slip of induction motors.	± 20% of the guaranteed slip.
6.	Inherent voltage regulation of d.c. generators, shunt or separate excitation.	±20% of the guaranteed regulation.
7.	Inherent voltage regulation of d.c. generators, shunt or compound excitation.	±20% of the guaranteed regulation, with a minimum of ±2% of the rated voltage.
8.	Starting current of induction motors with short-circuited rotor and specified starting apparatus.	±20% of the guaranteed starting current.
9.	Instantaneous short-circuit current of an alternator under specified conditions.	±30% of the guaranteed value.
10.	Steady short circuit current of an alternator at specified excitation.	±15% of the guaranteed value.
11.	Variation of speed of d.c shunt wound and compound wound motors (from no-load to full load).	±20% of the guaranteed variation. Minimum, ±20% of the rated speed.
12.	Starting torque of induction motors	
	(a) Motors without sliprings.	−10% of the guaranteed torque.
	(b) Slip-ring motors with automatic starting.	−10% of the guaranteed torque.
13.	Maximum torque or break down torque of induction motors.	−10% of the guaranteed torque, except that after allowing for the tolerance the excess torque of motors with a continuous or short-time rating shall be not less than 60%, and of the motors with an intermittent rating shall be not less than 100%.

Rating plates

Rating plates shall be marked with the appropriate items in the following list

1. The type of machine, whether motor or generator, also whether, for example, shunt, series, compound, squirrel cage, etc.
2. The class of rating and, if appropriate, the period or load factor.
3. The rated output (e.g. kW, kVA or h.p.)
4. The rated voltage.
5. The rated current.
6. Type of current (d.c. or a.c.)

7. For a.c. machines, the rated frequency and number of phases.

8. The rated speed range, in revolutions per minute.

9. The permissible overspeed, if applicable (*e.g.* turbine type and water turbine driven generators).

10. The class of insulation or the permissible temperature rise.

11. The number and date of the specification.

12. For a c. machines, the winding connections.

13. For a.c. machine, the power factor.

14. For synchronous machines or d.c machines with separate excitation, the rated excitation current and voltage.

15. For wound-rotor induction machines, the open-circuit voltage between slip rings and the slip-ring current.

16. For hydrogen cooled machine, the hydrogen pressure at rated output.

17. The manufacturer's name.

18. The manufacturer's serial number or identification mark.

INTERNATIONAL ELECTROTECHNICAL COMMISSION PUBLICATION 34-2

17 7. Recommendations on determination of efficiency of rotating electrical machines.

Efficiency. The efficiency of a rotating electrical machine is the ratio of the power output to the power input.

Determination of efficiency. The efficiency can be measured by many methods, where efficiency is determined by summation of losses, the schedules given below are followed. Unless otherwise specified, all I^2R losses shall be calculated at one of the temperatures given below

Classes A, E and B	75°C
Classes F and H	115°C.

D.C. machines. The following losses are included when calculating efficiency of d.c. machines.

1. *Exciting-circuit losses*

(*a*) Shunt I^2R loss

(*b*) Main rheostat loss

(*c*) Exciter loss.

2. *Losses independent of current*

(*d*) Core loss at no load and rated speed and rated terminal voltage

(*e*) Bearing friction loss

(*f*) Total windage loss

(*g*) Brush friction loss.

3. *Direct load loss*

(*h*) Change in core loss due to load

(*j*) I^2R loss in armature windings

(*k*) I^2R loss in windings in series with armature.

(*l*) Electrical loss in brushes. Total voltage drop in carbon or graphite brushes is 2 V and in metal carbon brushes 0·6 V.

4. *Stray load loss*

(*m*) Stray load loss in iron

(*n*) Stray load loss in conductors

(*o*) Additional brush losses.

Unless otherwise specified, it is assumed that the above losses (stray load losses) vary as square of the current and that their maximum value at maximum rated current is equal to 1 percent of basic output for uncompensated machines and 0.5 percent of rated output for compensated machines.

Polyphase induction machines. The following losses are be included when preparing a statement of efficiency of polyphase induction machines :

1. *Losses independent of current*

(*a*) Core loss at no load speed with rated terminal voltage and frequency

(*b*) Bearing friction loss

(*c*) Total windage loss

(*d*) Brush friction loss (included when brushes are not lifted).

2. *Direct load loss*

(*e*) I^2R loss in stator windings

(*f*) I^2R loss in rotor windings on load

(*g*) Summation of the I^2R loss in brushes and connectors, and brush contact loss. The voltage drop in all brushes of the same phase is taken as 1 V for carbon and graphite brushes and 0·3 V for metal carbon brushes.

3. *Stray load loss*

(*h*) Stray load loss in iron

(*j*) Stray load loss in conductors.

Unless otherwise specified, it is assumed that these stray load losses vary as square of primary current and their total value at full load is equal to 0·5 percent of rated output.

Polyphase synchronous machines. The following losses are to be included while computing the efficiency of polyphase synchronous machines.

1. *Exciting circuit loss*

(*a*) Field I^2R loss

(*b*) Main rheostat loss

(*c*) Electrical loss in brushes

(*d*) Exciter loss

2. *Losses independent of current*

(*e*) Core loss at no load and rated speed and rated terminal voltage

(*f*) Bearing friction loss

(*g*) Total windage loss

(*h*) Brush friction loss

3. *Direct load loss*

(*j*) I^2R loss in armature winding

4. *Stray load loss*

(*k*) Stray load loss in iron

(*l*) Stray load loss in conductors.

INTERNATIONAL ELECTROTECHNICAL COMMISSION PUBLICATION 34.3

17 8. Recommendations for preferred standard 3000 rev/min, 3-phase, 50 Hz turbine-type generators.

Air-cooled generators

1. Output. Rated output of air-cooled generators shall be as given in the table below :

MW	10	12	16	25	40	50
MVA	12·5	15	20	31·25	50	62·5

2. Power factor. The rated power factor at the generator terminals shall be 0·8 lagging.

3. Voltage. The manufacturer shall assign a rated voltage.

4. Voltage range. The voltage range at the generator terminals shall be $\pm$ 5 percent of the rated voltage assigned by the manufacturer.

5. Speed. For 50 Hz machines, the speed shall be 3000 r.p.m.

6. Short circuit ratio. The short-circuit ratio at rated MVA and at rated voltage shall be 0·55. The permitted tolerance is $\pm$0·05.

7. *Sub-transient reactance.* Sub-transient reactance as proved by a sudden three-phase short-circuit test at rated voltage at no load at the generator terminals, shall have a minimum value of 10 percent.

8. Stator windings. The stator windings shall be arranged for star connection with all six terminals brought out of the generator.

9. Generator insulation. Class *B* insulation shall be used.

10. Temperature-rise. Limits of temperature rise shall be in accordance with I.E.C. Publication 34-1.

11. Cooling air temperature. The generators shall be suitable for a maximum cooling-air temperature, measured at the inlet to the machine of 40°C.

12. Excitation voltage. The manufacturer shall declare the excitation voltage at the slip rings at rated MVA and power factor.

13. Generator cooling. The system of ventilation shall be closed air-circuit system.

14. Over-speed. Generator rotors shall be tested at 20 percent over-speed for 2 minutes.

15. Main exciter. The rated current shall be 110 percent of the excitation current at the rated output of the generator.

The nominal exciter response shall be not less than 0·5.

16. Exciter insulation. Exciter insulation may either be class *A* or class *B.*

Hydrogen-cooled generators

1. Output. Rated outputs of hydrogen-cooled generators shall be as given in the table.

MW	50	63	80	100	125
MVA	62·5	78·75	100	125	156·25

2. Hydrogen pressure. The rated hydrogen pressure shall not be less than 1·0 kg/cm² (—100 kN/m²) gauge.

Recommendations for power factor, voltage, voltage range are the same as for air-cooled generators.

8. Speed. For 50 Hz machines, the speed shall be 3,000 r.p.m.

Recommendations for short circuit ratio, sub-transient reactance and stator windings are same as for air cooled generators.

10. Generator terminals Terminal insulators shall withstand a 50 Hz dry test, in air, of not less than 4 times the rated voltage of the machine.

11. Generator insulation. Class *B* insulation shall be used. Dielectric tests shall be in accordance with I.E.C Publication 34-1.

12. Excitation voltage. The manufacturer shall declare the excitation voltage at the slip rings, at rated MVA and power factor.

Recommendations for over speed, main exciter, exciter insulation is same as for air cooled generators.

THREE PHASE INDUCTION MOTORS

17·9 Indian standard specifications for 3 phase induction motors. The Indian Standards Institution has laid down the following standards for three-phase induction motors (There are other standards also but we are primarily concerned with the following) :

(a) IS : 325-1961 Specifications for three phase induction motors.

(b) IS : 1231-1967 Dimensions of three phase foot mounted induction motors.

(c) IS : 4029-1967 Guide for testing three-phase induction motors.

Most of the information given in these standards has been covered in IEC Publication 34-1. However, additional information covered in these standards is being given here.

Types of enclosure. Following types of enclosures are considered as standard :

1 *Protected motor* (*P*).

Screen protected (*SP*)

Drip-proof motor (*DP*)

Splash proof motor (*SPLP*).

2. *Pipe or duct ventilated motor.*

A pipe or duct ventilated motor may be cooled by means of :

(*i*) Self-ventilation (*PV*).

(*ii*) Forced draught air supplied by external pressure (*PVFD*).

(*iii*) Induced draught air drawn through the machine by external means (*PVID*).

3. *Totally-enclosed motor* (*TE*).

A totally enclosed motor may be any of the following types :

(*i*) Totally enclosed fan cooled (*TEFC*).

(*ii*) Totally enclosed separately air cooled (*TESAC*).

(*iii*) Totally enclosed closed air circuit motor, *CACA* (cooler employing air) or *CACW* (cooler employing water).

4. *Weather-proof motor* (*WP*).

Performance.

Pull up torque of squirrel cage motors. With rated voltage and frequency applied to the terminals of the stator, squirrel cage induction motor shall be capable of running upto speed against a percentage of full load torque specified in the order.

Overloads in torque. The motors should be capable of withstanding the following overloads in torque, without injury :

(a) Motors with continuous rating upto and including 37 kW.	100 percent excess torque for 15 seconds.
(b) Motors with continuous rating above 37 kW upto and including 370 kW.	75 percent excess torque for 15 seconds.
(c) Motors with continuous rating above 370 kW.	60 percent excess for 15 seconds.
(d) Motors with short time rating.	100 percent excess torque for 15 seconds.

Tests The following are two types of tests.

(i) *Type tests.* Tests carried out on a type motor to varify confirmity to the performance requirements of this standard.

(ii) *Routine tests* Tests carried out on each motor to ascertain that it is electrically and mechanically sound

Type tests. The following shall constitute the type tests :

(a) Measurement of stator resistance and rotor resistance on slip ring motors.

(b) No load running of motor and reading of voltage, current, power input and speed.

(c) Open circuit voltage ratio on slip ring motors

(d) Reduced voltage running up test at no load to check the ability of motor to run upto full speed on load in both directions of rotation with $1/\sqrt{3}$ of the rated voltage applied.

(e) Locked rotor reading of voltage, current, power input and torque of squirrel cage motors.

(f) Full load reading of voltage, current, power input and slip.

(g) Temperature rise.

(h) Momentary overloads torque tests.

(j) Insulation resistance test (before and after high voltage test)

(k) High voltage test.

Routine tests.

(a) Insulation test (before high voltage test only).

(b) High voltage test.

(c) No load running of motor and reading of current in the three phases and of voltage.

(d) Locked rotor test at a suitable voltage (for squirrel cage motors only).

(e) Reduced voltage running up test.

(g) Open circuit voltage ratio test (for slip sing motors only).

Dimensions of foot mounted motors. IS : 1231—1967 specifies the dimensions for foot mounted three phase 50 Hz, a.c. squirrel cage motors of axle heights ranging from 63 mm to 315 mm.

Standard frame sizes. The fixing dimensions and shaft extensions for the frame sizes are given in this standard. (See Tables 17·22 to 17·26 with Fig. 17·1.) The frame designations consist of two parts, the first part giving values corresponding to the actual shaft heights and the second part giving letters indicating the frame lengths, the letters being '*S*' for short motors, '*M*' for medium length motors and '*L*' for long motors. The maximum torque values at continuous rating for different shaft diameters are also given in this standard. Ringed symbols refer to symbols which are specified in this standard. Unringed symbols are themselves standardised.

Terminal box. The terminal box should be located on the right-hand side of the motor when looking on the driving end of the motor. The terminal box should be situated with its centre line within a sector ranging from the top of the motor down to 10° below the horizontal centre line of motor. The space 10° below the centre line should only be utilized in special cases.

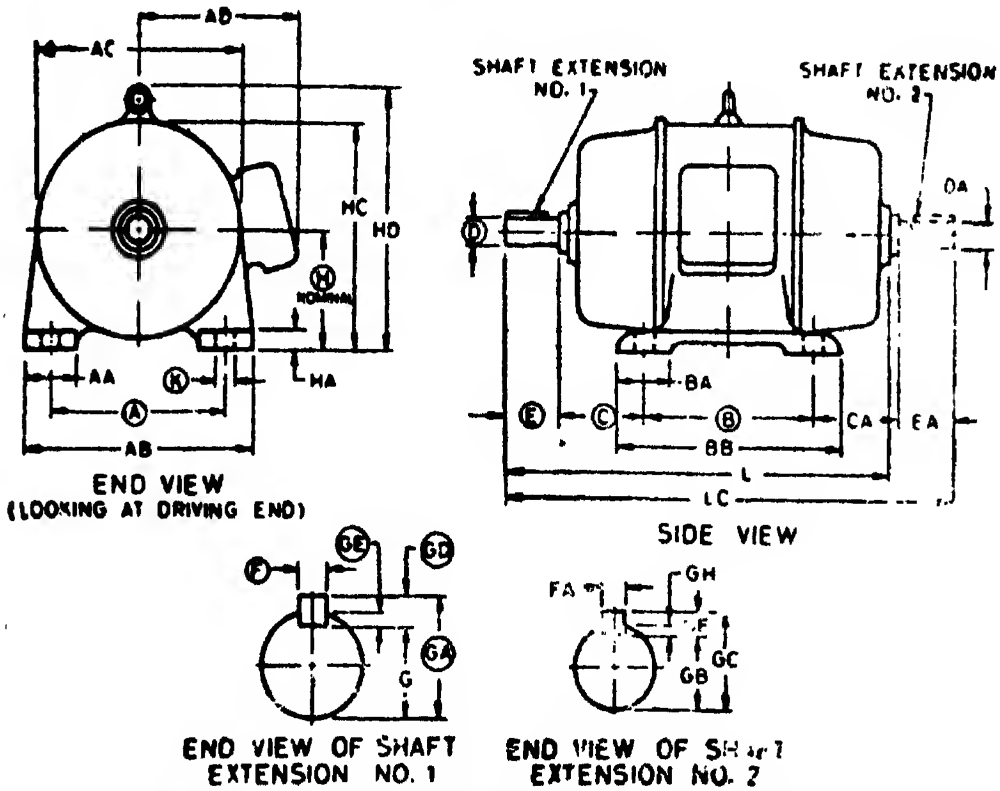

Fig. 17·1. Standardized and other dimensions for foot mounted motors.

Table 17·22. Dimensions for totally enclosed fan-cooled motors class E insulation.

(All dimensions in mm.)

No.	kW rating for synchronous r.p.m. 3000 & 1500	1000	750	Frame designation	H	A	B	C	K	Shaft extension diameter for synchronous r.p.m. 3000	1500 and below
1	0·06 & 0·09	—	—	56	56	90	71	36	6	9	9
2	0·12 & 0·18	—	—	63	63	100	80	40	7	11	11
3	0·25 & 0·37	—	—	71	71	112	90	45	7	14	14
4	0·55 & 0·75	0·37 & 0·55	—	80	80	125	103	50	9	19	19
5	1·1	0·75	0·37	90S	90	140	100	56	9	24	24
6	1·5	1·1	0·55	90L	90	140	125	56	9	24	24
7	2·2	1·5	0·75 & 1·1	100L	100	160	140	63	12	28	28
8	3·7	2·2	1·5	112M	112	190	140	70	12	28	28
9	5·5	—	2·2	132S	132	216	140	89	12	38	38
10	7·5	3·7 & 5·5	—	132M	132	216	178	89	12	38	38
11	11	7·5	3·7 & 5·5	160M	160	254	210	108	14	42	42
12	15	11	7·5	160L	160	254	254	108	14	42	42
13	18·5	—	—	180M	180	279	254	121	14	48	48
14	22	15	11	180L	180	279	279	121	14	48	48
15	30	18·5 & 22	15	200L	200	318	305	133	18	55	55
16	37	—	18·5	225S	225	356	286	149	18	55	60
17	45	30	22	225M	225	356	311	149	18	55	60
18	55	37	30	250M	250	406	349	168	22	60	65
19	75	45	37	280S	280	457	368	190	22	65	75
20	90	55	45	280M	280	457	419	190	22	65	75
21	110	75	55	315S	315	508	406	216	27	65	80
22	125	90	75	315M	315	508	457	216	27	65	80

17·23 Dimensions of motors with [illegible] are in mm.)

Note : The various sizes for this type of motor are the same as given in Table 17·22 upto S. No. 10. The other sizes are :

S. No.	kW rating for synchronous r.p.m. 3000 & 1500	1000	750	Frame designation	H	A	B	C	K	Shaft extension diameter for synchronous r.p.m. 3000	1500 and below
11	11	7·5	3·7 & 5·5	160M	160	254	210	108	14	48	48
12	15 & 18·5	11	7·5	160L	160	254	254	108	14	48	48
13	22	15	11	180M	180	279	241	121	14	55	55
14	30	18·5	15	180L	180	279	279	121	14	55	55
15	37	22	18·5	200M	200	318	267	133	18	60	60
16	45	30	22	200L	200	318	305	133	18	60	60
17	55	37	30	225M	225	356	311	149	18	60	65
18	75	45	37	250S	250	406	311	168	22	65	75
19	90	55	45	250M	250	406	349	168	22	65	75
20	110	75	55	280S	280	457	368	190	22	65	80
21	125	90	75	280M	280	457	419	190	22	65	80
22	160	110	90	315S	315	508	406	216	27	70	90
23	180 & 200	125	100	315M	315	508	457	216	27	70	90

Table 17·24. Dimensions for motors with protected enclosures class A insulation.
(All dimensions are in mm.)

S. No.	kW rating for synchronous r.p.m. 3000	1500	1000	750	Frame designation	H	A	B	C	K	Shaft extension diameter D
1	1·1	0·75	0·55	0·37	112S	112	190	114	70	12	22
2	1·5 & 2·2	1·1 & 1·5	0·75 & 1·1	0·55	112M	112	190	140	70	12	22
3	3·7	2·2	1·5	0·75 & 1·1	132S	132	216	140	89	12	28
4	5·5	3·7	2·2	1·5	132M	132	216	178	89	12	28
5	7·5	5·5	3·7	2·2	160M	160	254	210	108	14	38
6	11	7·5	5·5	3·7	160L	160	254	254	108	14	38
7	15	11	7·5	5·5	180M	180	279	241	121	14	42
8	18·5	15	—	7·5	180L	180	279	279	121	14	42
9	22	18·5	11	—	200M	200	318	267	133	18	48
10	30	22	15	11	200L	200	318	305	133	18	48
11	37	30	18·5	15	225S	225	356	286	149	18	55
12	45	37	22	18·5	225M	225	356	311	149	18	55
13	55	45	30	22	250S	250	406	311	168	22	65
14	—	55	37	30	250M	250	406	349	168	22	65
15	75	75	45	37	280S	280	457	368	190	22	75
16	90	90	55	45	280M	280	457	419	190	22	75

Table 17·25. Dimensions for totally enclosed fan cooled motors class A insulation.
(All dimensions are in mm.)

S. No.	kW rating for synchronous r.p.m. 3000	1500	1000	750	Frame designation	H	A	B	C	K Max	Shaft extension diameter
1	1·1	0·75	0·55	0·37	112S	112	190	114	70	12	22
2	1·5 & 2·2	1·1 & 1·5	0·75 & 1·1	0·55	112M	112	190	140	70	12	22
3	3·7	2·2	1·5	0·75 & 1·1	132S	132	216	140	89	12	28
4	5·5	3·7	2·2	1·5	132M	132	216	178	89	12	28
5	7·5	5·5	3·7	2·2	160M	160	254	210	108	14	38
6	11	7·5	5·5	3·7	160L	160	254	254	108	14	38
7	15	11	7·5	5.5	180M	180	279	241	121	14	42
8	18·5	15	—	7·5	180L	180	279	279	121	14	42

Phase displacement. The difference in phase between the primary and secondary current phasors, the direction of the phasors being so chosen that the angle is zero for a perfect transformer.

The phase displacement is said to be positive when the secondary current phasor leads the primary current phasor. It is usually expressed in minutes.

Accuracy Class. A classification assigned to a current transformer, the errors of which remain within specified limits under the prescribed conditions of use.

Burden. The impedance of secondary circuit expressed in ohm and power factor.

The burden is usually expressed as the apparent power in volt ampere absorbed at a specified power factor and at the rated secondary current.

Rated Burden. The value of the burden on which the accuracy requirements of this specification are based.

Ratings and performance requirements

Rated primary current. The values of the rated primary current shall be as given in Table 17·28.

Table 17·28. Rated primary current.

ampere	*ampere*	*ampere*	*ampere*	*ampere*
	10	100	1000	10000
0·5	12·5	125	1250	
1	15	150	1500	
2·5	20	200	2000	
5	25	250	2500	
	30	300	3000	
	40	400	4000	
	50	500	5000	
	60	600	6000	
	75	750	7500	
		800		

The values underlined above shall be preferred.

Rated secondary current. The value of rated secondary current shall be 5 A.

The secondary current rating of 2 and 1 A may also be used in some cases, as for example :

(*a*) if the number of secondary turns is low that the ratio cannot be adjusted within the requisite limits by addition or removal of one turn, or

(*b*) if the length of secondary connecting leads is such that the burden due to them at the higher secondary current would be excessive.

Standard frequency. The standard frequency shall be 50 Hz.

Temperature rise. The temperature rise of a current transformer winding when carrying a rated primary current, at a rated frequency and with rated burden shall not exceed

the approximate values given in Table 17·29. The temperature rise of the windings is limite by the lowest class of insulation either of the winding itself or of the surrounding medium i which it is embedded.

When the current transformer is fitted with a conservator tank, or has inert gas abov the oil, temperature-rise of the oil at the top of the tank or housing shall not exceed 55° C.

When the current transformer is not so fitted, the temperature rise of the oil at the to of the tank or housing shall not exceed 50°C.

The temperature rise measured on the external surface of the core and the othe metallic parts in contact with or adjacent to insulation shall not exceed the appropriate valu permitted for the adjacent part of the windings as given in Table 17·29.

Table 17·29. Limits of temperature rise of windings.

Class of insulation	*Maximum temperature rise* °*C*
All classes immersed in oil	60
All classes immersed in bituminous compound	50
Classes not immersed in oil or bituminous compound	
Y	45
A	60
E	75
B	85
F	110
H	135

Note. If current transformer is specified for service at altitudes exceeding 1000 m and tested at an altitude below 1000 m, the limits of the temperature rise given in Tabl 17·29 shall be reduced by the following amounts for each 100 m excess over 1000 m operatin altitude.

(*a*) Oil immersed transformers 0·4 per cent

(*b*) Dry-type transformers 0·5 per cent.

Indian Standard Specifications for current transformers

IS : 2705—1964

Part II Measuring Current Transformers

Accuracy Class.

Standard Accuracy Class. For measuring current transformers the accuracy class shall be designated by the highest permissible percentage current error at rated current for the accuracy class concerned. The standard accuracy classes for measuring transformers shall be 0·1, 0·2, 0·5, 1·3 and 5.

Limits of current error and phase displacement.

Classes 0·1 to 1·0. The current error and displacement at the rated frequency shall not exceed the values given in Table 17·30. When the secondary burden is any value from 25 per cent to 100 per cent of the rated burden.

Table 17·26. Shaft extension.

(All Dimensions in mm.)

D	*E*	*Maximum torque for continuous duty N-m*	*D*	*E*	*Maximum torque for continuous duty N-m*
7	16	0·25	42	110	125
9	20	0·63	48	110	200
11	23	1·25	55	110	355
14	30	2·8	60	140	450
16	40	4·5	65	140	630
18	40	7·1	70	140	800
19	40	9	75	140	1000
22	50	14	80	170	1250
24	50	18	85	170	1600
28	60	31·5	90	170	2000
32	80	50	95	170	2500
38	80	90	—	—	—

SINGLE PHASE INDUCTION MOTORS

17 10. **Indian standard specifications for single phase, small a.c. and universal motors.** The Indian standards institution has laid down the following standard for single phase small a c. and universal machines.

IS : 996—1964. (Amendment 1969)

This standard covers general purpose small single phase motors of the capacitor, split phase shaded pole and universal types, having outputs up to and including 1500 W and having windings with class *A*, class *E* or class *B* insulation.

Table 17·27. Standard frame sizes.

(All dimensions in mm.)

Frame designation	*H*	*A*	*B*	*C*	*K max.*	*D*	*E*	*F*	*G*
56	56	90	71	36	6	—	—	—	—
63	63	100	80	40	7	—	—	—	—
71	71	112	90	45	7	—	—	—	—
80	80	125	100	50	9	14	30	5	17·5
90 S	90	140	100	56	9	16	40	5	18·0
100 S	100	160	112	63	12	22	50	6	24·5
112 S	112	190	114	70	12	22	50	6	24·5
132 S	132	190	140	70	12	28	60	8	31·0

Types of enclosures. The various types of enclosures are :

(a) Protected motor

(i) Screen protected (SP)

(ii) Drip proof motor (DP)

(b) Totally enclosed motor (TE)

(c) Totally enclosed fan cooled motor (TEFC).

Standard Frame Sizes. The standard frame sizes are given in Table 17·27. Refer to Fig. 17·2).

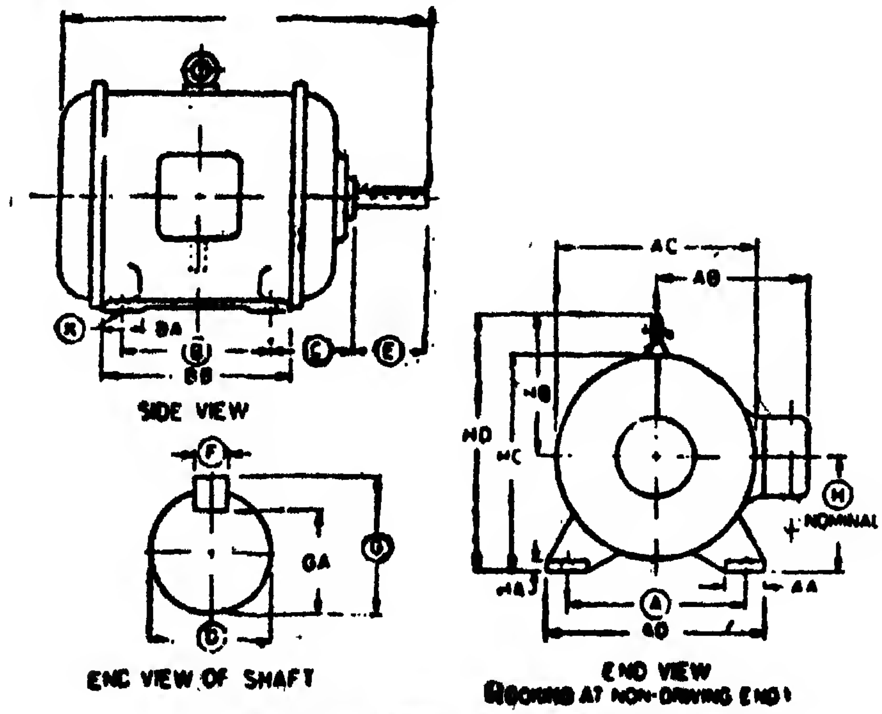

Fig. 17·2. Dimensions of single phase motors.

CURRENT TRANSFORMERS

17·11. Indian Standard Specification for current transformers.

IS : 2705—1964

Part I General Requirements

17·11·1. Terminology. For the purpose of this standard, the following definitions shall apply :

Rated transformation ratio. The ratio of the rated primary current to the rated secondary current.

Current error (Ratio error). The percentage error in the magnitude of the secondary current is defined by the following formula :

$$\text{Current error} = \frac{(K_n I_s - I_p) \times 100}{I_p} \text{ per cent}$$

where

K_n = the rated transformation ratio,

I_s = the actual secondary current when I_p is flowing,

and I_p = the actual primary current.

Table 17·30. Limits of error for accuracy classes 0·1 to 1·0.

Class	*Percentage current error at percentage of rated current*				*Phase displacement error in minutes at percentage of rated current*			
	10	*20*	*100*	*120*	*10*	*20*	*100*	*120*
0·1	±0·25	±0·2	±0·1	±0·1	±10	±8	±5	±5
0·2	±0·5	±0·35	±0·2	±0·2	±20	±15	±10	±10
0·5	±1·0	±0·75	±0·5	±0·5	±60	±45	±30	±30
1·0	±2·0	±1·5	±1·0	±1·0	±120	±90	±60	±60

Classes 3 and 5. The current error at rated frequency shall not exceed the values given in Table 17·31 when the secondary burden is any value from 50 per cent to 100 percent of the rated burden.

Table 17·31. Limits of error for accuracy classes 3 and 5.

Class	*Percentage current error at percentage of rated current*	
	50	*120*
3	±3	±3
5	±5	±5

Note. Limits of phase displacement are not specified for classes 3 and 5.